## **VICGRID94 Map Projection**

Updated 21/11/2000

### Introduction

The **VICGRID94** Map Projection was specified and adopted by Land Victoria on the 7th of February 2000 at a Land Information Group Technical Meeting in response to user need. **VICGRID94** is a projection created to cater for the needs of spatial data users with large regional area interests in and beyond the state of Victoria<sup>1</sup> and who also wish to use the Geocentric Datum of Australia 1994 (GDA94) as the underlying datum (rather than the Australian Geodetic Datum 1966 (AGD66) on which the original VICGRID was based).

It should be noted that the **VICGRID94** projection incorporates a different origin specification to VICGRID in order to avoid confusion between coordinates generated by the two projections. The northing false origin for **VICGRID94** has been adopted as 2,500,000 metres south of the intersection of the parallel of latitude 37° S and the central meridian rather than the 4,500,000 specified for the original VICGRID. The easting false origin remains unchanged at 2,500,000 metres west of the intersection of the parallel of latitude 37° S and the central meridian.

The formulae to compute Eastings and Northings for VICGRID / VICGRID94 are the same. However it should be noted that different ellipsoids apply to the datum's relating to VICGRID and VICGRID94. VICGRID uses the Australian Geodetic Datum 1966, which adopts the Australian National Spheroid (ANS). VICGRID94 uses the Geocentric Datum of Australia, which adopts the Geodetic Reference System 1980 (GRS80) ellipsoid. The constants for the semi major axis and inverse flattening are different for ANS and GRS80.

This document summarises the technical definition of the projection. The specifications in this document update and supersede the details contained within the version published on 8/3/2000

# **Specifications**

**VICGRID94** specifications are provided in the table below.

**Table 1 VICGRID94 Specifications** 

Name	VICGRID94				
Projection	Lambert's conformal conic projection				
Datum	Geocentric Datum of Australia 1994 (GDA94) using the				
	Geodetic Reference System 1980 ellipsoid (GRS80)				
	Semi major axis (a) 6,378,137 metres exactly				
	Inverse flattening (1/f) 298.257222101 metres				
Standard Parallels of Latitude	36° S and 38° S				
Central Meridian of Longitude	145° E				
Unit of measurement	Metre				
Coordinates defined by formulae	Refer to formulae for Lambert's Conformal, Conic Projection of				
for Easting and Northing	the GRS80 ellipsoid with Two Standard Parallels and				
	VICGRID94 coordinates allowing for change in false origin to				
	distinguish between VICGRID and VICGRID94 (refer to Origin				
	below)				
Origin	<b>2,500,000</b> metres west and <b>2,500,000</b> metres south of the				
	intersection of the parallel of latitude 37 ° S and the central				
	meridian				

<sup>&</sup>lt;sup>1</sup> VICGRID94 can be used to display Victoria, New South Wales, Tasmania and South Australia as a single entity.

### C Formulae

This section sets out the formulae for **Lambert's Conformal Conic Projection** of the **Geodetic Reference System 1980** ellipsoid (GRS80) with two standard parallels and **VICGRID94** coordinates.

The surface of the Earth, which is irregular and slightly squashed at the poles, may be represented by a smooth mathematical surface of revolution known as an *ellipsoid* (an ellipse rotated about its minor axis). The term *spheroid* is often used as an alternative to ellipsoid and for map projection purposes can be regarded as meaning the same.

An ellipsoid can be completely defined by stating the semi-major axis a and the flattening f. Other quantities for an ellipsoid may be defined as follows:

semi-minor axis of ellipsoid (b) 
$$b = a(1 - f)$$
eccentricity of ellipsoid (e) 
$$e = \sqrt{f(2 - f)}$$

For a point on the ellipsoid whose latitude is  $\phi$  the following maximum and minimum radii of curvature may be evaluated for that point:

$$n = \frac{a}{\left(1 - e^2 \sin^2 f\right)^{\frac{1}{2}}}$$

$$r = \frac{a(1 - e^2)}{\left(1 - e^2 \sin^2 f\right)^{\frac{3}{2}}}$$

In Lambert's conformal conic projection, parallels of latitude are unequally spaced arcs of concentric circles and meridians are equally spaced radial lines of the same circles, thereby cutting parallels at right angles.

For a point whose latitude  $\phi$  and longitude  $\lambda$  is known on the ellipsoid, the polar coordinates  $(r,\theta)$  of that point on the projection are

$$r = -c \left( m \tan q \right)^n \tag{C1}$$

$$q = -n(1 - I_0) \tag{C2}$$

where r is a positive radial distance and  $\theta$  is a radial angle measured positive east or negative west of a central meridian  $I_{\theta}$  as shown in Figure C1.

The variables m and q are

$$q = \left(\frac{p}{4} - \frac{f}{2}\right)_{radians} \tag{C3}$$

$$m = \left(\frac{1 + e\sin f}{1 - e\sin f}\right)^{\frac{e}{2}} \tag{C4}$$

where the latitude  $\phi$  is negative for the southern hemisphere. For standard parallels  $f_1$  and  $f_2$  the projection constants n and c are

$$n = \frac{\log(n_1 \cos f_1) - \log(n_2 \cos f_2)}{\log(m_1 \tan q_1) - \log(m_2 \tan q_2)}$$
 (C5)

$$c = \frac{v_1 \cos f_1}{n(m_1 \tan q_1)^n} \tag{C6}$$

**Notes:** 

For the southern hemisphere, c and n are negative constants and so a negative sign is shown in equations (C1) and (C2) to produce a positive radius r and a positive angle  $\theta$  east of the central meridian  $I_0$  for southern latitudes.

log means the natural logarithm and subscripts I and I refer to variables computed with latitudes I and I and I.

e is the eccentricity of the ellipsoid.

For a projection of southern latitudes, Cartesian coordinates X,Y whose origin (known as the *true* origin) is at the intersection of the parallel  $f_0$  and the central meridian  $I_0$  are

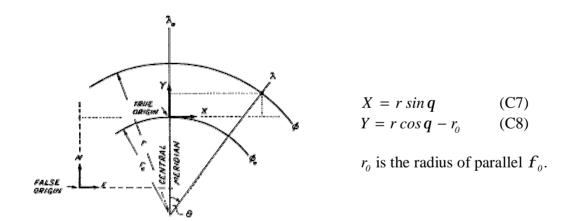


FIGURE C1 Cartesian coordinates (X, Y), polar coordinates (r, q), the *true origin* and the *false origin* of a conic projection of the southern hemisphere.

*VICGRID94* coordinates are derived from Lambert's conformal conic projection of latitudes and longitudes on the Geocentric Datum of Australia 1994 (GDA94) with standard parallels at 36°S. and 38°S. and a central meridian of 145°E.

The GDA defines the Geodetic Reference System 1980 ellipsoid (GRS80) having the following parameters:

Geodetic Reference System 1980

$$a = 6378137$$
 metres exactly  $f = \frac{1}{298.257222101}$ 

b = 6356752.314 metres

from which the following may be derived

$$e^2 = 0.006694380$$

e = 0.081819191

With the following standard parallels and the true origin of the projection as

Standard parallel 1  $f_1 = -36^{\circ}$ Standard parallel 2  $f_2 = -38^{\circ}$ X, Y coordinate origin at  $f_0 = -37^{\circ}$  and  $f_0 = 145^{\circ}$ 

together with the ANS parameters above, the projection constants have the numeric values

n = -0.601846105 c = -12849334.6 $r_0 = 8472630.5$ metres

The equations given above can be used to compute the Cartesian coordinates with respect to the true origin of Lambert's projection. To make all coordinates positive, a *false* origin is established which is the origin of *VICGRID94* coordinates. This false origin is 2,500,000 metres west and 2,500,000 metres south of the true origin (at  $f_0$  and  $I_0$ ) and *VICGRID94* coordinates E, N are defined as

$$E = X + 2.500.000$$
 metres (C9)

$$N = Y + 2,500,000 metres$$
 (C10)

The point scale factor (k) and the grid convergence  $(\gamma)$  are computed from

$$k = -\frac{rn}{n\cos f} \tag{C11}$$

$$g = -q \tag{C12}$$

**Notes:** The scale factor is positive and constant for all points with the same latitude. The negative sign accounts for n being negative in the southern hemisphere.

The grid convergence is considered to be positive west of the central meridian and negative east, which is the opposite sign convention to the polar angle  $\theta$ .

The *inverse formulae* for GDA94 coordinates  $(\phi,\lambda)$  given *VICGRID94* coordinates (E,N) and the constants a, e, c, n and  $r_0$  are as follows.

$$X = E - 2,500,000 (C13)$$

$$Y = N - 2,500,000 \tag{C14}$$

$$r = \sqrt{X^2 + \left(Y + r_0\right)^2} \tag{C15}$$

$$q = \tan^{-1} \left( \frac{X}{Y + r_0} \right) \tag{C16}$$

The latitude  $\phi$  may be solved by Newton's method of iteration when it is considered that the radius r is a function of  $\phi$  only. Equation C1 with substitutions for q and m can be written as

$$f(f) = -c \left\{ \left( \frac{p}{4} - \frac{f}{2} \right) \left( \frac{1 + e \sin f}{1 - e \sin f} \right)^{\frac{e}{2}} \right\}^{n} - r = 0$$

and Newton's iterative equation is

$$f_{i+1} = f_i - \frac{f(f_i)}{f'(f_i)}$$

where  $f(f_i)$  is the function and  $f'(f_i)$  is its derivative, both evaluated for the  $i^{th}$  latitude.

The iterative equation for the latitude  $\phi$ , written with simplified expressions for  $f(\mathbf{f}_i)$  and its derivative, is

$$f_{i+1} = f_i - \left\{ \frac{r_i - r}{n \, r_i \left( \frac{e^2 \cos f_i}{1 - e^2 \sin^2 f_i} - \frac{1}{\cos f_i} \right)} \right\}$$
 (C17)

and the equation for the longitude  $\lambda$  is

$$I = I_0 - \frac{q}{n} \tag{C18}$$

Note: In equation C17, an initial approximation of the latitude  $f_i$  can be taken as  $f_o$ .

#### D Test Data

This section contains five points that have been transformed from AGD66 to Vicgrid94 to assist users in the verification of software application. Five points have been chosen to provide examples of points lying at the extremes of the state.

	Datum – AGD66		Datum – Vicgrid	
Point Name	Latitude	Longitude	East	North
Willah PM 4	-34°29'46.7724"	141°59'14.8833"	2223143.321	4773459.258
Cobboboonee PM 14	-38°03'59.0913"	141°24'52.3647"	2185431.606	4375727.525
Goongerah PM 20	-37°23'45.2184"	148°46'38.6881"	2834353.246	4449435.092
Yarrawonga PM 49	-36°01'03.6403"	145°59'53.8910"	2589988.794	4608524.140
Frankston PM 396	-38°07'52.7666"	145°09'42.9142"	2514197.138	4374434.703

	Datum – GDA94		Datum – Vicgrid94	
Point Name	Latitude	Longitude	East	North
Willah PM 4	-34°29'41.3773"	141°59'19.5899"	2223259.175	2773628.391
Cobboboonee PM 14	-38°03'53.8007"	141°24'57.2580"	2185545.806	2375895.467
Goongerah PM 20	-37°23'39.6610"	148°46'43.1871"	2834469.388	2449602.655
Yarrawonga PM 49	-36°00'58.1475"	145°59'58.4589"	2590104.617	2608691.847
Frankston PM 396	-38°07'47.3418"	145°09'47.6172"	2514311.897	2374602.216

The points chosen here have been a part of the state geodetic network adjustment from AGD66 to GDA94 and are the official values as recorded in the Survey Marks Enquiry Service – SMES (21/11/00). The conversion of data from latitude and longitude to grid east and north has been performed using the AMGVG program, which incorporates the specifications for Vicgrid and Vicgrid94 and supports computation within coordinate systems.

AMGVG *only* supports coordinate transformations between geographical and Vicgrid **on the same datum** (AGD66 or GDA94).

GDA*it* has been developed for the transformation process **between** datums (AGD66 to GDA94).

To obtain copies of both AMGVG and GDAit please visit the Geodesy web site at, <a href="http://www.land.vic.gov.au/geodesy/">http://www.land.vic.gov.au/geodesy/</a>