UNIVERSITY AT ALBANY
STATE UNIVERSITY OF NEW YORK

CSI 436/536 (Spring 2025)

Machine Learning

Lecture 6: Evaluation Criteria

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Announcement

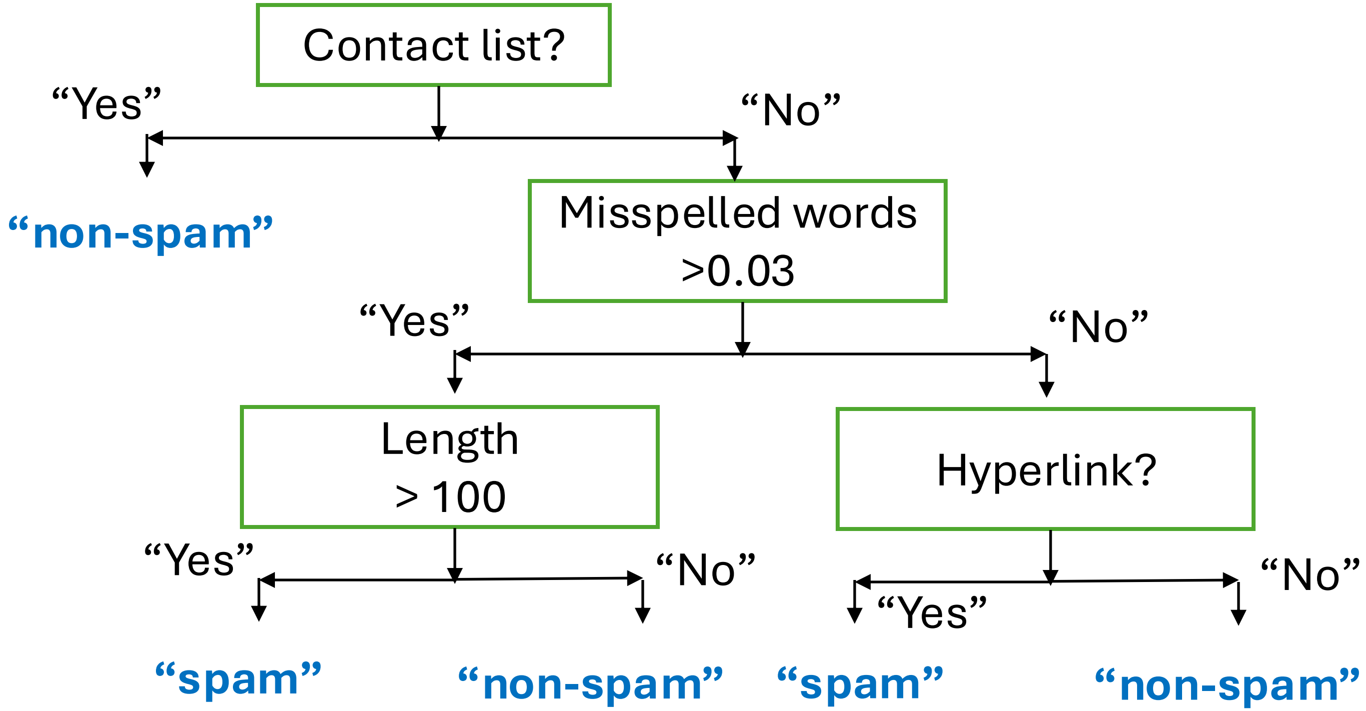
- TA office hour moved to podium!
  - New: Wed 11:45am-12:45pm at HU 25
  - Starting this week

- TA will give a tutorial on LaTeX and Python this Wed
  - LaTeX for your homework
  - Python for your course project

Recap: elements of machine learning

- Machine learning overview
  - Supervised learning
  - Unsupervised learning
  - Reinforcement learning
- Supervised learning: binary classification
  - Spam filtering
- Feature design and feature extraction
  - In contact list or not
  - Proportion of misspelled words
  - …
- Decision tree classifier

Recap: Decision tree



Recap: How is a decision tree specified?

- Parameters (built-in parameters of a model)
  - Which feature(s) to use when branching?
  - How to branch? Thresholding? Where to put the threshold?
  - Which label to assign at leaf nodes?

- Hyperparameters (parameters that you can set)
  - Max height of a decision tree?
  - Number of features the tree can use in each branch?

Today

- Linear classifier

- Performance metrics

- Feature transformation

Linear classifiers

- Model:
  - $\text{Score}(x) = w_0 + w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4$
  - $x_1 = 1(\text{has hyperlinks})$
  - $x_2 = 1(\text{on contact list})$
  - $x_3 = \text{proportion of misspelling}$
  - $x_4 = \text{length}$

Indicator function:

$$f(x) = 1(\text{condition}) = \begin{cases} 1, \text{if condition is true} \\ 0, \text{if condition is false} \end{cases}$$

Question: why do we need w_0?

Linear classifiers

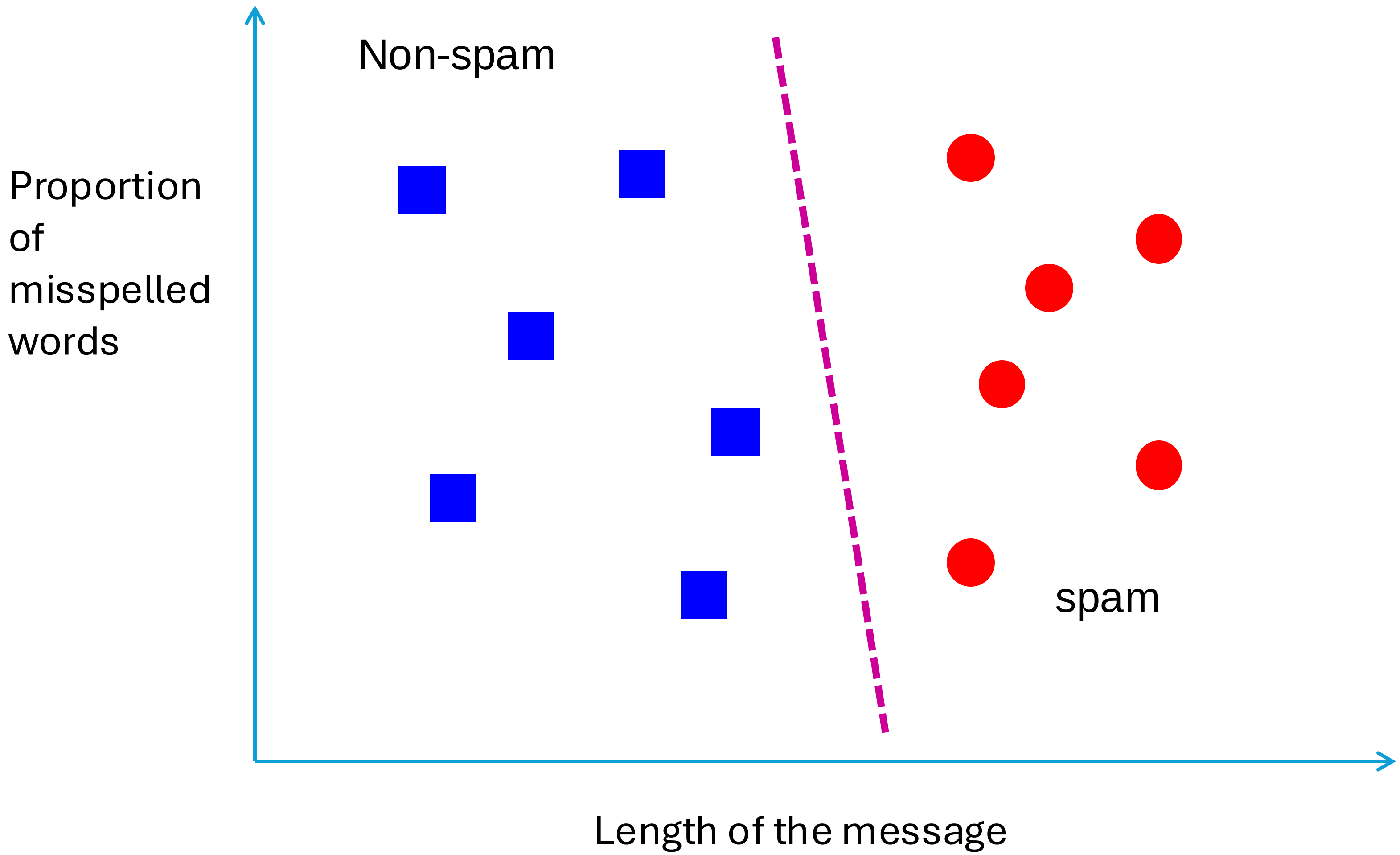
- Model:
  - $\text{Score}(x) = w_0 + w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4$
- A linear classifier:
  - $h(x) = \begin{cases} 1, \text{if Score}(x) \geq 0 \\ -1, \text{if Score}(x) < 0 \end{cases}$
  - A compact representation: $h(x) = \text{sign}(w^T[1; x])$

- Questions:
  - What are the parameters in a linear classifier?
  - Is there any hyperparameter?

Geometric view: Linear classifier is a decision line!

$$\{x | w_0 + w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4 > 0\}$$

The set of all ”emails” that will be classified as “Spams”



Family of classifiers: Hypothesis class

- Hypothesis class $\mathcal{H}$
  - A family of classifiers
  - Also known as “concept class”, “model”, “decision rule book”
  - “Linear classifiers” and “neural networks” are hypothesis classes.
  - Typically we want this family to be large and flexible.

- The task of machine learning:
  - A **selection problem** to find a

$$h \in \mathcal{H}$$

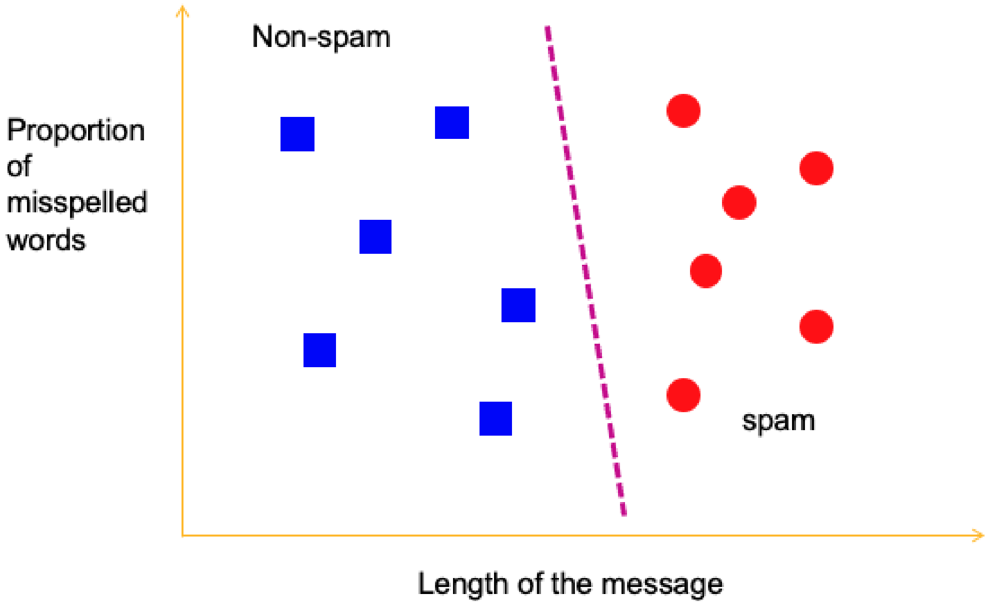
that “**works well**” on this problem.

We will use the following notation to denote a classifier (hypothesis) specified by a specific parameter choice w

$$h_w : \mathcal{X} \to \mathcal{Y}$$

- For any $x \in \mathcal{X}$
  - We can apply this classifier to get its predicted label
    $$\hat{y} = h_w(x)$$
  - The prediction doesn't have to be correct. It just need to be valid, i.e.,
    $$\hat{y} \in \mathcal{Y}$$

Learning linear classifiers

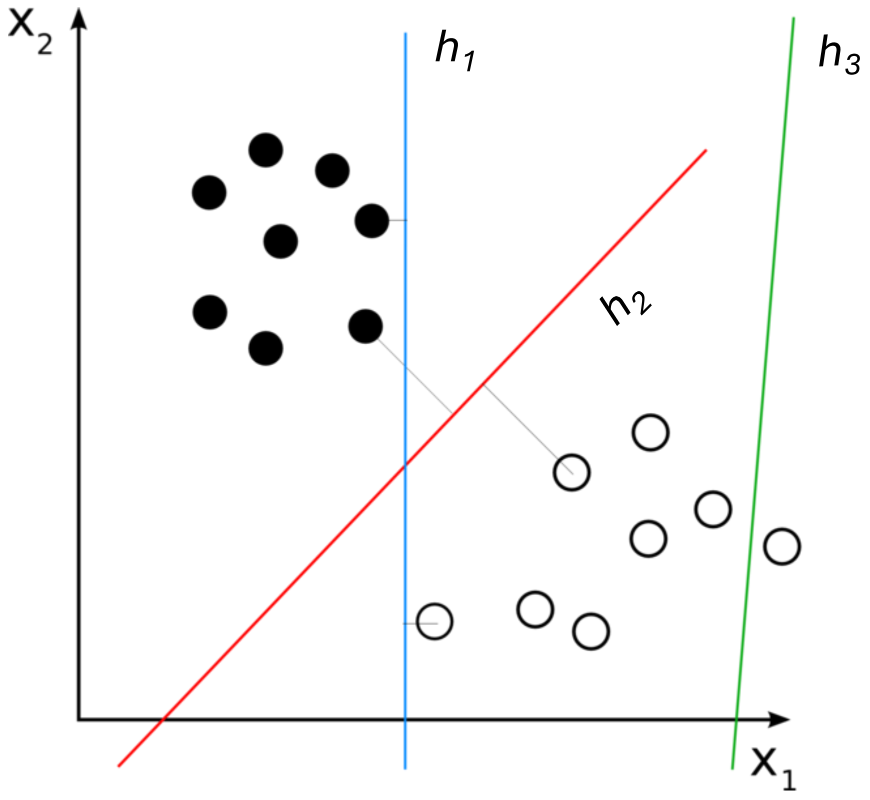


- Training data:

$$(x_1, y_1), \dots, (x_n, y_n) \in \mathcal{X} \times \mathcal{Y}$$

- There is a clean cut boundary that distinguishes “spams” from “non-spams”.
  - “Linearly separable” problem
  - Learning linear classifier: Finding vector w, such that the predictions of h_w is **consistent** with the observed training data.

Discussion: How can we evaluate a classifier (a spam filter)?



Which is better, h_1, h_2, h_3? Why?

Confusion matrix for binary classification

Actual class y ← Ground truth

Classifier output → Predicted class $\hat{y}$

Predicted class $\hat{y}$	1	0	
1	TP	FP Type I Error	*Estimated positive* $\hat{P}$
0	FN Type II Error	TN	*Estimated negative* $\hat{N}$
	Positive *P*	*Negative* *N*	TOTAL

TP – true positives

FP – false positives

TN – true negatives

FN – false negatives

Correct

Errors

$$TP + FN = P$$

$$FP + TN = N$$

$$TP + FP = \hat{P}$$

$$FN + TN = \hat{N}$$

$$P + N = TOTAL$$

$$\hat{P} + \hat{N} = TOTAL$$

In-class exercise: confusion matrix

Actual class ← Ground truth

y

Classifier output

Predicted class

$\hat{y}$

	1	0	
1	TP	FP Type I Error	*Estimated positive* $\hat{P}$
0	FN Type II Error	TN	*Estimated negative* $\hat{N}$
	Positive *P*	*Negative* *N*	TOTAL

$$\hat{y} = [1,1,1,1,0,0,0,1,1,1]$$
$$y = [1,0,0,0,0,1,1,0,0,0]$$

Key criteria

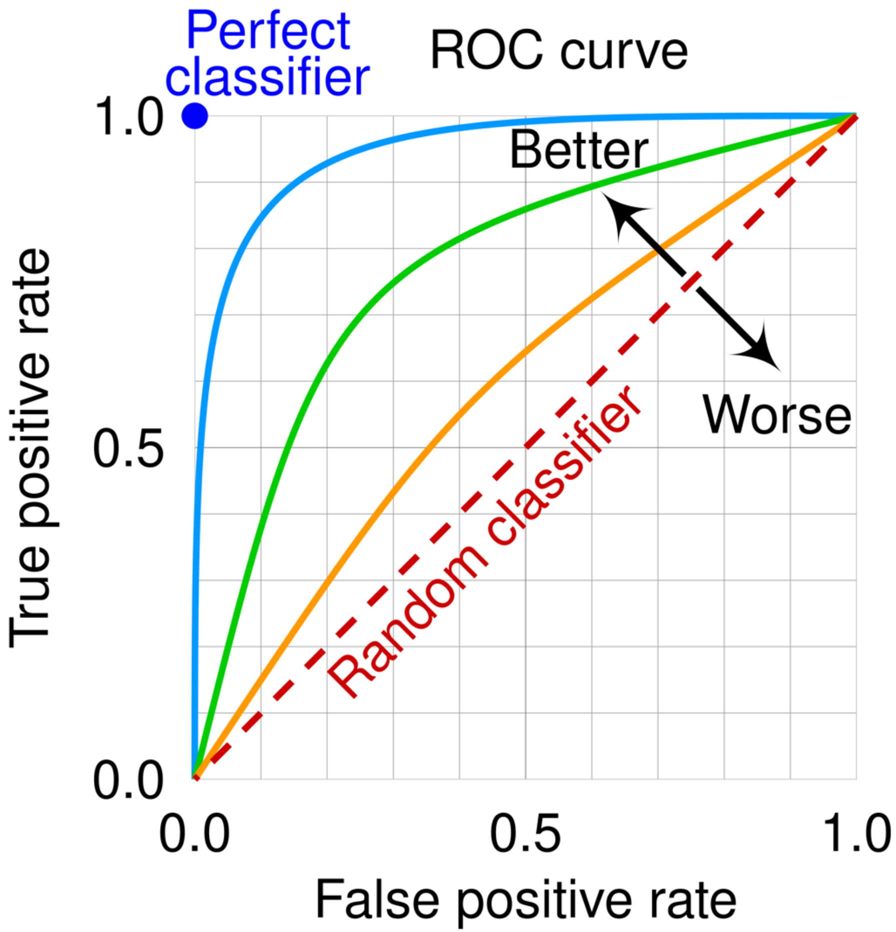
- $Accuracy = \frac{TP+TN}{\text{Total}}$
  - correct predictions / total
- $Precision = \frac{TP}{\widehat{P}}$
  - correctly predicted positive observations / total predicted positives
- $Recall = \frac{TP}{P}$
  - Correctly predicted positive observations / all actual positives
- $F1\ score = \frac{2 \cdot \text{Precision} \cdot \text{Recall}}{\text{Precision}+\text{Recall}}$

$$\hat{y} = [1,1,1,1,0,0,0,1,1,1]$$
$$y = [1,0,0,0,0,1,1,0,0,0]$$

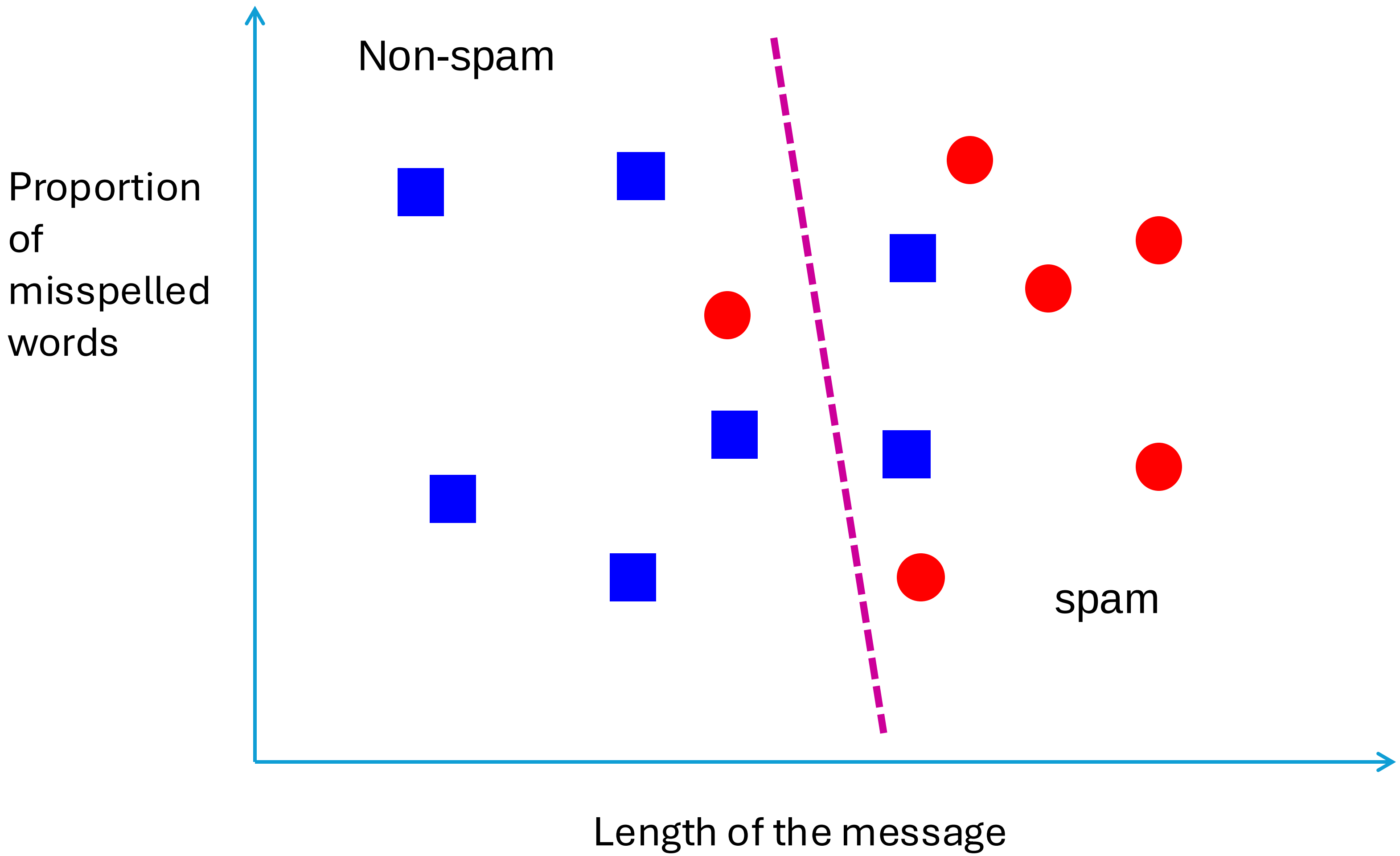
		Actual class		
	y	1	0	
Predicted class $\hat{y}$	1	TP	FP	*Estimated positive* $\widehat{P}$
	0	FN	TN	*Estimated negative* $\widehat{N}$
		Positive P	*Negative* N	TOTAL

Key criterion: AUC (Area Under the ROC Curve)

- ROC Cure:
  - Response Operator Characteristic (ROC) curve
  - False positive rate (FPR) $= \frac{FP}{N}$
  - True positive rate (TPR) $= \frac{TP}{P} = Recall$
- AUC:
  - Single number summary of any "**score function**"



In practice: many non-linearly separable case



How to learn LINEAR classifier in a non-linearly separable case?

- Training data:

$$(x_1, y_1), ..., (x_n, y_n) \in \mathcal{X} \times \mathcal{Y}$$

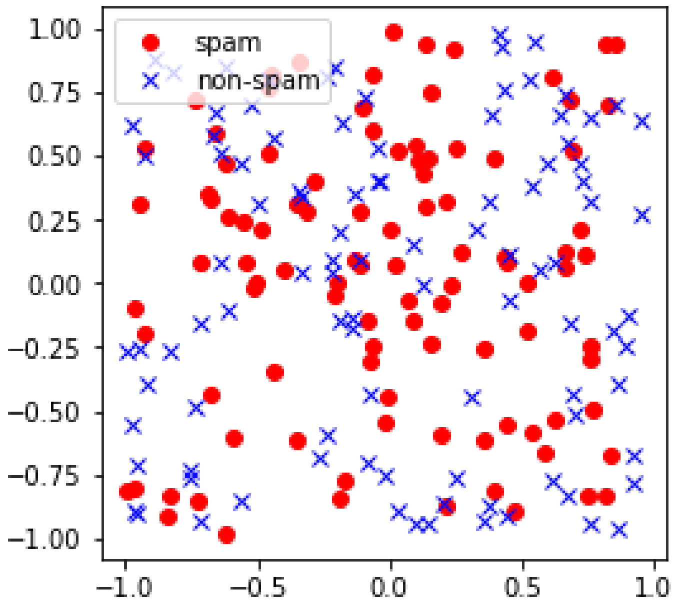
- Solving the following optimization problem:

$$\min_{w \in \mathbb{R}^d} \mathrm{Error}(w) = \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}(h_w(x_i) \neq y_i)$$

- Learning: Find the linear classifier that makes **the smallest number of mistakes** on the training data.

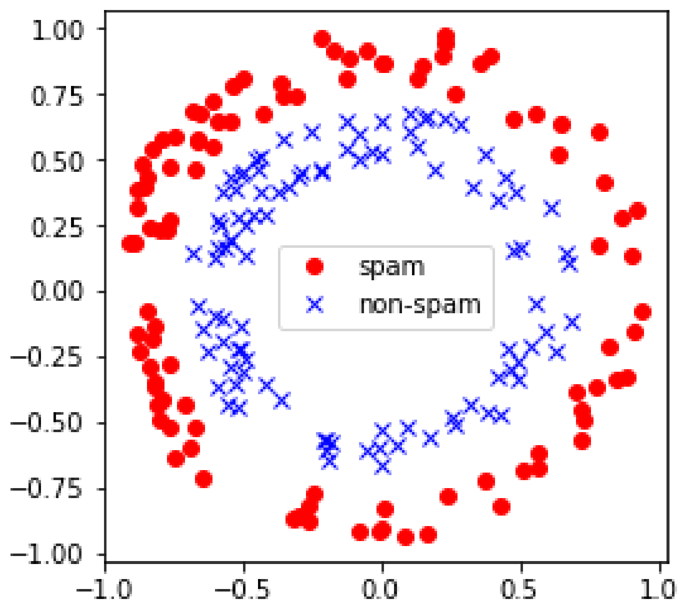
What happens if the linear classifier with the smallest number of mistakes still makes a mistake 49% of the time?

Case 1:



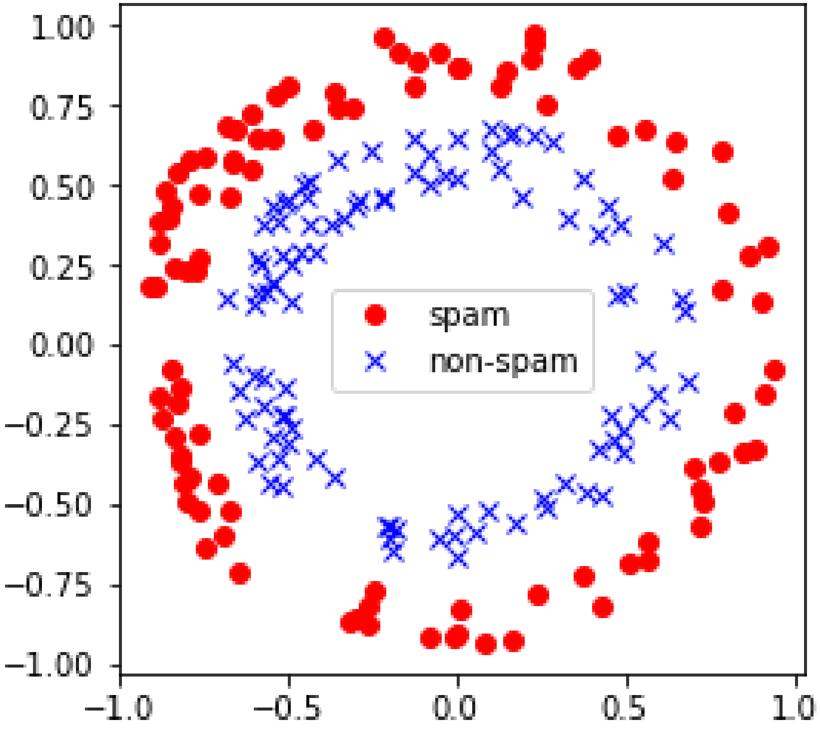
There is no information about the label in the features.
No classifier can do well.

Case 2:



There are some nonlinear classifier that works. But no linear classifiers will do better than random.

Example: Feature transformation



What we can do:

$$(\tilde{x_1}, \tilde{x_2}) = \left(\sqrt{x_1^2 + x_2^2}, \arctan(x_2/x_1)\right)$$

In the redefined space, the two classes are now linearly separable.