UNIVERSITY AT ALBANY
STATE UNIVERSITY OF NEW YORK

CSI 436/536 (Spring 2026)

Machine Learning

Lecture 8: Loss and Gradient Descent

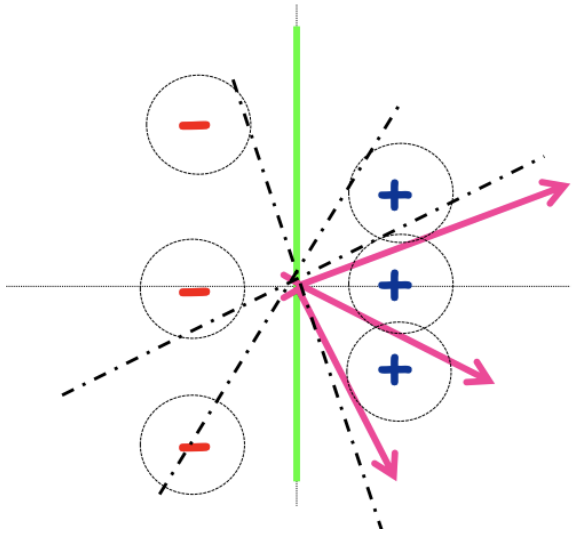
Chong Liu

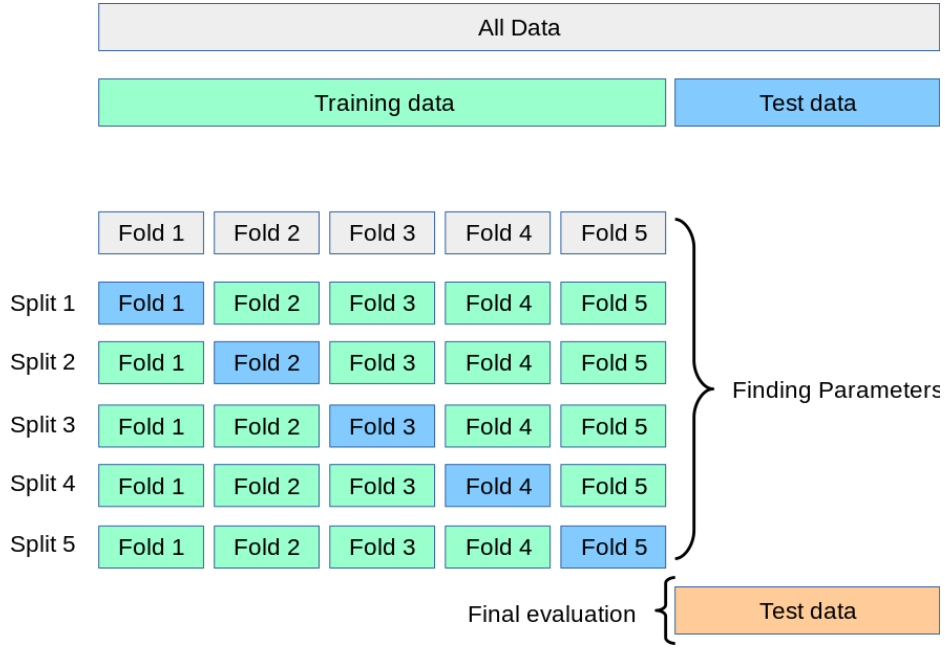
Department of Computer Science

Feb 18, 2026

Recap: linear classifier

- Problem of overfitting
  - Too dependent on training data
  - Bad on test data
- Data splitting methods:
  - Holdout
  - Cross validation
- Perceptron algorithm





Pseudo code / implementation for the perceptron algorithm

The Perceptron algorithm

Initialize $w_1 = [0, 0, ..., 0]$

while 1:

1. Gets feature vector x
2. Agent makes prediction $\hat{y} \leftarrow \text{sign}(w_t \cdot x)$
3. Reveals label y
4. **If** $\hat{y} \neq y$ **:**
   a. $w_{t+1} \leftarrow w_t + yx$
   b. $t \leftarrow t + 1$

Perceptron algorithm makes limited number of mistakes! (if the data is linearly separable)

Theorem (Novikoff, 1962): Assume $\|x\|_2 \leq R$ and there exists w^* such that every input satisfies that $\frac{|x \cdot w^*|}{\|w^*\|_2} \geq \gamma$ and $y = \text{sign}(x \cdot w^*)$. Then the total number of mistakes Perceptron makes is smaller than R^2/γ^2.

Remarks:

- The algorithm can run infinitely long.
- No assumption on data distribution.
- Every new data point to predict on is new.
- No hyperparameters to choose. The algorithm does not need to know R, γ.
- Can support infinitely many features!

Using Perceptron over and over

$$\min_{w \in \mathbb{R}^d} \text{Error}(w) = \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}(h_w(x_i) \neq y_i)$$

- Perceptron can take any sequence of data
- Loop over the data again and again until there is no mistakes.

```
while not converged:
    converged = true
    for over data set:
        make prediction
        If "Mistake":  1. Update weights.  2. set converged = false
```

- Converge in after at most $\frac{nR^2}{\gamma^2}$ inner loop iterations.

Limitations of the Perceptron Algorithm

- What if the margin is small?
    - The perceptron algorithm will be slower
- What if non-linearly separable?
    - The perceptron algorithm is known to not converge!

Today

- Learn how to train a machine learning classifier!
- Surrogate loss
- Continuous optimization
- Gradient Descent (GD)

Recap: Linear classifier

- Take input feature vector
  - $\text{Score}(x) = w_0 + w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4$
  - $x_1 = 1(\text{has hyperlinks})$
  - $x_2 = 1(\text{on contact list})$
  - $x_3 = \text{proportion of misspelling}$
  - $x_4 = \text{length}$
- Let label space be $\{-1,1\}$
- Linear classifier:
  - $h_w(x) = \begin{cases} 1, \text{if Score}(x) \geq 0 \\ -1, \text{if Score}(x) < 0 \end{cases}$

Key question: How to train linear classifier (find w)?

$$\min_{w \in \mathbb{R}^d} \text{Error}(w) = \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}(h_w(x_i) \neq y_i)$$

Discussion:

- 0-1 loss:

$$\mathbf{1}(\text{sign}(w^T x_i) \neq y_i)$$

- Training problem:

$$\min_{w \in \mathbb{R}^d} \text{Error}(w) = \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}(\text{sign}(w^T x_i) \neq y_i)$$

- How can you minimize 0-1 loss?

0-1 loss is unfortunately very hard to optimize

- 0-1 loss:

$$\mathbf{1}(\text{sign}(w^T x_i) \neq y_i)$$

- Training problem:

$$\min_{w \in \mathbb{R}^d} \text{Error}(w) = \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}(\text{sign}(w^T x_i) \neq y_i)$$

- Given n data points, the learner needs to check 2^n different configurations.
  - Why 2? Prediction matches / doesn't match label y.
  - It is known as NP-hard.
  - Highly inefficient when n is large.

Just “relax”: relaxing a hard problem into an easier one

$$\min_{w \in \mathbb{R}^d} \text{Error}(w) = \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}(\text{sign}(w^T x_i) \neq y_i)$$

$$\min_{w \in \mathbb{R}^d} \frac{1}{n} \sum_{i=1}^{n} \ell(w^T x_i, y_i).$$

New loss function is called “surrogate loss”

$$\min_{w \in \mathbb{R}^d} \text{Error}(w) = \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}(\text{sign}(w^T x_i) \neq y_i)$$

$$\min_{w \in \mathbb{R}^d} \frac{1}{n} \sum_{i=1}^{n} \ell(w^T x_i, y_i).$$

Key point: Choice of surrogate loss must satisfy

$$\mathbf{1}(\text{sign}(w^T x_i) \neq y_i) \leq \ell(w^T x_i, y_i)$$

- Discussion: why?

Loss functions

- 0-1 loss:

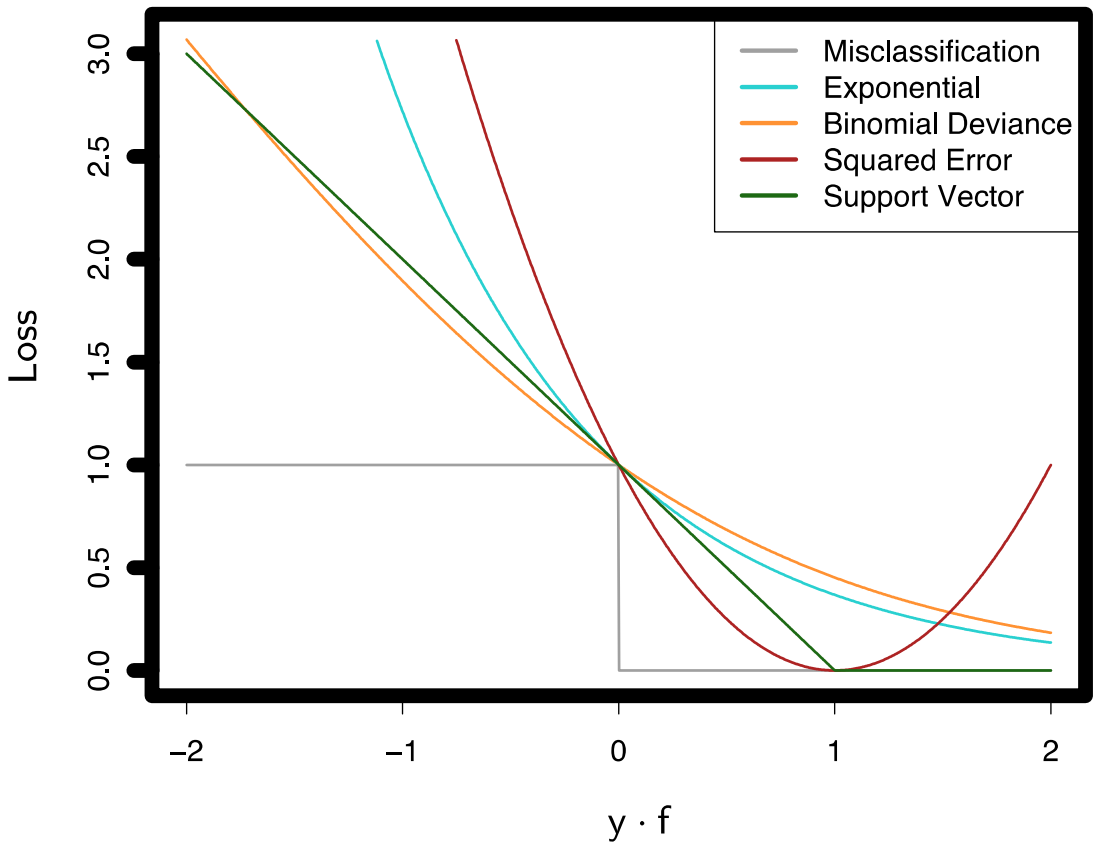
$$\mathbf{1}(h_w(x) \neq y) = \mathbf{1}(\text{sign}(S_w(x)) \neq y)$$

- Surrogate losses:
  - Logistic loss (binomial deviance):

$$\log_2(1 + \exp(-y \cdot S_w(x)))$$

  - Hinge loss (support vector):

$$\max(0, 1 - y \cdot S_w(x))$$



(Figure from Section 10.4 of "Elements of Statistical Learning")

In-class exercise: Intuition of the logistic loss

$$\log_2(1 + \exp(-y \cdot S_w(x)))$$

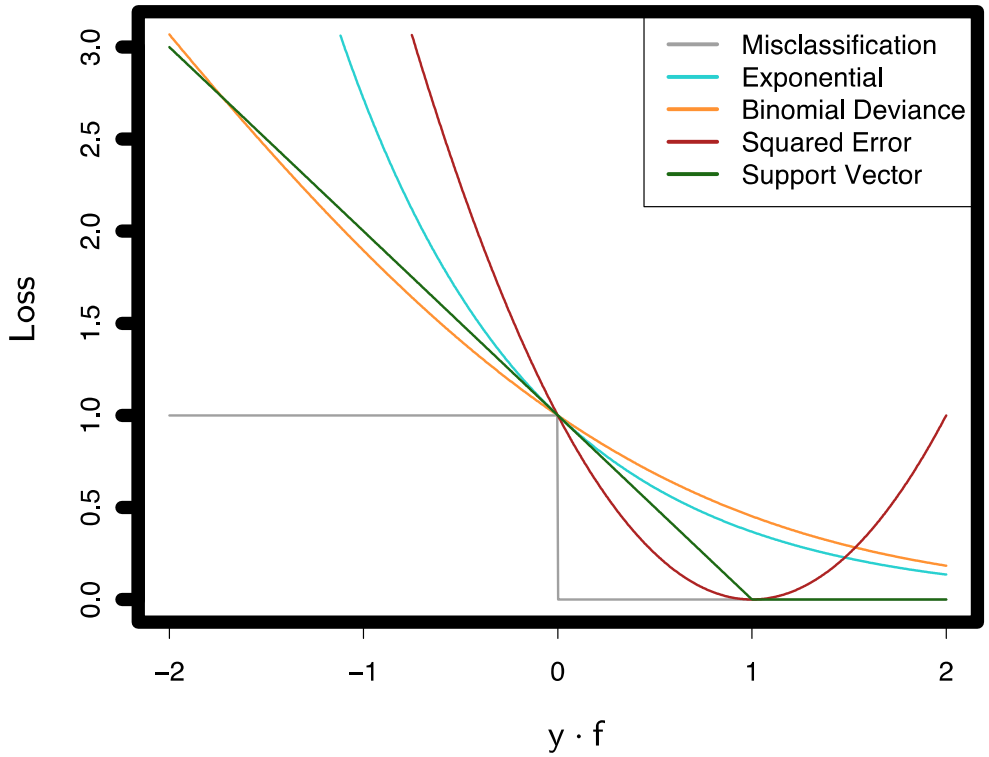
Try plotting the logistic loss as a function of $y \cdot S_w(x)$

1. What happens when the classifier predicts correctly?

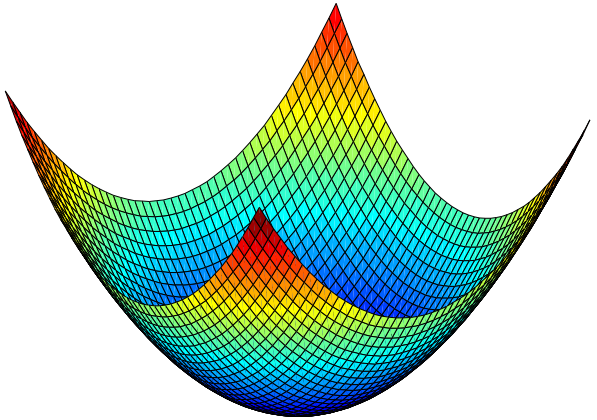
2. What happens when the classifier predicts incorrectly?

Which surrogate loss is easier to minimize?

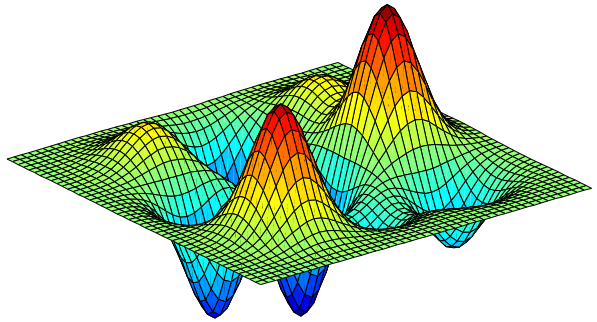
- Continuous
- Differentiable
  - Except hinge loss, i.e., loss used in “support vector machine (SVM)”
- Convex



Convex vs Nonconvex optimization



- Unique optimum: global/ local.



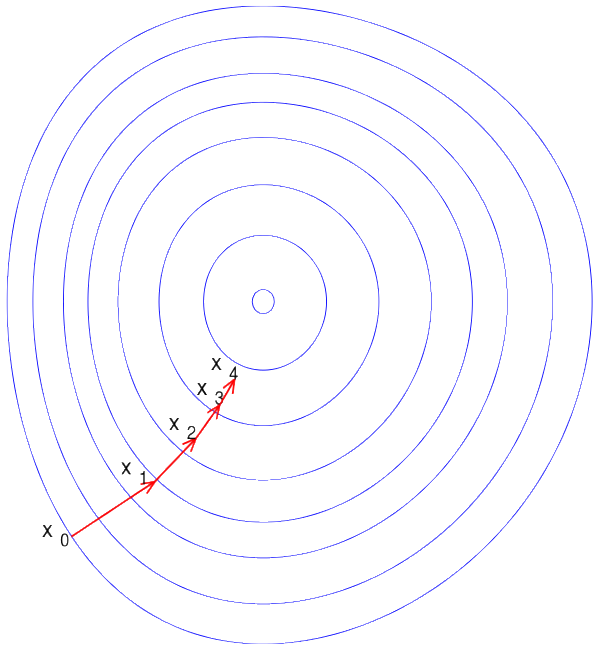
- Multiple local optima
- In high dimensions possibly exponential local optima

* Be careful: The surrogate loss being convex does not imply all ML problems using surrogate losses are convex. Linear classifiers are, but non-linear classifiers are usually not.

How do we optimize a continuously differentiable function in general?

- The problem: $\min_{\theta} f(\theta)$
- Gradient descent in iterations

$$\theta_{t+1} = \theta_t - \eta_t \nabla f(\theta_t)$$



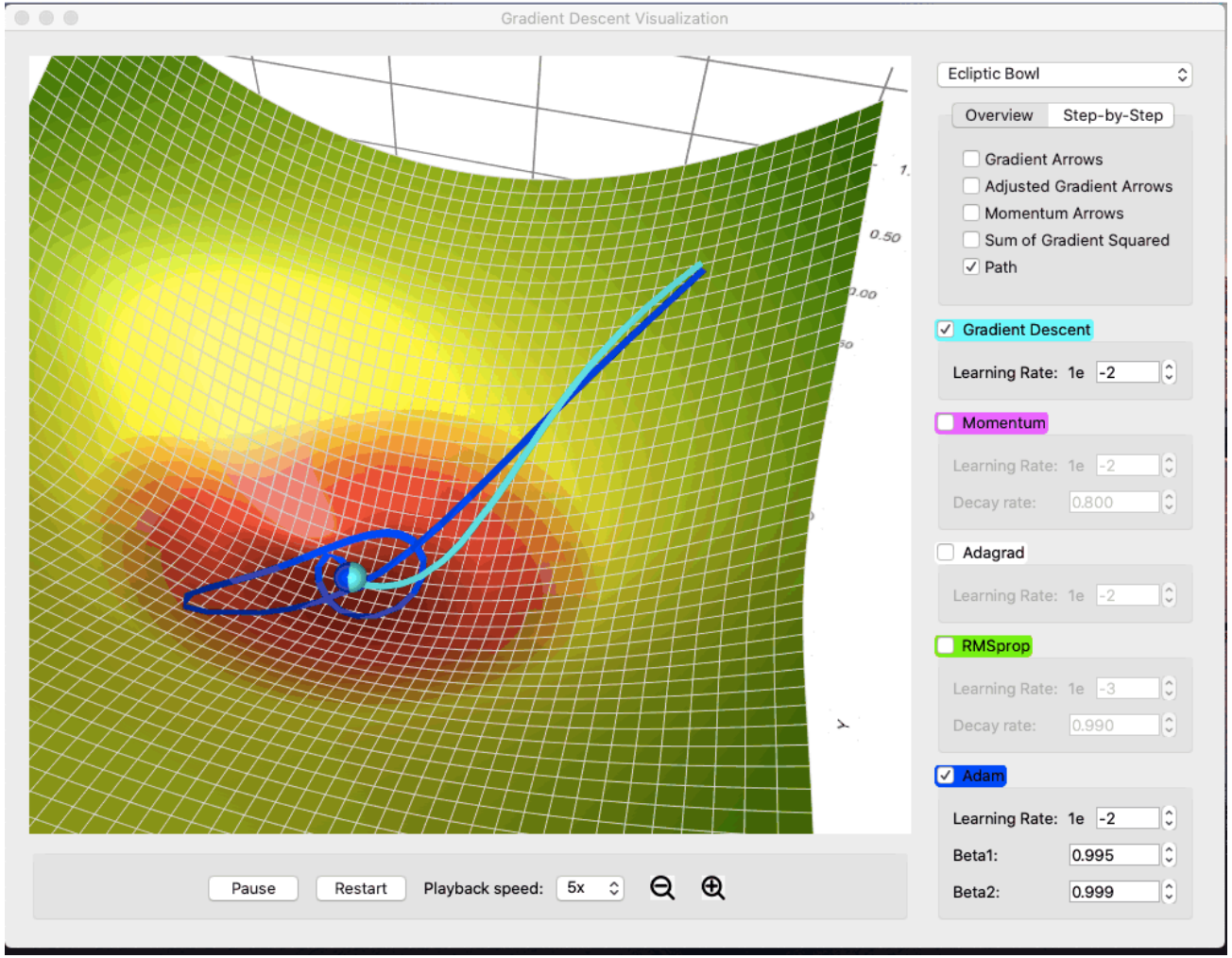
In-class exercise: gradient descent

- $\min f(x) = x^2$

1. Find x_2 given $x_0 = 2, \eta = 0.1$
2. Find x_2 given $x_0 = 2, \eta = 0.4$
3. Find x_2 given $x_0 = 4, \eta = 0.4$
4. Find x_2 given $x_0 = 2, \eta = 1.5$

Gradient Descent Demo in 2-D

- An excellent demo tool:
  - https://github.com/lilipads/gradient_descent_viz



Gradient of logistic loss for learning a linear classifier

- The function to minimize is

$$\min_{w \in \mathbb{R}^d} \frac{1}{n} \sum_{i=1}^{n} \log(1 + \exp(-y_i \cdot x_i^T w))$$

- In-class exercise: Calculate the gradient of loss function w.r.t w

$$\nabla f(w) = \frac{1}{n} \sum_{i=1}^{n} \frac{\exp(-y_i \cdot x_i^T w)}{1 + \exp(-y_i \cdot x_i^T w)} (-y_i x_i)$$

Hint:
- Apply the chain rule.
- d log(x) / dx = 1/x
- d exp(x) / dx = exp(x)

Drawback: Gradient Descent (GD) uses all data to do one update.

Key question: Is there an efficient way to optimize loss function?