UNIVERSITY AT ALBANY
STATE UNIVERSITY OF NEW YORK

CSI 436/536 (Spring 2025)

Machine Learning

Lecture 7: Linear Classifier

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Feb 17, 2025

Announcement

- HW1 due today.
- HW 2 will be released later today.

Recap: evaluation criteria

- Confusion matrix of binary classification
    - True Positive, False Positive, True Negative, False Negative
- Performance metrics
    - Accuracy
        - Very popular!
    - Precision / Recall / f1 score
    - AUC (Area Under Curve)
        - The larger area, the better performance
- Non-linearly separable problems
    - Feature transformation

Today

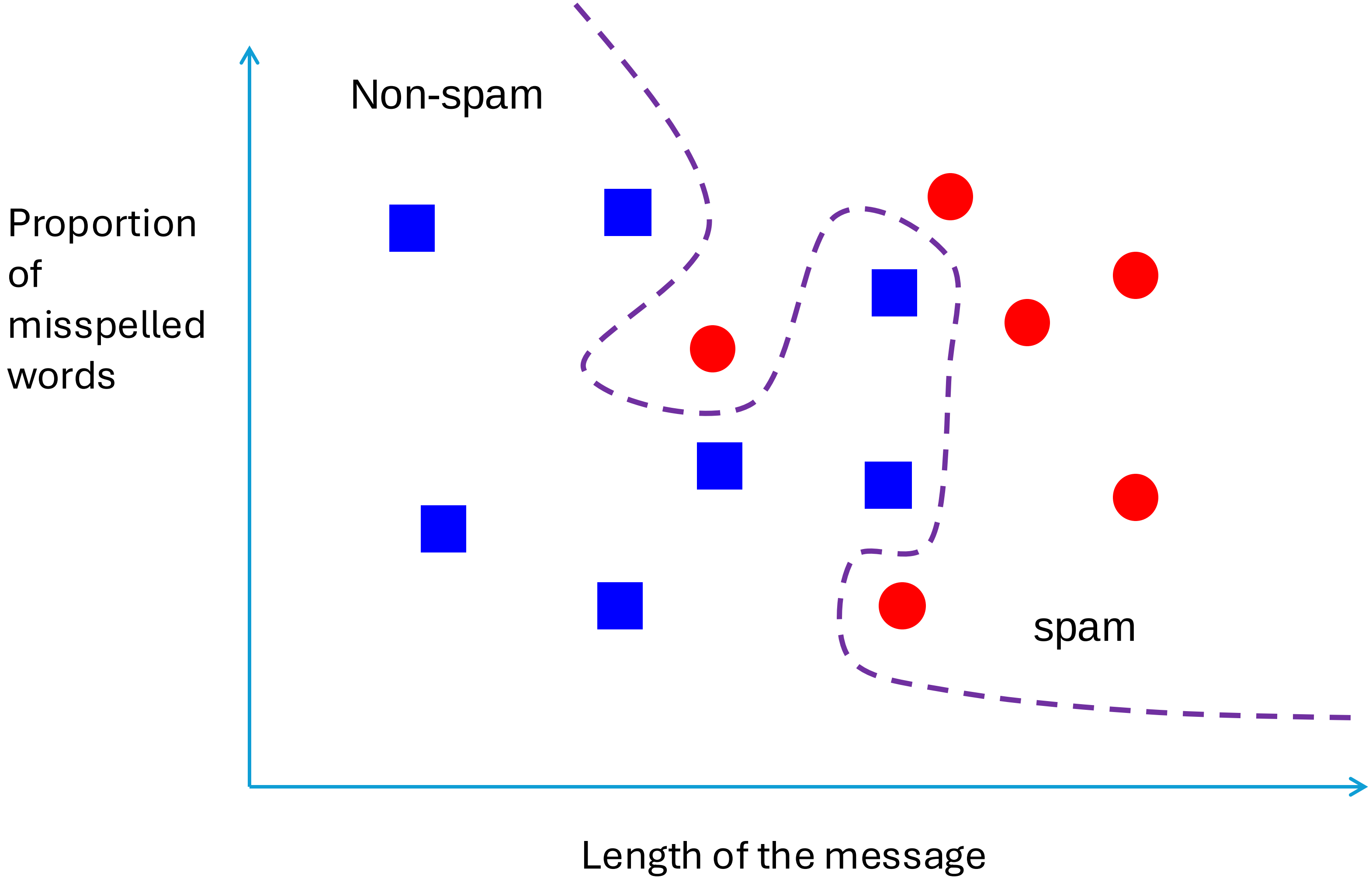
- Problem of overfitting

- Data splitting methods:
    - Holdout
    - Cross validation

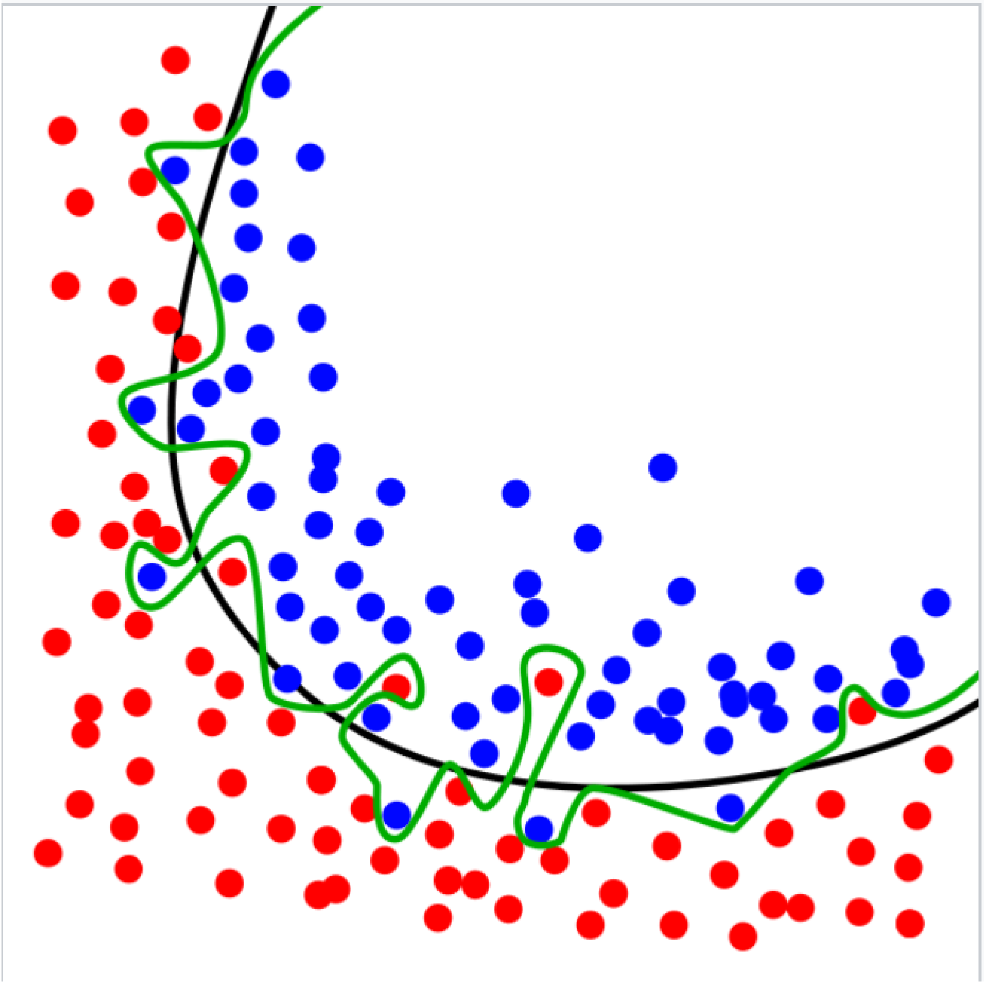
- Key questions about learning linear classifiers

- Perceptron algorithm

Non-linear decision boundary!



The problem of Overfitting

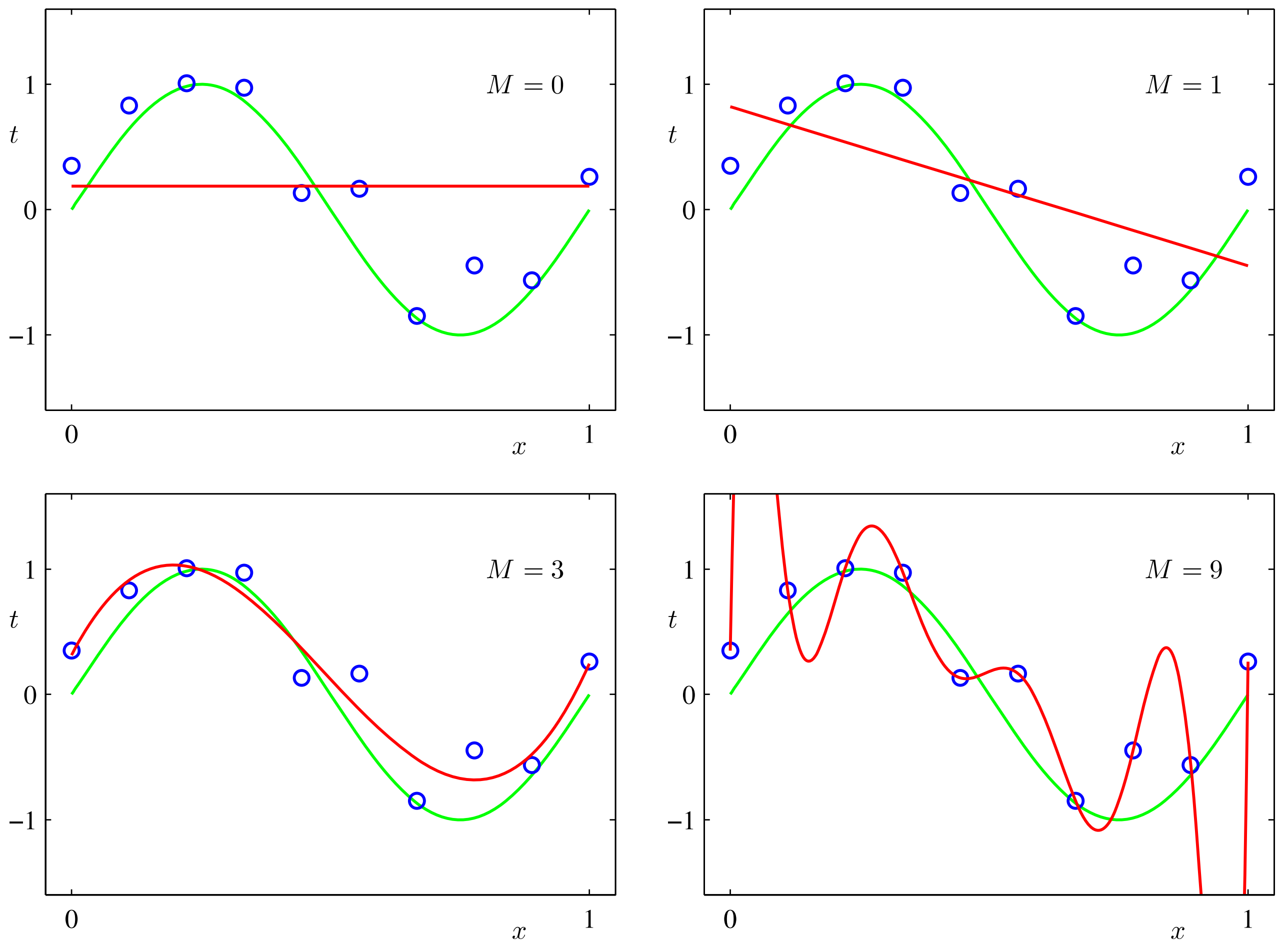


The green line represents an overfitted model.

1. Best follows the training data
2. Too dependent on training data
3. More likely to fail (higher error rate) than black line on new unseen test data

Discussion: examples of overfitting in our learning as human beings?

Another example of overfitting in the problem of "Curve Fitting"

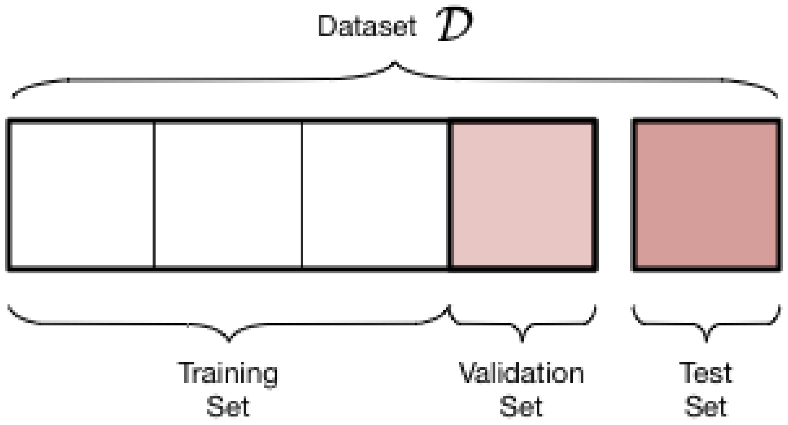


Fundamental question in ML:

The learner only sees the "training data" but ideally wants to do well on "new data"!

- The problem of generalization (from training to test).
- All performance metrics we learned before should be calculated on the new test data.
- The issue is that we don't have access to new test data…

Data splitting methods: Holdout

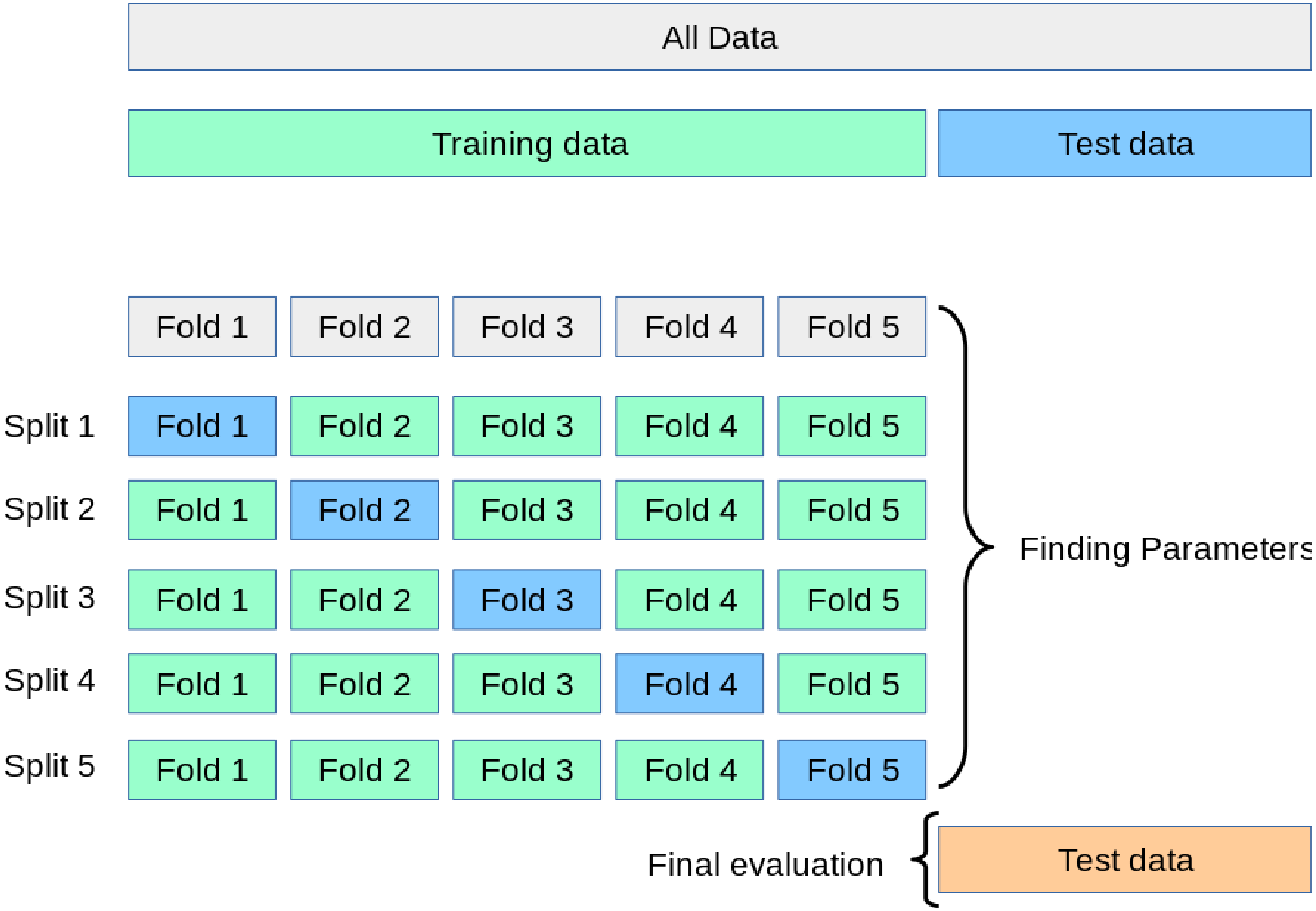


Validation set is used for hyperparameter search (also known as model-selection):

- choosing decision tree vs. linear classifier
- Select features, tune hyperparameters

Test set is used only once to report the final results.

Data splitting: 5-fold cross validation



	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5
Split 1	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5
Split 2	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5
Split 3	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5
Split 4	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5
Split 5	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5

https://scikit-learn.org/stable/modules/cross_validation.html

Checkpoint

- Deal with linearly non-separatable cases:
    - Feature transformation
    - Non-linear decision boundary
- Problem of overfitting
    - Too dependent on training data and may not good on test data
- Goal of machine learning
    - Learning == search for the best hypothesis in a hypothesis class
    - Ideally, we want to minimize “test error”
        - But all we have access to is the training data
        - We have a practical way --- data-splitting --- to evaluate a classifier

Recap: Linear classifier

- Take input feature vector
  - $\text{Score}(x) = w_0 + w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4$
  - $x_1 = 1(\text{has hyperlinks})$
  - $x_2 = 1(\text{on contact list})$
  - $x_3 = \text{proportion of misspelling}$
  - $x_4 = \text{length}$
- Let label space be $\{-1,1\}$
- Make prediction by thresholding a weighted average of the feature vector at 0:
  - $h_w(x) = \begin{cases} 1, \text{if Score}(x) \geq 0 \\ -1, \text{if Score}(x) < 0 \end{cases}$

Key questions about learning linear classifiers

$$\min_{w \in \mathbb{R}^d} \text{Error}(w) = \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}(h_w(x_i) \neq y_i)$$

1. Does a solution exist?

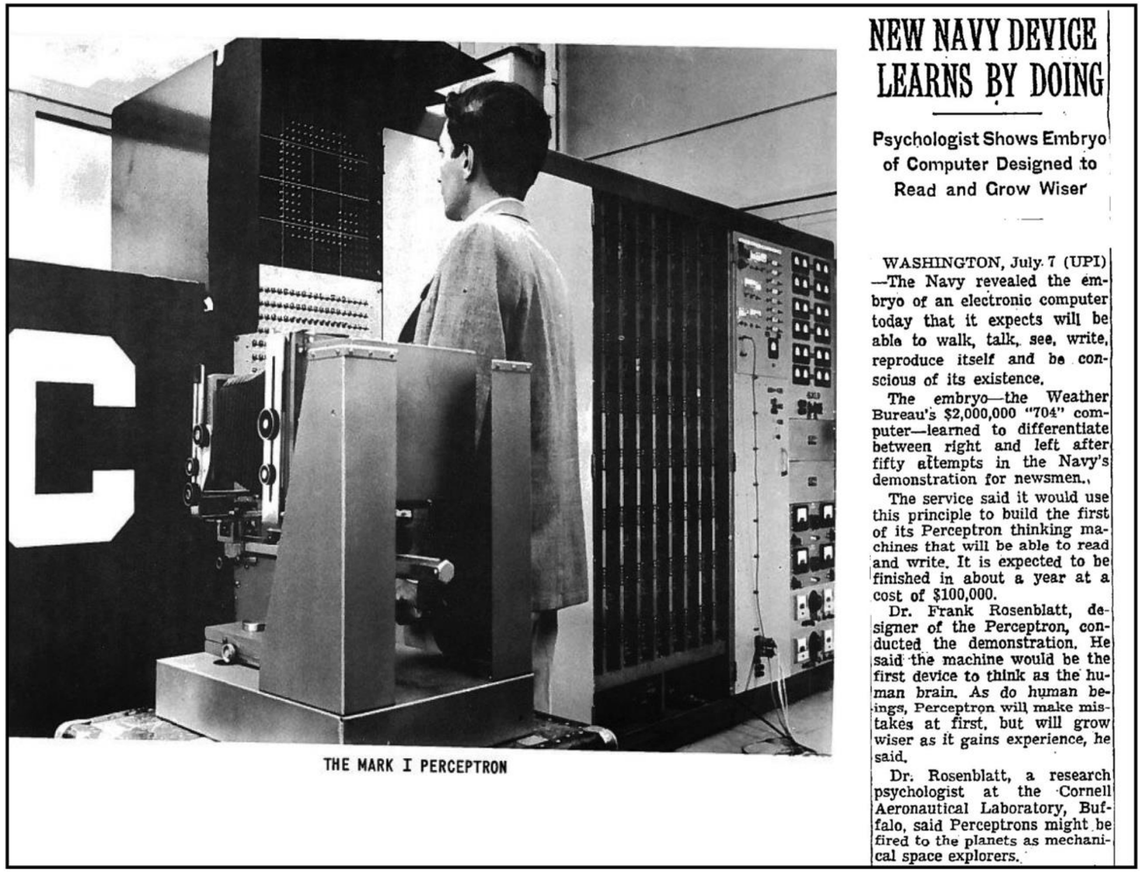
2. Is the solution unique?

3. Is there an efficient algorithm to find it?

4. How does it work on data points *not* used for training?

5. What are the assumptions needed?

Perceptron algorithm (Rosenblatt, 1957) for linear classifier



THE MARK I PERCEPTRON

NEW NAVY DEVICE LEARNS BY DOING

Psychologist Shows Embryo of Computer Designed to Read and Grow Wiser

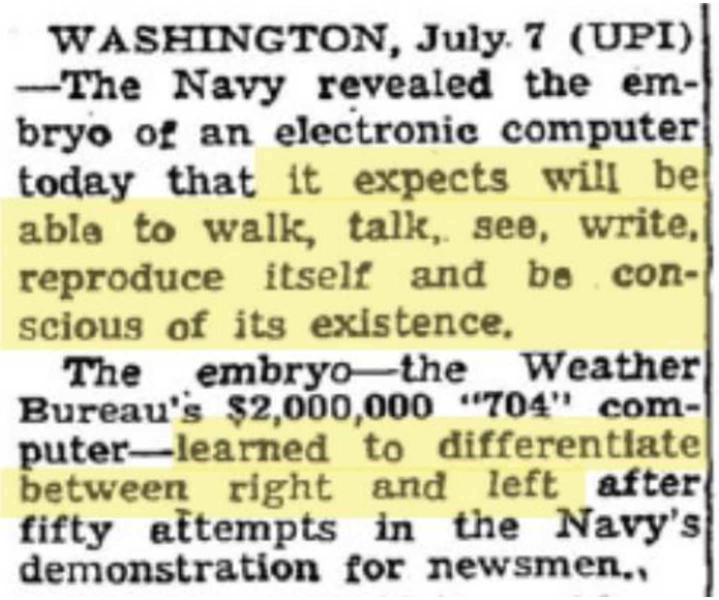
WASHINGTON, July 7 (UPI) —The Navy revealed the embryo of an electronic computer today that it expects will be able to walk, talk, see, write, reproduce itself and be conscious of its existence.

The embryo—the Weather Bureau's $2,000,000 "704" computer—learned to differentiate between right and left after fifty attempts in the Navy's demonstration for newsmen.,

The service said it would use this principle to build the first of its Perceptron thinking machines that will be able to read and write. It is expected to be finished in about a year at a cost of $100,000.

Dr. Frank Rosenblatt, designer of the Perceptron, conducted the demonstration. He said the machine would be the first device to think as the human brain. As do human beings, Perceptron will make mistakes at first, but will grow wiser as it gains experience, he said.

Dr. Rosenblatt, a research psychologist at the Cornell Aeronautical Laboratory, Buffalo, said Perceptrons might be fired to the planets as mechanical space explorers.



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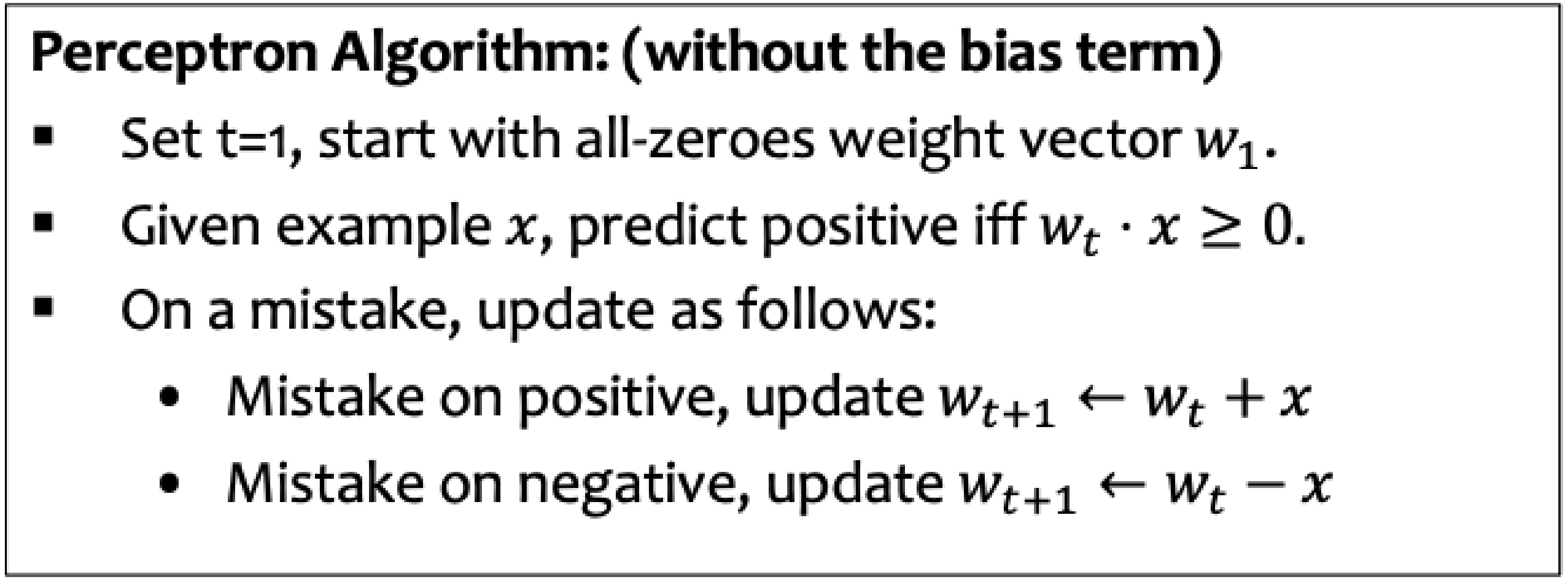
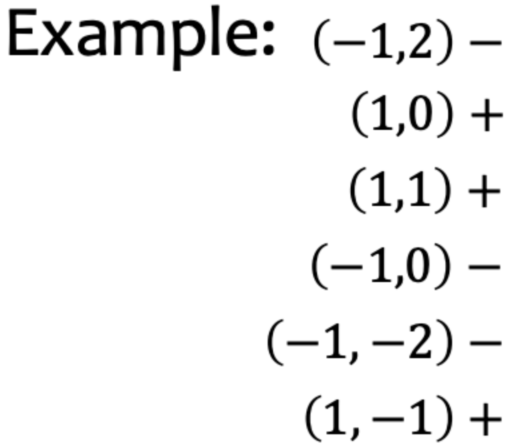
The perceptron algorithm takes an arbitrary sequence of inputs, predict on-the-fly, then update the weights if it makes a mistake!



Perceptron Algorithm: (without the bias term)

- Set t=1, start with all-zeroes weight vector w_1.
- Given example x, predict positive iff $w_t \cdot x \geq 0$.
- On a mistake, update as follows:
    - Mistake on positive, update $w_{t+1} \leftarrow w_t + x$
    - Mistake on negative, update $w_{t+1} \leftarrow w_t - x$

In-class exercise



Example: $(-1,2)\ -$

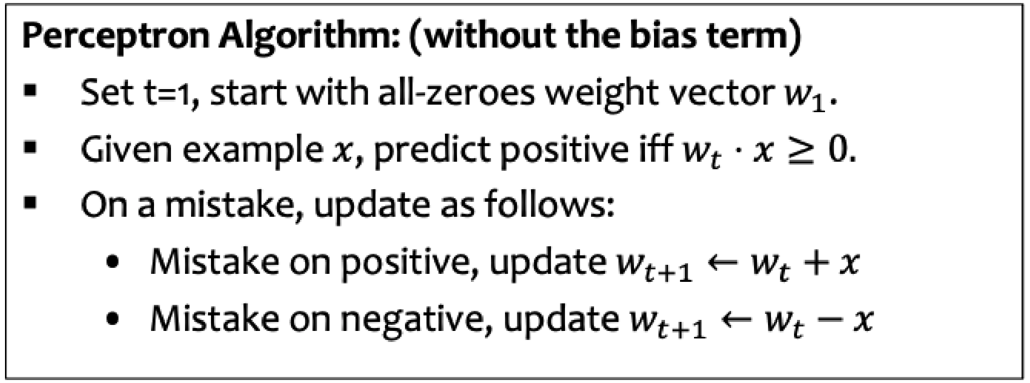
$(1,0)\ +$

$(1,1)\ +$

$(-1,0)\ -$

$(-1,-2)\ -$

$(1,-1)\ +$



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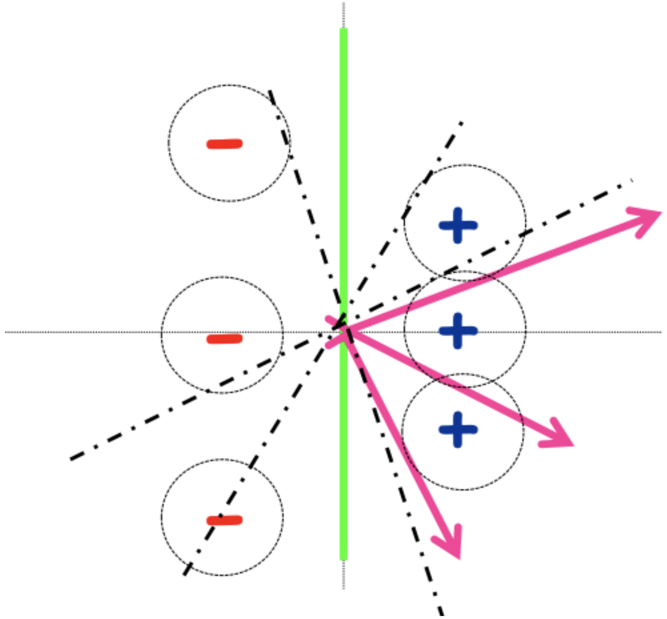
Solution

Example:

$(-1,2)$	$-$	✗
$(1,0)$	$+$	✓
$(1,1)$	$+$	✗
$(-1,0)$	$-$	✓
$(-1,-2)$	$-$	✗
$(1,-1)$	$+$	✓



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$$w_1 = (0,0)$$

$$w_2 = w_1 - (-1,2) = (1,-2)$$

$$w_3 = w_2 + (1,1) = (2,-1)$$

$$w_4 = w_3 - (-1,-2) = (3,1)$$

Pseudo code / implementation for the perceptron algorithm

The Perceptron algorithm

```
Initialize $w_1 = [0, 0, ..., 0]$
while $1$:
    1. Gets feature vector $x$
    2. Agent makes prediction $\hat{y} \leftarrow \text{sign}(w_t \cdot x)$
    3. Reveals label $y$
    4. If $\hat{y} \neq y$ :
        a. $w_{t+1} \leftarrow w_t + yx$
        b. $t \leftarrow t + 1$
```

Perceptron algorithm makes limited number of mistakes! (if the data is linearly separable)

Theorem (Novikoff, 1962): Assume $\|x\|_2 \leq R$ and there exists w^* such that every input satisfies that $\frac{|x \cdot w^*|}{\|w^*\|_2} \geq \gamma$ and $y = \text{sign}(x \cdot w^*)$. Then the total number of mistakes Perceptron makes is smaller than R^2/γ^2 .

Remarks:

- The algorithm can run infinitely long.
- No assumption on data distribution.
- Every new data point to predict on is new.
- No hyperparameters to choose. The algorithm does not need to know R, γ.
- Can support infinitely many features!

Using Perceptron over and over

$$\min_{w \in \mathbb{R}^d} \text{Error}(w) = \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}(h_w(x_i) \neq y_i)$$

- Perceptron can take any sequence of data
- Loop over the data again and again until there is no mistakes.

```
while not converged:
    converged = true
    for over data set:
        make prediction
        If "Mistake":  1. Update weights.  2. set converged = false
```

- Converge in after at most $\frac{nR^2}{\gamma^2}$ inner loop iterations.

Limitations of the Perceptron Algorithm

- What if the margin is small?
    - The perceptron algorithm will be slower

- What if non-linearly separable?
    - The perceptron algorithm is known to not converge!

Summary

- Problem of overfitting
    - Too dependent on training data and may not good on test data
- Goal of machine learning
    - Learning == search for the best hypothesis in a hypothesis class
        - Usually solved by optimization
    - Ideally, we want to minimize “test error”
        - But all we have access to is the training data
        - We have a practical way --- data-splitting --- to evaluate a classifier
- Perceptron
    - Algorithm to train a linear classifier
    - ML learns from mistakes
    - Doesn’t converge if non-linearly separable