Version Spaces

1. A version space describes a set of hypotheses. How is this set defined?

5 pts.

Decision Tree Learning

For the following questions, assume a top-down decision tree learning algorithm using information gain as we discussed it in class.

1. Correct or false: The ordering of the examples influences the decision tree that the algorithm returns. Justify your answer!

   3 pts.

2. Correct or false: If the examples have N attributes (all of which are binary), then the maximum number of nodes (with attribute tests) in a learned tree is $2^N - 1$. Justify your answer!

   3 pts.

3. Correct or false: Assume that there is at least one decision tree h with $Err_S(h) = 0$. The decision tree learning algorithm returns a tree that is consistent with the data and that has the minimum number of nodes.

   3 pts.

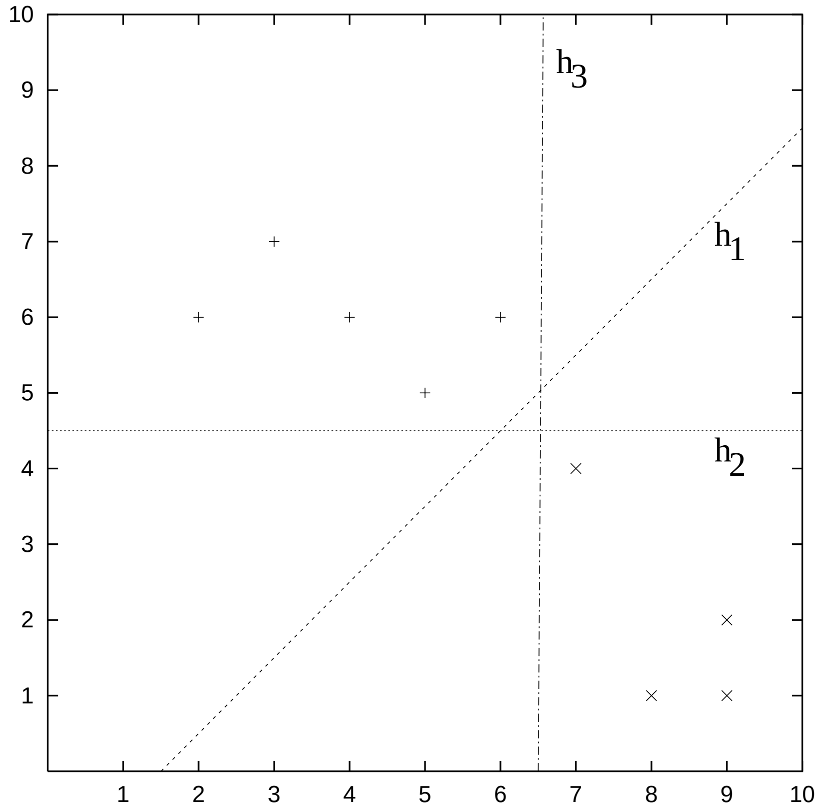
Perceptrons

1. Imagine the following situation: You train a perceptron (with $b = 0$) on a dataset and you notice that it made e updates. You know that all feature vectors $\vec{x}_i$ in the training set have Euclidean length one. What can you conclude about the (geometric) margin δ?

5 pts.

Optimal (Hard-Margin) Hyperplanes

The following graph has 9 points and three hyperplanes plotted into a coordinate system. “+” stands for a positive example, “x” for a negative example.



1. Which of the three hyperplanes is the maximum-margin hyperplane? Justify your answer by giving the definition of “maximum-margin hyperplane”!

3 pts.

2. Which of the points are support vectors? Mark them with “1” in the plot!

3 pts.

3. Which of the points are guaranteed to not be misclassified in leave-one-out testing? Mark them with "2" in the plot! Justify your answer with a one sentence explanation!

3 pts.

4. Indicate in the plot which distances are - by definition of maximum-margin hyperplanes - exactly equal to the size of the margin!

3 pts.

5. Given is a training sample $S = ((\vec{x}_1, y_1), ..., (\vec{x}_n, y_n)$ with $\vec{x}_i \in \Re^N$ and $y_i \in \{-1, +1\}$. Transform each vector $\vec{x}_i = (x_1, ..., x_N)$ to a vector $\vec{x}'_i = (x_1, ..., x_N, 0, ..., 0, 1, 0, ...0)$ with the 1 in the $(N+i)$-th position. Prove that this construction always leads to a linearly separable dataset (ie. $S = ((\vec{x}'_1, y_1), ..., (\vec{x}'_n, y_n)$ is linearly separable)!

10 pts.

Soft-Margin Support Vector Machines

1. Discuss two reasons for using a soft-margin SVM instead of a hard-margin SVM!

   6 pts.

2. How do training error and margin of a soft-margin SVM (generally) change for different values of C?

   6 pts.

   As a reminder, here is the primal soft-margin SVM optimization problem:

$$
\begin{aligned}
min_{\vec{w},b\vec{\xi}} \quad & \frac{1}{2}\vec{w}\cdot\vec{w} + C\sum_{i=1}^{n}\xi_i && (1)\\
s.t. \quad & y_1(\vec{w}\cdot\vec{x}_1 + b) \geq 1 - \xi_1 \text{ and } \xi_1 \geq 0 && (2)\\
& \dots && (3)\\
& y_n(\vec{w}\cdot\vec{x}_n + b) \geq 1 - \xi_n \text{ and } \xi_n \geq 0 && (4)
\end{aligned}
$$

7 Kernels

Let $K(\vec{x}_i, \vec{x}_j)$ be a kernel so that for all $\vec{x}_i$ and $\vec{x}_j$: $K(\vec{x}_i, \vec{x}_j) = \phi(\vec{x}_i) \cdot \phi(\vec{x}_j)$. $\phi(\vec{x})$ is called the image of $\vec{x}$.

1. Name two reasons for why it might be beneficial to use a kernel!

   6 pts.

2. Given two points $\vec{x}_i$ and $\vec{x}_j$ in input space, show how you can compute the Euclidian distance between their images in feature space $\phi(\vec{x}_i)$ and $\phi(\vec{x}_j)$ without computing ϕ explicitly.

   6 pts.