Lecture 32 : ImageNet is a Convolutional Neural Network (CNN), The Convolution Rule

$$(c \times d)_k = \sum_{i+j=k} c_i d_j \; x^i x^j = x^k = \sum c_i d_{k-i}$$

$$(c_0 + c_1 x + \dots + c_N x^N)(d_0 + d_1 x + \dots d_M x^M)$$

Convolution of functions

$$(f * g)(x) = \int_{t=-\pi}^{\pi} f(t) g(x-t)\, dt$$

$$(c \circledast d)_k = \sum_{i=0}^{N-1}{}^{\text{modulo}} c_i d_{k-i}$$ Cyclic convolution coefficient

Non periodic

$$\begin{bmatrix} t_0 & t_1 & t_2 & & \\ t_{-1} & t_0 & t_1 & t_2 & \\ t_{-2} & t_{-1} & t_0 & t_1 & t_2 \end{bmatrix}$$

Toeplitz

Periodic

$$\begin{bmatrix} c_0 & c_1 & c_2 & & \\ & c_0 & c_1 & c_2 & \\ c_2 & & c_0 & c_1 & c_2 \\ c_1 & c_2 & & c_0 & c_1 \\ c_1 & c_2 & & & c_0 \end{bmatrix}$$

Circulant

Eigenvectors of circulants = Columns of F

$$Fc = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & i^2 & i^3 \\ 1 & i^2 & i^4 & i^6 \\ 1 & i^3 & i^6 & i^9 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} = \text{Eigenvalues of } C$$

$$C = c_0 I + c_1 P + c_2 P^2 + c_3 P^3$$

Eigenvectors = cols of F

Eigenvalues = Fc

Convolution rule

$F(c \circledast d)$ Top row of CD = cyclic convolution = $c \circledast d$

Eigenvalues of CD = (eigenvalues of C)(eigenvalues of D)

$$F(c \circledast d) = (Fc) .* (Fd)$$

Eigenvectors of C equals eigenvectors of D = columns of F.

Hence, each eigenvalue $\lambda_k(CD) = \lambda_k(C)$ times $\lambda_k(D)$

=> Useful because of the FFT to multiply by F