



Lecture 33: Neural Nets and the Learning Function

Construction of Neural Nets

Learning Function $F(x, v)$

↑
to optimize the weights A_k and b_k

$$v_k = \text{ReLU}(A_k v + b_k)$$

Choose x to min Loss L

$$L = \frac{1}{N} \sum_{i=1}^N (F(x, v_i) - \text{true}_i)^2$$

Loss Functions

- ① Square loss = sum of $\| \cdot \|_2^2$ (regression)
- ② L^1 loss = sum of $\| \cdot \|_1$
- ③ Hinge loss $(-1, 1)$, (classifications)
- ④ Cross-entropy loss (neural nets)

Distance Matrices

Question: Know distances squared $\|x_i - x_j\|^2 = d_{ij}$

FIND positions x_i in \mathbb{R}^d (also find d)

$$\|x_i - x_j\|^2 = \text{given } d_{ij}$$

Given D matrix,

Find x matrix (positions)

$$d_{ij} = \|x_i - x_j\|^2 = (x_i, x_i) - (x_i, x_j) - (x_j, x_i) + (x_j, x_j) \text{ entries in } D$$

rank 1 matrix
cols. repeated

rank 1 matrix
all same rows

$$\begin{bmatrix} | & | & | & | & | \end{bmatrix} - 2 \overset{x^T x}{G} + \begin{bmatrix} \equiv \\ \equiv \\ \equiv \end{bmatrix}$$

$$x^T x = G = -\frac{1}{2} (D - 1d^T - d1^T)$$

$$\begin{matrix} \downarrow & \downarrow \\ \begin{bmatrix} | \\ | \\ | \end{bmatrix} [d] & [d] \begin{bmatrix} \dots & 1 \end{bmatrix} \end{matrix}$$

\Rightarrow Given $x^T x$ Find $x^{n \times n}$

① Eigenvalues of $x^T x = Q \Lambda Q^T \Rightarrow x = Q \sqrt{\Lambda} Q^T$

② Elimination on $x^T x = LDU \Rightarrow x^T x = LDL^T \Rightarrow x = \sqrt{D} L^T$
Cholesky