Lecture 24 : Linear Programming and Two-Person Games

Minimizing a function $F(x) = c_1x_1 + \dots + c_nx_n$

$$\min \quad c^Tx = c_1x_1 + \dots + c_nx_n$$

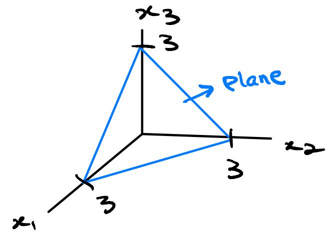
constraint on x $\quad \underset{m\times n}{A}\underset{n\times 1}{x} = b \quad m < n$

$x \geq 0$ means $x_1 \geq 0, \cdots, x_n \geq 0$

$\Big\}$ "feasible set" of x's

$\underline{\min}$ $x_1 + 2x_2 + 5x_3$

$x_1 + x_2 + x_3 = 3$



Ways to solve

① Simplex method

| corner → next corner

⇒ Minimum will be reached in one of the corners

② Interior method (Karmarkar)

Dual LP for $y_1, \cdots, y_m$

$$\max \quad b^Ty$$

$$A^Ty \leq c$$

Weak duality

$$\underset{\text{maximize}}{b^Ty} \leq \underset{\text{minimize}}{c^Tx}$$

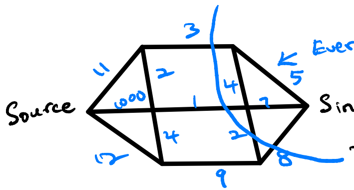
Proof

$$b^Ty = (x^TA^T)y = (A^Ty)^Tx \leq c^Tx$$

Strong duality

The maximum of y^Tb equals the minimum of c^Tx

max flow = min cut



Every edge is capacity

→ This is the minimum cut 14

∴ Max flow = min cut = 14

any flow ≤ capacity of any cut

x_{source}

x_{sink}

max $x_{sink} \leq 14$