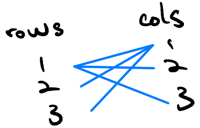
Lecture 30: Completing a Rank-One Matrix, Circulants!

$A = uv^T$

Example

$m = n = 3$ Condition: $m+n-1 = 5$ nonzeroes in 5 positions

$$\begin{bmatrix} x & x & x \\ x & & \\ x & & \end{bmatrix}$$



Does not form a self loop.

Hence, it can be a rank one matrix.

$$a_{22} = \frac{a_{12} a_{21}}{a_{11}}$$

Bipartite graph

Cyclic → diagonals circle around

Convolution → constant down each diagonal.

Circulant matrix $= c_0 I + c_1 P + c_2 P^2 + c_3 P^3$

$$C = \begin{bmatrix} 2 & 5 & 1 & 0 \\ 0 & 2 & 5 & 1 \\ 1 & 0 & 2 & 5 \\ 5 & 1 & 0 & 2 \end{bmatrix} \qquad P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \\ x_0 \\ x_1 \end{bmatrix}$$

C, D circulants

CD is also a circulant

Example: $P_{4\times4}$ circular shift $= \begin{matrix} (c_0 I + c_1 P + c_2 P^2 + c_3 P^3) \\ (d_0 I + d_1 P + d_2 P^2 + d_3 P^3) \end{matrix}$

$P^4 = I$

cyclic convolution

$(3, 1, 2) \circledast (4, 6, 1)$

$= (12, 22, 17, 13, 2)$

$= (25, 24, 17)$