# Lecture 16 : Derivatives of Inverse and Singular Values

$$\frac{dA^{-1}}{dt} = -A^{-1}\frac{dA}{dt}A^{-1} \qquad \frac{d\lambda}{dt} = y^T(t)\frac{dA}{dt}x(t)$$

$$\frac{d}{dt}A^2 = ? \qquad \frac{d\sigma}{dt} = ?$$

NOT $2A\frac{dA}{dt}$

$$\frac{(A+\Delta A)^2 - A^2}{\Delta t} = \frac{A(\Delta A) + (\Delta A)A + (\Delta A)^2}{\Delta t}$$

$$\frac{dA^2}{dt} = \boxed{A\frac{dA}{dt} + \frac{dA}{dt}A}$$

Derivative of $\frac{d\sigma}{dt}$

$$\boxed{\sigma(t) = u^TAv}$$

$Av = \sigma u$

$u^Tu = 1$

$v^Tv = 1$

$$\frac{du^T}{dt}u + u^T\frac{du}{dt} = \frac{d}{dt}(1) = 0$$

$$x^Ty = y^Tx$$

$$(\sigma u)^T v = (Au)^T v$$

$$\frac{d\sigma}{dt} = \frac{du^T}{dt}Av + u^T\frac{dA}{dt}v + u^TA\frac{dv}{dt}$$

$$= \sigma\frac{du^T}{dt}u \rightarrow 0 + u^T\frac{dA}{dt}v + \sigma u^T\frac{dv}{dt} \rightarrow 0$$

$$= \boxed{u^T\frac{dA}{dt}v}$$

$S \qquad \lambda_1 \geq \lambda_2 \geq \cdots$

$S + \theta u u^T \qquad \mu_1 \geq \mu_2 \geq \cdots$

positive change

$\mu_1 \geq \lambda_1 \geq \mu_2 \geq \lambda_2 \geq \cdots$

old new

Weyl's inequality for symmetric $S, T$

$$\boxed{\lambda_{i+j-1}(S+T) \leq \lambda_i(S) + \lambda_j(T)}$$

let $j=1$ : $\lambda_i(S+T) \leq \lambda_i(S) + \lambda_1(T)$

$\lambda_1 \to \lambda_{max}$

let $j=2$ : $\lambda_{i+1}(S+T) \leq \lambda_i(S) + \lambda_2(T)$

Nuclear Norm $\quad \|A\|_N = \sigma_1 + \cdots + \sigma_r$

like $\|v\|_1$

Completion (Netflix)

$$A = \begin{bmatrix} 3 & 2 & ? \\ 1 & ? & ? \\ ? & 4 & 6 \end{bmatrix}$$

Problem : put in missing data

Idea : Numbers that minimize $\|A\|_N$ are a good choice.

$\|v\|_1$ from $\|v\|_0 = \#$ nonzeros in $v$

$\to \sum |v_i|$

$\|A\|_N$ from $\|A\|_0 = \text{rank}(A)$