Lecture 15: Matrices $A(t)$ Depending on t, Derivative $= dA/dt$

Last time

- Change in A^{-1}

$(A - uv^T)^{-1} = \ldots$

This time

- Change in λ
- Change in σ

Very small change

$\frac{dA^{-1}}{dt}$ $\frac{d\lambda}{dt}$ $\frac{d\sigma}{dt}$

Why important?

- Matrices move, their inverses change, their λ's change, their σ's change.

Derivative of A^{-1}

$A(t)$ $A^{-1}(t)$

$\frac{dA}{dt}$ FIND $\frac{dA^{-1}}{dt}$

$B = A + \Delta A$

$$\boxed{B^{-1} - A^{-1} = B^{-1}(A - B)A^{-1}}$$

rank of this = rank of this

$$\frac{\Delta A^{-1}}{\Delta t} = (A + \Delta A)^{-1}\left(-\frac{\Delta A}{\Delta t}\right)A^{-1}$$

let $\Delta t \to 0$ Calculus

$$\boxed{\frac{dA^{-1}}{dt} = -A^{-1}\frac{dA}{dt}A^{-1}}$$

How about $\frac{d\lambda}{dt}$?

Facts $\quad A(t)\,x(t) = \lambda(t)\,x(t) \qquad y^T(t)\,A(t) = \lambda(t)\,y^T(x)$

$$Ax = x\Lambda \qquad y^TA = \Lambda y^T$$

normalize $\quad y^Tx = 1$

Formula 1

$$y^T(t)\,A(t)\,x(t) = \lambda(t)\,\underbrace{y^T(t)\,x(t)}_{=1}$$

Using product rule,

$$\frac{d\lambda}{dt} = \frac{dy^T}{dt}A\,x(t) + y^T(t)\frac{dA}{dt}x(t) + y^T(t)A\frac{dx}{dt}$$

$$= \lambda(t)\frac{dy^T}{dt}x(t) + \lambda(t)\,y^T(t)\frac{dx}{dt} + y^T(t)\frac{dA}{dt}x(t)$$

$$= \lambda(t)\Big[\underbrace{\frac{dy^T}{dt}x(t) + y^T(t)\frac{dx}{dt}}_{\text{This is derivative of } y^Tx=1 \implies 0}\Big] + y^T(t)\frac{dA}{dt}x(t)$$

$$= y^T(t)\frac{dA}{dt}x(t)$$

How does adding a true vector $(A + uv^T)$ affect λ's instead of differential equation in terms of ϵ just now?

$\lambda_j(S + uu^T) \,/\, \gamma_j(S)$

$\lambda_1 \geq \gamma_1 \geq \lambda_2 \geq \gamma_2 \geq \lambda_3 \geq \gamma_3 \geq \cdots$

λ_j is bigger than γ_j