Lecture 20 Definitions and Inequalities

Basic Probability $\quad p_1 \cdots p_n$

$x_1 \cdots x_n$

Expected Mean $\quad E[x] = p_1 x_1 + \cdots + p_n x_n$

Sample Mean $\quad m = \mu = \frac{1}{N}(x_1 + \cdots + x_N)$

Sample Variance $\quad s^2 = \frac{1}{N-1}\left[(x_1 - m)^2 + \cdots + (x_N - m)^2\right]$

Variance $\quad \sigma^2 = E[(x-m)^2] = E[x^2] - (E[x])^2$

Markov's inequality

applies when all $x_i \geq 0$

$$\mathrm{Prob}[x \geq a] \leq \frac{\bar{x}}{a} = \frac{\text{mean}}{a} = \frac{E[x]}{a}$$

Chebyshev's inequality

The probability of $|x(s) - \bar{x}| \geq a$ is at most $\frac{\sigma^2}{a^2}$

Proof:

Let $Y(s) = (x(s) - \bar{x})^2$

$E[Y] = \sigma^2$

$|x(s) - \bar{x}| \geq a$ square both side

$|x(s) - \bar{x}|^2 \geq a^2 \Rightarrow Y \geq a^2$

$$\boxed{\mathrm{Prob}(Y(s) \geq a^2) \leq \frac{\text{mean of } Y}{a^2} = \frac{\sigma^2}{a^2}}$$

Covariance Matrix

$$V = \sum_{\text{all } x_i, y_i} p_{ij} \begin{bmatrix} x_i - m_x \\ y_i - m_y \end{bmatrix} \begin{bmatrix} x_i - m_x & y_i - m_y \end{bmatrix}$$

$$E\left[(x - \bar{x})(x - \bar{x})^T \right] = \sum p_{ij} (x - \bar{x})(x - \bar{x})^T$$