STAB22

Chong Chen
University of Toronto, Scarborough
Department of Computer and Mathematical Sciences

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1 Recall

1.1 Testing hypothesis about proportions

- Determine the null hypothesis and the alternative hypothesis

$$H_0 : p = p_0$$
$$H_A : p \neq p_0$$

- Calculate a test statistic

$$z^{obs} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 * (1 - p_0)}{n}}}$$

- Determine the p-value
- Make a conclusion based on the p-value

1.2 Hypothesis Tests Errors

There are two types of errors in decisions:

- Type I Error: Rejection of H_0 when H_0 is true.
- Type II Error: Acceptance of H_0 when H_0 is false.

Table: Errors in Decisions

		Decision	
		Reject H_0	Do Not Reject H_0
Condition of H_0	H_0 True	Type I Error	Correct Decision
	H_0 False	Correct Decision	Type II Error

1.3 Significance Level α

If we reject H_0, we call this result statistically significant at level α.

Note: $P(\text{Type I error } |H_0) = \alpha$

1.4 Confidence Intervals and Hypothesis Tests

In general, a confidence interval with a confidence level of $C\%$ corresponds to a two-sided hypothesis test with an α level of (100 - C)%.

2 Tests and Confidence Intervals for the Population Mean

2.1 Example

A random sample of 25 New Yorkers were asked how much sleep they get per night. The sampled New Yorkers sleep 7.53 hours on average with standard deviation 0.77. Is there statistical evidence that the mean number of hours New Yorkers sleep per night is significantly larger than 7 hours and 30 minutes (7.5 hours)?

2.2 Hypothesis Test for the Mean

- Determine the null hypothesis and the alternative hypothesis

$$H_0 : \mu = \mu_0$$
$$H_A : \mu \neq \mu_0$$

- Calculate a test statistic

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$$

- Determine the p-value

  What would be the probability of observing a value at least as extreme as the observed test statistic if the null hypothesis were true?

  In this case, we have to compare t with a Student distribution with $n - 1$ degrees of freedom (d.f).

- Make a conclusion based on the p-value and the context of the problem.

  i.e. Reject the null hypothesis if the p-value is small.

3 Confidence Interval for the Mean

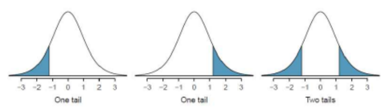
Confidence interval for μ is:

$$\bar{X} \pm t^*_{n-1} * \frac{s}{\sqrt{n}}$$

where t^*_{n-1} is the critical value.

3.1 Finding the Critical Values t^*_{n-1} with the t-Probability Table

one tail	0.100	0.050	0.025	0.010	0.005
two tails	0.200	0.100	0.050	0.020	0.010
df 1	3.08	6.31	12.71	31.82	63.66
2	1.89	2.92	4.30	6.96	9.92
3	1.64	2.35	3.18	4.54	5.84
4	1.53	2.13	2.78	3.75	4.60
5	1.48	2.02	2.57	3.36	4.03
6	1.44	1.94	2.45	3.14	3.71
7	1.41	1.89	2.36	3.00	3.50
8	1.40	1.86	2.31	2.90	3.36
9	1.38	1.83	2.26	2.82	3.25
10	1.37	1.81	2.23	2.76	3.17
11	1.36	1.80	2.20	2.72	3.11
12	1.36	1.78	2.18	2.68	3.05
13	1.35	1.77	2.16	2.65	3.01
14	1.35	1.76	2.14	2.62	2.98
15	1.34	1.75	2.13	2.60	2.95
16	1.34	1.75	2.12	2.58	2.92
17	1.33	1.74	2.11	2.57	2.90
18	1.33	1.73	2.10	2.55	2.88
19	1.33	1.73	2.09	2.54	2.86
20	1.33	1.72	2.09	2.53	2.85
21	1.32	1.72	2.08	2.52	2.83
22	1.32	1.72	2.07	2.51	2.82
23	1.32	1.71	2.07	2.50	2.81
24	1.32	1.71	2.06	2.49	2.80
25	1.32	1.71	2.06	2.49	2.79
150	1.29	1.66	1.98	2.35	2.61
200	1.29	1.65	1.97	2.35	2.60
300	1.28	1.65	1.97	2.34	2.59
400	1.28	1.65	1.97	2.34	2.59
500	1.28	1.65	1.96	2.33	2.59
∞	1.28	1.65	1.96	2.33	2.58



Example: For a sample size of $n = 25$,

- the critical value of a 95% CI for the mean is:
- a 95% CI for the mean is:
- the critical value of a 90% CI for the mean is:

3.2 Example cont'd

Find a 95% CI for the mean time New Yorkers sleep per night.

3.3 The Relationship between Intervals and Tests

Precisely, a level C confidence interval contains all of the possible null hypothesis values that would not be rejected by a two-sided hypothesis test at alpha level $1 - C$.

i.e. A 95% confidence interval matches a 0.05 level test for these data.

3.4 Interpretation

C% of intervals found in this way cover the true value.

4 Exercises

4.1 Exercise 1

You have sampled 25 students to find the mean SAT scores. A 95% confidence interval for the mean SAT score is 900 to 1100. Which of the following statements gives a valid interpretation of this interval?

(a) 95% of the 25 students have a mean score between 900 and 1100.

(b) 95% of the population of all students have a score between 900 and 1100.

(c) If this procedure were repeated many times, 95% of the sample means would be between 900 and 1100.

(d) If this procedure were repeated many times, 95% of the resulting confidence intervals would contain the true mean SAT score.

(e) If 100 samples were taken and a 95% confidence interval was computed, 5 of them would be in the interval from 900 to 1100.

4.2 Exercise 2

A manufacturer of light bulbs claims that their light bulbs have a mean life of 1520 hours with an unknown standard deviation. A random sample of 30 such bulbs is selected for testing. You want to test the null hypothesis that the population mean life is less than the manufacturer claims. If the sample produces a mean value of 1505 hours and a sample standard deviation of 86, what can you conclude? You may assume that all the necessary assumptions for the related test are satisfied by the data.

(a) We will reject the null hypothesis at alpha = 0.01

(b) We will reject the null hypothesis at alpha = 0.05

(c) We will reject the null hypothesis at alpha = 0.1

(d) We will reject the null hypothesis at alpha = 0.001

(e) We fail to reject the null hypothesis at alpha = 0.1

4.3 Exercise 3

Historically, the mean yield of corn in the United States has been 120 bushels per acre. A survey of 40 farmers this year gives a sample mean yield of 125 bushels per acre. In the past, the standard deviation of yields has been 12 bushels per acre.

Let μ be the mean yield of corn nationally for this year. Supposing that the past standard deviation is still correct, what is your P-value for testing a null hypothesis of $\mu = 120$ against an alternative of $\mu \neq 120$?

(a) 0.0041

(b) 0.0082

(c) between 0.01 and 0.02

(d) 2.64

(e) 125

4.4 Exercise 4

A hypothesis test at $\alpha = 0.05$ for the null hypothesis that $\mu = 30$ against the alternative that $\mu \neq 30$ has a P-value of 0.02. What can you say about the 95% confidence interval for μ?

(a) Since $\mu \neq 30$, 33 is inside the interval.

(b) 30 is inside the interval.

(c) 30 is outside the interval.

(d) There is no connection between the test and the confidence interval.