

# STAB22 TUT21

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## 1 Probability

### 1.1 Sample space

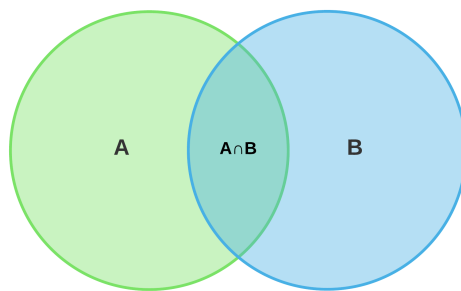
The sample space of a random phenomenon is the set of all possible outcomes. For example, if you toss a coin twice, the sample space is  $S = \{HH, HT, TH, TT\}$

### 1.2 Probability rules

- $P(A)$  is always between 0 and 1.
- If  $S$  is the sample space in a probability model, then  $P(S) = 1$ .
- The complement of an event  $A$ , written  $P(A^c)$ , is equal to  $1 - P(A)$ .

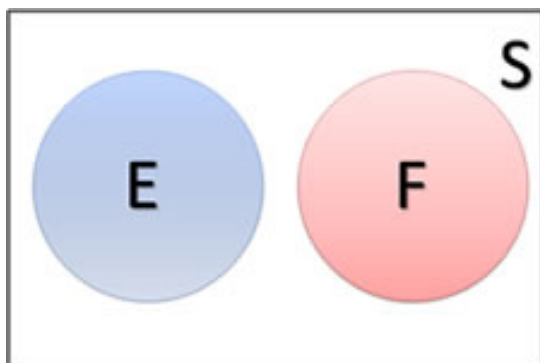
## 2 Disjoint events

### 2.1 Venn diagrams



$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

## 2.2 Disjointness



If  $E$  and  $F$  are **disjoint**, then  $P(E \text{ or } F) = P(E) + P(F)$ .

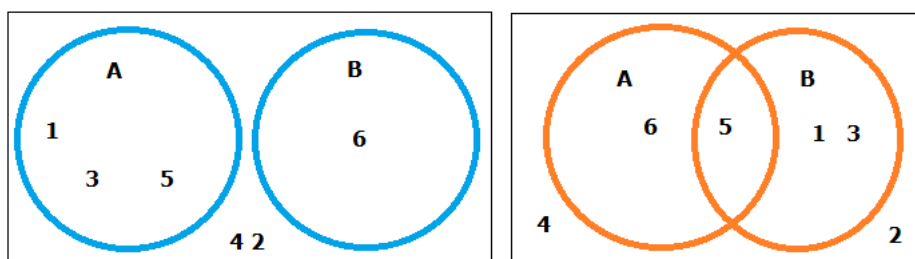
Example:

### Disjoint Events

**Event A:** Get an odd Number  
**Event B:** Get a 6

### Overlapping Events

**Event A:** Get a number over 4  
**Event B:** Get an odd number



## 3 Independent events

### 3.1 Conditional probabilities

$P(B|A) = P(B \text{ given } A)$  gives the probability of one event under the condition that another event has occurred. Such probabilities are called **conditional probabilities**.

When  $P(A) > 0$ , the conditional probability of  $B$  given  $A$  is

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

### 3.2 Independence

Two events  $A$  and  $B$  that both have positive probability are **independent** if  $P(A \text{ and } B) = P(A) * P(B)$

Example:

Toss a coin twice.

$A = \{\text{first toss is a head}\},$

$B = \{\text{second toss is a tail}\}$

## 4 Law of large numbers

The Law of Large Numbers (LLN) says that the relative frequency of some outcome reaches a limiting value as the number of trials becomes large.

NOTE In brief, the law of large numbers states that as the number of trials or observations increase, the actual or observed probability approaches the theoretical or expected value.

## 5 Examples

### 5.1

In a group of 100 people, 40 own a cat, 25 own a dog, and 15 own a cat and a dog. Find the probability that a person chosen at random, owns a dog, given that he owns a cat.

- (a) 0.375
- (b) 0.6
- (c) 0.25
- (d) 0.4
- (e) 0.15

### 5.2

The events  $A$  and  $B$  are such that  $P(A) = 2P(B)$ ,  $P(A \text{ and } B) = 0.08$ .  $A$  and  $B$  are independent. Find  $P(A \text{ or } B)$ .

- (a) 0.68
- (b) 0.6
- (c) 0.4
- (d) 0.52
- (e) 0