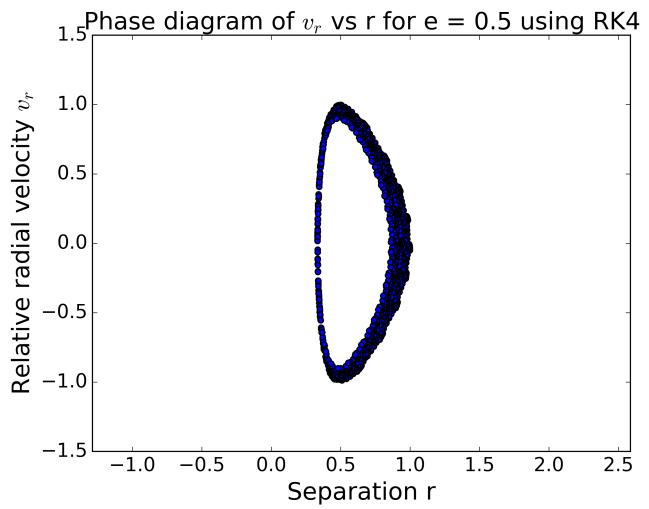
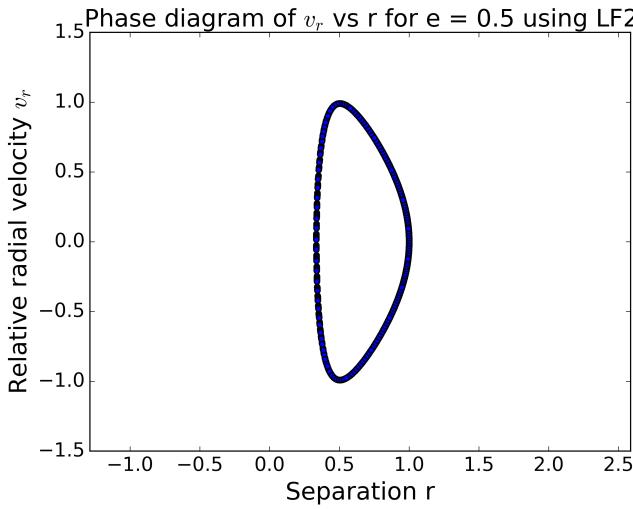
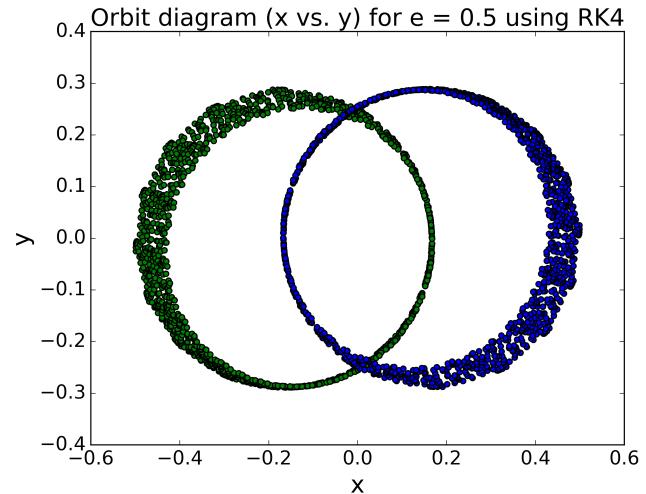
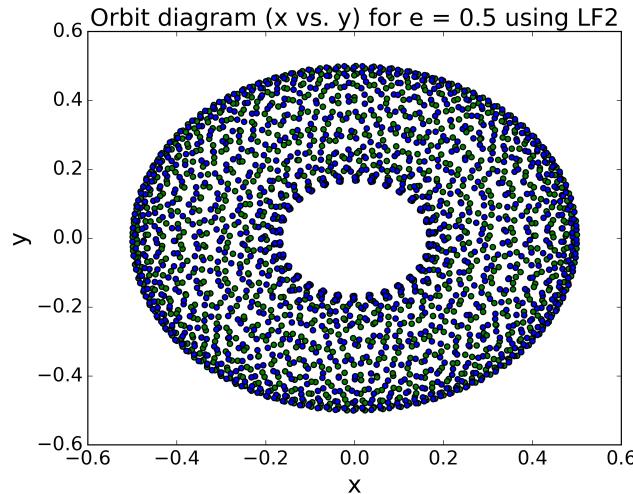


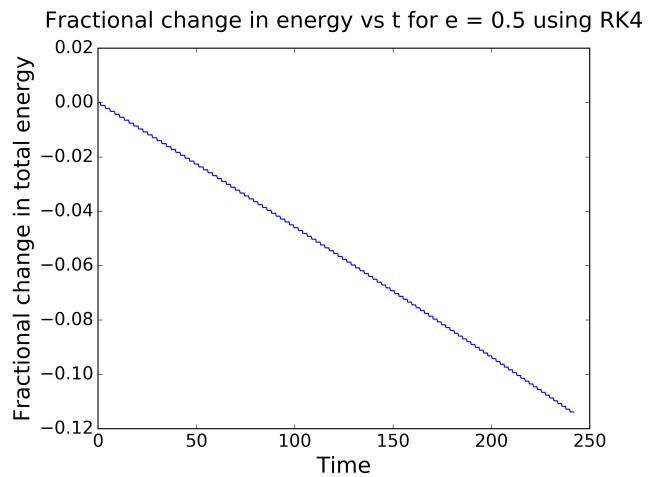
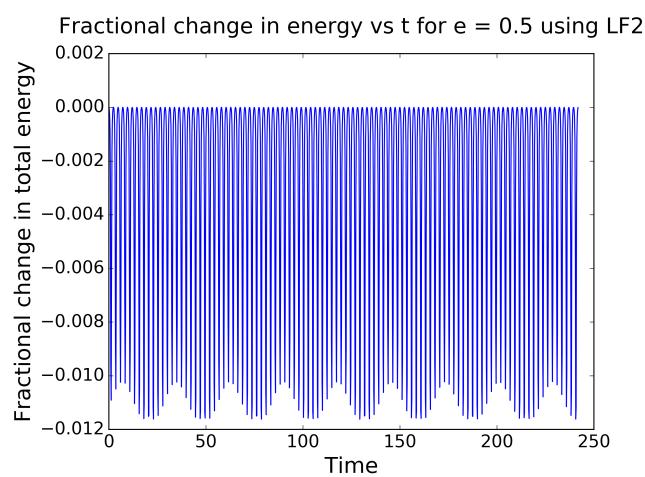
ASTR615 HW#4

Group 4: ChongChong He & Mohammed Khalil
November 21, 2017

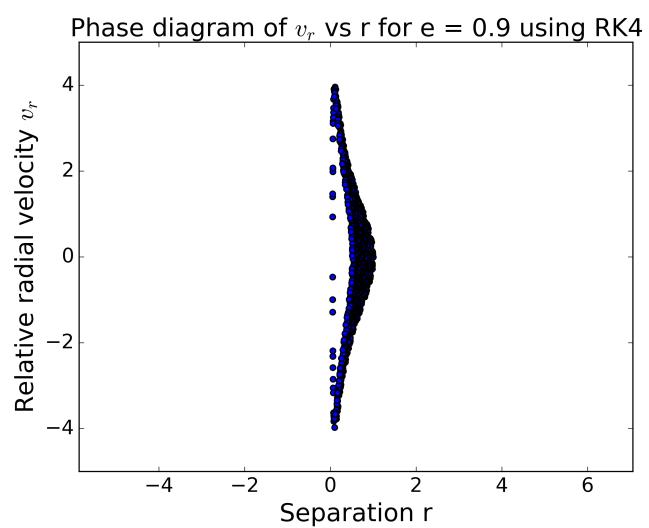
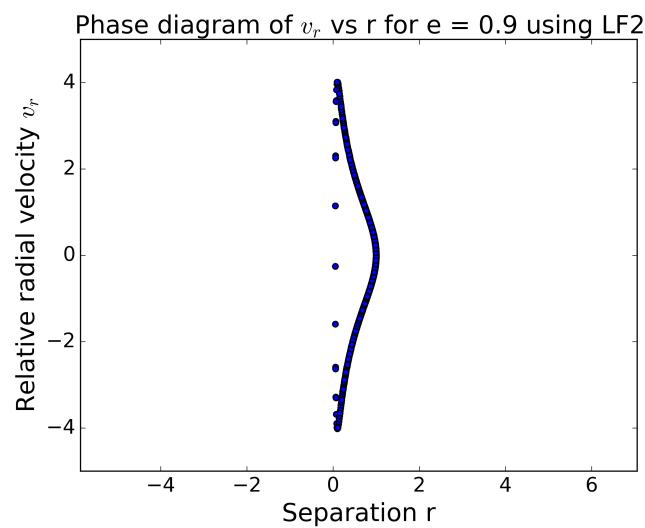
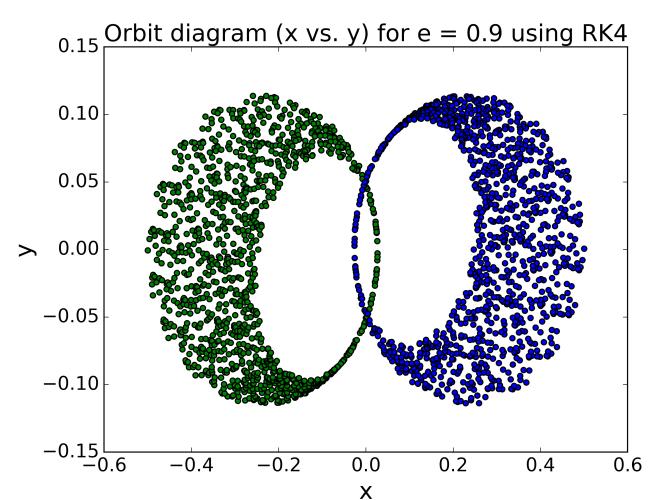
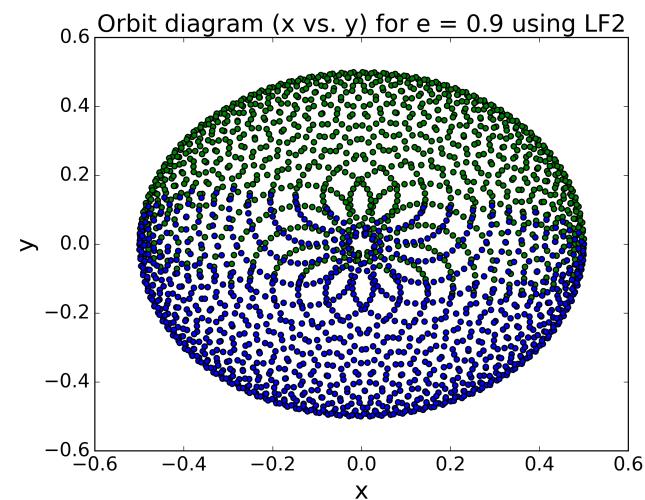
1 Problem 1

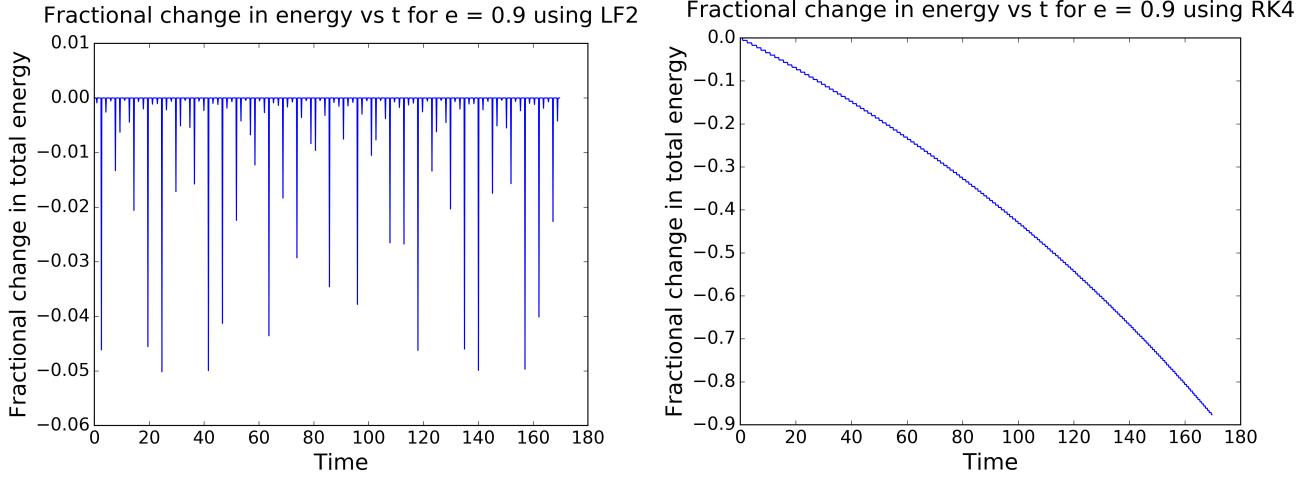
2nd order leapfrog vs. 4th order Runge-Kutta for $e = 0.5$





2nd order leapfrog vs. 4th order Runge-Kutta for $e = 0.9$





Comments

- Leapfrog is better at conserving energy. For $e = 0.9$, the energy error in RK4 reaches 90% after 100 periods. While in LF2, the fractional change in energy oscillates with a maximum error of 5%.
- This is also clear from the phase diagram; LF2 repeats almost exactly the same cycle, while RK4 produces larger errors since the separation decreases continually as the system loses energy.
- LF2 leads to larger phase error than RK4. After 100 periods, the phase error is almost $\pi/2$ for LF2 (at the given time step), while the phase remains almost the same for RK4.
- Therefore, leapfrog is more practical for N-body simulations with long simulation time, while Runge-Kutta is more accurate for small simulation time.

Problem 2

We perform a simulation of a cluster with Kroupa IMF.

Mass distribution

We implement in all of our simulations the Kroupa IMF:

$$\phi(m) \propto \begin{cases} m^{-1.3} & (0.08M_{\odot} < m < 0.5M_{\odot}) \\ 0.5m^{-2.3} & (0.5M_{\odot} < m < 100M_{\odot}) \end{cases} \quad (1)$$

after doing transformation we get

$$m = \begin{cases} -\frac{0.566179}{\sqrt[3]{1.7987 - x}(x^3 - 5.39611x^2 + 9.70599x - 5.81939)} & (0 < x < 0.760707) \\ \frac{0.166558}{(1.00024 - x)^{10/13}} & (0.760707 < x < 1) \end{cases} \quad (2)$$

Initial setup

The cluster is a specially uniformly distributed sphere with a radius of 1. The initial velocities are from a Gaussian distribution with a dispersion correspondent to a virial ratio of $\alpha \sim 0.4$. A virial ratio $\alpha < 0.5$ implies the system is bounded. The velocity dispersion crossing time is $t \approx 0.08$, so we use a step size of 0.001, i.e. 80 steps per course time.

1.1 Video

The video of this specific setup is *simulations/cluster03.mp4*.