ASTR615 HW#4

Group 4: ChongChong He & November 20, 2017

Problem 2

We perform a series of simulations of globular clusters.

Stellar Cluster

We did a few simulations with made-up initial conditions as well as initial conditions taken from a snapshot of a simulation from *RAMSES-RT*, an AMR MHD code with radiative transfer.

Mass distribution

We implement in all of our simulations the Kroupa IMF:

$$\phi(m) \propto \begin{cases} m^{-1.3} & (0.08M_\odot < m < 0.5M_\odot) \\ 0.5\, m^{-2.3} & (0.5M_\odot < m < 100M_\odot) \end{cases} \tag{1}$$

after doing transformation we get

$$m = \begin{cases} -\dfrac{0.566179}{\sqrt[3]{1.7987 - x}\,(x^3 - 5.39611x^2 + 9.70599x - 5.81939)} & (0 < x < 0.760707) \\ \dfrac{0.166558}{(1.00024 - x)^{10/13}} & (0.760707 < x < 1) \end{cases} \tag{2}$$

where x is uniform random numbers between 0 and 1. Figure 1 shows the histogram of the masses of the generated particles as compared to the analytical line.

Spacial and velocity distribution

The cluster is a specially uniformly distributed sphere with a radius of 1. The initial velocities are from a Gaussian distribution with a dispersion correspondent to a virial ratio of $\alpha \sim 0.4$. A virial ratio $\alpha < 0.5$ implies the system is bounded. The velocity dispersion crossing time is $t \approx 0.08$, so we use a step size of 0.001, i.e. 80 steps per crossing time.

Results

The video of this specific setup is *simulations/cluster03.mp4*.

Energy conservation and step sizes

Units and Time Scales

The units in our simulation are as follow: $[T] = 2.5395 \times 10^{15}\,\mathrm{s} \approx 80\,\mathrm{Myr}$, $[L] = 3.08 \times 10^{18}\,\mathrm{cm} \approx 1\,\mathrm{pc}$, $[M] = 6.7925 \times 10^{31}\,\mathrm{g} \approx 0.03416 M_\odot$. In this setup the gravitational constant is unity.

Imaging a cluster of stars from the outputs of *RAMSES* simulation which has ~ 1000 stars with Kroupa IMF located in a box of length $L = 50$. If this system is virialized, then $\sigma^2 = M_{\rm tot}/R$,

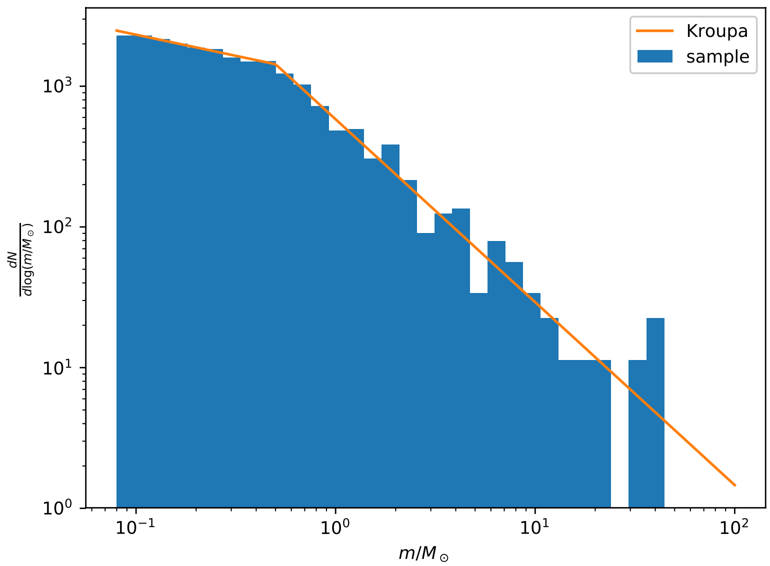

Figure 1: The mass distribution of the particles in our simulation compared to analytical Kroupa IMF lines.

from which we obtain $\sigma \approx 27$. Here I use a mean mass of $0.638\ M_\odot = 20$ (code unit) from Salpeter IMF and $R = 25$. Then, the crossing time of dispersion velocity is $t_\sigma = L/\sigma \approx 2$. If we choose a step size $\Delta t = t_\sigma/50 = 0.04$, i.e. 50 steps in a crossing time, then with 5000 steps we are able to simulate 100 crossing time, which is equal to 16 Gyr. This can be done with a PP n-body code on my laptop, and it will be even faster with our BH-tree n-body code.

Binary Problem

I have a try on the problem of binarity in our simulation. The basic idea is to check if two particles 1) are close enough to each other, and 2) $K + V < 0$, every n steps. If yes, we just replace these two particles with one star representing the motion of the center of mass. Binaries may further merge into trinaries and so on.

Realization of the Binary Problem

If two stars both with velocity σ are in virial equilibrium, i.e. $\alpha = K/|W| = 0.5$, the separation between them would be $d_{\text{virial}} = m/2\sigma^2 = 0.016$. However, the typical displacement of a particle in one step is $d_{\text{step}} = \sigma\Delta t \approx 1$, much greater than d_{virial}. Therefore our simulation is not able to identify binary stars. We need a step size ∼1000 times smaller to achieve the resolution of binary systems.

Searching for close encounters

We consider two stars in the center-of-mass frame. We defined the following two parameters:

- The *close-encounter parameter* α or γ which defines the criteria of being close enough to each other:

$$|\mathbf{r}_1 - \mathbf{r}_2| < d_{\text{close}} = \alpha \cdot d_{\text{virial}} = \gamma \cdot d_{\text{step}}. \tag{3}$$

Figure 2: The tree code fails to resolve the binary system after a few orbits. Here $\epsilon = 10^{-5}, \Delta t = 10^{-5}, 4000 steps per output$.

- The *escape parameter* β which confines the particles in a small region:

$$\frac{K}{|W|} < 1 - \beta^{-1}. \tag{4}$$

  This relation gives the largest separation between the two particles at any time,

$$d_{\text{max}} = \beta d_{\text{close}}, \tag{5}$$

  ignoring interactions with other particles [1]. When $d_{\text{max}} \ll d_{\text{step}}$ the two particles may be considered as a binary.

The solution to the binary problem then becomes balancing the typical particle separation $d_{\text{sepa}} = L/\sqrt[3]{N}$, the one-step displacement $d_{\text{step}} = \sigma \Delta t$, the close-encounter parameter α or γ, and the escape parameter β.

With $d_{\text{sepa}} \sim 5$, we set $\Delta t = 0.04$, $\alpha = 5$ and $\beta = 2$, which imply $d_{\text{step}} = 1$, $d_{\text{close}} = 0.08$, and $d_{\text{max}} = 0.16$.

Results

We artificially create a binary system and show that the binary is resolved with small step sizes but not resolved with large step sizes.

[1] This relation is obtained by solving equation $(1 - \beta^{-1})|V_0| + V_0 = 0 + V_1$ and $V_0 \propto 1/d_0$, $V_1 \propto 1/d_{\text{max}}$.