ASTR615 HW#4

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Problem 2: Simulations of Globular Clusters

We perform a series of simulations of globular clusters.

Mass distribution

We implement in all of our simulations the Kroupa IMF:

$$\phi(m) \propto \begin{cases} m^{-1.3} & (0.08M_\odot < m < 0.5M_\odot) \\ 0.5\, m^{-2.3} & (0.5M_\odot < m < 100M_\odot) \end{cases} \tag{1}$$

after doing transformation we get

$$m = \begin{cases} -\dfrac{0.566179}{\sqrt[3]{1.7987 - x}\,(x^3 - 5.39611x^2 + 9.70599x - 5.81939)} & (0 < x < 0.760707) \\ \dfrac{0.166558}{(1.00024 - x)^{10/13}} & (0.760707 < x < 1) \end{cases} \tag{2}$$

where x is uniform random numbers between 0 and 1.

Initial setup

The cluster is a specially uniformly distributed sphere with a radius of 1. The initial velocities are from a Gaussian distribution with a dispersion correspondent to a virial ratio of $\alpha \sim 0.4$. A virial ratio $\alpha < 0.5$ implies the system is bounded. The velocity dispersion crossing time is $t \approx 0.08$, so we use a step size of 0.001, i.e. 80 steps per crossing time. The video of this specific setup is *simulations/cluster03.mp4*.

Units and Time Scales

The units are: $[T] = 2.5395 \times 10^{15}\,\mathrm{s} \approx 80\,\mathrm{Myr}$, $[L] = 3.08 \times 10^{18}\,\mathrm{cm} \approx 1\,\mathrm{pc}$, $[M] = 6.7925 \times 10^{31}\,\mathrm{g} \approx 0.03416 M_\odot$. In this setup the gravitational constant is unity.

Suppose a cluster of stars from the results of *RAMSES* simulation which has ~ 1000 stars with Salpeter IMF in a box of length $L = 50$. If this stellar system is virialized, $\sigma^2 = M_{\mathrm{tot}}/R$, from which we obtain $\sigma \approx 27$. Here I use a mean mass of 0.638 $M_\odot = 20$ (code unit) from Salpeter IMF and $R = 25$. Then, the crossing time of dispersion velocity is $t_\sigma = L/\sigma \approx 2$. If we choose a step size $\Delta t = t_\sigma/50 = 0.04$, i.e. 50 steps in a course time, then with 5000 steps we are able to simulate 100 course time, which is equal to 16 Gyr. This can be done with a PP n-body code on my laptop.

Binary Problem

I am trying to conquer the problem of close encounters in our simulation. The basic idea is to check if two stars 1) are close enough to each other and 2) $K + V < 0$, every n steps. If yes, we just replace these two stars with one star at the center of mass. Binaries may further merge into trinaries, and so on so forth.

Realization of the Binary Problem

If two stars both with velocity σ are in virial equilibrium, i.e. $\alpha = K/|W| = 0.5$, the separation between them would be $d_{\text{virial}} = m/2\sigma^2 = 0.016$. However, the typical displacement of a particle in one step is $d_{\text{step}} = \sigma\Delta t \approx 1$, much greater than d_{virial}. Therefore our simulation is not able to identify binary stars. We need a step size $\sim$1000 times smaller to achieve the resolution of binary systems.

Searching for close encounters

We consider two stars in the center-of-mass frame. We defined the following two parameters:

- The *close-encounter parameter* α which confirms that they are close enough to each other so that we start to do further test:

$$d_{\text{close}} = |\mathbf{r}_1 - \mathbf{r}_2| < \alpha \cdot d_{\text{virial}}. \tag{3}$$

- The *escape parameter* β which confines the particles in a small region:

$$\frac{K}{|W|} < 1 - \beta^{-1}. \tag{4}$$

  This relation gives the largest separation between the two particles at any time,

$$d_{\max} = \beta d_{\text{close}}, \tag{5}$$

  ignoring interactions with other particles [1]. When $d_{\max} \ll d_{\text{step}}$ the two particles may be considered as a binary.

The solution to the binary problem then becomes balancing the typical particle separation $d_{\text{sepa}} = L/\sqrt[3]{N}$, the one-step displacement $d_{\text{step}} = \sigma\Delta t$, the close-encounter parameter α, and the escape parameter β.

With $d_{\text{sepa}} \sim 5$, we set $\Delta t = 0.04$, $\alpha = 5$ and $\beta = 2$, which imply $d_{\text{step}} = 1$, $d_{\text{close}} = 0.08$, and $d_{\max} = 0.16$.

[1]This relation is obtained by solving equation $(1 - \beta^{-1})|V_0| + V_0 = 0 + V_1$ and $V_0 \propto 1/d_0$, $V_1 \propto 1/d_{\max}$.