PHY 301

Quantum Mechanics

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Contents

1 Introduction

Quantum: quantities can vary by discrete amounts.

Mechanics: study of motion.

1.1 Franklin's Oil-Drop Experiment

Spilled a spoonful (2 ml) of oil on the surface of a lake and extended to about 2000 m^2, but not more. This experiment shows the **existent** and **size** of atoms.

$$V = Sh$$
$$h = \frac{V}{S} \sim \frac{2 \times 10^{-6}}{2 \times 10^{3}} \text{ m} \sim 10^{-9} \text{ m}$$

Atomic and molecular scales are **nanometric**

1.2 From Classical Mechanics to Quantum Mechanics

In classical mechanics, position is a function of time. **Deterministic**.

In quantum mechanics, the position of a particle is a random variable. **Probabilistic**.

2 Wave Function

2.1 Definition in 1-D Space

Definition 2.1 (Wave Function)**.** *For a small particle living in a **one-dimensional space**, the wave function Ψ is a complex-valued function of space and time:*

$$\Psi : \mathbf{R} \times \mathbf{R} \to \mathbb{C}$$
$$(x, t) \mapsto \Psi(x, t) \in \mathbf{C}$$

Remark 2.1. $\langle f(x) \rangle (t) = |\Psi(x,t)|^2$ *is the p.d.f. of finding the particle in position x at time t.*

Remark 2.2. $\langle F(x) \rangle (t) = \int_{-\infty}^{+\infty} |\Psi(x,t)|^2 dx$ *is the c.d.f of finding the particle between position $[-\infty, x]$ at time t.*

Remark 2.3. *Integration is over **space**, t is a **parameter**.*

2.2 Mean and Variance of the Position

These two statistics are expressed as integrals over the entire space. They are **deterministic functions of time**. Given wave function Ψ, then we have:

$$\langle E(x)\rangle(t) = \int_{-\infty}^{+\infty} x|\Psi(x,t)|^2 dx$$

$$\langle Var(x)\rangle(t) = \int_{-\infty}^{+\infty} (x - \langle E(x)\rangle(t))^2 |\Psi(x,t)|^2 dx$$