Calculation Report

1.Bill of Materials and Assembly Drawings

Figure 1 Isometric view: R4 Robotic Arm

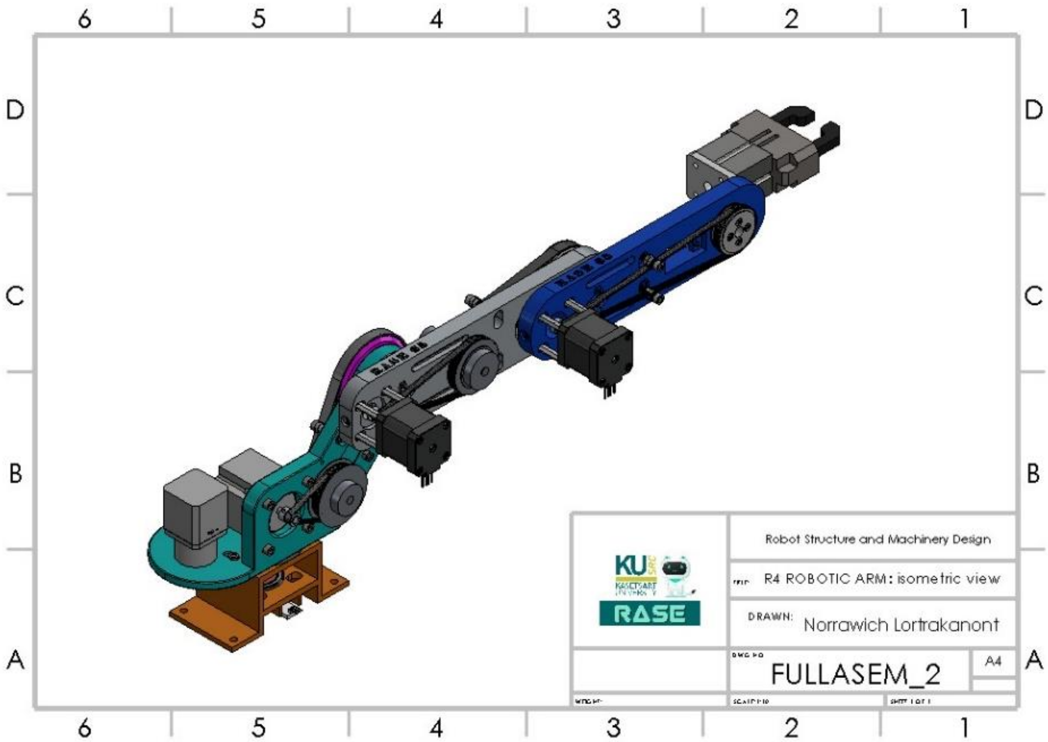
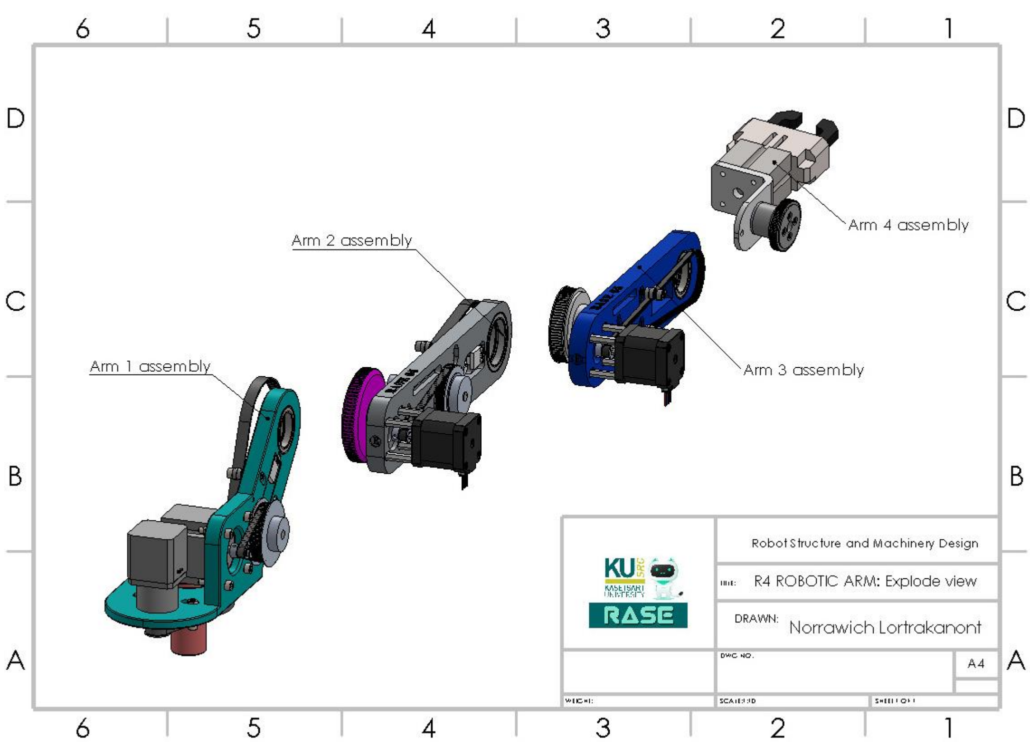
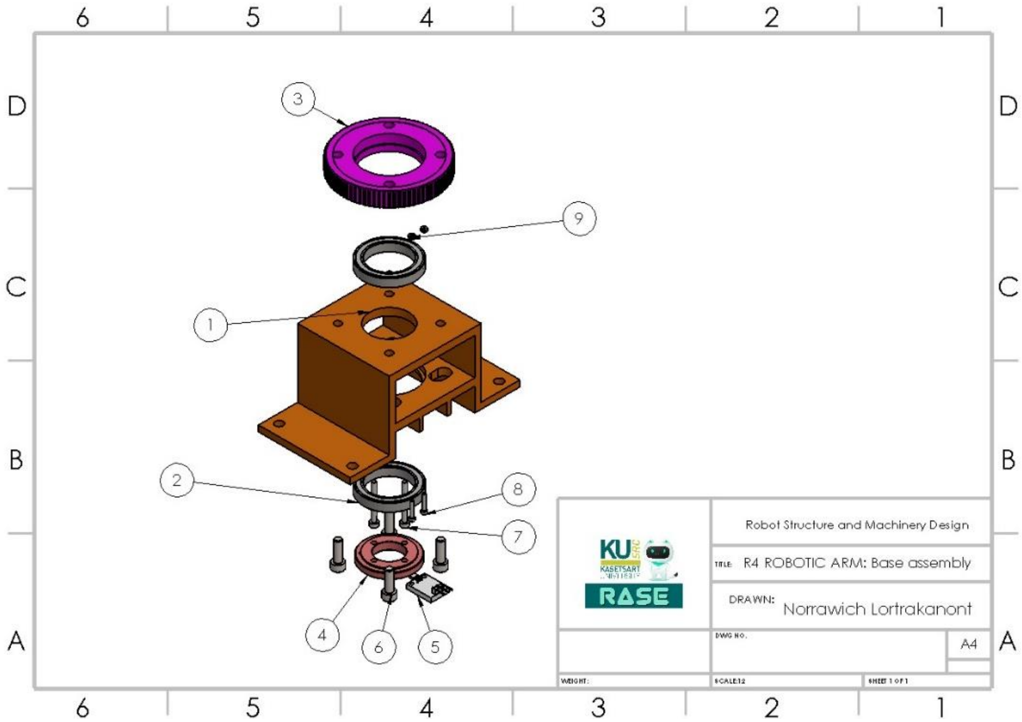


Figure 1 Explode view: R4 Robotic Arm

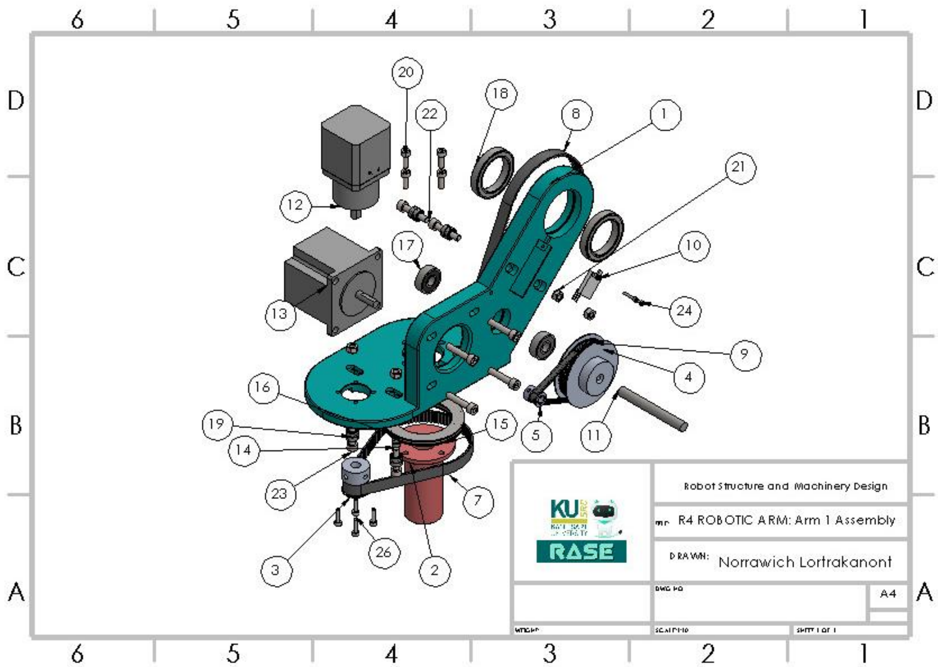


-Base Assembly



ITEM NO.	PART NAME	MATERIALS	QTY.
1	base	PLA+	1
2	5972K282 (bearing6806)	Stainless AISI 304	2
3	pulleyHTD3m80Tbase	PLA+	1
4	shaftJ1endPlate	PLA+	1
5	Magnetic Sensor	PCB: density of 1850 kg/m^3	1
6	ISO 4762 M5 x 16	Stainless AISI 304	4
7	M3x16	Stainless AISI 304	4
8	ISO 4762 M2 x 12 - 12N	Stainless AISI 304	2
9	ISO - 4032 - M2 - W - N	Stainless AISI 304	2

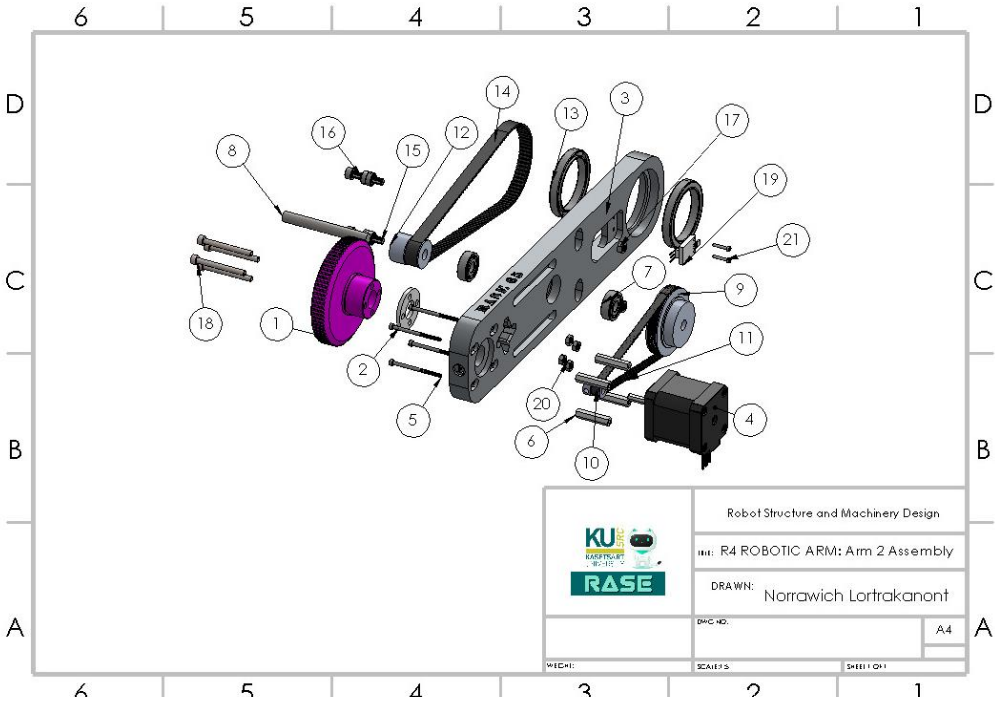
-Arm 1 Assembly



ITEM NO.	PART NUMBER	DESCRIPTION	QTY.
1	arm1	PLA+	1
2	shaftJ1	PLA+	1
3	Pulleyhtd3m20Tbore8	Aluminum Alloys 6061 Alloy	1
4	pulley2GT80T	Aluminum Alloys 6061 Alloy	1
5	pulley2GT16Tbore6.35	Aluminum Alloys 6061 Alloy	1
6	PulleyHTD3m15Tbore8	Aluminum Alloys 6061 Alloy	1
7	BeltHTD3m312L20T80T	Neoprene	1
8	BeltHTD3m363L15T80T	Neoprene	1
9	belt2GT210L	Neoprene	1
10	MagneticSensorV2	PCB: density of 1850 kg/m^3	1
11	shaft8x70	Stainless AISI 304	1
12	Nema17L40PG14	Nema17PG	1
13	Nema23_57x57x56	Nema23	1
14	BeltTensionSpacer	Stainless AISI 304	4
15	5909K79	Stainless AISI 304	2
16	5909K19	Stainless AISI 304	1
17	bearing608 8x22x7	Stainless AISI 304	2
18	5972K282	Stainless AISI 304	2
19	7804K106	Stainless AISI 304	8

20	ISO 4762 M3 x 12 - 12N	Stainless AISI 304	18
21	ISO - 4032 - M5 - W - N	Stainless AISI 304	10
22	M5x25	Stainless AISI 304	2
23	M5x20	Stainless AISI 304	2
24	M2x12	Stainless AISI 304	2
26	M3x12	Stainless AISI 304	4

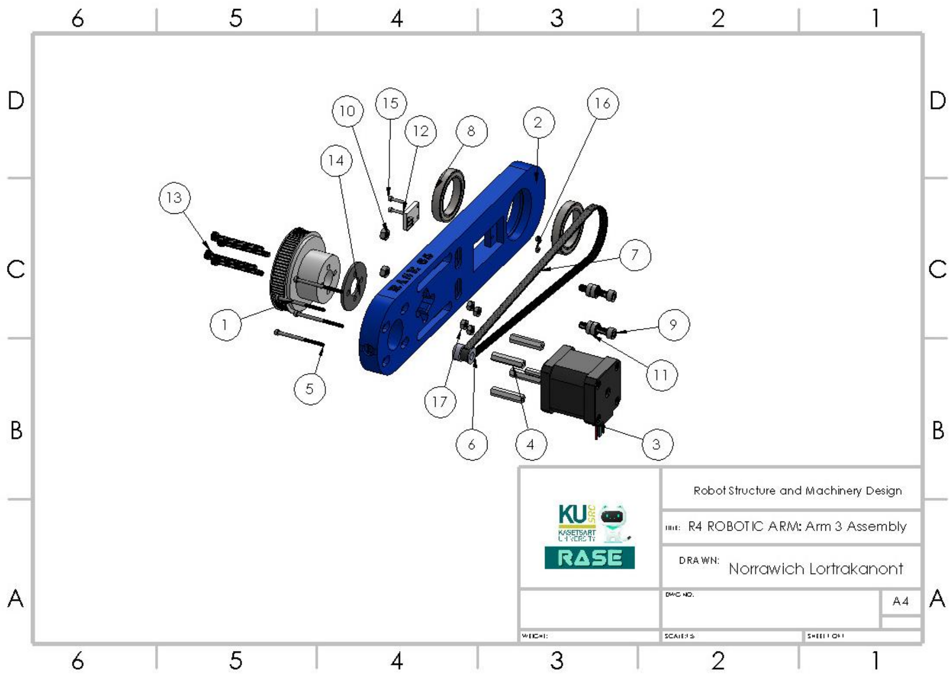
-Arm 2 Assembly



ITEM NO.	PART NAME	MATERIALS	QTY.
1	PulleyHtd3m80Tbore8	PLA+	1
2	bearingCover	PLA+	1
3	arm2.2	PLA+	1
4	17HE19-2004S.step	Nema17L60	1
5	91292A837 (M2.5x40)	Stainless AISI 304	4
6	spacer	PLA+	4
7	5972K91	Stainless AISI 304	2
8	8x70	Stainless AISI 304	1
9	pulley2_80teeths	Aluminum Alloys 6061 Alloy	1
10	pulleyGT2 16 bore 5mm	Aluminum Alloys 6061 Alloy	1
11	BELTY G2	Neoprene	1
12	Pulleyhtd3m20Tbore8	Aluminum Alloys 6061 Alloy	1
13	4668K161	Stainless AISI 304	2
14	Belt G3 2	Neoprene	1
15	91290A252 (M2.5x40)	Stainless AISI 304	2
16	7804K106 (M5x25)	Stainless AISI 304	4

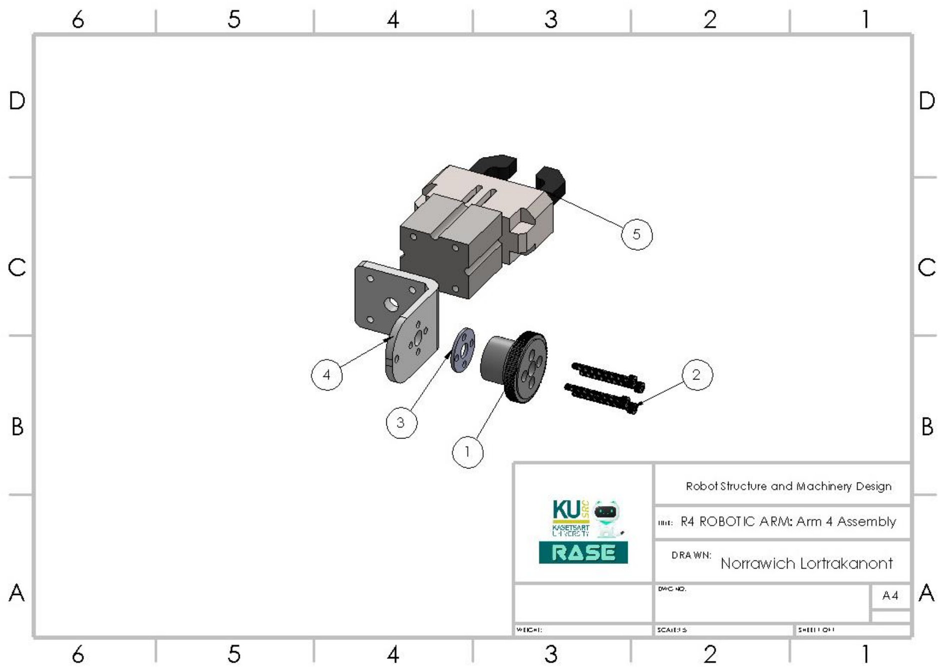
17	90685A041 (M5 nuts)	Stainless AISI 304	2
18	ISO 4762 M2 x 12 - 12N	Stainless AISI 304	6
19	MagneticSensorV2	PCB: density of 1850 kg/m^3	1
20	90685A039 (M4 nuts)	Stainless AISI 304	4
21	ISO - 4032 - M2 - W - N	Stainless AISI 304	2

-Arm 3 Assembly



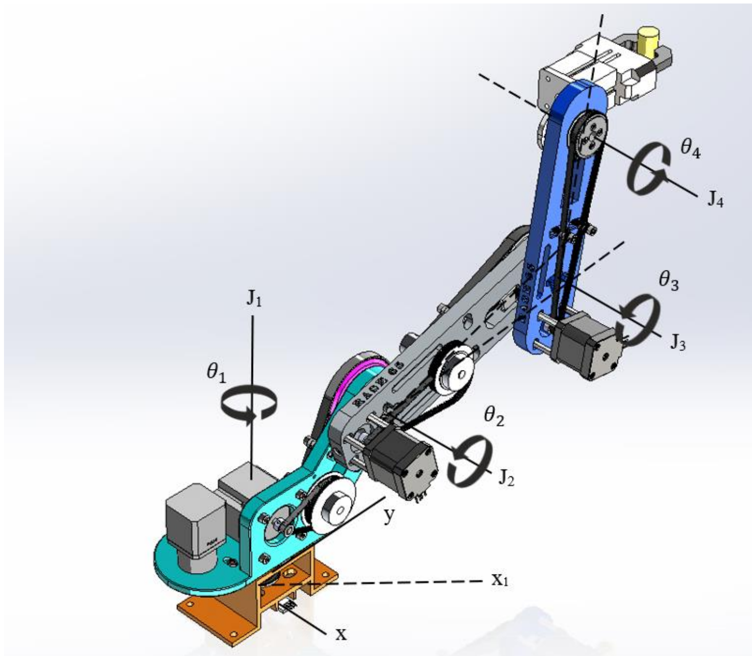
ITEM NO.	PART NUMBER	DESCRIPTION	QTY.
1	Pulley GT3 5	PLA+	1
2	Arm3	PLA+	1
3	17HE19-2004S_1.step	NE17L60	1
4	spacer	PLA+	4
5	91292A837 (M2.5x40)	Stainless AISI 304	4
6	pulleyGT2 16 bore 5mm	Aluminum Alloys 6061 Alloy	1
7	BELT_2GT_500L_16_60T	Neoprene	1
8	5972K282 (bearing 6808)	Stainless AISI 304	2
9	91290A252 (M5x25)	Stainless AISI 304	2
10	90685A041 (M5 nuts)	Stainless AISI 304	2
11	7804K106 (bearing105)	Stainless AISI 304	4
12	MagneticSensorV2	PCB: density of 1850 kg/m^3	1
13	91290A174 (M4x40)	Stainless AISI 304	4
14	flange1	PLA+	1
15	ISO 4762 M2 x 12 - 12N	Stainless AISI 304	2
16	ISO - 4032 - M2 - W - N	Stainless AISI 304	2
17	90685A039 (M4 nuts)	Stainless AISI 304	4

-Arm 4 Assembly

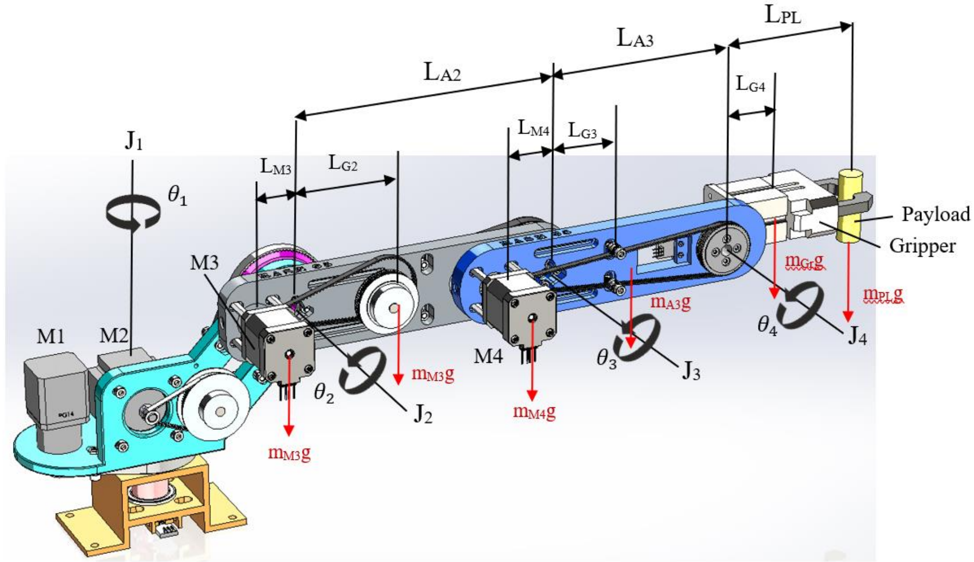


ITEM NO.	PART NUMBER	DESCRIPTION	QTY.
1	PulleyGT260T	PLA+	1
2	91290A174 (M4x40)	Stainless AISI 304	4
3	flange2	PLA+	1
4	Gripper_Holder	PLA+	1
5	Gripper	Use same mass as real gripper 190 g	1

2.Joint Configuration



3.Required Torque



Given:

Mass of motor and payload:

$$m_{M1} = 800\ g(Nema17\ stepper\ motor\ with\ planetary\ gearbox)$$
$$m_{M2} = 640\ g(Nema23\ stepper\ motor)$$
$$m_{M3} = 520\ g(Nema17\ stepper\ motor)$$
$$m_{M4} = 520\ g(Nema17\ stepper\ motor)$$
$$m_{PL} = 500\ g(Payload\ mass)$$

For calculation density:

$$density_{M1} = \frac{m}{V} = \frac{800}{15225.83} \times 10^6 = 52542 kg/m^3$$

$$density_{M2} = \frac{m}{V} = \frac{640}{152760} \times 10^6 = 4189.6\ kg/m^3$$

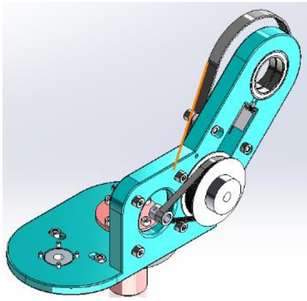
$$density_{M3} = \frac{m}{V} = \frac{520}{83856.73} \times 10^6 = 6201.05\ kg/m^3$$

$$density_{M4} = \frac{m}{V} = \frac{520}{83856.73} \times 10^6 = 6201.05\ kg/m^3$$

$$density_{payload} = \frac{m}{V} = \frac{500}{18849.55} \times 10^6 = 26525\ kg/m^3$$

Mass of arm:

Arm 1:



Report coordinate values relative to: Coordinate System1

Mass properties of FULLASEM_2
Configuration: Default
Coordinate system: Coordinate System1

Mass = 523.003630 grams

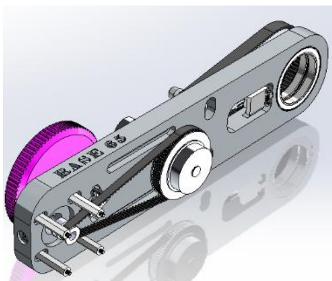
Volume = 376084.194771 cubic millimeters

Surface area = 172435.841606 square millimeters

Center of mass: (millimeters)
X = 12.030091
Y = 34.817043
Z = -23.337007

$$m_{A1} = 523\ g$$

Arm 2:



Report coordinate values relative to: Coordinate System2

Mass properties of FULLASEM_2
Configuration: Default
Coordinate system: Coordinate System2

Mass = 379.633397 grams

Volume = 383840.704063 cubic millimeters

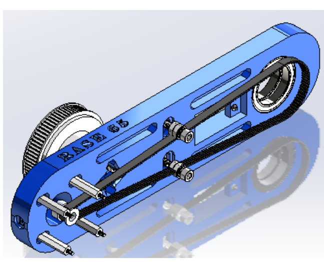
Surface area = 147136.100213 square millimeters

Center of mass: (millimeters)
X = 16.776349
Y = -0.074301
Z = -68.868455

$$m_{A2} = 379.63\ g$$

$$L_{G2} = 68.87\ mm$$

Arm 3:



Report coordinate values relative to: Coordinate System3

Mass properties of FULLASEM_2
Configuration: Default
Coordinate system: Coordinate System3

Mass = 255.411253 grams

Volume = 275235.349369 cubic millimeters

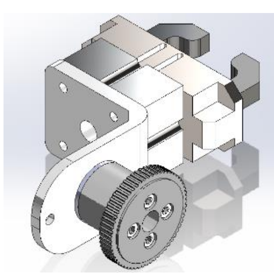
Surface area = 113735.377720 square millimeters

Center of mass: (millimeters)
X = 17.091556
Y = -0.074164
Z = -63.258862

$$m_{A3} = 255.41\ g$$

$$L_{G3} = 63.26\ mm$$

Gripper:



Report coordinate values relative to: Coordinate System4

Mass properties of FULLASEM_2
Configuration: Default
Coordinate system: Coordinate System4

Mass = 149.286502 grams

Volume = 156525.123271 cubic millimeters

Surface area = 46355.658699 square millimeters

Center of mass: (millimeters)
X = -26.296557
Y = -0.031966
Z = -44.423686

$$m_{GR} = 149.29\ g$$

$$L_{G4} = 44.42\ mm$$

Length of interest:

$$L_{A2} = 215.76 \quad mm$$
$$L_{A3} = 155.75 \quad mm$$
$$L_{PL} = 100 \quad mm$$
$$L_{M3} = 35.13 \quad mm$$
$$L_{M4} = 50 \ mm$$

Total mass:

$$m = m_{A1} + m_{A2} + m_{A3} + m_{M1} + m_{M2} + m_{M3} + m_{M4} + m_{Gr} + m_{PL}$$

$$m = 523 + 379.63 + 255.41 + 800 + 640 + 520 + 520 + 149.29 + 500$$
$$m = 4287.33 \ g$$

For determine load torque at each joint we can expressed as:

$$T_{L,J4} = g\left[m_{Gr}L_{G4} + m_{PL}L_{PL}\right]$$

$$T_{L,J3} = g\left[-m_{M4}L_{m4} + m_{A3}L_{G3} + m_{Gr}(L_{G4} + L_{A3}) + m_{PL}(L_{PL} + L_{A3})\right]$$

$$T_{L,J2} = g\left[m_{M3}(-L_{M3}) + m_{A2}(L_{G2}) + m_{M4}(-L_{M4} + L_{A2}) + m_{A3}(L_{G3} + L_{A2}) + m_{Gr}(L_{G4} + L_{A3} + L_{A2}) + m_{PL}(L_{PL} + L_{A3} + L_{A2})\right]$$

$$T_{L,J1} = \frac{2}{3}\mu m g\left(\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right)$$

$$T_{L,J4} = 9.81\left[(149.29 \times 44.42) + (500 \times 100)\right] \times 10^{-6} = 0.555 \ N \cdot m$$
$$T_{L,J3} = 9.81\left[-520 \times 50 + 255.41 \times 63.26 + 149.29(44.42 + 155.75) + 500(100 + 155.75)\right] \times 10^{-6} = 1.451 \ N \cdot m$$

$$T_{L,J2} = 9.81\left[\begin{array}{l} 520(-35.13) + 379.63(68.87) + 520(-50 + 215.76) + 255.41(63.26 + 215.76) \\ +149.29(44.42 + 155.75 + 215.76) + 500(100 + 155.75 + 215.76) \end{array}\right] \times 10^{-6} = 4.544 \ N \cdot m$$

$$T_{L,J1} = \frac{2}{3}(0.1)(4287.33 \times 10^{-3})(9.81)\left(\frac{0.06^3 - 0.045^3}{0.06^2 - 0.045^2}\right) = 0.222 \ N \cdot m$$

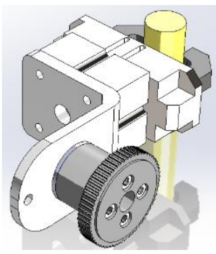
4.Acceleration torque:

$$\alpha = \frac{\omega}{t} = \frac{2\pi n}{60t}$$

$$t = 1s \ \ and \ n = 20\,rpm$$

$$\alpha = \frac{2\pi(20)}{60(1)} = 2.09 \ \ rad\,/\,s^2$$

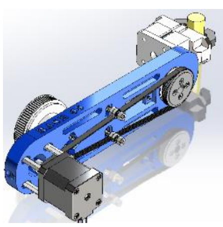
For finding I:



Taken at the output coordinate system. (Using positive tensor notation.)

Ixx = 5657104.661918	Ixy = 190091.497198	Ixz = 20
Iyx = 190091.497198	Iyy = 6295586.970854	Iyz = 52
Izx = 2047496.369167	Izy = 525334.367952	Izz = 10

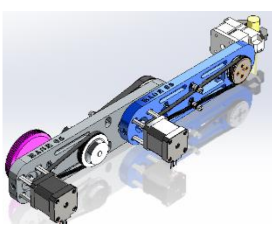
$$I_{j4} = 5657104.66 \times 10^{-9}\,kg\,/\,m^2$$



Taken at the output coordinate system. (Using positive tensor notation.)

Ixx = 43241117.104758	Ixy = 99749.585019	Ixz = 48
Iyx = 99749.585019	Iyy = 46558986.243787	Iyz = 13
Izx = 4805054.278227	Izy = 1336771.912198	Izz = 40

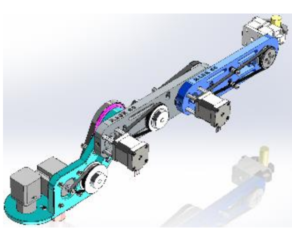
$$I_{j3} = 43241117.1 \times 10^{-9}\,kg\,/\,m^2$$



Taken at the output coordinate system. (Using positive tensor notation.)

Ixx = 177955167.927610	Ixy = 5686.280633	Ixz = -8
Iyx = 5686.280633	Iyy = 185644584.751970	Iyz = 24
Izx = -8120834.887845	Izy = 2499633.049543	Izz = 88

$$I_{j3} = 177955167.93 \times 10^{-9}\,kg\,/\,m^2$$



Taken at the output coordinate system. (Using positive tensor notation.)

Ixx = 335328157.093546	Ixy = 15554055.039428	Ixz = -34
Iyx = 15554055.039428	Iyy = 318910483.499752	Iyz = -7
Izx = -34286066.261658	Izy = -71139139.950234	Izz = 44

$$I_{j1} = 318910483.5 \times 10^{-9}\,kg\,/\,m^2$$

The acceleration torque can be express as:

$$T_{a,J4} = I_{J4}\alpha$$
$$T_{a,J3} = I_{J3}\alpha$$
$$T_{a,J2} = I_{J2}\alpha$$
$$T_{a,J1} = I_{J1}\alpha$$

$$T_{a,J4} = (5657104.66 \times 10^{-9})(2.09) = 0.012 \ N \cdot m$$
$$T_{a,J3} = (43241117.1 \times 10^{-9})(2.09) = 0.09 \ N \cdot m$$
$$T_{a,J2} = (177955167.93 \times 10^{-9})(2.09) = 0.37 \ N \cdot m$$
$$T_{a,J1} = (318910483.5 \times 10^{-9})(2.09) = 0.666 \ N \cdot m$$

5.Required torque:

The required torque can be determined from

$$T = SF(T_L + T_a)$$

Safety factor of SF = 1.7

$$T_{J4} = 1.7(0.555 + 0.012) = 0.96 \ N \cdot m$$
$$T_{J3} = 1.7(1.451 + 0.09) = 2.619 \ N \cdot m$$
$$T_{J2} = 1.7(4.544 + 0.37) = 8.354 \, N \cdot m$$
$$T_{J1} = 1.7(0.222 + 0.666) = 1.51 \ N \cdot m$$

6.Motor Selection and required gear ratio:

Joint J_4 and J_3:

HANPOSE – Stepper motor (Nema17)

Holding torque: 0.7 N.m, 1.7A

(17HS6401S / 42HS60) Black End

Motor mass: $m_{M3} = m_{M4} = 0.52$ kg

Link : https://www.zonemaker.com/product/190/hanpose-stepper-motor-nema17-%E0%B9%81%E0%B8%A3%E0%B8%87%E0%B8%9A%E0%B8%B4%E0%B8%94-70-n-cm-1-7a-17hs6401s-black-end

$$T_{M3} = T_{M4} = 0.4(0.7) = 0.280 \ N \cdot m$$

$$i_{4,required} = \frac{T_{J4}}{T_{M4}} = \frac{0.96}{0.280} = 3.43:1$$

$$i_{3,required} = \frac{T_{J3}}{T_{M3}} = \frac{2.619}{0.280} = 9.353:1$$

Joint J_2:

HANPOSE - Stepper Motor (Nema23)

Holding torque: 1.01N.m, 2.8A

Shaft 6.35mm (23HS5128-6.35) motor mass: $m_{M2} = 0.64$ kg

Link: https://www.zonemaker.com/product/3045/hanpose-stepper-motor-nema23-%E0%B9%81%E0%B8%A3%E0%B8%87%E0%B8%9A%E0%B8%B4%E0%B8%94-1-01n-m-2-8a-shaft-6-35mm-23hs5128-6-35

$$T_{M2} = 0.4(1.01) = 0.404 \ N \cdot m$$

$$i_{2,required} = \frac{T_{J2}}{T_{M2}} = \frac{8.354}{0.404} = 20.678:1$$

Joint J_1:

HANPOSE-Planetary Gearbox Ratio 5.18:1 Nema17 Stepper Motor (17HS2408S-PG5.18)

holding torque: 0.4 N.m.

motor and planetary gear box mass: $m_{M1} = 0.8$ kg

$$T_{M1} = 0.4(0.4) = 0.160 \ N \cdot m$$

$$i_{1,required} = \frac{T_{J1}}{T_{M1}} = \frac{0.222}{0.160} = 9.44:1$$

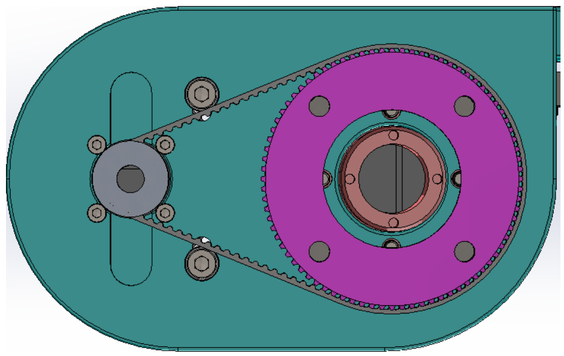
7.Power transmission selection:

Joint J_1:

Stage 1:

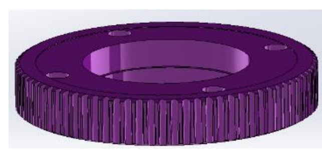
Planetary gearbox ratio: $i_{1\_1} = 5.18:1$

Stage 2:



$D_{1\_1} = \frac{20(3)}{\pi} = 19.10\ mm$

$D_{1\_2} = \frac{80(3)}{\pi} = 76.39\ mm$



Pulley belt distance calculation:

PL = 312 mm   D1 = 19.10 mm   D2 =76.39 mm

$$b = 2\times PL - \pi\times(D1+D2)$$

$$b = 2\times 312 - \pi\times(19.10+76.39)$$

$$b = 324\ mm$$

$$CD = \frac{(b+\sqrt{b^2 - 8\times(D2-D1)^2})}{8}$$

$$CD = \frac{(826+\sqrt{826^2 - 8\times(45.2-10.19)^2})}{8}$$

CD = 205.75 mm   Belt teeth = PL/3 = 104 mm

Planetary gearbox ratio: $i_{1\_2} = \frac{80}{20} = 4:1$

Conclusion:

J_1 Final gear ratio**:** $i_1 = i_{1\_1} \cdot i_{1\_2} = 5.18 \cdot \frac{N_{1\_2}}{N_{1\_1}} = 5.18 \cdot \frac{80}{20} = 20.72:1$

$i_1 > i_{1,required} = 20.72 > 9.44$

A gear ratio of $i_1 = 20.72:1$ can be operate the robotic arm's joint J_1

Source:

Pulley 1_1, HTM-3M, 20 teeth, 8 mm bore.

https://www.zonemaker.com/product/622/htd-3m-10mm-20-teeth-timing-pulley-bore-8mm

Pulley 1_2, HTM-3M, 80 teeth,

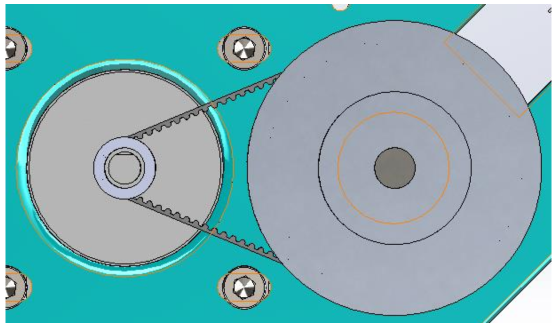
3D printed.

HTD-3M timing belt, 10 mm wide, with a closed loop length of 312 mm.

https://www.zonemaker.com/product/618/htd-3m-width-10mm-closed-loop-belt-length-312mm

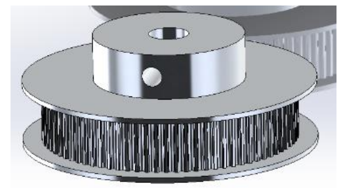
Joint J_2:

Stage 1:



$$D_{2\_1} = \frac{16(2)}{\pi} = 10.19\ mm$$

$$D_{2\_2} = \frac{80(2)}{\pi} = 50.93\ mm$$



Pulley belt distance calculation:

PL = 210 mm D1 = 10.19 mm D2 =50.93 mm

$$b = 2 \times PL - \pi \times (D1 + D2)$$

$$b = 2 \times 210 - \pi \times (10.19 + 50.93)$$

$$b = 228\ mm$$

$$CD = \frac{(b + \sqrt{b^2 - 8 \times (D2 - D1)^2})}{8}$$

$$CD = \frac{(228 + \sqrt{228^2 - 8 \times (50.93 - 10.19)^2})}{8}$$

CD = 75.6 mm Belt teeth = PL/2 = 105 mm

J_2 Stage 1 gear ratio: $i_{2\_1} = \frac{N_{2\_2}}{N_{2\_1}} = \frac{80}{16} = 5:1$

Source:

Pulley 2_1, 2GT, 16 teeth, 8 mm bore.

https://shopee.co.th/%F0%9F%94%A5-%E0%B8%9E%E0%B8%A3%E0%B9%89%E0%B8%AD%E0%B8%A1%E0%B8%AA%E0%B9%88%E0%B8%87-%F0%9F%94%A5-Pulley-2GT%E0%B8%9E%E0%B8%B9%E0%B9%80%E0%B8%A5%E0%B9%88-80%E0%B8%9F%E0%B8%B1%E0%B8%99-2GT-6-80-teeth-Timing-Pulley-BF-%E0%B8%AB%E0%B8%99%E0%B9%89%E0%B8%B2%E0%B8%81%E0%B8%A7%E0%B9%89%E0%B8%B2%E0%B8%87%E0%B8%AA%E0%B8%B2%E0%B8%A2%E0%B8%9E%E0%B8%B2%E0%B8%99-6-10-mm.-i.252491788.22318651118?xptdk=15cf7b14-b5c4-4dce-9e98-88e1b6cfbb9c

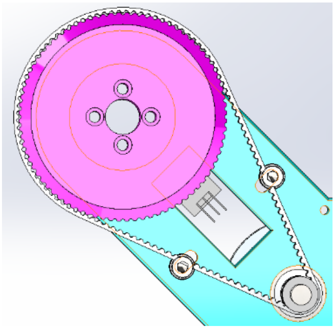
Pulley 2_2, 2GT, 80 teeth, 6.35 mm bore.

https://www.zonemaker.com/product/721/2gt-width-6mm-16-teeth-timing-pulley-bore-6-35mm

2GT timing belt, 6 mm wide, with a closed loop length of 210 mm.

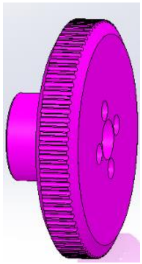
https://shopee.co.th/%F0%9F%94%A5-%E0%B8%9E%E0%B8%A3%E0%B9%89%E0%B8%AD%E0%B8%A1%E0%B8%AA%E0%B9%88%E0%B8%87-%F0%9F%94%A5%E0%B8%AA%E0%B8%B2%E0%B8%A2%E0%B8%9E%E0%B8%B2%E0%B8%99-2gt_6-%E0%B9%81%E0%B8%9A%E0%B8%9A%E0%B8%9B%E0%B8%B4%E0%B8%94-%E0%B9%80%E0%B8%AA%E0%B9%89%E0%B8%99%E0%B8%A3%E0%B8%AD%E0%B8%9A%E0%B8%A7%E0%B8%87-200-298mm.Timing-belt-width-6mm-%E0%B9%81%E0%B8%9A%E0%B8%9A-close-loop-%E0%B9%80%E0%B8%AA%E0%B9%89%E0%B8%99%E0%B8%A3%E0%B8%AD%E0%B8%9A%E0%B8%A7%E0%B8%87-200-298mm.-i.252491788.9463831265?xptdk=b3caed6c-7e29-43c6-8929-5e622752be9e

Stage 2:



$$D_{2\_3} = \frac{15(3)}{\pi} = 14.32\ mm$$

$$D_{2\_4} = \frac{88(3)}{\pi} = 84.03\ mm$$



Pulley belt distance calculation:

PL = 363 mm D1 = 14.32 mm D2 =84.03 mm

$$b = 2 \times PL - \pi \times (D1 + D2)$$

$$b = 2 \times 363 - \pi \times (14.32 + 84.03)$$

$$b = 417.02\ \ mm$$

$$CD = \frac{(b + \sqrt{b^2 - 8 \times (D2 - D1)^2})}{8}$$

$$CD = \frac{(417.02 + \sqrt{417.02^2 - 8 \times (84.03 - 14.32)^2})}{8}$$

CD = 98.06 mm Belt teeth = PL/3 = 121 mm

J_2 Stage 2 gear ratio: $i_{2\_2} = \dfrac{N_{2\_4}}{N_{2\_3}} = \dfrac{88}{15} = 5.86:1$

Source:

Pulley 2_3, HTM-3M, 15 teeth, 8 mm bore.

https://www.zonemaker.com/product/621/htd-3m-10mm-15-teeth-timing-pulley-bore-8mm

Pulley 2_4, HTM-3M, 80 teeth,

3D printed.

HTD-3M timing belt, 10 mm wide, with a closed loop length of 363 mm.

https://www.zonemaker.com/product/1920/htd-3m-width-10mm-closed-loop-belt-length-363mm

Conclusion:

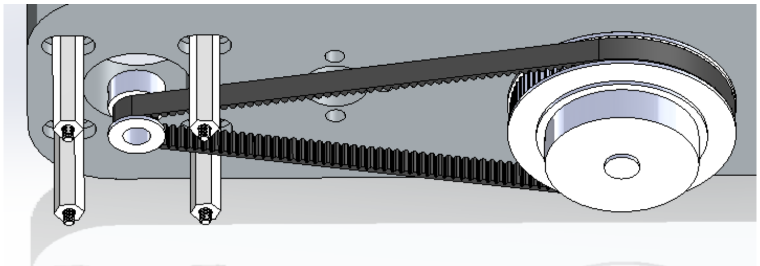
J_2 final gear ratio: $i_2 = i_{2\_1} \cdot i_{2\_2} = \dfrac{N_{2\_2}}{N_{2\_1}} \cdot \dfrac{N_{2\_4}}{N_{2\_3}} = \dfrac{80}{16} \cdot \dfrac{88}{15} = 5 \cdot 5.86 = 29.3:1$

$i_2 > i_{2,required} = 29.3 > 20.678$

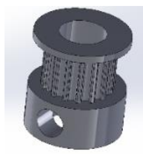
A gear ratio of $i_2 = 29.3:1$ can be operate the robotic arm's joint J_2

Joint J_3

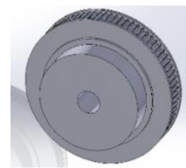
stage 1:



$$D_{3\_1} = \frac{16(2)}{\pi} = 10.19\,mm$$



$$D_{3\_2} = \frac{80(2)}{\pi} = 50.93\,mm$$



Pulley Distance calculation:

PL = 300 mm D1 = 10.19 mm D2 =50.93 mm

$$b = 2\times PL - \pi\times(D1 + D2)$$

$$b = 2\times 300 - \pi\times(10.19 + 50.93)$$

b = 407.98 mm

$$CD = \frac{(b + \sqrt{b^2 - 8\times(D2 - D1)^2})}{8}$$

$$CD = \frac{(407.98 + \sqrt{407.98^2 - 8\times(50.93 - 10.19)^2})}{8}$$

CD = 99.91 mm Belt teeth = PL/2 = 150 mm

J_3 stage 1 gear ratio: $i_{3\_1} = \frac{N_{3\_2}}{N_{3\_1}} = \frac{80}{16} = 5:1$

Source:

Pulley 3_1, 2GT, 16 teeth, 5 mm bore.

https://shopee.co.th/%F0%9F%94%A5-%E0%B8%9E%E0%B8%A3%E0%B9%89%E0%B8%AD%E0%B8%A1%E0%B8%AA%E0%B9%88%E0%B8%87-%F0%9F%94%A5Pulley-%E0%B8%9E%E0%B8%B9%E0%B9%80%E0%B8%A5%E0%B9%88-%E0%B8%9E%E0%B8%A5%E0%B8%B9%E0%B9%80%E0%B8%A5%E0%B9%88-2gt-20-%E0%B8%9F%E0%B8%B1%E0%B8%99-16-%E0%B8%9F%E0%B8%B1%E0%B8%99-%E0%B8%AB%E0%B8%99%E0%B9%89%E0%B8%B2%E0%B8%81%E0%B8%A7%E0%B9%89%E0%B8%B2%E0%B8%87%E0%B8%AA%E0%B8%B2%E0%B8%A2%E0%B8%9E%E0%B8%B2%E0%B8%99-6mm-.Pulley2GT20teeth-i.252491788.4833675066?xptdk=b41d5e09-b5d4-41ef-b3d9-6e628952ea57

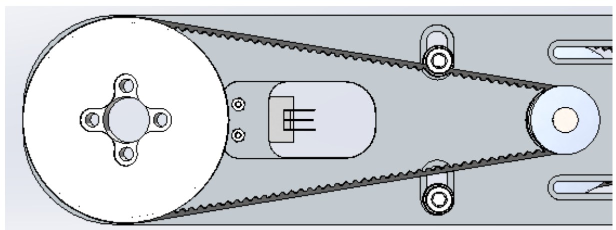
Pulley 3_2, 2GT, 80 teeth, 8 mm bore.

https://shopee.co.th/%F0%9F%94%A5-%E0%B8%9E%E0%B8%A3%E0%B9%89%E0%B8%AD%E0%B8%A1%E0%B8%AA%E0%B9%88%E0%B8%87-%F0%9F%94%A5Pulley-%E0%B8%9E%E0%B8%B9%E0%B9%80%E0%B8%A5%E0%B9%88-%E0%B8%9E%E0%B8%A5%E0%B8%B9%E0%B9%80%E0%B8%A5%E0%B9%88-2gt-20-%E0%B8%9F%E0%B8%B1%E0%B8%99-16-%E0%B8%9F%E0%B8%B1%E0%B8%99-%E0%B8%AB%E0%B8%99%E0%B9%89%E0%B8%B2%E0%B8%81%E0%B8%A7%E0%B9%89%E0%B8%B2%E0%B8%87%E0%B8%AA%E0%B8%B2%E0%B8%A2%E0%B8%9E%E0%B8%B2%E0%B8%99-6mm-.Pulley2GT20teeth-i.252491788.4833675066?xptdk=b41d5e09-b5d4-41ef-b3d9-6e628952ea57

2GT timing belt, 6 mm wide, with a closed loop length of 320 mm.

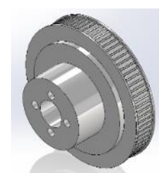
https://www.zonemaker.com/product/2167/belt-closed-loop-rubber-320-2gt-6-closed-loop-length-320mm-gt2-timing-belt-width-6mm-2

Stage 2:



$$D_{3\_3} = \frac{20(3)}{\pi} = 19.09mm$$

$$D_{3\_4} = \frac{68(3)}{\pi} = 64.93mm$$



Pulley Distance calculation:

PL = 417 mm D1 = 19.09 mm D2 =64.93 mm

$$b = 2 \times PL - \pi \times (D1 + D2)$$

$$b = 2 \times 417 - \pi \times (19.09 + 64.93)$$

b = 570.04 mm

$$CD = \frac{(b + \sqrt{b^2 - 8 \times (D2 - D1)^2})}{8}$$

$$CD = \frac{(570.04 + \sqrt{570.04^2 - 8 \times (64.93 - 19.09)^2})}{8}$$

CD = 140.64 mm Belt teeth = 417/3 = 139 mm

J_3 stage 2 Pulley system gear ratio: $i_{3\_2} = \frac{N_{3\_4}}{N_{3\_3}} = \frac{68}{20} = 3.4:1$

Source:

Pulley 3_3, HTM-3M, 20 teeth, 8 mm bore.

https://www.zonemaker.com/product/622/htd-3m-10mm-20-teeth-timing-pulley-bore-8mm

Pulley 3_4, HTD-3M, 68 teeth

3D printed.

HTD-3M timing belt, 10 mm wide, with a closed loop length of 417 mm.

https://www.zonemaker.com/product/2444/htd-3m-width-10mm-closed-loop-belt-length-417mm

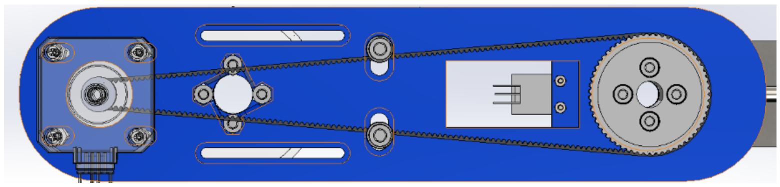
Conclusion:

J_3 final gear ratio: $i_3 = i_{3\_1} \cdot i_{3\_2} = \frac{N_{3\_2}}{N_{3\_1}} \cdot \frac{N_{3\_4}}{N_{3\_3}} = \frac{80}{16} \cdot \frac{68}{20} = 5 \cdot 3.4 = 17:1$

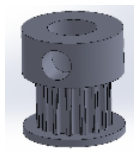
$i_3 > i_{3,required} = 17 > 9.353$

A gear ratio of $i_3 = 17:1$ can be operate the robotic arm's joint J_3

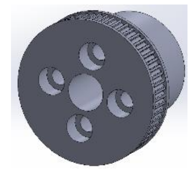
Joint J4:



$$D_{4\_1} = \frac{16(2)}{\pi} = 10.19\,mm$$



$$D_{4\_2} = \frac{71(2)}{\pi} = 45.2\,mm$$



Pulley belt distance calculation:

PL = 500 mm   D1 = 10.19 mm   D2 =45.2 mm

$$b = 2\times PL - \pi\times(D1 + D2)$$

$$b = 2\times 500 - \pi\times(10.19 + 45.2)$$

b = 826 mm

$$CD = \frac{(b + \sqrt{b^2 - 8\times(D2 - D1)^2})}{8}$$

$$CD = \frac{(826 + \sqrt{826^2 - 8\times(45.2 - 10.19)^2})}{8}$$

CD = 205.75 mm   Belt teeth = PL/2 = 250 mm

J4 Final gear ratio: $i_4 = \frac{N_{4\_2}}{N_{4\_1}} = \frac{71}{16} = 4.44:1$

Source:

Pulley 4_1, 2GT, 16 teeth, 5 mm bore.

https://www.zonemaker.com/product/720/2gt-width-6mm-16-teeth-timing-pulley-bore-5mm-2

Pulley 4_2, 2GT, 68 teeth

3D printed.

GT2 timing belt, 6 mm wide, with a closed loop length of 500 mm.

https://www.zonemaker.com/product/2168/belt-closed-loop-rubber-500-2gt-6-closed-loop-length-500mm-gt2-timing-belt-width-6mm-2

Conclusion:

J_4 Final gear ratio: $i_4 = \frac{N_{4\_2}}{N_{4\_1}} = \frac{71}{16} = 4.44:1$

$i_4 > i_{4,required} = 4.44 > 3.43$

A gear ratio of $i_4 = 4.44:1$ can be operate the robotic arm's joint J_3

4. Forward Kinematics

OA = 45.5 mm, AB = 36 mm, BC = 105.7 mm, α = 45 °,CD = L_{A2} = 202 mm, DE = L_{A3} = 160 mm, EF = L_{PL} = 10 mm

As show in Fig. 12, all coordinates on the x'-z plane can be expressed as

$O(0, 0, 0)$
$A(0, 0, OA)$

B

$$x_B = AB\cos\theta_1 \tag{12}$$
$$y_B = AB\sin\theta_1 \tag{13}$$
$$z_B = OA \tag{14}$$

C

$$x_C = x_B + BC\cos\alpha\cos\theta_1 \tag{15}$$
$$y_C = y_B + BC\cos\alpha\sin\theta_1 \tag{16}$$
$$z_C = z_B + BC\sin\alpha \tag{17}$$

D

$$x_D = x_C + CD\cos\theta_2\cos\theta_1 \tag{18}$$
$$y_D = y_C + CD\cos\theta_2\sin\theta_1 \tag{19}$$
$$z_D = z_C + CD\sin\theta_2 \tag{20}$$

E

$$x_E = x_D + DE\cos(\theta_2+\theta_3)\cos\theta_1 \tag{21}$$
$$y_E = y_D + DE\cos(\theta_2+\theta_3)\sin\theta_1 \tag{22}$$
$$z_E = z_D + DE\sin(\theta_2+\theta_3) \tag{23}$$

F

$$x_F = x_E + EF\cos(\theta_2+\theta_3+\theta_4)\cos\theta_1 \tag{24}$$
$$y_F = y_E + EF\cos(\theta_2+\theta_3+\theta_4)\sin\theta_1 \tag{25}$$
$$z_F = z_E + EF\sin(\theta_2+\theta_3+\theta_4) \tag{26}$$

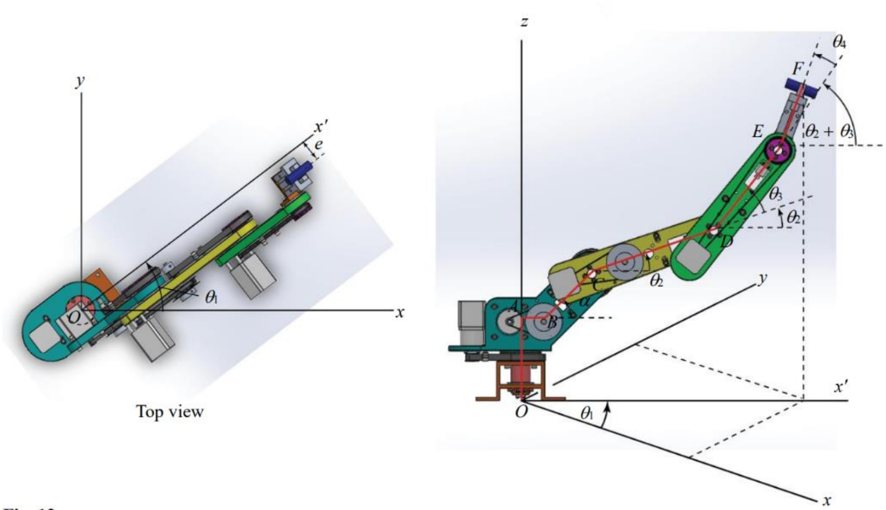


Fig. 12

5. Inverse Kinematics

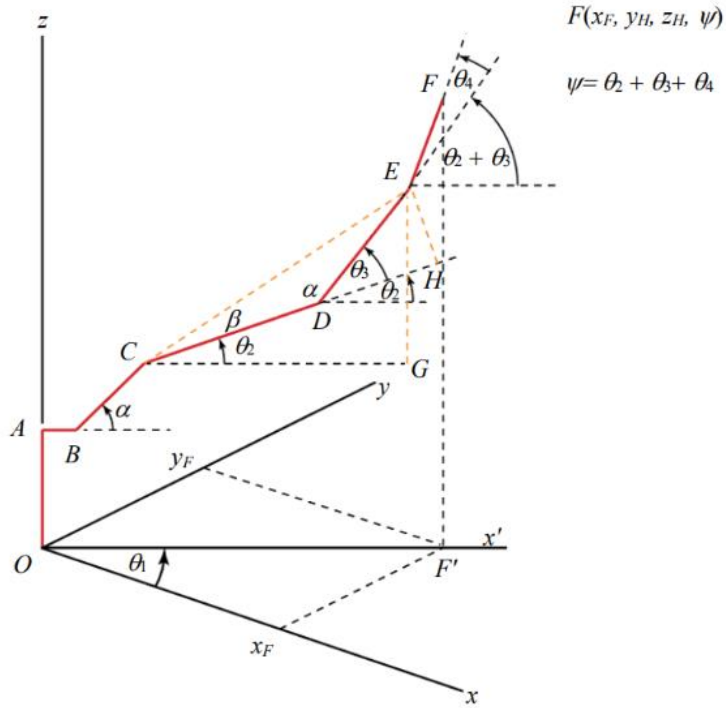


Fig. 13

As show in Fig. 13, given x_F, y_F, z_F, and ψ, we can express it as

$$\theta_1 = \tan^{-1}\frac{y_F}{x_H} \tag{27}$$

Using Eq. (24) and (26), given $\gamma = \theta_2 + \theta_3 + \theta_4$, we can write as

$$x_E = x_F - EF\cos\gamma\cos\theta_1 \tag{28}$$

$$y_E = y_F - EF\cos\gamma\sin\theta_1 \tag{29}$$

$$z_E = z_F - EF\sin\gamma \tag{30}$$

$$CG = CD\cos\theta_2 + DE\cos(\theta_2+\theta_3)$$

$$EG = CD\sin\theta_2 + DE\sin(\theta_2+\theta_3)$$

$$\begin{aligned} CG^2 + EG^2 &= [CD\cos\theta_2 + DE\cos(\theta_2+\theta_3)]^2 + [CD\sin\theta_2 + DE\sin(\theta_2+\theta_3)]^2 \\ &= CD^2\cos^2\theta_2 + DE^2[\cos(\theta_2+\theta_3)]^2 + 2(CD)(DE)\cos\theta_2\cos(\theta_2+\theta_3) \\ &\quad + CD^2\sin^2\theta_2 + DE^2[\sin(\theta_2+\theta_3)]^2 + 2(CD)(DE)\sin\theta_2\sin(\theta_2+\theta_3) \end{aligned}$$

$$\begin{aligned} CG^2 + EG^2 = {} & CD^2\cos^2\theta_2 + DE^2[\cos\theta_2\cos\theta_3 - \sin\theta_2\sin\theta_3]^2 + 2(CD)(DE)\cos\theta_2(\cos\theta_2\cos\theta_3 - \sin\theta_2\sin\theta_3) \\ & + CD^2\sin^2\theta_2 + DE^2[\sin\theta_2\cos\theta_3 + \cos\theta_2\sin\theta_3]^2 + 2(CD)(DE)\sin\theta_2(\sin\theta_2\cos\theta_3 + \cos\theta_2\sin\theta_3) \end{aligned}$$

$$
\begin{aligned}
CG^2 + EG^2 = {} & CD^2\cos^2\theta_2 + DE^2\cos^2\theta_2\cos^2\theta_3 + DE^2\sin^2\theta_2\sin^2\theta_3 - 2DE^2\sin\theta_2\sin\theta_3\cos\theta_2\cos\theta_3 \\
& + 2(CD)(DE)\cos^2\theta_2\cos\theta_3 - 2(CD)(DE)\sin\theta_3\sin\theta_2\cos\theta_2) \\
& + CD^2\sin^2\theta_2 + DE^2\sin^2\theta_2\cos^2\theta_3 + DE^2\sin^2\theta_3\cos^2\theta_2 + 2DE^2\sin\theta_2\sin\theta_3\cos\theta_3\cos\theta_2 \\
& + 2(CD)(DE)\sin^2\theta_2\cos\theta_3 + 2(CD)(DE)\sin\theta_2\sin\theta_3\cos\theta_2)
\end{aligned}
$$

$$
\begin{aligned}
CG^2 + EG^2 = {} & CD^2(\cos^2\theta_2 + \sin^2\theta_2) + DE^2(\cos^2\theta_3 + \sin^2\theta_3)(\sin^2\theta_2 + \cos^2\theta_2) \\
& + 2(CD)(DE)\cos\theta_3(\cos^2\theta_2 + \sin^2\theta_2) \\
CG^2 + EG^2 = {} & CD^2 + DE^2 + 2(CD)(DE)\cos\theta_3(\cos^2\theta_2 + \sin^2\theta_2)
\end{aligned}
$$

$$\theta_3 = \cos^{-1}\frac{CG^2 + EG^2 - CD^2 - DE^2}{2(CD)(DE)} \tag{31}$$

$$CG = x_E^2 + y_E^2 - AB - BC\cos\alpha \tag{32}$$

$$EG = z_E - OA - BC\sin\alpha \tag{33}$$

$$\beta = \tan^{-1}\frac{EH}{CH} = \tan^{-1}\frac{DE\sin\theta_3}{CD + DE\cos\theta_3} \tag{34}$$

$$\tan(\theta_2 + \beta) = \frac{EG}{CG}$$

$$\theta_2 = \tan^{-1}\frac{EG}{CG} - \beta \tag{35}$$

$$\theta_4 = \gamma - \theta_1 - \theta_2 - \theta_3 \tag{36}$$

8. Wiring Diagram