Online Appendix for Redistribution and the Monetary–Fiscal Policy Mix*

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Section A of this online appendix presents a tractable two agent model that permits analytical solutions. The flexible-price model analyzed in Section 2 of the main text is introduced as a special case of this model. We give more details to the derivation of the results in that section. Section B details the quantitative model presented in Section 3 of the main text.

A The simple model

A.1 Households

A.1.1 Ricardian household

There are Ricardian households of measure $1-\lambda$. These households, taking prices as given, choose $\{C_t^R, L_t^R, B_t^R\}$ to maximize

$$\sum_{t=0}^{\infty} \beta^t \left[\log C_t^R - \chi \frac{\left(L_t^R\right)^{1+\varphi}}{1+\varphi} \right]$$

subject to a standard No Ponzi condition, $\lim_{t\to\infty}\left[\beta^t \frac{1}{C_t^R}\left(\frac{B_t^R}{P_t}\right)\right] \geq 0$, and a sequence of flow budget constraints

$$C_t^R + \frac{B_t^R}{P_t} = R_{t-1}\frac{B_{t-1}^R}{P_t} + w_t L_t^R + \Psi_t^R - \tau_t^R,$$

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where C_t^R, L_t^R, B_t^R, Ψ_t^R, τ_t^R, P_t, w_t and R_t denote respectively consumption, hours, nominal government debt, real profits, lumpsum taxes, the price level, the real wage rate, and the nominal gross interest rate. The discount parameter and the inverse of the Frisch elasticity are denoted by $\beta \in (0,1)$ and $\varphi \geq 0$. The superscript, R, represents "Ricardian." The flow constraints can be written as

$$C_t^R + b_t^R = R_{t-1}\frac{1}{\Pi_t}b_{t-1}^R + w_t L_t^R + \Psi_t^R - \tau_t^R,$$

where $b_t^R = \frac{B_t^R}{P_t}$ is the real value of debt, and $\Pi_t = \frac{P_t}{P_{t-1}}$ is the gross rate of inflation.

Optimality conditions are given by the Euler equation, labor supply condition, and transversality condition (TVC):

$$\frac{C_{t+1}^R}{C_t^R} = \beta\frac{R_t}{\Pi_{t+1}}, \tag{A.1}$$

$$\chi\left(L_t^R\right)^{\varphi} C_t^R = w_t, \tag{A.2}$$

$$\lim_{t\to\infty}\left[\beta^t\frac{1}{C_t^R}\left(\frac{B_t^R}{P_t}\right)\right] = 0. \tag{A.3}$$

A.1.2 HTM Household

The hand-to-mouth (HTM) households, of measure λ, simply consume government transfers, s_t^H, every period

$$C_t^H = s_t^H,$$

and has no optimization problem to solve.

A.2 Firms

A.2.1 Final good producing firms

Perfectly competitive firms combine two types of intermediate composite goods $\{Y_{f,t}, Y_{s,t}\}$ to produce final consumption goods using a Cobb-Douglas production function

$$Y_t = \left(Y_{f,t}\right)^{1-\gamma}\left(Y_{s,t}\right)^{\gamma},$$

where the intermediate composites are given as

$$Y_{f,t} \equiv \left[\int_0^1 y_{f,t}(i)^{\frac{\theta-1}{\theta}}di\right]^{\frac{\theta}{\theta-1}} \quad \text{and} \quad Y_{s,t} \equiv \left[\int_0^1 y_{s,t}(i)^{\frac{\theta-1}{\theta}}di\right]^{\frac{\theta}{\theta-1}}.$$

Solving the standard cost minimization problems yields price indices of the form:

$$P_t = k^{-1} \left(P_{f,t}\right)^{1-\gamma} \left(P_{s,t}\right)^{\gamma},$$

$$P_{f,t} \equiv \left[\int_0^1 p_{f,t}(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}} \quad \text{and} \quad P_{s,t} \equiv \left[\int_0^1 p_{s,t}(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}},$$

where $k = (1-\gamma)^{1-\gamma}\gamma^{\gamma}$, and the demand functions for the intermediate goods:

$$Y_{f,t} = (1-\gamma)\left(\frac{P_{f,t}}{P_t}\right)^{-1} Y_t \quad \text{and} \quad Y_{s,t} = \gamma\left(\frac{P_{s,t}}{P_t}\right)^{-1} Y_t,$$
$$y_{f,t}(i) = \left(\frac{p_{f,t}(i)}{P_{f,t}}\right)^{-\theta} Y_{f,t} \quad \text{and} \quad y_{s,t}(i) = \left(\frac{p_{s,t}(i)}{P_{s,t}}\right)^{-\theta} Y_{s,t}.$$

A.2.2 Intermediate good producing firms

These firms produce goods using the linear production function

$$y_{f,t}(i) = l_{f,t}(i) \quad \text{and} \quad y_{s,t}(i) = l_{s,t}(i),$$

where $l_{f,t}(j)$ and $l_{s,t}(j)$ are labor hours employed by the firms. Firm i's real profits are given as

$$\Psi_{j,t}(i) = \frac{p_{j,t}(i)}{P_t} y_{j,t}(i) - w_t y_{j,t}(i) \quad \text{for } j = f \text{ and } s.$$

Firms in sector f set prices every period flexibly. The first order condition of these firms is given by

$$\frac{P_{f,t}}{P_t} = \frac{\theta}{\theta-1} w_t = \mu w_t,$$

where $\mu \equiv \frac{\theta}{\theta-1}$. Firms in sector s, in contrast, set their prices to the previous period price index P_{t-1}:

$$\frac{P_{s,t}}{P_t} = \frac{P_{t-1}}{P_t} = \Pi_t^{-1}.$$

A.2.3 Aggregation

First, we use the aggregate price index to obtain a Phillips curve relationship

$$1 = k^{-1}\left(\frac{P_{f,t}}{P_t}\right)^{1-\gamma}\left(\frac{P_{s,t}}{P_t}\right)^{\gamma} = k^{-1}\left(\mu w_t\right)^{1-\gamma}\left(\Pi_t^{-1}\right)^{\gamma}.$$

Solve for w_t to get

$$w_t = \mu^{-1} k^{\frac{1}{1-\gamma}} \Pi_t^{\frac{\gamma}{1-\gamma}} \qquad \text{(Phillips curve)}, \tag{A.4}$$

which shows the real wage depends positively on inflation, except for the flexible-price limit, $\gamma = 0$.

Aggregate hours are given as

$$L_t = \underbrace{\int l_{f,t}(j) dj}_{\equiv L_{f,t}} + \underbrace{\int l_{s,t}(j) dj}_{\equiv L_{s,t}}.$$

Since firms in each sector choose a common price, we have

$$\begin{aligned} y_{f,t}(j) &= Y_{f,t} \quad \text{and} \quad y_{s,t}(j) = Y_{s,t}, \\ l_{f,t}(j) &= L_{f,t} \quad \text{and} \quad l_{s,t}(j) = L_{s,t}. \end{aligned}$$

Aggregate profits are given by

$$\begin{aligned} \Psi_t &\equiv \int \Psi_{f,t}(i) di + \int \Psi_{s,t}(i) di \\ &= \left(\frac{P_{f,t}}{P_t} Y_{f,t} - w_t Y_{f,t} \right) + \left(\frac{P_{s,t}}{P_t} Y_{s,t} - w_t Y_{s,t} \right) \\ &= Y_t - w_t \left(Y_{f,t} + Y_{s,t} \right) \\ &= Y_t - w_t \left(L_{f,t} + L_{s,t} \right) \\ &\implies \Psi_t = Y_t - w_t L_t, \end{aligned}$$

Finally, the aggregate production function can be obtained as

$$\begin{aligned}
L_t &= \int l_{f,t}(i)di + \int l_{s,t}(i)di = L_{f,t} + L_{s,t} \\
&= (1-\gamma)\left(\frac{P_{f,t}}{P_t}\right)^{-1} Y_t + \gamma \left(\frac{P_{s,t}}{P_t}\right)^{-1} Y_t \\
&= (1-\gamma)\left(\mu w_t\right)^{-1} Y_t + \gamma \Pi_t Y_t \\
&= \left(\gamma^{\frac{\gamma}{1-\gamma}} \Pi_t^{\frac{\gamma}{1-\gamma}}\right)^{-1} Y_t + \gamma \Pi_t Y_t \\
&= \left[\left(\frac{1}{\gamma \Pi_t}\right)^{\frac{\gamma}{1-\gamma}} + \gamma \Pi_t\right] Y_t \\
&\Longrightarrow L_t = \Xi\left(\Pi_t\right) Y_t.
\end{aligned} \tag{A.5}$$

Notice that in the flexible-price limit, $\Xi\left(\Pi_t\right) = 1$, and output, Y_t, does not depend on inflation. Hours, L_t, therefore, is also independent from inflation in the absence of nominal rigidities. In general, however, inflation affects hours through Y_t and $\Xi\left(\Pi_t\right)$. Output Y_t is increasing in Π_t (as shown below).

A.3 Government

A.3.1 Flow budget constraint

The government issues one-period nominal debt B_t. Its budget constraint (GBC) is

$$\frac{B_t}{P_t} = R_{t-1}\frac{B_{t-1}}{P_t} - \tau_t + s_t,$$

where τ_t is taxes and s_t is transfers. It can be rewritten as

$$b_t = \frac{R_{t-1}}{\Pi_t} b_{t-1} - \tau_t + s_t. \tag{A.6}$$

Transfer, s_t, is exogenous and deterministic.

A.3.2 Policy rules

Monetary and fiscal policy rules are

$$\frac{R_t}{\bar{R}} = \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi}, \tag{A.7}$$

$$(\tau_t - \bar{\tau}) = \psi(b_{t-1} - \bar{b}), \tag{A.8}$$

where ϕ and ψ measure respectively the responsiveness of the policy instruments to inflation and government indebtedness. The steady state value of inflation, debt, and the exogenous variable, $\{\bar{\Pi}, \bar{b}, \bar{s}\}$, are set by policymakers and given exogenously.

A.3.3 Intertemporal budget constraint

For future use, we obtain the intertemporal GBC by combining the flow GBC and TVC. From the GBC (A.6), we have

$$b_t = R_{t-1} b_{t-1} \frac{1}{\Pi_t} - \tau_t + s_t \implies b_{t-1} = \frac{\Pi_t}{R_{t-1}} (b_t + \tau_t - s_t)$$

Iterating it forward leads to

$$b_{t-1} = \left(\frac{\Pi_t}{R_{t-1}} \frac{\Pi_{t+1}}{R_t} \cdots \frac{\Pi_{t+k-1}}{R_{t+k-2}} \frac{\Pi_{t+k}}{R_{t+k-1}}\right) b_{t+k} + \sum_{k=0}^{\infty} \left[\prod_{j=0}^{k} \frac{\Pi_{t+j}}{R_{t-1+j}}\right] (\tau_{t+k} - s_{t+k})$$

At $t = 0$

$$b_{-1} = \left(\frac{\Pi_0}{R_{-1}} \underbrace{\frac{\Pi_1}{R_0} \cdots \frac{\Pi_{k-1}}{R_{k-2}} \frac{\Pi_k}{R_{k-1}}}_{\beta^k \frac{C_0^R}{C_1^R} \frac{C_1^R}{C_2^R} \cdots \frac{C_{k-1}^R}{C_k^R}} \right) b_k + \sum_{i=0}^{k} \left[\prod_{j=0}^{i} \frac{\Pi_j}{R_{-1+j}}\right] (\tau_i - s_i),$$

where the discount factor is given as

$$\left[\prod_{j=0}^{i} \frac{\Pi_j}{R_{-1+j}}\right] = \frac{\Pi_0}{R_{-1}} \frac{C_0^R}{C_1^R} \frac{C_1^R}{C_2^R} \cdots \frac{C_{i-1}^R}{C_i^R} = \frac{\Pi_0}{R_{-1}} \beta^i \frac{C_0^R}{C_i^R}$$

In the limit, we have

$$b_{-1} = \underbrace{\frac{\Pi_0 C_0^R}{R_{-1}} \underbrace{\lim_{k\to\infty} \beta^k \frac{1}{C_k^R} b_k}_{\text{TVC}}}_{\to 0} + \frac{\Pi_0}{R_{-1}} \sum_{i=0}^{\infty} \beta^i \frac{C_0^R}{C_i^R} \left(\tau_i - s_i\right)$$

or

$$\frac{b_{-1}R_{-1}}{\Pi_0} = \sum_{i=0}^{\infty} \beta^i \frac{C_0^R}{C_i^R} \left(\tau_i - s_i\right). \tag{A.9}$$

The last equation is the intertemporal government budget constraint (IGBC).

A.4 Aggregation and the resource constraint

Aggregating the variables over the households yields

$$s_t = \lambda s_t^H$$

$$\tau_t = (1-\lambda)\,\tau_t^R$$

$$b_t = (1-\lambda)\,b_t^R$$

$$L_t = (1-\lambda)\,L_t^R$$

$$\Psi_t = (1-\lambda)\,\Psi_t^R$$

Combining household and government budget constraints gives

$$(1-\lambda)\,C_t^R + \lambda C_t^H = Y_t.$$

The resource constraint above, together with HTM household budget constraint, implies that output is simply divided between the two types of households as:∞∞

$$\begin{aligned} C_t^H &= \frac{1}{\lambda} s_t, \\ C_t^R &= \frac{1}{1-\lambda} Y_t - \frac{1}{1-\lambda} s_t. \end{aligned} \tag{A.10}$$

A.5 Solving the model

As in the main text, we solve the model, considering a redistribution program in which $\{s_t\}_{t=0}^{\infty}$ can have arbitrary values greater than $\bar{s}$ until time period T, and then $s_t = \bar{s}$ for $t \geq T+1$.

A.5.1 Output and consumption

As in the main text, we start with output. We use the household and firm optimality conditions to get

$$\chi \left(L_t^R\right)^{\varphi} C_t^R = w_t$$

$$\Longrightarrow \chi \left(\underbrace{\frac{1}{1-\lambda}\Xi\left(\Pi_t\right)Y_t}_{L_t}\right)^{\varphi} \left(\frac{1}{1-\lambda}Y_t - \frac{\omega}{1-\lambda}s_t\right) = \mu^{-1}k^{\frac{1}{1-\gamma}}\Pi_t^{\frac{\gamma}{1-\gamma}} \tag{A.11}$$

Equation (A.11) implicitly defines output as a function of transfers and inflation, the latter of which in turn is also a function of the entire schedule of transfers $\{s_t\}_{t=0}^{\infty}$. Once output is determined, Ricardian consumption is determined by Equation (A.10). We consider two special benchmarks, which helps us develop intuition for other in-between cases that are harder to solve.

Flexible prices. First, as in the main text, we shut down any effects of nominal rigidities. A perfectly competitive and flexible-price economy can be obtained by setting $\gamma = 0$ and $\mu = 1$ (as $\theta \to \infty$).

Equation (A.11) then simplifies to

$$\chi \left(\frac{1}{1-\lambda}Y_t\right)^{\varphi} \left(\frac{1}{1-\lambda}Y_t - \frac{1}{1-\lambda}s_t\right) = 1$$

$$\Longrightarrow Y_t = \chi^{-1}\left(1-\lambda\right)^{1+\varphi} Y_t^{-\varphi} + s_t,$$

Output (and other real variables) are now independent from inflation.

We can obtain the "transfer multiplier" using the implicit function theorem. Let

$$F(Y,s) \equiv Y_t - \chi^{-1}\left(1-\lambda\right)^{1+\varphi} Y_t^{-\varphi} - s_t$$

The derivative of Y with respect to s is

$$\frac{dY_t}{ds_t} = -\frac{F_s}{F_Y} = \frac{1}{1 + (1-\lambda)^{1+\varphi} \frac{\varphi}{\chi} Y_t^{-(1+\varphi)}}.$$

Notice that

$$0 \leq \frac{dY_t}{ds_t} \leq 1.$$

The Ricardian household consumption is

$$C_t^R = C^R(s_t) \equiv \frac{1}{1-\lambda} Y(s_t) - \frac{1}{1-\lambda} s_t.$$

The derivative is

$$\frac{dC^R(s_t)}{ds_t} = \frac{1}{1-\lambda}\left[\frac{dY(s_t)}{ds_t} - 1\right] \leq 0.$$

These are the results presented in the main text.

Sticky prices. We now consider the role of nominal rigidities. To this end, we assume perfectly elastic labor supply, $\varphi = 0$, which is a typical assumption in the early RBC literature. This assumption allows for an analytical characterization of the solution. It maximizes the wealth effects on labor supply and thus the multiplier. As a consequence, perfectly elastic labor supply eliminates the direct relationship between Ricardian consumption and transfers, which greatly simplifies the algebra.

We again use (A.11) to solve for output:

$$Y_t = (1-\lambda)(\chi\mu)^{-1}(\gamma\Pi_t)^{\frac{\gamma}{1-\gamma}} + s_t \tag{A.12}$$

The last equation shows output as a function of transfers and inflation. Unlike the case of flexible prices, the multiplier would in fact be greater if an increase in transfer generated inflation.

Ricardian consumption in this case is given as

$$C_t^R = C^R(\Pi_t) \equiv \frac{1}{1-\lambda} Y_t - \frac{1}{1-\lambda} s_t = (\chi\mu)^{-1}(\gamma\Pi_t)^{\frac{\gamma}{1-\gamma}},$$

which reveals that the Ricardian household consumption depends positively on inflation. Transfers no longer *directly* (and negatively) affect C_t^R. Consequently, and in contrast to the flexible-price case, an increase in s_t leads to an increase in C_t^R through the indirect channel (i.e., via Π_t) to the extent that transfers are inflationary.

General case. A more general case is difficult to obtain an analytical solution. If labor supply were imperfectly elastic ($\varphi > 0$) and prices were sticky, Ricardian consumption would depends negatively on transfer – controlling for inflation. An increase in transfer, therefore, has opposing effects on Ricardian consumption. On one hand, it generates inflation, which raises C_t^R due to nominal rigidity. On the other hand, it lowers C_t^R due to the redistributive role of transfer. So this is an intermediate case between the two benchmark setups above.

A.5.2 Inflation

We now turn to inflation determination given monetary, tax and transfer policies. As shown in the main text, the equilibrium time path of $\{\Pi_t, R_t, b_t, \tau_t\}$ satisfies the following conditions.

- Difference equations

$$\Pi_{t+1} = \frac{C_t^R}{C_{t+1}^R}\beta R_t$$

$$b_t = R_{t-1}b_{t-1}\frac{1}{\Pi_t} - \tau_t + s_t$$

$$\frac{R_t}{\bar{R}} = \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi}$$

$$(\tau_t - \bar{\tau}) = \psi(b_{t-1} - \bar{b})$$

- Terminal condition (TVC)

$$\lim_{t\to\infty}\left[\beta^t \frac{1}{C_t^R} b_t\right] = 0$$

- Initial conditions

$$b_{-1} \text{ and } R_{-1}.$$

We first solve for a steady state. Assume $s = \bar{s}$. The system of difference equation then simplifies to

$$\bar{R} = \beta^{-1}\bar{\Pi},$$

$$\bar{b} = \bar{b}\frac{\bar{R}}{\bar{\Pi}} - \bar{\tau} + \bar{s} \Rightarrow \bar{\tau} = \left(\beta^{-1} - 1\right)\bar{b} + \bar{s}.$$

So, $\bar{R}$ and $\bar{\tau}$ are determined given $\bar{s}$, $\bar{\Pi}$ and $\bar{b}$.

The system above can be simplified. First, as is well-known in this simple set-up, the Euler equation and Taylor rule can be combined to yield a non-linear difference equation in Π_t:

$$\Pi_{t+1} = \frac{C_t^R}{C_{t+1}^R} \beta R_t = \frac{C_t^R}{C_{t+1}^R} \beta \bar{R} \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\phi}.$$

Using the steady-state relation, $\bar{R} = \beta^{-1}\bar{\Pi}$, we obtain

$$\frac{\Pi_{t+1}}{\bar{\Pi}} = \frac{C_t^R}{C_{t+1}^R} \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\phi}.$$

This equation shows that, for given Π_t, an increase in r_t leads to a decrease in Π_{t+1}.

Second, we now simplify the GBC. Notice that the Euler equation implies

$$R_t = \beta^{-1} \frac{C_{t+1}^R}{C_t^R} \Pi_{t+1} \quad \text{for } t \geq 0$$
$$\Longrightarrow R_{t-1} = \beta^{-1} \frac{C_t^R}{C_{t-1}^R} \Pi_t \quad \text{for } t \geq 1$$

Use the above equation, the fiscal rule, and the steady-state relation, $\bar{\tau} = \left(\beta^{-1} - 1\right) \bar{b} + \bar{s}$, to obtain the budget constraint of the form (for $t \geq 1$):

$$\begin{aligned}
b_t &= R_{t-1} b_{t-1} \frac{1}{\Pi_t} - \tau_t + s_t \\
&= \beta^{-1} \frac{C_t^R}{C_{t-1}^R} \Pi_t b_{t-1} \frac{1}{\Pi_t} - \tau_t + s_t \\
&= \beta^{-1} \frac{C_t^R}{C_{t-1}^R} b_{t-1} - \bar{\tau} - \psi(b_{t-1} - \bar{b}) + s_t \\
&= \beta^{-1} \frac{C_t^R}{C_{t-1}^R} b_{t-1} - \left(\beta^{-1} - 1\right) \bar{b} - \psi(b_{t-1} - \bar{b}) + (s_t - \bar{s}),
\end{aligned}$$

which can be written as

$$(b_t - \bar{b}) = \left[\beta^{-1} \frac{C_t^R}{C_{t-1}^R} - \psi \right] (b_{t-1} - \bar{b}) + (s_t - \bar{s}) + \beta^{-1} \bar{b} \left[\frac{C_t^R}{C_{t-1}^R} - 1 \right] \quad \text{for } t \geq 1.$$

Now consider time-0 GBC. At $t = 0$, the Euler equation does not apply. We therefore have

$$b_0 = R_{-1} b_{-1} \frac{1}{\Pi_0} - \left[\bar{\tau} + \psi(b_{-1} - \bar{b})\right] + s_0$$

Again, use the steady state relation, $\bar{\tau} = \left(\beta^{-1} - 1\right) \bar{b} + \bar{s}$, to obtain

$$b_0 = \left(\frac{R_{-1}}{\Pi_0} - \psi\right) b_{-1} - \left(\beta^{-1} - 1 - \psi\right) \bar{b} + (s_0 - \bar{s})$$

Finally, for simplicity we assume $R_{-1} = \bar{R}$ and $b_{-1} = \bar{b}$. The system then simplifies to

$$\left(\frac{\Pi_{t+1}}{\bar{\Pi}}\right) = \frac{C_t^R}{C_{t+1}^R} \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi}, \tag{A.13}$$

$$\left(b_t - \bar{b}\right) = \left[\beta^{-1} \frac{C_t^R}{C_{t-1}^R} - \psi\right] (b_{t-1} - \bar{b}) + (s_t - \bar{s}) + \beta^{-1} \bar{b} \left[\frac{C_t^R}{C_{t-1}^R} - 1\right] \quad \text{for } t \geq 1 \tag{A.14}$$

$$\left(b_0 - \bar{b}\right) = \beta^{-1} \left(\frac{\bar{\Pi}}{\Pi_0} - 1\right) \bar{b} + (s_0 - \bar{s}) \quad \text{at } t = 0, \tag{A.15}$$

with the initial and terminal conditions.

Inflation determination under flexible prices. We first solve the model under flexible prices. In this case, $C_t^R = C^R(s_t)$, as shown above.

Monetary regime. Notice that, no matter what happens until time $T+1$, starting $T+2$, (A.14) becomes

$$\left(b_t - \bar{b}\right) = \left(\beta^{-1} - \psi\right) (b_{t-1} - \bar{b}).$$

If $\psi > 0$, debt b satisfies the TVC for all possible values of inflation (including Π_0) and regardless of monetary policy.

Inflation is solely determined by equation (A.13) which becomes

$$\left(\frac{\Pi_{t+1}}{\bar{\Pi}}\right) = \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi} \quad \text{for } t \geq T+1,$$

regardless of the history.

Suppose we are confined to find a bounded solution in the monetary regime ($\phi > 1$). In this case, we must have

$$\frac{\Pi_{T+1}}{\bar{\Pi}} = 1.$$

Otherwise, inflation would explode. Inflation before $T+1$ can then be solved backward using

$$\frac{\Pi_t}{\bar{\Pi}} = \left(\frac{\Pi_{t+1}}{\bar{\Pi}}\right)^{\frac{1}{\phi}} \left(\frac{C^R\left(s_{t+1}\right)}{C^R\left(s_t\right)}\right)^{\frac{1}{\phi}}.$$

That is,

$$\begin{aligned}
\frac{\Pi_T}{\bar{\Pi}} &= \left(\frac{C^R\left(\bar{s}\right)}{C^R\left(s_T\right)}\right)^{\frac{1}{\phi}} \\
\frac{\Pi_{T-1}}{\bar{\Pi}} &= \left(\left(\frac{C^R\left(\bar{s}\right)}{C^R\left(s_T\right)}\right)^{\frac{1}{\phi}}\right)^{\frac{1}{\phi}} \left(\frac{C^R\left(s_T\right)}{C^R\left(s_{T-1}\right)}\right)^{\frac{1}{\phi}} = \left(\frac{C^R\left(\bar{s}\right)}{C^R\left(s_T\right)}\right)^{\frac{1}{\phi^2}} \left(\frac{C^R\left(s_T\right)}{C^R\left(s_{T-1}\right)}\right)^{\frac{1}{\phi}} \\
\frac{\Pi_{T-2}}{\bar{\Pi}} &= \left(\left(\frac{C^R\left(\bar{s}\right)}{C^R\left(s_T\right)}\right)^{\frac{1}{\phi^2}} \left(\frac{C^R\left(s_T\right)}{C^R\left(s_{T-1}\right)}\right)^{\frac{1}{\phi}}\right)^{\frac{1}{\phi}} \left(\frac{C^R\left(s_{T-1}\right)}{C^R\left(s_{T-2}\right)}\right)^{\frac{1}{\phi}} \\
&= \left(\frac{C^R\left(\bar{s}\right)}{C^R\left(s_T\right)}\right)^{\frac{1}{\phi^3}} \left(\frac{C^R\left(s_T\right)}{C^R\left(s_{T-1}\right)}\right)^{\frac{1}{\phi^2}} \left(\frac{C^R\left(s_{T-1}\right)}{C^R\left(s_{T-2}\right)}\right)^{\frac{1}{\phi}} \\
&\vdots \\
\frac{\Pi_0}{\bar{\Pi}} &= \left(\frac{C^R\left(\bar{s}\right)}{C^R\left(s_T\right)}\right)^{\frac{1}{\phi^{T+1}}} \left(\frac{C^R\left(s_T\right)}{C^R\left(s_{T-1}\right)}\right)^{\frac{1}{\phi^T}} \cdots \left(\frac{C^R\left(s_1\right)}{C^R\left(s_0\right)}\right)^{\frac{1}{\phi}} \\
&= C^R\left(\bar{s}\right)^{\frac{1}{\phi^{T+1}}} \left[\frac{1}{C^R\left(s_T\right) C^R\left(s_{T-1}\right) \cdots C^R\left(s_0\right)}\right]^{\frac{1}{\phi}}.
\end{aligned}$$

An interesting example is a one-time increase in transfer ($s_0 > \bar{s}$ and $s_t = \bar{s}$ afterwards). In the bounded solution, this raises the rate of inflation by:

$$\frac{\Pi_0}{\bar{\Pi}} = \left(\frac{C^R\left(\bar{s}\right)}{C^R\left(s_0\right)}\right)^{\frac{1}{\phi}},$$

and subsequently $\Pi_t = \bar{\Pi}$ (for $t \geq 1$). Notice that the effect of transfer on inflation is purely transitory in the monetary regime.

Given the time path of inflation, we can solve for debt. Debt at $t = 0$ is given by

$$\begin{aligned} b_0 &= \left[\left(\frac{\bar{\Pi}}{\Pi_0} - 1 \right) \beta^{-1} + 1 \right] \bar{b} + (s_0 - \bar{s}) \\ &= \left[\left(\left(\frac{C^R(s_0)}{C^R(\bar{s})} \right)^{\frac{1}{\phi}} - 1 \right) \beta^{-1} + 1 \right] \bar{b} + (s_0 - \bar{s}) \end{aligned}$$

An increase in s_0 has two opposing effects on b_0. It directly increases b_0 as reflected in the last term, $(s_0 - \bar{s})$. On the other hand, there exists an indirect effect which lowers b_0 as an increase in s_0 raises inflation Π_0. The net effect depends on parameterization. In the following periods, $\{b_t\}$ is given by

$$\begin{aligned} \left(b_1 - \bar{b}\right) &= \left[\beta^{-1} \frac{C^R(\bar{s})}{C^R(s_0)} - \psi \right] (b_0 - \bar{b}) + \beta^{-1} \bar{b} \left[\frac{C^R(\bar{s})}{C^R(s_0)} - 1 \right], \\ \left(b_t - \bar{b}\right) &= \left[\beta^{-1} - \psi \right] (b_{t-1} - \bar{b}) \quad \text{for } t \geq 2. \end{aligned}$$

Fiscal regime. We now consider the flip side of the policy space: $\psi \leq 0$ and $\phi < 1$. Consider the GBC at time $T + 2$:

$$\left(b_{T+2} - \bar{b}\right) = \left(\beta^{-1} - \psi\right) \left(b_{T+1} - \bar{b}\right).$$

Suppose $b_{T+1} \neq \bar{b}$. This violates the TVC and thus cannot be an equilibrium because $\left(\beta^{-1} - \psi\right) \geq \beta^{-1}$. It thus has to be that $b_{T+1} = \bar{b}$ – if a solution exists.

Now look at the GBC at time $T + 1$

$$\left(b_{T+1} - \bar{b}\right) = \left[\beta^{-1} \underbrace{\frac{C^R(s_{T+1})}{C^R(s_T)}}_{\frac{C^R(\bar{s})}{C^R(s_T)}} - \psi \right] (b_T - \bar{b}) + \underbrace{(s_{T+1} - \bar{s})}_{=0} + \beta^{-1} \bar{b} \left[\underbrace{\frac{C^R(s_{T+1})}{C^R(s_T)}}_{\frac{C^R(\bar{s})}{C^R(s_T)}} - 1 \right]. \tag{A.16}$$

Substituting out debt backwards yields

$$
\begin{aligned}
\left(b_{T+1}-\bar{b}\right) = {} & \left(b_0-\bar{b}\right)\prod_{j=1}^{T+1}\left[\beta^{-1}\frac{C^R\left(s_j\right)}{C^R\left(s_{j-1}\right)}-\psi\right] \\
& + \sum_{k=1}^{T}\left(s_k-\bar{s}\right)\prod_{j=k+1}^{T+1}\left[\beta^{-1}\frac{C^R\left(s_j\right)}{C^R\left(s_{j-1}\right)}-\psi\right] \\
& + \sum_{k=1}^{T}\beta^{-1}\bar{b}\left[\frac{C^R\left(s_k\right)}{C^R\left(s_{k-1}\right)}-1\right]\prod_{j=k+1}^{T+1}\left[\beta^{-1}\frac{C^R\left(s_j\right)}{C^R\left(s_{j-1}\right)}-\psi\right]+\beta^{-1}\bar{b}\left[\frac{C^R\left(\bar{s}\right)}{C^R\left(s_T\right)}-1\right].
\end{aligned}
$$

Using the equilibrium property that $b_{T+1}=\bar{b}$, we can solve for b_0:

$$
\begin{aligned}
-\left(b_0-\bar{b}\right) = {} & \sum_{k=1}^{T}\left(s_k-\bar{s}\right)\frac{\prod_{j=k+1}^{T+1}\left[\beta^{-1}\frac{C^R\left(s_j\right)}{C^R\left(s_{j-1}\right)}-\psi\right]}{\prod_{j=1}^{T+1}\left[\beta^{-1}\frac{C^R\left(s_j\right)}{C^R\left(s_{j-1}\right)}-\psi\right]} \\
& + \sum_{k=1}^{T}\beta^{-1}\bar{b}\left[\frac{C^R\left(s_k\right)}{C^R\left(s_{k-1}\right)}-1\right]\frac{\prod_{j=k+1}^{T+1}\left[\beta^{-1}\frac{C^R\left(s_j\right)}{C^R\left(s_{j-1}\right)}-\psi\right]}{\prod_{j=1}^{T+1}\left[\beta^{-1}\frac{C^R\left(s_j\right)}{C^R\left(s_{j-1}\right)}-\psi\right]}+\frac{\beta^{-1}\bar{b}\left[\frac{C^R\left(\bar{s}\right)}{C^R\left(s_T\right)}-1\right]}{\prod_{j=1}^{T+1}\left[\beta^{-1}\frac{C^R\left(s_j\right)}{C^R\left(s_{j-1}\right)}-\psi\right]}
\end{aligned}
$$

Let

$$
\begin{aligned}
\Omega_k & \equiv \left\{\prod_{j=1}^{k}\left[\beta^{-1}\frac{C^R\left(s_j\right)}{C^R\left(s_{j-1}\right)}-\psi\right]\right\}^{-1}, \\
\Omega_0 & \equiv 1.
\end{aligned}
$$

We can then rewrite the equation above as

$$
\left(b_0-\bar{b}\right) = -\sum_{k=1}^{T}\Omega_k\left(s_k-\bar{s}\right)-\beta^{-1}\bar{b}\sum_{k=1}^{T+1}\Omega_k\left[\frac{C^R\left(s_k\right)}{C^R\left(s_{k-1}\right)}-1\right], \tag{A.17}
$$

which shows the value of b_0 required to generate $b_t=\bar{b}$ for $t\geq T+1$. Given b_0, debt in the ensuing periods is then determined by (A.14).

Let us now turn to inflation. In order to obtain Π_0 necessary to generate b_0 in (A.17), we look at the GBC at $t = 0$:

$$b_0 - \bar{b} = \left(\frac{\bar{\Pi}}{\Pi_0} - 1\right)\beta^{-1}\bar{b} + (s_0 - \bar{s}).$$

Substitute out $\left(b_0 - \bar{b}\right)$ using (A.17), and solve for Π_0 to obtain

$$-\sum_{k=1}^{T}\Omega_k (s_k - \bar{s}) - \beta^{-1}\bar{b}\sum_{k=1}^{T+1}\Omega_k\left[\frac{C^R(s_k)}{C^R(s_{k-1})} - 1\right] = \left(\frac{\bar{\Pi}}{\Pi_0} - 1\right)\beta^{-1}\bar{b} + (s_0 - \bar{s}).$$

$$\implies \frac{\Pi_0}{\bar{\Pi}} = \frac{1}{1 - \frac{\beta}{\bar{b}}\sum_{k=0}^{T}\Omega_k (s_k - \bar{s}) - \sum_{k=1}^{T+1}\Omega_k\left[\frac{C^R(s_k)}{C^R(s_{k-1})} - 1\right]},$$

which shows that Π_0 rises when current and/or future transfers increase. Subsequently, inflation follows (A.13), converging to $\bar{\Pi}$.

As before, consider the case of a one-time increase in s_0. Then inflation at time 0 is given by

$$\frac{\Pi_0}{\bar{\Pi}} = \frac{1}{1 - \frac{\beta}{\bar{b}}(s_0 - \bar{s}) - \Omega_1\left[\frac{C^R(\bar{s})}{C^R(s_0)} - 1\right]} = \left\{1 - \frac{\beta}{\bar{b}}(s_0 - \bar{s}) - \frac{\left[\frac{C^R(\bar{s})}{C^R(s_0)} - 1\right]}{\left[\beta^{-1}\frac{C^R(\bar{s})}{C^R(s_0)} - \psi\right]}\right\}^{-1}. \tag{A.18}$$

One can easily show that Π_0 is increasing in s_0. A *sufficient* condition is that:

$$g(s_0) \equiv \frac{\left[\frac{C^R(\bar{s})}{C^R(s_0)} - 1\right]}{\left[\beta^{-1}\frac{C^R(\bar{s})}{C^R(s_0)} - \psi\right]}$$

is increasing in s_0. Consider the derivative:

$$\begin{aligned}\frac{dg(s_0)}{ds_0} &\equiv \frac{-\frac{C^R(\bar{s})C^{R\prime}(s_0)}{C^R(s_0)^2}\left[\frac{C^R(\bar{s})}{C^R(s_0)} - \psi\beta\right] + \left[\frac{C^R(\bar{s})}{C^R(s_0)} - 1\right]\frac{C^R(\bar{s})C^{R\prime}(s_0)}{C^R(s_0)^2}}{\beta\left[\beta^{-1}\frac{C^R(\bar{s})}{C^R(s_0)} - \psi\right]^2} \\ &= \frac{-\frac{C^R(\bar{s})C^{R\prime}(s_0)}{C^R(s_0)^2}\left[1 - \psi\beta\right]}{\beta\left[\beta^{-1}\frac{C^R(\bar{s})}{C^R(s_0)} - \psi\right]^2},\end{aligned}$$

which is positive when $C^{R\prime}(s_0) < 0$.

Alternatively, one can solve the model using the IGBC. Equation (A.9) implies

$$\Pi_0 = \frac{b_{-1}R_{-1}}{\sum_{i=0}^{\infty} \beta^i \frac{C^R(s_0)}{C^R(s_i)} (\tau_i - s_i)}.$$

We consider a plausible case where $\psi = 0$.[1] We then have

$$\begin{aligned}
\frac{\Pi_0}{\bar{\Pi}} &= \frac{\bar{b}\beta^{-1}}{\sum_{i=0}^{\infty} \beta^i \frac{C^R(s_0)}{C^R(s_i)} \left(\beta^{-1} - 1\right) \bar{b} - \sum_{i=0}^{\infty} \beta^i \frac{C^R(s_0)}{C^R(s_i)} (s_i - \bar{s})} \\
&= \frac{1}{(1-\beta) \sum_{i=0}^{\infty} \beta^i \frac{C^R(s_0)}{C^R(s_i)} - \frac{\beta}{\bar{b}} (s_0 - \bar{s})} \\
&= \frac{1}{1 - \frac{\beta}{\bar{b}} (s_0 - \bar{s}) - \beta \left[1 - \frac{C^R(s_0)}{C^R(\bar{s})}\right]}.
\end{aligned} \tag{A.19}$$

This coincides with (A.18) when $\psi = 0$.

Inflationary effects of the redistribution policy In Proposition 1, we show that under a mild sufficient condition, the redistribution policy is more inflationary under the fiscal regime than under the monetary regime.

Proposition 1. *The redistribution policy is more inflationary on impact under the fiscal regime than under the monetary regime if debt-to-GDP ratio is sufficiently low.*

Proof. Let's consider the case that transfers increase only for one period: $s_0 > \bar{s}$ and $s_t = \bar{s}$ for $t \geq 1$. First, using equation (A.16) at $T = 0$, we can obtain the initial debt level under the fiscal regime, b_0^F, ensuring that $b_1 = \bar{b}$:

$$\frac{b_0^F - \bar{b}}{\bar{b}} = -\frac{\frac{1}{\beta}\frac{\bar{C}^R}{C_0^R} - \frac{1}{\beta}}{\frac{1}{\beta}\frac{\bar{C}^R}{C_0^R} - \psi} < 0.$$

We can also obtain the initial debt level under the monetary regime, b_0^M, using equations

[1]Cases in which $\psi < 0$ are implausible and difficult to solve using IGBC as τ_i in the equation is endogenous.

(A.13) and (A.15):

$$\begin{aligned}\frac{b_0^M - \bar{b}}{\bar{b}} &= \left(\frac{\bar{\Pi}}{\Pi_0} - 1\right)\frac{1}{\beta} + \frac{s_0 - \bar{s}}{\bar{b}} \\ &= \left(\left(\frac{C_0^R}{\bar{C}^R}\right)^{\frac{1}{\phi}} - 1\right)\frac{1}{\beta} + \frac{s_0 - \bar{s}}{\bar{b}} \\ &\geq \left(\frac{C_0^R - \bar{C}^R}{\bar{C}^R}\right)\frac{1}{\beta} + \frac{s_0 - \bar{s}}{\bar{b}}.\end{aligned}$$

Here the second equality holds since $C_1^R = \bar{C}^R$ and $\Pi_1 = \bar{\Pi}$ under the monetary regime. Notice that equation (A.15) implies that if $\frac{b_0^M - \bar{b}}{\bar{b}} > 0$, then $b_0^M > b_0^F$ and thus $\Pi_0^F > \Pi_0^M$. We want to find a sufficient condition for $\frac{b_0^M - \bar{b}}{\bar{b}} > 0$. Note that from the solution of C_0^R and $\bar{C}^R$, we can derive

$$\frac{C_0^R - \bar{C}^R}{\bar{C}^R} = \frac{Y_0 - \bar{Y} - (s_0 - \bar{s})}{\bar{Y} - \bar{s}}$$

Then,

$$\begin{aligned}\frac{b_0^M - \bar{b}}{\bar{b}} &\geq \left(\frac{C_0^R - \bar{C}^R}{\bar{C}^R}\right)\frac{1}{\beta} + \frac{s_0 - \bar{s}}{\bar{b}} \\ &= \left(\frac{Y_0 - \bar{Y}}{\bar{Y} - \bar{s}}\right)\frac{1}{\beta} + (s_0 - \bar{s})\left(\frac{1}{\bar{b}} - \frac{1}{\beta}\frac{1}{\bar{Y} - \bar{s}}\right)\end{aligned}$$

Here the first term is positive since $Y_0 > \bar{Y}$ and $\bar{Y} > \bar{s}$. Thus, $\frac{b_0^M - \bar{b}}{\bar{b}} > 0$ if the second term is positive, i.e.,

$$\frac{\bar{b}}{\bar{Y}} < \beta\left(1 - \frac{\bar{s}}{\bar{Y}}\right).$$

□

Inflation determination under sticky prices. We now solve the model under sticky prices. In this case, $C_t^R = C^R(\Pi_t)$ rather than $C_t^R = C^R(s_t)$.[2]

[2] In the general case (which we do not consider here), $C_t^R = C^R(\Pi_t, s_t)$.

Monetary regime. As in the flexible-price case, we focus on a bounded solution. Notice that the inverse of consumption growth is given by

$$\frac{C^R(\Pi_t)}{C^R(\Pi_{t+1})} = \left(\frac{\Pi_t}{\Pi_{t+1}}\right)^{\frac{\gamma}{1-\gamma}}.$$

Equation (A.13) thus can be written as

$$\left(\frac{\Pi_{t+1}}{\bar{\Pi}}\right) = \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi(1-\gamma)+\gamma}. \tag{A.20}$$

When $\tilde{\phi} = \phi(1-\gamma) + \gamma > 1$ ($\Longleftrightarrow \phi > 1$), the solution for non-explosive gross inflation is

$$\frac{\Pi_t}{\bar{\Pi}} = 1 \quad \text{for all } t \geq 0.$$

In other words, transfers does not generate inflation in the monetary regime.

Given the constant rate of inflation, (A.14) and (A.15) becomes

$$\begin{aligned}
\left(b_t - \bar{b}\right) &= \left[\beta^{-1} - \psi\right]\left(b_{t-1} - \bar{b}\right) + \left(s_t - \bar{s}\right) \\
\left(b_0 - \bar{b}\right) &= \left(s_0 - \bar{s}\right)
\end{aligned}$$

If $\psi > 0$, debt b satisfies the TVC for all possible values of inflation and regardless of monetary policy.

Fiscal regime. We let $\tilde{\phi} \equiv \phi(1-\gamma) + \gamma < 1$ (or $\phi < 1$). This condition generates bounded inflation for any given Π_0 – as indicated by (A.20). To pin down Π_0, it is easier to use the IGBC (A.9) in this case; we obtain

$$\frac{\Pi_0}{\bar{\Pi}} = \frac{\beta^{-1}\bar{b}}{\sum_{i=0}^{\infty} \beta^i \left(\frac{\Pi_0}{\bar{\Pi}}\right)^{\frac{\gamma}{1-\gamma}\left(1-\tilde{\phi}^i\right)} \left(\tau_i - s_i\right)}.$$

Once again, we consider the plausible case where $\psi = 0$. We then obtain

$$\frac{\Pi_0}{\bar{\Pi}} = \frac{\beta^{-1}\bar{b}}{\sum_{i=0}^{\infty} \beta^i \left(\frac{\Pi_0}{\bar{\Pi}}\right)^{\frac{\gamma}{1-\gamma}\left(1-\tilde{\phi}^i\right)} \left(\bar{\tau} - s_i\right)} \tag{A.21}$$

$$= \frac{1}{\sum_{i=0}^{\infty} \beta^i \left(\frac{\Pi_0}{\bar{\Pi}}\right)^{\frac{\gamma}{1-\gamma}\left(1-\tilde{\phi}^i\right)} \left[(1-\beta) - \frac{\beta}{\bar{b}}\left(s_i - \bar{s}\right)\right]}.$$

Equation (A.21) implicitly defines Π_0 as a function of transfers. Equilibrium Π_0 can be obtained as a fixed point of the equation.

For intuition, consider a one-time increase in transfer. Equation (A.21) then can be written as:

$$\frac{\Pi_0}{\bar{\Pi}} = \frac{1}{(1-\beta)\sum_{i=0}^{\infty} \beta^i \left(\frac{\Pi_0}{\bar{\Pi}}\right)^{\frac{\gamma}{1-\gamma}\left(1-\tilde{\phi}^i\right)} - \frac{\beta}{\bar{b}}\left(s_0 - \bar{s}\right)} \tag{A.22}$$

It is easy to show that Π_0 is increasing in s_0. Compared to the flexible-price case, however, inflation does not increase as much in this sticky-price case. The reason is that the real interest rate

$$r_t = \beta^{-1}\frac{C^R\left(\Pi_{t+1}\right)}{C^R\left(\Pi_t\right)} = \beta^{-1}\left(\frac{\Pi_0}{\bar{\Pi}}\right)^{-\frac{\gamma(1-\tilde{\phi})}{1-\gamma}\tilde{\phi}^t}$$

is decreasing in Π_0. Therefore an increase in Π_0 now exerts a downward pressure on real value of debt in the ensuing periods, which implies that a smaller increase in inflation is necessary to stabilize debt.

We now formally show the claim that Π_0 is increasing in s_0 using the implicit function theorem. Let

$$F\left(\Pi_0, s_0\right) \equiv f\left(\Pi_0\right) - g\left(\Pi_0, s_0\right) = 0$$

where

$$f\left(\Pi_0\right) = \frac{\Pi_0}{\bar{\Pi}}$$

$$g\left(\Pi_0, s_0\right) = \frac{1}{(1-\beta)\sum_{i=0}^{\infty} \beta^i \left(\frac{\Pi_0}{\bar{\Pi}}\right)^{\frac{\gamma}{1-\gamma}\left(1-\tilde{\phi}^i\right)} - \frac{\beta}{\bar{b}}\left(s_0 - \bar{s}\right)}.$$

Then the derivative is given by

$$\frac{d\Pi_0}{ds_0} = -\frac{F_s}{F_{\Pi_0}} = \frac{\overset{+}{g_{s_0}}}{\underset{+}{f_{\Pi_0}} - \underset{-}{g_{\Pi_0}}} > 0.$$

In the flexible-price limit ($\gamma = 0$), the function g does not depend on inflation. Inflation at time 0 responds more as $g_{\Pi_0} = 0$; it is given by

$$\frac{\Pi_0}{\bar{\Pi}} = \frac{1}{1 - \frac{\beta}{\bar{b}}\left(s_0 - \bar{s}\right)},$$

which coincides with the previous solution in (A.19) under perfectly elastic labor supply.

A.5.3 Comparison of the two regimes under sticky prices

The results on inflation are qualitatively similar to those obtained in the flexible-price case. The fiscal regime produces more persistent and greater inflation, compared to the monetary regime. In fact, the latter regime does not generate inflation at all.

B Quantitative model

B.1 Model setup

There are two-sectors: Ricardian and hand to mouth. Labor is immobile across these two sectors. Each sector produces a distinct good, which is in turn produced in differentiated varieties. Firms in both sectors are owned by the Ricardian household.

B.1.1 Ricardian sector

Households. There are Ricardian (R) households of measure $1 - \lambda$. The optimization problem of this type households is to

$$\max_{\{C_t^R, L_t^R, \frac{B_t^R}{P_t^R}\}} \sum_{t=0}^{\infty} \beta^t \exp(\eta_t^{\xi}) \left[\frac{\left(C_t^R\right)^{1-\sigma}}{1-\sigma} - \chi \frac{\left(L_t^R\right)^{1+\varphi}}{1+\varphi} \right]$$

subject to a standard No-ponzi-game constraint and sequence of flow budget constraints

$$C_t^R + b_t^R = R_{t-1} \frac{1}{\Pi_t^R} b_{t-1}^R + \left(1 - \tau_{L,t}^R\right) w_t^R L_t^R + \Psi_t^R,$$

where σ is the coefficient of relative risk aversion, η_t^{ξ} is a preference shock, C_t^R is consumption, L_t^R is labor supply, $b_t^R = \frac{B_t^R}{P_t^R}$ is the real value of government issued debt, Π_t^R is inflation, R_{t-1} is the nominal interest rate, w_t^R is the real wage, and Ψ_t^R is real profits (this household owns firms in both sectors). We introduce a labor tax, $(1-\tau_{L,t}^R)$, which constitutes one way in which the government finances transfers to the Hand-to-mouth household.

Note that as we make clear below, we set up the model generally so that there could be two "CPI" indices in the economy, due to different baskets. So here, we are deflating nominal variables by the "CPI" index of the Ricardian household (defined as P_t^R).

Three optimality conditions are given by the Euler equation, (distorted) labor supply condition, and TVC.

$$\left(\frac{\exp(\eta_t^{\xi})C_t^R}{\exp(\eta_{t+1}^{\xi})C_{t+1}^R}\right)^{-\sigma} = \beta\frac{R_t}{\Pi_{t+1}^R},$$

$$\chi\left(L_t^R\right)^{\varphi}\left(C_t^R\right)^{\sigma} = \left(1-\tau_{L,t}^R\right)w_t^R,$$

$$\lim_{t\to\infty}\left[\beta^t\left(C_t^R\right)^{-\sigma}\left(\frac{B_t^R}{P_t^R}\right)\right] = 0.$$

Here, C_t^R is a CES/Armington-type aggregator ($\varepsilon > 0$) of the consumption good produced in the R and HTM sectors.

$$C_t^R = \left[(\alpha_R)^{\frac{1}{\varepsilon}}\left(C_{R,t}^R\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\alpha_R)^{\frac{1}{\varepsilon}}\left(\exp(\zeta_{H,t})C_{H,t}^R\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where $C_{R,t}^R$ and $C_{H,t}^R$ are R-household's demand for R-sector and for HTM-sector goods, respectively. $\zeta_{H,t}$ is demand shocks for HTM goods This gives the following optimal price index and demand functions from a standard static expenditure minimization problem

$$P_t^R = \left[\alpha_R\left(P_{R,t}^R\right)^{1-\varepsilon} + (1-\alpha_R)\left(\frac{P_{H,t}^R}{\exp(\zeta_{H,t})}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}},$$

$$\frac{C_{R,t}^R}{C_t^R} = \alpha_R\left(\frac{P_{R,t}^R}{P_t^R}\right)^{-\varepsilon}, \frac{C_{H,t}^R}{C_t^R} = (1-\alpha_R)\left(\exp\left(\zeta_{H,t}\right)\right)^{\varepsilon-1}\left(\frac{P_{H,t}^R}{P_t^R}\right)^{-\varepsilon}.$$

Let us define for future use one of the relative prices

$$S_{R,t} \equiv \left(\frac{P_{R,t}^{R}}{P_{t}^{R}} \right).$$

Within each sector, there is monopolistic competition, as we make clear with the firm's problem. Thus, $C_{R,t}^{R}$ and $C_{H,t}^{R}$ in turn are Dixit-Stiglitz aggregators of a continuum of varieties. That is, with $\theta > 1$,

$$C_{R,t}^{R} = \left[\int_{0}^{1} \left(C_{R,t}^{R}(i) \right)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, C_{H,t}^{R} = \left[\int_{0}^{1} \left(C_{H,t}^{R}(i) \right)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

and

$$P_{R,t}^{R} = \left[\int_{0}^{1} \left(P_{R,t}^{R}(i) \right)^{1-\theta} di \right]^{1-\theta}, P_{H,t}^{R} = \left[\int_{0}^{1} \left(P_{H,t}^{R}(i) \right)^{1-\theta} di \right]^{1-\theta}$$

where

$$\frac{C_{R,t}^{R}(i)}{C_{R,t}^{R}} = \left(\frac{P_{R,t}^{R}(i)}{P_{R,t}^{R}} \right)^{-\theta}, \frac{C_{H,t}^{R}(i)}{C_{H,t}^{R}} = \left(\frac{P_{H,t}^{R}(i)}{P_{H,t}^{R}} \right)^{-\theta}.$$

There is no price discrimination across sectors for varieties, and we will impose the law of one price later.

Firms. Firms in the R-sector produce differentiated varieties using the linear production function

$$Y_{R,t}(i) = L_{t}^{R}(i)$$

and set prices according to Calvo friction. Flow (real) profts are given by

$$\Psi_{R,t}(i) = \frac{P_{R,t}^{R*}(i) Y_{R,t}(i)}{P_{t}^{R}} - w_{t}^{R} L_{t}^{R}(i)$$

Profit maximization problem of firms that get to adjust prices is given by

$$\max \sum_{s=0}^{\infty} \left(\omega^{R} \beta \right)^{s} \left(\frac{C_{t+s}^{R}}{C_{t}^{R}} \right)^{-\sigma} \left[\left(\frac{P_{R,t}^{R*}(i)}{P_{R,t+s}^{R}} \right) S_{R,t+s} - w_{t+s}^{R} \right] \left(\frac{P_{R,t}^{R*}(i)}{P_{R,t+s}^{R}} \right)^{-\theta} Y_{R,t+s}.$$

Notice that no price discrimination (with notation introduced later, $P_{RR,t}(i) = P_{HR,t}(i)$) allows us to write the demand directly in terms of $Y_{R,t}(i) = \left(\frac{P_{R,t}^{R}(i)}{P_{R,t}^{R}} \right)^{-\theta} Y_{R,t}$. Relative prices, $S_{R,t}$, show up here, because of a different price levels of the good and CPI of this sector, where we use CPI to deflate wages in the household problem. This is clear from the flow profit experssion above. Moreover, linearity of production function gives marginal cost as w_t^R.

Optimal first-order conditions is given by:

$$P_{R,t}^{R*}(i) = \left(\frac{\theta}{\theta-1}\right) \frac{\sum_{s=0}^{\infty} \left(\omega^R \beta\right)^s \left(\frac{C_{t+s}^R}{C_t^R}\right)^{-\sigma} \left[w_{t+s}^R \left(\frac{1}{P_{R,t+s}^R}\right)^{-\theta}\right] Y_{R,t+s}}{\sum_{s=0}^{\infty} \left(\omega^R \beta\right)^s \left(\frac{C_{t+s}^R}{C_t^R}\right)^{-\sigma} \left[\left(\frac{1}{P_{R,t+s}^R}\right)^{1-\theta} S_{R,t+s}\right] Y_{R,t+s}}.$$

We can rewrite this optimal condition in terms of law of motions of prices as following:

$$P_{R,t}^{R*}(i) = \left(\frac{\theta}{\theta-1}\right) \frac{Z_{1,t}^R}{Z_{2,t}^R}$$

where

$$Z_{1,t}^R = w_t^R \left(P_{R,t}^R\right)^{\theta} Y_{R,t} + \omega^R \beta \left(\frac{C_{t+1}^R}{C_t^R}\right)^{-\sigma} Z_{1,t+1}^R$$

$$Z_{2,t}^R = S_{R,t} \left(P_{R,t}^R\right)^{\theta-1} Y_{R,t} + \omega^R \beta \left(\frac{C_{t+1}^R}{C_t^R}\right)^{-\sigma} Z_{2,t+1}^R.$$

B.1.2 Non-Recardian sector

Households. There are *HTM* households of measure λ. *HTM* household does not have endogenous labor supply decision. It's labor endowment is exogenously fixed and can change with the Covid-shock. The *HTM* household then consumes, every period, wage income and government transfers

$$C_t^H = w_t^H \overline{L^H}(1+\eta_t^{\xi}) + \left(\frac{P_t^R}{P_t^H}\right) s_t^H.$$

where η_t^{ξ} is HTM labor supply shock. Note that relative price appears in transfers as for transfers/govt variables we use the Ricardian household CPI as deflator. We define the "real exchange rate" across sectors as

$$Q_t \equiv \left(\frac{P_t^H}{P_t^R}\right)$$

The utility function of the HTM then is (again, labor supply is inelastic)

$$\frac{\left(C_t^H\right)^{1-\sigma}}{1-\sigma}$$

and the aggregate consumption and the sector-specific goods are

$$C_t^H = \left[(\alpha_H)^{\frac{1}{\varepsilon}} \left(\exp\left(\zeta_{H,t}\right) C_{H,t}^H\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\alpha_H)^{\frac{1}{\varepsilon}} \left(C_{R,t}^H\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where, again, $\zeta_{H,t}$ is a demand shock for *HTM*-goods. This gives the following optimal price index and demand functions from a standard static expenditure minimization problem

$$P_t^H = \left[(\alpha_H)\left(\frac{P_{H,t}^H}{\exp\left(\zeta_{H,t}\right)}\right)^{1-\varepsilon} + (1-\alpha_H)\left(P_{R,t}^H\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

$$\frac{C_{H,t}^H}{C_t^H} = \alpha_H \left(\exp\left(\zeta_{H,t}\right)\right)^{\varepsilon-1}\left(\frac{P_{H,t}^H}{P_t^H}\right)^{-\varepsilon}, \quad \frac{C_{R,t}^H}{C_t^H} = (1-\alpha_H)\left(\frac{P_{R,t}^H}{P_t^H}\right)^{-\varepsilon}$$

Let us define for future use one of the relative prices

$$S_{H,t} \equiv \frac{P_{H,t}^H}{P_t^H}.$$

This implies that

$$Q_t S_{H,t} = \frac{P_{H,t}^H}{P_t^R}$$

which will be useful later.

Within each sector, there is monopolistic competition, as we make clear with the firm's problem. Thus, $C_{HH,t}$ and $C_{HR,t}$ in turn are Dixit-Stiglitz aggregators of a continuum of varieties. That is, with $\theta > 1$,

$$C_{H,t}^{H} = \left(\int_0^1 \left(C_{H,t}^{H}(i) \right)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \; C_{R,t}^{H} = \left(\int_0^1 \left(C_{R,t}^{H}(i) \right)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$$

$$P_{H,t}^{H} = \left(\int_0^1 \left(P_{H,t}^{H}(i) \right)^{1-\theta} di \right)^{\frac{1}{1-\theta}}, \; P_{R,t}^{H} = \left(\int_0^1 \left(P_{R,t}^{H}(i) \right)^{1-\theta} di \right)^{\frac{1}{1-\theta}}$$

$$C_{H,t}^{H}(i) = \left(\frac{P_{H,t}^{H}(i)}{P_{H,t}^{H}} \right)^{-\theta} C_{H,t}^{H}, \; C_{R,t}^{H}(i) = \left(\frac{P_{R,t}^{H}(i)}{P_{R,t}^{H}} \right)^{-\theta} C_{R,t}^{H}.$$

There is no price discrimination across sectors for varieties, and we will impose the law of one price later.

Firms. Firms in the HTM sector produce differentiated varieties using the linear production function

$$Y_{H,t}(i) = L_t^H(i)$$

and set prices according to Calvo friction. Flow (real, in terms of CPI of Ricardian household) profts are given by

$$\Psi_{H,t}(i) = \frac{P_{HH,t}^{*}(i) Y_{H,t}(i)}{P_t^R} - \frac{P_t^H}{P_t^R} w_t^H L_t^H(i)$$

Profit maximization problem of firms that get to adjust prices is given by (they are owned by R households)

$$\max \sum_{s=0}^{\infty} \left(\omega^H \beta \right)^s \left(\frac{C_{t+s}^R}{C_t^R} \right)^{-\sigma} \left[\left(\frac{P_{H,t}^{H*}(i)}{P_{H,t+s}^{H}} \right) Q_{t+s} S_{H,t+s} - Q_{t+s} w_{t+s}^H \right] \left(\frac{P_{H,t}^{H*}(i)}{P_{H,t+s}^{H}} \right)^{-\theta} Y_{H,t+s}.$$

Relative prices, $Q_t S_{H,t} = \frac{P_{H,t}^H}{P_t^R}$, show up here, because of a different price levels of the good and CPI of this sector, where we use CPI to deflate wages in the household problem. Moreover, real exchange rate also shows up as we deflate the real profits by the Ricardian household's CPI as they own the firms. This is clear from the flow profit expression above. Moreover, linearity of production function gives marginal cost as w_t^R. Firms' optimal first-order condition is given by:

$$P_{H,t}^{H*}(i) = \left(\frac{\theta}{\theta-1}\right) \frac{\sum_{s=0}^{\infty} \left(\omega^H \beta\right)^s \left(\frac{C_{t+s}^R}{C_t^R}\right)^{-\sigma} \left[Q_{t+s} w_{t+s}^H \left(\frac{1}{P_{H,t+s}^H}\right)^{-\theta}\right] Y_{H,t+s}}{\sum_{s=0}^{\infty} \left(\omega^H \beta\right)^s \left(\frac{C_{t+s}^R}{C_t^R}\right)^{-\sigma} \left[\left(\frac{1}{P_{H,t+s}^H}\right)^{1-\theta} Q_{t+s} S_{H,t+s}\right] Y_{H,t+s}}$$

We can rewrite it in terms of law of motions of prices as following:

$$P_{H,t}^{H*}(i) = \left(\frac{\theta}{\theta-1}\right) \frac{Z_{1,t}^H}{Z_{2,t}^H},$$

where

$$Z_{1,t}^H = Q_t w_t^H \left(P_{H,t}^H\right)^{\theta} Y_{H,t} + \omega^H \beta \left(\frac{C_{t+1}^R}{C_t^R}\right)^{-\sigma} Z_{1,t+1}^H$$

$$Z_{2,t}^H = Q_t S_{H,t} \left(P_{H,t}^H\right)^{\theta-1} Y_{H,t} + \omega^H \beta \left(\frac{C_{t+1}^R}{C_t^R}\right)^{-\sigma} Z_{2,t+1}^H.$$

B.1.3 Law of one price

There is no pricing to market on varieties across sectors. Thus, law of one price holds for each variety. This is implicitly already imposed while writing the price-setting problem of the firms. This means

$$P_{R,t}^R(i) = P_{R,t}^H(i), \quad P_{H,t}^H(i) = P_{H,t}^R(i)$$

and correspondingly the various sector specific prices (but not the CPI prices) are also equalized.

$$P_{R,t}^R = P_{R,t}^H, \quad P_{H,t}^H = P_{H,t}^R$$

B.1.4 Government

Government budget constraint is (deflating by CPI of the Ricardian household)

$$B_t + T_t^L = R_{t-1} B_{t-1} - P_t^R \tau_t + P_t^R s_t$$

where

$$T_t^L = (1-\lambda)\,\tau_{L,t}^R P_t^R w_t^R L_t^R + \lambda \tau_{L,t}^H P_t^H w_t^H \overline{L^H}(1+\eta_t^{\xi}).$$

Transfer, s_t, is exogenous and deterministic.

Monetary and fiscal policy rules are (we have a choice on how to define the inflation the government targets; here it is just a weighted average)

$$\begin{aligned}
\frac{R_t}{\bar{R}} &= \left(\frac{(1-\lambda)\,\Pi_t^R + (\lambda)\,\Pi_t^H}{\bar{\Pi}}\right)^{\phi}, \\
\tau_t - \bar{\tau} &= \psi(b_{t-1} - \bar{b}), \\
\tau_{L,t}^R - \bar{\tau}_L^R &= \psi_L(b_{t-1} - \bar{b})
\end{aligned}$$

where h_t is a deterministic exogenous variable. The steady state value of inflation, debt, and the exogenous variables, $\bar{\Pi}$, $\bar{b}$, $\bar{s}$ and $\bar{h}$ are policy choices and given exogenously.

We use the parameter $\omega \in [0,1]$ to measure the fraction of transfers given to the HTM households. We therefore have

$$s_t^H = \frac{\omega}{\lambda} s_t \quad \text{and} \quad s_t^R = \frac{(1-\omega)}{(1-\lambda)} s_t,$$

that is, each HTM household receives $\frac{\omega}{\lambda} s_t$.

B.1.5 Market clearing, aggregation, resource constraints

Notice that

$$s_t = (1-\lambda)\, s_t^R + \lambda s_t^H$$

$$\tau_t = (1-\lambda)\, \tau_t^R + \lambda \tau_t^H$$

$$b_t = (1-\lambda)\, b_t^R + \lambda b_t^H$$

$$L_t = (1-\lambda)\, L_t^R + \lambda \overline{L^H}(1+\eta_t^{\xi})$$

$$\Psi_t = (1-\lambda)\, \Psi_t^R + \lambda \Psi_t^H$$

In our benchmark model, $\tau_t^H = b_t^H = \Psi_t^H = 0$.

Labor market clear conditions are:

$$(1-\lambda)\, L_t^R = \int L_{R,t}\,(i)\, di, \;\; \lambda \overline{L^H}(1+\eta_t^{\xi}) = \int L_{H,t}\,(i)\, di$$

To derive an aggregate resource constraint, we combine households' budget constraints and government budget constraint:

$$(1-\lambda) C_t^R + \lambda Q_t C_t^H = \int \left(\frac{P_{H,t}(i)}{P_t^R} Y_{H,t}(i) + \frac{P_{R,t}(i)}{P_t^R} Y_{R,t}(i) \right) di.$$

Define an aggregate consumption, C_t, as

$$C_t = (1-\lambda) C_t^R + \lambda Q_t C_t^H = \int \frac{P_{R,t}(i)}{P_t^R} Y_{R,t}(i)\, di + \int \frac{P_{H,t}(i)}{P_t^R} Y_{H,t}(i)\, di$$

Note that from the law of one price,

$$\begin{aligned} Y_{R,t}(i) &= (1-\lambda) C_{R,t}^R(i) + \lambda C_{R,t}^H(i) = \left(\frac{P_{R,t}(i)}{P_{R,t}} \right)^{-\theta} Y_{R,t} \\ Y_{H,t}(i) &= (1-\lambda) C_{H,t}^R(i) + \lambda C_{H,t}^H(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\theta} Y_{H,t}. \end{aligned}$$

where

$$\begin{aligned} Y_{R,t} &= (1-\lambda) C_{R,t}^R + \lambda C_{R,t}^H \\ Y_{H,t} &= (1-\lambda) C_{H,t}^R + \lambda C_{H,t}^H \end{aligned}$$

Then,

$$\begin{aligned} C_t &= \int \frac{P_{R,t}(i)}{P_t^R} Y_{R,t}(i)\, di + \int \frac{P_{H,t}(i)}{P_t^R} Y_{H,t}(i)\, di \\ &= \frac{P_{R,t}}{P_t^R} \int \left(\frac{P_{R,t}(i)}{P_{R,t}} \right)^{1-\theta} Y_{R,t} di + \left(\exp\left(\zeta_{H,t}\right)\right)^{\theta-1} \frac{P_{H,t}}{P_t^R} \int \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{1-\theta} Y_{H,t} di \\ &= S_{R,t} Y_{R,t} + S_{H,t} Q_t Y_{H,t} \end{aligned}$$

To derive an aggregate sectoral output, we aggregate firms' product function:

$$\begin{aligned} \int L_t^R(i)\, di &= Y_{R,t} \int \left(\frac{P_{R,t}(i)}{P_{R,t}} \right)^{-\theta} di \\ \int L_t^H(i)\, di &= Y_{H,t} \int \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\theta} di \end{aligned}$$

Each sectoral market clears:

$$(1-\lambda)\,L_t^R = Y_{R,t}\Xi_{R,t}, \;\; \lambda\overline{L^H}(1+\eta_t^{\xi}) = Y_{H,t}\Xi_{H,t}$$

where $\Xi_{R,t}$ and $\Xi_{H,t}$ are price dispersion terms which are given by:

$$\Xi_{R,t} = \left(1-\omega^R\right)\left(\frac{P_{R,t}^*}{P_{R,t}}\right)^{-\theta} + \omega^R\left(\pi_{R,t}\right)^{\theta}\Xi_{R,t-1}$$

$$\Xi_{H,t} = \left(1-\omega^H\right)\left(\frac{P_{H,t}^*}{P_{H,t}}\right)^{-\theta} + \omega^H\left(\pi_{H,t}\right)^{\theta}\Xi_{H,t-1}$$

Lastly, we derive law of motions of each sector's inflation:

$$\left(P_{H,t}\right)^{1-\theta} = \left(\int_0^1 \left(P_{H,t}\left(i\right)\right)^{1-\theta} di\right)$$

$$\left(\pi_{H,t}\right)^{1-\theta} = \left(1-\omega^H\right)\left(\frac{P_{H,t}^*}{P_{H,t}}\right)^{1-\theta}\left(\pi_{H,t}\right)^{1-\theta} + \omega^H$$

$$\left(\pi_{R,t}\right)^{1-\theta} = \left(1-\omega^R\right)\left(\frac{P_{R,t}^*}{P_{R,t}}\right)^{1-\theta}\left(\pi_{R,t}\right)^{1-\theta} + \omega^R$$

B.2 System of equilibrium conditions

- Ricardian HH - Intertemporal EE

$$\exp\left(\eta_t^{\xi}\right)\left(C_t^R\right)^{-\sigma} = \beta\frac{R_t}{\pi_{t+1}^R}\exp\left(\eta_{t+1}^{\xi}\right)\left(C_{t+1}^R\right)^{-\sigma} \tag{B.1}$$

- Ricardian HH - Intra-temporal EE

$$\chi\left(L_t^R\right)^{\varphi}\left(C_t^R\right)^{\sigma} = \left(1-\tau_{L,t}^R\right)w_t^R \tag{B.2}$$

- Ricardian HH - Phillips curve 1

$$\frac{P_{R,t}^*}{P_{R,t}} = \left(\frac{\theta}{\theta-1}\right)\frac{\tilde{Z}_{1,t}^R}{\tilde{Z}_{2,t}^R} \tag{B.3}$$

- Ricardian HH - Phillips curve 2

$$\tilde{Z}^R_{1,t} = w^R_t Y_{R,t} + \omega^R \beta \left(\frac{C^R_{t+1}}{C^R_t} \right)^{-\sigma} \tilde{Z}^R_{1,t+1} \left(\pi_{R,t+1} \right)^{\theta} \tag{B.4}$$

- Ricardian HH - Phillips curve 3

$$\tilde{Z}^R_{2,t} = S_{R,t} Y_{R,t} + \omega^R \beta \left(\frac{C^R_{t+1}}{C^R_t} \right)^{-\sigma} \tilde{Z}^R_{2,t+1} \left(\pi_{R,t+1} \right)^{\theta-1} \tag{B.5}$$

- HTM HH - Budget constraint

$$C^H_t = \left(1 - \tau^H_{L,t} \right) w^H_t L^H_t + \left(\frac{1}{Q_t} \right) s^H_t \tag{B.6}$$

- HTM HH - Phillips curve 1

$$\frac{P^*_{H,t}}{P_{H,t}} = \left(\frac{\theta}{\theta - 1} \right) \frac{\tilde{Z}^H_{1,t}}{\tilde{Z}^H_{2,t}} \tag{B.7}$$

- HTM HH - Phillips curve 2

$$\tilde{Z}^H_{1,t} = Q_t w^H_t Y_{H,t} + \omega^H \beta \left(\frac{C^R_{t+1}}{C^R_t} \right)^{-\sigma} \tilde{Z}^H_{1,t+1} \left(\pi_{H,t+1} \right)^{\theta} \tag{B.8}$$

- HTM HH - Phillips curve 3

$$\tilde{Z}^H_{2,t} = Q_t S_{H,t} Y_{H,t} + \omega^H \beta \left(\frac{C^R_{t+1}}{C^R_t} \right)^{-\sigma} \tilde{Z}^H_{2,t+1} \left(\pi_{H,t+1} \right)^{\theta-1} \tag{B.9}$$

- Output R sector

$$Y_{R,t} = (1 - \lambda) C^R_{R,t} + \lambda C^H_{R,t} \tag{B.10}$$

- Output H sector

$$Y_{H,t} = (1 - \lambda) C^R_{H,t} + \lambda C^H_{H,t} \tag{B.11}$$

- Consumption 1

$$C_{R,t}^{R} = \alpha_R \left(S_{R,t}\right)^{-\varepsilon} C_t^R \tag{B.12}$$

- Consumption 2

$$C_{H,t}^{R} = \left(\exp\left(\zeta_{H,t}\right)\right)^{\varepsilon-1} \left(1-\alpha_R\right) \left(S_{H,t} Q_t\right)^{-\varepsilon} C_t^R \tag{B.13}$$

- Consumption 3

$$C_{H,t}^{H} = \left(\exp\left(\zeta_{H,t}\right)\right)^{\varepsilon-1} \alpha_H \left(S_{H,t}\right)^{-\varepsilon} C_t^H \tag{B.14}$$

- Consumption 4

$$C_{R,t}^{H} = \left(1-\alpha_H\right) \left(S_{R,t} \frac{1}{Q_t}\right)^{-\varepsilon} C_t^H \tag{B.15}$$

- Resource constraint

$$C_t = S_{R,t} Y_{R,t} + Q_t S_{H,t} Y_{H,t} \tag{B.16}$$

- Aggregate output 1

$$\left(1-\lambda\right) L_t^R = Y_{R,t} \Xi_{R,t} \tag{B.17}$$

- Price dispersion 1

$$\Xi_{R,t} = \left(1-\omega^R\right) \left(\frac{P_{R,t}^*}{P_{R,t}}\right)^{-\theta} + \omega^R \left(\pi_{R,t}\right)^{\theta} \Xi_{R,t-1} \tag{B.18}$$

- Aggregate output 2

$$\lambda L_t^H = Y_{H,t} \Xi_{H,t} \tag{B.19}$$

- Price dispersion 2

$$\Xi_{H,t} = \left(1-\omega^H\right) \left(\frac{P_{H,t}^*}{P_{H,t}}\right)^{-\theta} + \omega^H \left(\pi_{H,t}\right)^{\theta} \Xi_{H,t-1} \tag{B.20}$$

- Aggregate price index 1

$$(\pi_{R,t})^{1-\theta} = \left(1-\omega^R\right)\left(\frac{P^*_{R,t}}{P_{R,t}}\right)^{1-\theta}(\pi_{R,t})^{1-\theta} + \omega^R \tag{B.21}$$

- Aggregate price index 2

$$(\pi_{H,t})^{1-\theta} = \left(1-\omega^H\right)\left(\frac{P^*_{H,t}}{P_{H,t}}\right)^{1-\theta}(\pi_{H,t})^{1-\theta} + \omega^H \tag{B.22}$$

- GBC

$$b_t + T^L_t = R_{t-1}\frac{b_{t-1}}{\pi^R_t} - \tau_t + s_t \tag{B.23}$$

- Labor income tax

$$T^L_t = \lambda \tau^H_{L,t} Q_t w^H_t L^H_t + (1-\lambda)\,\tau^R_{L,t} w^R_t L^R_t \tag{B.24}$$

- Transfer

$$s_t : \text{ exogenous} \tag{B.25}$$

- MP rule

$$\frac{R_t}{\bar{R}} = \left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi} \tag{B.26}$$

where $\pi_t = (1-\lambda)\,\pi^R_t + \lambda\pi^H_t$, $\bar{\pi} = (1-\lambda)\,\bar{\pi}^R + \lambda\bar{\pi}^H$.

- Relative prices relationship

$$1 = \left(\alpha_R - \left(\frac{1-\alpha_R}{\alpha_H}\right)(1-\alpha_H)\right)(S_{R,t})^{1-\varepsilon} + \left(\frac{1-\alpha_R}{\alpha_H}\right)(Q_t)^{1-\varepsilon} \tag{B.27}$$

$$S_{H,t} = \exp\left(\zeta_{H,t}\right)\left(\frac{1-\alpha_R\,(S_{R,t})^{1-\varepsilon}}{1-\alpha_R}\left(\frac{1}{Q_t}\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} \tag{B.28}$$

If symmetry: $(1-\alpha_R = \alpha_H)$, then

$$Q_t = 1$$

$$S_{H,t} = \exp\left(\zeta_{H,t}\right)\left(\frac{1-\alpha_R\,(S_{R,t})^{1-\varepsilon}}{(1-\alpha_R)}\right)^{\frac{1}{1-\varepsilon}}$$

- Inflation relationship

$$\pi_t^H = \frac{Q_t}{Q_{t-1}} \pi_t^R \tag{B.29}$$

$$\left(\pi_t^R\right)^{1-\varepsilon} = \frac{\left(\pi_{R,t}\pi_{H,t}\right)^{1-\varepsilon}}{\alpha_R \left(S_{R,t}\right)^{1-\varepsilon} \left(\pi_{H,t}\right)^{1-\varepsilon} + \left(1 - \alpha_R \left(S_{R,t}\right)^{1-\varepsilon}\right) \left(\pi_{R,t}\right)^{1-\varepsilon}} \tag{B.30}$$

$$\left(\pi_t^H\right)^{1-\varepsilon} = \frac{\left(\pi_{R,t}\pi_{H,t}\right)^{1-\varepsilon}}{\alpha_H \left(S_{H,t}\right)^{1-\varepsilon} \left(\pi_{R,t}\right)^{1-\varepsilon} + \left(1 - \alpha_H \left(S_{H,t}\right)^{1-\varepsilon}\right) \left(\pi_{H,t}\right)^{1-\varepsilon}} \tag{B.31}$$

- Tax rules

$$\tau_t - \bar{\tau} = \psi \left(b_{t-1} - \bar{b}\right) \tag{B.32}$$

$$\tau_{L,t}^R - \bar{\tau}_L^R = \psi_L \left(b_{t-1} - \bar{b}\right) \tag{B.33}$$

- Transfer sharing rule:

$$s_t^H = \frac{\xi}{\lambda} s_t \tag{B.34}$$

$$s_t^R = \frac{1-\xi}{1-\lambda} s_t \tag{B.35}$$

- HTM labor supply shock

$$L_t^H = \bar{L}^H (1 + \eta_t^H) \tag{B.36}$$