Dynamic Inattention, the Phillips Curve and Forward Guidance

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Motivation: Forward Guidance and Inflation Dynamics

Monetary policy presumably plays a key role in shaping [inflation] expectations ... by providing guidance about the FOMC's objectives for inflation in the future. Even so, economists' understanding of exactly how and why inflation expectations change over time is limited.

Janet Yellen (Sept. 2017)

- Objective of forward guidance:
  - affect the economy today through news about future policy.
- Two natural questions:
  - Do price setters pay attention to the news about future policy?
    - Yes
  - If so, do their prices respond to such news?
    - Yes
- More generally, how are price setters' expectations formed and how do they affect inflation dynamics?

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Overview

- Two types of Phillips curves for inflation dynamics:
  - Sticky/Noisy information Phillips curves:

$$\pi_t = \tilde{\mathbb{E}}_{t-1}[\pi_t + \alpha \Delta y_t] + \alpha \frac{\lambda}{1-\lambda} y_t,$$

    - ⋆ criticized for not being forward looking.
  - Sticky price models:

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \gamma y_t,$$

    - ⋆ ciricized for forward guidance puzzle.
- We micro-found a new Phillips curve that
  - captures the effects of expectations about future on inflation,
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Environment

- A measure of firms $i \in [0, 1]$.
- Firms' flow nominal profit depends on their own price, aggregate price and output:

$$\Pi_{i,t} = \Pi(P_{i,t}, P_t, Y_t)$$
$$\approx -(p_{i,t} - mc_t)^2 + \text{terms independent of } p_{i,t}$$

where small letters are log-deviations from steady state and

$$mc_t = p_t + \alpha y_t.$$

- Here:
  - y_t is output gap.
  - p_t is the aggregate price: $p_t = \int_0^1 p_{i,t} di$.
- Firms are rationally inattentive.

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Firms' Problem Given an Information Structure

- For any t, let S_i^t be i's information set at time t.
- i's pricing problem is

$$L_0^i \equiv \min_{p_{i,t}:S_i^t \to \mathbb{R}} \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t (p_{i,t} - mc_t)^2 | S_i^0\right].$$

- Solution:

$$p_{i,t}(S_i^t) = \mathbb{E}[mc_t|S_i^t]$$

and

$$L_0^i = \sum_{t=0}^{\infty} \beta^t var(mc_t|S_i^t).$$

- Kalman filtering:

$$\Delta p_{i,t} = \mathbb{E}[\Delta mc_t|S_i^{t-1}] + k_t^i(s_{i,t} - \mathbb{E}[s_{i,t}|S_i^{t-1}])$$

- Need to characterize what kind of signals firms choose to see.

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Endogenous Information Acquisition

- Firm i wakes up at time t with S_i^{t-1}.
- Chooses $S_{i,t} \subset \mathcal{S}_t$ subject to cost $C(S_{i,t}|S_i^{t-1})$ and forms $S_i^t = S_i^{t-1} \cup S_{i,t}$.
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$$L_t(S_i^{t-1}) = \min_{S_{i,t} \subset \mathcal{S}_t} \{ \underbrace{var(mc_t|S_i^t)}_{\text{gain from information}} + \underbrace{C(S_{i,t}|S_i^{t-1})}_{\text{cost of information}} + \beta L_{t+1}(S_i^t)\}$$

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- Today's signals end up in tomorrow's information set:
  - information has continuation value.

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- A forward looking firm cares about mc_t, mc_{t+1}, mc_{t+2}, ...
- These are subject to shocks that might not have been realized at time t.
- So if $\mathbb{E}_t^f[.]$ captures availability of infromation at t, firms can learn about

$$mc_t,\ \mathbb{E}_t^f[mc_{t+1}],\ \mathbb{E}_t^f[mc_{t+2}],\ \ldots$$

- It is optimal for firm to consider signals of the following form at time t:

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Properties of the Cost of Information

- Data Processing Inequality (DPI) in information theory:
  - for $\{s_1, s_2\} \subset \mathcal{S}_t$, seeing a combination of them is less costly than seeing both

$$C(as_1 + bs_2 | S^{t-1}) \leq C(s_1, s_2 | S^{t-1})$$

Proposition

Every firm observes only one signal at any time.

- Intuition:
  - Price is a linear combination of signals.
  - So instead of seeing signals separately and paying a high cost, the firm would like to see the combination.

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Properties of the Cost of Information

- The marginal cost of learning more about any $\mathbb{E}_t^f[mc_{t+\tau}]$ is increasing.

Proposition

Optimal signals are forward looking $(b_{\tau>0} \neq 0)$

$$s_{i,t} = \sum_{j=0}^{\infty} \beta^j b_j \mathbb{E}_t^f[mc_{t+j}] + \sigma_s^i e_{i,t}$$

- The agent is forward looking and wants to know about

$$mc_t, \mathbb{E}_t^f[mc_{t+1}], \mathbb{E}_t^f[mc_{t+2}], \ldots$$

Marginal benefit is decreasing with horizon while marginal cost is increasing with precision $\Rightarrow$ Information smoothing.

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Results: the Phillips Curve

- Recall, $mc_t = p_t + \alpha y_t$.
- In sticky/noisy information models:

$$\pi_t = \tilde{\mathbb{E}}_{t-1}[\pi_t + \alpha \Delta y_t] + \alpha \frac{\lambda}{1-\lambda} y_t$$

  - sticky information: λ is the fraction that update their information.
  - noisy information: λ is the Kalman gain.

- Under dynamic inattention:

$$\pi_t = \tilde{\mathbb{E}}_{t-1}[\pi_t + \alpha \Delta y_t] + \alpha \delta_0 y_t - \sum_{\tau=1}^{\infty} \beta^\tau \delta_\tau \tilde{\mathbb{FE}}_t[\pi_{t+\tau} + \alpha \Delta y_{t+\tau}]$$

where $\tilde{\mathbb{FE}}[x] \equiv \tilde{\mathbb{E}}_t[x] - \mathbb{E}^f_t[x]$ is the forecast error of firms relative to a fully informed agent.

Results: the Phillips Curve

- Recall, $mc_t = p_t + \alpha y_t$.
- In sticky/noisy information models:

$$\pi_t = \tilde{\mathbb{E}}_{t-1}[\pi_t + \alpha \Delta y_t] + \alpha \frac{\lambda}{1-\lambda} y_t$$

  - sticky information: λ is the fraction that update their information.
  - noisy information: λ is the Kalman gain.

- Under dynamic inattention:

$$\begin{aligned}\pi_t = &\tilde{\mathbb{E}}_{t-1}[\pi_t + \alpha \Delta y_t] + \alpha \delta_0 y_t \\ &- \sum_{\tau=1}^{\infty} \beta^\tau \delta_\tau \tilde{\mathbb{FE}}_t[\pi_{t+\tau} + \alpha \Delta y_{t+\tau}]\end{aligned}$$

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$$\pi_t = \tilde{\mathbb{E}}_{t-1}[\pi_t + \alpha\Delta y_t] + \alpha\delta_0 y_t$$
$$- \sum_{\tau=1}^{\infty} \beta^\tau \delta_\tau \mathbb{F}\tilde{\mathbb{E}}_t[\pi_{t+\tau} + \alpha\Delta y_{t+\tau}]$$

Remarks:

1. This imbeds the noisy information Phillips curve when $\beta = 0$.
2. Inflation is affected by expectations about future, but in a different way than sticky price models:
    1. $\pi_t = \beta\mathbb{E}_t[\pi_{t+1}] + \kappa y_t$: inflation is increasing in expected inflation.
    2. dynamic inattention: inflation is decreasing in forecast errors.

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Results: Estimating the Phillips curve

- Estimate the Phillips curve using GMM

$$
\begin{aligned}
\pi_t = \tilde{\mathbb{E}}_{t-1}[\pi_t + \alpha \Delta y_t] + \alpha \delta_0 y_t \\
- \sum_{\tau=1}^{T} \beta^\tau \delta_\tau \mathbb{F}\tilde{\mathbb{E}}_t[\pi_{t+\tau} + \alpha \Delta y_{t+\tau}] + \xi_t
\end{aligned}
$$

1. Use Survey of Professional Forecasters as proxy for firms' forecasts.
2. Instrument forecast revisions for forecast errors. (Coibion Gorodnichenko (2015))
3. Null hypothesis: $\delta_\tau \neq 0$ for $\tau > 0$.

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Results: Estimating the Phillips curve

	π_t	
	(1)	(2)
	GDP Deflator (72Q1-16Q4)	CPI (81Q3-16Q4)
$\tilde{\mathbb{E}}_{t-1}[\pi_t + \alpha \Delta y_t]$	1.00 ***	1.01 ***
	(0.01)	(0.14)
αy_t	1.28 **	0.67 ***
	(0.50)	(0.10)
$\tilde{\mathbb{FE}}_t[\pi_{t+1} + \alpha \Delta y_{t+1}]$	0.42 ***	0.16 ***
	(0.05)	(0.03)
$\tilde{\mathbb{FE}}_t[\pi_{t+2} + \alpha \Delta y_{t+2}]$	0.21 ***	-0.31 ***
	(0.05)	(0.03)
$\tilde{\mathbb{FE}}_t[\pi_{t+3} + \alpha \Delta y_{t+3}]$	0.11 ***	-0.17 ***
	(0.02)	(0.04)

Newey-West robust standard errors in parentheses

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Example: One Period Ahead News

- Suppose $\mathsf{mc_t} = \mathsf{mc_{t-1}} + \mathsf{u_{t-1}}$
- Shocks are announced one period ahead.
- How much do agents pay attention to this news and react?
- Under myopic inattention$(\beta = 0)$:

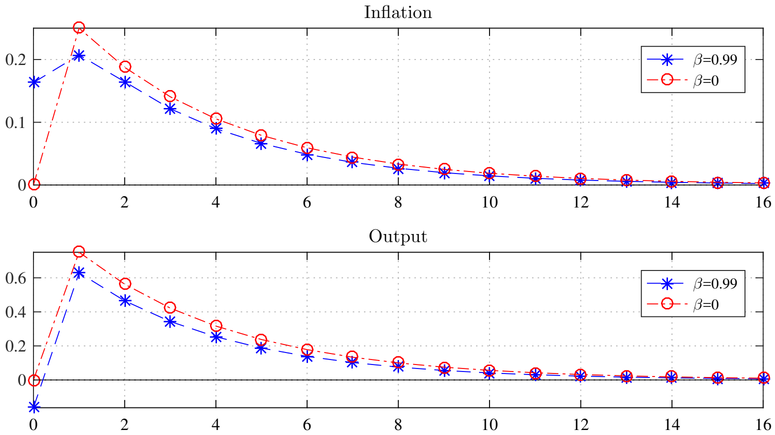
$$\mathsf{s_{i,t}} = \mathsf{mc_t} + \mathsf{e_{i,t}}$$

- Notice that in this case $\mathsf{u_t} \perp \mathsf{S_i^t}$: myopic firms completely ignore news about future.
- Under dynamic inattention$(\beta > 0)$:

$$\mathsf{s_{i,t}} = \mathsf{mc_t} + \gamma \mathsf{mc_{t+1}} + \mathsf{e_{i,t}}$$

Example: One Period Ahead News

- Under dynamic inattention inflation responds to the news shock.
- Output falls on impact because marginal cost is fixed by assumption, which is relaxed in GE.



Example: General Equilibrium

A Three Equation Model

- Dynamic Phillips curve:

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- Dynamic IS curve:

$$y_t = \mathbb{E}_t^f[y_{t+1}] - \sigma^{-1}(i_t - \mathbb{E}_t^f[\pi_{t+1}])$$

- Taylor rule:

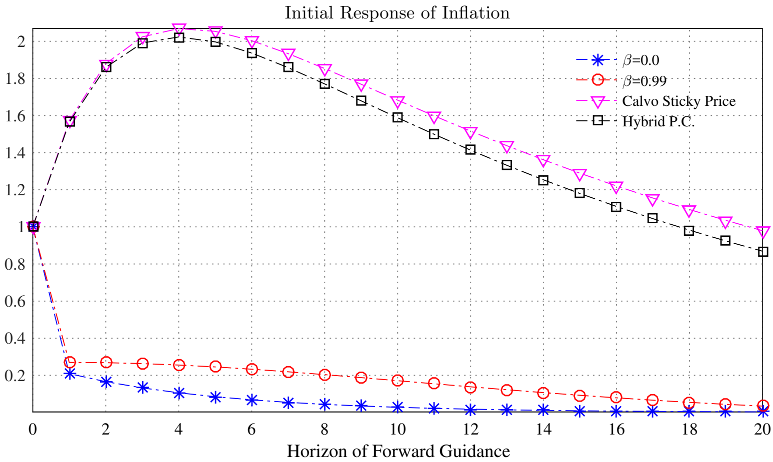
$$i_t = \rho i_{t-1} + (1-\rho)\left(\phi_\pi \pi_t + \phi_y y_t\right) + u_{t-k}$$

where k is the horizon of forward guidance.

Example: General Equilibrium

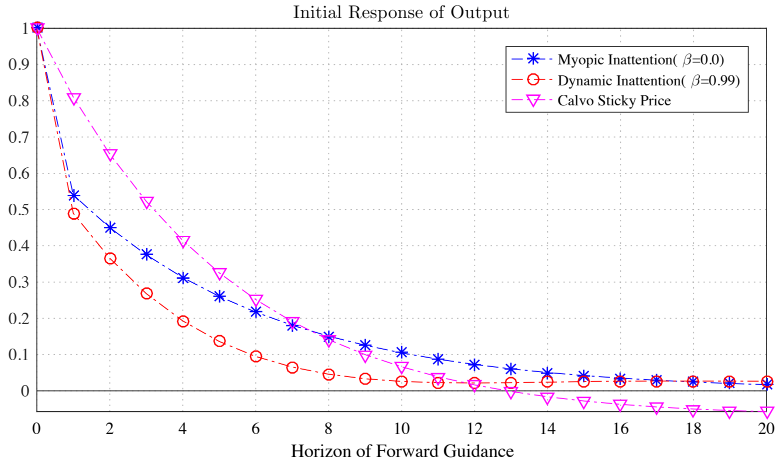
Forward Guidance Puzzle

- Impact response of inflation is decreasing in horizon of forward guidance



Example: General Equilibrium

4-period ahead Forward Guidance Shock



Conclusion

- Showed that firms have information smoothing incentives:
  - they pay attention to news about future,
  - and incorporate such news in their current prices.
- Derived and estimated a new micro founded Phillips curve:
  - inflation is forward looking in contrast to other models of information rigidity.
  - no forward guidance puzzle despite inflation being forward looking.

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4-period ahead Forward Guidance Shock