Macroeconomic Effects of Capital Tax Rate Changes*

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Abstract

We study aggregate and distributional effects of a permanent reduction in the capital tax rate in a dynamic equilibrium model with capital-skill complementarity. Such a tax reform leads to expansionary long-run aggregate effects, but is coupled with an increase in the skill premium. Moreover, the expansionary long-run aggregate effects are smaller when distortionary labor or consumption tax rates have to increase to finance the capital tax rate cut. An extension to a model with heterogeneous households shows that consumption inequality increases in the long-run. We study transition dynamics and show that short-run effects depend critically on the monetary policy response: whether the central bank allows inflation to directly facilitate government debt stabilization and how inertially it raises interest rates. We provide both analytical and numerical results with an extensive sensitivity analysis.

JEL Classification: E62, E63, E52, E58, E31

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1 Introduction

The macroeconomic effects of permanent capital tax cuts have recently become a subject of widespread discussion, spurred by the recent U.S. tax reform that reduced the corporate tax rate from 35% to 21%. Several questions have been raised. What are the long-run and the short-run effects on output, investment, and consumption? What are the distributional consequences in terms of wage, income, and consumption inequality? Will such a large tax cut be self-financing? How does the monetary policy response matter for the short-run effects of a capital tax cut? Given the nature of these questions, it is useful to pursue an analysis through the lens of a quantitative dynamic model. Moreover, since the tax reform is large-scale, it is imperative to consider general equilibrium effects.

This paper addresses these questions using a quantitative dynamic equilibrium model that features capital-skill complementarity. We show analytically in a simplified model and numerically in the quantitative model that capital tax cuts, as expected, have expansionary long-run aggregate effects on the economy. In particular, with a permanent reduction of the capital tax rate from 35% to 21%, in our baseline calibration, output in the new steady state, compared to the initial steady state, is greater by 3.8%, structure investment by 19.7%, equipment investment by 6.8%, and consumption by 2.1%. Moreover, skilled wages increase by 3.4%, unskilled wages by 2.7%, and both skilled and unskilled hours increase.

The mechanism for aggregate effects is well understood. A reduction in the capital tax rate leads to a decrease in the rental rate of capital, raising demand for capital by firms. This stimulates investment and capital accumulation. A larger amount of capital stock, in turn, makes workers more productive, raising wages and hours. Finally, given the increase in the factors of production, output increases, which also increases consumption in the long-run.

These estimates above are obtained in a scenario in which the government has the ability to finance the capital tax cuts in a completely non-distorting way by cutting back lump-sum transfers. When the government has to rely on distortionary labor or consumption taxes, the effectiveness of capital tax cuts is smaller. We show this result again both analytically in a simplified model and numerically in the quantitative model. For instance, in our baseline calibration, a permanent reduction of the capital tax rate from 35% to 21% requires an increase in the labor tax rate by 2.7% points.[1] Then, output in the new steady state, compared to the initial steady state, is greater by 1.96%, equipment investment by 4.96%, structures investment by 17.61%, and consumption by 0.31%. The reason for the smaller boost in aggregate variables is a decline of the after-tax wages, by 0.24% for skilled workers and 0.90% for unskilled workers. This in turn leads to a decrease in labor hours in the long-run, for both skilled and unskilled workers.

Our baseline quantitative macroeconomic model features skill heterogeneity and equipment capital-skill complementarity, which generate important distributional implications. As mentioned above, skilled wages rise relatively more, which leads to a rise in the skill premium of 1.06% points under lump-sum transfer adjustment. This long-run rise in wage inequality is driven by the rise in

[1] We keep debt-GDP ratio the same between the initial and the new steady-state. Debt-GDP ratio, however, is allowed to deviate from the steady-state level along the transition path, when we study short-run effects.

equipment capital, which raises skilled wages as there is equipment capital-skill complementarity. In addition, a measure of income inequality, the ratio of after-tax capital income to labor income, increases. Furthermore, in an extended model with heterogeneous households, consumption inequality also increases in the long-run. In fact, unskilled consumption decreases in the long-run as a result of a decrease in transfers in this extended model.

In addition to the aggregate expansion getting muted with distortionary tax increases and long-run increase in some inequality measures, another caveat comes about from analyzing transition dynamics in the model as the economy evolves from the initial steady-state to the new steady-state. During the transition, the economy experiences a decline in not only consumption, as a result of the need for financing greater capital accumulation, but also output. This holds even though lump-sum transfers finance the capital tax rate cut.[2]

The most important aspect of transition dynamics that we highlight is on the need to analyze monetary and fiscal policy adjustments jointly. This is because the short-run effects depend critically on the monetary policy response: whether the central bank allows inflation to directly facilitate government debt stabilization and how inertially it raises interest rates. In the situation where the government only has access to distortionary labor taxes, we consider the central bank directly allowing inflation to facilitate government debt stabilization along the transition. In this interesting scenario, the rise of inflation in the short-run completely negates any short-run contraction in output. Similar results hold when the central bank raises interest rates in an inertial way, even with lump-sum transfer adjustment.

Our paper is related to several strands to the literature. We focus entirely on a positive analysis, assessing the macroeconomic effects of a given reduction in the capital tax rate, but it is related to classic normative analysis of the optimal capital tax rate in Chamley (1986) and Judd (1985). Additionally, our analysis of the central bank allowing inflation to directly facilitate debt stabilization when the government has access to only distortionary labor taxes is related to the normative analysis in Sims (2001). We implement this scenario using a rules-based positive description of interest rate policy, as in Leeper (1991), Sims (1994), and Woodford (1994) for instance.[3]

In terms of analyzing the long-run effects of changes in the capital tax rate in an equilibrium macroeconomic model, our paper is closest to Trabandt and Uhlig (2011) and the more recent work of Barro and Furman (2018) that analyzes the U.S. tax reform. Compared to this literature, our model features capital-skill complementarity, following Krusell *et al.* (2000), such that wage inequality issues can be analyzed. We also show in detail, both analytically and numerically, how the effects are different depending on whether non-distortionary or distortionary sources of government financing are available. Finally, we study transition dynamics as well, highlighting that it is imperative to model monetary and fiscal policy adjustments jointly for determining short-run

[2]The short-run fall in output is a result of costly price and investment adjustment. This fall is stronger under distortionary labor tax rate adjustment, as is natural. While studying transition dynamics under distortionary tax rate adjustment, we model a very smooth change in the tax rate.

[3]In this case, the central bank does not follow the Taylor principle. Bhattarai, Lee, and Park (2016) analytically characterize the effects of such a case in a model with sticky prices.

effects.

There is by now a fairly large literature in the dynamic stochastic general equilibrium modeling tradition that assesses the effects of changes in distortionary tax rate changes and of fiscal policy generally. For instance, among others, Forni, Monteforte, and Sessa (2009) study transmission of various fiscal policy, including government spending and transfer changes in a quantitative model. Sims and Wolff (2017) additionally study state-dependent effects of tax rate changes. These papers often study effects of transitory and small changes in the tax rate while our main focus is on the long-run effects of a permanent reduction in the capital tax rate, and then on an analysis of full (nonlinear) transition dynamics following a fairly large reduction. Additionally, we provide several analytical results that help illustrate the key mechanisms on the long-run effects, while in the quantitative part, we use a model that can assess distributional consequences. Finally, our work is also motivated by the study of effects of government spending and how that depends on the monetary policy response, as highlighted recently by Christiano, Eichenbaum, and Rebelo (2011) and Woodford (2011).

While we are motivated by the particular recent U.S. episode of a permanent tax rate change, generally, our paper is influenced also by a large literature that empirically assesses the macroeconomic effects of tax policy. In particular, various identification strategies, such as narrative (Romer and Romer (2010)) and statistical (Blanchard and Perrotti (2002), Mountford and Uhlig (2009)) have been used to assess equilibrium effects of tax changes. Relatedly, House and Shapiro (2008) study a particular case of change in investment tax incentive. The effects on aggregate variables that we find using a calibrated equilibrium model is consistent with this work, although these papers have generally focused either explicitly on temporary tax policies or do not explicitly separate out permanent changes from transitory ones. We also use our model to assess several distributional effects following a permanent capital tax rate cut.

2 Model

We now present the baseline model, which is a standard neoclassical equilibrium framework augmented with two types of workers (skilled and unskilled) and two types of capital (structures and equipment). We introduce equipment capital-skill complementarity following Krusell *et al.* (2000), and a skill premium arises endogenously in the model. This framework allows us to study both aggregate and wage inequality implications of a capital tax rate change in a unified way. The model also features adjustment costs, in investment and nominal pricing, to enable a realistic study of transition dynamics. Pricing frictions also enable an analysis of the role of monetary policy, and in particular interest rate policy, for the transition dynamics.[4]

2.1 Private Sector

We start by describing the maximization problems of the private sector.

[4]In an extension, heterogeneous households are introduced to study consumption inequality.

2.1.1 Households

There are two types of households who supply skilled labor (type s) and unskilled labor (type u), respectively. The measure of type-i household for $i \in \{s, u\}$ is denoted by N^i. The type-i household's problem is to

$$\max_{\left\{C_t^i, H_t^i, B_t^i, I_{b,t}^i, I_{e,t}^i, K_{b,t+1}^i, K_{e,t+1}^i, V_{t+1}^i\right\}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U\left(C_t^i, H_t^i\right) \right\}$$

subject to the flow budget constraint

$$\begin{aligned}
\left(1+\tau_t^C\right) P_t C_t^i + P_t I_{b,t}^i + P_t I_{e,t}^i + B_t^i + E_t Q_{t,t+1} V_{t+1}^i = & \left(1-\lambda_{\tau^H}^i \tau_t^H\right) W_t^i H_t^i + R_{t-1} B_{t-1}^i + V_t^i \\
& + \left(1-\tau_t^K\right) R_t^{K,b} K_{b,t}^i + \left(1-\tau_t^K\right) R_t^{K,e} K_{e,t}^i \\
& + \lambda_b \tau_t^K P_t I_{b,t}^i + \lambda_e \tau_t^K P_t I_{e,t}^i \\
& + P_t \frac{\chi_\Phi^i}{N^i} \Phi_t + P_t \frac{\chi_S^i}{N^i} S_t,
\end{aligned}$$

where E_t is the mathematical expectation operator, C_t^i is consumption, H_t^i is hours, and $I_{b,t}^i$ and $I_{e,t}^i$ are investment in the capital stock of structures and equipment denoted by $K_{b,t}^i$ and $K_{e,t}^i$, respectively. Households trade one-period state-contingent nominal securities V_{t+1}^i at price $Q_{t,t+1}$ in period t so as to fully insure against idiosyncratic risks. Thus, there is complete consumption insurance in the model. They trade nominal risk-less one-period government bonds B_t^i as well. Type-i households are paid a fraction χ_Φ^i of the aggregate profits Φ_t from the firms and a fraction χ_S^i of the aggregate lump-sum transfers S_t from the government.[5] The aggregate price level is P_t, W_t^i is the nominal wage for type-i households, R_t is the nominal one-period interest rate, and $R_t^{K,b}$ and $R_t^{K,e}$ are the rental rate of capital structures and equipment, respectively.

The government levies taxes on consumption, labor income, and capital income with tax rates τ_t^C, τ_t^H, and τ_t^K, respectively. The parameter $\lambda_{\tau^H}^i$ is introduced to allow for differential effective labor tax rates on the two types of households and λ_b and λ_e are the rates of expensing of capital investment in structures and equipment, respectively. The discount factor is β.

The evolutions of the two types of capital stock are described by

$$\begin{aligned}
K_{b,t+1}^i &= \left(1-d_b\right) K_{b,t}^i + \left(1-\mathcal{S}\left(\frac{I_{b,t}^i}{I_{b,t-1}^i}\right)\right) I_{b,t}^i, \\
K_{e,t+1}^i &= \left(1-d_e\right) K_{e,t}^i + \left(1-\mathcal{S}\left(\frac{I_{e,t}^i}{I_{e,t-1}^i}\right)\right) I_{e,t}^i q_t,
\end{aligned}$$

[5] Due to complete markets (or equivalently, a large family who supplies both types of labor) in the baseline model, the share of profits or fraction of transfers allocated to a particular type of household does not matter regardless of how the capital tax rate cuts are financed.

where q_t is the relative price between investment in capital structures and equipment and d_b and d_e are the rates of depreciation of the capital stock invested in structures and equipment, respectively.[6]

The period utility $U(C_t, H_t)$ and investment adjustment cost $\mathcal{S}\left(\frac{I_t}{I_{t-1}}\right)$ have standard properties, which are detailed later.

2.1.2 Firms

The model has final goods firms and intermediate goods firms.

Final goods firms Perfectly competitive final goods firms produce aggregate output Y_t by combining a continuum of differentiated intermediate goods, indexed by $i \in [0,1]$, using the CES aggregator given by $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}$, where $\theta > 1$ is the elasticity of substitution between intermediate goods. The corresponding optimal price index P_t for the final good is $P_t = \left(\int_0^1 P_t(i)^{1-\theta} di\right)^{\frac{1}{1-\theta}}$, where $P_t(i)$ is the price of intermediate goods and the optimal demand for $Y_t(i)$ is

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t. \tag{1}$$

The final good is used for private and government consumption as well as investment in capital structures and equipment.

Intermediate goods firms Monopolistically competitive intermediate goods firms indexed by i produce output using a CRS production function $F(.)$

$$Y_t(i) = F\left(A_t, K_{b,t}(i), K_{e,t}(i), L_{s,t}(i), L_{u,t}(i)\right) \tag{2}$$

where A_t is an exogenous stochastic process that represents technological progress, with its gross growth rate given by $a_t \equiv \frac{A_t}{A_{t-1}} = \bar{a}$.[7] . As we describe later, we follow Krusell *et al.* (2000) in functional form assumptions on $F(.)$, which is a nested CES formulation, and parameterizations of the elasticities of substitution across factors such that it features (equipment) capital-skill complementarity. Firms rent capital and hire labor in economy wide perfectly competitive factor markets. Intermediate good firms also face price adjustment costs $\Xi\left(\frac{P_t(i)}{P_{t-1}(i)}\right) Y_t$ that has standard properties detailed later.

Intermediate good firms problem is to

$$\max_{\{P_t(i), Y_t(i), L_{s,t}(i), L_{u,t}(i), K_{e,t}(i), K_{b,t}(i)\}} \quad E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{\Lambda_0} P_t \Phi_t(i) \right\}$$

subject to (1) and (2), where Λ_t is the marginal utility of nominal income and flow profits $\Phi_t(i)$ is

[6] As we describe in detail later, this relative price is exogenous to ensure balanced growth in the model.

[7] Steady-state of a variable x is denoted by $\overline{x}$ throughout. As we discuss later, we restrict preferences and technology such that the model is consistent with balanced growth.

given by

$$\Phi_t(i) = \frac{P_t(i)}{P_t} Y_t(i) - \frac{W_t^s}{P_t} L_{s,t}(i) - \frac{W_t^u}{P_t} L_{u,t}(i) - \frac{R_t^{K,e}}{P_t} K_{e,t}(i) - \frac{R_t^{K,b}}{P_t} K_{b,t}(i) - \Xi\left(\frac{P_t(i)}{P_{t-1}(i)}\right) Y_t.$$

Note that there is a skill premium in the model, which we define as the wage of skilled labor relative to that of unskilled labor, that is, $\frac{W_t^s}{W_t^u}$. Given the CRS production function and the assumption of perfectly competitive factor markets, the factor prices are equal to marginal products of each factor multiplied by firms' marginal costs. Moreover, as we show in detail later, if capital-skill complementarity exists, the skill premium increases in the amount of equipment capital when the quantities of the two types of labor inputs are held fixed. It is also increasing in the ratio of unskilled to skilled labor.

2.2 Government

We now describe the constraints on the government and how it determines monetary and fiscal policy.

2.2.1 Government budget constraint

The government flow budget constraint, written by expressing fiscal variables as ratio of output, is given by

$$\frac{B_t}{P_t Y_t} + \tau_t^C \frac{C_t}{Y_t} + \tau_t^H \left(\lambda_{\tau^H}^s \frac{W_t^s}{P_t Y_t} L_{s,t} + \lambda_{\tau^H}^u \frac{W_t^u}{P_t Y_t} L_{u,t}\right) + \tau_t^K \left(\frac{R_t^{K,b}}{P_t Y_t} K_{b,t} - \lambda_b I_{b,t} + \frac{R_t^{K,e}}{P_t Y_t} K_{e,t} - \lambda_e I_{e,t}\right)$$
$$= R_{t-1} \frac{B_{t-1}}{P_{t-1} Y_{t-1}} \frac{1}{\pi_t} \frac{Y_{t-1}}{Y_t} + \frac{G_t}{Y_t} + \frac{S_t}{Y_t}$$

where $B_t = \sum_{i \in \{s,u\}} N^i B_t^i$, $S_t = \sum_{i \in \{s,u\}} N^i S_t^i$, $S_t^i = \frac{\chi_S^i}{N^i} S_t$ and G_t is government spending on the final good.[8]

2.2.2 Monetary policy

Monetary policy is given by a simple interest-rate feedback rule

$$\frac{R_t}{\bar{R}} = \left[\frac{R_{t-1}}{\bar{R}}\right]^{\rho^R} \left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi}\right]^{(1-\rho^R)} \tag{3}$$

where $\phi \geq 0$ is the feedback parameter on inflation $\left(\pi_t = \frac{P_t}{P_{t-1}}\right)$, $0 \leq \rho^R < 1$ governs interest rate smoothing, $\bar{R}$ is the steady-state value of R_t, and $\bar{\pi}$ is the steady-state value of π_t. When $\phi > 1$, the

[8]We introduce government spending in the model for a realistic calibration. As we discuss later, government spending-to-GDP ratio is held fixed throughout in the model.

standard case, the Taylor principle is satisfied. When $\phi < 1$, which we will also consider, inflation response will play a direct role in government debt stabilization along the transition.

2.2.3 Fiscal policy

We consider a one-time permanent change in the capital tax rate τ_t^K in period 0, when the economy is in the initial steady-state. In order to isolate the effects of the capital tax rate cut, $\frac{G_t}{Y_t}$ is kept unchanged from its initial steady-state value in all periods. The debt-to-GDP ratio, $\frac{B_t}{P_t Y_t}$, may deviate from the initial steady-state in the short run but will converge back to the initial steady-state in the long-run, through appropriate changes in fiscal instruments.

We will study both long-run effects of such permanent changes in the capital tax rate, as well as in extensions, full transition dynamics as the economy evolves towards the new steady-state. We consider the following fiscal policy adjustments that ensure that in the long-run, debt-to-GDP stays at the same level as the initial level through appropriate adjustment of fiscal instruments, while ensuring that distortionary tax rates adjust smoothly during the transition. First, only lump-sum transfers adjust to maintain $\frac{B_t}{P_t Y_t}$ constant at each point in time.[9] Second, only labor tax rates τ_t^H adjust following the simple feedback rule

$$\tau_t^H - \bar{\tau}_{new}^H = \rho^H \left(\tau_{t-1}^H - \bar{\tau}_{new}^H\right) + \left(1 - \rho^H\right) \psi^H \left(\frac{B_{t-1}}{P_{t-1} Y_{t-1}} - \overline{\frac{B}{PY}}\right) \tag{4}$$

where $\psi^H \geq 1 - \beta$ is the feedback parameter on outstanding debt, $0 \leq \rho^H < 1$ governs labor tax rate smoothing, $\bar{\tau}_{new}^H$ is the new steady-state value of τ_t^H, and $\overline{\frac{B}{PY}}$ is the (initial and new) steady-state value of $\frac{B_t}{P_t Y_t}$. Third, only consumption tax rates τ_t^C adjust following the simple feedback rule

$$\tau_t^C - \bar{\tau}_{new}^C = \rho^C \left(\tau_{t-1}^C - \bar{\tau}_{new}^C\right) + \left(1 - \rho^C\right) \psi^C \left(\frac{B_{t-1}}{P_{t-1} Y_{t-1}} - \overline{\frac{B}{PY}}\right) \tag{5}$$

where $\psi^C \geq 1 - \beta$ is the feedback parameter on outstanding debt, $0 \leq \rho^C < 1$ governs consumption tax rate smoothing, and $\bar{\tau}_{new}^C$ is the new steady-state value of τ_t^C.

For transition dynamics, the behavior of the monetary authority generally matters. In the three cases above, we have the monetary policy rule (3) satisfying the Taylor principle, $\phi > 1$, which thereby, implies that inflation plays no direct role in government debt stabilization. We consider a fourth case to highlight the role of monetary policy response to inflation for transition dynamics. In this case, labor taxes adjust, but not sufficiently enough, as $0 < \psi^H < 1 - \beta$, and inflation partly plays a direct role in government debt stabilization, as $\phi < 1$. The monetary and labor tax rules are still given by (3) and (4), but now with these restrictions on the feedback parameters. Thus, in this fourth case, we allow debt stabilization, (only) along the transition, to occur partly through distortionary labor taxes and partly through inflation.[10]

[9] Since transfers are lump-sum and there is complete risk-sharing, the time-path of transfers does not matter, and so we just use a simple formulation.

[10] An analogous consumption tax rule, with $0 < \psi^C < 1 - \beta$, generates similar results and is thus omitted here.

2.3 Equilibrium

The equilibrium definition is standard, given the maximization problems of the private sector and the monetary and fiscal policy described above. Moreover, we consider a symmetric equilibrium across firms, where all firms set the same price and produce the same amount of output. We also have perfect risk sharing across households. Goods, asset, and factor markets clear in equilibrium.[11]

The economy features balanced growth. As we describe below, we use standard assumptions on preferences that ensure balanced growth. Moreover, since our production function features two types of capital and capital-skill complementarity, we impose an additional assumption on the growth rate of q_t, the exogenous relative price between investment in capital structures and equipment. Generally, we normalize variables growing along the balanced growth path by the level of technology. Fiscal variables, as mentioned above, are normalized by output. We use the notation, for instance, $\tilde{Y}_t = \frac{Y_t}{\gamma^t}$ and $\tilde{b}_t = \frac{B_t}{P_t Y_t}$ to denote these stationary variables where γ is the growth rate of output. We also use the notation T_t^C, T_t^H, and T_t^K to denote (real) consumption, labor, and capital tax revenues. Nominal variables are denoted in real terms in small case letters, for instance, $w_t = \frac{W_t}{P_t}$. All the equilibrium conditions are derived and given in detail in the Appendix.

3 Long-Run Results

We now present our main results. We start with the parameterization and then discuss the long-run effects, analytically (in a simplified model) and numerically (in the baseline model), of permanent changes in the capital tax rate. As we mentioned above in Section 2.2.3, we consider three different fiscal policy to ensure that the government debt-to-GDP ratio is at the same level in the long-run. The first is by (non-distortionary) transfer adjustment, which we take as the starting point. We then look at how a distortionary adjustment of labor tax rate and consumption tax rate alters results.

3.1 Parameterization

We use the following functional forms for preferences and technology

$$U(C_t^i, H_t^i) \equiv \log C_t^i - \bar{\omega}^i \frac{\left(H_t^i\right)^{1+\varphi}}{1+\varphi},$$

$$F\left(A_t, K_{b,t}, K_{e,t}, L_{s,t}, L_{u,t}\right) \equiv A_t \left(K_{b,t}\right)^{\alpha} \left[\mu L_{u,t}^{\sigma} + (1-\mu)\left(\lambda \left(K_{e,t}\right)^{\rho} + (1-\lambda)\left(L_{s,t}\right)^{\rho}\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\alpha}{\sigma}},$$

Moreover, while we consider these various fiscal/monetary adjustment scenarios to investigate how results depend on different policy choices, our analysis is not normative as in the Ramsey policy tradition.

[11]The aggregate market clearing condition for goods is then given by

$$C_t + I_{b,t} + I_{e,t} + G_t + \Xi\left(\frac{P_t}{P_{t-1}}\right) Y_t = Y_t.$$

Moreover, in the baseline model, marginal utility growth rates are equalized across the skilled and unskilled households.

and standard functional forms for the investment and price adjustment costs

$$\mathcal{S}\left(\frac{I_t}{I_{t-1}}\right) \equiv \frac{\xi}{2}\left(\frac{I_t}{I_{t-1}} - \gamma\right)^2, \; \Xi\left(\frac{P_t}{P_{t-1}}\right) \equiv \frac{\kappa}{2}\left(\frac{P_t}{P_{t-1}} - \bar{\pi}\right)^2.$$

The utility function is a standard one and consistent with balanced growth. The production function $F(\cdot)$ is a nested CES structure used in Krusell *et al.* (2000). This implies that equipment capital and skilled labor have the same elasticity of substitution against unskilled labor, given by $1/(1-\sigma)$. The elasticity of substitution between equipment capital and skilled labor is $1/(1-\rho)$. Capital-skill complementarity exists when $\sigma > \rho$. The parameters μ and λ govern income shares. Note that when $\rho \to 0$, the production function reduces to a standard Cobb-Douglas formulation, which we will use for analytical results. Suppose that the gross growth rate of A_t is $\bar{a} = \gamma^{1-\alpha}$. We assume that q_t, the exogenous relative price between consumption (structures) and equipment investment, grows at rate $\gamma_q = 1/\gamma$, which leads to balanced growth of the model. It follows that all the growing variables grow at rate γ.[12]

Table 1 contains numerical values we used for the parameters of the model. The parameterization is standard, and we provide references or justification for values we pick from the literature in Table 1. In the baseline, as given above, we use separable preferences that imply log utility and then calibrate a modest, unit Frisch elasticity of labor supply ($\frac{1}{\varphi}= 1$).[13] For the production function elasticity of substitution parameters, we use the estimates in Krusell *et al.* (2000) ($\sigma = 0.401, \rho = -0.495$). This parameterization implies (equipment) capital-skill complementarity. We also follow Krusell *et al.* (2000) in matching the income share of structure ($\alpha = 0.117$) as well as the depreciation rates of the two types of capital. Finally, for the income share of equipment and unskilled labor, we pick parameter values to get a steady-state labor share of 0.56 (Elsby, Hobijn, and Sahin (2013)) and steady-state skill premium of 60% (Krusell *et al.* (2000)).

Additionally, across various fiscal adjustment scenarios and preference and technology functions specifications, we normalize hours for skilled labor to be 0.330 and hours for unskilled labor to be 0.307 in the initial steady-state by appropriately adjusting the scaling parameters $\bar{\omega}^s$ and $\bar{\omega}^u$. We follow the calibration of Lindquist (2004) for this choice of steady-state hours as well as the fraction of skilled labor (N^s=0.5).

The steady state of the fiscal variables such as the debt-to-GDP ratio, the government spending-to-GDP ratio, and the taxes-to-GDP ratio, is matched to their respective long-run values in the data. The Appendix describes this data in detail. We then calibrate the steady-state markup to obtain a 35% capital tax rate initially. The implied initial levels of labor tax rate and consumption tax rates are 12.8% and 0.9% respectively. For the effective expensing rates of the two types of capital, we use the estimates in Barro and Furman (2018), which implies higher expensing of

[12] King, Plosser, and Rebelo (2002) describes the required restrictions on preferences and technology in the standard neoclassical model. Balanced growth with capital-skill complementarity in the production function was shown in Maliar and Maliar (2011), who pointed out the need to have an exogenous path for relative price between consumption (structures) and equipment and restrictions on the growth rate.

[13] We will show detailed comparative statics with respect to the Frisch elasticity, given that distortions of labor supply decisions are a key component of our analytical results.

structure investment. For the parameter governing the incidence of labor tax rate on the two types of workers, we set equal weights in the baseline ($\lambda^s_{\tau^H} = \lambda^u_{\tau^H}$=1) and show results when we vary these tax rate weights. In our baseline, we assume that the profit shares for skilled labor is 1 (χ^s_Φ) and the transfers share for unskilled labor is 1 (χ^u_S). That is, all profits are distributed to skilled labor and all transfers are distributed to unskilled labor.[14]

Finally, for transition dynamics, the parameterization of price and investment adjustment costs, as well as policy rules matters. We use estimates from Ireland (2000) and Smets and Wouters (2007) for price and investment adjustment cost respectively. For the policy rule parameters, we use estimates from Bhattarai, Lee, and Park (2016) for baseline and then some variations around those for other policy regimes. In extensions and in the Appendix, we also show results where we consider several variations around our baseline parameterization of important utility and production function parameters.

3.2 Analytical results of a simplified model

We now present several analytical results that help clarify the mechanisms regarding long-run aggregate effects. For this, we simplify the model presented above such that it converges to a standard business cycle model. In particular, we first assume $\rho \to 0$ to get a nested version of the model with a Cobb-Douglas production function. Then, we assume the two share parameters to be zero, $\mu = \alpha = 0$, and the fraction of skilled households to be 1, $N^S = 1$. In this case, we now have one type of capital $K_{e,t}$ and one type of labor $L_{s,t}$ and a standard Cobb-Douglas production function that implies a unit elasticity of substitution between capital and labor. In the analytical results below, we then drop subscripts e and s for variables. We also for simplicity do not have expensing of the tax rate. While there is no skill premium in this simple model, these analytical results on aggregate effects are relevant as not only do they show the mechanisms, but also because as we show later, for aggregate effects, our baseline model with capital-skill complementarity has very similar predictions to the simpler case presented here.

3.2.1 Lump-sum transfer adjustment

We start with the case where lump-sum transfers adjust to finance the capital tax rate cut. Capital tax cuts, as expected, have expansionary long-run effects on the economy. It is useful to state as an assumption a mild restriction on government spending in steady-state as given below.[15]

Assumption 1. $\bar{\tilde{G}} < 1 - \frac{\theta-1}{\theta}\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\left(1-\bar{\tau}^K\right) = 1 - \frac{1}{\lambda}\left(\frac{\bar{\tilde{I}}}{\bar{\tilde{Y}}}\right)$ *in the initial steady-state.*

[14]In the baseline model, these parameters do not matter regardless of how the capital tax rate cuts are financed. In an extension where we consider heterogenous households, these parameters matter for the aggregate effects of the capital tax rate cuts.

[15]This restriction is very mild, and is just to ensure that government spending in steady-state is not very high. For instance, for a case of not an unrealistically high markup and separable preferences, this holds for any reasonable parameterization of government spending in steady-state.

Then, we can show that a permanent capital tax rate cut leads to an increase in output, consumption, investment, and wages, and a decline in the rental rate of capital in the model. We state this formally below in Lemma 1.

Lemma 1. *Fix $\bar{\tau}^H$ and $\bar{\bar{b}}$. With lump-sum transfer adjustment,*

1. Rental rate of capital is increasing, while capital to hours ratio, wage, hours, capital, investment, and output are decreasing in $\bar{\tau}^K$.

2. Under Assumption 1, consumption is also decreasing in $\bar{\tau}^K$.

Proof. See Appendix C.2. □

Intuition for this result is well-understood. A reduction in the capital tax rate leads to a decrease in the rental rate of capital, raising firms' demand for capital. This stimulates investment and capital accumulation. The capital-to-labor ratio increases as a result. A larger amount of capital stock, in turn, makes workers more productive, raising wages and hours. Given the increase in the factors of production, output increases, which also raises consumption unless the steady-state ratio of government spending-to-GDP is unrealistically very high, as ruled out by Assumption 1.[16]

Additionally, we can also derive an exact solution for the change in macroeconomic quantities and factor prices, as well as an approximate solution for small changes in the capital tax rates that are intuitive to understand and sign. We state this formally below in Proposition 1. Note that the results below are in terms of changes from the original steady-state.

Proposition 1. *Let $\bar{\tau}^K_{new} = \bar{\tau}^K + \Delta\left(\bar{\tau}^K\right)$. With lump-sum transfer adjustment, relative changes of various variables from their initial steady-states are:*

$$\frac{\bar{r}^K_{new}}{\bar{r}^K} = \left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{-1}, \qquad \frac{\bar{\bar{w}}_{new}}{\bar{\bar{w}}} = \left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{\frac{\lambda}{1-\lambda}},$$

$$\left(\frac{\bar{\bar{K}}_{new}/\bar{H}_{new}}{\bar{\bar{K}}/\bar{H}}\right) = \left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{\frac{1}{1-\lambda}}, \qquad \frac{\bar{H}_{new}}{\bar{H}} = \left(1 + \Omega\Delta\left(\bar{\tau}^K\right)\right)^{-\frac{1}{1+\varphi}},$$

$$\frac{\bar{\bar{K}}_{new}}{\bar{\bar{K}}} = \frac{\bar{\bar{I}}_{new}}{\bar{\bar{I}}} = \left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{\frac{1}{1-\lambda}} \frac{\bar{H}_{new}}{\bar{H}}, \qquad \frac{\bar{\bar{Y}}_{new}}{\bar{\bar{Y}}} = \left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{\frac{\lambda}{1-\lambda}} \frac{\bar{H}_{new}}{\bar{H}},$$

and

$$\frac{\bar{\bar{C}}_{new}}{\bar{\bar{C}}} = \left(1 + \frac{\bar{\bar{I}}}{\bar{H}}\left(\frac{\bar{\bar{C}}}{\bar{H}}\left(1-\bar{\tau}^K\right)\right)^{-1}\right)\frac{\bar{\bar{Y}}_{new}}{\bar{\bar{Y}}}$$

where $\Omega = \bar{\omega}\bar{H}^{1+\varphi}\frac{\lambda}{1-\lambda}\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H} > 0$. Moreover, for small changes in the capital tax rate

[16] In such a case, government consumption or investment crowds out private consumption.

$\Delta\left(\bar{\tau}^{K}\right)$, *the percent changes of these variables from their initial steady-states are:*

$$\ln\left(\frac{\bar{\tau}^{K}_{new}}{\bar{\tau}^{K}}\right)=\frac{\Delta\left(\bar{\tau}^{K}\right)}{1-\bar{\tau}^{K}},\qquad \ln\left(\frac{\bar{\tilde{w}}_{new}}{\bar{\tilde{w}}}\right)=-\left(\frac{\lambda}{1-\lambda}\frac{\Delta\left(\bar{\tau}^{K}\right)}{1-\bar{\tau}^{K}}\right),$$

$$\ln\left(\frac{\bar{\tilde{K}}_{new}/\bar{H}_{new}}{\bar{\tilde{K}}/\bar{H}}\right)=-\left(\frac{1}{1-\lambda}\frac{\Delta\left(\bar{\tau}^{K}\right)}{1-\bar{\tau}^{K}}\right),\qquad \ln\left(\frac{\bar{H}_{new}}{\bar{H}}\right)=-\mathcal{M}_{H}\Delta\left(\bar{\tau}^{K}\right),$$

$$\ln\left(\frac{\bar{\tilde{K}}_{new}}{\bar{\tilde{K}}}\right)=\ln\left(\frac{\bar{\tilde{I}}_{new}}{\bar{\tilde{I}}}\right)=-\mathcal{M}_{K}\Delta\left(\bar{\tau}^{K}\right),\qquad \ln\left(\frac{\bar{\tilde{Y}}_{new}}{\bar{\tilde{Y}}}\right)=-\mathcal{M}_{Y}\Delta\left(\bar{\tau}^{K}\right),$$

and

$$\ln\left(\frac{\bar{\tilde{C}}_{new}}{\bar{\tilde{C}}}\right)=-\mathcal{M}_{C}\Delta\left(\bar{\tau}^{K}\right),$$

where $\mathcal{M}_H=\frac{\Omega}{1+\varphi}$, $\mathcal{M}_K=\frac{1}{(1-\lambda)(1-\bar{\tau}^K)}+\frac{\Omega}{1+\varphi}>0$, $\mathcal{M}_Y=\frac{\lambda}{(1-\lambda)(1-\bar{\tau}^K)}+\frac{\Omega}{1+\varphi}>0$ *and* $\mathcal{M}_C=\mathcal{M}_Y-\frac{\bar{\tilde{I}}}{\bar{H}}\left(\frac{\bar{\tilde{C}}}{\bar{H}}\left(1-\bar{\tau}^K\right)\right)^{-1}$. *Under Assumption 1,* $\mathcal{M}_C>0$.

Proof. See Appendix C.3. □

Proposition 1 provides a simple representation of the model solution that helps us understand the mechanism for aggregate variables even further. As is standard, the effects on factor prices and capital to labor ratio depend only on the production side parameters. For the level of aggregate quantities (output, consumption and investment), however, the proposition shows that the key step, in the aforementioned channel, is in fact how labor hours respond, $\frac{H_{new}}{H}$.[17] This implies that preference parameters, in particular, the Frisch elasticity of labor supply, generally matter for the effectiveness of a capital tax cut. In fact, it is clear that since hours in the initial steady-state is less than 1, the capital tax elasticity of hours, $\mathcal{M}_H$, is decreasing in φ, and thus hours increase more with higher Frisch elasticity. Moreover, given the importance of hours response, the proposition naturally leads us to a conjecture that a capital tax cut would have a smaller effect if the labor tax rate needed to adjust, which we prove formally in the next subsection. Finally, the solution also reveals that the effectiveness of a tax reform depends on the economy's current tax rates. When the economy is initially farther away from the non-distortionary case (i.e. when $\bar{\tau}^K$, $\bar{\tau}^H$, and $\bar{\tau}^C$ are currently high), a given capital tax cut will have a stronger long-run effect.

3.2.2 Labor tax rate and consumption tax rate adjustment

We next discuss the case where distortionary tax rates increase to finance the capital tax rate cut. We first derive results where labor tax rate increases in the long-run to finance the permanent capital tax rate cuts. Overall, compared to the previous case of lump-sum transfer adjustment, the model predicts qualitatively similar long-run effects on most of the variables – except for labor hours and for after-tax wages. Quantitatively, however, the macroeconomic effects are expected to

[17] Output for example, increases by the same amount (in percentage from the initial steady-state) as pre-tax labor income.

be smaller because of distortions created by the labor tax rate increase. In fact, for small changes in the capital tax rate, we have analytical results below on exactly how small these effects are and what parameters determine the differences.

Once again, a mild restriction on steady-state government spending is assumed as given below.

Assumption 2. *$\bar{\tilde{G}} < 1 - \frac{\theta-1}{\theta}\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)} = 1 - \frac{1}{\lambda(1-\bar{\tau}^K)}\left(\frac{\bar{\tilde{I}}}{\bar{\tilde{Y}}}\right)$ in the initial steady-state.*

Then, we can show that a permanent capital tax rate cut, financed by an increase in the labor tax rate, leads to an increase in the capital-to-hours ratio and in (pre-tax) wages and a decrease in the rental rate of capital, as before.[18] In contrast to the lump-sum transfer case, however, hours now decline in the new steady-state.

Lemma 2. *Fix $\bar{\tilde{S}}$ and $\bar{\tilde{b}}$. With labor tax rate adjustment,*

1. Rental rate of capital is increasing, while capital to hours ratio and wage are decreasing in $\bar{\tau}^K$.

2. Under Assumption 2, hours are increasing in $\bar{\tau}^K$.

Proof. See Appendix C.4. □

We next show analytically the required adjustment in labor tax rate in the new steady-state as well as the approximate solution for small changes in the capital tax rates that are intuitive to understand and sign. We state the results formally below in Proposition 2.

Proposition 2. *Let $\bar{\tau}^K_{new} = \bar{\tau}^K + \Delta\left(\bar{\tau}^K\right)$. With labor tax rate adjustment,*

1. New steady-state labor tax rate is given by $\bar{\tau}^H_{new} = \bar{\tau}^H + \Delta\left(\bar{\tau}^H\right)$ where

$$\Delta\left(\bar{\tau}^H\right) = -\frac{\lambda}{1-\lambda}\left(1 + \bar{\tau}^C\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\right)\Delta\left(\bar{\tau}^K\right).$$

2. For small changes in the capital tax rate $\Delta\left(\bar{\tau}^K\right)$, relative changes of rental rate, wage, after-tax wage, capital to hours ratio and hours from their initial steady-states are:

$$\ln\left(\frac{\bar{r}^K_{new}}{\bar{r}^K}\right) = \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K},\ \ln\left(\frac{\bar{\tilde{K}}_{new}/\bar{H}_{new}}{\bar{\tilde{K}}/\bar{H}}\right) = -\frac{1}{1-\lambda}\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K},$$

$$\ln\left(\frac{\bar{\tilde{w}}_{new}}{\bar{\tilde{w}}}\right) = -\frac{\lambda}{1-\lambda}\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K},\ \ln\left(\frac{\left(1-\bar{\tau}^H_{new}\right)\bar{\tilde{w}}_{new}}{\left(1-\bar{\tau}^H\right)\bar{\tilde{w}}}\right) = \mathcal{M}_W\Delta\left(\bar{\tau}^K\right),$$

and

$$\ln\left(\frac{\bar{H}_{new}}{\bar{H}}\right) = \mathcal{M}_{H,\tau^H}\Delta\left(\bar{\tau}^K\right),$$

[18] In fact the entire response of capital-to-hours, rental rate of capital, and (pre-tax) wages are the same between transfer and labor tax rate adjustment.

where $\mathcal{M}_{H,\tau^H} = \frac{1-\bar{\tilde{G}}+\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\left(\bar{\tilde{T}}^C+\bar{\tilde{T}}^H+\bar{\tilde{T}}^K-\frac{\theta-1}{\theta}\right)}{(1+\varphi)\frac{1-\lambda}{\lambda}(1-\bar{\tau}^H)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}$ *and* $\mathcal{M}_W = \frac{\lambda\left(1+\bar{\tau}^C\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}-\frac{1-\bar{\tau}^H}{1-\bar{\tau}^K}\right)}{(1-\lambda)(1-\bar{\tau}^H)}$. *Under Assumption 2,* $\mathcal{M}_{H,\tau^H} > 0$. *Moreover,* $\mathcal{M}_W > 0$ *if and only if* $1+\bar{\tau}^C\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right) > \frac{1-\bar{\tau}^H}{1-\bar{\tau}^K}$.

Proof. See Appendix C.5. □

The required adjustment in the labor tax rate is approximately given by the ratio of the capital to labor input in the production function, as the government is keeping debt-to-GDP constant and hence has to compensate the loss of capital tax revenue-to-GDP with gains in labor tax revenue. One interesting result on the approximate solution is that the effects on *after-tax* wage rate depends on initial level of labor tax rate relative to the other tax rates. Intuitively, a further increase in labor tax rate (to finance a capital tax cut,) when it is sufficiently high already, lowers *after-tax* wage rate. Moreover, again, hours fall, which is the result we highlight given that it is qualitatively different.[19] Additionally, note that the (absolute) capital tax elasticity of hours, $\mathcal{M}_{H,\tau^H}$ in Proposition 2, decreases in φ, and thus hours fall more with a higher Frisch elasticity.

We next discuss the case where consumption tax rate increases in the long-run to finance the permanent capital tax rate cuts. Overall, the results are very similar to the labor tax rate adjustment case as both these distortionary source of taxes affect the consumption-leisure choice in a similar way. Thus, first, we can show that a permanent capital tax rate cut, financed by an increase in the consumption tax rate rate, leads to an increase in the capital-to-hours ratio and wages and a decrease in the rental rate of capital, as before for both transfer and labor tax rate adjustment, as well as a decrease in hours, as before for labor tax rate adjustment.

Lemma 3. *Fix* $\bar{\tilde{S}}$ *and* $\bar{\tilde{b}}$. *With consumption tax rate adjustment,*

1. *Rental rate of capital is increasing, while capital to hours ratio and wage are decreasing in* $\bar{\tau}^K$.
2. *Hours are increasing in* $\bar{\tau}^K$.

Proof. See Appendix C.6. □

Then, we can also show analytically the required adjustment in consumption tax rate in the new steady-state as well as the approximate solution for small changes in the capital tax rates that are intuitive to understand and sign. We state the results formally below in Proposition 3.The economic mechanisms are very similar to the labor tax rate change scenario that we described in detail above, where here as well, hours decline.

Proposition 3. *Let* $\bar{\tau}^K_{new} = \bar{\tau}^K + \Delta\left(\bar{\tau}^K\right)$. *With consumption tax rate adjustment,*

[19] For this case, because of opposite movement of hours and capital to hours ratio, it is not possible to provide intuitive results on the levels of variables such as output and consumption.

1. New steady-state consumption tax rate is given by $\bar{\tau}^C_{new} = \bar{\tau}^C + \Delta\left(\bar{\tau}^C\right)$ *where*

$$\Delta\left(\bar{\tau}^C\right) = -\left(1 + \frac{\bar{a} - (1-d)}{\frac{\bar{a}}{\beta} - (1-d)}\bar{\tau}^C\right)\frac{\Theta_C \Delta\left(\bar{\tau}^K\right)}{1 + \left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\Theta_C\Delta\left(\bar{\tau}^K\right)}.$$

with $\Theta_C = \frac{\lambda \bar{mc}}{\left(1-\bar{\tilde{G}}\right) - \frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}(1-\bar{\tau}^K)} > 0.$

2. For small changes in the capital tax rate $\Delta\left(\bar{\tau}^K\right)$*, relative changes of rental rate, wage, after-tax wage, capital to hours ratio and hours from their initial steady-states are:*

$$\ln\left(\frac{\bar{r}^K_{new}}{\bar{r}^K}\right) = \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K},\ \ln\left(\frac{\bar{\tilde{K}}_{new}/\bar{H}_{new}}{\bar{\tilde{K}}/\bar{H}}\right) = -\frac{1}{1-\lambda}\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K},\ \ln\left(\frac{\bar{\tilde{w}}_{new}}{\bar{\tilde{w}}}\right) = -\frac{\lambda}{1-\lambda}\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}$$

and

$$\ln\left(\frac{\bar{H}_{new}}{\bar{H}}\right) = \mathcal{M}_{H,\tau^C}\Delta\bar{\tau}^K,$$

where $\mathcal{M}_{H,\tau^C} = \frac{1}{1+\varphi}\frac{\lambda\bar{mc}}{(1+\bar{\tau}^C)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\left(1 - \frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right) > 0.$

Proof. See Appendix C.7. □

Finally, we are also able to compare analytically the change in macroeconomic quantities as a result of the capital tax rate cut for the three fiscal adjustment cases. We do this for the small capital tax rate adjustment approximation and prove in Proposition 4 that the increase in output, capital, investment, consumption, and hours increase by more under adjustment in lump-sum transfers compared to labor tax rate adjustment.[20] Moreover, the differences in these changes for output, investment, consumption, and hours are given by the same amount. This constant difference depends intuitively and precisely on the labor supply parameter for a given change in the tax rates. A higher Frisch elasticity ($\frac{1}{\varphi}$) makes workers more responsive to labor tax rates, thereby generating greater distortions, which in turn, magnifies the difference. The two fiscal adjustments produce the same outcomes only if labor supply is completely inelastic ($\frac{1}{\varphi} = 0$). Moreover, as is intuitive, higher is the initial level of the labor tax rate, bigger is the difference. Thus, for the same change in the labor tax rate, if the initial labor tax rate is higher, the increase in output, investment, consumption, and hours will be relatively smaller.

Proposition 4. *Let* $\bar{\tau}^K_{new} = \bar{\tau}^K + \Delta\left(\bar{\tau}^K\right)$*,* $\bar{\tau}^H_{new} = \bar{\tau}^H + \Delta\left(\bar{\tau}^H\right)$*. Denote* $\bar{X}^T_{new}$ *and* $\bar{X}^L_{new}$ *as the new steady-state variables in transfer adjustment case and in labor tax rate adjustment case, respectively.*

[20] This result does not require Assumption 2. That is, it holds regardless of whether hours increase or decrease following a capital tax rate cut. Additionally, as seen above wages and rental rates are the same across the two fiscal adjustments, as shown in Proposition 1 and 2, and so we do not present these obvious results in Proposition 4.

For small changes in the capital tax rate $\Delta\left(\bar{\tau}^K\right)$, *for* $X \in \left\{\tilde{C}, \tilde{K}, \tilde{I}, \tilde{Y}, H\right\}$, *we get*

$$\ln\left(\frac{\bar{X}^T_{new}}{\bar{X}^L_{new}}\right) = -\Theta\Delta\left(\bar{\tau}^K\right) = \frac{1}{1+\varphi}\left(\frac{1}{1-\bar{\tau}^H}\right)\Delta\left(\bar{\tau}^H\right)$$

where $\Theta = \frac{1}{1+\varphi}\left(\frac{1}{1-\bar{\tau}^H}\right)\frac{\lambda}{1-\lambda}\left(1+\bar{\tau}^C\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right) > 0$. *In other words, generally, output, capital, investment, consumption and hours increase by more in the transfer adjustment case than in the labor tax rate adjustment case when capital tax rate is cut.*

Proof. See Appendix C.8. □

For completeness, we also show similar comparisons for the consumption tax rate adjustment case in Propositions 5 and 6, which is relegated to the Appendix to conserve space.

3.3 Numerical results of baseline model

We now present numerical results of the baseline model with capital-skill complementarity, as presented in Section 2, and as parameterized in Table 1. One main finding is that worker and capital heterogeneity in the model, while certainly generating new distributional implications, have little aggregate effects. The analytical results in the previous section thus serve as a useful benchmark for economic intuition for aggregate variables. The first set of numerical results are summarized in Figure 1.[21] While our focus is on a reduction of the capital tax rate from 35% to 21%, which are clearly shown with colored dots in the Figure, we show the entire range of tax rate changes for completeness.

We start with the case of transfer adjustment. For a reduction of the capital tax rate from 35% to 21%, output increases by 3.8% relative to the initial steady state, structure investment by 19.7%, equipment investment by 6.8%, and consumption by 2.1%.[22] Moreover, skilled wages increase by 3.4%, unskilled wages by 2.7%, skilled hours increase to 0.334 from 0.330, and unskilled hours to 0.310 from 0.307. In terms of financing, as shown in Figure 1, a decrease in the capital tax rate reduces total (tax) revenues-to-GDP ratio (driven by decrease in capital tax revenue-to-GDP ratio), which is financed by a decline in transfers-to-GDP ratio from 1.0% to -0.5%.[23]

The mechanisms behind aggregate effects are the same as described in the previous section of the simple model. In fact, to make this transparent, in Figure 2, we explicitly show the comparison with a nested model where there is a Cobb-Douglas production function, everything else the same. As

[21] We show population weighted aggregates, and note that due to complete markets, marginal utilities are equalized across the two types of households.

[22] For comparison, Barro and Furman (2018) predict that the long-run increase in output will be 3.1% for a permanent capital tax rate cut from 38% to 26%.

[23] Note that this result is obtained not only because output (i.e. the denominator) increases. In fact, the total tax revenues also decline. In particular, there is a significant decrease in capital tax revenues (about 42% decline relative to the initial steady state), which is only partially offset by an increase in consumption and labor tax revenues. The government therefore finances such a deficit by taking resources away from the household: transfers decline by roughly 151% of the initial steady-state. There is a "Laffer curve" for capital tax revenues but the capital tax revenue starts to decline at very high and empirically irrelevant range, such as above 90% in our baseline calibration.

is clear, the aggregate effects are extremely similar, with output increase of 3.6% and consumption increase of 2.0%. We also show results based on another nested model, where there is a general CES production function, but not capital-skill complementarity. Again, the aggregate effects are very similar. One way to obtain intuition is to look at how hours respond, as suggested by Proposition 1. As shown in Figure 2, skilled hours increase by more, but unskilled hours increase by less, than they would in the absence of skill complementarity. These two countervailing forces contribute to a small differential in aggregate output.

We now turn to distributional implications. First, why does structure investment increase more than equipment investment? Quantitatively, the major reason is the higher expensing rate on structure investment in our calibration. Qualitatively, a role is also played by the fact that in the production function, the elasticity of substitution between equipment investment and skilled hours make them complements.[24]

Second, and more interestingly, as mentioned above, skilled wages increase by more compared to unskilled, and thus, the skill premium or wage inequality increases following a capital tax rate cut. In particular, the skill premium goes up by 1.06% points.[25] To get intuition, in our model, we can express the skill premium as

$$\frac{W_t^s}{W_t^u} = \frac{(1-\mu)(1-\lambda)}{\mu} \left(\lambda \left(\frac{K_{e,t}}{L_{s,t}} \right)^{\rho} + (1-\lambda) \right)^{\frac{\sigma-\rho}{\rho}} \left(\frac{L_{u,t}}{L_{s,t}} \right)^{1-\sigma} .$$

Thus, if a capital-skill complementarity ($\sigma > \rho$) exists, as in our model calibration, the skill premium increases in the amount of equipment capital when the quantities of the two types of labor inputs are held fixed. This mechanism drives our result on the skill premium. Also, note that the skill premium in increasing in the ratio of unskilled-to-skilled labor. This however declines in our experiment. Thus, the main force behind the increase in the skill premium is the increase in equipment capital, and in particular, the increase in the equipment-to-skilled labor ratio. Finally, income inequality, measured by the the ratio of after-tax capital-to-labor income, unambiguously increases – although both types of income increase. The increase in wage and income inequality can be considered as caveats to the effectiveness of the capital tax rate cut in our model, even when lump-sum transfers are allowed to finance the tax cut.

Next, we discuss the results when labor tax rate increases to finance the capital tax rate cut. Figure 1 shows that in the long-run, to finance the reduction of the capital tax rate from 35% to 21%, labor tax rates have to increase from 22.8% to 25.5%. The same mechanism for aggregate variables as we described for the transfer adjustment case works, and moreover the capital-to-hours ratio, (pre-tax) wages on both skilled and unskilled and rental rate on both types of capital change by the same amount as before under the baseline specification. There continues to be an expansion in output, investment, and consumption as a result of the capital tax rate cut.

[24] We show in more detail in sensitivity analysis, the role played by different degrees of expensing and elasticities of substitution.

[25] In the Appendix C.11, we show an analytical result on how the skill premium increases with the capital tax rate cut in our model.

The increase in output, investment, and consumption is however, less under labor tax rate adjustment – as is consistent with what we proved in Proposition 4 above for small changes in the simplified model. In particular, for the baseline experiment of a reduction of the capital tax rate from 35% to 21%, output increases by 1.96%, equipment investment by 4.96%, structures investment by 17.61%, and consumption by 0.31%. The reason for the smaller boost in aggregate variables is a decline of the after-tax wages, by 0.24% for skilled workers and 0.90% for unskilled workers. This in turn leads to a decrease in labor hours in the long-run, from 0.330 to 0.3282 for skilled workers and from 0.307 to 0.3047 for skilled workers. These are the first major qualitative differences from the lump-sum transfer adjustment case, as also highlighted by Proposition 2. The decrease in hours dampens the expansionary effect of capital tax cuts on output, consumption, and investment.

Furthermore, in addition to the smaller expansionary effects, with labor tax adjustment, there continues to be cost in terms of inequality. First, the skill premium (which is the same regardless of pre- or post-tax measures as the labor tax rate is the same on the two types of labor) increases as before. Our measure of income inequality continues to increases, but in fact by more here compared to transfer adjustment, as after-tax labor income now decreases.

We also analyze the case when consumption tax rate increases in the long-run to finance the capital tax rate cut. Figure 1 shows that in the long-run, to finance the reduction of the capital tax rate from 35% to 21%, consumption tax rates have to increase from 1.3% to 3.5%. Generally, as we emphasized before in analytical results for the simple model, the effects are qualitatively similar to labor tax rate adjustment, with the main distortion again coming in labor supply decisions (here more concentrated on the unskilled), which leads to a smaller expansionary effect.

3.4 Extensions

We now consider several extensions, including a model variant with heterogeneous households.

3.4.1 Heterogeneous households

We now consider an extension to heterogeneous households, a model with a hand-to-mouth household. In particular, the unskilled household is hand-to-mouth and consumes wage income plus government transfers every period. The skilled worker still has access to government bond markets and makes dynamic, optimal consumption and savings decisions. The model thus does not have complete consumption insurance across the two types of households and the details are in Appendix B. In our baseline, we assume that the profit shares for skilled labor (χ^s_Φ) is 1 and the transfers share for unskilled labor (χ^u_S) is 1.[26]

The results for long-run effects under transfer adjustment for this model are in Figure 3. Since transfers decline to finance the capital tax rate cut, and transfers fall on the unskilled, as is to be expected, consumption of the unskilled falls. Consumption of the skilled continues to rise, as in the

[26] We discuss how results might change with alternate assumptions later in a sensitivity analysis.

baseline. This implies that now consumption inequality, as measured by relative consumption of the skilled vs. the unskilled increases in the long-run. Additionally, these consumption responses cause strong wealth effects on labor supply now, unlike the baseline case. Thus, we see that hours of the unskilled households increase while those of the skilled decline. The increase in labor supply from the unskilled then leads to a much more muted increase in their wages, compared to the baseline, with a tiny increase. On the flip side, wages of the skilled increases by more now. This means that wage inequality, as measured by the skill premium, increases. In this extension therefore, inequality increases get worse.

On the aggregate side, the effects on output and investment are very similar to the baseline model. For a reduction of the capital tax rate from 35% to 21%, output increases by 4.2% relative to the initial steady state (compared to 3.8% in the baseline model), structure investment by 20.2% (compared to 19.7% in the baseline model), and equipment investment by 6.3% (compared to 6.8% in the baseline model). The slightly higher output effects are driven by the increased labor supply of the unskilled worker, while the slightly smaller increase in equipment investment is a result of reduced labor supply of the skilled worker combined with equipment capital-skill complementarity.

3.4.2 Sensitivity analysis and other results

We now present some additional results on sensitivity analysis and extensions. All the results from this sub-section are in Appendix E.

First, we present comparative statics result with respect to Frisch elasticity of labor supply. This is an important parameter, given that different source of financing imply different labor supply response, as we highlighted in the analytical results based on the simple model. Given the baseline parameterization of a unit Frisch elasticity, we now show results for a higher and lower Frisch elasticity. Figure A.1 shows the results under transfer adjustment, where consistent with Proposition 1, we find that output effects are higher with higher Frisch elasticity due to a stronger hours response. Figure A.2 shows the results under labor tax rate adjustment, where consistent with Proposition 2, we find that output effects are lower with higher Frisch elasticity due to a stronger negative hours response. In the range for the values we consider here, the Figures show that the results overall are quantitatively not different for aggregate output.

Finally, in Figures A.3-A.4, we compare across the two fiscal adjustment for a given Frisch elasticity. Consistent with Proposition 4, the difference between the two cases is bigger for a higher Frisch elasticity. In our baseline calibration of a unit Frisch elasticity, we pointed out above that output increases by 3.8% under lump-sum transfer adjustment and by 1.96% under labor tax rate adjustment. Here, with a Frisch elasticity of 4, output increases by 4.4% under lump-sum transfer adjustment and by 1.5% under labor tax rate adjustment while with a Frisch elasticity of 0.5, output increases by 3.4% under lump-sum transfer adjustment and by 2.2% under labor tax rate adjustment.

Second, for the case of labor tax rate adjustment in the baseline model, we consider a different tax schedule across worker types. Note that in the baseline, for the parameter governing the incidence of labor tax rate on the two types of workers, we set equal weights in the baseline ($\lambda^s_{\tau H} = \lambda^u_{\tau H}$=1), which is arguably a realistic starting-point. In Figure A.5 we show results when we vary these parameters. In particular, consider the case where the skilled workers only pay the labor tax ($\lambda^s_{\tau H} = 1$, $\lambda^u_{\tau H} = 0$). This can be considered a progressive tax regime. In such as case, we see that the after-tax skill premium declines, while the boom in output is also reduced. Thus, wage inequality decrease comes at a cost of lower expansion, with consumption in fact falling in the long-run.

We note however, that letting the tax incidence fall more on the unskilled workers does not lead to a bigger aggregate effect. In fact, Figure A.5 shows that while that case ($\lambda^s_{\tau H} = 0.1$, $\lambda^u_{\tau H} = 1$) certainly leads to an increase in wage inequality, it actually goes together with a lower aggregate effect. The aggregate output and consumption effects depend on labor supply responses of the two types of workers, which get distorted to a varying degree with the changing incidence of the labor tax rates and affected in equilibrium from consumption response (as it affects the marginal utility of consumption).

Third, we present additional results in the model with heterogeneous households. We want to first point out that in this model, clearly the assumptions made on how profits and transfers are distributed across the two types of households makes a non-trivial difference for distributional variables. While the assumptions we made in the baseline case are arguably realistic, where the skilled workers get the profits stream while the unskilled/hand-to-mouth workers get government transfers, in Figure A.6, we show the long-run results under various other combinations of these distributions. For instance, if the skilled worker get both the profits and (cut in) transfers, then it leads to a decline in consumption inequality, in sharp contrast to the baseline case. The results also show that aggregate effects are relatively similar across the various possibilities for profits and transfer distributions.

Then we present two comparative statics result that are useful to interpret the long-run effects, especially when it comes to the differential increase in structure and equipment investment following a capital tax rate decrease in our baseline model. First, we show how results depend on different rates of expensing in Figure A.7. Note that in the baseline calibration, structure investment is expensed at a higher rate in our calibration, in line with the data. Figure A.7 shows that if the expensing rates were to be the same, then the long-run increases in investment of the two types of capital would also be more similar. For instance, if both are expensed at the rate of 0.338, then equipment investment increases in the long-run by 16.7% (compared to baseline of 6.8%), while structure investment increases by 22.0% (compared to baseline of 19.7%). It also follows that in such a case, as equipment investment increases by more, the rise in skill premium increases more than the baseline. So in this case, for wage inequality implications, our calibration can be regarded as conservative.

Second, we show how results depend on changing the elasticity of substitution between equipment capital and skilled labor in Figure A.8. Note that in the production function, the elasticity of substitution between equipment capital and skilled labor is given as $1/(1-\rho)$. As to be expected, a lower elasticity of substitution, making equipment and skilled labor even stronger complements, reduces the long-run increase in equipment investment. This is another reason why in our baseline case, equipment investment increases less than structure investment in the long-run, following a permanent capital tax rate decrease.

4 Transition Dynamics

We now discuss transition dynamics associated with a permanent capital tax rate cut, from 35% to 21%. Thus, we trace the evolution of the economy as it transitions from the initial steady-state to the new steady-state. Studying transition dynamics is important as we find that it typically takes a quite long time, around 80 quarters, for the economy to converge to a new steady-state following a permanent reduction in the capital rate. This allows us in particular to analyze short-run effects, which are the focus here. Compared to the long-run analysis in the previous section, we also pay a special attention to the role of the monetary policy, which can be potentially important due to imperfect price adjustments in the short-run. An overall theme we highlight thus in this section is how a joint analysis of monetary and fiscal policy is essential to understand the short-run effects to a permanent capital tax rate change.

4.1 Four different fiscal/monetary adjustments

We start with the baseline parameterization and version of the model, and now consider four different fiscal/monetary policy adjustment, as described in Section 2.2.3. In particular, a new policy response that we consider here is one where inflation plays a partial role in debt stabilization. The parameterization for the various policy regime parameters is in Table 1. The results for all three policy regimes are shown in Figure 4.

4.1.1 Lump-sum transfer adjustment

Once again, the starting point is the case of non-distortionary transfer adjustment. What makes the short-run distinct from the long-run is that in principle, capital tax cuts can now generate a contractionary effect during the transition periods.

The model dynamics can be best understood as depicting transition dynamics when the capital stock is initially below the new steady-state. As mentioned before, a reduction in the capital tax rate leads to a decrease in the rental rate of capital, thereby facilitating capital accumulation via more investment. In the short-run, to finance this increase of investment, consumption in fact declines for many periods. Given this postponement of consumption, combined with sticky prices, output also falls temporarily, before rising towards the high new steady-state. The temporary contraction in output is a result of sticky prices and investment adjustment costs, which renders output (partially) demand-determined and markups countercyclical in the model.

Moreover, the temporary fall in output (which is coupled with increased capital stock), leads to fall in hours of both skilled and unskilled workers. Finally, inflation is determined by forward looking behavior of firms that face adjustment costs. In particular, inflation depends on current and future real marginal costs, which are a function of wages and capital rental rate. As wage dynamics matter more and wages drop in the short-run, the path of inflation roughly follows that of wages. The decrease in wages is driven by both supply and demand forces. The drop in consumption and the rise in marginal utility of consumption raise the supply of hours for a given wage rate. On the other hand, demand declines as firms produce a smaller amount of output as discussed above.

In terms of inequality, the skill premium increases slightly in the short-run and slowly converges to the new steady state. The capital to labor income ratio also increases in the short-run, above the new long-run level. Moreover, the long-run positive effects of capital tax cuts come at the expense of short-run decline of labor tax revenue– even under lump-sum transfer adjustments. Furthermore, the decrease in labor income requires a larger adjustment of transfers. Transfers fall sharply and in fact, go negative.

4.1.2 Labor tax rate adjustment

Next, we analyze the case of labor tax rate increases. Here, labor tax rate evolves according to the tax rate rule, (4), given in Section 2.2.3. Overall, model dynamics are qualitatively similar to those in the benchmark. We still see capital accumulation, achieved by increased investment and postponement of consumption, which in turn also causes output to fall with sticky prices. Quantitatively, however, the drop in consumption and output is larger in this case compared to the lump-sum transfer adjustment case. As in the lump-sum transfer adjustment case, delayed consumption decreases hours by lowering firms' labor demand. In addition, increased labor tax rate decreases hours even further by discouraging workers from supplying labor. Consequently, hours in equilibrium fall much more, of both the skilled and the unskilled. This in turn amplifies the short-run contraction in consumption and output.

4.1.3 Labor tax rate and inflation adjustment

Finally, we analyze the case where labor tax rates increase, but not by enough, and inflation partly plays a role in government debt stabilization, as described in Section 2.2.3.[27] The main difference now compared to the pure labor tax adjustment analysis is that there is a short-run burst of inflation to help stabilize debt. This increase in inflation, as the model has nominal rigidities, helps counteract the short-run contractionary effects. This in fact increases output, investment, hours and wages in the short-run, while leading to a stronger drop in consumption. In addition, a more gradual increase in labor tax rates contributes to the lack of contraction in output. In fact, even for wage inequality, such a monetary and fiscal policy regime is quite beneficial as it reduces the skill premium.

[27] Note in particular that in this case, the monetary policy rule (3) does not satisfy the Taylor principle, which is coupled with a low response of the tax rate in the tax rule (4). Clearly, we can analyze a similar fiscal adjustment case where inflation plays a role in debt stabilization even with lump-sum transfer adjustment. When non distortionary sources of revenue is possible, allowing inflation to play a role in debt stabilization might not be a very insightful experiment and so we do not emphasize this.

4.1.4 Consumption tax rate adjustment

For completeness, we also study transition dynamics for the case of consumption tax rate adjustment. To keep Figure 4 uncluttered, we show the results in Figure A.9 in Appendix E. The transition dynamics associated with the labor tax rate adjustment and consumption tax rate are very similar.

4.2 Role of monetary policy smoothing

We now highlight another mechanism through which modeling the details of monetary policy reaction matters for the transition dynamics in a non-trivial way. Figure 5 shows comparative statics with respect to the interest rate smoothing parameter in the monetary policy rule (ρ^R), for the transfer adjustment case. As is clear, transition dynamics depend clearly on how inertially the central bank adjusts the nominal rate. In fact, with a high enough smoothing, there is no longer a contraction in output and a fall in hours, unlike the case in Figure 4. In such a case, the contraction in consumption is also muted, while inflation actually increases in the short-run. The main driving force is that with high interest rate smoothing, the policy rate rises less along the transition, which leads to a lower contraction throughout.

4.3 Transition dynamics with heterogeneous households

We finally present transition dynamics in the model with heterogeneous households (as described in Section 3.4.1), under otherwise baseline parameter values and transfer adjustment. Figure A.10 in the Appendix shows that consumption inequality increases throughout the transition, with a large decline along the transition in consumption of the unskilled. This is because of the large dynamic decline in transfers. Because of the effects on marginal utility of the unskilled, they work more, unlike the baseline case with perfect consumption insurance. On the other hand, introducing such heterogeneity has little effect on the transition dynamics of aggregate output, mirroring the long-run results in Section 3.3.

5 Conclusion

We study aggregate and distributional effects of a permanent reduction in the capital tax rate in a dynamic equilibrium model with capital-skill complementarity. Such a tax reform leads to expansionary long-run aggregate effects, but is coupled with an increase in the skill premium. Moreover, the expansionary long-run aggregate effects are lower when distortionary labor or consumption tax rates have to increase to finance the capital tax rate cut. An extension to a model with heterogeneous households shows that consumption inequality also increases in the long-run.

We study transition dynamics and show that there are contractionary effects in the short-run, with a fall not just in consumption but also output, coupled with increases in wage inequality. We show however, that for transition dynamics it is imperative to study fiscal and monetary policy jointly. This is because the short-run effects depend critically on the monetary policy response: whether the central bank allows inflation to directly facilitate government debt stabilization and how inertially it raises interest rates.

Introducing some additional forms of heterogeneity is a potentially important extension. Our analysis of the short-run and the long-run suggests that the proposed tax reform will have heterogeneous effects on different generations. Thus, exploring generational heterogeneity is a particularly interesting avenue for future research. Introducing firm heterogeneity and financing constraints, similar to the household heterogeneity extension, might also be an interesting avenue for future research.

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6 Tables and figures

Table 1: Calibration

	Value	Description	References
Households			
β	0.9975	Time preference	Smets and Wouters (2007)
φ	1.0	Inverse of Frisch elasticity of labor supply	Smets and Wouters (2007)
$\bar{\omega}^s$	7.14	Labor supply dis-utility parameter	Steady-state $\bar{H}^s = 0.33$
$\bar{\omega}^u$	4.75		Steady-state $\bar{H}^u = 0.31$
d_e	0.031	Equipment capital depreciation	Krusell *et al.*(2000)
d_b	0.014	Structures capital depreciation	Krusell *et al.*(2000)
ξ	4.0	Investment adjustment cost	Smets and Wouters (2007)
N^s	0.5	Fraction of skilled labor	Lindquist (2004)
Firms			
σ	0.401	EoS between unskilled labor and equipment	Krusell *et al.*(2000)
ρ	-0.495	EoS between skilled labor and equipment	Krusell *et al.*(2000)
α	0.117	Structures capital Income share	Krusell *et al.*(2000)
λ	0.35	Equipment capital income share	Steady-state labor share: 56% (Elsby, Hobijn, and Şahin (2013))
μ	0.345	Unskilled labor income share	Steady-state skill premium: 60% (Krusell *et al.*(2000))
κ	50	Quadratic price adjustment cost	Ireland (2000)
θ	4.0	Elasticity of substitution between goods	Steady-state Markup: 33%
γ	1.0054	Long-run growth rate of output	Bhattarai, Lee, and Park (2016)
q_0	0.95	Relative price of structure to equipment capital	Maliar and Maliar (2011)
Government(Fiscal/Monetary Policy)			
$\bar{\tilde{b}}$	0.363	SS debt to GDP ratio	Data(See Appendix D)
$\bar{\tilde{G}}$	0.161	SS government spending to GDP ratio	Data(See Appendix D)
$\bar{\tilde{T}}^C$	0.009	SS consumption tax revenue to GDP ratio	Data(See Appendix D)
$\bar{\tilde{T}}^H$	0.128	SS labor tax revenue to GDP ratio	Data(See Appendix D)
$\lambda^s_{\tau^H}$	1.0	Effective rate of labor tax on skilled workers	
$\lambda^u_{\tau^H}$	1.0	Effective rate of labor tax on unskilled workers	
λ_b	0.812	Effective expensing rate of structure investment	Barro and Furman (2018)
λ_e	0.338	Effective expensing rate of equipment investment	Barro and Furman (2018)
χ^s_Φ	1	Fraction of profit distribution to skilled worker	
χ^s_S	0	Fraction of transfers distribution to skilled worker	
ρ^R	0.0	Interest rate smoothing parameter	
ϕ	1.5	Inflation feedback parameter under Taylor Principle	Bhattarai, Lee, and Park (2016)
	0.5	Inflation feedback parameter when inflation helps debt adjustment	
ρ^S	0.9	Transfers smoothing when transfers adjust	
	0.0	Otherwise	
ρ^H	0.9	Labor tax rate smoothing when labor tax rate adjusts	
	0.0	Otherwise	
ρ^C	0.9	Consumption tax rate smoothing when consumption tax rate adjusts	
	0.0	Otherwise	
ψ^H	0.0	Transfers or consumption tax rate adjust	
	0.05	Labor tax rate response to debt	
	0.002	Labor tax rate response when inflation helps debt adjustment	
ψ^C	0.0	Transfers or labor tax rate adjust	
	0.05	Consumption tax rate response to debt	

Figure 1: Long-run Effects of Permanent Capital Tax Rate Changes

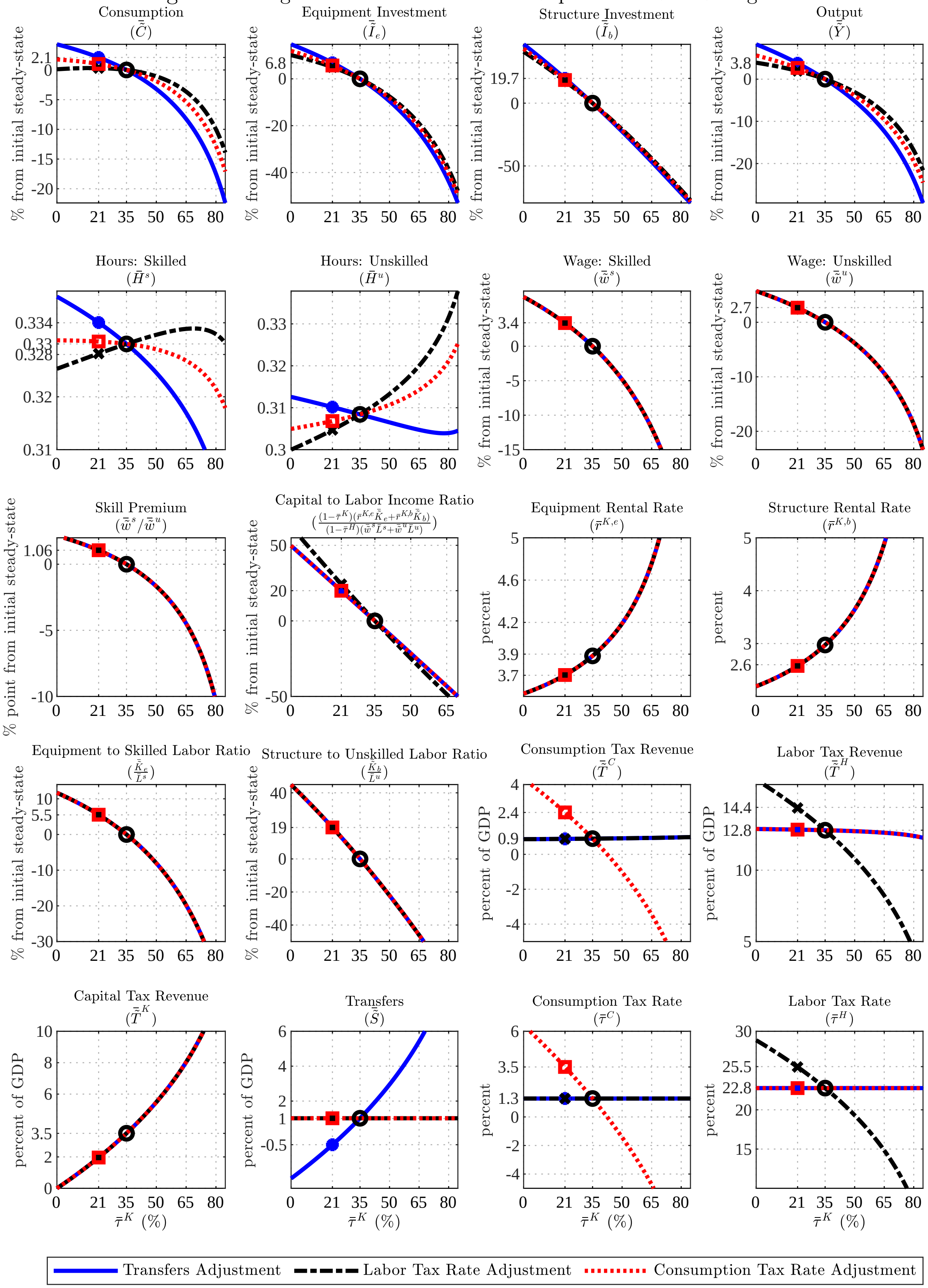


Figure 2: Long-run Effects of Permanent Capital Tax Rate Changes (Comparison with Nested Models)

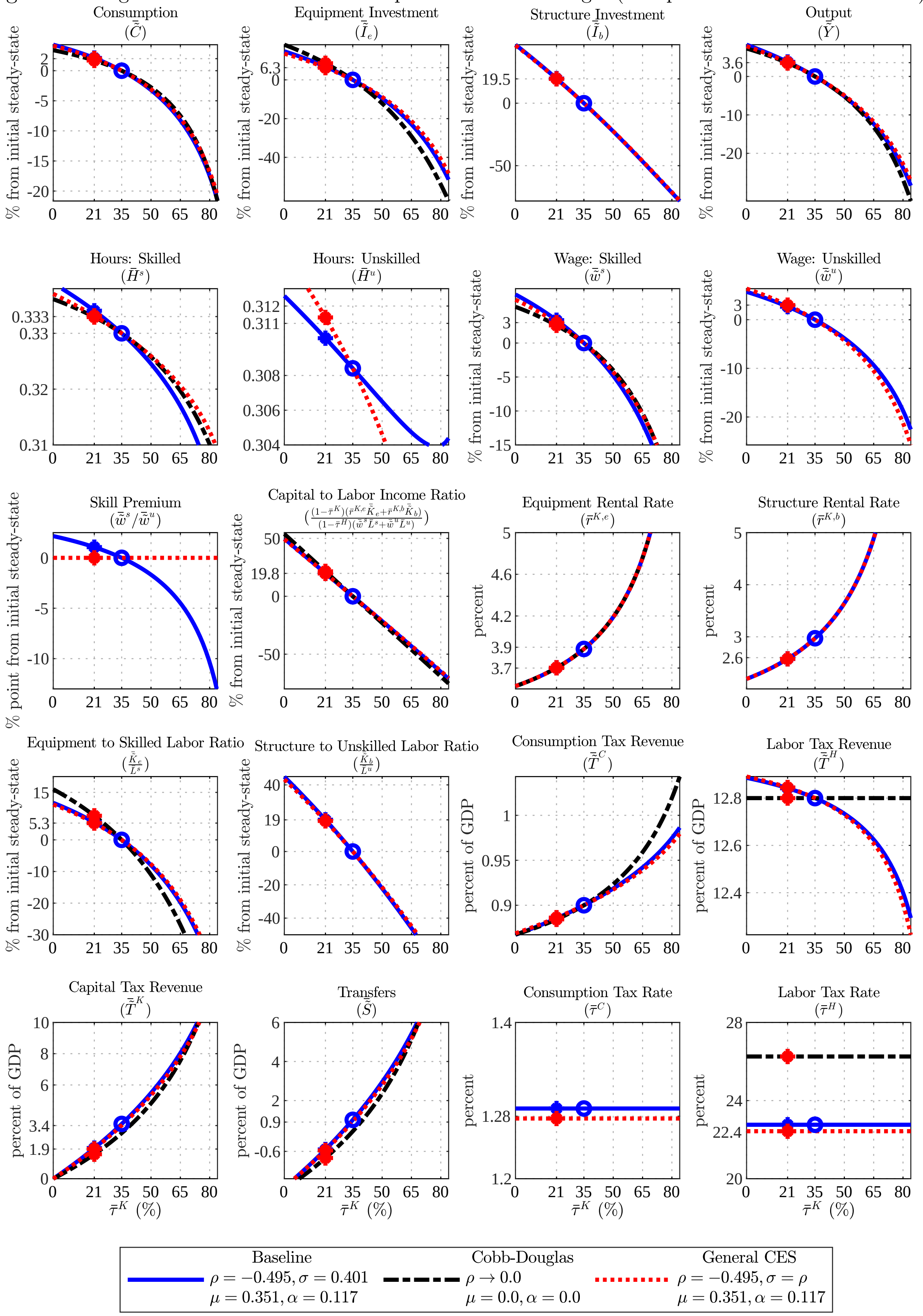


Figure 3: Long-run Effects of Permanent Capital Tax Rate Changes (Comparison with Heterogenous Households Model)

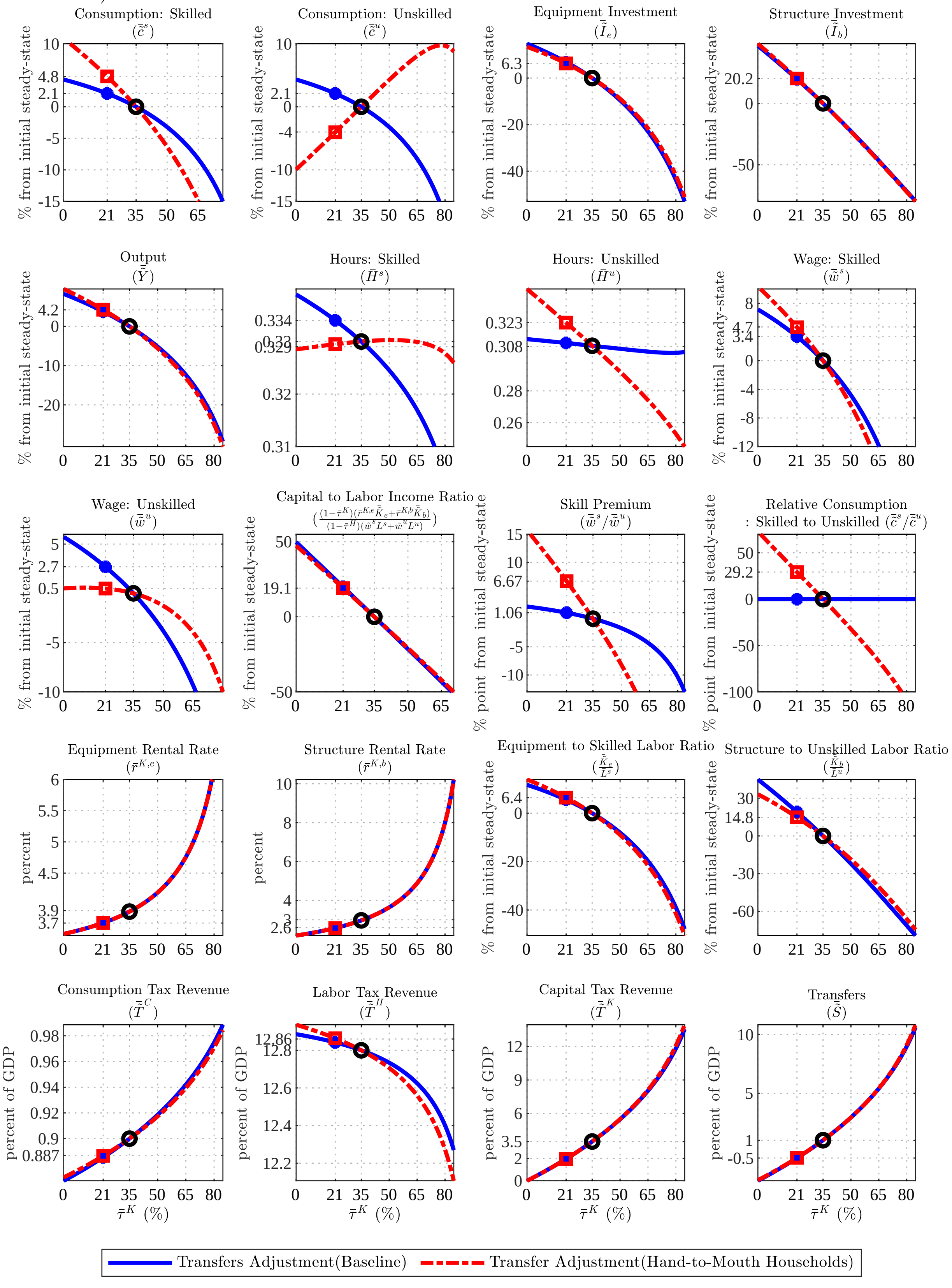


Figure 4: Transition Dynamics of Permanent Capital Tax Rate Changes Under Various Policy Regimes

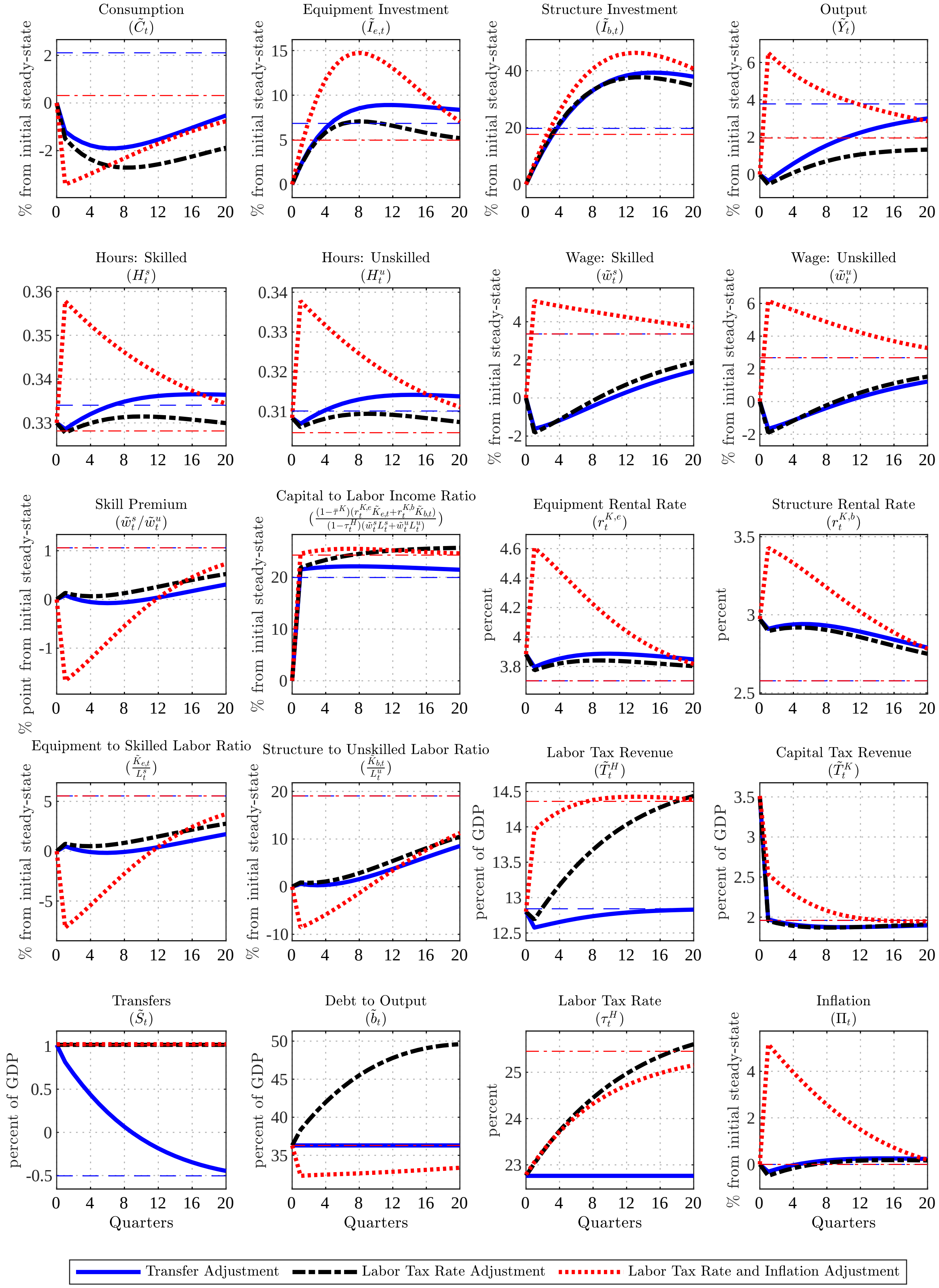
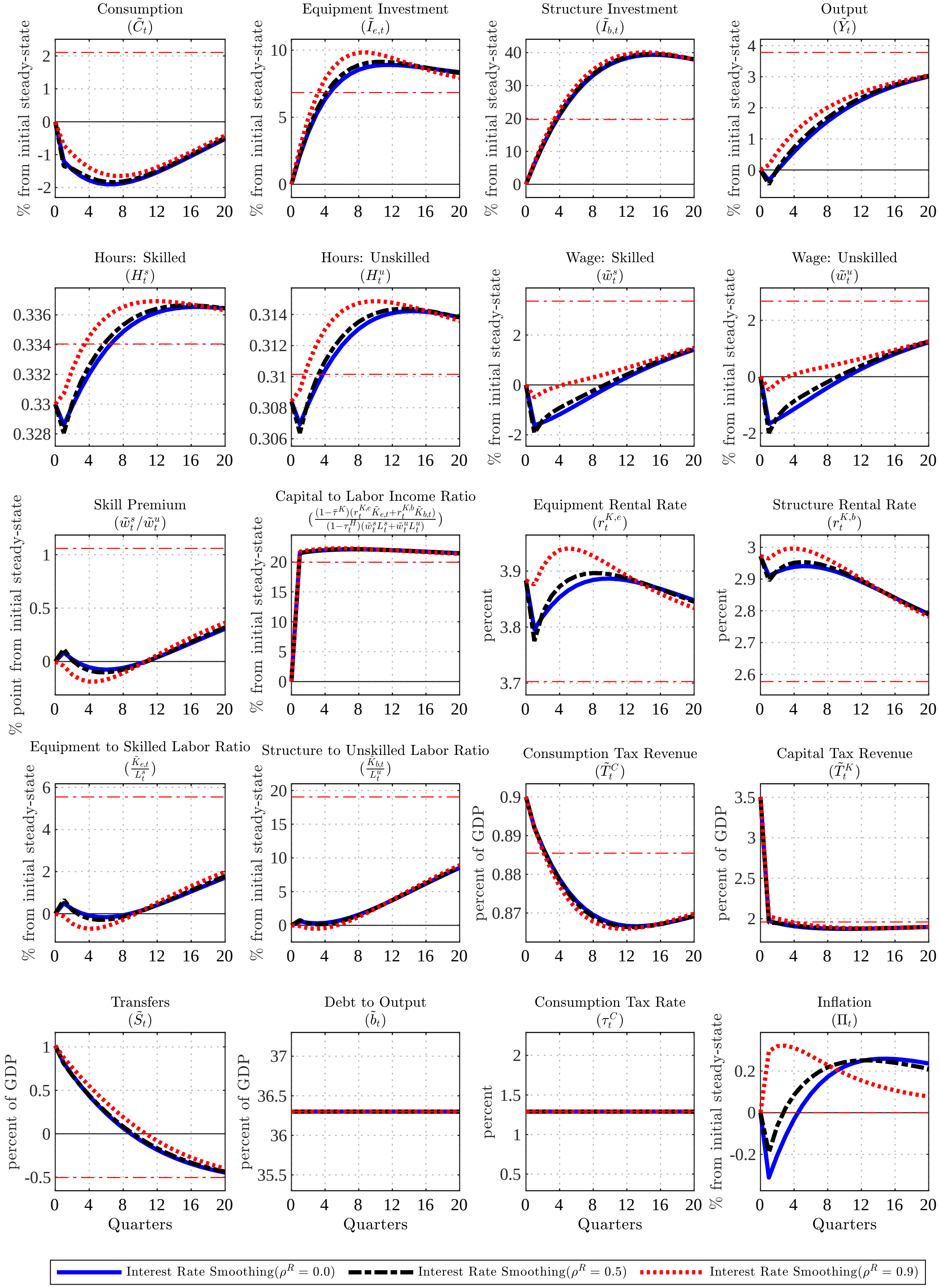


Figure 5: Transition Dynamics of Permanent Capital Tax Rate Changes (Transfers Adjustment, Smoothing in Monetary Policy)



A Model

A.1 Households

There are two types of agents: skilled(s) and unskilled(u). We assume there is no population growth. We denote N^i by the mass of i-type agents for $i \in \{s, u\}$. There are two types of capital stocks: capital structures ($K_{b,t}$) and capital equipment ($K_{e,t}$). The economy has two sectors: one sector produces consumption goods and capital structures, and the other sector produces capital equipment. Both sectors use the same technology; however, there is a technology factor specific to the capital equipment sector. We aggregate the production of the two sectors by introducing an exogenous relative price between consumption (structures) and equipment, q_t. We assume that q_t grows a constant rate γ_q, that is $q_t = q_0\gamma_q^t$. Households' maximization problem is as follows:

$$
\begin{aligned}
\max_{\left\{C_t^i, H_t^i, B_t^i, I_{b,t}^i, I_{e,t}^i, K_{b,t+1}^i, K_{e,t+1}^i, V_{t+1}^i\right\}} \quad & E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\log C_t^i - \bar{\omega}^i \frac{\left(H_t^i\right)^{1+\varphi}}{1+\varphi} \right] \right\} \\
\text{s.t.} \quad & \left(1+\tau_t^C\right) P_t C_t^i + P_t I_{b,t}^i + P_t I_{e,t}^i + B_t^i + E_t Q_{t,t+1} V_{t+1}^i \\
& = \left(1 - \lambda_{\tau^H}^i \tau_t^H\right) W_t^i H_t^i + R_{t-1} B_{t-1}^i + V_t^i \\
& + \left(1-\tau_t^K\right) R_t^{K,b} K_{b,t}^i + \left(1-\tau_t^K\right) R_t^{K,e} K_{e,t}^i \\
& + \lambda_b \tau_t^K P_t I_{b,t}^i + \lambda_e \tau_t^K P_t I_{e,t}^i \\
& + P_t \frac{\chi_\Phi^i}{N^i} \Phi_t + P_t \frac{\chi_S^i}{N^i} S_t, \\
K_{b,t+1}^i = \left(1-d_b\right) K_{b,t}^i + & \left(1 - \mathcal{S}\left(\frac{I_{b,t}^i}{I_{b,t-1}^i}\right)\right) I_{b,t}^i \\
K_{e,t+1}^i = \left(1-d_e\right) K_{e,t}^i + & \left(1 - \mathcal{S}\left(\frac{I_{e,t}^i}{I_{e,t-1}^i}\right)\right) I_{e,t}^i q_t
\end{aligned}
$$

where E is the expectation operator, C_t^i is consumption, H_t^i is hours, and $I_{b,t}^i$ and $I_{e,t}^i$ are capital structure investment and capital equipment investment, respectively. Similarly, $K_{b,t}^i$ and $K_{e,t}^i$ are the capital stock of structures and equipment, respectively. Households trade at time t one-period state-contingent nominal securities V_{t+1}^i at price $Q_{t,t+1}$, and hence fully insure against idiosyncratic risk. Thus, there is complete consumption insurance in the model. B_t^i is nominal risk-less one-period government bonds, Φ_t is profits from firms, and χ_Φ^i is the share of profits for i-type households.

P_t is the aggregate price level, W_t^i is nominal wage for type-i agent, and R_t is the nominal one-period interest rate. Moreover, $R_{b,t}^K$ and $R_{e,t}^K$ are the rental rate of capital invested in structures and equipment, respectively, while q_t is the relative price between consumption (structures) and equipment. S_t^i is lump-sum transfers from the government and χ_Φ^i is the fraction of the transfers for i-type households. τ_t^C is the tax rate on consumption, τ_t^H is the tax rate on wage income, and τ_t^K is the tax rate on capital income. The parameter $\lambda_{\tau^H}^i$ governs the (relative) effective labor tax rate on the two types of agents. β is the discount factor and d_b and d_e are the rates of depreciation of the capital stock invested in structures and equipment, respectively. Moreover, λ_b and λ_e are the rates of expensing of the capital stock invested in structures and equipment, respectively.

A.2 Firms

A.2.1 Final goods firms

Competitive final goods firms produce aggregate output Y_t by combining a continuum of differentiated intermediate goods using a CES production function $Y_t = \left(\int_0^1 Y_t\left(i\right)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}$, where θ is the elasticity of substitution between intermediate goods indexed by i. The corresponding optimal price index P_t for the final good is $P_t = \left(\int_0^1 P_t\left(i\right)^{1-\theta} di\right)^{\frac{1}{1-\theta}}$, where $P_t(i)$ is the price of intermediate goods and the optimal demand for $Y_t\left(i\right)$ is

$$Y_t\left(i\right) = \left(\frac{P_t\left(i\right)}{P_t}\right)^{-\theta} Y_t. \tag{6}$$

A.2.2 Intermediate goods firms

Intermediate goods firms indexed by i produce output using a CRS production function

$$Y_t\left(i\right) = A_t K_{b,t}^{\alpha}\left(i\right)\left[\mu L_{u,t}^{\sigma}\left(i\right) + \left(1-\mu\right)\left(\lambda K_{e,t}^{\rho}\left(i\right) + \left(1-\lambda\right) L_{s,t}^{\rho}\left(i\right)\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\alpha}{\sigma}} \tag{7}$$

where A_t represents exogenous economy-wide technological progress. The gross growth rate of technology is given by $a_t \equiv \frac{A_t}{A_{t-1}} = \bar{a}$. Firms rent capital and hire labor in economy wide competitive factor markets. Intermediate good firms also face price adjustment cost $\Xi\left(\frac{P_t(i)}{P_{t-1}(i)}\right) Y_t$ that has standard properties.

Firms problem is to

$$\max_{\{P_t(i),Y_t(i),H_t(i),K_t(i)\}} E_0\left\{\sum_{t=0}^{\infty}\beta^t\frac{\Lambda_t}{\Lambda_0}P_t\Phi_t\left(i\right)\right\}$$

subject to (6) and (7), where Λ_t is the marginal utility of nominal income and flow profits$\Phi_t\left(i\right)$ is given by

$$\Phi_t\left(i\right) = \frac{P_t\left(i\right)}{P_t}Y_t\left(i\right) - \frac{W_t^u}{P_t}L_{u,t}\left(i\right) - \frac{W_t^s}{P_t}L_{s,t}\left(i\right) - \frac{R_t^{K,b}}{P_t}K_{b,t}\left(i\right) - \frac{R_t^{K,e}}{P_t}K_{e,t}\left(i\right) - \Xi\left(\frac{P_t\left(i\right)}{P_{t-1}\left(i\right)}\right)Y_t.$$

A.3 Government

A.3.1 Government budget constraint

The government flow budget constraint, written by expressing fiscal variables as ratio of output, is given by

$$\frac{B_t}{P_tY_t} + \left(\frac{T_t^C}{Y_t} + \frac{T_t^H}{Y_t} + \frac{T_t^{K,b}}{Y_t} + \frac{T_t^{K,e}}{Y_t}\right) = R_{t-1}\frac{B_{t-1}}{P_{t-1}Y_{t-1}}\frac{1}{\pi_t}\frac{Y_{t-1}}{Y_t} + \frac{G_t}{Y_t} + \frac{S_t}{Y_t}$$

where

$$
\begin{aligned}
B_t &= \sum_{i\in\{s,u\}} N^i B_t^i \\
T_t^C &= \sum_{i\in\{s,u\}} N^i \tau_t^C C_t^i \\
T_t^H &= \tau_t^H \sum_{i\in\{s,u\}} \frac{W_t^i}{P_t} \lambda_{\tau^H}^i N^i H_t^i \\
T_t^{K,b} &= \tau_t^K \sum_{i\in\{s,u\}} N^i \left(\frac{R_t^{K,b}}{P_t} K_{b,t}^i - \lambda_b I_{b,t}^i \right) \\
T_t^{K,e} &= \tau_t^K \sum_{i\in\{s,u\}} N^i \left(\frac{R_t^{K,e}}{P_t} K_{e,t}^i - \lambda_e I_{e,t}^i \right) \\
S_t &= \sum_{i\in\{s,u\}} N^i S_t^i \\
N^s S_t^s &= \chi_S^s S_t \\
N^u S_t^u &= \chi_S^u S_t = (1-\chi_S^s)\, S_t
\end{aligned}
$$

and G_t is government spending on the final good.

A.3.2 Monetary policy

Monetary policy is given by a simple interest-rate feedback rule

$$
\frac{R_t}{\bar{R}} = \left[\frac{R_{t-1}}{\bar{R}}\right]^{\rho^R} \left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi}\right]^{(1-\rho^R)}
$$

where $\phi \geq 0$ is the feedback parameter on inflation $\left(\pi_t = \frac{P_t}{P_{t-1}}\right)$, $\bar{R}$ is the steady-state value of R_t, and $\bar{\pi}$ is the steady-state value of π_t. When $\phi > 1$, the standard case, the Taylor principle is satisfied. When $\phi < 1$, which we will also consider, inflation response will play a direct role in government debt stabilization along the transition.

A.3.3 Fiscal policy

We consider a one-time permanent change in the capital tax rate τ_t^K in period 0, when the economy is in the initial steady-state. In order to isolate the effects of the capital tax rate cut, $\frac{G_t}{Y_t}$ is kept unchanged from its initial steady-state value in all periods. The debt-to-GDP ratio, $\frac{B_t}{P_t Y_t}$, may deviate from the initial steady-state in the short run but will converge back to the initial steady-state in the long-run, through appropriate changes in fiscal instruments.

We will study both long-run effects of such permanent changes in the capital tax rate, as well as in extensions, full transition dynamics as the economy evolves towards the new steady-state. We consider the following fiscal policy adjustments that ensure that in the long-run, debt-to-GDP stays at the same level as the initial level through appropriate adjustment of fiscal instruments, while ensuring that distortionary tax rates adjust smoothly during the transition. First, only lump-sum transfers adjust to maintain $\frac{B_t}{P_t Y_t}$ constant at each point in time.[28] Second, only labor tax rates τ_t^H

[28]Since transfers are lump-sum and there is complete risk-sharing, the time-path of transfers does not matter, and so we just use a simple formulation.

adjust following the simple feedback rule

$$\tau_t^H - \bar{\tau}_{new}^H = \rho^H \left(\tau_{t-1}^H - \bar{\tau}_{new}^H\right) + \left(1-\rho^H\right)\psi^H \left(\frac{B_{t-1}}{P_{t-1}Y_{t-1}} - \overline{\frac{B}{PY}}\right)$$

where $\psi^H \geq 1-\beta$ is the feedback parameter on outstanding debt, $0 \leq \rho^H < 1$ governs labor tax rate smoothing, $\bar{\tau}_{new}^H$ is the new steady-state value of τ_t^H, and $\overline{\frac{B}{PY}}$ is the (initial and new) steady-state value of $\frac{B_t}{P_tY_t}$. Third, only consumption tax rates τ_t^C adjust following the simple feedback rule

$$\tau_t^C - \bar{\tau}_{new}^C = \rho^C \left(\tau_{t-1}^C - \bar{\tau}_{new}^C\right) + \left(1-\rho^C\right)\psi^C \left(\frac{B_{t-1}}{P_{t-1}Y_{t-1}} - \overline{\frac{B}{PY}}\right)$$

where $\psi^C \geq 1-\beta$ is the feedback parameter on outstanding debt, $0 \leq \rho^C < 1$ governs consumption tax rate smoothing, and $\bar{\tau}_{new}^C$ is the new steady-state value of τ_t^C.

For transition dynamics, the behavior of the monetary authority generally matters. In the three cases above, we have the monetary policy rule satisfying the Taylor principle, $\phi > 1$, which thereby, implies that inflation plays no direct role in government debt stabilization. We consider a fourth case to highlight the role of monetary policy response to inflation for transition dynamics. In this case, labor taxes adjust, but not sufficiently enough, as $0 < \psi^H < 1-\beta$, and inflation partly plays a direct role in government debt stabilization, as $\phi < 1$. Thus, in this fourth case, we allow debt stabilization, (only) along the transition, to occur partly through distortionary labor taxes and partly through inflation.

A.4 Market Clearing

$$N^s\left(C_t^s + I_{b,t}^s + I_{e,t}^s\right) + N^u\left(C_t^u + I_{b,t}^u + I_{e,t}^u\right) + G_t + \Xi\left(\frac{P_t}{P_{t-1}}\right)Y_t = Y_t$$
$$\int_0^1 L_{u,t}(i)\,di = N^u H_t^u$$
$$\int_0^1 L_{s,t}(i)\,di = N^s H_t^s$$
$$\int_0^1 K_{b,t}(i)\,di = N^s K_{b,t}^s + N^u K_{b,t}^u$$
$$\int_0^1 K_{e,t}(i)\,di = N^s K_{e,t}^s + N^u K_{e,t}^u$$
$$\int_0^1 \Phi_t(i)\,di = N^s \Phi_t^s + N^u \Phi_t^u$$

$$K_{b,t+1}^u = (1-d_b)K_{b,t}^u + \left(1 - \mathcal{S}\left(\frac{I_{b,t}^s}{I_{b,t-1}^s}\right)\right)I_{b,t}^u$$
$$K_{b,t+1}^s = (1-d_b)K_{b,t}^s + \left(1 - \mathcal{S}\left(\frac{I_{b,t}^s}{I_{b,t-1}^s}\right)\right)I_{b,t}^s$$
$$K_{e,t+1}^u = (1-d_e)K_{e,t}^u + \left(1 - \mathcal{S}\left(\frac{I_{e,t}^u}{I_{e,t-1}^u}\right)\right)I_{e,t}^u q_t$$
$$K_{e,t+1}^s = (1-d_e)K_{e,t}^s + \left(1 - \mathcal{S}\left(\frac{I_{e,t}^s}{I_{e,t-1}^s}\right)\right)I_{e,t}^s q_t$$

A.5 Nonlinear Equilibrium Conditions

In this section, we derive the equilibrium conditions that are necessary to solve the model.

A.5.1 Firms

- Production function

$$Y_t(i) = A_t K_{b,t}^{\alpha}(i)\left[\mu L_{u,t}^{\sigma}(i) + (1-\mu)\left(\lambda K_{e,t}^{\rho}(i) + (1-\lambda) L_{s,t}^{\rho}(i)\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\alpha}{\sigma}}$$

- Cost minimization: capital-labor ratio

$$R_t^{K,b} = \alpha MC_t(i) A_t K_{b,t}^{\alpha-1}(i)\left[\mu L_{u,t}^{\sigma}(i) + (1-\mu)\left(\lambda K_{e,t}^{\rho}(i) + (1-\lambda) L_{s,t}^{\rho}(i)\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\alpha}{\sigma}} = \alpha MC_t(i)\frac{Y_t(i)}{K_{b,t}(i)}$$

$$R_t^{K,e} = (1-\alpha) MC_t(i)\frac{Y_t(i)}{K_{e,t}(i)}\left(\frac{(1-\mu)\left(\lambda K_{e,t}^{\rho}(i) + (1-\lambda) L_{s,t}^{\rho}(i)\right)^{\frac{\sigma}{\rho}}}{\mu L_{u,t}^{\sigma}(i) + (1-\mu)\left(\lambda K_{e,t}^{\rho}(i) + (1-\lambda) L_{s,t}^{\rho}(i)\right)^{\frac{\sigma}{\rho}}}\right)\left(\frac{\lambda K_{e,t}^{\rho}(i)}{\lambda K_{e,t}^{\rho}(i) + (1-\lambda) L_{s,t}^{\rho}(i)}\right)$$

$$W_t^u = (1-\alpha) MC_t(i)\frac{Y_t(i)}{L_{u,t}(i)}\left(\frac{\mu L_{u,t}^{\sigma}(i)}{\mu L_{u,t}^{\sigma}(i) + (1-\mu)\left(\lambda K_{e,t}^{\rho}(i) + (1-\lambda) L_{s,t}^{\rho}(i)\right)^{\frac{\sigma}{\rho}}}\right)$$

$$W_t^s = (1-\alpha) MC_t(i)\frac{Y_t(i)}{L_{s,t}(i)}\left(\frac{(1-\mu)\left(\lambda K_{e,t}^{\rho}(i) + (1-\lambda) L_{s,t}^{\rho}(i)\right)^{\frac{\sigma}{\rho}}}{\mu L_{u,t}^{\sigma}(i) + (1-\mu)\left(\lambda K_{e,t}^{\rho}(i) + (1-\lambda) L_{s,t}^{\rho}(i)\right)^{\frac{\sigma}{\rho}}}\right)\left(\frac{(1-\lambda) L_{s,t}^{\rho}(i)}{\lambda K_{e,t}^{\rho}(i) + (1-\lambda) L_{s,t}^{\rho}(i)}\right)$$

- Skill-premium

$$\frac{W_t^s}{W_t^u} = \frac{(1-\mu)(1-\lambda)}{\mu}\left(\lambda\left(\frac{K_{e,t}(i)}{L_{s,t}(i)}\right)^{\rho} + (1-\lambda)\right)^{\frac{\sigma-\rho}{\rho}}\left(\frac{L_{u,t}(i)}{L_{s,t}(i)}\right)^{1-\sigma}$$

$$\frac{R_t^{K,e}}{R_t^{K,b}} = \frac{1-\alpha}{\alpha}\left(\frac{(1-\mu)\left(\lambda K_{e,t}^{\rho}(i) + (1-\lambda) L_{s,t}^{\rho}(i)\right)^{\frac{\sigma}{\rho}}}{\mu L_{u,t}^{\sigma}(i) + (1-\mu)\left(\lambda K_{e,t}^{\rho}(i) + (1-\lambda) L_{s,t}^{\rho}(i)\right)^{\frac{\sigma}{\rho}}}\right)\left(\frac{\lambda K_{e,t}^{\rho}(i)}{\lambda K_{e,t}^{\rho}(i) + (1-\lambda) L_{s,t}^{\rho}(i)}\right)\frac{K_{b,t}(i)}{K_{e,t}(i)}$$

$$\frac{W_t^s}{R_t^{K,e}} = \frac{1-\lambda}{\lambda}\left(\frac{K_{e,t}(i)}{L_{s,t}(i)}\right)^{1-\rho}$$

- Profit maximization:

$$\max_{P_t(i)} \; E_0\sum_{t=0}^{\infty}\beta^t\frac{\Lambda_t}{\Lambda_0}P_t\left[\left(\frac{P_t(i)}{P_t} - \frac{MC_t}{P_t}\right)Y_t(i) - \Xi\left(\frac{P_t(i)}{P_{t-1}(i)}\right)Y_t\right]$$

where

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t$$

The first-order condition is:

$$\Xi'\left(\frac{P_t(i)}{P_{t-1}(i)}\right)\frac{P_t}{P_{t-1}(i)}Y_t = \left((1-\theta) + \theta\frac{MC_t}{P_t(i)}\right)\left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t$$
$$+ \beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}\Xi'\left(\frac{P_{t+1}(i)}{P_t(i)}\right)\frac{P_{t+1}P_{t+1}(i)}{(P_t(i))^2}Y_{t+1}$$

- Profit

$$\Phi_t(i) = \frac{P_t(i)}{P_t} Y_t(i) - \frac{W_t^u}{P_t} L_{u,t}(i) - \frac{W_t^s}{P_t} L_{s,t}(i) - \frac{R_t^{K,b}}{P_t} K_{b,t}(i) - \frac{R_t^{K,e}}{P_t} K_{e,t}(i) - \Xi\left(\frac{P_t(i)}{P_{t-1}(i)}\right) Y_t$$

A.5.2 Households

- Maximization Problem:

$$\begin{aligned}
& E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\log C_t^i - \bar{\omega}^i \frac{\left(H_t^i\right)^{1+\varphi}}{1+\varphi} \right] \right\} \\
& - E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \Lambda_t^i \left\{ \left(1+\tau_t^C\right) P_t C_t^i + P_t I_{b,t}^i + Q_{t,t+1} V_{t+1}^i + \frac{P_t}{q_t} I_{e,t}^i + B_t^i \right\} \right\} \\
& + E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \Lambda_t^i \left\{ \left(1-\lambda_{\tau^H}^i \tau_t^H\right) W_t^i H_t^i + V_{t+1}^i + R_{t-1} B_{t-1}^i + P_t \Phi_t^i + P_t S_t^i \right\} \right\} \\
& + E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \Lambda_t^i \left\{ \left(1-\tau_t^K\right) R_t^{K,b} K_{b,t}^i + \left(1-\tau_t^K\right) R_t^{K,e} K_{e,t}^i + \lambda_b \tau_t^K P_t I_{b,t}^i + \lambda_e \tau_t^K P_t I_{e,t}^i \right\} \right\} \\
& + E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \Psi_{b,t}^i \left\{ \left(1-d_b\right) K_{b,t}^i + \left(1 - \mathcal{S}\left(\frac{I_{b,t}^i}{I_{b,t-1}^i}\right)\right) I_{b,t}^i - K_{b,t+1}^i \right\} \right\} \\
& + E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \Psi_{e,t}^i \left\{ \left(1-d_e\right) K_{e,t}^i + \left(1 - \mathcal{S}\left(\frac{I_{e,t}^i}{I_{e,t-1}^i}\right)\right) I_{e,t}^i q_t - K_{e,t+1}^i \right\} \right\}
\end{aligned}$$

- FOCs:

$$\begin{aligned}
C_t^i &: \; P_t \Lambda_t^i \left(1+\tau^C\right) = \frac{1}{C_t^i} \\
H_t^i &: \; \Lambda_t^i \left(1-\lambda_{\tau^H}^i \tau_t^H\right) W_t^i = \bar{\omega} \left(H_t^i\right)^{\varphi} \\
B_t^i &: \; \Lambda_t^i = \beta R_t E_t \left\{\Lambda_{t+1}^i\right\} \\
V_{t+1}^i &: \; Q_{t,t+1} = \beta \left\{ \frac{\Lambda_{t+1}^i}{\Lambda_t^i} \right\} \\
K_{b,t+1}^i &: \; \Psi_{b,t}^i = \beta E_t \left\{ \left(1-d_b\right) \Psi_{b,t+1}^i + \left(1-\tau_{t+1}^K\right) R_{t+1}^{K,b} \Lambda_{t+1}^i \right\} \\
I_{b,t}^i &: \; \left(1-\lambda_b \tau_t^K\right) P_t \Lambda_t^i = \Psi_{b,t}^i \left(1 - \mathcal{S}\left(\frac{I_{b,t}^i}{I_{b,t-1}^i}\right) - \mathcal{S}'\left(\frac{I_{b,t}^i}{I_{b,t-1}^i}\right) \frac{I_{b,t}^i}{I_{b,t-1}^i} \right) \\
& \quad + \beta E_t \left\{ \Psi_{b,t+1}^i \left(\frac{I_{b,t+1}^i}{I_{b,t}^i}\right)^2 \mathcal{S}'\left(\frac{I_{b,t+1}^i}{I_{b,t}^i}\right) \right\} \\
K_{e,t+1}^i &: \; \Psi_{e,t}^i = \beta E_t \left\{ \left(1-d_e\right) \Psi_{e,t+1}^i + \left(1-\tau_{t+1}^K\right) R_{t+1}^{K,e} \Lambda_{t+1}^i \right\} \\
I_{e,t}^i &: \; \left(1-\lambda_e \tau_t^K\right) P_t \Lambda_t^i = q_t \Psi_{e,t}^i \left(1 - \mathcal{S}\left(\frac{I_{e,t}^i}{I_{e,t-1}^i}\right) - \mathcal{S}'\left(\frac{I_{e,t}^i}{I_{e,t-1}^i}\right) \frac{I_{e,t}^i}{I_{e,t-1}^i} \right) \\
& \quad + \beta E_t \left\{ q_{t+1} \Psi_{e,t+1}^i \left(\frac{I_{e,t+1}^i}{I_{e,t}^i}\right)^2 \mathcal{S}'\left(\frac{I_{e,t+1}^i}{I_{e,t}^i}\right) \right\}
\end{aligned}$$

- Capital accumulation:

$$
\begin{aligned}
K_{b,t+1}^{u} &= (1-d_b)\,K_{b,t}^{u} + \left(1-\mathcal{S}\left(\frac{I_{b,t}^{u}}{I_{b,t-1}^{u}}\right)\right) I_{b,t}^{u}\\
K_{b,t+1}^{s} &= (1-d_b)\,K_{b,t}^{s} + \left(1-\mathcal{S}\left(\frac{I_{b,t}^{s}}{I_{b,t-1}^{s}}\right)\right) I_{b,t}^{s}\\
K_{e,t+1}^{u} &= (1-d_e)\,K_{e,t}^{u} + \left(1-\mathcal{S}\left(\frac{I_{e,t}^{u}}{I_{e,t-1}^{u}}\right)\right) I_{e,t}^{u}\\
K_{e,t+1}^{s} &= (1-d_e)\,K_{e,t}^{s} + \left(1-\mathcal{S}\left(\frac{I_{e,t}^{s}}{I_{e,t-1}^{s}}\right)\right) I_{e,t}^{s}
\end{aligned}
$$

A.5.3 Government and Market Clearing

- Government budget constraint

$$
\frac{B_t}{P_t Y_t} + \left(\frac{T_t^C + T_t^H + T_t^{K,b} + T_t^{K,e}}{Y_t}\right) = R_{t-1}\frac{B_{t-1}}{P_{t-1}Y_{t-1}}\frac{1}{\pi_t}\frac{Y_{t-1}}{Y_t} + \frac{G_t}{Y_t} + \frac{S_t}{Y_t}
$$

where

$$
\begin{aligned}
B_t &= \sum_{i\in\{s,u\}} N^i B_t^i\\
T_t^C &= \sum_{i\in\{s,u\}} N^i \tau_t^C C_t^i\\
T_t^H &= \tau_t^H \sum_{i\in\{s,u\}} \frac{W_t^i}{P_t}\lambda_{\tau^H}^i N^i H_t^i\\
T_t^{K,b} &= \tau_t^K \sum_{i\in\{s,u\}} N^i \left(\frac{R_t^{K,b}}{P_t}K_{b,t}^i - \lambda_b I_{b,t}^i\right)\\
T_t^{K,e} &= \tau_t^K \sum_{i\in\{s,u\}} N^i \left(\frac{R_t^{K,e}}{P_t}K_{e,t}^i - \lambda_e I_{e,t}^i\right)\\
S_t &= \sum_{i\in\{s,u\}} N^i S_t^i\\
N^s S_t^s &= \chi_S^s S_t\\
N^u S_t^u &= \chi_S^u S_t = (1-\chi_S^s)\,S_t
\end{aligned}
$$

- Profit distribution

$$
\begin{aligned}
\Phi_t &= \sum_{i\in\{s,u\}} N^i \Phi_t^i\\
N^s \Phi_t^s &= \chi_\Phi^s \Phi_t\\
N^u \Phi_t^u &= \chi_\Phi^u \Phi_t = (1-\chi_\Phi^s)\,\Phi_t
\end{aligned}
$$

- Resource constraint:

$$
N^s\left(C_t^s + I_{b,t}^s + I_{e,t}^s\right) + N^u\left(C_t^u + I_{b,t}^u + I_{e,t}^u\right) + G_t + \Xi\left(\frac{P_t}{P_{t-1}}\right)Y_t = Y_t
$$

A.6 Stationary Equilibrium

We consider a symmetric equilibrium across firms, where all firms set the same price and produce the same amount of output.

A.6.1 Notations

A balanced growth path can be achieved if $\frac{A_t}{A_{t-1}} = \bar{a} = \gamma^{1-\alpha}$and $\frac{q_t}{q_{t-1}} = \gamma_q = \frac{1}{\gamma}$, i.e. $\gamma_q \gamma = 1$. In this case, the growth rate of output $\frac{Y_t}{Y_{t-1}} = \gamma$.

$$\begin{aligned}
\text{Quantities :} & \tilde{C}_t^i = \frac{C_t^i}{\gamma^t},\ \tilde{I}_{b,t}^i = \frac{I_{b,t}^i}{\gamma^t},\ \tilde{I}_{e,t}^i = \frac{I_{e,t}^i}{\gamma^t},\ \tilde{K}_{b,t}^i = \frac{K_{b,t}^i}{\gamma^t},\ \tilde{K}_{e,t}^i = \frac{K_{e,t}^i}{(\gamma_q\gamma)^t} \text{ for } i \in \{s,u\} \\
& \tilde{Y}_t = \frac{Y_t}{\gamma^t},\ \tilde{K}_{b,t} = \frac{K_{b,t}}{\gamma^t},\ \tilde{K}_{e,t} = \frac{K_{e,t}}{(\gamma_q\gamma)^t} \\
\text{Prices :} & \tilde{w}_t^s = \frac{W_t^s}{P_t\gamma^t},\ \tilde{w}_t^u = \frac{W_t^u}{P_t\gamma^t},\ r_t^{K,b} = \frac{R_t^{K,b}}{P_t},\ r_t^{K,e} = \frac{R_t^{K,e}}{P_t\gamma^t} \\
& \tilde{\Phi}_t = \frac{\Phi_t}{\gamma^t},\ \tilde{\Phi}_t^i = \frac{\Phi_t^i}{\gamma^t},\ \pi_t = \frac{P_t}{P_{t-1}},\ mc_t = \frac{MC_t}{P_t} \\
\text{Fiscal variables:} & \tilde{b}_t = \frac{B_t}{P_tY_t},\ \tilde{G}_t = \frac{G_t}{Y_t},\ \tilde{T}_t^C = \frac{T_t^C}{Y_t},\ \tilde{T}_t^H = \frac{T_t^H}{Y_t} \\
& \tilde{T}_t^{K,b} = \frac{T_t^{K,b}}{Y_t},\ \tilde{T}_t^{K,e} = \frac{T_t^{K,e}}{Y_t},\ \tilde{S}_t = \frac{S_t}{Y_t},\ \tilde{S}_t^i = \frac{S_t^i}{Y_t} \\
\text{Multipliers:} & \tilde{\Lambda}_t^i = \gamma^t P_t \Lambda_t^i,\ \tilde{\Psi}_{b,t}^i = \gamma^t \Psi_{b,t}^i,\ \tilde{\Psi}_{e,t}^i = \Psi_{e,t}^i
\end{aligned}$$

A.6.2 Stationary Equilibrium Conditions

- Production function (Let $A_0 = 1$)

$$\tilde{Y}_t = \tilde{K}_{b,t}^{\alpha}\left[\mu L_{u,t}^{\sigma} + (1-\mu)\left(\lambda\tilde{K}_{e,t}^{\rho} + (1-\lambda)L_{s,t}^{\rho}\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\alpha}{\sigma}} \tag{8}$$

- Cost minimization

$$r_t^{K,b} = \alpha mc_t \frac{\tilde{Y}_t}{\tilde{K}_{b,t}} \tag{9}$$

$$r_t^{K,e} = (1-\alpha)mc_t\frac{\tilde{Y}_t}{\tilde{K}_{e,t}}\left(\frac{(1-\mu)\left(\lambda\tilde{K}_{e,t}^{\rho} + (1-\lambda)L_{s,t}^{\rho}\right)^{\frac{\sigma}{\rho}}}{\mu L_{u,t}^{\sigma} + (1-\mu)\left(\lambda\tilde{K}_{e,t}^{\rho} + (1-\lambda)L_{s,t}^{\rho}\right)^{\frac{\sigma}{\rho}}}\right)\left(\frac{\lambda\tilde{K}_{e,t}^{\rho}}{\lambda\tilde{K}_{e,t}^{\rho} + (1-\lambda)L_{s,t}^{\rho}}\right) \tag{10}$$

$$\tilde{w}_t^u = (1-\alpha)mc_t\frac{\tilde{Y}_t}{L_{u,t}}\left(\frac{\mu L_{u,t}^{\sigma}}{\mu L_{u,t}^{\sigma} + (1-\mu)\left(\lambda\tilde{K}_{e,t}^{\rho} + (1-\lambda)L_{s,t}^{\rho}\right)^{\frac{\sigma}{\rho}}}\right) \tag{11}$$

$$\tilde{w}_t^s = (1-\alpha)mc_t\frac{\tilde{Y}_t}{L_{s,t}}\left(\frac{(1-\mu)\left(\lambda\tilde{K}_{e,t}^{\rho} + (1-\lambda)L_{s,t}^{\rho}\right)^{\frac{\sigma}{\rho}}}{\mu L_{u,t}^{\sigma} + (1-\mu)\left(\lambda\tilde{K}_{e,t}^{\rho} + (1-\lambda)L_{s,t}^{\rho}\right)^{\frac{\sigma}{\rho}}}\right)\left(\frac{(1-\lambda)L_{s,t}^{\rho}}{\lambda\tilde{K}_{e,t}^{\rho} + (1-\lambda)L_{s,t}^{\rho}}\right) \tag{12}$$

- Skill-premium

$$\frac{\tilde{w}_t^s}{\tilde{w}_t^u} = \frac{(1-\mu)(1-\lambda)}{\mu} \left(\lambda \left(\frac{\tilde{K}_{e,t}}{L_{s,t}} \right)^{\rho} + (1-\lambda) \right)^{\frac{\sigma-\rho}{\rho}} \left(\frac{L_{u,t}}{L_{s,t}} \right)^{1-\sigma}$$

- Firms' maximization: Phillips Curve

$$\Xi'(\pi_t)\pi_t = ((1-\theta)+\theta mc_t) + \beta E_t \frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t} \frac{\tilde{Y}_{t+1}}{\tilde{Y}_t} \Xi'(\pi_{t+1})\pi_{t+1} \tag{13}$$

- Profit

$$\tilde{\Phi}_t = \tilde{Y}_t - \tilde{w}_t^u L_{u,t} - \tilde{w}_t^s L_{s,t} - r_t^{K,b} \tilde{K}_{b,t} - r_t^{K,e} \tilde{K}_{e,t} - \Xi(\pi_t)\tilde{Y}_t \tag{14}$$

- Households
  - Marginal Utilities:

$$\tilde{\Lambda}_t^i \left(1+\tau_t^C\right) = \frac{1}{\tilde{C}_t^i} \tag{15}$$

$$\tilde{\Lambda}_t^i \left(1-\lambda_{\tau^H}^i \tau_t^H\right) \tilde{w}_t^i = \bar{\omega}^i \left(H_t^i\right)^{\varphi}$$

  - FOCs and Capital Accumulation

$$\tilde{w}_t^i \frac{1-\lambda_{\tau^H}^i \tau_t^H}{1+\tau_t^C} = \bar{\omega}^i \tilde{C}_t^i \left(H_t^i\right)^{\varphi} \tag{16}$$

$$\tilde{\Lambda}_t^i = \frac{\beta}{\gamma} R_t E_t \left\{ \tilde{\Lambda}_{t+1}^i \frac{1}{\pi_{t+1}} \right\} \tag{17}$$

$$Q_{t,t+1} = \frac{\beta}{\gamma} \frac{\tilde{\Lambda}_{t+1}^i}{\tilde{\Lambda}_t^i} \frac{1}{\pi_{t+1}} \tag{18}$$

$$\tilde{\Psi}_{b,t}^i = \frac{\beta}{\gamma} E_t \left\{ (1-d_b)\tilde{\Psi}_{b,t+1}^i + \left(1-\tau_{t+1}^K\right) r_{t+1}^{K,b} \tilde{\Lambda}_{t+1}^i \right\} \tag{19}$$

$$\begin{aligned} \left(1-\lambda_b \tau_t^K\right)\tilde{\Lambda}_t^i &= \tilde{\Psi}_{b,t}^i \left(1 - \mathcal{S}\left(\frac{\tilde{I}_{b,t}^i}{\tilde{I}_{b,t-1}^i}\gamma \right) - \mathcal{S}'\left(\frac{\tilde{I}_{b,t}^i}{\tilde{I}_{b,t-1}^i}\gamma \right) \frac{\tilde{I}_{b,t}^i}{\tilde{I}_{b,t-1}^i}\gamma \right) \\ &+ \frac{\beta}{\gamma} E_t \left\{ \tilde{\Psi}_{b,t+1}^i \left(\frac{\tilde{I}_{b,t+1}^i}{\tilde{I}_{b,t}^i}\gamma \right)^2 \mathcal{S}'\left(\frac{\tilde{I}_{b,t+1}^i}{\tilde{I}_{b,t}^i}\gamma \right) \right\} \end{aligned} \tag{20}$$

$$\tilde{\Psi}_{e,t}^i = \beta E_t \left\{ (1-d_e)\tilde{\Psi}_{e,t+1}^i + \left(1-\tau_{t+1}^K\right) r_{t+1}^{K,e} \tilde{\Lambda}_{t+1}^i \right\} \tag{21}$$

$$\begin{aligned} \left(1-\lambda_e \tau_t^K\right)\frac{1}{q_0}\tilde{\Lambda}_t^i &= \tilde{\Psi}_{e,t}^i \left(1 - \mathcal{S}\left(\frac{\tilde{I}_{e,t}^i}{\tilde{I}_{e,t-1}^i}\gamma \right) - \mathcal{S}'\left(\frac{\tilde{I}_{e,t}^i}{\tilde{I}_{e,t-1}^i}\gamma \right) \frac{\tilde{I}_{e,t}^i}{\tilde{I}_{e,t-1}^i}\gamma \right) \\ &+ \frac{\beta}{\gamma} E_t \left\{ \tilde{\Psi}_{e,t+1}^i \left(\frac{\tilde{I}_{e,t+1}^i}{\tilde{I}_{e,t}^i}\gamma \right)^2 \mathcal{S}'\left(\frac{\tilde{I}_{e,t+1}^i}{\tilde{I}_{e,t}^i}\gamma \right) \right\} \end{aligned} \tag{22}$$

$$\gamma \tilde{K}_{b,t+1}^i = (1-d_b)\tilde{K}_{b,t}^i + \left(1 - \mathcal{S}\left(\frac{\tilde{I}_{b,t}^i}{\tilde{I}_{b,t-1}^i}\gamma \right) \right) \tilde{I}_{b,t}^i \tag{23}$$

$$\tilde{K}_{e,t+1}^i = (1-d_e)\tilde{K}_{e,t}^i + \left(1 - \mathcal{S}\left(\frac{\tilde{I}_{e,t}^i}{\tilde{I}_{e,t-1}^i}\gamma \right) \right) \tilde{I}_{e,t}^i q_0 \tag{24}$$

- Budget constraint

$$\left(1+\tau_t^C\right)\tilde{C}_t^s+\tilde{I}_{b,t}^s+\tilde{I}_{e,t}^s+\tilde{b}_t^s\tilde{Y}_t=\left(1-\lambda_{\tau^H}^s\tau_t^H\right)\tilde{w}_t^sH_t^s+R_{t-1}\tilde{b}_{t-1}^s\tilde{Y}_{t-1}\frac{1}{\gamma\pi_t}$$
$$+\left(1-\tau_t^K\right)r_t^{K,b}\tilde{K}_{b,t}^s+\left(1-\tau_t^K\right)r_t^{K,e}\tilde{K}_{e,t}^s+\frac{\chi_\Phi^s}{N^s}\tilde{\Phi}_t+\frac{\chi_S^s}{N^s}\tilde{S}_t\tilde{Y}_t$$
$$+\lambda_b\tau_t^K\tilde{I}_{b,t}^s+\lambda_e\tau_t^K\tilde{I}_{e,t}^s$$

- Resource constraint

$$N^s\left(\tilde{C}_t^s+\tilde{I}_{b,t}^s+\tilde{I}_{e,t}^s\right)+N^u\left(\tilde{C}_t^u+\tilde{I}_{b,t}^u+\tilde{I}_{e,t}^u\right)=\left(1-\tilde{G}_t-\Xi\left(\pi_t\right)\right)\tilde{Y}_t \tag{25}$$

- Market clearing

$$\tilde{C}_t=N^s\tilde{C}_t^s+N^u\tilde{C}_t^u \tag{26}$$
$$L_{u,t}=N^uH_t^u \tag{27}$$
$$L_{s,t}=N^sH_t^s \tag{28}$$
$$\tilde{K}_{b,t}=N^s\tilde{K}_{b,t}^s+N^u\tilde{K}_{b,t}^u \tag{29}$$
$$\tilde{K}_{e,t}=N^s\tilde{K}_{e,t}^s+N^u\tilde{K}_{e,t}^u \tag{30}$$
$$\tilde{I}_{b,t}=N^s\tilde{I}_{b,t}^s+N^u\tilde{I}_{b,t}^u \tag{31}$$
$$\tilde{I}_{e,t}=N^s\tilde{I}_{e,t}^s+N^u\tilde{I}_{e,t}^u \tag{32}$$

- Government budget constraint

$$\tilde{b}_t+\tilde{T}_t^C+\tilde{T}_t^H+\tilde{T}_t^{K,b}+\tilde{T}_t^{K,e}=R_{t-1}\tilde{b}_{t-1}\frac{1}{\pi_t\gamma}\frac{\tilde{Y}_{t-1}}{\tilde{Y}_t}+\tilde{G}_t+\tilde{S}_t \tag{33}$$

where

$$\tilde{T}_t^C=\tau^C\frac{\tilde{C}_t}{\tilde{Y}_t},\ \tilde{T}_t^H=\tau_t^H\sum_{i\in s,u}\left(\lambda_{\tau^H}^i\tilde{w}_t^i\frac{L_{i,t}}{\tilde{Y}_t}\right),$$
$$\tilde{T}_t^{K,b}=\tau_t^K\left(r_t^{K,b}\frac{\tilde{K}_{b,t}}{\tilde{Y}_t}-\lambda_b\frac{\tilde{I}_{b,t}}{\tilde{Y}_t}\right),\ \tilde{T}_t^{K,e}=\tau_t^K\left(r_t^{K,e}\frac{\tilde{K}_{e,t}}{\tilde{Y}_t}-\lambda_e\frac{\tilde{I}_{e,t}}{\tilde{Y}_t}\right)$$

- Fiscal Policy Rules

$$\tau_t^H-\bar{\tau}_{new}^H=\rho^H\left(\tau_{t-1}^H-\bar{\tau}_{new}^H\right)+\left(1-\rho^H\right)\psi^H\left(\tilde{b}_{t-1}-\bar{\tilde{b}}\right) \tag{34}$$
$$\tau_t^C-\bar{\tau}_{new}^C=\rho^C\left(\tau_{t-1}^C-\bar{\tau}_{new}^C\right)+\left(1-\rho^C\right)\psi^C\left(\tilde{b}_{t-1}-\bar{\tilde{b}}\right) \tag{35}$$
$$\tau_t^K=\begin{cases}\bar{\tau}^K & \text{if } t=0\\ \bar{\tau}_{New}^K & \text{if } t>0\end{cases} \tag{36}$$
$$\tilde{G}_t=\bar{\tilde{G}} \tag{37}$$

- Monetary policy

$$\frac{R_t}{\bar{R}}=\left[\frac{R_{t-1}}{\bar{R}}\right]^{\rho^R}\left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi}\right]^{\left(1-\rho^R\right)} \tag{38}$$

- Government debt to GDP (or transfers):

$$\begin{cases} \tilde{b}_t = \bar{\tilde{b}} & \text{if adjusting transfers} \\ \tilde{S}_t = \bar{\tilde{S}} & \text{if adjusting labor tax rate} \end{cases} \tag{39}$$

A.7 Steady State

Recall that in steady-state, $\mathcal{S}(\gamma) = \mathcal{S}'(\gamma) = 0$:

From (13), we get

$$\bar{mc} = \frac{\theta - 1}{\theta}.$$

From (19) and (20), we get

$$\bar{r}^{K,b} = \frac{\frac{\gamma}{\beta} - (1 - d_b)}{1 - \bar{\tau}^K} \tag{40}$$

$$\bar{r}^{K,e} = \frac{1}{q_0} \frac{\frac{1}{\beta} - (1 - d_e)}{1 - \bar{\tau}^K} \tag{41}$$

From the production function, we get

$$\frac{\bar{\tilde{Y}}}{\bar{L}_s} = \left(\frac{\bar{\tilde{K}}_b}{\bar{L}_s}\right)^{\alpha} \left[\mu \left(\frac{\bar{L}_u}{\bar{L}_s}\right)^{\sigma} + (1-\mu) \left(\lambda \left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho} + (1-\lambda)\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\alpha}{\sigma}} \tag{42}$$

From firms' optimality conditions, we get

$$\bar{\tilde{w}}^s = (1-\lambda)(1-\mu)(1-\alpha)\,\bar{mc} \left(\frac{\bar{r}^{K,b}}{\alpha \bar{mc}}\right)^{1-\frac{\sigma}{1-\alpha}} \left(\frac{\bar{\tilde{K}}_b}{\bar{L}_s}\right)^{1-\sigma} \left(\lambda \left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho} + (1-\lambda)\right)^{\frac{\sigma}{\rho}-1} \tag{43}$$

$$\bar{\tilde{w}}^u \left(\frac{\bar{L}_u}{\bar{L}_s}\right)^{1-\sigma} = \mu(1-\alpha)\,\bar{mc} \left(\frac{\bar{r}^{K,b}}{\alpha \bar{mc}}\right)^{\frac{1-\alpha-\sigma}{1-\alpha}} \left(\frac{\bar{\tilde{K}}_b}{\bar{L}_s}\right)^{1-\sigma} \tag{44}$$

$$\bar{r}^{K,e} \left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{1-\rho} = \lambda(1-\mu)(1-\alpha)\,\bar{mc} \left(\frac{\bar{r}^{K,b}}{\alpha \bar{mc}}\right)^{1-\frac{\sigma}{1-\alpha}} \left(\frac{\bar{\tilde{K}}_b}{\bar{L}_s}\right)^{1-\sigma} \left(\lambda \left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho} + (1-\lambda)\right)^{\frac{\sigma}{\rho}-1} \tag{45}$$

$$\left(\frac{\bar{r}^{K,b}}{\alpha \bar{mc}}\right) = \left(\frac{\tilde{K}_b}{\bar{L}_s}\right)^{\alpha-1} \left[\mu \left(\frac{\bar{L}_u}{\bar{L}_s}\right)^{\sigma} + (1-\mu) \left(\lambda \left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho} + (1-\lambda)\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\alpha}{\sigma}} \tag{46}$$

Also, from skilled-household's budget constraint, we have budget constraint

$$\begin{aligned}
\left(1+\bar{\tau}^C\right) \frac{\bar{\tilde{C}}^s}{\bar{\tilde{Y}}} &= \left(1 - \lambda^s_{\tau H} \bar{\tau}^H\right) \bar{\tilde{w}}^s \frac{\bar{H}^s}{\bar{\tilde{Y}}} + \left(\left(1-\bar{\tau}^K\right) \bar{r}^{K,b} + ((1-d_b) - \gamma)\right) \frac{\bar{\tilde{K}}^s_b}{\bar{\tilde{Y}}} \\
&+ \left(\left(1-\bar{\tau}^K\right) \bar{r}^{K,e} - \frac{d_e}{q_0}\right) \frac{\bar{\tilde{K}}^s_e}{\bar{\tilde{Y}}} + \frac{\chi_\Phi}{N^s} \frac{\bar{\tilde{\Phi}}}{\bar{\tilde{Y}}} + \left(\frac{1}{\beta} - 1\right) \bar{\tilde{b}}^s + \frac{\chi_S}{N^s} \bar{\tilde{S}} \\
&+ \bar{\tau}^K \left(\lambda_b \bar{\tilde{I}}^s_b + \lambda_e \bar{\tilde{I}}^s_e\right)
\end{aligned}$$

where the profit is

$$\frac{\bar{\tilde{\Phi}}}{\bar{\tilde{Y}}} = 1 - \bar{\tilde{w}}^u \frac{\bar{L}_u}{\bar{\tilde{Y}}} - \bar{\tilde{w}}^s \frac{\bar{L}_s}{\bar{\tilde{Y}}} - \bar{r}^{K,b} \frac{\bar{\tilde{K}}_b}{\bar{\tilde{Y}}} - \bar{r}^{K,e} \frac{\bar{\tilde{K}}_e}{\bar{\tilde{Y}}}$$

and the transfer is

$$\bar{\tilde{S}} = \bar{\tau}^C \frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}} + \bar{\tau}^H \left(\sum_{i\in\{s,u\}} \lambda^i_{\tau^H} \bar{\tilde{w}}^i \frac{\bar{L}_i}{\bar{\tilde{Y}}} \right) + \bar{\tau}^K \bar{r}^{K,b} \left(\frac{\bar{\tilde{K}}_b}{\bar{\tilde{Y}}} - \lambda_b \frac{\bar{\tilde{I}}_b}{\bar{\tilde{Y}}} \right) + \bar{\tau}^K \bar{r}^{K,e} \left(\frac{\bar{\tilde{K}}_e}{\bar{\tilde{Y}}} - \lambda_e \frac{\bar{\tilde{I}}_e}{\bar{\tilde{Y}}} \right) - \left(\left(\frac{1}{\beta} - 1 \right) \bar{\tilde{b}} + \bar{\tilde{G}} \right)$$

From (23) and (24) for both types of households, we get

$$(\gamma - (1-d_b)) \frac{\bar{\tilde{K}}^s_b}{\bar{H}^s} = \frac{\bar{\tilde{I}}^s_b}{\bar{H}^s} \tag{47}$$

$$d_e \frac{\bar{\tilde{K}}^s_e}{\bar{H}^s} = \frac{\bar{\tilde{I}}^s_e}{\bar{H}^s} q_0 \tag{48}$$

$$(\gamma - (1-d_b)) \frac{\bar{\tilde{K}}^u_b}{\bar{H}^u} = \frac{\bar{\tilde{I}}^u_b}{\bar{H}^u} \tag{49}$$

$$d_e \frac{\bar{\tilde{K}}^u_e}{\bar{H}^u} = \frac{\bar{\tilde{I}}^u_e}{\bar{H}^u} q_0 \tag{50}$$

From households' intra-temporal Euler equations, we have

$$\frac{\tilde{\Lambda}^s \left(1 - \lambda^s_{\tau^H} \bar{\tau}^H\right) \bar{\tilde{w}}^s}{\tilde{\Lambda}^u \left(1 - \lambda^u_{\tau^H} \bar{\tau}^H\right) \bar{\tilde{w}}^u} = \frac{\bar{\omega}^s}{\bar{\omega}^u} \left(\frac{\bar{H}^s}{\bar{H}^u} \right)^{\varphi} \tag{51}$$

$$\bar{\tilde{w}}^S \left(\frac{1 - \lambda^s_{\tau^H} \bar{\tau}^H}{1 + \bar{\tau}^C} \right) = \bar{\omega}^S \left(\bar{\tilde{C}}^S \right) \left(\bar{H}^s \right)^{\varphi} \tag{52}$$

$$\Lambda_{s,u} = \frac{\bar{\tilde{\Lambda}}^s}{\bar{\tilde{\Lambda}}^u} = \frac{\bar{\tilde{C}}^u}{\bar{\tilde{C}}^s} \tag{53}$$

$$\bar{\tilde{C}}^s = \frac{1}{(N^u \Lambda + N^s)} \bar{\tilde{C}}$$

$$\bar{\tilde{C}}^u = \frac{\Lambda}{(N^u \Lambda + N^s)} \bar{\tilde{C}}$$

From the market clearing conditions, we have

$$\frac{\bar{\tilde{K}}_b}{\bar{L}_s} = \frac{\bar{\tilde{K}}^s_b}{\bar{H}^s} + \frac{\bar{L}_u}{\bar{L}_s} \frac{\bar{\tilde{K}}^u_b}{\bar{H}^u} \tag{54}$$

$$\frac{\bar{\tilde{K}}_e}{\bar{L}_s} = \frac{\bar{\tilde{K}}^s_e}{\bar{H}^s} + \frac{\bar{L}_u}{\bar{L}_s} \frac{\bar{\tilde{K}}^u_e}{\bar{H}^u} \tag{55}$$

From (25), we get

$$\frac{\bar{\tilde{C}}}{\bar{L}_s} + (\gamma - (1-d_b)) \frac{\bar{\tilde{K}}_b}{\bar{L}_s} + \frac{d_e}{q_0} \frac{\bar{\tilde{K}}_e}{\bar{L}_s} = \left(1 - \tilde{G}\right) \frac{\bar{\tilde{Y}}}{L_s} \tag{56}$$

The nominal interest rate is obtained from Euler equation (17)

$$\bar{R} = \frac{\gamma \bar{\pi}}{\beta}.$$

We fix steady-state hours for skilled labor $\bar{H}^s = 0.33$ by assuming the skilled works 40 hours per week and $\bar{H}^u = 0.93 * \bar{H}^s$(Skilled workers work 7% more than low-skilled worker).

B Model with Incomplete Markets

B.1 Households and Firms

- Skilled households make saving/investment decisions and own the entire capital in the economy

$$\max_{\left\{C_t^s, H_t^s, B_t^s, I_{b,t}^s, I_{e,t}^s, K_{b,t+1}^s, K_{e,t+1}^s\right\}} \quad E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\log C_t^s - \bar{\omega}^s \frac{\left(H_t^s\right)^{1+\varphi}}{1+\varphi} \right] \right\}$$

$$\begin{aligned}
\text{s.t.} \quad & \left(1+\tau_t^C\right) P_t C_t^s + P_t I_{b,t}^s + P_t I_{e,t}^s + B_t^s \\
& = \left(1 - \lambda_{\tau^H}^s \tau_t^H\right) W_t^s H_t^s + R_{t-1} B_{t-1}^s + \left(1-\tau_t^K\right) R_t^{K,b} K_{b,t}^s + \lambda_b \tau_t^K P_t I_{b,t} \\
& + \left(1-\tau_t^K\right) R_t^{K,e} K_{e,t}^s + \lambda_e \tau_t^K P_t I_{e,t} + P_t \Phi_t^s + P_t S_t^s \\
& K_{b,t+1}^s = \left(1-d_b\right) K_{b,t}^s + \left(1 - \mathcal{S}\left(\frac{I_{b,t}^s}{I_{b,t-1}^i}\right)\right) I_{b,t}^s \\
& K_{e,t+1}^s = \left(1-d_e\right) K_{e,t}^s + \left(1 - \mathcal{S}\left(\frac{I_{e,t}^s}{I_{e,t-1}^s}\right)\right) I_{e,t}^s q_t
\end{aligned}$$

- Unskilled households are hand-to-mouth households, that is, they consume their disposable income every period.

$$\max_{\left\{C_t^u, H_t^u\right\}} \quad \log C_t^u - \bar{\omega}^u \frac{\left(H_t^u\right)^{1+\varphi}}{1+\varphi}$$

$$\text{s.t.} \quad \left(1+\tau_t^C\right) P_t C_t^u = \left(1 - \lambda_{\tau^H}^u \tau_t^H\right) W_t^u H_t^u + P_t \Phi_t^u + P_t S_t^u$$

- Firms' problem are identical to that of the baseline model with complete markets.

B.2 Government Budget Constraint, Monetary Policy, and Fiscal Policy

- The government flow budget constraint, written by expressing fiscal variables as ratio of output, is given by

$$\frac{B_t}{P_t Y_t} + \left(\frac{T_t^C}{Y_t} + \frac{T_t^H}{Y_t} + \frac{T_t^{K,b}}{Y_t} + \frac{T_t^{K,e}}{Y_t}\right) = R_{t-1} \frac{B_{t-1}}{P_{t-1} Y_{t-1}} \frac{1}{\pi_t} \frac{Y_{t-1}}{Y_t} + \frac{G_t}{Y_t} + \frac{S_t}{Y_t}$$

where

$$
\begin{aligned}
B_t &= \sum_{i \in \{s,u\}} N^i B_t^i \\
T_t^C &= \tau^C \sum_{i \in \{s,u\}} N^i C_t^i = \tau^C C_t \\
T_t^H &= \tau_t^H \sum_{i \in \{s,u\}} \left(N^i \lambda_{\tau^H}^i \frac{W_t^i}{P_t} H_t^i \right) \\
T_t^{K,b} &= N^s \tau_t^K \left(\frac{R_t^{K,b}}{P_t} K_{b,t}^s - \lambda_b I_{b,t} \right) \\
T_t^{K,e} &= N^s \tau_t^K \left(\frac{R_t^{K,e}}{P_t} K_{e,t}^s - \lambda_e I_{e,t} \right) \\
S_t &= \sum_{i \in \{s,u\}} N^i S_t^i \\
N^s S_t^s &= \chi_S^s S_t \\
N^u S_t^u &= \chi_S^u S_t = \left(1 - \chi_S^s\right) S_t
\end{aligned}
$$

- Profit distribution

$$
\begin{aligned}
\Phi_t &= \sum_{i \in \{s,u\}} N^i \Phi_t^i \\
N^s \Phi_t^s &= \chi_\Phi^s \Phi_t \\
N^u \Phi_t^u &= \chi_\Phi^u \Phi_t = \left(1 - \chi_\Phi^s\right) \Phi_t
\end{aligned}
$$

- Monetary policy and fiscal policy rules are identical to those under the model with complete markets.

B.3 Market Clearing

$$
\begin{aligned}
N^s \left(C_t^s + I_{b,t}^s + I_{e,t}^s \right) + N^u C_t^u + G_t + \Xi \left(\frac{P_t}{P_{t-1}} \right) Y_t &= Y_t \\
\int_0^1 L_{u,t}(i)\, di &= N^u H_t^u \\
\int_0^1 L_{s,t}(i)\, di &= N^s H_t^s \\
\int_0^1 K_{b,t}(i)\, di &= N^s K_{b,t}^s \\
\int_0^1 K_{e,t}(i)\, di &= N^s K_{e,t}^s \\
\int_0^1 \Phi_t(i)\, di &= N^s \Phi_t^s + N^u \Phi_t^u
\end{aligned}
$$

$$
\begin{aligned}
K_{b,t+1}^s &= (1 - d_b) K_{b,t}^s + \left(1 - \mathcal{S} \left(\frac{I_{b,t}^s}{I_{b,t-1}^s} \right) \right) I_{b,t}^s \\
K_{e,t+1}^s &= (1 - d_e) K_{e,t}^s + \left(1 - \mathcal{S} \left(\frac{I_{e,t}^s}{I_{e,t-1}^s} \right) \right) I_{e,t}^s q_t
\end{aligned}
$$

B.4 Stationary Equilibrium

We consider a symmetric equilibrium across firms, where all firms set the same price and produce the same amount of output. Given nonlinear equilibrium conditions, we detrend variables to specify stationary equilibrium conditions. All the notations are the same as before. We now state all the stationary equilibrium equations under incomplete markets.

- Production function:(Let $A_0 = 1$)

$$\tilde{Y}_t = \tilde{K}_{b,t}^{\alpha} \left[\mu L_{u,t}^{\sigma} + (1-\mu) \left(\lambda \tilde{K}_{e,t}^{\rho} + (1-\lambda) L_{s,t}^{\rho} \right)^{\frac{\sigma}{\rho}} \right]^{\frac{1-\alpha}{\sigma}}$$

- Cost minimization:

$$r_t^{K,b} = \alpha mc_t \frac{\tilde{Y}_t}{\tilde{K}_{b,t}}$$

$$r_t^{K,e} = (1-\alpha) mc_t \frac{\tilde{Y}_t}{\tilde{K}_{e,t}} \left(\frac{(1-\mu) \left(\lambda \tilde{K}_{e,t}^{\rho} + (1-\lambda) L_{s,t}^{\rho} \right)^{\frac{\sigma}{\rho}}}{\mu L_{u,t}^{\sigma} + (1-\mu) \left(\lambda \tilde{K}_{e,t}^{\rho} + (1-\lambda) L_{s,t}^{\rho} \right)^{\frac{\sigma}{\rho}}} \right) \left(\frac{\lambda \tilde{K}_{e,t}^{\rho}}{\lambda \tilde{K}_{e,t}^{\rho} + (1-\lambda) L_{s,t}^{\rho}} \right)$$

$$\tilde{w}_t^u = (1-\alpha) mc_t \frac{\tilde{Y}_t}{L_{u,t}} \left(\frac{\mu L_{u,t}^{\sigma}}{\mu L_{u,t}^{\sigma} + (1-\mu) \left(\lambda \tilde{K}_{e,t}^{\rho} + (1-\lambda) L_{s,t}^{\rho} \right)^{\frac{\sigma}{\rho}}} \right)$$

$$\tilde{w}_t^s = (1-\alpha) mc_t \frac{\tilde{Y}_t}{L_{s,t}} \left(\frac{(1-\mu) \left(\lambda \tilde{K}_{e,t}^{\rho} + (1-\lambda) L_{s,t}^{\rho} \right)^{\frac{\sigma}{\rho}}}{\mu L_{u,t}^{\sigma} + (1-\mu) \left(\lambda \tilde{K}_{e,t}^{\rho} + (1-\lambda) L_{s,t}^{\rho} \right)^{\frac{\sigma}{\rho}}} \right) \left(\frac{(1-\lambda) L_{s,t}^{\rho}}{\lambda \tilde{K}_{e,t}^{\rho} + (1-\lambda) L_{s,t}^{\rho}} \right)$$

- skill-premium

$$\frac{\tilde{w}_t^s}{\tilde{w}_t^u} = \frac{(1-\mu)(1-\lambda)}{\mu} \left(\lambda \left(\frac{\tilde{K}_{e,t}}{L_{s,t}} \right)^{\rho} + (1-\lambda) \right)^{\frac{\sigma-\rho}{\rho}} \left(\frac{L_{u,t}}{L_{s,t}} \right)^{1-\sigma}$$

- Firms' maximization: Phillips Curve

$$\Xi'(\pi_t)\pi_t = ((1-\theta) + \theta mc_t) + \beta E_t \frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t} \frac{\tilde{Y}_{t+1}}{\tilde{Y}_t} \Xi'(\pi_{t+1}) \pi_{t+1}$$

- Profit

$$\tilde{\Phi}_t = \tilde{Y}_t - \tilde{w}_t^u L_{u,t} - \tilde{w}_t^s L_{s,t} - r_t^{K,b} \tilde{K}_{b,t} - r_t^{K,e} \tilde{K}_{e,t} - \Xi(\pi_t) \tilde{Y}_t$$

- Budget constraint

$$\begin{aligned}
\left(1+\tau_t^C\right) \tilde{C}_t^s + \tilde{I}_{b,t}^s + \tilde{I}_{e,t}^s + \tilde{b}_t^s \tilde{Y}_t = & \left(1 - \lambda_{\tau^H}^s \tau_t^H\right) \tilde{w}_t^s H_t^s + R_{t-1} \tilde{b}_{t-1}^s \tilde{Y}_{t-1} \frac{1}{\gamma \pi_t} \\
& + \left(1-\tau_t^K\right) r_t^{K,b} \tilde{K}_{b,t}^s + \left(1-\tau_t^K\right) r_t^{K,e} \tilde{K}_{e,t}^s + \frac{\chi_\Phi^s}{N^s} \tilde{\Phi}_t + \frac{\chi_S^s}{N^s} \tilde{S}_t \tilde{Y}_t \\
& + \lambda_b \tau_t^K \tilde{I}_{b,t}^s + \lambda_e \tau_t^K \tilde{I}_{e,t}^s
\end{aligned}$$

- Households

– Marginal Utilities:

$$\tilde{\Lambda}_t^s \left(1+\tau_t^C\right) = \frac{1}{\tilde{C}_t^s}$$
$$\tilde{\Lambda}_t^s \left(1-\lambda_{\tau^H}^s \tau_t^H\right) \tilde{w}_t^s = \bar{\omega}^s \left(H_t^s\right)^{\varphi}$$

– FOCs and Capital Accumulation

$$\begin{aligned}
\tilde{w}_t^s \frac{1-\lambda_{\tau^H}^s \tau_t^H}{1+\tau_t^C} &= \bar{\omega}^s \left(\tilde{C}_t^s\right) \left(H_t^s\right)^{\varphi} \\
\tilde{\Lambda}_t^s &= \frac{\beta}{\gamma} R_t E_t \left\{\tilde{\Lambda}_{t+1}^s \frac{1}{\pi_{t+1}}\right\} \\
\tilde{\Psi}_{b,t}^s &= \frac{\beta}{\gamma} E_t \left\{\left(1-d_b\right) \tilde{\Psi}_{b,t+1}^s + \left(1-\tau_{t+1}^K\right) r_{t+1}^{K,b} \tilde{\Lambda}_{t+1}^s\right\} \\
\left(1-\lambda_b \tau_t^K\right) \tilde{\Lambda}_t^s &= \tilde{\Psi}_{b,t}^s \left(1-\mathcal{S}\left(\frac{\tilde{I}_{b,t}^s}{\tilde{I}_{b,t-1}^s}\gamma\right) - \mathcal{S}'\left(\frac{\tilde{I}_{b,t}^s}{\tilde{I}_{b,t-1}^s}\gamma\right) \frac{\tilde{I}_{b,t}^s}{\tilde{I}_{b,t-1}^s}\gamma\right) \\
&\quad + \frac{\beta}{\gamma} E_t \left\{\tilde{\Psi}_{b,t+1}^s \left(\frac{\tilde{I}_{b,t+1}^s}{\tilde{I}_{b,t}^s}\gamma\right)^2 \mathcal{S}'\left(\frac{\tilde{I}_{b,t+1}^s}{\tilde{I}_{b,t}^s}\gamma\right)\right\} \\
\tilde{\Psi}_{e,t}^s &= \beta E_t \left\{\left(1-d_e\right) \tilde{\Psi}_{e,t+1}^s + \left(1-\tau_{t+1}^K\right) r_{t+1}^{K,e} \tilde{\Lambda}_{t+1}^s\right\} \\
\left(1-\lambda_e \tau_t^K\right) \frac{1}{q_0} \tilde{\Lambda}_t^s &= \tilde{\Psi}_{e,t}^s \left(1-\mathcal{S}\left(\frac{\tilde{I}_{e,t}^s}{\tilde{I}_{e,t-1}^s}\gamma\right) - \mathcal{S}'\left(\frac{\tilde{I}_{e,t}^s}{\tilde{I}_{e,t-1}^s}\gamma\right) \frac{\tilde{I}_{e,t}^s}{\tilde{I}_{e,t-1}^s}\gamma\right) \\
&\quad + \frac{\beta}{\gamma} E_t \left\{\tilde{\Psi}_{e,t+1}^s \left(\frac{\tilde{I}_{e,t+1}^s}{\tilde{I}_{e,t}^s}\gamma\right)^2 \mathcal{S}'\left(\frac{\tilde{I}_{e,t+1}^s}{\tilde{I}_{e,t}^s}\gamma\right)\right\} \\
\gamma \tilde{K}_{b,t+1}^s &= \left(1-d_b\right) \tilde{K}_{b,t}^s + \left(1-\mathcal{S}\left(\frac{\tilde{I}_{b,t}^s}{\tilde{I}_{b,t-1}^i}\gamma\right)\right) \tilde{I}_{b,t}^s \\
\tilde{K}_{e,t+1}^s &= \left(1-d_e\right) \tilde{K}_{e,t}^s + \left(1-\mathcal{S}\left(\frac{\tilde{I}_{e,t}^s}{\tilde{I}_{e,t-1}^i}\gamma\right)\right) \tilde{I}_{e,t}^s q_0
\end{aligned}$$

- Hand-to-mouth households

$$\begin{aligned}
\tilde{w}_t^u \frac{1-\lambda_{\tau^H}^u \tau_t^H}{1+\tau_t^C} &= \bar{\omega}^u \left(\tilde{C}_t^u\right) \left(H_t^u\right)^{\varphi} \\
\left(1+\tau_t^C\right) \tilde{C}_t^u &= \left(1-\lambda_{\tau^H}^u \tau_t^H\right) \tilde{w}_t^u H_t^u + \tilde{\Phi}_t^u + \tilde{S}_t^u \tilde{Y}_t
\end{aligned}$$

- Resource constraint

$$\tilde{C}_t + N^s \left(\tilde{I}_{b,t}^s + \tilde{I}_{e,t}^s\right) = \left(1-\tilde{G}_t - \Xi\left(\pi_t\right)\right) \tilde{Y}_t$$

- Market clearing

$$\begin{aligned}
\tilde{C}_t &= N^s \tilde{C}_t^s + N^u \tilde{C}_t^u \\
L_{u,t} &= N^u H_t^u \\
L_{s,t} &= N^s H_t^s \\
\tilde{K}_{b,t} &= N^s \tilde{K}_{b,t}^s \\
\tilde{K}_{e,t} &= N^s \tilde{K}_{e,t}^s
\end{aligned}$$

- Government budget constraint

$$\tilde{b}_t + \tilde{T}_t^C + \tilde{T}_t^H + \tilde{T}_t^{K,b} + \tilde{T}_t^{K,e} = R_{t-1}\tilde{b}_{t-1}\frac{1}{\pi_t\gamma}\frac{\tilde{Y}_{t-1}}{\tilde{Y}_t} + \tilde{G}_t + \tilde{S}_t$$

where

$$\tilde{T}_t^C = \tau^C\frac{\tilde{C}_t}{\tilde{Y}_t},\ \tilde{T}_t^H = \tau_t^H\sum_{i\in\{s,u\}}\left(\lambda_{\tau^H}^i\tilde{w}_t^i\frac{L_{i,t}}{\tilde{Y}_t}\right),$$
$$\tilde{T}_t^{K,b} = \tau_t^K\left(r_t^{K,b}\frac{\tilde{K}_{b,t}}{\tilde{Y}_t} - \lambda_b\frac{\tilde{I}_{b,t}}{\tilde{Y}_t}\right),\ \tilde{T}_t^{K,e} = \tau_t^K\left(r_t^{K,e}\frac{\tilde{K}_{e,t}}{\tilde{Y}_t} - \lambda_e\frac{\tilde{I}_{e,t}}{\tilde{Y}_t}\right)$$

- Fiscal Policy Rules

$$\tau_t^H - \bar{\tau}_{new}^H = \rho^H\left(\tau_{t-1}^H - \bar{\tau}_{new}^H\right) + \left(1-\rho^H\right)\psi^H\left(\tilde{b}_{t-1} - \bar{\tilde{b}}\right)$$
$$\tau_t^C - \bar{\tau}_{new}^C = \rho^C\left(\tau_{t-1}^C - \bar{\tau}_{new}^C\right) + \left(1-\rho^C\right)\psi^C\left(\tilde{b}_{t-1} - \bar{\tilde{b}}\right)$$
$$\tau_t^K = \begin{cases}\bar{\tau}^K & \text{if } t=0\\ \bar{\tau}_{New}^K & \text{if } t>0\end{cases}$$
$$\tilde{G}_t = \bar{\tilde{G}}$$

- Monetary policy

$$\frac{R_t}{\bar{R}} = \left[\frac{R_{t-1}}{\bar{R}}\right]^{\rho^R}\left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi}\right]^{(1-\rho^R)}$$

- Government debt to GDP (or transfers):

$$\begin{cases}\tilde{b}_t = \bar{\tilde{b}} & \text{if adjusting transfers}\\ \tilde{S}_t = \bar{\tilde{S}} & \text{if adjusting labor tax rate}\end{cases}$$

C Proofs of Propositions

C.1 Steady-state equilibrium equations for a nested version of the model

We assume $\mu = 0$, $\alpha = 0$, and $\rho \to 0$ to get a nested version of the model with a Cobb-Douglas production function. In this case, to have balanced growth, the growth rate of output is the same with the growth rate of technology,$\gamma = \bar{a}$, and the growth rate of relative price (γ_q) is 1. Let $q_0 = 1$. Also, let the fraction of skilled workers $N^S = 1$ and set $\chi_\Phi^s = 1$ and $\chi_S^s = 1$ for profit and transfers distributions. Then, in this economy, we have one type of capital $K_{e,t}$ and one type of labor $L_{s,t}$. We derive steady-state equilibrium equations for this nested version of the model and drop subscripts e and s.

- Marginal cost

$$\bar{mc} = \frac{\theta-1}{\theta}.$$

- Capital rental rate

$$\bar{r}^K = \frac{\frac{\bar{a}}{\beta} - (1-d)}{1 - \bar{\tau}^K} \tag{57}$$

- Production function

$$\frac{\bar{\tilde{Y}}}{\bar{H}} = \left(\frac{\bar{\tilde{K}}}{\bar{H}}\right)^{\lambda} \tag{58}$$

- Wages and capital-to-labor ratio

$$\frac{\bar{\tilde{K}}}{\bar{H}} = \left(\frac{\bar{r}^K}{\lambda \bar{m}c}\right)^{\frac{1}{\lambda-1}} \tag{59}$$

$$\begin{aligned}
\bar{\tilde{w}} &= (1-\lambda)\,\bar{m}c\left(\frac{\bar{\tilde{K}}}{\bar{H}}\right)^{\lambda} \\
&= (1-\lambda)\,(\lambda)^{\frac{\lambda}{1-\lambda}}\,(\bar{m}c)^{\frac{1}{1-\lambda}}\,\left(\bar{r}^K\right)^{\frac{\lambda}{\lambda-1}}
\end{aligned} \tag{60}$$

- Resource constraint

$$\frac{\bar{\tilde{C}}}{\bar{H}} = \left(1 - \bar{\tilde{G}}\right)\frac{\bar{\tilde{Y}}}{\bar{H}} - \frac{\bar{\tilde{I}}}{\bar{H}}. \tag{61}$$

- Profit

$$\frac{\bar{\tilde{\Phi}}}{\bar{\tilde{Y}}} = 1 - \bar{\tilde{w}}\frac{\bar{H}}{\bar{\tilde{Y}}} - \bar{r}^K\frac{\bar{\tilde{K}}}{\bar{\tilde{Y}}}$$

- Transfer

$$\bar{\tilde{S}} = \left(1 - \frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}} - \bar{\tilde{G}} + \bar{\tilde{T}}^C + \bar{\tilde{T}}^H + \bar{\tilde{T}}^K. \tag{62}$$

- The consumption, labor income and capital income tax rates are respectively given as:

$$\bar{\tau}^C = \frac{\bar{\tilde{T}}^C}{\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}},\ \bar{\tau}^H = \frac{1}{\bar{\tilde{w}}}\frac{\bar{\tilde{T}}^H}{\frac{\bar{H}}{\bar{\tilde{Y}}}},\ \bar{\tau}^K = \frac{1}{\bar{r}^K}\frac{\bar{\tilde{T}}^K}{\frac{\bar{\tilde{K}}}{\bar{\tilde{Y}}}}.$$

- Intra-temporal Euler equation

$$\bar{H} = \left(\frac{1}{\bar{\omega}\left(\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H}\frac{\bar{\tilde{C}}}{\bar{H}}\frac{1}{\bar{\tilde{w}}}\right)}\right)^{\frac{1}{1+\varphi}}. \tag{63}$$

- Investment

$$\frac{\bar{\tilde{I}}}{\bar{H}} = \frac{\bar{\tilde{K}}}{\bar{H}}\left(\bar{a} - (1-d)\right). \tag{64}$$

- Nominal interest rate

$$\bar{R} = \frac{\bar{a}\bar{\pi}}{\beta}.$$

C.2 Proof of Lemma 1

Proof. From (57) and (60), we get

$$\begin{aligned}
\frac{\partial \bar{r}^K}{\partial \bar{\tau}^K} &= \frac{\bar{r}^K}{1-\bar{\tau}^K} > 0 \\
\frac{\partial \bar{\tilde{w}}}{\partial \bar{\tau}^K} &= -\left(\frac{\bar{\tilde{w}}}{\bar{r}^K}\right)\left(\frac{\lambda}{1-\lambda}\right)\frac{\partial \bar{r}^K}{\partial \bar{\tau}^K} < 0.
\end{aligned}$$

Let $\bar{\tilde{k}} = \frac{\bar{\tilde{K}}}{\bar{H}}$ and $\bar{\tilde{y}} = \frac{\bar{\tilde{Y}}}{\bar{H}}$. From (59) and (58), we get

$$\begin{aligned}
\frac{\partial \bar{\tilde{k}}}{\partial \bar{\tau}^K} &= -\frac{\bar{\tilde{k}}}{\bar{r}^K}\frac{1}{1-\lambda}\frac{\partial \bar{r}^K}{\partial \bar{\tau}^K} < 0 \\
\frac{\partial \bar{\tilde{y}}}{\partial \bar{\tau}^K} &= \lambda\left(\frac{\bar{\tilde{y}}}{\bar{\tilde{k}}}\right)^{\frac{1}{\varepsilon}}\frac{\partial \bar{\tilde{k}}}{\partial \bar{\tau}^K} < 0
\end{aligned}$$

Combining (60) and (61) with (63), we rewrite the steady-state hours as

$$\bar{H} = \left(\bar{\omega}\left(\frac{1}{1-\lambda}\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H}\left(\frac{1-\bar{\tilde{G}}}{\bar{mc}} - \lambda\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right)\right)\right)^{-\frac{1}{1+\varphi}}.$$

Then, the partial derivative with respect to capital tax rate is

$$\frac{\partial \bar{H}}{\partial \bar{\tau}^K} = -\frac{\bar{H}^{2+\varphi}}{1+\varphi}\left(\bar{\omega}\frac{\lambda}{1-\lambda}\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H}\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right) < 0.$$

Now, we find the partial derivatives of levels of variables. For capital, investment and output, we can easily verify that

$$\begin{aligned}
\frac{\partial \bar{\tilde{K}}}{\partial \bar{\tau}^K} &= \bar{H}\frac{\partial \bar{\tilde{k}}}{\partial \bar{\tau}^K} + \bar{\tilde{k}}\frac{\partial \bar{H}}{\partial \bar{\tau}^K} < 0 \\
\frac{\partial \bar{\tilde{I}}}{\partial \bar{\tau}^K} &= \frac{\partial \bar{\tilde{K}}}{\partial \bar{\tau}^K}\left(\bar{a}-(1-d)\right) < 0 \\
\frac{\partial \bar{\tilde{Y}}}{\partial \bar{\tau}^K} &= \bar{H}\frac{\partial \bar{\tilde{y}}}{\partial \bar{\tau}^K} + \bar{\tilde{y}}\frac{\partial \bar{H}}{\partial \bar{\tau}^K} < 0
\end{aligned}$$

For consumption, combining (58) and (64) with (61), we get

$$\bar{\tilde{C}} = \left(\left(1-\bar{\tilde{G}}\right)\frac{\bar{\tilde{Y}}}{\bar{H}} - \frac{\bar{\tilde{I}}}{\bar{H}}\right)\bar{H}$$
$$= \left(\bar{mc}\frac{\lambda\left(1-\bar{\tau}^K\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)^{\frac{\lambda}{1-\lambda}}\left[\left(1-\bar{\tilde{G}}\right) - \lambda\bar{mc}\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right]\bar{H}.$$

Then, the partial derivative of consumption with respect to capital tax rate is

$$\frac{\partial\bar{\tilde{C}}}{\partial\bar{\tau}^K} = -\left(\frac{\lambda}{1-\lambda}\frac{1}{1-\bar{\tau}^K}\right)\left(\frac{\lambda\bar{mc}\left(1-\bar{\tau}^K\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)^{\frac{\lambda}{1-\lambda}}\left[\left(1-\bar{\tilde{G}}\right) - \bar{mc}\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right]\bar{H} + \frac{\bar{\tilde{C}}}{\bar{H}}\frac{\partial\bar{H}}{\partial\bar{\tau}^K}.$$

Under Assumption 1, we find $\frac{\partial\bar{\tilde{C}}}{\partial\bar{\tau}^K} < 0$. □

C.3 Proof of Proposition 1

Proof. Let $\bar{\tau}^K_{new} = \bar{\tau}^K + \Delta\left(\bar{\tau}^K\right)$. From (57), we get

$$\frac{\bar{r}^K_{new}}{\bar{r}^K} = \left(\frac{1-\bar{\tau}^K}{1-\bar{\tau}^K_{new}}\right) = \left(1-\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{-1}$$

From (60), (59) and (58), we get

$$\frac{\bar{\tilde{w}}_{new}}{\bar{\tilde{w}}} = \left(1-\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{\frac{\lambda}{1-\lambda}}$$
$$\frac{\bar{\tilde{k}}_{new}}{\bar{\tilde{k}}} = \left(1-\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{\frac{1}{1-\lambda}}$$
$$\frac{\bar{\tilde{y}}_{new}}{\bar{\tilde{y}}} = \left(1-\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{\frac{\lambda}{1-\lambda}}.$$

Combining (60) and (61) with (63), we get

$$\frac{\bar{H}_{new}}{\bar{H}} = \left(\frac{\frac{1}{1-\lambda}\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H}\left(\frac{1-\bar{\tilde{G}}}{\bar{mc}} - \lambda\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)\right)}{\frac{1}{1-\lambda}\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H}\left(\frac{1-\bar{\tilde{G}}}{\bar{mc}} - \lambda\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right)}\right)^{-\frac{1}{1+\varphi}}$$
$$= \left(1+\bar{\omega}\bar{H}^{1+\varphi}\frac{\lambda}{1-\lambda}\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H}\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\Delta\left(\bar{\tau}^K\right)\right)^{-\frac{1}{1+\varphi}}$$
$$= \left(1+\Omega\Delta\left(\bar{\tau}^K\right)\right)^{-\frac{1}{1+\varphi}}$$

where $\Omega = \bar{\omega}\bar{H}^{1+\varphi}\frac{\lambda}{1-\lambda}\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H} > 0$.

Now, we find changes of levels of variables. For capital, investment and output, we can easily verify that

$$\frac{\bar{\tilde{K}}_{new}}{\bar{\tilde{K}}} = \frac{\bar{\tilde{k}}_{new}}{\bar{\tilde{k}}}\frac{\bar{H}_{new}}{\bar{H}} = \left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{\frac{1}{1-\lambda}}\left(1+\Omega\Delta\left(\bar{\tau}^K\right)\right)^{-\frac{1}{1+\varphi}}$$

$$\frac{\bar{\tilde{I}}_{new}}{\bar{\tilde{I}}} = \frac{\bar{\tilde{K}}_{new}}{\bar{\tilde{K}}} = \left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{\frac{1}{1-\lambda}}\left(1+\Omega\Delta\left(\bar{\tau}^K\right)\right)^{-\frac{1}{1+\varphi}}$$

and

$$\frac{\bar{\tilde{Y}}_{new}}{\bar{\tilde{Y}}} = \left(\frac{\bar{\tilde{k}}_{new}}{\bar{\tilde{k}}}\right)^{\lambda}\frac{\bar{H}_{new}}{\bar{H}} = \left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{\frac{\lambda}{1-\lambda}}\left(1+\Omega\Delta\left(\bar{\tau}^K\right)\right)^{-\frac{1}{1+\varphi}}.$$

For consumption, combining (58) and (64) with (61), we get

$$\begin{aligned}
\frac{\bar{\tilde{C}}_{new}}{\bar{\tilde{C}}} &= \frac{\left(\bar{mc}\frac{\lambda\left(1-\bar{\tau}^K_{new}\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)^{\frac{\lambda}{1-\lambda}}\left[\left(1-\bar{\tilde{G}}\right)-\lambda\bar{mc}\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)\right]\bar{H}_{new}}{\left(\bar{mc}\frac{\lambda\left(1-\bar{\tau}^K\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)^{\frac{\lambda}{1-\lambda}}\left[\left(1-\bar{\tilde{G}}\right)-\lambda\bar{mc}\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right]\bar{H}} \\
&= \left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{\frac{\lambda}{1-\lambda}}\left(1+\frac{\lambda\bar{mc}\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}}{\left(1-\bar{\tilde{G}}\right)-\lambda\bar{mc}\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)}\Delta\left(\bar{\tau}^K\right)\right)\left(1+\Omega\Delta\left(\bar{\tau}^K\right)\right)^{-\frac{1}{1+\varphi}} \\
&= \left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{\frac{\lambda}{1-\lambda}}\left(1+\frac{\bar{\tilde{I}}}{\bar{H}}\left(\frac{\bar{\tilde{C}}}{\bar{H}}\left(1-\bar{\tau}^K\right)\right)^{-1}\Delta\left(\bar{\tau}^K\right)\right)\left(1+\Omega\Delta\left(\bar{\tau}^K\right)\right)^{-\frac{1}{1+\varphi}}.
\end{aligned}$$

Now for small changes in the capital tax rate $\Delta\left(\bar{\tau}^K\right)$, the percent changes of rental rate, wages, capital to hours ratio, output to hours ratio from their initial steady-states are:

$$\ln\left(\frac{\bar{r}^K_{new}}{\bar{r}^K}\right) = -\ln\left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right) \simeq \left(\frac{1}{1-\bar{\tau}^K}\right)\Delta\left(\bar{\tau}^K\right).$$

$$\ln\left(\frac{\bar{\tilde{w}}_{new}}{\bar{\tilde{w}}}\right) = \frac{\lambda}{1-\lambda}\ln\left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right) \simeq -\left(\frac{\lambda}{1-\lambda}\frac{1}{1-\bar{\tau}^K}\right)\Delta\left(\bar{\tau}^K\right)$$

$$\ln\left(\frac{\bar{\tilde{k}}_{new}}{\bar{\tilde{k}}}\right) = \frac{1}{1-\lambda}\ln\left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right) \simeq -\left(\frac{1}{1-\lambda}\frac{1}{1-\bar{\tau}^K}\right)\Delta\left(\bar{\tau}^K\right)$$

$$\ln\left(\frac{\bar{\tilde{y}}_{new}}{\bar{\tilde{y}}}\right) = \frac{\lambda}{1-\lambda}\ln\left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right) \simeq -\left(\frac{\lambda}{1-\lambda}\frac{1}{1-\bar{\tau}^K}\right)\Delta\left(\bar{\tau}^K\right)$$

and

$$\ln\left(\frac{\bar{H}_{new}}{\bar{H}}\right) = -\frac{1}{1+\varphi}\ln\left(1+\Omega\Delta\left(\bar{\tau}^K\right)\right) \simeq -\frac{1}{1+\varphi}\left(1+\Omega\right)\Delta\left(\bar{\tau}^K\right).$$

The percent changes of levels of capital and investment from their initial steady-states are:

$$\ln\left(\frac{\bar{\tilde{K}}_{new}}{\bar{\tilde{K}}}\right)=\ln\left(\frac{\bar{\tilde{I}}_{new}}{\bar{\tilde{I}}}\right)\simeq-\left(\frac{1}{(1-\lambda)\left(1-\bar{\tau}^K\right)}+\frac{\Omega}{1+\varphi}\right)\Delta\left(\bar{\tau}^K\right)$$
$$=-\mathcal{M}_K\Delta\left(\bar{\tau}^K\right)$$

where $\mathcal{M}_K=\frac{1}{(1-\lambda)(1-\bar{\tau}^K)}+\frac{\Omega}{1+\varphi}>0$. Also, the percent change of output from the initial steady-state is:

$$\ln\left(\frac{\bar{\tilde{Y}}_{new}}{\bar{\tilde{Y}}}\right)\simeq-\left(\frac{\lambda}{(1-\lambda)\left(1-\bar{\tau}^K\right)}+\frac{\Omega}{1+\varphi}\right)\Delta\left(\bar{\tau}^K\right)$$
$$=-\mathcal{M}_Y\Delta\left(\bar{\tau}^K\right)$$

where $\mathcal{M}_Y=\frac{\lambda}{(1-\lambda)(1-\bar{\tau}^K)}+\frac{\Omega}{1+\varphi}>0$.

Finally, the percent change of consumption from its initial steady-state is:

$$\begin{aligned}\ln\left(\frac{\bar{\tilde{C}}_{new}}{\bar{\tilde{C}}}\right)&\simeq-\left[\frac{\lambda}{(1-\lambda)\left(1-\bar{\tau}^K\right)}+\frac{\Omega}{1+\varphi}-\frac{\lambda\bar{mc}\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}}{\left(1-\bar{\tilde{G}}\right)-\lambda\bar{mc}\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)}\right]\Delta\left(\bar{\tau}^K\right)\\&=-\frac{\lambda}{1-\lambda}\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}^{\eta}}{\beta}-(1-d)}\right)\times\\&\left(\frac{\bar{\omega}\bar{H}^{1+\varphi}\left(\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H}\right)}{1+\varphi}+\frac{\left(1-\bar{\tilde{G}}\right)-\bar{mc}\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\left(1-\bar{\tau}^K\right)}{\frac{(\bar{a}-(1-d))\left(1-\bar{\tau}^K\right)}{\frac{\bar{a}}{\beta}-(1-d)}\left(\left(1-\bar{\tilde{G}}\right)-\frac{\lambda\bar{mc}(\bar{a}-(1-d))\left(1-\bar{\tau}^K\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)}\right)\Delta\left(\bar{\tau}^K\right)\\&=-\mathcal{M}_C\Delta\left(\bar{\tau}^K\right).\end{aligned}$$

Notice that under Assumption 1, the numerator of the second term in the large bracket is greater than zero. Thus, we have $\mathcal{M}_C=\mathcal{M}_Y-\frac{\bar{\tilde{I}}}{\bar{H}}\left(\frac{\bar{\tilde{C}}}{\bar{H}}\left(1-\bar{\tau}^K\right)\right)^{-1}>0$. □

C.4 Proof of Lemma 2

Proof. Notice that rental rate of capital, wage, capital to hours ratio, and output to hours ratio are the same with the lump-sum transfers adjustment case in C.2.

To show hours are increasing in $\bar{\tau}^K$, we rewrite (62) as the following:

$$\begin{aligned}\bar{\tilde{S}}&=\left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}}-\bar{\tilde{G}}+\bar{\tau}^C\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}+\bar{\tau}^H\bar{\tilde{w}}\frac{\bar{H}}{\bar{\tilde{Y}}}+\bar{\tau}^K\bar{r}^K\frac{\bar{\tilde{K}}}{\bar{\tilde{Y}}}\\&=\left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}}-\bar{\tilde{G}}+\bar{\tau}^C\left[\left(1-\bar{\tilde{G}}\right)-\frac{\lambda\bar{mc}\left(\bar{a}-(1-d)\right)}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right]+\bar{\tau}^H(1-\lambda)\bar{mc}+\bar{\tau}^K\lambda\bar{mc}\end{aligned}$$

$$1-\bar{\tau}^H=\frac{(1-\lambda)\bar{mc}-\bar{\tilde{S}}+\left[\left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}}-\bar{\tilde{G}}+\bar{\tau}^C\left(\left(1-\bar{\tilde{G}}\right)-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right)+\bar{\tau}^K\lambda\bar{mc}\right]}{(1-\lambda)\bar{mc}}$$

Then, from (63), we get

$$\bar{H} = \left(\frac{\bar{\omega}\left(1+\bar{\tau}^C\right)\left(1-\bar{\tilde{G}}-\frac{\lambda \bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right)}{(1-\lambda)\,\bar{mc}-\bar{\tilde{S}}+\left[\left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}}-\bar{\tilde{G}}+\bar{\tau}^C\left(\left(1-\bar{\tilde{G}}\right)-\frac{\lambda \bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right)+\bar{\tau}^K\lambda\bar{mc}\right]} \right)^{-\frac{1}{1+\varphi}} \tag{65}$$

Taking a partial derivative with respect to capital tax rate gives:

$$\begin{aligned}
\frac{\partial \bar{H}}{\partial \bar{\tau}^K} &= \frac{\bar{H}^{-\varphi}}{1+\varphi}\left[\frac{\bar{\tau}^C\frac{\lambda \bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}+\lambda\bar{mc}}{\bar{\omega}\left(1+\bar{\tau}^C\right)\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))\left(1-\bar{\tau}^K\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)}-\frac{\bar{\omega}\left(1+\bar{\tau}^C\right)\frac{\lambda \bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}(1-\lambda)\,\bar{mc}\left(1-\bar{\tau}^H\right)}{\left(\bar{\omega}\left(1+\bar{\tau}^C\right)\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))\left(1-\bar{\tau}^K\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\right)^2}\right] \\
&= \frac{\frac{1}{1+\varphi}\bar{H}^{-\varphi}}{\bar{\omega}\left(1+\bar{\tau}^C\right)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\left[\bar{\tau}^C\frac{\lambda\bar{mc}\left(\bar{a}-(1-d)\right)}{\frac{\bar{a}}{\beta}-(1-d)}+\lambda\bar{mc}-\frac{(1-\lambda)\,\bar{mc}\left(1-\bar{\tau}^H\right)\frac{\lambda \bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}}{\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\right] \\
&= \frac{\frac{1}{1+\varphi}\bar{H}^{-\varphi}\frac{\lambda \bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}}{\bar{\omega}\left(1+\bar{\tau}^C\right)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\left[\frac{\bar{\tau}^C\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}+\frac{\frac{\bar{a}}{\beta}-(1-d)}{(\bar{a}-(1-d))}\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}-(1-\lambda)\,\bar{mc}\left(1-\bar{\tau}^H\right)}{\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\right] \\
&= \frac{\frac{1}{1+\varphi}\bar{H}^{-\varphi}\frac{\lambda \bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}}{\bar{\omega}\left(1+\bar{\tau}^C\right)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\left[\frac{\bar{\tau}^C\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}+\bar{\tau}^H\bar{\tilde{w}}\frac{\bar{H}}{\bar{\tilde{Y}}}+\frac{\frac{\bar{a}}{\beta}-(1-d)}{(\bar{a}-(1-d))}\left(\left(1-\bar{\tilde{G}}\right)-\frac{\bar{\tilde{K}}}{\bar{\tilde{Y}}}\left(\bar{a}-(1-d)\right)\right)-(1-\lambda)\,\bar{mc}}{\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\right] \\
&= \frac{\frac{1}{1+\varphi}\bar{H}^{-\varphi}\frac{\lambda \bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}}{\bar{\omega}\left(1+\bar{\tau}^C\right)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\left[\frac{\bar{\tau}^C\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}+\bar{\tau}^H\bar{\tilde{w}}\frac{\bar{H}}{\bar{\tilde{Y}}}+\bar{\tau}^K\frac{\bar{\tilde{K}}}{\bar{\tilde{Y}}}+\frac{\frac{\bar{a}}{\beta}-(1-d)}{(\bar{a}-(1-d))}\left(1-\bar{\tilde{G}}\right)-\bar{mc}}{\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\right] \\
&= \frac{\frac{1}{1+\varphi}\bar{H}^{-\varphi}\frac{1}{\left(1-\bar{\tau}^K\right)}\frac{\bar{\tilde{I}}}{\bar{\tilde{C}}}}{\bar{\omega}\left(1+\bar{\tau}^C\right)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\left[\bar{\tilde{T}}^C+\bar{\tilde{T}}^K+\bar{\tilde{T}}^H+\frac{\frac{\bar{a}}{\beta}-(1-d)}{(\bar{a}-(1-d))}\left(\left(1-\bar{\tilde{G}}\right)-\frac{\bar{mc}\left(\bar{a}-(1-d)\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\right]
\end{aligned}$$

Under Assumption 2, $\frac{\partial \bar{H}}{\partial \bar{\tau}^K} > 0$. □

C.5 Proof of Proposition 2

Proof. Let $\bar{\tau}^K_{new} = \bar{\tau}^K + \Delta\left(\bar{\tau}^K\right)$. Under the labor tax adjustment case, the steady-state labor tax rate is:

$$\bar{\tau}^H = \frac{\bar{\tilde{S}}-\left[\left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}}-\bar{\tilde{G}}+\bar{\tilde{T}}^C+\bar{\tilde{T}}^K\right]}{\bar{\tilde{w}}\frac{\bar{H}}{\bar{\tilde{Y}}}}$$

Then, after capital tax rate changes, the new steady-state labor tax rate is given by:

$$\begin{aligned}
\bar{\tau}^H_{new} &= \frac{\bar{\tilde{S}}-\left[\left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}}-\bar{\tilde{G}}+\bar{\tau}^C\left(\left(1-\bar{\tilde{G}}\right)-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)\right)+\bar{\tau}^K_{new}\lambda\bar{mc}\right]}{(1-\lambda)\,\bar{mc}} \\
&= \bar{\tau}^H-\frac{\lambda}{1-\lambda}\left(1+\bar{\tau}^C\frac{\left(\bar{a}-(1-d)\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\Delta\left(\bar{\tau}^K\right) \qquad (66) \\
&= \bar{\tau}^H+\Delta\left(\bar{\tau}^H\right).
\end{aligned}$$

Notice that the relative changes of rental rate, wage and capital to hours ratio from their initial steady-states are the same with the lump-sum transfers adjustment case. For after-tax wage, from (60) and (66), we get

$$\ln\left(\frac{\left(1-\bar{\tau}^{H}_{new}\right)\bar{\tilde{w}}_{new}}{\left(1-\bar{\tau}^{H}\right)\bar{\tilde{w}}}\right)=\ln\left(\left(\left(1+\frac{\lambda}{1-\lambda}\frac{\Delta\left(\bar{\tau}^{K}\right)}{\left(1-\bar{\tau}^{H}\right)}\left(1+\bar{\tau}^{C}\frac{\left(\bar{a}-(1-d)\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\right)\left(1-\frac{\Delta\left(\bar{\tau}^{K}\right)}{1-\bar{\tau}^{K}}\right)^{\frac{\lambda}{1-\lambda}}\right).$$
$$\simeq\frac{\lambda}{1-\lambda}\left(\frac{1}{1-\bar{\tau}^{H}}\right)\left(\left(1+\bar{\tau}^{C}\frac{\left(\bar{a}-(1-d)\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)-\frac{1-\bar{\tau}^{H}}{1-\bar{\tau}^{K}}\right)\Delta\left(\bar{\tau}^{K}\right)$$
$$=\mathcal{M}_{W}\Delta\left(\bar{\tau}^{K}\right)$$

where $\mathcal{M}_{W}>0$ if $\left(1+\bar{\tau}^{C}\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)>\frac{1-\bar{\tau}^{H}}{1-\bar{\tau}^{K}}$. For hours, from (65), we get

$$\frac{\bar{H}_{new}}{\bar{H}}=\left(\frac{\frac{(1-\lambda)\bar{mc}-\bar{\tilde{S}}+\left[\left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}}-\bar{\tilde{G}}+\bar{\tau}^{C}\left(\left(1-\bar{\tilde{G}}\right)-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^{K}-\Delta\bar{\tau}^{K}\right)\right)+\left(\bar{\tau}^{K}+\Delta\left(\bar{\tau}^{K}\right)\right)\lambda\bar{mc}\right]}{\bar{\omega}\left(1+\bar{\tau}^{C}\right)\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^{K}-\Delta\left(\bar{\tau}^{K}\right)\right)\right)}}{\frac{(1-\lambda)\bar{mc}-\bar{\tilde{S}}+\left[\left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}}-\bar{\tilde{G}}+\bar{\tau}^{C}\left(\left(1-\bar{\tilde{G}}\right)-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^{K}\right)\right)+\bar{\tau}^{K}\lambda\bar{mc}\right]}{\bar{\omega}\left(1+\bar{\tau}^{C}\right)\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^{K}\right)\right)}}\right)^{\frac{1}{1+\varphi}}$$
$$=\left(\frac{1+\left(\frac{\lambda\bar{mc}+\frac{\bar{\tau}^{C}}{1-\bar{\tau}^{K}}\frac{\bar{\tilde{I}}}{\bar{\tilde{Y}}}}{\left(1-\bar{\tau}^{H}\right)(1-\lambda)\bar{mc}}\right)\Delta\left(\bar{\tau}^{K}\right)}{1+\left(\frac{\frac{1}{1-\bar{\tau}^{K}}\frac{\bar{\tilde{I}}}{\bar{\tilde{Y}}}}{1-\bar{\tilde{G}}-\frac{\bar{\tilde{I}}}{\bar{\tilde{Y}}}}\right)\Delta\left(\bar{\tau}^{K}\right)}\right)^{\frac{1}{1+\varphi}}$$

Then, for small changes of capital tax rate $\Delta\left(\bar{\tau}^{K}\right)$, we get:

$$\ln\left(\frac{\bar{H}_{new}}{\bar{H}}\right)$$
$$=\frac{\frac{1}{1+\varphi}\left(\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)}{1-\bar{\tilde{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))\left(1-\bar{\tau}^{K}\right)}{\frac{\bar{a}}{\beta}-(1-d)}}\left[\frac{\bar{\tau}^{C}\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}+\frac{\frac{\bar{a}}{\beta}-(1-d)}{\bar{a}-(1-d)}\left(1-\bar{\tilde{G}}\right)-\left(1-\bar{\tau}^{K}\right)\frac{\bar{\tilde{K}}}{\bar{\tilde{Y}}}\bar{r}^{K}-(1-\lambda)\bar{mc}\left(1-\bar{\tau}^{H}\right)}{(1-\lambda)\bar{mc}\left(1-\bar{\tau}^{H}\right)}\right]\Delta\left(\bar{\tau}^{K}\right)$$
$$=\frac{\frac{1}{1+\varphi}\left(\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)}{1-\bar{\tilde{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))\left(1-\bar{\tau}^{K}\right)}{\frac{\bar{a}}{\beta}-(1-d)}}\left[\frac{\bar{\tau}^{C}\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}+\bar{\tau}^{K}\bar{r}^{K}\frac{\bar{\tilde{K}}}{\bar{\tilde{Y}}}+\bar{\tau}^{H}\bar{\tilde{w}}\frac{\bar{H}}{\bar{\tilde{Y}}}+\frac{\frac{\bar{a}}{\beta}-(1-d)}{\bar{a}-(1-d)}\left(\left(1-\bar{\tilde{G}}\right)-\frac{\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)}{(1-\lambda)\bar{mc}\left(1-\bar{\tau}^{H}\right)}\right]\Delta\left(\bar{\tau}^{K}\right)$$
$$=\frac{\frac{1}{1+\varphi}\frac{\lambda}{1-\lambda}\left(\bar{a}-(1-d)\right)}{\left(1-\bar{\tau}^{H}\right)\left(\frac{\bar{a}}{\beta}-(1-d)\right)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\left[\bar{\tilde{T}}^{C}+\bar{\tilde{T}}^{H}+\bar{\tilde{T}}^{K}+\frac{\frac{\bar{a}}{\beta}-(1-d)}{\bar{a}-(1-d)}\left(\left(1-\bar{\tilde{G}}\right)-\frac{\bar{mc}\left(\bar{a}-(1-d)\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\right]\Delta\left(\bar{\tau}^{K}\right)$$
$$=\frac{\frac{1}{1+\varphi}\frac{\lambda}{1-\lambda}\left(\bar{a}-(1-d)\right)}{\left(1-\bar{\tau}^{H}\right)\left(\frac{\bar{a}}{\beta}-(1-d)\right)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\left[\bar{\tilde{T}}+\frac{\frac{\bar{a}}{\beta}-(1-d)}{\bar{a}-(1-d)}\left(\left(1-\bar{\tilde{G}}\right)-\frac{\bar{mc}\left(\bar{a}-(1-d)\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\right]\Delta\left(\bar{\tau}^{K}\right)$$
$$=\frac{1}{1+\varphi}\frac{\lambda}{1-\lambda}\left[\frac{\left(1-\bar{\tilde{G}}\right)+\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\left(\bar{\tilde{T}}-\bar{mc}\right)}{\left(1-\bar{\tau}^{H}\right)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\right]\Delta\left(\bar{\tau}^{K}\right)$$
$$=\mathcal{M}_{H}\Delta\left(\bar{\tau}^{K}\right)$$

where $\mathcal{M}_{H}>0$ under Assumption 2. $\square$

C.6 Proof of Lemma 3

Proof. Notice that rental rate of capital, wage, capital to hours ratio, and output to hours ratio are the same with the lump-sum transfers adjustment case in C.2.

To show hours are increasing in $\bar{\tau}^K$, we rewrite (62) as the following:

$$\begin{aligned}
\bar{\tilde{S}} &= \left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}} - \bar{\tilde{G}} + \bar{\tau}^C\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}} + \bar{\tau}^H\bar{\tilde{w}}\frac{\bar{H}}{\bar{\tilde{Y}}} + \bar{\tau}^K\bar{r}^K\frac{\tilde{K}}{\tilde{Y}} \\
&= \left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}} - \bar{\tilde{G}} + \bar{\tau}^C\left[\left(1-\bar{\tilde{G}}\right) - \frac{\lambda\bar{mc}\left(\bar{a}-(1-d)\right)}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right] + \bar{\tau}^H\left(1-\lambda\right)\bar{mc} + \bar{\tau}^K\lambda\bar{mc}
\end{aligned}$$

$$1+\bar{\tau}^C = \frac{\left(1-\bar{\tilde{G}}\right) - \frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right) + \bar{\tilde{S}} - \left[\left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}} - \bar{\tilde{G}} + \bar{\tau}^H\left(1-\lambda\right)\bar{mc} + \bar{\tau}^K\lambda\bar{mc}\right]}{\left(1-\bar{\tilde{G}}\right) - \frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)}$$

Then, from (63), we get

$$\begin{aligned}
\bar{H} &= \left(\bar{\omega}\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H}\frac{\bar{\tilde{C}}}{\bar{H}}\frac{1}{\bar{\tilde{w}}}\right)^{-\frac{1}{1+\varphi}} \\
&= \left(\frac{\bar{\omega}\left(\left(1-\bar{\tilde{G}}\right) - \frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right) + \bar{\tilde{S}} - \left[\left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}} - \bar{\tilde{G}} + \bar{\tau}^H\left(1-\lambda\right)\bar{mc} + \bar{\tau}^K\lambda\bar{mc}\right]\right)}{\left(1-\lambda\right)\bar{mc}\left(1-\bar{\tau}^H\right)}\right)^{-\frac{1}{1+\varphi}}
\end{aligned}$$

Taking a partial derivative with respect to capital tax rate gives:

$$\frac{\partial\bar{H}}{\partial\bar{\tau}^K} = \frac{\bar{\omega}\bar{H}^{2+\varphi}}{1+\varphi}\frac{\lambda}{\left(1-\lambda\right)\left(1-\bar{\tau}^H\right)}\left(1-\frac{\left(\bar{a}-(1-d)\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right) > 0.$$

□

C.7 Proof of Proposition 3

Proof. Let $\bar{\tau}^K_{new} = \bar{\tau}^K + \Delta\left(\bar{\tau}^K\right)$. Under the consumption tax adjustment case, the steady-state consumption tax rate is:

$$\bar{\tau}^C = \frac{\bar{\tilde{S}} - \left[\left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}} - \bar{\tilde{G}} + \bar{\tilde{T}}^H + \bar{\tilde{T}}^K\right]}{\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}$$

Then, after capital tax rate changes, the new steady-state consumption tax rate is given by:

$$\begin{aligned}
\bar{\tau}^C_{new} &= \frac{\bar{\tilde{S}} - \left[\left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}} - \bar{\tilde{G}} + \bar{\tau}^H\left(1-\lambda\right)\bar{mc} + \bar{\tau}^K\lambda\bar{mc} + \Delta\left(\bar{\tau}^K\right)\lambda\bar{mc}\right]}{\left(1-\bar{\tilde{G}}\right) - \frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^K_{new}\right)} \\
&= \frac{\bar{\tau}^C - \frac{\lambda\bar{mc}}{\left(1-\bar{\tilde{G}}\right) - \frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^K\right)}\Delta\left(\bar{\tau}^K\right)}{1 + \frac{\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}}{\left(1-\bar{\tilde{G}}\right) - \frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^K\right)}\Delta\left(\bar{\tau}^K\right)} \\
&= \bar{\tau}^C + \Delta\left(\bar{\tau}^C\right)
\end{aligned}$$

Then,

$$\Delta\left(\bar{\tau}^{C}\right)=-\left(1+\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\bar{\tau}^{C}\right)\frac{\Theta_{C}\Delta\left(\bar{\tau}^{K}\right)}{1+\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\Theta_{C}\Delta\left(\bar{\tau}^{K}\right)}$$

where $\Theta_{C}=\frac{\lambda\bar{mc}}{\left(1-\bar{\tilde{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^{K}\right)}=\bar{r}^{K}\frac{\bar{\tilde{K}}}{\bar{\tilde{C}}}>0$.

Notice that the relative changes of rental rate, wage and capital to hours ratio from their initial steady-states are the same with the lump-sum transfers adjustment case. For hours, we get

$$\begin{aligned}
\frac{\bar{H}_{new}}{\bar{H}}&=\left(\frac{\left(1-\bar{\tilde{G}}\right)-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^{K}_{new}\right)+\bar{\tilde{S}}-\left[\left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}}-\bar{\tilde{G}}+\bar{\tau}^{H}\left(1-\lambda\right)\bar{mc}+\bar{\tau}^{K}_{new}\lambda\bar{mc}\right]}{\left(1-\bar{\tilde{G}}\right)-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^{K}\right)+\bar{\tilde{S}}-\left[\left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}}-\bar{\tilde{G}}+\bar{\tau}^{H}\left(1-\lambda\right)\bar{mc}+\bar{\tau}^{K}\lambda\bar{mc}\right]}\right)^{-\frac{1}{1+\varphi}}\\
&=\left(1-\frac{\left(1-\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)\lambda\bar{mc}\Delta\left(\bar{\tau}^{K}\right)}{\left(1-\bar{\tilde{G}}\right)-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^{K}\right)+\bar{\tilde{S}}-\left[\left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}}-\bar{\tilde{G}}+\bar{\tau}^{H}\left(1-\lambda\right)\bar{mc}+\bar{\tau}^{K}\lambda\bar{mc}\right]}\right)^{-\frac{1}{1+\varphi}}\\
&=\left(1-\frac{\left(1-\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)\lambda\bar{mc}}{\left(1+\bar{\tau}^{C}\right)\left(\left(1-\bar{\tilde{G}}\right)-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^{K}\right)\right)}\Delta\left(\bar{\tau}^{K}\right)\right)^{-\frac{1}{1+\varphi}}\\
&=\left(1-\frac{\left(1-\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)\lambda\bar{mc}}{\left(1+\bar{\tau}^{C}\right)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\Delta\left(\bar{\tau}^{K}\right)\right)^{-\frac{1}{1+\varphi}}
\end{aligned}$$

Then, for small changes of capital tax rate $\Delta\left(\bar{\tau}^{K}\right)$, we get:

$$\begin{aligned}
\ln\left(\frac{\bar{H}_{new}}{\bar{H}}\right)&=\frac{1}{1+\varphi}\frac{\lambda\bar{mc}}{\left(1+\bar{\tau}^{C}\right)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\left(1-\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)\Delta\left(\bar{\tau}^{K}\right)\\
&=\mathcal{M}_{H,\tau^{C}}\Delta\left(\bar{\tau}^{K}\right)
\end{aligned}$$

where $\mathcal{M}_{H,\tau^{C}}=\frac{1}{1+\varphi}\frac{\lambda\bar{mc}}{\left(1+\bar{\tau}^{C}\right)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\left(1-\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)>0$.

Now, we find changes of levels of variables. For capital, investment and output, we can easily verify that

$$\frac{\bar{\tilde{K}}_{new}}{\bar{\tilde{K}}}=\frac{\bar{\tilde{k}}_{new}}{\bar{\tilde{k}}}\frac{\bar{H}_{new}}{\bar{H}}=\left(1-\frac{\Delta\left(\bar{\tau}^{K}\right)}{1-\bar{\tau}^{K}}\right)^{\frac{1}{1-\lambda}}\left(1-\frac{\left(1-\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)\lambda\bar{mc}}{\left(1+\bar{\tau}^{C}\right)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\Delta\left(\bar{\tau}^{K}\right)\right)^{-\frac{1}{1+\varphi}}$$

Now for small changes in the capital tax rate $\Delta\left(\bar{\tau}^{K}\right)$, the percent changes of rental rate, wages, capital to hours ratio, output to hours ratio from their initial steady-states are:

$$
\begin{aligned}
\ln\left(\frac{\bar{\tilde{K}}_{new}}{\bar{\tilde{K}}}\right) = \ln\left(\frac{\bar{\tilde{I}}_{new}}{\bar{\tilde{I}}}\right) &\simeq -\frac{1}{1-\lambda}\left(1-\left(1-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\frac{1}{1+\varphi}\frac{\left(1-\bar{\tau}^K\right)\bar{\tilde{w}}\bar{H}}{\left(1+\bar{\tau}^C\right)\bar{\tilde{C}}}\right)\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K} \\
&= -\frac{1}{1-\lambda}\left(1-\left(1-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\left(\frac{1-\bar{\tau}^K}{1-\tau_t^H}\right)\frac{\bar{\omega}H_t^{1+\varphi}}{1+\varphi}\right)\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K} \\
&= -\frac{1}{1-\lambda}\left(1-\frac{1}{1+\varphi}\frac{1-\lambda}{\lambda}\frac{1}{1+\bar{\tau}^C}\left(\frac{\left(1-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\lambda\bar{m}c\left(1-\bar{\tau}^K\right)}{\left(1-\bar{\tilde{G}}\right)-\frac{\lambda\bar{m}c(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)}\right)\right)\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}
\end{aligned}
$$

□

C.8 Proof of Proposition 4

Proof. From (57), (60) and (59), we get

$$
\begin{aligned}
\frac{\bar{\tilde{K}}}{\bar{H}} &= \frac{\bar{\tilde{w}}}{\bar{r}^K}\frac{\lambda}{1-\lambda} \\
&= \left(\frac{\lambda\bar{m}c}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right)^{\frac{1}{1-\lambda}}.
\end{aligned}
$$

The amount of changes in capital to hours ratio to the capital tax cut is the same in both lump-sum transfers adjustment case and labor tax rate case. In a similar way, we know that output to hours ratio, investment to hours ratio, and consumption to hours ratio change by the same amount in both cases. Thus, all the magnitudes of changes in macro quantities to capital tax cuts are determined by the hours responses. Now, we compare the changes in hours to capital tax rate changes under the transfers adjustment case with the changes under the labor tax rate adjustment case. Notice that the initial steady-states are the same in both cases. Let $\bar{H}^T_{new}$ and $\bar{H}^L_{new}$ denote the steady-state hours after the capital tax changes in the transfers adjustment case and in the labor tax rate adjustment case, respectively. Then, from (63) and (66), we get

$$
\begin{aligned}
\frac{\bar{H}^T_{new}}{\bar{H}^L_{new}} &= \left(\frac{\frac{\bar{\omega}}{(1-\lambda)\bar{m}c}\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H}\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{m}c(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)\right)}{\frac{\bar{\omega}}{(1-\lambda)\bar{m}c}\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H_{new}}\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{m}c(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)\right)}\right)^{-\frac{1}{1+\varphi}} \\
&= \left(1-\frac{\Delta\left(\bar{\tau}^H\right)}{1-\bar{\tau}^H}\right)^{-\frac{1}{1+\varphi}} \\
&= \left(1+\frac{\lambda}{1-\lambda}\left(\frac{1}{1-\bar{\tau}^H}\right)\left(1+\bar{\tau}^C\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)\Delta\left(\bar{\tau}^K\right)\right)^{-\frac{1}{1+\varphi}}.
\end{aligned}
$$

For small changes in capital tax rate $\Delta\left(\bar{\tau}^K\right)$, we get

$$
\begin{aligned}
\ln\left(\frac{\bar{H}^T_{new}}{\bar{H}^L_{new}}\right) &= -\frac{1}{1+\varphi}\frac{\lambda}{1-\lambda}\left(\frac{1}{1-\bar{\tau}^H}\right)\left(1+\bar{\tau}^C\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)\Delta\left(\bar{\tau}^K\right) \\
&= -\Theta\Delta\left(\bar{\tau}^K\right)
\end{aligned}
$$

where $\Theta = \frac{1}{1+\varphi}\frac{\lambda}{1-\lambda}\left(\frac{1}{1-\bar{\tau}^H}\right)\left(1+\bar{\tau}^C\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right) > 0$. Then, for the levels of output, consumption, capital and investment, the differences are the same: that is,

$$\ln\left(\frac{\bar{\tilde{Y}}^T_{new}}{\bar{\tilde{Y}}^L_{new}}\right) = \ln\left(\frac{\bar{\tilde{C}}^T_{new}}{\bar{\tilde{C}}^L_{new}}\right) = \ln\left(\frac{\bar{\tilde{K}}^T_{new}}{\bar{\tilde{K}}^L_{new}}\right) = \ln\left(\frac{\bar{\tilde{I}}^T_{new}}{\bar{\tilde{I}}^L_{new}}\right) = \ln\left(\frac{\bar{H}^T_{new}}{\bar{H}^L_{new}}\right) = -\Theta\Delta\left(\bar{\tau}^K\right).$$

□

C.9 Relative Changes of Variables in the Consumption Tax Rate Adjustment Case

Proposition 5. *Let $\bar{\tau}^K_{new} = \bar{\tau}^K + \Delta\left(\bar{\tau}^K\right)$. Denote $\bar{X}^T_{new}$ and $\bar{X}^C_{new}$ as the new steady-state variables in transfer adjustment case and in consumption tax rate adjustment case, respectively. For small changes in the capital tax rate $\Delta\left(\bar{\tau}^K\right)$, for $X \in \left\{\tilde{C}, \tilde{K}, \tilde{I}, \tilde{Y}, H\right\}$, we get*

$$\ln\left(\frac{\bar{X}^T_{new}}{\bar{X}^C_{new}}\right) = -\Theta^T_C\Delta\left(\bar{\tau}^K\right)$$

where $\Theta^T_C > 0$ if and only if $\bar{\tilde{G}} < 1 - \lambda\frac{\theta-1}{\theta}\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)$.

Proof. Let $\bar{H}^T_{new}$ and $\bar{H}^C_{new}$ denote the steady-state hours after the capital tax changes in the transfers adjustment case and in the consumption tax rate adjustment case, respectively. Then, we get

$$\begin{aligned}
\frac{\bar{H}^T_{new}}{\bar{H}^C_{new}} &= \left(\frac{\frac{\bar{\omega}}{(1-\lambda)\bar{m}c}\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H}\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{m}c(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)\right)}{\frac{\bar{\omega}}{(1-\lambda)\bar{m}c}\frac{1+\bar{\tau}^C_{new}}{1-\bar{\tau}^H}\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{m}c(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)\right)}\right)^{-\frac{1}{1+\varphi}} \\
&= \left(1+\frac{\Delta\left(\bar{\tau}^C\right)}{1+\bar{\tau}^C}\right)^{\frac{1}{1+\varphi}} \\
&= \left(1-\left(\frac{1+\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\bar{\tau}^C}{1+\bar{\tau}^C}\right)\frac{\tilde{\Delta}\left(\bar{\tau}^K\right)}{1+\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\tilde{\Delta}\left(\bar{\tau}^K\right)}\right)^{\frac{1}{1+\varphi}}
\end{aligned}$$

For small changes in capital tax rate $\Delta\left(\bar{\tau}^K\right)$, we get

$$\ln\left(\frac{\bar{H}^T_{new}}{\bar{H}^C_{new}}\right) = -\frac{1}{1+\varphi}\left(\frac{1+\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\bar{\tau}^C}{1+\bar{\tau}^C}\right)\left(\frac{\lambda\bar{m}c}{\left(1-\bar{\tilde{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{m}c\left(1-\bar{\tau}^K_{new}\right)}\right)\Delta\left(\bar{\tau}^K\right)$$

Then, for the levels of output, consumption, capital and investment, the differences are the same:

that is,

$$
\begin{aligned}
\ln\left(\frac{\bar{\tilde{Y}}^T_{new}}{\bar{\tilde{Y}}^C_{new}}\right) = \ln\left(\frac{\bar{\tilde{C}}^T_{new}}{\bar{\tilde{C}}^C_{new}}\right) = \ln\left(\frac{\bar{\tilde{K}}^T_{new}}{\bar{\tilde{K}}^C_{new}}\right) = \ln\left(\frac{\bar{\tilde{I}}^T_{new}}{\bar{\tilde{I}}^C_{new}}\right) &= \ln\left(\frac{\bar{H}^T_{new}}{\bar{H}^C_{new}}\right) \\
&= \frac{1}{1+\varphi}\frac{\Delta\left(\bar{\tau}^C\right)}{1+\bar{\tau}^C} \\
&= -\frac{1}{1+\varphi}\left(\frac{1+\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\bar{\tau}^C}{1+\bar{\tau}^C}\right)\left(\frac{\lambda\bar{mc}}{\left(1-\bar{\tilde{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^K_{new}\right)}\right)\Delta\left(\bar{\tau}^K\right) \\
&= -\mathcal{M}^T_C\Delta\left(\bar{\tau}^K\right).
\end{aligned}
$$

Then, $\mathcal{M}^T_C = \frac{1}{1+\varphi}\left(\frac{1+\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\bar{\tau}^C}{1+\bar{\tau}^C}\right)\left(\frac{\lambda\bar{mc}}{\left(1-\bar{\tilde{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^K_{new}\right)}\right) > 0$ if

$$
\begin{aligned}
\bar{\tilde{G}} &< 1-\lambda\frac{\theta-1}{\theta}\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right) \\
&\approx 1-0.15 = 0.85
\end{aligned}
$$

in our calibration. In this case, capital tax cut is more expansionary in the transfer adjustment case than in the consumption tax rate adjustment case. □

Proposition 6. *Let $\bar{\tau}^K_{new} = \bar{\tau}^K + \Delta\left(\bar{\tau}^K\right)$. Denote $\bar{X}^L_{new}$ and $\bar{X}^C_{new}$ as the new steady-state variables in labor tax rate adjustment case and in consumption tax rate adjustment case, respectively. For small changes in the capital tax rate $\Delta\left(\bar{\tau}^K\right)$, for $X \in \left\{\tilde{C}, \tilde{K}, \tilde{I}, \tilde{Y}, H\right\}$, we get*

$$
\ln\left(\frac{\bar{X}^L_{new}}{\bar{X}^C_{new}}\right) = \Theta^L_C\Delta\left(\bar{\tau}^K\right)
$$

where $\Theta^L_C > 0$ if and only if $\bar{\tilde{G}} < 1-\lambda\frac{\theta-1}{\theta}\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right) - (1-\lambda)\frac{\theta-1}{\theta}\frac{1-\bar{\tau}^H}{1+\bar{\tau}^C}$.

Proof. Let $\bar{H}^L_{new}$ and $\bar{H}^C_{new}$ denote the steady-state hours after the capital tax changes in the labor tax adjustment case and in the consumption tax adjustment case, respectively. Then, we get

$$
\begin{aligned}
\frac{\bar{H}^L_{new}}{\bar{H}^C_{new}} &= \left(\frac{\frac{\bar{\omega}}{(1-\lambda)\bar{mc}}\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H_{new}}\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)\right)}{\frac{\bar{\omega}}{(1-\lambda)\bar{mc}}\frac{1+\bar{\tau}^C_{new}}{1-\bar{\tau}^H}\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)\right)}\right)^{-\frac{1}{1+\varphi}} \\
&= \left(\frac{1+\bar{\tau}^C_{new}}{1+\bar{\tau}^C}\frac{1-\bar{\tau}^H_{new}}{1-\bar{\tau}^H}\right)^{\frac{1}{1+\varphi}} \\
&= \left(\left(1+\frac{\Delta\left(\bar{\tau}^C\right)}{1+\bar{\tau}^C}\right)\left(1-\frac{\Delta\left(\bar{\tau}^H\right)}{1-\bar{\tau}^H}\right)\right)^{\frac{1}{1+\varphi}} \\
&= \left(\left(1-\frac{1+\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\bar{\tau}^C}{1+\bar{\tau}^C}\frac{\lambda\bar{mc}}{\left(1-\bar{\tilde{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^K_{new}\right)}\Delta\left(\bar{\tau}^K\right)\right)\left(1+\frac{1+\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\bar{\tau}^C}{1-\bar{\tau}^H}\frac{\lambda}{1-\lambda}\Delta\left(\bar{\tau}^K\right)\right)\right)^{\frac{1}{1+\varphi}}
\end{aligned}
$$

For small changes in capital tax rate $\Delta\left(\bar{\tau}^{K}\right)$, we get

$$
\begin{aligned}
\ln\left(\frac{\bar{H}^{L}_{new}}{\bar{H}^{C}_{new}}\right) &= \frac{1}{1+\varphi}\left(\frac{\Delta\left(\bar{\tau}^{C}\right)}{1+\bar{\tau}^{C}} - \frac{\Delta\left(\bar{\tau}^{H}\right)}{1-\bar{\tau}^{H}}\right) \\
&= \frac{1}{1+\varphi}\left(1+\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\bar{\tau}^{C}\right)\left(\frac{1}{1-\bar{\tau}^{H}}\frac{\lambda}{1-\lambda} - \frac{1}{1+\bar{\tau}^{C}}\frac{\lambda\bar{mc}}{\left(1-\bar{\tilde{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^{K}_{new}\right)}\right)\Delta\left(\bar{\tau}^{K}\right) \\
&= \frac{1}{1+\varphi}\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{1+\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\bar{\tau}^{C}}{1-\bar{\tau}^{H}}\right)\left(1-\frac{1-\bar{\tau}^{H}}{1+\bar{\tau}^{C}}\frac{(1-\lambda)\,\bar{mc}}{\left(1-\bar{\tilde{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^{K}_{new}\right)}\right)\Delta\left(\bar{\tau}^{K}\right)
\end{aligned}
$$

Then, for the levels of output, consumption, capital and investment, the differences are the same: that is,

$$
\begin{aligned}
\ln\left(\frac{\bar{\tilde{Y}}^{L}_{new}}{\bar{\tilde{Y}}^{C}_{new}}\right) = \ln\left(\frac{\bar{\tilde{C}}^{L}_{new}}{\bar{\tilde{C}}^{C}_{new}}\right) = \ln\left(\frac{\bar{\tilde{K}}^{L}_{new}}{\bar{\tilde{K}}^{C}_{new}}\right) = \ln\left(\frac{\bar{\tilde{I}}^{L}_{new}}{\bar{\tilde{I}}^{C}_{new}}\right) = \ln\left(\frac{\bar{H}^{L}_{new}}{\bar{H}^{C}_{new}}\right) &= \frac{1}{1+\varphi}\left(\frac{\Delta\left(\bar{\tau}^{C}\right)}{1+\bar{\tau}^{C}} - \frac{\Delta\left(\bar{\tau}^{H}\right)}{1-\bar{\tau}^{H}}\right) \\
&= \frac{1}{1+\varphi}\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{1+\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\bar{\tau}^{C}}{1-\bar{\tau}^{H}}\right)\left(1-\frac{1-\bar{\tau}^{H}}{1+\bar{\tau}^{C}}\frac{(1-\lambda)\,\bar{mc}}{\left(1-\bar{\tilde{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^{K}_{new}\right)}\right)\Delta\left(\bar{\tau}^{K}\right) \\
&= \mathcal{M}^{L}_{C}\Delta\left(\bar{\tau}^{K}\right).
\end{aligned}
$$

Notice that $\mathcal{M}^{L}_{C} = \frac{1}{1+\varphi}\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{1+\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\bar{\tau}^{C}}{1-\bar{\tau}^{H}}\right)\left(1-\frac{1-\bar{\tau}^{H}}{1+\bar{\tau}^{C}}\frac{(1-\lambda)\bar{mc}}{\left(1-\bar{\tilde{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^{K}_{new}\right)}\right) > 0$ if

$$
\begin{aligned}
\bar{\tilde{G}} &< 1-\lambda\frac{\theta-1}{\theta}\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^{K}_{new}\right)-(1-\lambda)\frac{\theta-1}{\theta}\frac{1-\bar{\tau}^{H}}{1+\bar{\tau}^{C}} \\
&\approx 1-0.4976 = 0.5023
\end{aligned}
$$

in our baseline calibration. In this case, capital tax cut is more expansionary in the consumption tax rate adjustment case than in the labor tax rate adjustment case. □

C.10 Changes in Output with Infinite Frisch Elasticity

How do the changes in output to the capital tax rate vary with the different Frisch elasticity parameters under labor tax rate adjustment case? Notice that from (65), we get

$$
\begin{aligned}
\ln\left(\frac{\bar{\tilde{Y}}_{new}}{\bar{\tilde{Y}}}\right) &= \ln\left(\frac{\bar{\tilde{y}}_{new}}{\bar{\tilde{y}}}\right) + \ln\left(\frac{\bar{H}_{new}}{\bar{H}}\right). \\
&= -\left[\left(\frac{\lambda}{1-\lambda}\frac{1}{1-\bar{\tau}^{K}}\right) - \mathcal{M}_{H}\right]\Delta\left(\bar{\tau}^{K}\right)
\end{aligned}
$$

where $\mathcal{M}_H = \frac{\frac{1}{1+\varphi}\left(\frac{\lambda}{(1-\lambda)}\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)\left[\tilde{\bar{T}}+\frac{\frac{\bar{a}^{\eta}}{\beta}-(1-d)}{(\bar{a}-(1-d))}\left(1-\tilde{\bar{G}}-\frac{\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}^{\eta}}{\beta}-(1-d)}\right)\right]}{(1-\bar{\tau}^H)\left(1-\tilde{\bar{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}(1-\bar{\tau}^K)\right)}$ and $\tilde{\bar{T}} = \tilde{\bar{T}}^C + \tilde{\bar{T}}^H + \tilde{\bar{T}}^K$. Then, we can rewrite $\ln\left(\frac{\bar{Y}_{new}}{\bar{Y}}\right)$ as

$$-\frac{\ln\left(\frac{\bar{Y}_{new}}{\bar{Y}}\right)}{\Delta\left(\bar{\tau}^K\right)} = \frac{\lambda}{1-\lambda}\left(\frac{1}{1-\bar{\tau}^K} - \frac{\frac{1}{1+\varphi}\left(\frac{\lambda}{(1-\lambda)}\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)\left[\tilde{\bar{T}}+\frac{\frac{\bar{a}}{\beta}-(1-d}{(\bar{a}-(1-d))}\left(1-\tilde{\bar{G}}-\frac{\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)\right]}{\left(1-\tilde{\bar{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right)\left(1-\bar{\tau}^H\right)}\right).$$

Notice that under Assumption 2, the second term in the RHS is positive. Thus, $-\frac{\ln\left(\frac{\bar{Y}_{new}}{\bar{Y}}\right)}{\Delta(\bar{\tau}^K)}$ is increasing in φ and it has a lower bound at $\frac{1}{\varphi} = \infty$. That is, for $\Delta\left(\bar{\tau}^K\right) < 0$,

$$\frac{\lambda}{1-\lambda}\left[\frac{1}{1-\bar{\tau}^K} - \frac{\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(\tilde{\bar{T}}^C+\tilde{\bar{T}}^H+\tilde{\bar{T}}^K-\bar{mc}\right)+\left(1-\tilde{\bar{G}}\right)}{\left(1-\tilde{\bar{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right)\left(1-\bar{\tau}^H\right)}\right] \leq -\frac{\ln\left(\frac{\bar{Y}_{new}}{\bar{Y}}\right)}{\Delta\left(\bar{\tau}^K\right)} < \frac{\lambda}{1-\lambda}\left(\frac{1}{1-\bar{\tau}^K}\right).$$

The lower bound is:

$$\begin{aligned}
&\frac{\lambda}{1-\lambda}\left(\frac{1}{1-\bar{\tau}^K} - \frac{\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(\tilde{\bar{T}}^C+\tilde{\bar{T}}^H+\tilde{\bar{T}}^K-\bar{mc}\right)+\left(1-\tilde{\bar{G}}\right)}{\left(1-\tilde{\bar{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))(1-\bar{\tau}^K)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\left(1-\bar{\tau}^H\right)}\right)\\
=&\frac{\lambda}{1-\lambda}\left(\frac{1}{1-\bar{\tau}^K} - \frac{\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(\bar{\tau}^C\frac{\tilde{\bar{C}}}{\tilde{\bar{Y}}}+\bar{\tau}^H(1-\lambda)\bar{mc}+\bar{\tau}^K\lambda\bar{mc}-\bar{mc}\right)+\left(1-\tilde{\bar{G}}\right)}{\left(1-\tilde{\bar{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))(1-\bar{\tau}^K)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\left(1-\bar{\tau}^H\right)}\right)\\
=&\frac{\left(\frac{1-\bar{\tau}^H}{1-\bar{\tau}^K}-\left(1+\frac{(\bar{a}-(1-d))\bar{\tau}^C}{\frac{\bar{a}}{\beta}-(1-d)}\right)\right)\left(1-\tilde{\bar{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))\left(1-\bar{\tau}^K\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)+\frac{(\bar{a}-(1-d))(1-\lambda)\bar{mc}}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^H\right)}{\frac{1-\lambda}{\lambda}\left(1-\tilde{\bar{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))(1-\bar{\tau}^K)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\left(1-\bar{\tau}^H\right)}\\
=&\frac{\lambda}{1-\lambda}\left[\frac{1}{1-\bar{\tau}^K} - \frac{1+\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\bar{\tau}^C}{1-\bar{\tau}^H} + \left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\frac{(1-\lambda)\bar{mc}}{\left(\frac{\tilde{\bar{C}}}{\tilde{\bar{Y}}}\right)}\right]\\
=&\frac{\lambda}{1-\lambda}\left(\frac{1}{1-\bar{\tau}^H}\right)\left(\left(1+\frac{1-\lambda}{\lambda}\frac{\tilde{\bar{I}}}{\tilde{\bar{C}}}\right)\frac{1-\bar{\tau}^H}{1-\bar{\tau}^K} - \left(1+\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\bar{\tau}^C\right)\right)
\end{aligned}$$

Thus, the change in output is positive even under the infinite Frisch elasticity if $\frac{1-\bar{\tau}^H}{1-\bar{\tau}^K} > 1+\bar{\tau}^C$.

C.11 Analytical Results with Capital-Skill Complementarity

Equipment and Structure Capital Rental Rates

From equations (40) and (41), we have

$$\frac{\partial\bar{r}^{K,e}}{\partial\bar{\tau}^K} = \frac{\bar{r}^{K,e}}{1-\bar{\tau}^K} > 0,\ \frac{\partial\bar{r}^{K,b}}{\partial\bar{\tau}^K} = \frac{\bar{r}^{K,b}}{1-\bar{\tau}^K} > 0.$$

Equipment Capital to Skilled Labor Ratio

We can combine equations (45) and (46) to get

$$
\begin{aligned}
\left(\bar{r}^{K,e}\right)^{\frac{1}{1-\alpha}}\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{1-\rho} &= \frac{\lambda\left(1-\mu\right)\left(\frac{1-\alpha}{\alpha}\right)\left(\alpha\bar{mc}\right)^{\frac{1}{1-\alpha}}}{\left(q_0\frac{\frac{\gamma}{\beta}-(1-d_b)}{\frac{1}{\beta}-(1-d_e)}\right)^{\frac{\alpha}{1-\alpha}}}\left(\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho}+\left(1-\lambda\right)\right)^{\frac{\sigma}{\rho}-1} \\
&\times\left[\mu\left(\frac{\bar{L}_u}{\bar{L}_s}\right)^{\sigma}+\left(1-\mu\right)\left(\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho}+\left(1-\lambda\right)\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\sigma}{\sigma}}.
\end{aligned}
$$

Combining equations (43), (44) and (51), we derive unskilled to skilled labor ratio $(\bar{L}_u/\bar{L}_s)$,

$$
\begin{aligned}
\left(\frac{\bar{L}_u}{\bar{L}_s}\right) &= \left(\frac{\left(1-\lambda\right)\left(1-\mu\right)}{\mu}\frac{\tilde{\Lambda}_{s,u}}{\frac{\bar{\omega}^s}{\bar{\omega}^u}\left(\frac{N^u}{N^s}\right)^{\varphi}}\frac{\left(1-\lambda^s_{\tau^H}\bar{\tau}^H\right)}{\left(1-\lambda^u_{\tau^H}\bar{\tau}^H\right)}\left(\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho}+\left(1-\lambda\right)\right)^{\frac{\sigma}{\rho}-1}\right)^{\frac{1}{\sigma-1-\varphi}} \\
&= \left(\frac{\left(1-\lambda\right)\left(1-\mu\right)}{\mu}\frac{\tilde{\Lambda}_{s,u}}{\frac{\bar{\omega}^s}{\bar{\omega}^u}\left(\frac{N^u}{N^s}\right)^{\varphi}}\frac{\left(1-\lambda^s_{\tau^H}\bar{\tau}^H\right)}{\left(1-\lambda^u_{\tau^H}\bar{\tau}^H\right)}\right)^{\frac{1}{\sigma-\varphi-1}}\left(\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho}+\left(1-\lambda\right)\right)^{\frac{(\sigma-\rho)}{\rho(\sigma-\varphi-1)}}.
\end{aligned}
$$

Let's combine the above two equations to get a nonlinear equation which determines steady-state equipment capital to skilled labor ratio $(\bar{\tilde{K}}_e/\bar{L}_s)$ which is a function of equipment capital rental rate $(\bar{r}^{K,e})$,

$$
\begin{aligned}
\left(\bar{r}^{K,e}\right)^{\frac{1}{1-\alpha}} &= \chi_A\left[\mu\chi_C\left(F\right)^{\frac{\sigma(\sigma-\rho)}{\rho(\sigma-\varphi-1)}}+\left(1-\mu\right)\left(F\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\sigma}{\sigma}}\left(F\right)^{\frac{\sigma}{\rho}-1}\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho-1} \\
&= \chi_A\left[\mu\chi_C\left(F\right)^{\frac{\sigma}{\rho}\left(\frac{\varphi(\sigma-\rho)}{(1-\sigma)(1-\sigma+\varphi)}\right)}+\left(1-\mu\right)\left(F\right)^{\frac{\sigma}{\rho}\left(\frac{1-\rho}{1-\sigma}\right)}\right]^{\frac{1-\sigma}{\sigma}}\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho-1}
\end{aligned}
\tag{67}
$$

where

$$
\chi_A = \frac{\lambda\left(1-\mu\right)\left(\frac{1-\alpha}{\alpha}\right)\left(\alpha\bar{mc}\right)^{\frac{1}{1-\alpha}}}{\left(q_0\frac{\frac{\gamma}{\beta}-(1-d_b)}{\frac{1}{\beta}-(1-d_e)}\right)^{\frac{\alpha}{1-\alpha}}} > 0
$$

$$
\chi_C = \left(\frac{\left(1-\lambda\right)\left(1-\mu\right)}{\mu}\frac{\tilde{\Lambda}_{s,u}}{\frac{\bar{\omega}^s}{\bar{\omega}^u}\left(\frac{N^u}{N^s}\right)^{\varphi}}\frac{\left(1-\lambda^s_{\tau^H}\bar{\tau}^H\right)}{\left(1-\lambda^u_{\tau^H}\bar{\tau}^H\right)}\right)^{\frac{\sigma}{\sigma-\varphi-1}} > 0
$$

and

$$
F\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right) = \lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho}+\left(1-\lambda\right) > 0.
$$

Taking a partial derivative of (67) with respect to capital tax rate $\bar{\tau}^K$, we have

$$\begin{aligned}
\frac{1}{1-\alpha}\left(\bar{r}^{K,e}\right)^{\frac{\alpha}{1-\alpha}}\frac{\partial\left(\bar{r}^{K,e}\right)}{\partial\bar{\tau}^K} =& \frac{(1-\sigma)\left(\bar{r}^{K,e}\right)^{\frac{1}{1-\alpha}}}{F}\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho-1}\frac{\partial}{\partial\bar{\tau}^K}\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right) \\
&\times\left(\frac{\left(\frac{\varphi(\sigma-\rho)}{(1-\sigma)(1-\sigma+\varphi)}\right)\mu\chi_C\left(F\right)^{\frac{\sigma}{\rho}\left(\frac{\varphi(\sigma-\rho)}{(1-\sigma)(1-\sigma+\varphi)}\right)}+\left(\frac{1-\rho}{1-\sigma}\right)(1-\mu)\left(F\right)^{\frac{\sigma}{\rho}\left(\frac{1-\rho}{1-\sigma}\right)}}{\mu\chi_C\left(F\right)^{\frac{\sigma}{\rho}\left(\frac{\varphi(\sigma-\rho)}{(1-\sigma)(1-\sigma+\varphi)}\right)}+(1-\mu)\left(F\right)^{\frac{\sigma}{\rho}\left(\frac{1-\rho}{1-\sigma}\right)}}\right) \\
&+(\rho-1)\left(\bar{r}^{K,e}\right)^{\frac{1}{1-\alpha}}\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{-1}\frac{\partial}{\partial\bar{\tau}^K}\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right) \\
\frac{1}{1-\alpha}\frac{\partial\left(\bar{r}^{K,e}\right)}{\partial\bar{\tau}^K} =& -\frac{\bar{r}^{K,e}}{F}\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{-1}\frac{\partial}{\partial\bar{\tau}^K}\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right) \\
&\times\left[\left(\frac{(1-\sigma)\frac{(1-\rho+\varphi)}{(1-\sigma+\varphi)}\mu\chi_C\left(F\right)^{\frac{\sigma}{\rho}\left(\frac{\varphi(\sigma-\rho)}{(1-\sigma)(1-\sigma+\varphi)}\right)}}{\mu\chi_C\left(F\right)^{\frac{\sigma}{\rho}\left(\frac{\varphi(\sigma-\rho)}{(1-\sigma)(1-\sigma+\varphi)}\right)}+(1-\mu)\left(F\right)^{\frac{\sigma}{\rho}\left(\frac{1-\rho}{1-\sigma}\right)}}\right)\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho}+(1-\rho)(1-\lambda)\right]
\end{aligned}$$

Thus

$$\begin{aligned}
\frac{\partial}{\partial\bar{\tau}^K}\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right) =& -\frac{F}{1-\alpha}\frac{1}{\bar{r}^{K,e}}\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\left[(1-\sigma)\frac{(1-\rho+\varphi)}{(1-\sigma+\varphi)}\left(\frac{\mu\chi_C}{\mu\chi_C+(1-\mu)F^{\frac{\sigma}{\rho}\left(\frac{1-\rho+\varphi}{1-\sigma+\varphi}\right)}}\right)\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho}+(1-\rho)(1-\lambda)\right]^{-1}\frac{\partial\left(\bar{r}^{K,e}\right)}{\partial\bar{\tau}^K} \\
&< 0.
\end{aligned}$$

Unskilled to skilled labor

Combining equations (43), (44) and (51), we derive unskilled to skilled labor ratio $(\bar{L}_u/\bar{L}_s)$,

$$\begin{aligned}
\left(\frac{\bar{L}_u}{\bar{L}_s}\right) &= \left(\frac{(1-\lambda)(1-\mu)}{\mu}\frac{\tilde{\Lambda}_{s,u}}{\frac{\bar{\omega}^s}{\bar{\omega}^u}\left(\frac{N^u}{N^s}\right)^{\varphi}}\frac{\left(1-\lambda^s_{\tau^H}\bar{\tau}^H\right)}{\left(1-\lambda^u_{\tau^H}\bar{\tau}^H\right)}\right)^{\frac{1}{\sigma-\varphi-1}}\left(\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho}+(1-\lambda)\right)^{\frac{(\sigma-\rho)}{\rho(\sigma-\varphi-1)}} \\
&= \chi_c^{\frac{1}{\sigma}}\left(\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho}+(1-\lambda)\right)^{\frac{(\sigma-\rho)}{\rho(\sigma-\varphi-1)}} \qquad (68)
\end{aligned}$$

where $\chi_c = \left(\frac{(1-\lambda)(1-\mu)}{\mu}\frac{\tilde{\Lambda}_{s,u}}{\frac{\bar{\omega}^s}{\bar{\omega}^u}\left(\frac{N^u}{N^s}\right)^{\varphi}}\frac{\left(1-\lambda^s_{\tau^H}\bar{\tau}^H\right)}{\left(1-\lambda^u_{\tau^H}\bar{\tau}^H\right)}\right)^{\frac{\sigma}{\sigma-\varphi-1}} > 0$. Then,

$$\begin{aligned}
\frac{\partial}{\partial\bar{\tau}^K}\left(\frac{\bar{L}_u}{\bar{L}_s}\right) &= -\left(\frac{\sigma-\rho}{1-\sigma+\varphi}\right)\frac{\bar{L}_u}{\bar{L}_s}\left(\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho}+(1-\lambda)\right)^{-1}\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho-1}\frac{\partial}{\partial\bar{\tau}^K}\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right) \\
&> 0
\end{aligned}$$

Skill premium

Combining equations (43), (44) and (68), we derive the skill premium

$$\begin{aligned}
\frac{\bar{\tilde{w}}^s}{\bar{\tilde{w}}^u} &= \frac{(1-\lambda)(1-\mu)}{\mu}\frac{\left(\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho}+(1-\lambda)\right)^{\frac{\sigma}{\rho}-1}}{\left(\frac{\bar{L}_u}{\bar{L}_s}\right)^{\sigma-1}} \\
&= \frac{(1-\lambda)(1-\mu)}{\mu}\chi_c^{\frac{1-\sigma}{\sigma}}\left(F\right)^{\frac{\sigma-\rho}{\rho}\left(\frac{\varphi}{1-\sigma+\varphi}\right)}
\end{aligned}$$

where $\chi_c = \left(\frac{(1-\lambda)(1-\mu)}{\mu} \frac{\tilde{\Lambda}_{s,u}}{\frac{\bar{\omega}^s}{\bar{\omega}^u}\left(\frac{N^u}{N^s}\right)^{\varphi}} \frac{\left(1-\lambda^s_{\tau^H}\bar{\tau}^H\right)}{\left(1-\lambda^u_{\tau^H}\bar{\tau}^H\right)} \right)^{\frac{\sigma}{\sigma-\varphi-1}} > 0$ and $F\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right) = \lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho} + (1-\lambda) > 0.$

Then,

$$
\begin{aligned}
\frac{\partial}{\partial \bar{\tau}^K}\left(\frac{\bar{\tilde{w}}^s}{\bar{\tilde{w}}^u}\right) &= (\sigma-\rho)\left(\frac{\varphi}{1-\sigma+\varphi}\right)\frac{\bar{\tilde{w}}^s}{\bar{\tilde{w}}^u}\frac{1}{F}\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho-1}\frac{\partial}{\partial \bar{\tau}^K}\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right) \\
&< 0.
\end{aligned}
$$

D Data Appendix

We calibrate the steady-state fiscal variables using US quarterly data for the post-Volcker period from 1982:Q4 to 2008:Q2.

D.1 Debt and spending data

We use the following definitions for our debt and spending variables:

- Government debt = market value of privately held gross federal debt;
- Government expenditures = government consumption;

Note that we use a single price level, GDP deflator, for both variables.

The market value of privately held gross federal debt series was obtained from Federal Reserve Bank of Dallas and the government consumption data series was taken from National Income and Product Accounts (NIPA) tables.

D.2 Tax data

We follow a method originally based on Jones (2002). Additionally, we use the tax revenues of the federal government and local property taxes.

We use federal taxes on production and imports (lines 4 of NIPA Table 3.2) for consumption tax revenues. Let this be T^C.

The average personal income tax rate is computed to get both capital tax revenues and labor tax revenues. We first compute the average personal income tax rate as

$$\tau^P = \frac{IT}{W + PRI/2 + CI}$$

where IT is the personal current tax revenues (line 3 of NIPA Table 3.2), W is wage and salary accruals (line 3 of NIPA Table 1.12), PRI is proprietor's income (line 9 of NIPA Table 1.12), and CI is capital income, which is the sum of rental income (line 12 of NIPA Table 1.12), corporate profits (line 13 of NIPA Table 1.12), interest income (line 18 of NIPA Table 1.12), and $PRI/2$. We here regard half of proprietor's income as wage labor income and the other half as capital income.

Then the capital tax revenue is

$$T^K = \tau^P CI + CT + PT$$

where CT is taxes on corporate income (line 7 of NIPA Table 3.2), and PT is property taxes (line 8 of NIPA Table 3.3). In NIPA, home owners are thought of as renting their houses to themselves and thus property taxes are included as taxes on rental income or capital income. The labor tax revenue is computed

$$T^H = \tau^P (W + PRI/2) + CSI$$

where CSI is contributions for government social insurance (line 11 of NIPA Table 3.2).

E Appendix Figures

Figure A.1: Long-run Effects of Permanent Capital Tax Rate Changes under Transfer Adjustment with Different Frisch Elasticity

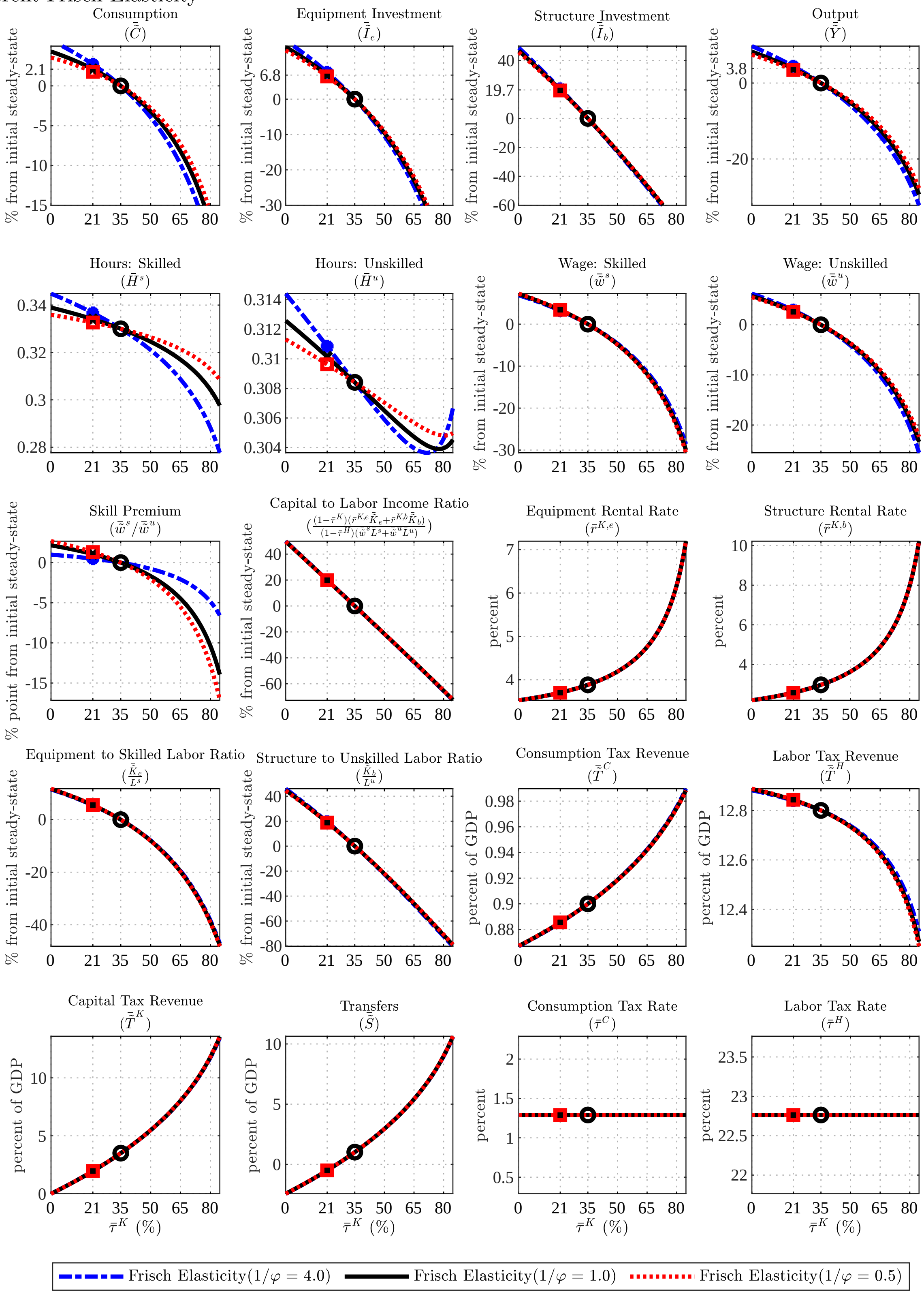


Figure A.2: Long-run Effects of Permanent Capital Tax Rate Changes under Labor Tax Rate Adjustment with Different Frisch Elasticity

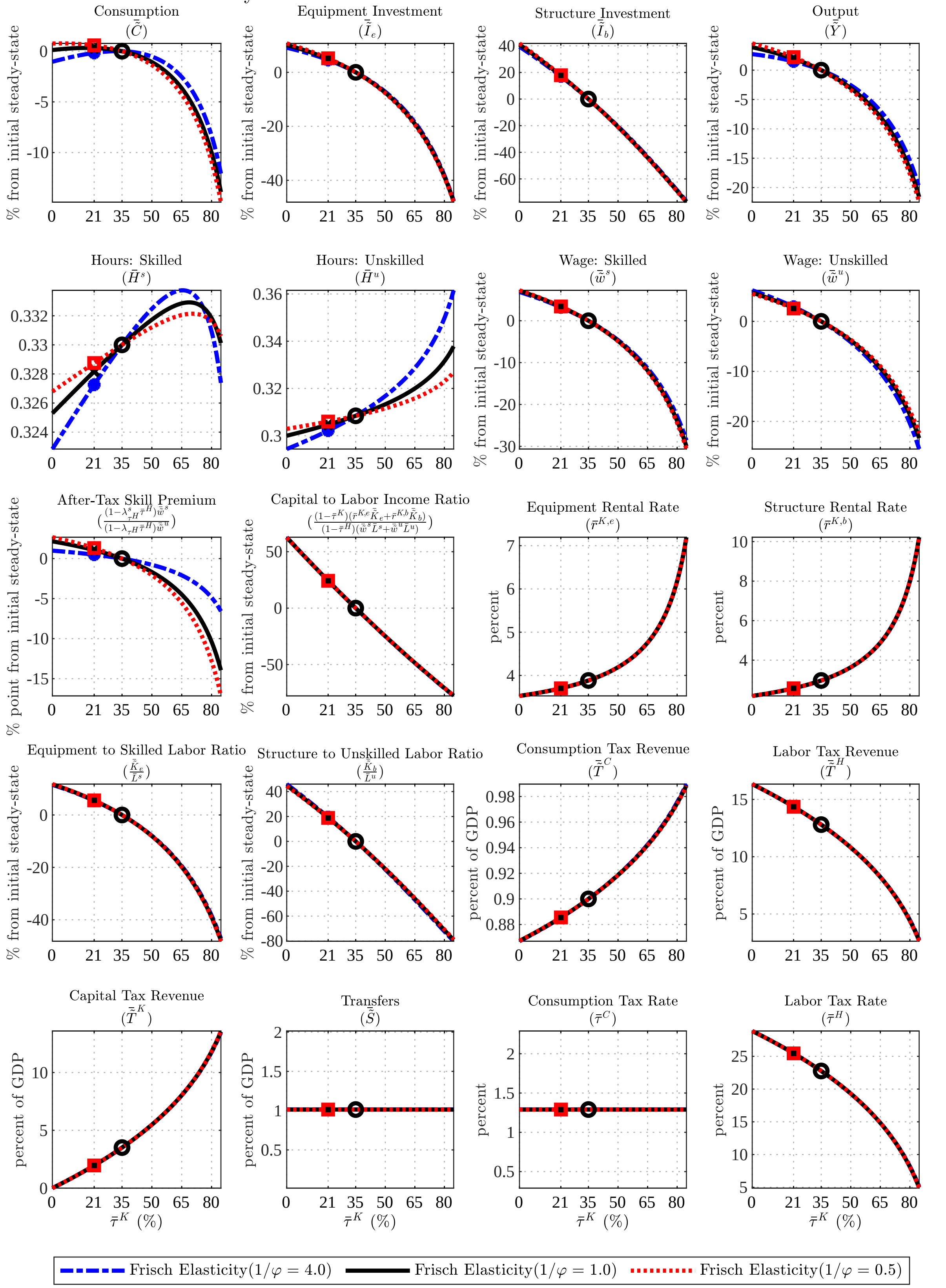


Figure A.3: Long-run Effects of Permanent Capital Tax Rate Changes under Transfer Adjustment and Labor Tax Rate Adjustment

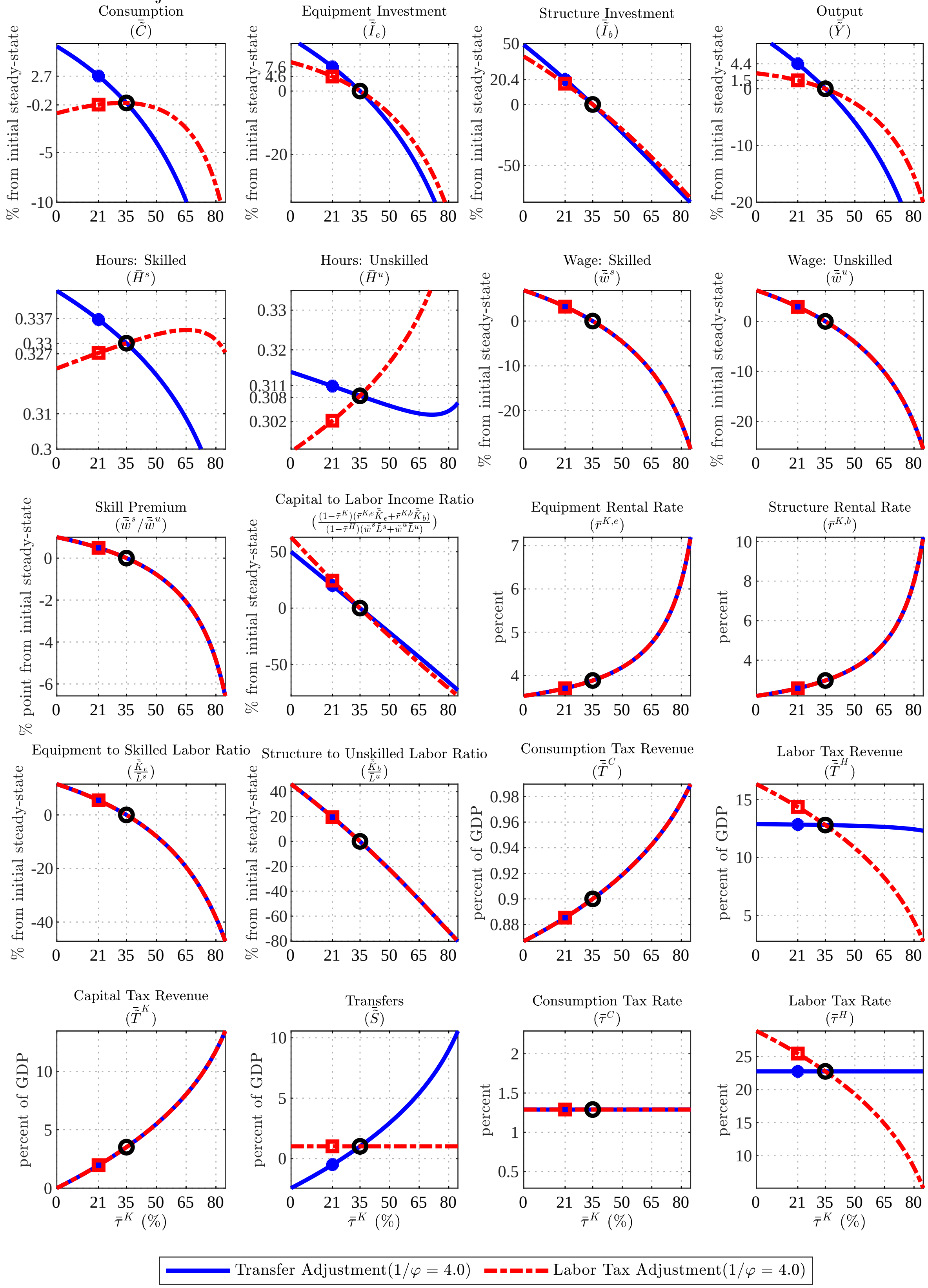


Figure A.4: Long-run Effects of Permanent Capital Tax Rate Changes under Transfer Adjustment and Labor Tax Rate Adjustment

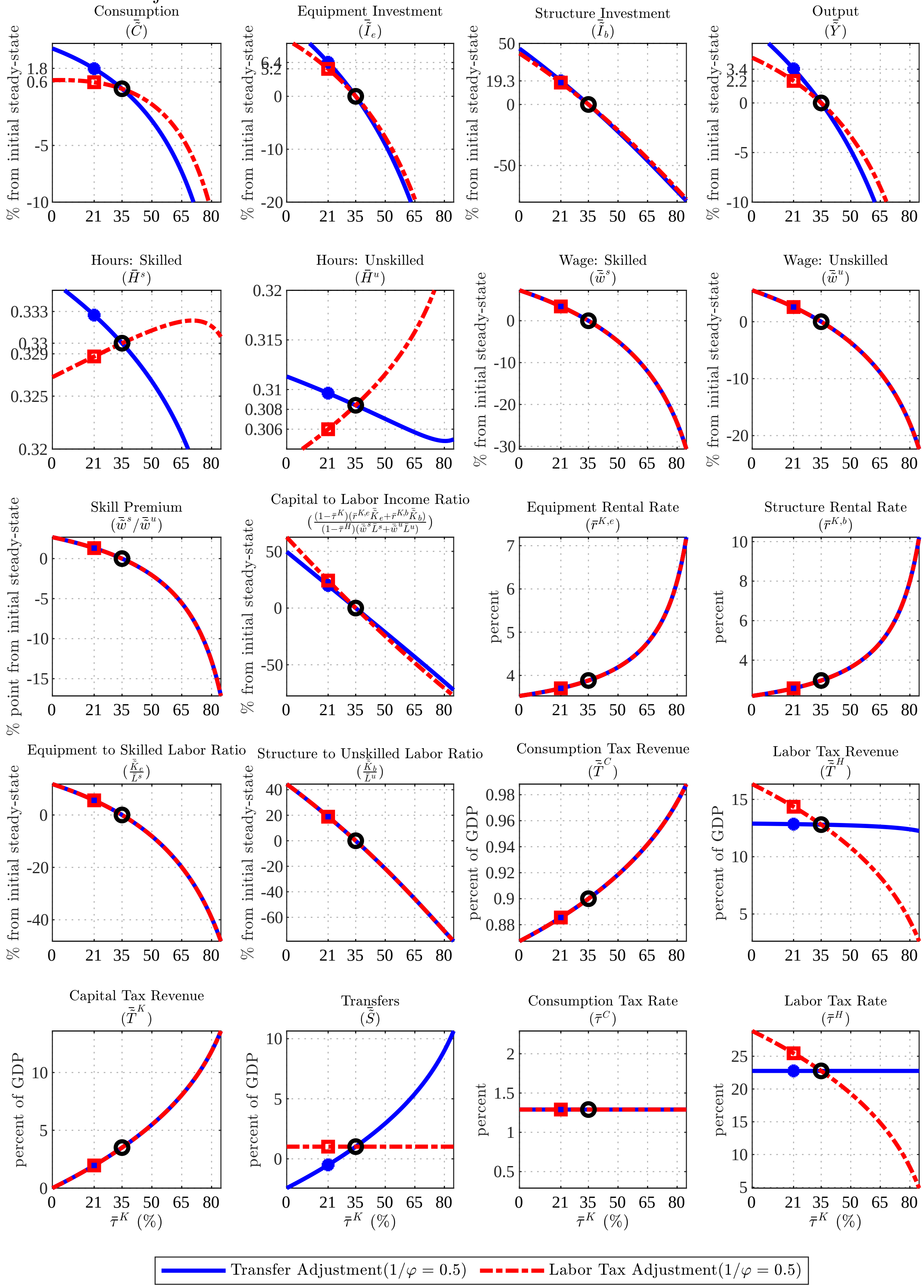


Figure A.5: Long-run Effects of Permanent Capital Tax Rate Changes (Progressivity of Labor Taxes)

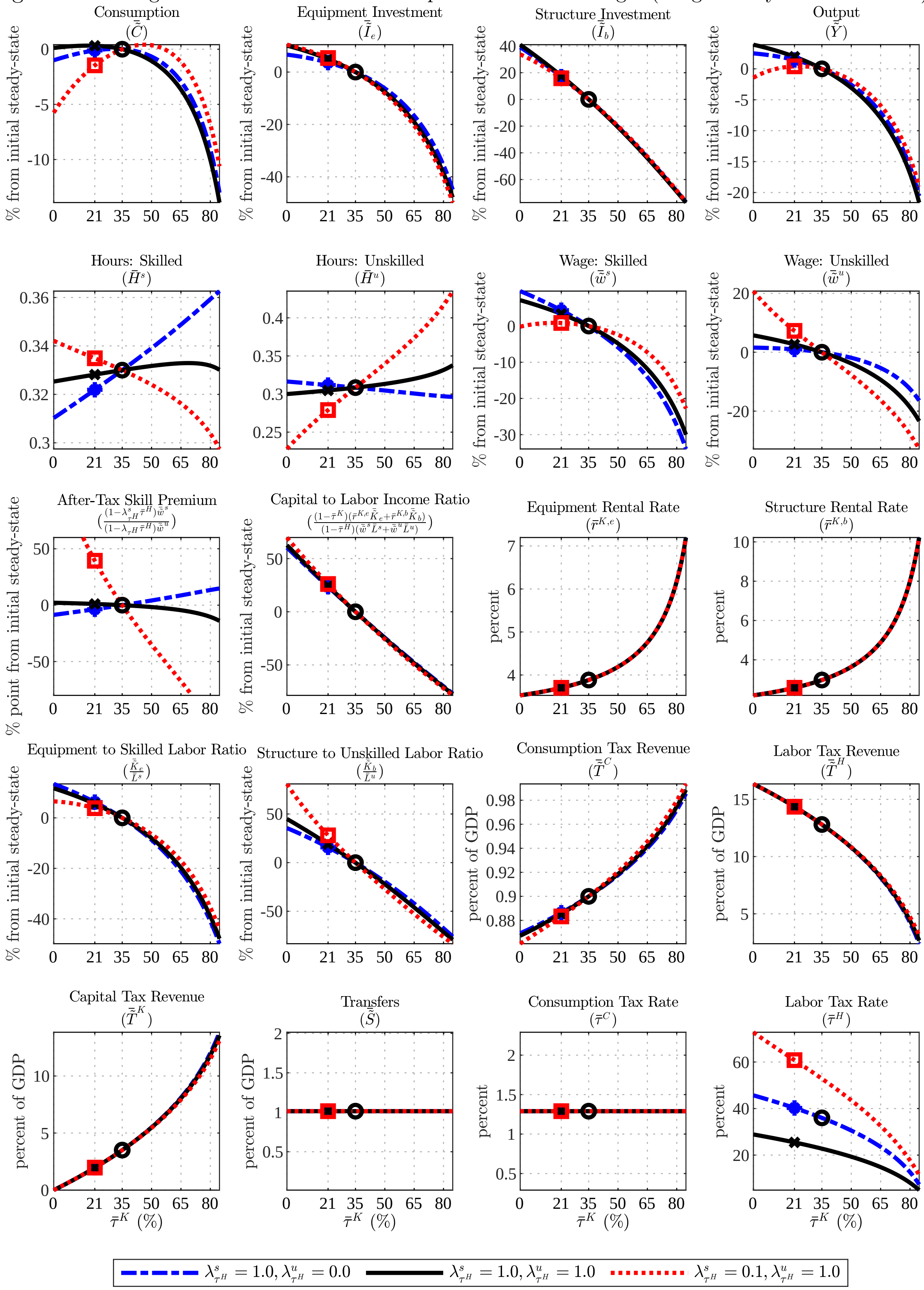


Figure A.6: Long-run Effects of Permanent Capital Tax Rate Changes (Incomplete Market Model with Different Profits and Transfer Distribution Rules)

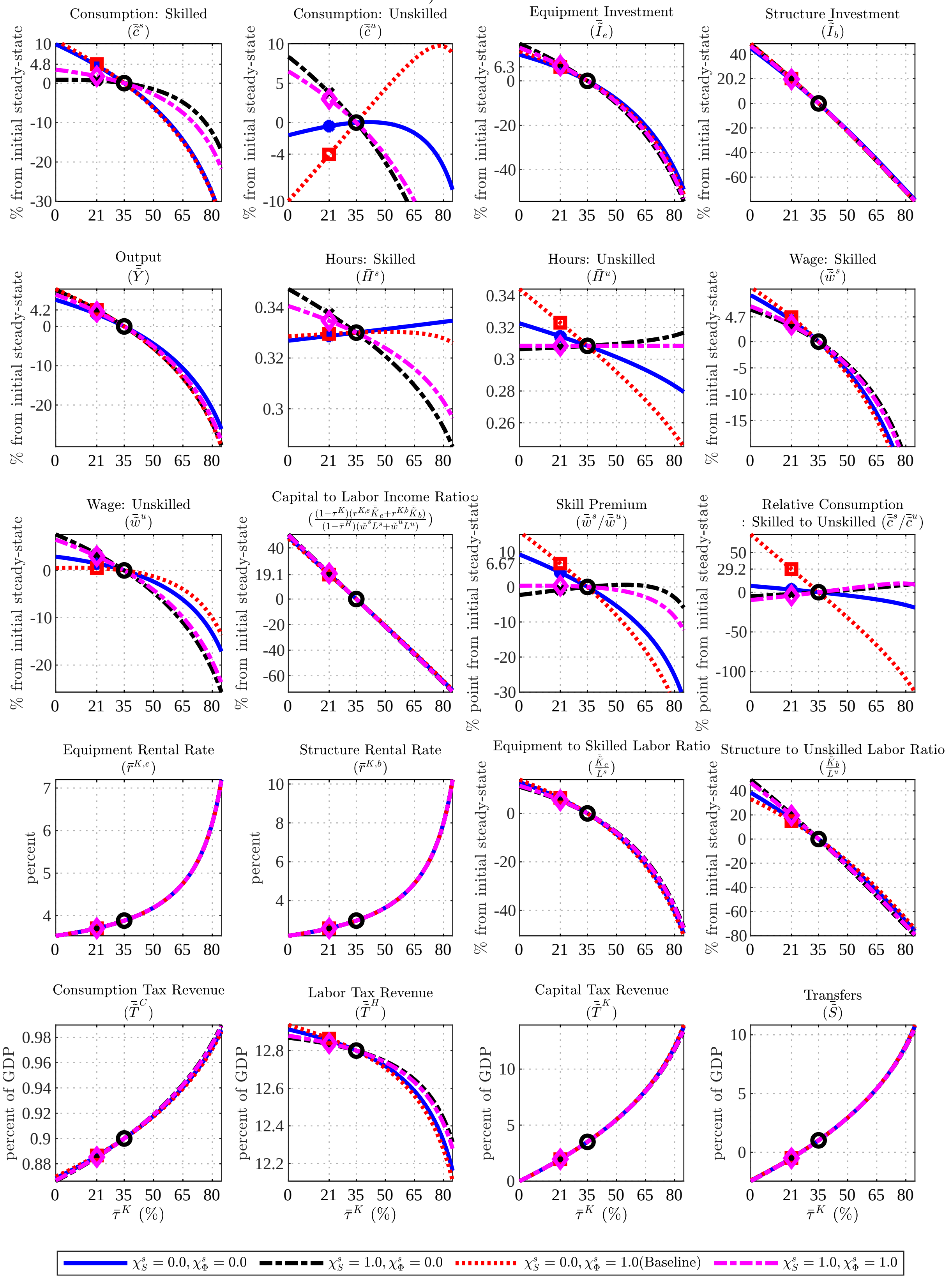


Figure A.7: Long-run Effects of Permanent Capital Tax Rate Changes Under Transfer Adjustment with Different Expensing Rules

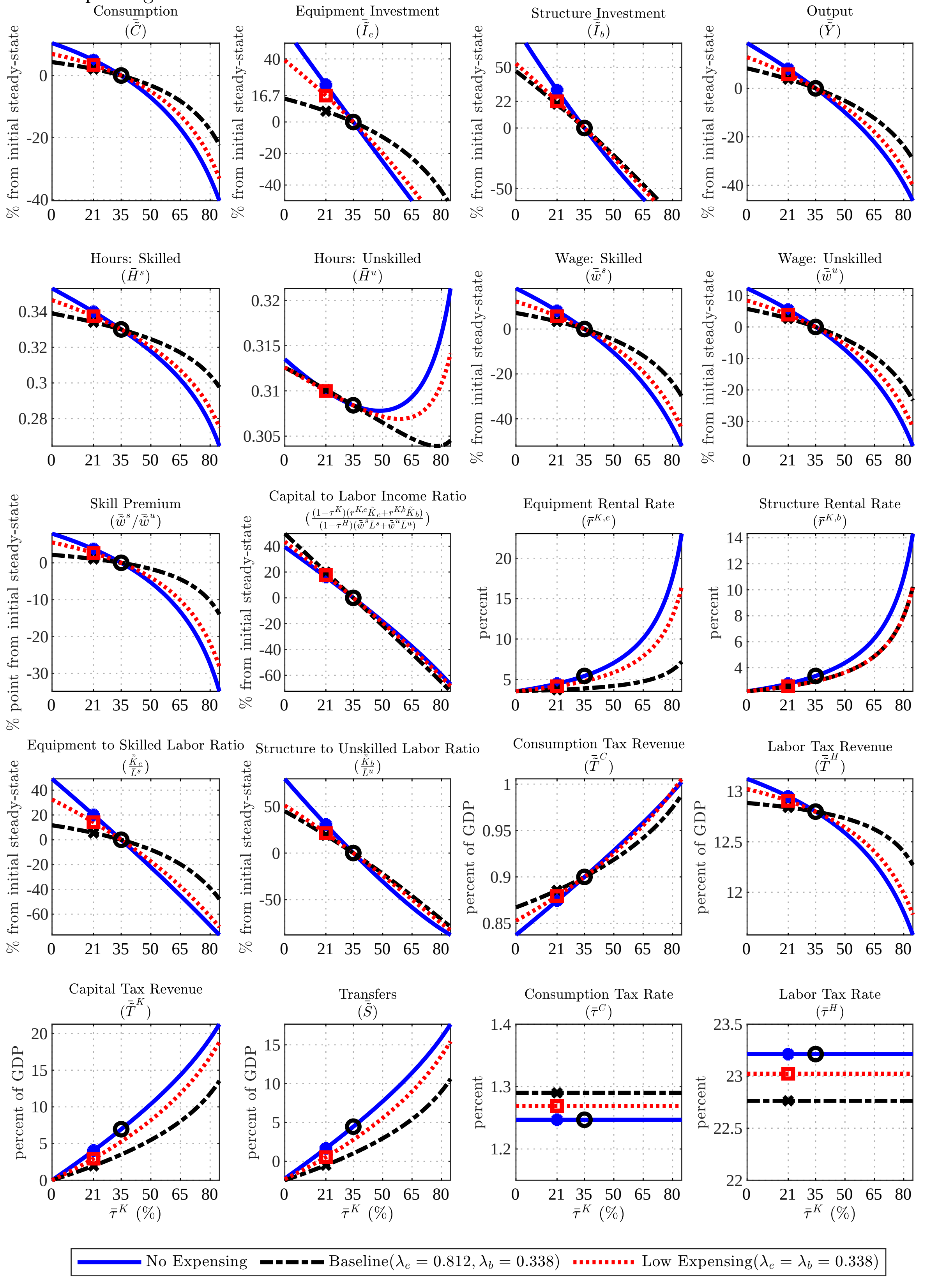


Figure A.8: Long-run Effects of Permanent Capital Tax Rate Changes under Transfer Adjustment with Different Elasticity of Substitution between Skilled and Equipment Capital

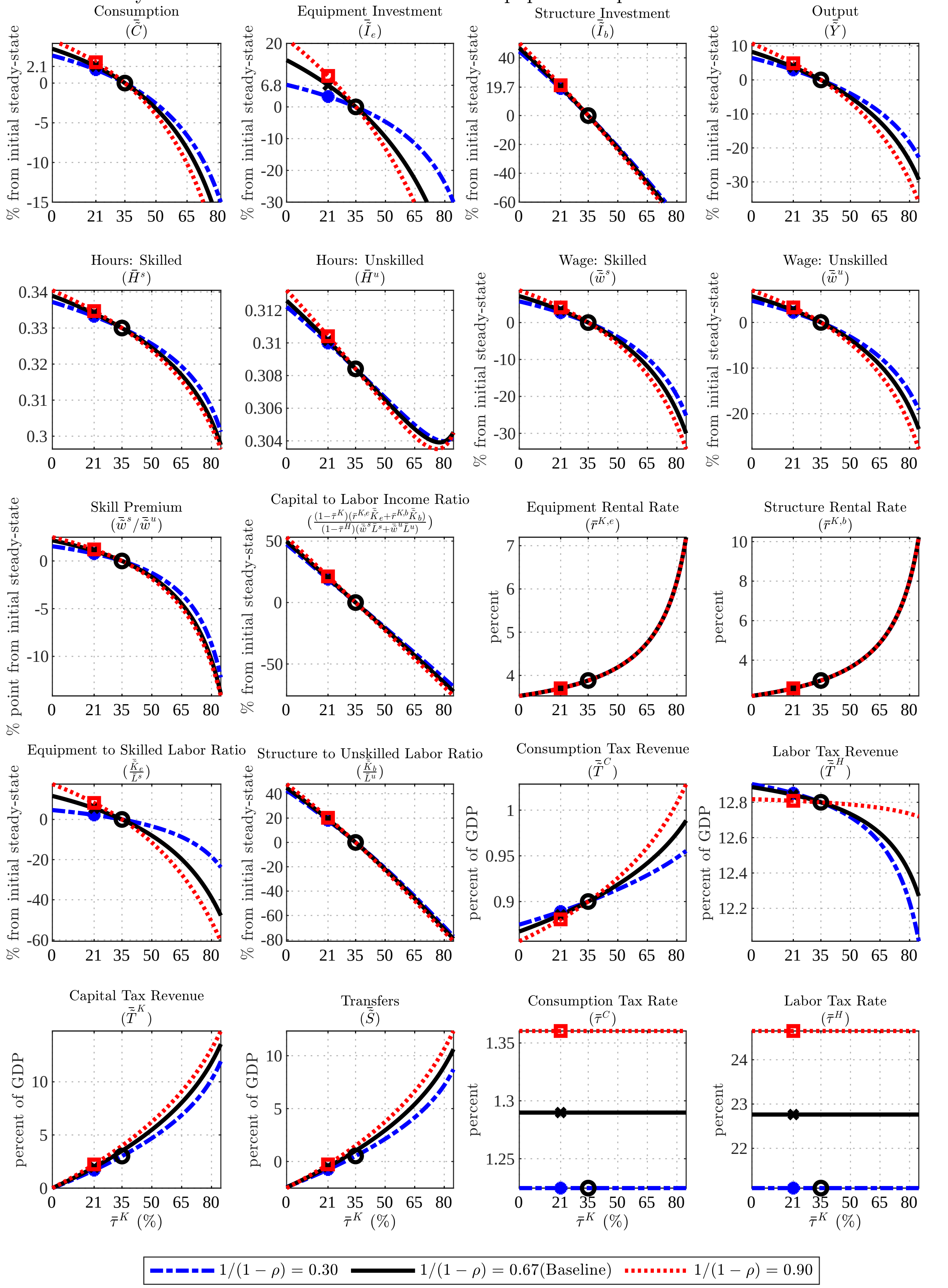


Figure A.9: Transition Dynamics Under Labor Tax Rate and Consumption Tax Rate Adjustment

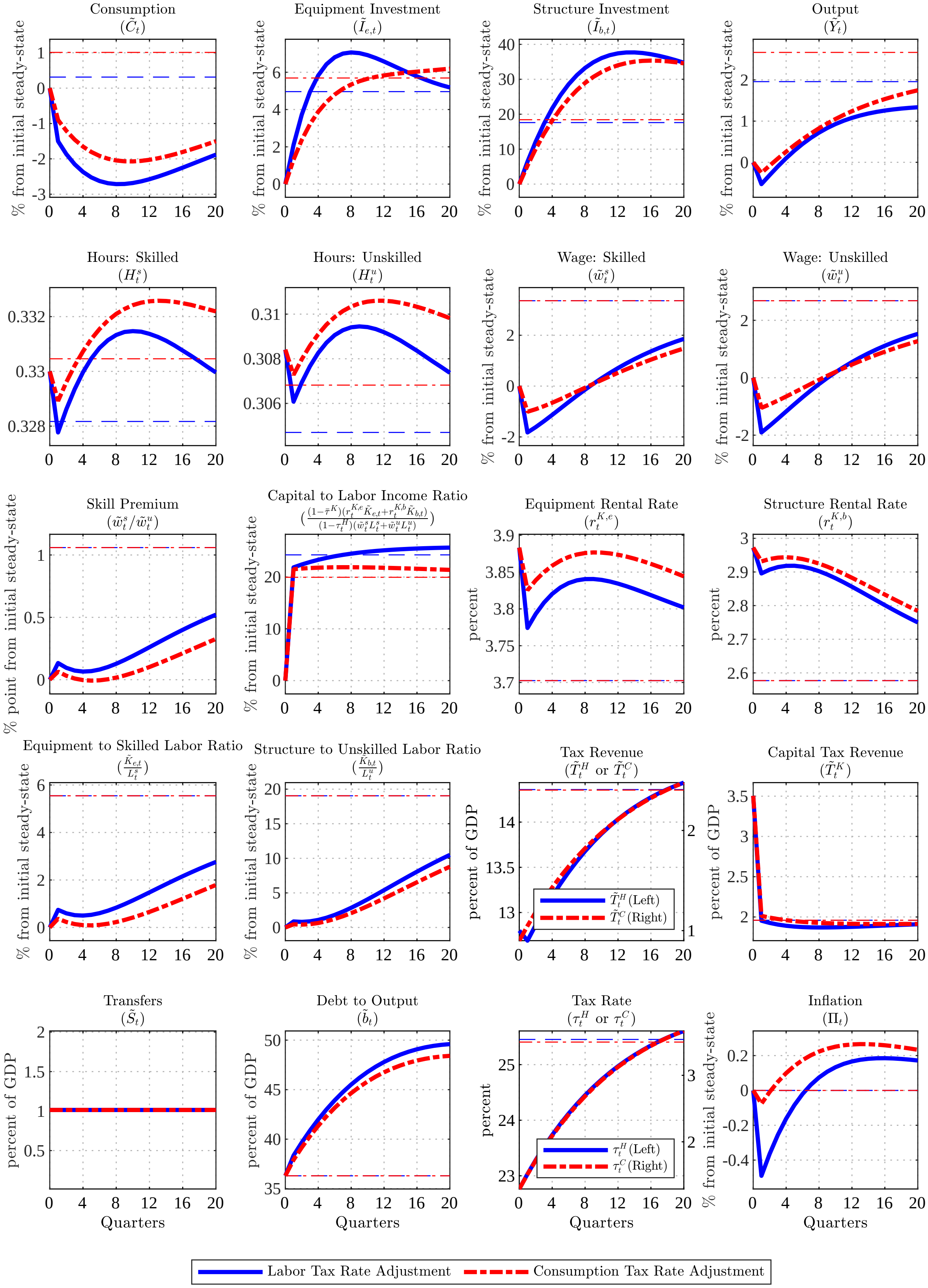


Figure A.10: Transition Dynamics of Permanent Capital Tax Rate Changes Under Transfer Adjustment with Incomplete Markets