Online Appendix for

"Rational Inattention, Menu Costs, and Multi-Product Firms:

Micro Evidence and Aggregate Implications" *

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A Computing the Equilibrium

In this appendix, I describe a second order approximation to a firm's profit function. Also, I give a recursive formulation of two-product firms' problem that I studied in Section 4.

A.1 Quadratic Approximation to a Firm's Profit Function

Firm i produces N products indexed by j in monopolistic competitive markets. Its demand for good j is given by

$$Y_{i,j,t} = A_{i,j,t}^{\varepsilon-1} \left(\frac{P_{i,j,t}}{P_{j,t}}\right)^{-\varepsilon} \left(\frac{P_{j,t}}{P_t}\right)^{-\gamma} Y_t,$$

where P_t is the price of aggregate output Y_t, $P_{j,t}$ is the price of good j, γ is the constant elasticity of substitution across different firms that produce the same good, and ε is the constant elasticity of substitution across different goods. Then, the firm's profit function is

$$\begin{aligned}\Pi_{i,t} &= \sum_{j=1}^{N} \left(P_{i,j,t} - W_t A_{i,j,t}\right) Y_{i,j,t} \\ &= \sum_{j=1}^{N} \left(P_{i,j,t} - W_t A_{i,j,t}\right) A_{i,j,t}^{\varepsilon-1} \left(\frac{P_{i,j,t}}{P_{j,t}}\right)^{-\varepsilon} \left(\frac{P_{j,t}}{P_t}\right)^{-\gamma} Y_t,\end{aligned}$$

where

$$P_t = \left(\frac{1}{N}\sum_{j=1}^{N} P_{j,t}^{1-\gamma}\right)^{\frac{1}{1-\gamma}}, \quad P_{j,t} = \left(\int_0^1 \left(P_{i,j,t}\right)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}.$$

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Define firm i's markup for good j, $\mu_{i,j,t} = \frac{P_{i,j,t}}{W_t A_{i,j,t}}$. Then, the profit function can be written as a function of the firm's markups for each good:

$$\begin{aligned}
\Pi_{i,t} &= \sum_{j=1}^{N} \left(P_{i,j,t} - W_t A_{i,j,t}\right) A_{i,j,t}^{\varepsilon-1} \left(\frac{P_{i,j,t}}{P_{j,t}}\right)^{-\varepsilon} \left(\frac{P_{j,t}}{P_t}\right)^{-\gamma} Y_t \\
&= \sum_{j=1}^{N} \left(\mu_{i,j,t} - 1\right) \left(\mu_{i,j,t}\right)^{-\varepsilon} \left(W_t\right)^{1-\varepsilon} \left(P_{j,t}\right)^{\varepsilon-\gamma} \left(P_t\right)^{\gamma} Y_t.
\end{aligned}$$

Let $R_{i,j,t}$ be the revenue from good j:

$$\begin{aligned}
R_{i,j,t} &= P_{i,j,t} A_{i,j,t}^{\varepsilon-1} \left(\frac{P_{i,j,t}}{P_{j,t}}\right)^{-\varepsilon} \left(\frac{P_{j,t}}{P_t}\right)^{-\gamma} Y_t \\
&= \mu_{i,j,t} \left(\mu_{i,j,t}\right)^{-\varepsilon} \left(W_t\right)^{1-\varepsilon} \left(P_{j,t}\right)^{\varepsilon-\gamma} \left(P_t\right)^{\gamma} Y_t.
\end{aligned}$$

A second order approximation to the profit function around the optimal frictionless markup, $\mu_j^* = \frac{\varepsilon}{\varepsilon-1}$, yields

$$\begin{aligned}
\Pi\left(\left\{\mu_{j,t}\right\}_{j=1}^{N}\right) &= \Pi\left(\left\{\mu_j^*\right\}_{j=1}^{N}\right) + \frac{1}{2} \sum_{j=1}^{N} \left.\frac{\partial^2 \Pi_t}{\partial \mu_{j,t}^2}\right|_{\left\{\mu_{j,t}=\mu_j^*\right\}_{j=1}^{N}} \left(\frac{\mu_{j,t} - \mu_j^*}{\mu_j^*}\right)^2 \left(\mu_j^*\right)^2 \\
&= \Pi\left(\left\{\mu_j^*\right\}_{j=1}^{N}\right) + \frac{1}{2} \sum_{j=1}^{N} \left.\frac{\partial^2 \Pi_t}{\partial \mu_{j,t}^2}\right|_{\left\{\mu_{j,t}=\mu_j^*\right\}_{j=1}^{N}} \left(\hat{\mu}_{j,t}\right)^2 \left(\mu_j^*\right)^2,
\end{aligned}$$

where $\hat{\mu}_{j,t} = \log\left(\mu_{j,t}\right) - \log\left(\mu_j^*\right)$ is the realized markup-gap. Then, given the CES demand and the constant returns to scale technology, we can express the expected losses that arise from frictions (both nominal and informational) relative to the frictionless case, expressed as a fraction of pergood revenue:

$$\mathcal{L} \equiv \mathbb{E}\left[\frac{\Pi\left(\{\mu_{j,t}\}_{j=1}^{N}\right) - \Pi\left(\{\mu_j^*\}_{j=1}^{N}\right) - \tilde{\theta}\mathbf{1}_{\{\text{for any } j,\ p_{i,j,t}\neq p_{i,j,t-1}\}} - \tilde{\psi}\mathcal{I}(s_{i,t};\{\{A_{i,j,t}\}_{j=1}^{N}, W_t\}|S_i^{t-1})}{R\left(\mu_j^*\right)}\middle| S^{t-1}\right]$$

$$= \mathbb{E}\Bigg[\frac{1}{2}\frac{1}{R\left(\mu_j^*\right)}\sum_{j=1}^{N}\frac{\partial^2\Pi_t}{\partial\mu_{j,t}^2}\Bigg|_{\{\mu_{j,t}=\mu_j^*\}_{j=1}^{N}}\left(\hat{\mu}_{j,t}\right)^2\left(\mu_j^*\right)^2 - \theta\mathbf{1}_{\{\text{for any } j,\ p_{i,j,t}\neq p_{i,j,t-1}\}} - \psi\mathcal{I}(s_{i,t};\{\{A_{i,j,t}\}_{j=1}^{N}, W_t\}|S_i^{t-1})\Bigg| S^{t-1}\Bigg]$$

$$= \mathbb{E}\Bigg[\frac{1}{2}\frac{\Pi\left(\{\mu_j^*\}_{j=1}^{N}\right)}{R\left(\mu_j^*\right)}\sum_{j=1}^{N}\frac{\left(\mu_j^*\right)^2\left.\frac{\partial^2\Pi_t}{\partial\mu_{j,t}^2}\right|_{\{\mu_{j,t}=\mu_j^*\}_{j=1}^{N}}}{\Pi\left(\{\mu_j^*\}_{j=1}^{N}\right)}\left(\hat{\mu}_{j,t}\right)^2 - \theta\mathbf{1}_{\{\text{for any } j,\ p_{i,j,t}\neq p_{i,j,t-1}\}} - \psi\mathcal{I}(s_{i,t};\{\{A_{i,j,t}\}_{j=1}^{N}, W_t\}|S_i^{t-1})\Bigg| S^{t-1}\Bigg],$$

where

$$\left.\frac{\partial^2\Pi_t}{\partial\mu_{j,t}^2}\right|_{\{\mu_{j,t}=\mu_j^*\}_{j=1}^{N}} = \varepsilon\left(\mu_j^*\right)^{-\varepsilon-2}\left[\left(\varepsilon+1\right)\left(\mu_j^*-1\right)-2\mu_j^*\right]\left(\bar{W}\right)^{1-\varepsilon}\left(\bar{P}_j\right)^{\varepsilon-\gamma}\left(\bar{P}\right)^{\gamma}\bar{Y}$$
$$= -\varepsilon\left(\mu_j^*\right)^{-\varepsilon-2}\left(\bar{W}\right)^{1-\varepsilon}\left(\bar{P}_j\right)^{\varepsilon-\gamma}\left(\bar{P}\right)^{\gamma}\bar{Y},$$

$$\Pi\left(\{\mu_j^*\}_{j=1}^{N}\right) = \sum_{j=1}^{N}\left(\mu_j^*-1\right)\left(\mu_j^*\right)^{-\varepsilon}\left(\bar{W}\right)^{1-\varepsilon}\left(\bar{P}_j\right)^{\varepsilon-\gamma}\left(\bar{P}\right)^{\gamma}\bar{Y},$$

and

$$R\left(\mu_j^*\right) = \left(\mu_j^*\right)^{1-\varepsilon}\left(\bar{W}\right)^{1-\varepsilon}\left(\bar{P}_j\right)^{\varepsilon-\gamma}\left(\bar{P}\right)^{\gamma}\bar{Y}.$$

Notice that I express the cost of change price, $\tilde{\theta}$, as a fraction θ of the steady state frictionless revenue from selling one of N products, that is $\tilde{\theta} = \theta R(\mu_j^*)$. Similarly, the marginal cost of information processing is $\tilde{\psi} = \psi R(\mu_j^*)$.

Then, the loss function is

$$\mathcal{L} = \mathbb{E}\Bigg[\frac{1}{2}\frac{\Pi\left(\left\{\mu_j^*\right\}_{j=1}^N\right)}{R\left(\mu_j^*\right)}\sum_{j=1}^N \frac{\left(\mu_j^*\right)^2 \left.\frac{\partial^2 \Pi_t}{\partial \mu_{j,t}^2}\right|_{\left\{\mu_{j,t}=\mu_j^*\right\}_{j=1}^N}}{\Pi\left(\left\{\mu_j^*\right\}_{j=1}^N\right)}\left(\hat{\mu}_{j,t}\right)^2$$

$$- \theta \mathbf{1}_{\{\text{for any } j,\ p_{i,j,t}\neq p_{i,j,t-1}\}} - \psi \mathcal{I}(s_{i,t}; \{\{A_{i,j,t}\}_{j=1}^N, W_t\}|S_i^{t-1})\Bigg|S^{t-1}\Bigg]$$

$$= -\mathbb{E}\Bigg[\varepsilon\frac{1}{2}\left(\frac{\sum_{j=1}^N\left(\mu_j^*-1\right)\left(\mu_j^*\right)^{-\varepsilon}\left(\bar{W}\right)^{1-\varepsilon}\left(\bar{P}_j\right)^{\varepsilon-\gamma}\left(\bar{P}\right)^{\gamma}\bar{Y}}{\left(\mu_j^*\right)^{1-\varepsilon}\left(\bar{W}\right)^{1-\varepsilon}\left(\bar{P}_j\right)^{\varepsilon-\gamma}\left(\bar{P}\right)^{\gamma}\bar{Y}}\right)$$

$$\times \frac{\sum_{j=1}^N\left(\mu_j^*\right)^{-\varepsilon}\left(\bar{W}\right)^{1-\varepsilon}\left(\bar{P}_j\right)^{\varepsilon-\gamma}\left(\bar{P}\right)^{\gamma}\bar{Y}}{\sum_{j=1}^N\left(\mu_j^*-1\right)\left(\mu_j^*\right)^{-\varepsilon}\left(\bar{W}\right)^{1-\varepsilon}\left(\bar{P}_j\right)^{\varepsilon-\gamma}\left(\bar{P}\right)^{\gamma}\bar{Y}}\left(\hat{\mu}_{j,t}\right)^2$$

$$+ \theta \mathbf{1}_{\{\text{for any } j,\ p_{i,j,t}\neq p_{i,j,t-1}\}} + \psi \mathcal{I}(s_{i,t}; \{\{A_{i,j,t}\}_{j=1}^N, W_t\}|S_i^{t-1})\Bigg|S^{t-1}\Bigg].$$

Now, assume $\varepsilon = \gamma$ (or by symmetry across product industry due to there are no common industry specific shocks). Then, the second order approximation of the firm's profit function is

$$\mathcal{L} = \mathbb{E}\Bigg[-\varepsilon\frac{1}{2}\left(\frac{1}{\mu_j^*}\right)\sum_{j=1}^N\left(\hat{\mu}_{i,j,t}\right)^2 + \theta \mathbf{1}_{\{\text{for any } j,\ p_{i,j,t}\neq p_{i,j,t-1}\}} + \psi \mathcal{I}(s_{i,t}; \{\{A_{i,j,t}\}_{j=1}^N, W_t\}|S_i^{t-1})\Bigg|S^{t-1}\Bigg].$$

Let $p_{i,j,t}^* = \log(W_t) + \log(A_{i,j,t})$ be the log deviation of (frictionless) optimal price of good j from its non-stochastic steady state. Then, we define firm i's true price gap of good j as

$$\hat{\mu}_{i,j,t} = p_{i,j,t} - p_{i,j,t}^*,$$

where $p_{i,j,t}$ is the log deviation of the price of good j from its non-stochastic steady state.[1] Then, I derive a second order approximation of firms' loss function:

$$\mathcal{L} = \mathbb{E}\Bigg[-\varepsilon\frac{1}{2}\left(\frac{1}{\mu_j^*}\right)\sum_{j=1}^N\left(p_{i,j,t} - p_{i,j,t}^*\right)^2 + \theta \mathbf{1}_{\{\text{for any } j,\ p_{i,j,t}\neq p_{i,j,t-1}\}} + \psi \mathcal{I}(s_{i,t}; \{\{A_{i,j,t}\}_{j=1}^N, W_t\}|S_i^{t-1})\Bigg|S^{t-1}\Bigg]$$

$$= -\mathbb{E}\Bigg[B\sum_{j=1}^N\left(p_{i,j,t} - p_{i,j,t}^*\right)^2 + \theta \mathbf{1}_{\{\text{for any } j,\ p_{i,j,t}\neq p_{i,j,t-1}\}} + \psi \mathcal{I}(s_{i,t}; \{\{A_{i,j,t}\}_{j=1}^N, W_t\}|S_i^{t-1})\Bigg|S^{t-1}\Bigg],$$

where $B = \frac{\varepsilon-1}{2}$ is a slope of profit curve.

[1] The true price gap $\hat{\mu}_{i,j,t}$ can be also written as a markuk gap, $\hat{\mu}_{i,j,t} = \log(\mu_{i,j,t}/\mu_j^*)$, which is a the log deviation of the current markup to the non-stochastic steady state markup $\mu_j^* = \frac{\varepsilon}{\varepsilon-1}$.

A.2 A Recursive Formulation

At the beginning of period t, firm i takes its initial information set, S_i^{t-1}, as given and chooses a set of optimal signals, $s_{i,t}$, subject to a cost of information processing, $\psi\mathcal{I}(s_{i,t}; \{p^*_{i,j,t}\}_{j=1}^N|S_i^{t-1}|S_i^{t-1})$, where ψ is a marginal cost of a bit of information and $\mathcal{I}(\cdot)$ is Shannon's mutual information function. Notice that I replace the idiosyncratic shocks and the nominal wage in Shannon's mutual information function with the firm's frictionless optimal price since all the firms need to know is its frictionless optimal price after deriving the firms' loss function. It then forms a new information set, $S_i^t = S_i^{t-1} \cup s_{i,t}$, and sets its new prices, $\{p_{i,j,t}\}_{j=1}^N$, based on that. The firm pays a menu cost, θ, if it decides to change any prices. Otherwise, the firm waits for the next period.

Formally, after taking a quadratic approximation of firm i's profit function around non-stochastic steady state and deriving a loss function from the suboptimal prices, the firm's problem can be written as:

$$\min_{\{\{p_{i,j,t}\}_{j=1}^N, s_{i,t}\}_{t\geq 0}} \mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\left\{B\sum_{j=1}^N (p_{i,j,t}-p^*_{i,j,t})^2 + \theta\mathbf{1}_{\{\text{for any } j,\ p_{i,j,t}\neq p_{i,j,t-1}\}} + \psi\mathcal{I}(s_{i,t}; \{p^*_{i,j,t}\}_{j=1}^N|S_i^{t-1})\right\}\middle| S_i^{-1}\right]$$

$$\text{s.t.} \quad S_i^t = S_i^{t-1} \cup s_{i,t}.$$

I assume that the set of available signals satisfies that 1) the firm chooses $N+1$ independent signals for each shock, 2) each signal is Gaussian, and 3) all noise in signals is idiosyncratic and independent. Then, the set of optimal signals about both the idiosyncratic good-specific shocks and the monetary shock can be written as:

$$\begin{aligned}\mathcal{S}^a_{i,j,t} &= \{a_{i,j,t} + \eta^a_{i,j,t}\xi^a_{i,j,t} : \eta^a_{i,j,t} \geq 0,\ \xi^a_{i,j,t} \sim N(0,1)\}, \text{ for } j = 1,2,\cdots,N,\\ \mathcal{S}^m_{i,t} &= \{m_t + \eta^m_{i,t}\xi^m_{i,t} : \eta^m_{i,t} \geq 0,\ \xi^m_{i,t} \sim N(0,1)\}.\end{aligned}$$

Here, rational inattention errors, $\xi^a_{i,j,t}$ and $\xi^m_{i,t}$, are independent across firms. At each time t, given S^{t-1}, firm i chooses its optimal signals $s^a_{i,j,t} \in \mathcal{S}^a_{i,j,t}$ for $j = 1,2,\cdots,N$, and $s^m_{i,t} \in \mathcal{S}^m_{i,t}$ subject to the cost of information processing. Then, the firm's new information set evolves as $S_i^t = S_i^{t-1} \cup s_{i,t}$ where $s_{i,t} = \{\{s^a_{i,j,t}\}_{j=1}^N, s^m_{i,t}\}$. Now, the firms' problem is virtually identical to the problem I studied in Section 3.

Let $x_{j,t} = p_{j,t} - \mathbb{E}[p^*_{j,t}|S^t]$ be firm's *perceived* price gap about product j and

$$z^a_{j,t} = \mathbb{E}\left[\left(a_{j,t} - \mathbb{E}_t[a_{j,t}|S^t]\right)^2\middle|S^t\right], \quad z^m_t = \mathbb{E}\left[\left(m_t - \mathbb{E}_t[m_t|S^t]\right)^2\middle|S^t\right]$$

be subjective uncertainty about the j-good specific shock and the aggregate shock, respectively. I rewrite the recursive formation of the firms' problem as:

$$
\begin{aligned}
&V(\{x_{j,-1}\}_{j=1}^N,\{z_{j,-1}^a\}_{j=1}^N, z_{-1}^m) = \\
&\max_{\{\{z_j^a\}_{j=1}^N, z^m\}} \mathbb{E}\Bigg[\max\left\{ V^I\left(\{x_j\}_{j=1}^N, \{z_j^a\}_{j=1}^N, z^m\right), V^C\left(\{z_j^a\}_{j=1}^N, z^m\right) \right\} \\
&\qquad\qquad - \frac{\psi}{2}\left(\sum_{j=1}^N \log_2\left(\frac{z_{j,-1}^a+\sigma_a^2}{z_j^a}\right) + \log_2\left(\frac{z_{-1}^m+\sigma_m^2}{z^m}\right)\right) \Bigg| S^{-1} \Bigg] \\
&\text{s.t.} \qquad 0 \le z_j^a \le z_{j,-1}^a + \sigma_a^2, \quad \forall j = 1, 2, \cdots, N, \\
&\qquad\quad\;\; 0 \le z^m \le z_{-1}^m + \sigma_m^2,
\end{aligned}
$$

where

$$
V^I\left(\{x_j\}_{j=1}^N, \{z_j^a\}_{j=1}^N, z^m\right) = -B\sum_{j=1}^N\left(x_j^2 + z_j^a + z^m\right) + \beta V\left(\{x_j\}_{j=1}^N, \{z_j^a\}_{j=1}^N, z^m\right)
$$

$$
\text{with } \mathbf{x} \sim N\left(\mathbf{x}_{-1}, \Sigma\right), \text{ and}
$$

$$
V^C\left(\{z_j^a\}_{j=1}^N, z^m\right) = \max_{\{y_j\}_{j=1}^N} -B\sum_{j=1}^N\left(y_j^2 + z_j^a + z^m\right) - \theta + \beta V\left(\{y_j\}_{j=1}^N, \{z_j^a\}_{j=1}^N, z^m\right).
$$

Here $\mathbf{x} = \{x_1, x_2, \cdots, x_N\}'$ and

$$
\Sigma_t(j,k) = \begin{cases} z_{-1}^m + \sigma_m^2 - z^m & \text{if } j \neq k \\ z_{j,-1}^a + \sigma_a^2 - z_j^a + z_{-1}^m + \sigma_m^2 - z^m & \text{if } j = k. \end{cases}
$$

A.3 Definition of Equilibrium

Given exogenous processes for idiosyncratic good-specific shocks $\{\{\{a_{i,j,t}\}_{j=1}^N\}_{i\in[0,1]}\}_{t\geq 0}$, a general equilibrium of the economy consists of an allocation for the representative household, $\Omega^H \equiv \{C_t, \{C_{i,j,t}\}_{j=1}^N, L_t, B_t\}_{t=0}^\infty$, an allocation for every firm $i \in [0,1]$ given the initial set of signals, $\Omega_i^F \equiv \{s_{i,t} \in \mathcal{S}_{i,t}, \{P_{i,j,t}, L_{i,j,t}, Y_{i,j,t}\}_{j=1}^N\}_{t=0}^\infty$, a set of prices $\{\{P_{j,t}\}_{j=1}^N, P_t, R_t, W_t\}_{t=0}^\infty$, and a stationary distribution over firms' states such that

1. given the set of prices and $\left\{\Omega_i^F\right\}_{i\in[0,1]}$, the household's allocation solves its problem as specified in Equation (5) ;

2. given the set of prices and Ω^H, and the implied labor supply and output demand, firms' allocations solve their problem as specified in Equation (7) ;

3. given the set of prices, Ω^H, and $\left\{\Omega_i^F\right\}_{i\in[0,1]}$, $\{M_t \equiv P_tC_t\}_{t\geq 0}$ satisfies the monetary policy rule specified in Equation (9) ;

4. all markets clear: for all $t \geq 0$,

$$Y_{i,j,t} = C_{i,j,t},\ \forall i \in [0,1], j = 1, 2, \cdots, N,\ \ L_t = \int (\sum_{j=1}^{N} L_{i,j,t}) di,\ \ B_t = 0\,;$$

5. the stationary distribution is consistent with actions.

B A Rationally Inattentive Firm's Problem without Menu Costs ($\theta = 0$)

In this appendix, I solve a rationally inattentive firm's problem without menu costs. This problem is similar to the one studied in Pasten and Schoenle (2016) with one main difference. They solve the problem by assuming that the cost of information is not discounted and optimizing at the long-run steady-state for information structure. Here, I assume that the firm discounts future costs of information at the same discount rate as their payoffs and solve the dynamic information acquisition problem. This setup is also similar to Afrouzi and Yang (2021a) that study the dynamic multivariate rational inattention problem. One difference is that I assume that the set of available signals are partitioned into two subsets, one for signals about idiosyncratic shocks and the other for signals about aggregate shocks, that are independent each other.

Without menu costs, the firm can change its prices and collapses the price gaps to zero whenever it wants, i.e. there is no micro rigidity in price setting. In this case, the firm's prior price gaps are no longer its state variables, and thus the firm's problem is deterministic. Then, in a recursive formation, the firm's problem is:

$$\begin{aligned}
V\left(\{z^a_{j,-1}\}_{j=1}^N, z^m_{-1}\right) = \max_{\{\{z^a_j\}_{j=1}^N, z^m\}} & - B\sum_{j=1}^{N}\left(z^a_j + z^m\right) + \beta V\left(\{z^a_j\}_{j=1}^N, z^m\right) \\
& - \frac{\psi}{2}\left(\sum_{j=1}^{N}\log_2\left(\frac{z^a_{j,-1}+\sigma_a^2}{z^a_j}\right) + \log_2\left(\frac{z^m_{-1}+\sigma_m^2}{z^m}\right)\right) \\
\text{s.t.} \qquad & 0 \leq z^a_j \leq z^a_{j,-1} + \sigma_a^2, \quad \forall j = 1, 2, \cdots, N, \\
& 0 \leq z^m \leq z^m_{-1} + \sigma_m^2,
\end{aligned}$$

Notice that with $\psi > 0$, the constraints $z^a_j \geq 0$ and $z^m_j \geq 0$ will not bind. The first order necessary conditions are:

$$
\begin{aligned}
\partial z_j^a: \quad & -B + \frac{\psi}{2\log 2}\frac{1}{z_j^a} + \beta V_{z_j^a}\left(\{z_j^a\}_{j=1}^N, z^m\right) - \phi_j = 0, \ \ \forall\, j \in \{1, 2, \cdots, N\}, \\
\partial z^m: \quad & -BN + \frac{\psi}{2\log 2}\frac{1}{z^m} + \beta V_{z^m}\left(\{z_j^a\}_{j=1}^N, z^m\right) - \phi_m = 0, \\
& V_{z_{j,-1}^a}\left(\{z_{j,-1}^a\}_{j=1}^N, z_{-1}^m\right) = -\frac{\psi}{2\log 2}\frac{1}{z_{j,-1}^a + \sigma_a^2} \ \ \forall\, j \in \{1, 2, \cdots, N\}, \\
& V_{z_{-1}^m}\left(\{z_{j,-1}^a\}_{j=1}^N, z_{-1}^m\right) = -\frac{\psi}{2\log 2}\frac{1}{z_{-1}^m + \sigma_m^2},
\end{aligned}
$$

and complementarity slackness conditions where $\{\phi_j\}_{j=1}^N$ and ϕ_m are Lagrangian multipliers for no-forgetting constraints. The no-forgetting constraints will bind when the marginal cost of information processing is high enough. Here, I focus on interior solutions where the constraints do not bind.

When the standard deviation of idiosyncratic good-specific shocks is the same, then subjective uncertainty about idiosyncratic shocks is also the same across all goods. Then, the optimal subjective uncertainty about both shocks satisfies:

$$
\begin{aligned}
B =& \frac{\psi}{2\log 2}\left(\frac{1}{z_j^a} - \beta\frac{1}{z_j^a + \sigma_a^2}\right), \ \ \forall\, j \in \{1, 2, \cdots, N\} \\
B \cdot N =& \frac{\psi}{2\log 2}\left(\frac{1}{z^m} - \beta\frac{1}{z^m + \sigma_m^2}\right).
\end{aligned} \tag{B.1}
$$

Several interesting characteristics emerge. First, the firm's optimal subjective uncertainty is constant while it is time-varying with menu costs. Second, subjective uncertainty increases in the size of marginal cost of information processing, ψ, and the size of shocks, σ_a^2, while it decreases in the slope of profit function , B, and the time preference parameter, β. Third, optimal subjective uncertainty about idiosyncratic shocks is independent of the number of products that the firm produces. Fourth, optimal subjective uncertainty about aggregate shocks is *decreasing* in the number of products ($\frac{\partial z^m(N)}{\partial N} < 0$) and $\lim_{N\to\infty} z^m(N) = 0$.

B.1 Real Effects of Monetary Policy Shocks

Let $\tilde{z}^m = \frac{z^m}{\sigma_m^2}$ be firms' subjective uncertainty relative to the variance of monetary policy shocks. Then I can rewrite Equation (B.1) as:

$$
\frac{1}{\tilde{z}^m} - \beta\frac{1}{\tilde{z}^m + 1} = \sigma_m^2\frac{BN}{\psi}(2\log 2) \tag{B.2}
$$

Given parameters, firms' subjective uncertainty about monetary shocks decreases in their number of products.

Now, the size of price changes for good j is given by:

$$\begin{aligned}\Delta p_{i,j,t} =& \mathcal{K}^a(N)\left(a_{i,j,t-1} - a_{i,j,t-1|t-1} + \varepsilon_{i,j,t} + \eta_{i,j,t}\right) \\ &+ \mathcal{K}^m(N)\left(m_{t-1} - m_{i,t-1|t-1} + \varepsilon_{m,t} + \eta_{i,m,t}\right),\end{aligned}$$

where

$$\begin{aligned}a_{i,jt} - a_{i,j,t|t} &= (1-\mathcal{K}^a(N))\left(a_{i,j,t-1} - a_{i,j,t-1|t-1} + \varepsilon_{i,j,t}\right) - \mathcal{K}^a(N)\eta_{i,j,t}, \\ m_t - m_{i,t|t} &= (1-\mathcal{K}^m(N))\left(m_{t-1} - m_{i,t-1|t-1} + \varepsilon_{m,t}\right) - \mathcal{K}^m(N)\eta_{i,m,t},\end{aligned}$$

and Kalman gains are:

$$\mathcal{K}^a(N) = \frac{1}{1+\tilde{z}^a_j(N)}, \;\; \mathcal{K}^m(N) = \frac{1}{1+\tilde{z}^m(N)}.$$

Let $p_{j,t} = \int p_{i,j,t} di$. Let $p_{j,t}^{\text{NoMP}}$ and $p_{j,t}^{\text{MP}}$ be price level at time t without and with monetary shocks, respectively. Then, since all noise in signals, $\eta_{i,j,t}$, is idiosyncratic and independent, we have

$$\begin{aligned}p_{j,t}^{\text{NoMP}} &= p_{j,t-1}^{\text{NoMP}} + \Delta p_{j,t}^{\text{NoMP}} \\ &= p_{j,t-1}^{\text{NoMP}} + \left(\mathcal{K}^a(N)\int\left(a_{i,j,t-1} - a_{i,j,t-1|t-1}\right)di + \mathcal{K}^m(N)\int\left(m_{t-1} - m_{i,t-1|t-1}\right)di\right)\end{aligned}$$

and

$$\begin{aligned}p_{j,t}^{\text{MP}} &= p_{j,t-1}^{\text{MP}} + \Delta p_{j,t}^{\text{MP}} \\ &= p_{j,t-1}^{\text{MP}} + \left(\mathcal{K}^a(N)\int\left(a_{i,j,t-1} - a_{i,j,t-1|t-1}\right)di + \mathcal{K}^m(N)\int\left(m_{t-1} - m_{i,t-1|t-1}\right)di + \varepsilon_{m,t}\right)\end{aligned}$$

Then, Notice that by symmetry across goods, we have $p_t = p_{j,t}$ for all j. Define an impulse response of aggregate price to a monetary shock as the gap between the prices with and without the monetary shock, that is,

$$IRF_t^P = p_t^{\text{MP}} - p_t^{\text{NoMP}}.$$

Assume at time 0, there is a monetary shock, $\varepsilon_{m,0}$. Then,

$$\begin{aligned}IRF_0^P &= \mathcal{K}^m(N)\varepsilon_{m,0} \\ IRF_1^P &= \mathcal{K}^m(N)\varepsilon_{m,0} + \mathcal{K}^m(N)\left(1-\mathcal{K}^m(N)\right)\varepsilon_{m,0} \\ IRF_2^P &= \mathcal{K}^m(N)\varepsilon_{m,0} + \mathcal{K}^m(N)\left(1-\mathcal{K}^m(N)\right)\varepsilon_{m,0} + \mathcal{K}^m(N)\left(1-\mathcal{K}^m(N)\right)^2\varepsilon_{m,0} \\ &\vdots \\ IRF_t^P &= \mathcal{K}^m(N)\left(\frac{1-\left(1-\mathcal{K}^m(N)\right)^{t+1}}{1-\left(1-\mathcal{K}^m(N)\right)}\right)\varepsilon_{m,0} \\ &= \left(1-\left(1-\mathcal{K}^m(N)\right)^{t+1}\right)\varepsilon_{m,0}\end{aligned}$$

and output responses are given by

$$\begin{aligned} IRF_t^Y &= \varepsilon_{m,0} - IRF_t^P \\ &= (1-\mathcal{K}^m(N))^{t+1}\varepsilon_{m,0}. \end{aligned}$$

Let a cumulative impulse response of output as a function of the number of product, $M(N)$ be the area under the impulse response function of output. Then,

$$\begin{aligned} \mathcal{M}(N) &= \int_0^\infty IRF_t^Y dt = \int_0^\infty (1-\mathcal{K}^m(N))^{t+1}\varepsilon_{m,0}dt \\ &= -\frac{(1-\mathcal{K}^m(N))}{\log(1-\mathcal{K}^m(N))}\varepsilon_{m,0} \\ &= -\frac{\left(\frac{\tilde{z}_m(N)}{1+\tilde{z}_m(N)}\right)}{\log\left(\frac{\tilde{z}_m(N)}{1+\tilde{z}_m(N)}\right)}\varepsilon_{m,0}. \end{aligned}$$

where $\tilde{z}_m(N)$ is a solution of Equation (B.2) as a function of N. Notice that

$$\frac{\partial\mathcal{M}(N)}{\partial N} = -\underbrace{\frac{\frac{\partial\tilde{z}^m(N)}{\partial N}}{(1+\tilde{z}^m(N))^2}}_{<0} \times \underbrace{\frac{1}{\log\left(\frac{\tilde{z}_m(N)}{1+\tilde{z}_m(N)}\right)}}_{<0} \times \underbrace{\left(1-\frac{1}{\log\left(\frac{\tilde{z}_m(N)}{1+\tilde{z}_m(N)}\right)}\right)}_{>0} < 0.$$

Although N is an arbitrary integer number, here I assume $\mathcal{M}(N)$ is continuously differentiable with respect to N. Appendix Figure G.2 shows that both subjective uncertainty about monetary shocks and cumulative responses of output to monetary shocks decreases in number of products that firms produce.

C Computational Procedures for the Two-Product Model

I use the method of value function iteration to solve the two-product firms' optimization problem. There are 5 state variables for the problem: prior perceived price gap for two products, subjective uncertainty about each good-specific shock, and prior subjective uncertainty about monetary policy shocks. Since this problem is non-convex optimization problem with occasionally binding constraints, it should b e solved numerically.

The most computationally burdensome part is to compute firms' expected future values since 1) tomorrow's perceived price gaps are stochastic variables when firms do not change prices today, and 2) the distribution of these price gaps has a mean vector $(x_{1,-1}, x_{2,-1})'$, which is firms' state variable, and a covariance matrix, Σ, which is firms' choice variable. Standard approximation methods for the transition probability of states, such as Tauchen approximation method, are not applicable since the approximation errors are quite large. I compute expected value of the firms' value functions using Gauss-Legendre quadrature which is an explicit numerical integration technique.

I solve the firms' problem and the value function and the optimal policy functions using the following procedure:

1. Construct grids for individual state variables, such as prior of perceived price gaps for each product, $x_{1,-1}$, $x_{2,-1}$, prior subjective uncertainty about two good-specific shocks, $z^a_{1,-1}$, $z^a_{2,-1}$, and prior subjective uncertainty about monetary shocks, z^m_{-1}. I use 21 grids for $x_{1,-1}$, $x_{2,-1}$, and 16 grids for $z^a_{1,-1}$, $z^a_{2,-1}$, and z^m_{-1}. The ranges of $x_{1,-1}$ and $x_{2,-1}$, are $[-1.5\sqrt{\theta/B}, 1.5\sqrt{\theta/B}]$ where θ is the size of menu costs and B is the slope of firms' profit curve.[2] More grid points are assigned around inaction bands. $z^a_{1,-1}$, $z^a_{2,-1}$, and z^m_{-1} are equally spaced in the range of [0 0.004].

2. Construct the abscissas, $\{\tilde{x}_i\}_{i=1}^{N_q}$, and weights, $\{\tilde{w}_i\}_{i=1}^{N_q}$, of the Gauss-Legendre quadrature with $N_q = 500$ points.

3. Solve the individual value functions at each grid point. In this step, I obtain the optimal decision rules for subjective uncertainty about both good-specific shocks and monetary shocks,

$$g^a_1(x_{1,-1}, x_{2,-1}, z^a_{1,-1}, z^a_{2,-1}, z^m_{-1}),\ g^a_2(x_{1,-1}, x_{2,-1}, z^a_{1,-1}, z^a_{2,-1}, z^m_{-1}),\ g^m(x_{1,-1}, x_{2,-1}, z^a_{1,-1}, z^a_{2,-1}, z^m_{-1}),$$

   and the value function, $V(x_{1,-1}, x_{2,-1}, z^a_{1,-1}, z^a_{2,-1}, z^m_{-1})$. The detailed steps are as follows:

   (a) Make an initial guess for the value functions, V_0, for all grid points.

   (b) Solve firms' optimization problem and compute V_1. Notice that the problem can be

[2] In fact $[-\sqrt{\frac{\theta}{B}}, \sqrt{\frac{\theta}{B}}]$ is the inaction bands for myopic firms ($\beta = 0$). In this regard, I have more conservative ranges of grids for prior price gaps.

written:

$$
\begin{aligned}
V_1\Big(&x_{1,-1}, x_{2,-1}, z^a_{1,-1}, z^a_{2,-1}, z^m_{,-1}\Big) = \\
&\max_{\{z^a_1, z^a_2, z^m\}} - B\left(z^a_1 + z^a_2 + 2z^m\right) - \frac{\psi}{2}\left(\sum_{j=1}^{2} \log_2\left(\frac{z^a_{j,-1} + \sigma^2_a}{z^a_j}\right) + \log_2\left(\frac{z^m_{-1} + \sigma^2_m}{z^m}\right)\right) \\
&\qquad + \int_{(x_1,x_2)} \max\Big\{ - B(x_1^2 + x_2^2) + \beta V_0\left(x_1, x_2, z^a_1, z^a_2, z^m\right), \\
&\qquad\qquad - \theta + \beta V_0\left(0, 0, z^a_1, z^a_2, z^m\right) \Big\} dF((x_1, x_2); (x_{1,-1}, x_{2,-1}), \Sigma) \\
\text{s.t.} \quad & 0 \le z^a_j \le z^a_{j,-1} + \sigma^2_a, \quad \forall j = 1, 2 \\
& 0 \le z^m \le z^m_{-1} + \sigma^2_m \\
& \Sigma_t(j,k) = \begin{cases} z^m_{-1} + \sigma^2_m - z^m & \text{if } j \neq k \\ z^a_{j,-1} + \sigma^2_a - z^a_j + z^m_{-1} + \sigma^2_m - z^m & \text{if } j = k. \end{cases}
\end{aligned}
$$

where $F((x_1, x_2); (x_{1,-1}, x_{2,-1}), \Sigma)$ is a joint normal distribution with mean $(x_{1,-1}, x_{2,-1})$ and covariance matrix Σ.

(c) If V_0 and V_1 are close enough for each grid point, and go to the next step. Otherwise, update the value functions ($V_0 = V_1$), and go back to (a).

(d) Simulate the model with a large number of firms to obtain a stationary distribution, $G(x_1, x_2, z^a_1, z^a_2, z^m)$, over firm states $(x_1, x_2, z^a_1, z^a_2, z^m)$. Simulation algorithm is described in Appendix C.1.

(e) Compute aggregate variables.

C.1 Simulation Algorithm for the Two-Product Model

I simulate the two-good version of the baseline model with 100,000 firms for 5,000 periods.

1. Set initial $x_{i,1,t-1}$, $x_{i,2,t-1}$, $z^a_{i,1,t-1}$, $z^a_{i,2,t-1}$, and $z^m_{i,t-1}$.

2. Generate random numbers for the shocks $\varepsilon^m_t \sim N(0, \sigma^2_m)$, $\varepsilon^a_{i,1,t} \sim N(0, \sigma^2_a)$, and $\varepsilon^a_{i,2,t} \sim N(0, \sigma^2_a)$

3. Find $z^m_{i,t}$, $z^a_{i,1,t}$, and $z^a_{i,2,t}$, given policy functions,

$$
\begin{aligned}
g^a_1 &\left(x_{i,1,t-1}, x_{i,2,t-1}, z^a_{i,1,t-1}, z^a_{i,2,t-1}, z^m_{i,t-1}\right) \\
g^a_2 &\left(x_{i,1,t-1}, x_{i,2,t-1}, z^a_{i,1,t-1}, z^a_{i,2,t-1}, z^m_{i,t-1}\right) \\
g^m &\left(x_{i,1,t-1}, x_{i,2,t-1}, z^a_{i,1,t-1}, z^a_{i,2,t-1}, z^m_{i,t-1}\right).
\end{aligned}
$$

4. Calculate standard deviations of signal noises and Kalman gains from

$$z_{i,t}^{m} = \left(1 - \mathcal{K}_{i,t}^{m}\right)\left(z_{i,t-1}^{m} + \sigma_m^2\right)$$
$$z_{i,1,t}^{a} = \left(1 - \mathcal{K}_{i,1,t}^{a}\right)\left(z_{i,1,t-1}^{a} + \sigma_1^2\right)$$
$$z_{i,2,t}^{a} = \left(1 - \mathcal{K}_{i,2,t}^{a}\right)\left(z_{i,2,t-1}^{a} + \sigma_1^2\right)$$

and

$$\mathcal{K}_{i,t}^{m} = \frac{z_{i,t-1}^{m} + \sigma_m^2}{z_{i,t-1}^{m} + \sigma_m^2 + \eta_{i,m,t}^2},\ \mathcal{K}_{i,1,t}^{a} = \frac{z_{i,1,t-1}^{a} + \sigma_1^2}{z_{i,1,t-1}^{a} + \sigma_1^2 + \eta_{i,1,t}^2},\ \mathcal{K}_{i,2,t}^{a} = \frac{z_{i,2,t-1}^{a} + \sigma_2^2}{z_{i,2,t-1}^{a} + \sigma_2^2 + \eta_{i,2,t}^2}.$$

Then

$$\eta_{i,m,t}^2 = \frac{z_{i,t}^{m}\left(z_{i,t-1}^{m} + \sigma_m^2\right)}{z_{i,t-1}^{m} + \sigma_m^2 - z_{i,t}^{m}},\ \eta_{i,1,t}^2 = \frac{z_{i,1,t}^{a}\left(z_{i,1,t-1}^{a} + \sigma_1^2\right)}{z_{i,1,t-1}^{a} + \sigma_1^2 - z_{i,1,t}^{a}},\ \eta_{i,2,t}^2 = \frac{z_{i,2,t}^{a}\left(z_{i,2,t-1}^{a} + \sigma_1^2\right)}{z_{i,2,t-1}^{a} + \sigma_1^2 - z_{i,2,t}^{a}}$$

5. Generate random numbers for signal noises $\xi_{i,m,t} \sim \mathcal{N}\left(0, \eta_{i,m,t}^2\right)$, $\xi_{i,1,t} \sim \mathcal{N}\left(0, \eta_{i,1,t}^2\right)$, $\xi_{i,2,t} \sim \mathcal{N}\left(0, \eta_{i,2,t}^2\right)$.

6. Calculate the perceived gap(markup) **after observing their signals** at t

$$\begin{aligned} x_{i,1,t} &= x_{i,1,t-1} - \left[\mathcal{K}_{i,t}^{m}\left(s_{i,t}^{m} - m_{i,t-1|t-1}\right) + \mathcal{K}_{i,1,t}^{a}\left(s_{i,1,t}^{a} - a_{i,1,t-1|t-1}^{a}\right)\right] \\ &= x_{i,1,t-1} - \Big[\mathcal{K}_{i,t}^{m}\left(m_{t-1} - m_{i,t-1|t-1} + \varepsilon_t^{m} + \xi_{i,m,t}\right) \\ &\qquad + \mathcal{K}_{i,1,t}^{a}\left(a_{i,1,t-1}^{a} - a_{i,1,t-1|t-1}^{a} + \varepsilon_{i,1,t}^{a} + \xi_{i,1,t}\right)\Big] \end{aligned}$$

$$\begin{aligned} x_{i,2,t} &= x_{i,2,t-1} - \left[\mathcal{K}_{i,t}^{m}\left(s_{i,t}^{m} - m_{i,t-1|t-1}\right) + \mathcal{K}_{i,2,t}^{a}\left(s_{i,2,t}^{a} - a_{i,2,t-1|t-1}^{a}\right)\right] \\ &= x_{i,2,t-1} - \Big[\mathcal{K}_{i,t}^{m}\left(m_{t-1} - m_{i,t-1|t-1} + \varepsilon_t^{m} + \xi_{i,m,t}\right) \\ &\qquad + \mathcal{K}_{i,2,t}^{a}\left(a_{i,2,t-1}^{a} - a_{i,2,t-1|t-1}^{a} + \varepsilon_{i,2,t}^{a} + \xi_{i,2,t}\right)\Big] \end{aligned}$$

where

$$a_{i,1,t} - a_{i,1,t|t} = \left(1 - \mathcal{K}_{i,1,t}^{a}\right)\left(a_{i,1,t-1} - a_{i,1,t-1|t-1}\right) + \varepsilon_{i,1,t}^{a} - \mathcal{K}_{i,1,t}^{a}\left(\varepsilon_{i,1,t}^{a} + \xi_{i,1,t}\right)$$
$$a_{i,2,t} - a_{i,2,t|t} = \left(1 - \mathcal{K}_{i,2,t}^{a}\right)\left(a_{i,2,t-1} - a_{i,2,t-1|t-1}\right) + \varepsilon_{i,2,t}^{a} - \mathcal{K}_{i,2,t}^{a}\left(\varepsilon_{i,2,t}^{a} + \xi_{i,2,t}\right)$$
$$m_t - m_{i,t|t} = \left(1 - \mathcal{K}_{i,t}^{m}\right)\left(m_{t-1} - m_{i,t-1|t-1}\right) + \varepsilon_t^{m} - \mathcal{K}_{i,t}^{m}\left(\varepsilon_t^{m} + \xi_{i,m,t}\right)$$

with given $a_{i,1,-1} - a_{i,1,-1|-1} = 0$, $a_{i,2,1} - a_{i,2,-1|-1} = 0$, and $m_{-1} - m_{i,-1|-1} = 0$.

7. Price changes: for $j \in \{1,2\}$,

$$\Delta p_{i,j,t} = \begin{cases} 0 & \text{if } -\theta + \beta V\left(0, 0, z^a_{i,1,t}, z^a_{i,2,t}, z^m_{i,t}\right) \\ & \qquad \leq -\left[(x_{i,1,t})^2 + (x_{i,2,t})^2\right] + \beta V\left(x_{i,1,t}, x_{i,2,t}, z^a_{i,1,t}, z^a_{i,2,t}, z^m_{i,t}\right) \\ -x_{i,j,t} & \text{if } -\theta + \beta V\left(0, 0, z^a_{i,1,t}, z^a_{i,2,t}, z^m_{i,t}\right) \\ & \qquad > -\left[(x_{i,1,t})^2 + (x_{i,2,t})^2\right] + \beta V\left(x_{i,1,t}, x_{i,2,t}, z^a_{i,1,t}, z^a_{i,2,t}, z^m_{i,t}\right) \end{cases}$$

8. True markup: for $j \in \{1,2\}$,

$$\begin{aligned} \mu_{i,j,t} &= p_{i,j,t} - m_t - a_{i,j,t} \\ &= \Delta p_{i,j,t} + x_{i,j,t-1} - \left(m_{t-1} - m_{i,t-1|t-1}\right) - \left(a_{i,j,t-1} - a_{i,j,t-1|t-1}\right) - \varepsilon^m_t - \varepsilon^a_{i,j,t} \end{aligned}$$

where $a_{i,j,-1} - a_{i,j,-1|-1} = 0$ and $m_{-1} - m_{i,-1|-1} = 0$.

D Distribution of Price Changes

Appendix Figure G.3 shows the distribution of price changes in the single-product model and in a two-product model. As a comparison, I also plot the distribution of price changes in the menu-cost-only model with single-product firms (yellow bar). All three models are calibrated to match the same frequency and size of price changes. In the baseline single-product model (blue bar), there are no small price changes because price changes occur when firms believe that their price is outside of their inaction bands. However, the kurtosis of the distribution in the baseline model is higher than that in the menu-cost-only model, and there is a relatively small fraction of firms around the inaction bands in the baseline model compared with the menu-cost-only model.[3] Notice that firms in the baseline model have different inaction bands depending on their subjective uncertainty, while firms in the menu-cost-only model have the same inaction bands (black vertical line). The heterogeneity in firms' subjective uncertainty makes the distribution of price changes in the baseline model more dispersed than that in the menu-cost-only model.

In contrast to the single-product model, the baseline two-product model generates both small and large price changes. When a two-product firm believes that one of its prices is far away from its perceived optimal price, the firm pays a fixed menu cost to change its price. As additional price changes are free after paying this menu cost, the firm also changes the price of the other product, even if it is still close to the perceived optimal price. Thus, the economy with two-product firms can have a large fraction of small price changes and a higher kurtosis of the price change distribution.[4]

[3] The red vertical lines are the average inaction bands across all firms in the baseline single-product model. Because both models have the same frequency and size of price changes, the average inaction bands in both models are similar.

[4] Standard menu cost models with two-product firms can also generate small price changes through the same

This motive based on economies of scope in menu cost technology weakens selection effects of price changes, which act as a strong force to reduce monetary non-neutrality in a standard menu cost model such as Golosov and Lucas (2007).

E Discussion on Model Assumptions

In this appendix, I discuss two key assumptions in my baseline model with both menu costs and rational inattention. First, I provide some evidence of firm-specific menu costs from the survey data. Second, I discuss implications and limitations of the assumptions about the set of available information.

Firm-Specific Menu Costs. In the baseline two-good version of the model, economies of scope in price setting emerge from the existence of firm-specific menu costs. Previous literature found ample evidence of the firm-specific menu costs for multi-product firm. For example, recent work by Stella (2018) and Letterie and Nilsen (2016) directly estimate various types of adjustment costs for the multi-product firms and find that there are sizable component of costs from firm-specific menu costs.[5] In the New Zealand survey data, I also find some evidence of firm-specific menu costs. Managers were asked about how typical it is to synchronize price reviews and price changes across multiple products sold by their firms. They report that on average 75% of their price changes and price review decisions are synchronized within their firms.

While the firm-specific fixed cost implies perfect within-firm synchronization of price changes, the data show that firms synchronize their price changes partially. Bonomo et al. (2019a) also find partial synchronization using Israel retail price data and show that even small departures from full synchronization in menu costs models substantially weaken monetary non-neutrality. This implies that introducing a product-specific menu cost in my baseline model would weaken economies of scope in price setting. In this case, the real effects of monetary shocks in the two-good version of the model will be much smaller than those in the single-product model.

economies of scope motive (e.g. Midrigan, 2011; Bhattarai and Schoenle, 2014; Alvarez and Lippi, 2014). However, the baseline two-product model has a more dispersed distribution of price changes than the menu cost only models with two-product firms because, again, firms' optimal inaction bands are a function of their subjective uncertainty. Appendix Figure G.4 shows a comparison of the distribution of price changes in the baseline two-product model with that in the menu cost only model with two-product firms.

[5] Bhattarai and Schoenle (2014) and Lach and Tsiddon (2007) test the implications of menu cost models with a single fixed menu cost and find that micro price data support the existence of economies of scope for multi-product firms. Moreover, Lach and Tsiddon (1996) Fisher and Konieczny (2000), and Midrigan (2011) find that price changes within multi-product firms are highly synchronized, suggesting the existence of firm-specific menu costs.

Set of Available Information. In my main model analysis, I assume that the set of available signals has three properties. First, the firm chooses $N + 1$ independent signals for each shock, implying that paying attention to aggregate conditions and paying attention to good-specific idiosyncratic conditions are separate activities. Although this assumption is often made in the rational inattention literature, such as Maćkowiak and Wiederholt (2009) and Pasten and Schoenle (2016), it might be suboptimal for firms to choose to observe independent signals. In fact, Afrouzi and Yang (2021a) show that in LQG setting rational inattention models (without menu costs), the number of signals that firms choose to observe are no more than the number of actions. In my model, the number of actions is N as firms choose N prices for their goods, implying that firms might waste their resources to observe additional signals.

Second, I assume that every signal is Gaussian. Gaussian signals are optimal when the underlying shocks are Gaussian and firms' objective function is quadratic. However, in my model, firms' objective is not quadratic due to menu costs. Recent rational inattention literature considers models with general objective functions with some assumptions of a simple stochastic process, a static setup or finite actions and states (Matějka, 2015; Jung et al., 2019; Steiner et al., 2017). Solving fully non-linear dynamic problems under rational inattention is computationally demanding as firms' state variable is an infinitely dimensional object if the shocks are continuously distributed. The assumption of Gaussian signals is for tractability.

Third, I assume that all noise in signals is idiosyncratic and independent. This assumption is without loss of generality since I consider Shannon's mutual information as the cost of information (Denti, 2015; Afrouzi, 2020).

F Additional Evidence on Models Predictions

In this appendix, I show two additional kinds of evidence that support the key predictions of the baseline model. First, I show using the New Zealand survey data that the empirical distribution of desired price changes has a fat tail. Second, I show that in the survey data, firms with greater uncertainty are more likely to delay their price changes.

F.1 Evidence on the Leptokurtic Distribution of Desired Price Changes.

One key result of this paper is to show that selection in information processing endogenously leads to a fat-tail distribution of desired price changes. I find that this result is empirically consistent with what we observe in the survey data. In the second wave of the survey data, firms' managers were asked how much they would like to change the price of their main product if it was free to change its price in three months. The answer gives firms' desired price changes in three months. To construct a model-consistent measure of desired price changes, I define an inflation-adjusted desired price changes as the gap between the desired price changes and their inflation expectations in three months. The left panel of Appendix Figure G.9 shows that the distribution of desired price changes has a cluster near zero, while some desired price changes are very far away from zero. The distribution of desired price changes exhibits kurtosis around 5, implying that the survey supports the fat-tail distribution of desired price changes. The baseline model with both rational inattention and menu costs endogenously captures this distributional characteristic without an assumption that the distribution of good-specific shocks is leptokurtic.[6]

F.2 Subjective Uncertainty and the Duration of Price Changes.

Another characteristic of firms' pricing rule is that firms' optimal inaction bands depend on their subjective uncertainty about the underlying shocks. When firms are more uncertain, the inaction bands are wider, implying that the wait-and-see effects are present in firms' optimal price-setting decision. I directly test this implication using the New Zealand survey data. Firms asked to assign probabilities (from 0 to 100) to the different outcomes for growth rates of unit sales of their main product over the next 12 months. I calculate the standard deviation—which is a measure of firms' subjective uncertainty—surrounding firms' sales forecast using the implied probability distribution. The right panel of Appendix Figure G.9 shows that the firms are shorter duration of next price changes when they are less uncertain about their future sales. I regress the duration of firms' expected next price changes on their subjective uncertainty about future sales growth. Appendix Table G.5 shows that firms that have greater uncertainty expect a longer duration before their next price changes. This finding is consistent with the prediction of the baseline model. When firms are more uncertain about their fundamentals, they are reluctant to change their prices. Instead, firms want to wait until they acquire more information to resolve their own uncertainty about their fundamentals.

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[6]The previous literature on menu costs often assumes that this fat-tail distribution of idiosyncratic shocks weakens selection effects of price changes. Midrigan (2011) supports this assumption by providing evidence of excess kurtosis in the distribution of markup gaps in U.S. retail data.

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G Appendix Tables and Figures

Appendix Table G.1: Summary Statistics for Number of Products

Industries	Obs.	Mean	Median	Std. Dev.	Max.
Total	712	67.4	9	234.2	2115
Total without Retail/Wholesale Trade	627	9.55	7	8.47	48
– Manufacturing	278	9.57	8	7.75	39
– Professional and Financial Services	276	7.95	7	6.09	35
– Other Services	37	14.49	13	11.63	48
– Construction and Transportation	36	8.42	5	8.92	40

Notes: This table reports summary statistics for firms' number of products by sectors. The number of products of each firm is measured from answers to the following question in the second wave of New Zealand Firms' Expectation Survey: "*In addition to your main product or product line, how many other products do you sell?*" See Coibion et al. (2018) for details about the survey data. Moments are calculated using sampling weights.

Appendix Table G.2: Summary Statistics for Inflation Backcast Errors by Industries

Industries	Quartile 1		Quartile 2		Quartile 3		Quartile 4	
	N	Mean (S.D.)	N	Mean (S.D.)	N	Mean (S.D.)	N	Mean (S.D.)
Total	1-5	5.01 (4.56)	6-9	5.83 (4.93)	10-16	3.89 (5.20)	>16	2.48 (2.99)
Total without Retail/Wholesale	1-4	5.23 (4.10)	5-8	6.30 (5.03)	9-15	4.40 (5.57)	>15	3.54 (3.62)
– Manufacturing	1-5	1.52 (2.19)	6-9	1.70 (1.86)	10-15	2.25 (2.74)	>15	2.46 (2.56)
– Professional and Financial Services	1-4	6.42 (3.17)	5-8	6.16 (4.65)	9-13	7.00 (3.57)	>13	5.51 (3.71)
– Other Services	1-6	2.29 (1.97)	7-15	0.72 (0.46)	16-22	0.76 (0.52)	>22	0.90 (0.51)
– Construction and Transportation	1-3	7.46 (5.06)	4-5	7.38 (4.55)	6-9	10.82 (5.34)	>9	7.36 (7.59)

Notes: This table reports summary statistics for firms' (absolute) backcast errors about aggregate inflation by quartiles of the distribution of the number of products in each industry. The backcast errors are the absolute value of firm errors about past 12 month inflation from Wave #1 survey. Moments are calculated using sampling weights.

Appendix Table G.3: Number of Products and Knowledge about Aggregate Inflation (All Industries)

	(1)	(2)	(3)	(4)
Panel A. Dependent variable: Inflation backcast errors				
log(number of products)	-0.405***	-0.102***	-0.205***	-0.057*
	(0.069)	(0.030)	(0.063)	(0.033)
Observations	673	658	506	495
R-squared	0.229	0.835	0.273	0.896
Panel B. Dependent variable: Willingness to pay for professional inflation forecasts				
log(number of products)	-1.968	2.359*	-2.945*	3.871***
	(1.782)	(1.268)	(1.532)	(1.162)
Observations	438	434	378	372
R-squared	0.102	0.626	0.171	0.662
Firm-level controls	Yes	Yes	Yes	Yes
Industry fixed effects		Yes		Yes
Manager controls			Yes	Yes

Notes: This table reports results for the Huber robust regression. Dependent variables are the absolute value of firm errors about past 12 month inflation from Wave #1 survey (Panel A) and firms' willingness to payment for professional forecaster's forecasts about future inflation from Wave #4 (Panel B). Firm-level controls include log of firms' age, log of firms' employment, foreign trade share, number of competitors, firms' beliefs about price difference from competitors, and the slope of the profit function. Industry fixed effects include dummies for 17 sub-industries. Manager controls include the age of the respondent (each firm's manger), education, income, and tenure at the firm. Sample weights are applied to all specifications. Robust standard errors (clustered at the 3-digit ANZ SIC level) are reported in parentheses. ***, **, * denotes statistical significance at 1%, 5%, and 10% levels respectively.

Appendix Table G.4: Number of Products and Knowledge about Nominal GDP Growth

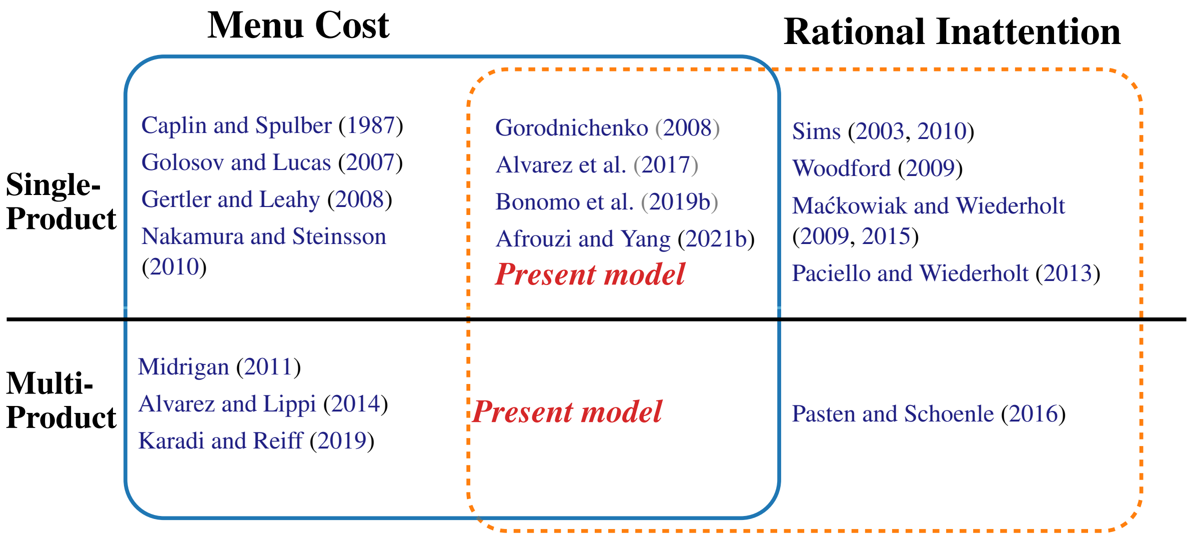
	(1)	(2)	(3)	(4)
Dependent variable: Backcast errors about nominal GDP growth rate				
Number of products	-0.041***	-0.020***	-0.035***	-0.017*
	(0.012)	(0.007)	(0.011)	(0.009)
Observations	390	378	334	326
R-squared	0.375	0.615	0.412	0.610
Firm-level controls	Yes	Yes	Yes	Yes
Industry fixed effects		Yes		Yes
Manager controls			Yes	Yes

Notes: This table reports results for the Huber robust regression. Dependent variable is the absolute value of firm errors about the growth rate of nominal GDP from Wave #4 survey. Firms' perceived growth rate of nominal GDP are calculated by taking the summation of firms' belief about current inflation and the real GDP growth rate in New Zealand. Firm-level controls include log of firms' age, log of firms' employment, foreign trade share, number of competitors, firms' beliefs about price difference from competitors, and the slope of the profit function. Industry fixed effects include dummies for 14 sub-industries excluding retail and wholesale trade sectors. Manager controls include the age of the respondent (each firm's manger), education, income, and tenure at the firm. Sample weights are applied to all specifications. Robust standard errors (clustered at the 3-digit ANZ SIC level) are reported in parentheses. ***, **, * denotes statistical significance at 1%, 5%, and 10% levels respectively.

Appendix Table G.5: Subjective Uncertainty and Expected Duration of Price Changes

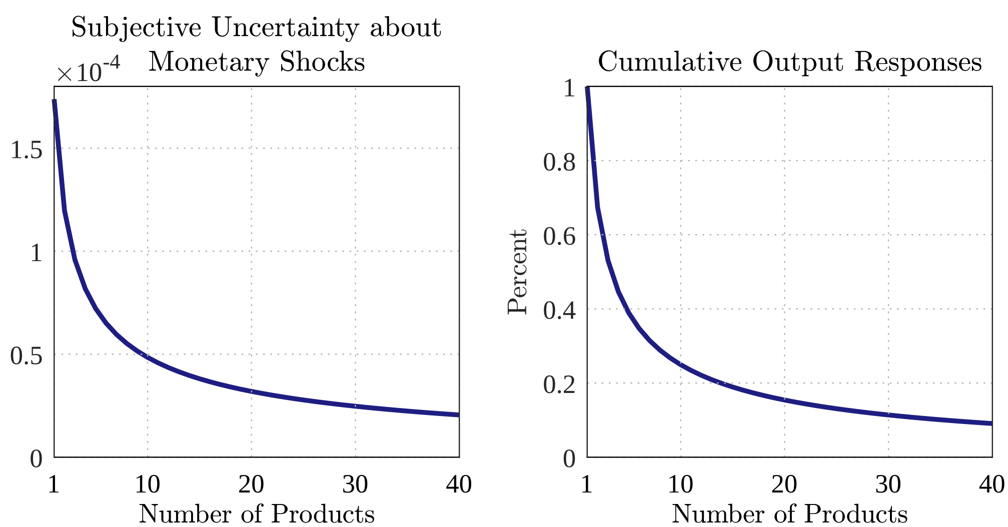
	(1)	(2)	(3)	(4)
Dependent variable: Duration of expected next price changes				
Standard deviation of the growth rate	0.132***	0.104**	0.153***	0.160**
of sales over the next 12 months	(0.039)	(0.042)	(0.054)	(0.065)
Observations	583	591	443	442
R-squared	0.323	0.697	0.342	0.436
Firm-level controls	Yes	Yes	Yes	Yes
Industry fixed effects		Yes		Yes
Manager controls			Yes	Yes

Notes: This table reports results for the Huber robust regression. Dependent variable is the duration of expected next price changes from Wave #2. The regressor is the standard deviation implied by the reported probability distribution for the growth rates of unit sales of firms' main product over the next 12 months. Firm-level controls include log of firms' age, log of firms' employment, the number of competitors, and log of firms' number of products. Industry fixed effects include dummies for 14 sub-industries excluding retail and wholesale trade sectors. Manager controls include the age of the respondent (each firm's manger), education, income, and tenure at the firm. Sample weights are applied to all specifications. Robust standard errors (clustered at the 3-digit ANZ SIC level) are reported in parentheses. ***, **, * denotes statistical significance at 1%, 5%, and 10% levels respectively.



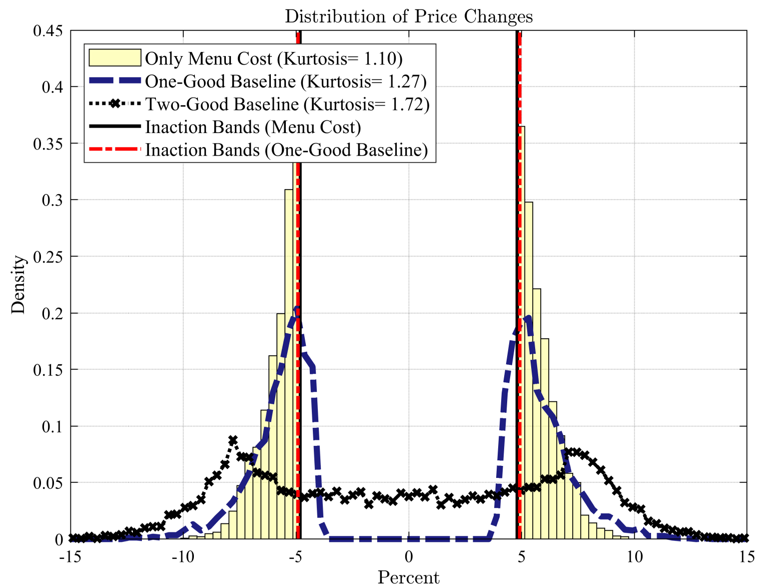
Appendix Figure G.1: Classification of Models

Notes: This figure shows classification of models in the literature on menu costs and rational inattention.



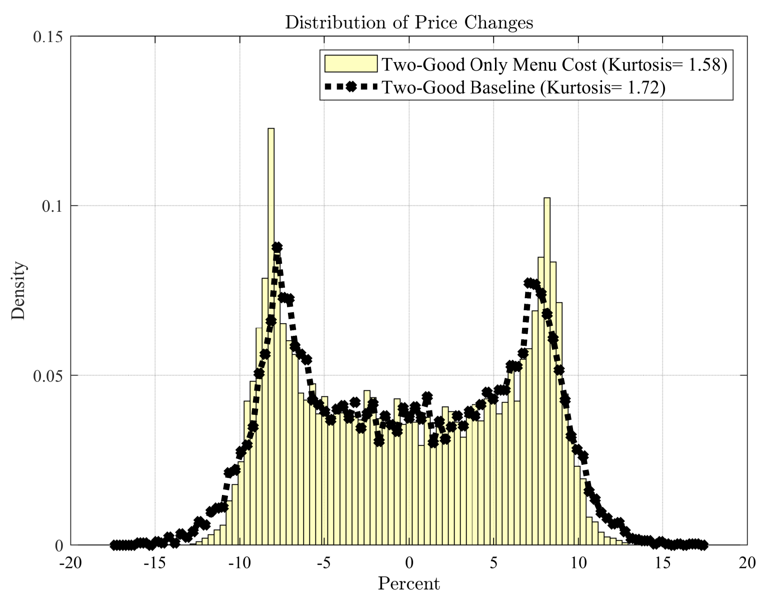
Appendix Figure G.2: Subjective Uncertainty and Cumulative Output Responses in Rational Inattention Only Models with Different Numbers of Products

Notes: The left panel plots firms' subjective uncertainty about monetary shocks in the rational inattention only model with different numbers of products. The right panel plots cumulative output responses to a one standard deviation monetary shock in the rational inattention only models with different numbers of products sold by firms. The cumulative output response in the single-product model is normalized to one.



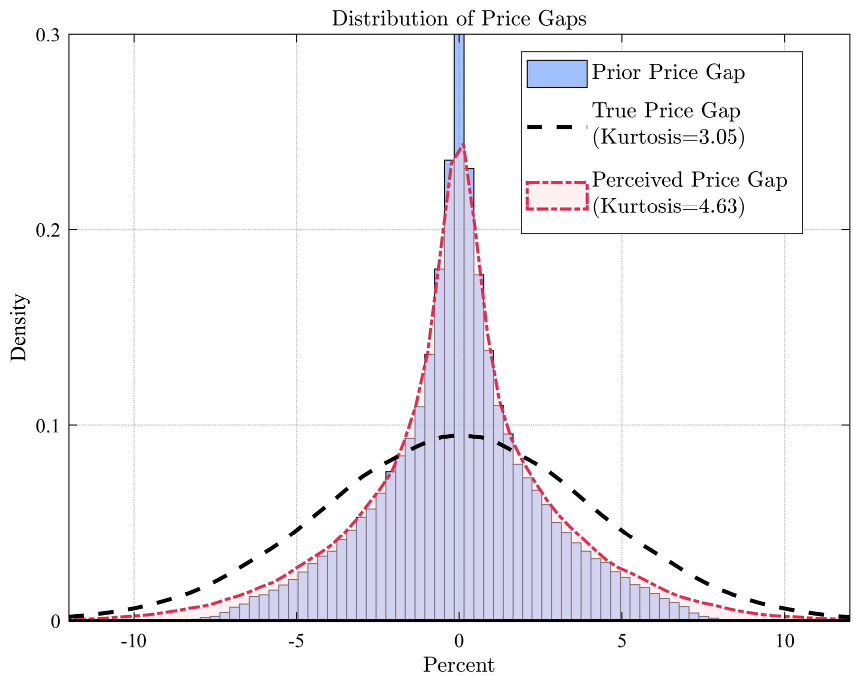
Appendix Figure G.3: Distributions of Price Changes

Notes: This figure plots the distribution of price changes in the single-product menu cost model with perfect information (yellow bar), that in the baseline single-product model (blue dashed line), and that in the baseline two-product model (black line with cross markers). Black vertical lines are the inaction bands for firms in the menu cost model with perfect information. In this model, every firms have the same inaction bands. Red vertical dash-dot lines are the average of inaction bands across firms in the baseline single-product model.



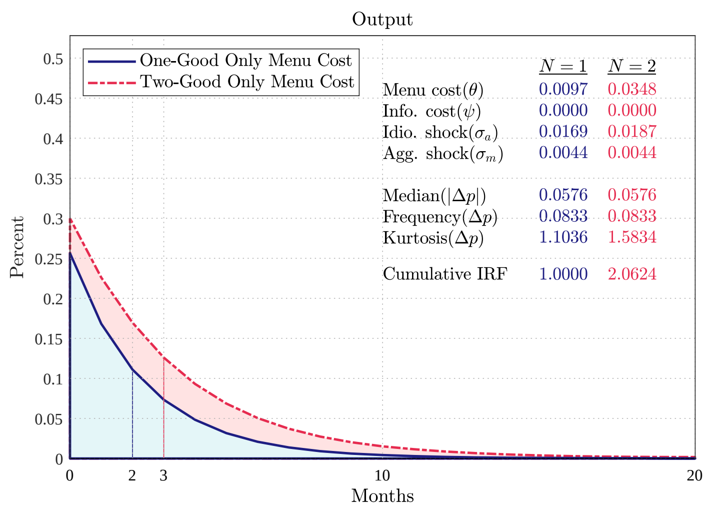
Appendix Figure G.4: Distribution of Price Changes in the Two-Product Models

Notes: This figure plots the distribution of price changes in the two-product menu cost model with perfect information (yellow bar) and that in the two-good version of the baseline model (black line with circle markers).



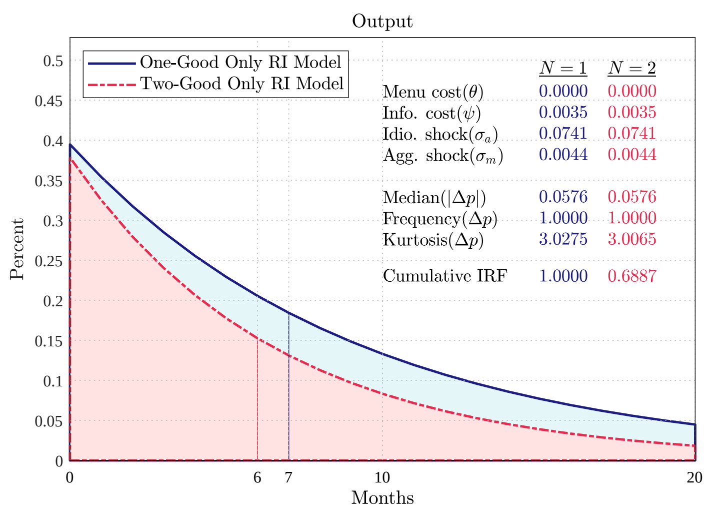
Appendix Figure G.5: Distribution of True and Perceived Price Gaps in the Two-Product Model

Notes: This figure plots distributions of price gaps in the two-good version of the baseline model. Blue bar graph shows the distribution of firms' *prior* about their price gaps at the beginning of period. Black dashed line shows the distribution of firms' true price gaps after their Gaussian shocks realized. Firms choose their optimal signals about the shocks and form a new posterior about their (frictionless) optimal price. Red dash-dot line shows the distribution of the *posterior* of perceived price gaps.



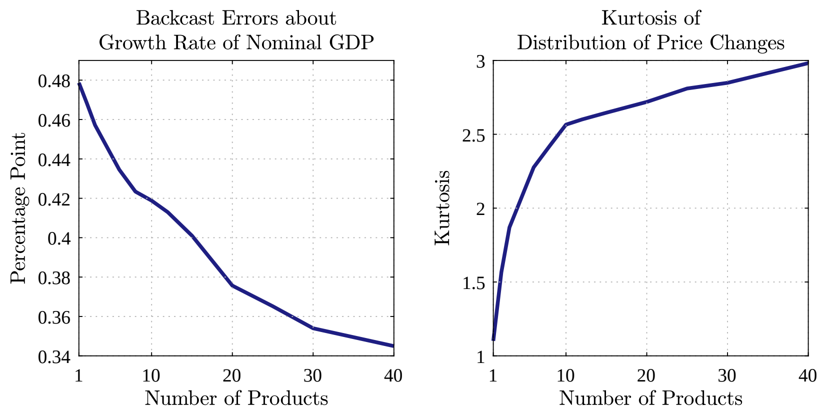
Appendix Figure G.6: IRFs of Output in the Menu Costs Models with Perfect Information

Notes: This figure plots impulse responses of output to a one standard deviation monetary shock in the one-good and two-good versions of menu cost models with perfect information. Cumulative IRFs refers to area under the responses of output. I normalize the cumulative output response in the one-good version of the model as one.



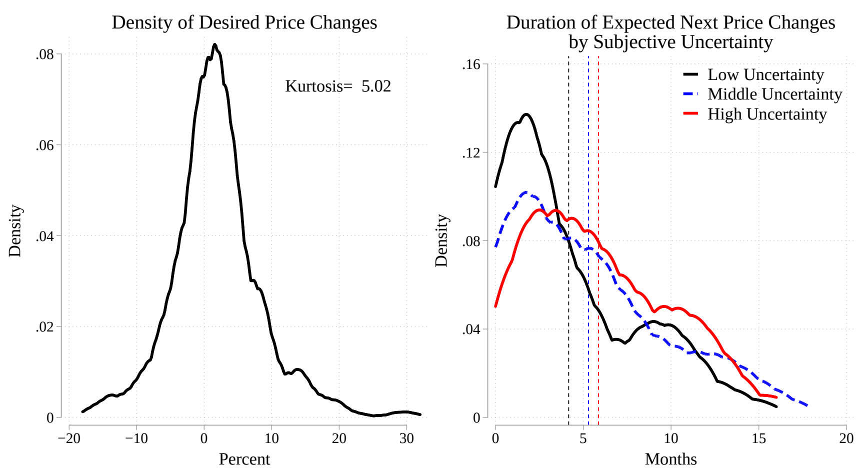
Appendix Figure G.7: IRFs of Output in the Rational Inattention Models without Menu Costs

Notes: This figure plots impulse responses of output to a one standard deviation monetary shock in the one-good and two-good versions of rational inattention models without menu costs. Cumulative IRFs refers to area under the responses of output. I normalize the cumulative output response in the one-good version of the model as one.



Appendix Figure G.8: Backcast Errors and Kurtosis of Price Changes in the Simplified Models

Notes: This figure shows shows the backcast errors of firms about the growth rate of nominal GDP (left panel) and the kurtosis of distribution of price changes (right panel) in the simplified version of the baseline models with different numbers of products sold by firms. See Section 4.7 for details.



Appendix Figure G.9: Distribution of Desired Price Changes and Duration of Expected Price Changes in the Survey Data

Notes: This left panel shows the distribution of desired price changes in the second wave of the survey data. Firms' managers in the survey were asked how much they would like to change the price of their main product if it was free to change its price in three months. The answer gives firms' desired price changes in three months. I define the inflation-adjusted desired price changes as the gap between firms' desired price changes in three months and their three-month ahead inflation expectations. The right panel shows the distribution of firms' expected duration of their next price changes by their degree of subjective uncertainty. The subjective uncertainty is measured by the standard deviation implied by the reported probability distribution for the growth rates of unit sales of firms' main product over the next 12 months. See Appendix F for details.