Rational Inattention, Menu Costs, and Multi-Product Firms: Micro Evidence and Aggregate Implications*

Choongryul Yang†

University of Texas at Austin

November 13, 2019

< Link to latest version >

Abstract

How do information and nominal rigidities affect multi-product firms? Using a firm-level survey from New Zealand, I show that firms that produce more goods have both better information about aggregate inflation, and more frequent but smaller price changes. To assess the aggregate implications of these firm-level findings, I develop a dynamic general equilibrium menu cost model with rationally inattentive multi-product firms. I parameterize the model to be consistent with the micro-evidence and show that the interaction of the menu cost and rational inattention frictions gives rise to a novel selection effect: firms that adjust prices have better information about underlying shocks than non-adjusters. This selection effect leads to an endogenous leptokurtic distribution of desired price changes that amplifies monetary non-neutrality. As a result, the real effects of monetary policy shocks in the one-good version of the model are nearly as large as those in the Calvo model. In the two-good version of the model, firms optimally choose to have better information about aggregate inflation than in the one-good version of the model, leading to a 20% reduction in the real effects of monetary shocks.

JEL Classification: E31, E32, E37, E52, E70

Keywords: Inflation expectations, Monetary non-neutrality, Rational inattention, Menu costs, Multi-product firms, Economies of scope

---

*I would like to thank my advisors Olivier Coibion and Saroj Bhattarai for their guidance and support. I am grateful to Hassan Afrouzi, Chris Boehm, Carlos Carvalho, Stefano Eusepi, Cooper Howes, Andreas Mueller, Ernesto Pasten, Claudia Sahm, Raphael Schoenle, and seminar participants at Boston Fed, MEG 2019, and UT Austin for helpful comments and suggestion. Part of this paper was written at the Federal Reserve Bank of Boston while I was a dissertation intern. I acknowledge the Texas Advanced Computing Center (TACC) at UT Austin for providing HPC resources that have contributed to the simulation results reported within this paper.

†Department of Economics, University of Texas at Austin, 2225 Speedway C3100, Austin, TX, 78712 U.S.A. Email: c.yang@utexas.edu

1 Introduction

The nature of price setting and firms' expectations formation have important implications for the transmission of monetary policy shocks. The extensive empirical literature studying detailed microdata on firms' beliefs and pricing behavior has found that many firms are not fully aware of macroeconomic conditions and change their prices infrequently.[1] Over time, economic models have embedded these realistic features of price setting and firms' expectations formation, such as costly price adjustment and a lack of awareness of economic conditions, to study the ability of monetary policy to stimulate the economy. Most of those models, however, assume firms produce only one product.[2] This assumption comes from either the view that multi-product firms are approximately a collection of single-product firms or a lack of empirical evidence regarding multi-product firms' economic decisions. This paper challenges this view by providing new empirical evidence and developing a new theoretical model in which multi-product pricing has important implications for monetary policy transmission.

In the first part of this paper, I seek answers to the following empirical question: How are firms' price setting and information acquisition decisions related to their product scope? In particular, I explore whether there is empirical evidence on economies of scope in multi-product firms such that firms with a greater number of products have lower costs when changing prices and acquiring information about the economy. If so, firms' product scope would affect their decisions regarding price setting and information acquisition and thus has important implications for the ability of monetary policy to stimulate the economy. The second part of this paper then investigates how firms' product scope affects the transmission of monetary policy shocks when both adjusting prices and acquiring information are costly. To do this, I develop a general equilibrium model disciplined by micro-evidence and seek answers to the following two theoretical questions. First, how do economies of scope in multi-product firms quantitatively affect the monetary policy transmission? Second, regardless of firms' product scope, is there any new insight about how the interaction between nominal and informational rigidities affects monetary non-neutrality?

This paper builds on a growing literature studying how firms form their expectations and process information. For example, Kumar et al. (2015), Coibion et al. (2018a), and Afrouzi (2019b)

[1] See, for instance, Klenow and Malin (2010) and Nakamura and Steinsson (2008, 2013) for comprehensive reviews about micro price stickiness. Also, see Kumar et al. (2015), Coibion et al. (2018a), Boneva et al. (2019), and Afrouzi (2019b) for the evidence on pervasive inattention on the firm side to macroeconomic variables.

[2] Notable exceptions are Midrigan (2011) and Alvarez and Lippi (2014), who study menu cost models with multi-product firms, and Pasten and Schoenle (2016), who build a rational inattention model with multi-product firms.

study various factors that determine how much attention firms devote to tracking macroeconomic conditions.[3] My new empirical contribution to this literature is to document that the number of products that firms produce is an important determinant of firm-level inattention to macroeconomic conditions. I find that firms with greater product scope have better information about aggregate economic conditions. Moreover, I investigate how firms' product scope affects their price-setting decisions and show that firms with greater product scope have more frequent but smaller price changes. The joint characterization of the relationship between firms' product scope and their decisions regarding both price setting and information acquisition complements previous literature studying microdata on multi-product pricing (e.g. Bhattarai and Schoenle, 2014; Stella, 2018; Bonomo et al., 2019a).

This paper also contributes to our understanding of monetary non-neutrality using a new general equilibrium model with both nominal and informational rigidities in a world of multi-product pricing. This model builds on two strands of monetary models with multi-product firms to capture the micro-evidence on economies of scope. First, menu cost models with multi-product firms, such as Midrigan (2011) and Alvarez and Lippi (2014), exhibit economies of scope in price setting: firms with a greater number of products have more frequent but smaller price changes, as they can change the prices of all their products by paying a single fixed cost. Second, rational inattention models with multi-product firms, such as Pasten and Schoenle (2016), show economies of scope in information processing: firms with a greater number of products have a large incentive to acquire and process information about aggregate shocks. My theoretical contribution is to combine all three elements—menu costs, rational inattention, and multi-product firms—in a unified framework to study the ability of monetary policy to stimulate the economy.[4] This model, disciplined by micro-evidence, serves to quantitatively study how monetary non-neutrality is affected 1) by firms' product scope and 2) by the interaction between menu costs and rational inattention regardless of firms' product scope.

My starting point is to explore the empirical characteristics of multi-product firms in terms of their price setting and information acquisition decisions. To do so, I use a representative survey of New Zealand firms' macroeconomic beliefs. Inattention among firms to macroeconomic conditions is pervasive. For example, on average, firms overstate aggregate inflation over the prior 12 months by 4.5 percentage points. Moreover, firms in the survey change their prices infrequently,

[3] See also Frache and Lluberas (2019), Grasso and Ropele (2018), and Coibion et al. (2019).

[4] Appendix Figure A.1 shows how my model fits within the literature on menu costs and rational inattention models.

and there is a great deal of heterogeneity in the frequency and size of price changes. For example, an expected duration until subsequent price changes is 12 months on average, though this varies substantially across sectors.

My empirical analysis documents that firms' product scope is systematically related to their inattention to macroeconomic conditions and price-setting decisions. First, I find that firms with a greater number of goods are better informed about aggregate inflation. Firms make systematically smaller errors about recent values of aggregate inflation when they produce a greater number of products. Moreover, firms with greater product scope are also willing to pay more for information about future inflation. This finding implies that firms with more products have incentives to process more information about macroeconomic conditions. To the best of my knowledge, these results are the first empirical evidence documenting differential information acquisition decisions of firms based on the number of products they sell.[5] Second, I show that firms with a greater number of products have more frequent but smaller price changes. This finding is consistent with the previous empirical literature using different micro price data.[6] Jointly, these results illustrate that the scope of products sold by firms affects both their information acquisition and price-setting decisions.

What are the aggregate implications of the micro-evidence that I show above for the ability of monetary policy to stimulate the economy? To answer this question, I develop a new model that captures the behavior of firms in the survey. Specifically, I assume that it is costly for firms to observe and process information regarding underlying shocks. Firms optimally choose their information set given the costs of information. This captures the pervasive inattention among firms in the survey. Second, firms have to pay a fixed menu cost to reset their prices, which leads to the infrequent price changes observed in the survey data. I further assume that this fixed cost is independent of how many prices firms change. This assumption introduces economies of scope in price setting: firms with greater product scope change their prices more frequently and by smaller amounts since the average costs of changing prices are lower for them. Finally, I assume that firms face two types of shocks—idiosyncratic good-specific shocks and aggregate monetary shocks—and the marginal cost of processing information is independent of firms' number of products. This

[5] I further provide experimental evidence that supports this result. When firms are given new information about aggregate inflation, firms with greater product scope update their expectations by less as they already have more precise prior beliefs. See Section 2.1.2 for details.

[6] Using the U.S. PPI microdata, Bhattarai and Schoenle (2014) find that multi-product firms change their prices more frequently and by smaller amounts. Parker (2017) also finds the similar findings using New Zealand data. Stella (2018) estimates common menu costs for multi-product firms and finds substantial economies of scope in price setting.

assumption introduces economies of scope in information processing: firms with greater product scope want to learn more about the monetary shocks since information about them can be used to price all their goods.

I embed this setup of firm decision making into a full-fledged dynamic general equilibrium model and study the macroeconomic implications for monetary non-neutrality. I use three target moments from the survey data to discipline model parameters: the frequency and size of price changes and the slope of a backcast errors curve on the number of products. The first two help to calibrate the menu cost parameter and size of idiosyncratic shocks, while the third helps to calibrate the informational cost parameter. There are two main focuses of the quantitative analysis using this general equilibrium model. First, I explore how the interaction between rational inattention and menu costs affects firms' optimal decisions and, therefore, how the economy responds to monetary shocks, regardless of firms' product scope. To do this, I compare the output responses to monetary shocks in the one-good version of my model with those in the standard menu cost only model, such as Golosov and Lucas (2007). Second, I show how firms' product scope affects monetary non-neutrality through economies of scope in multi-product firms by comparing the macroeconomic dynamics to monetary shocks in one-good vs. two-good versions of my model.

My first theoretical finding is that the baseline one-good version of the model generates large real effects of monetary policy shocks that are seven times larger than those in the standard menu cost model and are nearly as large as those in the Calvo sticky price model. Standard menu cost models have small and short-lived real effects of monetary shocks due to the strong selection effects of price changes: an expansionary monetary shock triggers numerous price increases and offsets many price decreases.[7] The extent of the selection effects depends on the underlying distribution of firms' desired price changes. When the distribution is Gaussian, a large mass of prices is around adjustment margins, leading to large price selection effects. Previous menu cost models often assumed that the distribution of idiosyncratic shocks has excess kurtosis and a fat tail, so the majority of desired price changes are near zero while some of them are very far from zero (e.g. Gertler and Leahy, 2008; Midrigan, 2011; Vavra, 2013; Karadi and Reiff, 2019; Baley and Blanco, 2019). My new contribution to this literature is to show that the interaction between menu costs and rational inattention frictions can endogenously generate the distribution of firms' desired price changes with excess kurtosis. I show that this leptokurtic distribution of desired price

[7] Gagnon et al. (2013) study the effect of large inflationary shocks on the timing of price changes using Mexican CPI data and find direct support for a selection effect. Carvalho and Kryvtsov (2018) finds evidence of strong price selection across goods and services using detailed micro-level consumer price data for the UK, the US, and Canada.

changes weakens the selection effects of price changes, amplifying the impact response of output to monetary shocks by 30% in my baseline one-good version of the model compared to the standard menu cost models with fully informed firms.

How does the interaction between nominal and informational rigidities generate the leptokurtic distribution of desired price changes? As highlighted in previous models with both information and nominal rigidities, when both acquiring information and adjusting prices are costly, firms' optimal price-setting rules depend on their own subjective uncertainty (e.g. Gorodnichenko, 2008; Afrouzi, 2019a). Firms with large uncertainty regarding underlying shocks have a wider inaction region for pricing decisions, since they want to wait and see until they get more information to resolve their uncertainty.[8] This leads to another kind of selection effect regarding information processing: price adjusters have better information than non-adjusters. A new finding of this paper is that this selection effect in information processing endogenously leads to a distribution of firms' desired price changes with the majority near zero and some far away from zero, or in other words, the distribution is leptokurtic. Consider an economy with a large number of one-product firms.[9] At the beginning of each period, firms' prior beliefs about their price gaps are all within their inaction bands, implying a high kurtosis in the distribution of *prior* price gaps. After being hit by Gaussian idiosyncratic shocks, the distribution of *true* price gaps is Gaussian. Firms then all choose their optimal signals and update their estimates of price gaps, but they do not do so in the same way. Firms that think it is unlikely that they will need to change prices have little incentive to collect much new information: they choose to remain quite uninformed. In contrast, firms that think they are close to the boundaries of their inaction region have a high incentive to collect information and therefore choose to become more informed. This results in a leptokurtic distribution of *posterior* price gaps, or equivalently, their desired price changes.[10]

The second theoretical finding is that the real effects of monetary policy shocks decrease by 20% in the two-good version of the model compared to the one-good version of the baseline model. As I discuss above, the two-good version of the model exhibits two types of economies

[8]Real option value and the "wait-and-see" rule has been studied in the literature on the effects of steady-state uncertainty (e.g. Dixit and Pindyck, 1994; Abel and Eberly, 1999) or second moment uncertainty shocks (e.g. Bloom, 2009; Vavra, 2013; Gilchrist et al., 2014; Bloom et al., 2018). Unlike this literature, the interaction between information and nominal rigidities makes firms' optimal pricing rules depend on their *own* subjective uncertainty.

[9]While the mechanism I explain here also operates in the two-good version of the model, the one-good version of the model gives a clear comparison to the standard menu cost models with one-product firms, such as Golosov and Lucas (2007).

[10]In Section 5.2, I provide evidence on the leptokurtic distribution of desired price changes using the New Zealand survey data. Moreover, I show that firms with greater subjective uncertainty expect longer duration of their subsequent price changes. This is a direct evidence on the "wait-and-see" rule in firms' price-setting decisions.

of scope in multi-product firms. First, there are economies of scope in price setting. Since paying the menu cost allows firms to change the prices of all of their goods simultaneously, there are many small and large price changes in the two-good version of the model. This weakens selection effects of price changes, as emphasized by Midrigan (2011) and Alvarez and Lippi (2014), and should tend to amplify the real effects of monetary policy shocks in the two-good version of the model. Second, I also find significant economies of scope in information processing: the value of information about aggregate shocks increases with firms' product scope. Under my benchmark calibration, firms in the two-good version of the model have better information and lower uncertainty about monetary policy shocks than firms in the one-good version of the model. Since multi-product firms learn about monetary policy shocks rapidly, this force will tend to reduce the real effects of nominal shocks, as highlighted by Pasten and Schoenle (2016). The quantitative analysis shows that cumulative output effects are smaller in the two-good version of the model than the one-good version of the model. This implies that the scope motive in information processing quantitatively dominates its effect on pricing decisions, thereby leading to reduced effects of nominal shocks on economic activity as we move to a multi-product environment.

My results are robust to introducing higher numbers of products in the model. I simplify the baseline model by assuming that firms' information acquisition decisions are independent of their price-setting decisions. I solve the simplified version of the model with an arbitrary number of products to explore whether my main results can be extended with higher numbers of products. Although this assumption eliminates the interesting interaction between nominal and informational rigidities, the simplified version of the model shares the core predictions of the baseline model. Firms with greater product scope have better information about aggregate shocks. Moreover, the kurtosis of the distribution of price changes increases with the number of products firms produce. I calibrate each model with different numbers of products to match the same frequency and size of price changes and study impulse response analysis to monetary policy shocks. Consistent with the results from the full baseline model, I find that the cumulative output responses in the simplified version of the model decrease with the number of products sold by firms. This implies that in the class of rational inattention models, the ratio of kurtosis and the frequency of price changes might not be a sufficient statistic, which is derived by Alvarez et al. (2016), for the real effects of monetary shocks.

The paper is organized as follows. Section 2 empirically evaluates how firms' attentiveness to aggregate inflation and price-setting behavior are related to their number of products. In Section

3, I develop a new menu cost model with rationally inattentive multi-product firms that captures the behavior of firms in the survey, and I study firms' optimal decisions regarding information acquisition and price setting. In Section 4, I extend my model to show how product scope affects firm behavior and monetary policy transmission. In Section 5, I study an extension of the baseline model to show the robustness of my results and provide additional evidence that supports the model's predictions using the survey data. Section 6 concludes the paper.

2 Empirical Evidence

In this section, I empirically explore how firms' product scope relates to 1) their attentiveness to aggregate economic conditions and 2) the frequency and size of their price changes. To this end, I use a quantitative survey of firms' expectations toward macroeconomic conditions in New Zealand.[11] The survey was conducted in multiple waves among a random sample of firms in New Zealand with broad sectoral coverage. There are two novel empirical contributions in this paper relative to the previous studies that have used the same survey.[12] First, I show that firms producing a greater number of products are better informed about current aggregate inflation. Second, I document that both the duration and average size of their price changes decrease with the number of products firms produce.

2.1 Number of Products and Knowledge of Aggregate Inflation

I use the second wave of the survey, which was implemented between February and April 2014, to identify the number of products firms produce. In the survey, firms' managers were asked the following question:

> **"In addition to your main product or product line, how many other products do you sell?"**
>
> *Answer*: .... products

Appendix Table A.1 shows the summary statistics regarding firms' number of products by industry, for which the median number is 9, while it is 7 when firms in retail and wholesale trade sectors are excluded. In the baseline regressions, I exclude these retail and wholesale trade firms

[11] See Coibion et al. (2018a) and Kumar et al. (2015) for a comprehensive description of the survey.

[12] Several papers use the data to characterize how firms form their expectation. For example, Afrouzi (2019b) shows that strategic complementarity decreases with competition, and documents that firms with more competitors have more certain posteriors about the aggregate inflation. Also, Coibion et al. (2018b) evaluate the relation between first-order and higher-order expectations of firms, including how they adjust their beliefs in response to a variety of information treatments.

since their strategies for pricing and information processing are likely to be different from those of firms in other sectors, such as manufacturing and service industries.[13] In the data, a large fraction of firms (about 18% of all firms) sell only one product or have one product line when compared to other studies.[14] There are two reasons behind the large fraction of single-product firms in this survey. First, the firms included in the survey were relatively small, with the average amount of firm employees being about 31 and the largest firm having about 600 employees. Second, the survey question is about the number of products or product lines of a firm. Since there might be several similar types of products in a product line, this question captures firms' perception of the unit of their product scope. In fact, I find that the average of firms' output share of main product (or product line) is about 60% excluding single-product firms, implying that firms define their unit of product scope a bit broadly.

2.1.1 Number of Products and Attentiveness to Aggregate Inflation

I first investigate the relationship between firms' number of products and their attentiveness to current aggregate economic conditions. Firms' attentiveness to aggregate economic conditions, referred to as backcast error about aggregate inflation, is defined to capture firms' knowledge about the current aggregate economy. Given that recent aggregate economic conditions are largely observable in real-time, I define the backcast error as the absolute values of the difference between the actual past 12 months of aggregate inflation and managers' corresponding beliefs from the survey.[15] As documented in Coibion et al. (2018a), firms are not well-informed about the current aggregate inflation, making 4.5% backcast errors on average.[16]

In addition to that, I find firms' backcast errors are related to their number of products.[17] Figure 1 shows there is a clear positive relationship between the number of products firms produce and their attentiveness to aggregate inflation. The left panel shows that firms with a smaller product scope produce larger backcast errors, on average. In the right panel, I show the relationship

[13] Including retail and wholesale trade firms in the sample does not change the baseline results that I show below. See, for example, Appendix Table A.3.

[14] For example, Bhattarai and Schoenle (2014) document that 98% of all prices are set by firms with more than one good in the microdata that underlie the calculation of the U.S. PPI.

[15] Consumer Price Index (CPI) is used to calculate the actual past 12 month aggregate inflation. The baseline results are quantitatively similar when I use GDP deflator or Produce Price Index to calculate the actual inflation rate.

[16] One might have a concern that whether firms do not know what the inflation means. However, Kumar et al. (2015) document that 86% of firm managers in the survey could correctly explain what inflation means and they believed that statistical agencies were credible in measuring price changes. Coibion et al. (2018a) also highlight that the large errors are not driven by specific language about the definition of inflation used in the survey.

[17] See Appendix Table A.2 for the summary statistics of the firms' backcast error about aggregate inflation by the quartiles of firms' product scope within different industries.

between firms' product scope and their willingness to pay for professional forecasts about future inflation from the fourth wave of the survey. The latter is another measure of firms' incentives to be attentive to the aggregate economy. Here, I find a positive correlation between the number of products firms produce and their willingness to pay for professional forecasts regarding future aggregate inflation. This implies that firms with larger product scopes are likely to pay more attention to aggregate conditions.

One potential concern is that this negative correlation between the number of products firms produce and their knowledge of current aggregate inflation is driven by other firm-level characteristics. For example, as big firms are likely to have a larger product scope and a larger capacity to process information, it might seem that the negative correlation stems from the size of the firm rather than its product scope.[18] To address this, I regress firms' inattention to inflation, as measured by 1) their absolute errors in regard to recent inflation rates and 2) their willingness to pay for professional future inflation forecasts, using a log of firms' number of products controlling for firm-level characteristics, such as a log of firms' age, a log of total employment, foreign trade share, firms' number of competitors, their beliefs about price differences relative to their competitors, and the slope of the profit function.[19] Column (1) of Table 1 shows that firms with a large number of products are likely to make small errors about current aggregate inflation and are also more willing to pay more for professional forecasts about future aggregate inflation. Column (2) shows the significant negative correlation after controlling for industry fixed effects

Another potential concern is that the survey respondents, here the managers of firms, have different abilities or incentives to pay attention to aggregate economic conditions.[20] To address this issue, in Column (3), I report the regression results after controlling for managers' characteristics, such as age, education, income-level, and amount of tenure years at their firms. Again, after controlling for manager characteristics, I find a negative correlation between firms' number of products and their knowledge of or attentiveness to aggregate inflation.[21]

[18]Kaihatsu and Shiraki (2016) show that size of firms significantly affects differences in their inflation expectations. Also, Frache and Lluberas (2019) find that large firms have lower forecast errors about aggregate inflation than small firms.

[19]The slope of a firm's profit function is calculated as the ratio of by how much a firm could increase its profit (as a percent of revenue) if it could reset its price freely at the time of the survey relative to the percent price change the firm would implement if it could reset its price freely at the time of the survey.

[20]For example, Tanaka et al. (2019) show that managers' GDP forecasting ability is linked to their management ability and experience.

[21]In Appedix Table A.5, I show that firms' backcast errors about the growth rate of nominal GDP also decrease in their number of products. In the general equilibrium model I study in Section 4, I calibrate the information cost parameter to match the slope coefficient of the regression of the backcast errors about the growth rate of nominal GDP. See Section 4.3 for details.

2.1.2 Experimental Evidence

In this subsection, I further provide two kinds of experimental evidence that firms with greater number of products have better information about aggregate inflation. Firms revise their expectations when they receive new information, and the revision in expectations leads to change in firms' decisions. I show that when firms are given new information about aggregate inflation, those with a greater number of products revise their expectations by less, and they barely change their actions when compared to firms without new information.

Information Updates I first ask how firms differentially revise their expectations depending on their number of products when they are given new information. Suppose firm i has a prior belief about aggregate inflation that is normally distributed with mean μ_i and precision τ_i. Then, each firm receives a common signal s that is also normally distributed with precision ψ_s. Firms update their beliefs in a Bayesian manner and get posterior p_i:

$$p_i = \mu_i + \frac{\psi_s}{\psi_s + \tau_i}(s - \mu_i). \tag{1}$$

Firms update their beliefs by more when the signal has greater precision or their prior precision is smaller. This implies that if firms with a greater number of products have better information about aggregate inflation, they revise their beliefs by less given the precision of the signal. I can test this implication more formally using the following regression:

$$p_i = c + \beta\mu_i + \gamma N_i^{\text{product}} \times \mu_i + \delta N_i^{\text{product}} + \varepsilon_i,$$

where N_i^{product} is a measure of firm i's number of products and the constant term absorbs the common signal. I would expect $\gamma > 0$ if firms with a greater number of products revise their expectations by less.

In the fourth wave of the survey, firms were asked to assign probabilities to the different outcomes for future inflation, from which I can compute their mean forecasts. This mean forecast is firms' prior beliefs (μ_i) about future inflation. Randomly assigned firms were given additional information about aggregate inflation, which is the common signal about future inflation (s).[22] Af-

[22]There were five group of firms which receive information about inflation: 1) the most recent realization of inflation, 2) the target inflation of the central bank, 3) the most recent professional forecaster of inflation, 4) both the central bank target and the professional forecast, and 5) the average inflation forecast of other firms in the survey. One piece of additional information is given to firms in each group. See Section 5.1 in Coibion et al. (2018a) for details of the

ter receiving this information, firms were asked for a point forecast about future inflation, which corresponds to their posterior beliefs (p_i).

Table 2 shows the regression results with different specifications. All regressions include fixed effects for each treatment group. Column (1) shows that firms revise their beliefs toward the signal, which is consistent with Bayesian updating.[23] Column (2) shows the regression result using a dummy variable, which is one if a firm sells four or more products, for the interaction term. The positive coefficient on the interaction term implies that firms with less than four products revise their expectations about inflation by more when they receive new information. Column (3) shows that this result is robust to the use of a continuous variable (a log of firms' number of products). Moreover, including firms' size and the interaction term between size and prior does not change the results (Column 4). Finally, I use a proxy for firms' number of products: share of total product value for the main product (S_y^{main}). This share should be one if firms only sell one good and is likely to be small when firms sell many products. I use the output share of non-main product, $1 - S_y^{\text{main}}$, as a proxy for firms' number of products. Column (5) shows that the interaction coefficient is significantly positive, implying that firms with greater product scope might update their information by less when they receive new information.

In sum, the sensitivity of firms' inflation beliefs to new information decreases with their number of products, which is consistent with the previous finding that firms with a greater number of products have better information about aggregate inflation.

Treatment Effects of Providing Information Since firms' revision of expectations differs based on their product scope when given new information, one might expect that firms also change their actions differently. I take one more step to test this implication. In the fifth wave of the survey, firms are asked about their plans for changing their future investments, employment, prices, and wages over the next six months. Then, a randomly selected subset of firms were told about the Reserve Bank of New Zealand (RBNZ)'s inflation target. After six months, the follow-up wave asked firms about the outcomes of each of these variables during the previous six months. The treatment effects of providing new information can be measured by the gap between these actual outcomes and their previous forecasts, which captures the extent to which their actions deviated from their ex-ante forecasts. If firms with a greater number of products have better information

experiment.

[23]This is clearly seen when Equation (1) is rewritten as $p_i - \mu_i = \frac{\psi_s}{\psi_s + \tau_i}(s - \mu_i)$. Then, the estimated coefficient captures $\beta = 1 - \frac{\psi_s}{\psi_s + \tau_i}$.

about aggregate inflation, they would not further revise their expectations and do not change their actions relative to the control group, which did not receive information about the central bank's inflation target.

Table 3 shows the treatment effects of information about RBNZ's inflation target. First, the treated firms with four or less products immediately decrease their inflation expectations relative to the control firms with the same number of products. This is consistent with the previous finding regarding information updating. After sixth months, the firms with four or less products in the treatment group exhibited prices and investments below their ex-ante expectations than firms in the control group. In contrast, I find no significant deviation in the actions of the information-treated firms with more than four products than firms in the control group. This is another evidence that firms with a greater number of products have better information about aggregate inflation, and thus, they do not strongly revise their expectations nor change their actions when given new information.

2.2 Number of Products and Size and Frequency of Price Changes

In this subsection, I document the relationship between firms' number of products and the frequency and size of their price changes. Firms' managers were asked the following question:

> **"Please report when and by how much you expect to next change the price of your main product and your second main product.** Please provide a numerical answer in months for the durations (e.g. "0" for within the next month, 1 for one month from now, ...) and a percentage answer for the size of the price change (e.g. "+10%" for a 10% increase in price or "-10%" for a 10% decrease)

After controlling for firms' characteristics and their incentives for changing their prices, this question quantifies the frequency and size of firms' price changes.

Table 4 shows the relationship between firms' number of products and the frequency and size of their price changes. Panel A shows that, after controlling for firm-level characteristics, the duration of price changes is negatively correlated with the number of products. This means that firms with greater product scope change their prices more frequently. This negative correlation is even stronger when I control for managers' characteristics. In Panel B, I also present that after controlling for industry fixed effects, there is a negative correlation between the number of products firms produce and the size of their price changes. This shows that, conditional on price change, firms with greater product scope change their prices by smaller amounts.

Previous studies have also found negative correlations between firms' number of products and the duration and size of their price changes. For example, using microdata that underlies the calculation of the U.S. PPI, Bhattarai and Schoenle (2014) show that firms with larger product scopes are more likely to change their prices frequently, and, conditional on price changes, they change by smaller amounts. Parker (2017) also identified this negative correlation between firms' product scopes and the duration and size of their price changes using another New Zealand firms' survey data from Statistics New Zealand in 2010.

2.3 Summary and Relation to Monetary Models

In this section, I find two stylized facts about how firms' information acquisition about aggregate inflation and price-setting decisions are related to their number of products such that:

1. firms with a greater number of products have better information about aggregate inflation;
2. firms with a greater number of products have more frequent but smaller price changes.

Can existing monetary models with multi-product firms explain both empirical findings? In fact, the empirical finding 1 is consistent with the prediction of rational inattention models with multi-product firms, such as Pasten and Schoenle (2016). In the presence of both goods-specific shocks and aggregate shocks, multi-product firms want to be more informed about the aggregate shocks, since the aggregate shocks will affect the marginal costs of all their products. Thus, this model implies a negative correlation between firms' number of products and their inattentiveness to aggregate shocks.

On the other hand, the empirical finding 2 is consistent with the prediction of menu cost models with multi-product firms, such as Midrigan (2011) and Alvarez and Lippi (2014). When firms can change all their prices by paying a firm-specific fixed cost, then firms with a greater number of products are likely to change their prices more frequently and by smaller amounts.

However, those two models are not capable to explain both findings simultaneously by assumption. Rational inattention models assume flexible prices to focus on the effects of information rigidity while menu cost models assume perfect information to focus on the effects of nominal rigidity. Those two assumptions are not consistent with the empirical evidence that firms change their prices infrequently and they are not fully aware of macroeconomic conditions.[24]

[24] Appendix Table A.4 shows that other models with nominal or information rigidities also cannot explain those two stylized facts I find in this section. For example, Calvo or Taylor sticky price models imply that the frequency and size

Moreover, previous literature finds a contradictory implication of firms' product scope for monetary non-neutrality. In menu cost models, the effects of monetary shocks on output *increase* with the firms' number of products, since selection effects of price changes from menu cost technology decrease with the firms' number of products as shown in Alvarez and Lippi (2014).[25] In contrast, in rational inattention models, the real effects of monetary shocks *decrease* with the number of products firms produce since firms with greater product scope have better information about monetary policy shocks.[26]

In sum, neither model with multi-product firms can account for the empirical relationship between firms' product scope and their decisions regarding both price setting and information acquisition, and they have contradictory implications regarding firms' product scope for monetary non-neutrality. This calls for a new model that is disciplined by the empirical findings from microdata in order to study the macroeconomic implications of monetary non-neutrality. This is the goal of the following sections.

3 Price Setting with Menu Costs for a Rationally Inattentive Multi-Product Firm

In this section, I develop a menu cost model for a rationally inattentive multi-product firm. Before constructing a fully-fledged dynamic general equilibrium model in the next section, I consider a decision problem of a rationally inattentive firm that faces both product-specific shocks and an aggregate shock. The firm chooses a set of optimal signals about the underlying shocks, and pays a single fixed cost (i.e. a menu cost) to change all its prices. The goal of this section is to show how the firm's endogenous information choices are affected by the presence of menu costs, and how its information acquisition and price setting decisions are affected by the firm's product scope .

First, to show the interaction between menu costs and rational inattention frictions, I characterize a single-product firm's optimal decision rules. When information is costly, the rationally

of price changes are independent of the number of products since the frequency of price changes are exogenously given in the model. Similarly, sticky and noisy information models imply that the attentiveness to aggregate conditions is independent of firms' number of products since information acquisition is exogenous in those models. Observational cost models such as Alvarez et al. (2015) also imply the independence results since it is assumed that firms use steady-state policy rules which are not affected by aggregate shocks in the model.

[25] Alvarez et al. (2016) show that in menu cost models, the cumulative output response to a monetary shock increases in the number of products, N, and is given by $\frac{3N}{N+2}$.

[26] In Appendix B.1, I show that in a rational inattention model, the cumulative response of output to a monetary shock is only a function of firms' average subjective uncertainty about the monetary shock and decreases firms' number of products.

inattentive firm should decide how much to acquire new information about its marginal costs. Moreover, given menu costs, the firm decides whether it changes its price, based on its information set. I find that the firm's optimal price setting decisions exhibit the *wait-and-see* property: the more the firm is uncertain about the underlying shocks, the less it is likely to change its price. Since the firm optimally decides its subjective uncertainty, this implies that the timing of price changes has a selection feature such that it is more likely that the firm changes its price when it has more information and less uncertainty about the underlying shocks. This interaction of nominal and informational rigidities plays a key role in amplifying monetary non-neutrality in the dynamic general equilibrium model that I will discuss in the next section.

Second, given both menu costs and rational inattention frictions, I investigate how a two-product firm is different from a single-product firm in terms of its information and price setting decisions to highlight economies of scope motives in both price changes and information processing. I show that the two-product firm optimally chooses to be more informed about the aggregate shock than the single-product firm since information about the aggregate shock can be utilized for its pricing decisions for all goods. Moreover, due to the single fixed cost of price changes, the two-product firm changes all its prices at the same time. Then, the two-product economy will generate small price changes which are absent in the single-product economy. These two motives of economies of scope are also key elements for monetary non-neutrality in the general equilibrium model with multi-product firms.

3.1 A Rationally Inattentive Firm's Problem

Consider a multi-product firm that produces N goods, indexed by $j = 1, 2, \cdots, N$. The firm sets its price of good j, $p_{j,t}$, to match a (frictionless) optimal price, $p^*_{j,t}$.[27] Suppose its optimal price of good j consists of two components, a good-specific shock, $a_{j,t}$, and an aggregate shock, m_t:

$$p^*_{j,t} = a_{j,t} + m_t.$$

[27]Small letters denote log deviation from (firctionless) steady state. When we consider the firm as a monopolistically competitive producer, its optimal reset price of good j without any frictions is a constant markup over the marginal cost: $P^*_{j,t} = \mu \times MC_{j,t}$. Then the log deviation of optimal price is that of marginal cost: $p^*_{j,t} = mc_{j,t}$.

I assume that both shocks follow random walk processes:[28]

$$
\begin{aligned}
a_{j,t} = & \quad a_{j,t-1} + \varepsilon^a_{j,t}, \quad \varepsilon^a_{j,t} \sim N(0, \sigma_a^2) \ \text{ for } \ j = 1, 2, \cdots, N, \\
m_t = & \quad m_{t-1} + \varepsilon^m_t, \quad \varepsilon^m_t \sim N(0, \sigma_m^2),
\end{aligned}
$$

where $\varepsilon^a_{j,t}$ and ε^m_t are independent and identically distributed. A flow loss of the firm in profits is the sum of the distance between its price of each good and the (frictionless) optimal price:[29]

$$
B \sum_{j=1}^{N} \left(p_{j,t} - p^*_{j,t} \right)^2,
$$

where B captures the concavity of the firm's profit function with respect to each prices.

This firm is rationally inattentive. At the beginning of each period, the firm has to choose how precisely it wants to observe its current set of (frictionless) optimal prices subject to a cost of information processing . Formally, at time t, the firm chooses a set of signals about both good-specific and aggregate shocks from a set of available signals, $\mathcal{S}_t = \{\mathcal{S}^a_{j,t}\}_{j=1}^N \cup \mathcal{S}^m_t$, such that

$$
\begin{aligned}
\mathcal{S}^a_{j,t} &= \{a_{j,t} + \eta_{j,t}\xi^a_{j,t} : \eta_{j,t} \geq 0, \ \xi^a_{j,t} \sim N(0,1)\}, \ \text{ for } \ j = 1, 2, \cdots, N, \\
\mathcal{S}^m_t &= \{m_t + \eta_{m,t}\xi^m_t : \eta_{m,t} \geq 0, \ \xi^m_t \sim N(0,1)\},
\end{aligned}
$$

where $\{\xi^a_{j,t}\}_{j=1}^N$ and ξ^m_t are the firm's rational inattention errors. Let S^{t-1} be the firm's information set at the beginning of period t before it receives new signals about its frictionless optimal prices. At each time t, given S^{t-1}, the firm chooses a set of its signals $s^a_{j,t} \in \mathcal{S}^a_{j,t}$ for $j = 1, 2, \cdots, N$, and $s^m_t \in \mathcal{S}^m_t$ subject to the cost of information processing. Then, the firm's information set evolves as follows:

$$
S^t = S^{t-1} \cup s_t,
$$

where $s_t = \{\{s^a_{j,t}\}_{j=1}^N, s^m_t\}$. This implies that the firm does not forget information over time. This "no-forgetting constraint" implies that the current information choice has a continuation value and thus the optimal information choice is a solution of a dynamic information acquisition problem.

I assume that the cost of information is linear in Shannon's mutual information function. The

[28] Random walk process of the underlying shock is a common assumption in the menu costs literature since it simplifies the firm's problem by making it chooses its price gaps, which are defined by the difference between the frictionless and the actual prices. See, among others, Barro (1972), Tsiddon (1993), and Alvarez and Lippi (2014).

[29] While I take this as an assumption, this loss function can also be derived as a second order approximation to a twice differentiable profit function around the non-stochastic steady state.

firm pays ψ units of its (per-good) revenue for every bit of expected reduction in uncertainty, where uncertainty is measured by entropy. Denote this cost as $\psi\mathcal{I}(s_t; \{p^*_{j,t}\}_{j=1}^N | S^{t-1})$, which will be defined later in this section. At each period, based on its optimal choice of information, the firm chooses whether to change its prices. I assume that the firm can change all its prices by paying a single fixed cost, θ. This cost is independent of the number of prices the firm changes.[30]

Figure 2 shows the timing of events for the firm's problem: at the beginning of period t, the firm starts with a priori information set, S^{t-1}, and forms a prior over its optimal prices at that time. Then it chooses a new set of signals, s_t, subject to the cost of information processing and updates its information set, S^t. Given this time t information set, the firm decides whether to change its prices and pays the fixed cost θ or to wait until the next period without changing its prices. If the firm decides to change its prices, it also chooses how much it changes the prices. Thus, the firm optimally chooses a set of signals about the underlying shocks and prices ($p_{j,t}$) over time, contingent on the evolution of its beliefs.

Formally, the firm's problem is as follows:[31]

$$\begin{aligned}\min_{\{\{p_{j,t}\}_{j=1}^N, s_t\}_{t=0}^\infty} \mathbb{E}\Bigg[\sum_{t=0}^\infty \beta^t \Bigg(& \underbrace{B\sum_{j=1}^N \left(p_{j,t} - p^*_{j,t}\right)^2}_{\textbf{loss from suboptimal prices}} + \underbrace{\theta \mathbf{1}_{\{\text{for any } j,\ p_{j,t}\neq p_{j,t-1}\}}}_{\textbf{cost of price changes}} \\ & + \underbrace{\psi\mathcal{I}\left(s_t; \{p^*_{j,t}\}_{j=1}^N | S^{t-1}\right)}_{\textbf{cost of information processing}} \Bigg) \Big| S^{-1}\Bigg] \end{aligned} \tag{2}$$

$$\begin{aligned} \text{s.t.} \quad & p^*_{j,t} = a_{j,t} + m_t, \quad \forall j = 1, 2, \cdots, N, \\ & S^t = S^{t-1} \cup s_t, \quad S^{-1} \text{ is given,} \end{aligned}$$

where $\mathbf{1}_{\{\text{for any } j,\ p_{j,t}\neq p_{j,t-1}\}}$ is an indicator function which is one if it changes any one of its prices.

Cost of Information Processing. The cost of information processing is linear in Shannon's mutual information function. Let $\mathcal{H}(X|Y)$ be a conditional entropy of a random variable of X given

[30] In Section E, I discuss some evidence of the firm-specific menu costs using the survey data. I also discuss the implications of adding product-specific menu costs in the model.

[31] Besides the existence of menu costs, this problem is different from the previous rational inattention models in LQG settings, such as Maćkowiak and Wiederholt (2015) or Pasten and Schoenle (2016), which solve the problem by assuming that the cost of information is not discounted and optimizing at the long-run steady-state for information structure. I assume that the firm discounts future costs of information at the same discount rate as their payoffs and solve the dynamic information acquisition problem. See, for instance, Afrouzi and Yang (2019), for the detailed discussion of solutions for the dynamic rational inattention problem in LQG setups.

knowledge of Y. The firm's flow cost of information at time t is $\psi\mathcal{I}(s_t; \{p^*_{j,t}\}_{j=1}^N | S^{t-1})$, where

$$\mathcal{I}(s_t; \{p^*_{j,t}\}_{j=1}^N | S^{t-1}) = \mathcal{H}(\{p^*_{j,t}\}_{j=1}^N | S^{t-1}) - \mathbb{E}[\mathcal{H}(\{p^*_{j,t}\}_{j=1}^N | S^t) | S^{t-1}]$$

is the reduction in uncertainty about its (frictionless) optimal prices that the firm experiences by observing the set of signals, s_t, given its prior information set, S^{t-1}, and ψ is the marginal cost of a bit of information.

Let $z^a_{j,t} \equiv var(a_{j,t}|S^t)$ and $z^m_t \equiv var(m_t|S^t)$ be the firm's subjective uncertainty about the j-good specific shock and that about the aggregate shock, respectively. Then, I can rewrite the cost of information processing at time t in terms of $\{z^a_{j,t}\}_{j=1}^N$ and $z^m_t \equiv var(m_t|S^t)$:

$$\begin{aligned}
\mathcal{I}\left(s_t; \{p^*_{j,t}\}_{j=1}^N | S^{t-1}\right) &= \sum_{j=1}^N \mathcal{I}\left(s^a_{j,t}; a_{j,t} | S^{t-1}\right) + \mathcal{I}\left(s^m_t; m_t | S^{t-1}\right) \\
&= \frac{1}{2}\left(\sum_{j=1}^N \log_2\left(\frac{z^a_{j,t-1} + \sigma^2_a}{z^a_{j,t}}\right) + \log_2\left(\frac{z^m_{t-1} + \sigma^2_m}{z^m_t}\right)\right),
\end{aligned} \tag{3}$$

where $\{z^a_{j,-1}\}_{j=1}^N$ and z^m_{-1} are given. The first equality follows from the fact that the underlying shocks are independent and the firm observes independent signals about them. The second equality holds since the firm observes Gaussian signals about the underlyinig shocks which are also Gaussian. Moreover, in this setup, I can rewrite the no-forgetting constraint, $S^t = S^{t-1} \cup s_t$, in terms of the firm's subjective uncertainty:

$$\begin{aligned}
&0 \leq z^a_{j,t} \leq z^a_{j,t-1} + \sigma^2_a \ \text{ for } \ j = 1, 2, \cdots, N, \\
&0 \leq z^m_t \leq z^m_{t-1} + \sigma^2_m.
\end{aligned}$$

This reformulation shows that the cost of information processing is directly related to how much each firm reduces its subjective uncertainty about the good-specific shocks and the aggregate shock, given their priors uncertainty about those shocks. If the marginal cost of information processing, ψ, is zero, the firm would like to choose zero subjective uncertainty about both underlying shocks. Since it is costly for the firm to reduce large amount of uncertainty about the underlying shocks when $\psi > 0$, it optimally chooses to observe less precise signals and to be optimally uncertain about the underlying shocks.

Recursive Formulation of the Firm's Problem. I reformulate the firm's problem (2) in a recursive form to characterize its optimal decision rules and simulate the model numerically. Let $x_{j,t} = p_{j,t} - \mathbb{E}[p^*_{j,t}|S^t]$ be the firm's *perceived* price gap of product j. Then, the loss from suboptimal prices can be decomposed into two components:

$$\mathbb{E}\left[\left(p_{j,t} - p^*_{j,t}\right)^2 \middle| S^t\right] = \underbrace{z^a_{j,t} + z^m_t}_{\substack{\text{contemporaneous loss from} \\ \textbf{imperfect information}}} + \underbrace{x^2_{j,t}}_{\substack{\text{contemporaneous loss from} \\ \textbf{nominal rigidities}}}, \tag{4}$$

where

$$z^a_{j,t} = \mathbb{E}\left[\left(a_{j,t} - \mathbb{E}_t[a_{j,t}|S^t]\right)^2 \middle| S^t\right], \quad z^m_t = \mathbb{E}\left[\left(m_t - \mathbb{E}_t[m_t|S^t]\right)^2 \middle| S^t\right]$$

are subjective uncertainty about the j-good specific shock and that about the aggregate shock, respectively.

On the one hand, if there is no informational cost, $\psi = 0$, then the firm chooses zero subjective uncertainty and thus there is no contemporaneous loss from imperfect information. In this case, the firm's problem is identical to the problem in a standard menu cost model with multi-product firms. On the other hand, if there is no menu cost, $\theta = 0$, then the firm can always adjust its prices freely and thus will choose to zero *perceived* price gaps, $x_{j,t} = 0$ for all $j = 1, 2, \cdots, N$. In this case, the firm's problem is identical to the problem in a standard rational inattention model with multi-product firms.

Notice that the perceived price gaps, $\{x_{j,t}\}$, are the firm's choice variables when it decides to change its prices. However, if the firm does not want to change its prices, then the perceived price gaps are stochastic variables which evolve according to

$$\mathbf{x}_t \sim N(\mathbf{x}_{t-1}, \Sigma_t),$$

where $\mathbf{x}_t = \{x_{1,t}, x_{2,t}, \cdots, x_{N,t}\}'$ and

$$\Sigma_t(j,k) = \begin{cases} z^m_{t-1} + \sigma^2_m - z^m_t & \text{if } j \neq k \\ z^a_{j,t-1} + \sigma^2_a - z^a_{j,t} + z^m_{t-1} + \sigma^2_m - z^m_t & \text{if } j = k. \end{cases} \tag{5}$$

Given (3), (4), and (5), I reformulate the firm's problem (2) in a recursive form with $2N + 1$

state variables and occasionally binding constraints:

$$V\left(\{x_{j,-1}\}_{j=1}^{N},\{z_{j,-1}^{a}\}_{j=1}^{N},z_{-1}^{m}\right)=\max_{\{\{z_j^a\}_{j=1}^N,z^m\}}\mathbb{E}\Bigg[\max\left\{V^{I}\left(\{x_j\}_{j=1}^{N},\{z_j^a\}_{j=1}^{N},z^m\right),V^{C}\left(\{z_j^a\}_{j=1}^{N},z^m\right)\right\}$$
$$-\frac{\psi}{2}\left(\sum_{j=1}^{N}\log_2\left(\frac{z_{j,-1}^{a}+\sigma_a^2}{z_j^a}\right)+\log_2\left(\frac{z_{-1}^{m}+\sigma_m^2}{z^m}\right)\right)\Bigg|S^{-1}\Bigg],$$
$$\text{s.t.}\qquad 0\le z_j^a\le z_{j,-1}^a+\sigma_a^2,\quad \forall j=1,2,\cdots,N,$$
$$0\le z^m\le z_{-1}^m+\sigma_m^2,$$

where

$$V^{I}\left(\{x_j\}_{j=1}^{N},\{z_j^a\}_{j=1}^{N},z^m\right)=-B\sum_{j=1}^{N}\left(x_j^2+z_j^a+z^m\right)+\beta V\left(\{x_j\}_{j=1}^{N},\{z_j^a\}_{j=1}^{N},z^m\right)$$
$$\text{with } \mathbf{x}\sim N\left(\mathbf{x}_{-1},\Sigma\right),\text{ and}$$
$$V^{C}\left(\{z_j^a\}_{j=1}^{N},z^m\right)=\max_{\{y_j\}_{j=1}^N}-B\sum_{j=1}^{N}\left(y_j^2+z_j^a+z^m\right)-\theta+\beta V\left(\{y_j\}_{j=1}^{N},\{z_j^a\}_{j=1}^{N},z^m\right).$$

Here $V^I(\{x_j\}_{j=1}^N,\{z_j^a\}_{j=1}^N,z^m)$ represents the firm's value of not changing its prices. Similarly, $V^C(\{z_j^a\}_{j=1}^N,z^m)$ is the firm's value of changing its prices.

3.2 Decision Rules

In this section, I describe key properties of the firm's optimal decision rules. First, because of the quadratic objective function and the symmetry of normal distribution, the value function is also symmetric around the null vector for the perceived price gaps. Second, given optimal choices of subjective uncertainty about the good-specific shocks ($\{z_{j,t}^a\}$) and that about the aggregate shock (z_t^m), the value function is decreasing in the absolute values of perceived price gaps. These two properties imply that given optimal choices of subjective uncertainty, the firm chooses to have zero perceived price gaps for all their goods whenever it decides to change its prices by paying the menu cost, θ. Then, the value function of the firm which changes its prices can be written as:

$$V^{C}\left(\{z_j^a\}_{j=1}^{N},z^m\right)=-B\sum_{j=1}^{N}\left(z_j^a+z^m\right)-\theta+\beta V\left(\{0\},\{z_j^a\}_{j=1}^{N},z^m\right).$$

Since the firm's problem is a non-convex optimization problem in its price setting decision and there are occasionally binding no-forgetting constraints for its choices of subjective uncertainty, it needs to be solved numerically.[32] Using the method of value function iteration, I solve the problems of two types of firms: a single-product firm and a two-product firm. I first investigate how the interaction between menu costs and rational inattention frictions affects the single-product firm by characterizing its optimal information acquisition and price setting decisions. Then, I show how economies of scope motives in both price setting and information processing affect the two-product firm by comparing with the single-product firm.

3.2.1 A Single-Product Firm

I first consider a single-product firm's optimal decision rules.[33] Here I drop the j-index because the firm produces only one product.

Optimal Information Acquisition. Optimal policy functions for subjective uncertainty about the good-specific shock are presented in Figure 3. Given the firm's prior subjective uncertainty (z^a_{t-1}), the firm chooses a high posterior uncertainty (z^a_t) when it believes its current price is not far from the frictionless optimal price. In particular, the right panel of Figure 3 shows that when its prior uncertainty is low enough and its prior price gap is close to zero, the no-forgetting constraint binds and the firm does not acquire new information. The amount of information acquisition increases in both the firm's prior uncertainty (z^a_{t-1}) and the distance between its current price and the the frictionless optimal price ($|x_{t-1}|$). In other words, the firm has a large incentive to collect and process information when the firm is quite uncertain about the realization of the underlying shocks and thinks that it is likely that it will need to change prices. This is because potential losses from mistakes in the price setting decisions are large if the firm thinks that it is likely that it will need to change its price. The firm could make wrong decisions either by paying the menu cost and changing the price when it was not supposed to do or by choosing not to change its price when it should.

Price-Setting Decision Given the Optimal Information Choices. After choosing optimal signals about the underlying shocks and forming the new information set, the firm decides whether

[32] The most computationally burdensome part is to compute firms' expected future values. When the firm does not change prices, its price gaps are stochastic variables that are jointly normally distributed with a mean vector, $\mathbf{x}_{-1}$, which is the firm's state variable, and a covariance matrix, Σ, which is its choice variable. Standard approximation methods for the transition probability of states, such as Tauchen (1986), are not applicable as approximation errors are quite large. I compute expected value of the firm' value functions using an explicit numerical integration technique. See Appendix C for the detailed description.

[33] To illustrate this example, I set $\theta = 0.0093, \psi = 0.007, \sigma_a = 0.0189$, and $\sigma_m = 0.0065$. These are parameters that I calibrate when I solve a general equilibrium single-product model in the next section.

to change its price, based on the new information set. Due to the fixed menu cost of adjusting prices, the firm adopts S-s rules in setting its prices, like in standard menu cost models. There are adjustment thresholds (s, S) such that if the firm's *perceived* price gap is greater than S, it pays the fixed cost and decreases its price to the level of frictionless optimal. Similarly, the firm increases its price and collapses the perceived gap to zero when it believes its price gap is less than s. The adjustment thresholds are the firm's inaction bands. As we can see in the upper right panel of Figure 5, one interesting feature in this model is that the inaction bands are time-varying. Formally, let $\hat{x}_t$ be the firm's posterior belief about its perceived price gap *after* observing the new optimal signals and *before* changing its price at time t. Then,

$$\begin{aligned}\hat{x}_t &= p_{t-1} - \mathbb{E}[a_t + m_t|S^t] \\ &= x_{t-1} - \{\mathcal{K}_t^a(s_t^a - \mathbb{E}[a_t|S^{t-1}]) + \mathcal{K}_t^m(s_t^m - \mathbb{E}[m_t|S^{t-1}])\},\end{aligned}$$

where $\mathcal{K}_t^a$ and $\mathcal{K}_t^m$ are the optimal Kalman gains for the good-specific shock and for the aggregate shock, respectively. Higher value of the Kalman gains imply that the firm chooses to observe more precise signals about the underlying hsocks. Given the firm's optimal choice of subjective uncertainty, z_t^a and z_t^m, there exists an adjustment threshold $\tilde{x}(z_t^a, z_t^m) \geq 0$ such that

$$-\tilde{x}_t(z_t^a, z_t^m)^2 + \beta V\left(B\tilde{x}_t(z_t^a, z_t^m), z_t^a, z_t^m\right) = -\theta + \beta V\left(0, z_t^a, z_t^m\right).$$

The firm will change its price if $|\hat{x}_t| > \tilde{x}_t(z_t^a, z_t^m)$. Then, the perceived price gap at the end of period t, x_t, is

$$x_t = \begin{cases} \hat{x}_t & \text{if } |\hat{x}_t| \leq \tilde{x}_t(z_t^a, z_t^m) \\ 0 & \text{if } |\hat{x}_t| > \tilde{x}_t(z_t^a, z_t^m). \end{cases}$$

Figure 4 shows the inaction bands, $(-\tilde{x}_t(\cdot, z_t^m), \tilde{x}_t(\cdot, z_t^m))$, for the various values of z_t^m. Notice that the inaction bands in a myopic model ($\beta = 0$) are constant and given by $(-\sqrt{\theta/B}, \sqrt{\theta/B})$ since the firm's subjective uncertainty is no longer a state variable for the firm's problem. With $\beta > 0$, the inaction bands vary with the firm's subjective uncertainty. When the firm is more uncertain about the underlying shocks, the inaction bands are wider. This makes sense since when the firm is uncertain about the underlying shocks, it is optimal to wait and see until it gets more information about them. As a result, as shown in Figure 5, the firm is likely to change its price when its subjective uncertainty is low and thus the inaction bands are narrow.

The main implication of the interaction between menu costs and rational inattention frictions is that the firm is likely to be more informed about the underlying shocks when it changes its price than when it does not. As it will be clear in the next section, in a general equilibrium model with a large number of firms, this interaction leads to a selection effects of information processing such that price adjusters are more informed about the underlying shocks than non-adjusters.[34]

Another interesting characteristic of the firm's pricing rule is that the firm makes mistakes in both timing and size of price changes. Since the firm's pricing decision is based on its *belief* about the price gap ($p_t - \mathbb{E}[p_t^*|S^t]$) rather than the *true* price gap ($p_t - p_t^*$), the firm changes its price by a wrong amount when it decides to change. Moreover, it makes mistakes on the timing of its price changes. The firm changes its price when it is not supposed to do, or it does not adjust the price when it should. Figure 5, for example, shows that in period 20, the firm decreases its price when the true price gap (a red dashed line) is well-within the inaction bands while in period 30, the firm does not change its price when the true price gap is outside of the inaction bands.

3.2.2 A Two-Product Firm

Now, I consider the two-product firm's optimal information acquisition and price setting decision. In fact, the two-product firm shares the same characteristics about its optimal decision rules with the single-product firm that I discussed above. However, two interesting economies of scope motives emerge in the two-product firm's optimal choices—economies of scope in price changes through the menu cost technology and economies of scope in information processing through the rational inattention friction.

Economies of Scope in Price Changes. Figure 6 shows a two-product firm's information acquisition and price setting behavior in the model simulation. Like a single-product firm, the inaction bands of the two-product firm also depend on its subjective uncertainty. The main difference in the two-product firm's price setting decision is that the price change of one of its products depends on the perceived price gap of the other product. For example, the right upper and lower panels of

[34]Models with both menu costs and observational costs such as Alvarez et al. (2011) and Bonomo et al. (2019b) also have similar implication. In these models, a firm has to pay a fixed cost to acquire full-information about the underlying shocks and can change its price by paying a fixed menu cost. Optimal pricing rule implies that the firm only changes its price when it pays the observational cost. This is an extreme case of the selection effect of information processing in a sense that the firm has full information when it changes its price while it does not acquire new information al all otherwise. Gorodnichenko (2008) also shows that firms have an incentive to buy an additional signal prior to changing prices in a model with menu costs and endogenous information choice.

Figure 6 show that when the perceived price gap of the second product is large, the inaction bands for the first product are narrow. This implies that the timing of its price changes within the firm is synchronized and, more importantly there are both large and small price changes. This is called economies of scope in price changes from the menu cost technology. If the firm decides to pay the menu cost to change one of its prices, then the price change of the other products is free for the firm. As a result, given the same size of menu cost, the two-product firm is likely to change its prices more frequently than the single-product firm. Moreover, since the additional price change is free and thus there are many small price changes, the two-product firm changes its prices, on average, by a smaller amount than the single-product firm. As I show in Section 2.2, these results are consistent with the empirical finding 2 that firms producing more goods have more frequent but smaller price changes. As highlighted in the menu cost literature, the economies of scope in price changes will weaken selection effects of price changes by letting some price changes be random (e.g. Midrigan 2011, Alvarez and Lippi 2014). Then the weak selection effects of price changes in an economy with a large number of multi-product firms will lead to an amplified real effect of monetary shocks.

Economies of Scope in Information Processing. The two-product firm is also different from the single-product firm in terms of its optimal information acquisition about the aggregate shock. In particular, I find that given the same marginal cost of information processing, the two-product firm is more informed about the aggregate shock than the single-product firm. Since the firm's optimal prices for all goods are affected by the aggregate shock, the value of information about the aggregate shock will be higher if the firm produces more products. This relationship is clearly seen if we consider the firm's problem without menu costs ($\theta = 0$).[35] In this case, the firm's optimal subjective uncertainty about the aggregate shock satisfies the following FOC:

$$B \cdot N = \frac{\psi}{2 \log 2} \left(\frac{1}{z_t^m} - \beta \frac{1}{z_t^m + \sigma_m^2} \right), \tag{6}$$

where z_t^m is decreasing in the number of products, N.

With menu costs ($\theta > 0$), I also find economies of scope motive in information processing of the two-product firm. The average subjective uncertainty about the aggregate shock for the two-product firm is 25% smaller than that for the single-product firm. As I show in Section 2.1, this finding is consistent with the empirical finding 1 that firms that produce more goods have

[35] See Appendix B for the optimal solutions for the rationally inattentive firm's problem without menu costs.

better information about aggregate inflation. Since the two-product firm has lower subjective uncertainty about the aggregate shock than the single-product firm, it is more likely that the two-product firm responds more strongly to monetary policy shocks by learning about them more rapidly and therefore changing their prices more rapidly. In the economy with large number of firms, this scope motive in information processing is likely to act as a strong force to weaken monetary non-neutrality.

In sum, the two-product firm's information acquisition and price setting decisions show two economies of scope motives that work in opposite direction for monetary non-neutrality. The question is how quantitatively large each scope motive is. To draw the implications of firms' product scope for monetary non-neutrality in the model with both menu costs and rational inattention friction, we need to extend the model in a general equilibrium setup and discipline it with micro-evidence. This is the goal of the next section.

4 A Dynamic General Equilibrium Model

In this section, I extend the menu cost model with rationally inattentive multi-product firms of Section 3 to a dynamic general equilibrium model. The model is disciplined using micro-evidence of Section 2 and then used for quantitative analysis on the transmission of monetary shocks. There are two main focuses of this analysis. First, I investigate how the interaction between rational inattention and menu costs affects the distribution of firms' desired price changes as well as the distribution of subjective uncertainty. I show these two distributions are important determinants of monetary non-neutrality. Second, I compare the two-good version of the model with the one-good version of the model to study how multi-product pricing affects the real effects of monetary shocks through economies of scope motives in both information processing and price setting that I studied in the previous section.

4.1 Environment

The economy is populated by a representative household and a unit measure of monopolistically competitive firms. Each firm sells N products. I first discuss the household problem and then present the firm problem and define equilibrium.

Households. The representative household consumes a Dixit-Stiglitz aggregate consumption, C_t, of a basket of multiple goods $j \in \{1, 2, \cdots, N\}$ purchased from firms $i \in [0, 1]$, and supplies labor L_t to maximize the expected lifetime utility with a discount factor $\beta \in (0, 1)$.

The representative household's problem is

$$\max_{\{\{C_{i,j,t}\}_{j=1}^{N}, C_t, L_t, B_t\}_{t \geq 0}} \quad \mathbb{E}_0^f \left[\sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \right], \tag{7}$$

subject to

$$\int \Big(\sum_{j=1}^{N} P_{i,j,t} C_{i,j,t} \Big) di + B_t \leq R_{t-1} B_{t-1} + W_t L_t + \Pi_t, \quad \text{for all } t,$$

where

$$C_t = \left(\frac{1}{N} \sum_{j=1}^{N} C_{j,t}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}, \quad C_{j,t} = \left(\int \left(A_{i,j,t} C_{i,j,t} \right)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Here $\mathbb{E}_t^f [\cdot]$ is the full information rational expectation operator at time t. Since the main purpose of this paper is to study the effects of nominal rigidity and rational inattention among firms, I assume that the household is fully informed about all prices and wages. B_t is the demand for nominal bond and R_{t-1} is the nominal interest rate.[36] L_t is the labor supply of the household, W_t is the nominal wage, and Π_t is the aggregate profit from the firms. C_t is the aggregator over the consumption for differentiated goods and $C_{j,t}$ is an aggregator over the consumption of good j. $A_{i,j,t}$ is the quality of the good j produced by firm i. Higher $A_{i,j,t}$ increases the marginal utility of consumption for that good while it also increases the production cost for that good, as I describe below.[37] ε is the constant elasticity of substitution across different firms that produce the same good and γ is the constant elasticity of substitution across different goods.

The above formulation implies that a demand for good j produced by firm i is

$$C_{i,j,t} = A_{i,j,t}^{\varepsilon-1} \left(\frac{P_{i,j,t}}{P_{j,t}} \right)^{-\varepsilon} \left(\frac{P_{j,t}}{P_t} \right)^{-\gamma} C_t,$$

where P_t is the price of aggregate consumption bundle C_t, and $P_{j,t}$ is the price index for good j.

[36] Natural borrowing limits on the demand for nominal bond are imposed to rule out Ponzi schemes.

[37] The assumption that idiosyncratic shocks affect both the cost at which a good is sold and the household's marginal utility for the good is common in the menu cost literature to reduce the dimensionality of the state space and thus the computational burden. See, for instance, Midrigan (2011), Alvarez and Lippi (2014), Mongey et al. (2017), and Karadi and Reiff (2019).

These prices are given by

$$P_t = \left(\frac{1}{N} \sum_{j=1}^{N} P_{j,t}^{1-\gamma} \right)^{\frac{1}{1-\gamma}}, \quad P_{j,t} = \left(\int_0^1 \left(\frac{P_{i,j,t}}{A_{i,j,t}} \right)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}.$$

Firms. There is a measure one of firms, indexed by i, that operate in monopolistically competitive markets. Each firm produces N goods, indexed by j. Firms take wages and demands for their goods as given, and choose their prices $\{P_{i,j,t}\}_{j=1}^{N}$ based on their information set, S_i^t, at that time. After setting their prices, firms then hire labor from a competitive labor market and produce the realized level of demands that their prices induce with a production function for good j,

$$Y_{i,j,t} = \frac{1}{A_{i,j,t}} L_{i,j,t},$$

where $L_{i,j,t}$ is firm i's demand for labor for producing good j. Notice that higher quality products require extra labor input. I assume that shocks to $A_{i,j,t}$ are independently and identically distributed and the log of the j-good specific shock, $a_{i,j,t} \equiv \log(A_{i,j,t})$, follows a random walk process:

$$\log(A_{i,j,t}) = \log(A_{i,j,t-1}) + \varepsilon_{i,j,t}^{a}, \;\; \varepsilon_{i,j,t}^{a} \sim N(0, \sigma_a^2) \;\text{ for }\; j = 1, 2, \cdots, N.$$

Then, firm i's nominal profit from sales of all goods at prices $\{P_{i,j,t}\}_{j=1}^{N}$ is given by

$$\Pi_{i,t}(\{P_{i,j,t}, A_{i,j,t}, P_{j,t}\}_{j=1}^{N}, W_t, P_t, Y_t) = \sum_{j=1}^{N} \left(P_{i,j,t} - W_t A_{i,j,t} \right) A_{i,j,t}^{\varepsilon-1} \left(\frac{P_{i,j,t}}{P_{j,t}} \right)^{-\varepsilon} \left(\frac{P_{j,t}}{P_t} \right)^{-\gamma} Y_t, \tag{8}$$

where Y_t is the nominal aggregate demand.

At each period, firms optimally decide their prices and signals subject to costs of changing prices and of processing information. First, changing price entails a fixed cost, $\tilde{\theta}$. I express this cost as a fraction θ of the steady state frictionless revenue from selling one of N products. This cost is incurred when at least one price is changed and is independent of the total number of prices that the firm adjusts. Second, firms are rationally inattentive in a sense that they choose their optimal information set by taking into account the cost of obtaining and processing information. At the beginning of period t, firm i wakes up with its initial information set, S_i^{t-1}. Then it chooses optimal signals, $s_{i,t}$, from a set of available signals, $\mathcal{S}_{i,t}$, subject to the cost of information which is linear in Shannon's mutual information function. Denote $\tilde{\psi}$ as the marginal cost of information processing. Again, I express this marginal cost as a fraction ψ of the steady state frictionless revenue from selling one of N products. Firm i forms a new information set, $S_i^t = S_i^{t-1} \cup s_{i,t}$, and sets its new prices, $\{P_{i,j,t}\}_{j=1}^N$, based on that.

The firm i chooses a set of signals to observe over time $(s_{i,t} \in \mathcal{S}_{i,t})_{t=0}^{\infty}$ and a pricing strategy that maps the set of its prices at $t-1$ and its information set at t to its optimal price at any given period, $P_{i,j,t} : (\{P_{i,j,t-1}\}_{j=1}^N, S_i^t) \to \mathbb{R}$ where $S_i^t = S_i^{t-1} \cup s_{i,t} = S_i^{-1} \cup \{s_{i,\tau}\}_{\tau=0}^t$ is the firm's information set at time t. Then, the firm i's problem is to maximize the net present value of its life time profits given an initial information set:

$$\max_{\{s_{i,t} \in \mathcal{S}_{i,t}, \{P_{i,j,t}(\{P_{i,k,t-1}\}_{k=1}^N, S_i^t)\}_{j=1}^N\}_{t\geq 0}} \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \Lambda_t \left\{ \Pi_{i,t}(\{P_{i,j,t}, A_{i,j,t}, P_{j,t}\}_{j=1}^N, W_t, P_t, Y_t) \right.\right. \tag{9}$$
$$\left.\left. - \tilde{\theta}\mathbf{1}_{\{\text{for any } j,\ P_{i,j,t} \neq P_{i,j,t-1}\}} - \tilde{\psi}\mathcal{I}(s_{i,t}; \{\{A_{i,j,t}\}_{j=1}^N, W_t\} | S_i^{t-1}) \right\} \middle| S_i^{-1} \right]$$
$$\text{s.t.} \quad S_i^t = S_i^{t-1} \cup s_{i,t},$$

where $\Lambda_t = \frac{U_{c,t}/P_t}{U_{c,0}/P_0}$ and $\mathcal{I}(s_{i,t}; \{\{A_{i,j,t}\}_{j=1}^N, W_t\} | S_i^{t-1})$ is Shannon's mutual information function.[38]

Monetary Policy. For simplicity, I assume that the nominal spending must be equal to the money supply:

$$\int \Big(\sum_{j=1}^N P_{i,j,t} C_{i,j,t} \Big) di = P_t C_t = M_t, \tag{10}$$

and the log of money supply, $m_t \equiv \log(M_t)$, follows a random walk process:

$$m_t = m_{t-1} + \varepsilon_t^m, \quad \varepsilon_t^m \sim N(0, \sigma_m^2), \tag{11}$$

where ε_t^m is an independently and identically distributed normal disturbance.

Definition of Equilibrium. Given exogenous processes for idiosyncratic good-specific shocks $\{\{\{a_{i,j,t}\}_{j=1}^N\}_{i\in[0,1]}\}_{t\geq 0}$, a general equilibrium of the economy consists of an allocation for the representative household,

$$\Omega^H \equiv \left\{ C_t, \{C_{i,j,t}\}_{j=1}^N, L_t, B_t \right\}_{t=0}^{\infty},$$

[38]While I implicitly assume that firms' signals are informative only about the idiosyncratic good-specific shocks and aggregate nominal wage, this is an optimal signal structure after taking the second-order approximation to the firms' profit function as I do when solving the problem. Up to the second order approximation, only the aggregate nominal wage and the idiosyncratic shocks matter for the firm's profit function. Thus this assumption is without loss of generality.

an allocation for every firm $i \in [0,1]$ given the initial set of signals,

$$\Omega_i^F \equiv \left\{ s_{i,t} \in \mathcal{S}_{i,t}, \{P_{i,j,t}, L_{i,j,t}, Y_{i,j,t}\}_{j=1}^N \right\}_{t=0}^{\infty},$$

a set of prices $\left\{ \{P_{j,t}\}_{j=1}^N, P_t, R_t, W_t \right\}_{t=0}^{\infty}$, and a stationary distribution over firms' states such that

1. given the set of prices and $\{\Omega_i^F\}_{i\in[0,1]}$, the household's allocation solves its problem as specified in Equation (7) ;

2. given the set of prices and Ω^H, and the implied labor supply and output demand, firms' allocation solve their problem as specified in Equation (9) ;

3. given the set of prices, Ω^H, and $\{\Omega_i^F\}_{i\in[0,1]}$, $\{M_t \equiv P_t C_t\}_{t\geq 0}$ satisfies the monetary policy rule specified in Equation (11) ;

4. all markets clear: for all $t \geq 0$,

$$\begin{aligned} Y_{i,j,t} =& C_{i,j,t}, \qquad \text{for all } i \in [0,1] \text{ and } j = 1, 2, \cdots, N, \\ L_t =& \int \left(\sum_{j=1}^N L_{i,j,t} \right) di, \\ B_t =& 0\,; \end{aligned}$$

5. and, the stationary distribution is consistent with actions.

4.2 Computing the Equilibrium

I assume the representative household's preferences of the form $U(C,L) = \log(C) - L$. The log-utility implies that the intertemporal optimal condition relates the nominal interest rate to the law of motion of the aggregate demand. This enables to formulate monetary policy in terms of either the nominal interest rates or the aggregate nominal demand. Moreover, linear disutility in labor ensures that the nominal wage is proportional to the nominal aggregate demand, $W_t = P_t C_t = M_t$. This closely follows Golosov and Lucas (2007) and Midrigan (2011) and makes monetary shocks translate one-for-one into changes in the firms' nominal marginal cost.

Firms' profit function (8) implies that without any frictions in price setting and in information processing, firm i's frictionless optimal price of good j, $P_{i,j,t}^*$, is a constant markup over its nominal marginal cost:

$$P_{i,j,t}^{*} = \frac{\varepsilon}{\varepsilon - 1} W_t A_{i,j,t}.$$

Let $\mu_{i,j,t} = \bar{\mu} \frac{P_{i,j,t}}{P_{i,j,t}^{*}}$ be the firm i's true price gap for good j, which is a gap between the actual price and the frictionless optimal price. Here $\bar{\mu} = \frac{\varepsilon}{\varepsilon-1}$ is a non-stochastic steady state level of the price gap. Then, the firm i's nominal flow profit at time t can be written as a function of the set of these price gaps:

$$\sum_{j=1}^{N} \left(\mu_{i,j,t} - 1\right) \left(\mu_{i,j,t}\right)^{-\varepsilon} \left(W_t\right)^{1-\varepsilon} \left(P_{j,t}\right)^{\varepsilon-\gamma} \left(P_t\right)^{\gamma} Y_t. \tag{12}$$

To solve the firm's problem, I take the second order approximation to the firms' profit function (12) and derive firms' losses from sub-optimal pricing following the rational inattention literature (e.g. Maćkowiak and Wiederholt 2009, Pasten and Schoenle 2016, Afrouzi 2019b).[39] I assume that the set of available signals, $\mathcal{S}_{i,t}$, has the following properties. First, the firm chooses $N+1$ independent signals for each shock, implying that paying attention to aggregate conditions and paying attention to good-specific idiosyncratic conditions are separate activities. Second, each signal is Gaussian. Third, all noise in signals is idiosyncratic and independent.[40] The second order approximation reduces the state space of the problem from an entire distribution to its covariance matrix. Moreover, since the signals that firms choose are Gaussian and the objective function is quadratic, it enables us to focus on a Gaussian posterior. Under these assumptions, each firm's problem is virtually identical to that studied in Section 3.1.

I compare two economies with $N = 1$ and $N = 2$.[41] I use the method of value function iteration to solve firms' problem and simulate the economy with a large number of firms for a long period to make sure that the economy reaches a stationary distribution over firms' states. I describe the computational details in Appendix C.

[39] See Appendix A for the detailed derivation of the second-order approximation and the recursive representation for firms' problem.

[40] See Appendix E for the discussion on the implications and limitations of these assumptions.

[41] Since the number of state variables are increasing linearly in the number of products as $2N+1$, a model with more than two-product is hard to solve. A two-product economy is also considered as a baseline in Midrigan (2011) and Karadi and Reiff (2019). Moreover, in the New Zealand survey data, the average of main product's share of total output value is about 60% excluding single-product firms. This implies that a two-product firm is a good benchmark for a multi-product firm. In Section 5.1, I solve models with any arbitrary number of products under some simplifying assumptions.

4.3 Calibration and Parameterization

In the numerical exercises, I set the monthly discount factor to $\beta = 0.96^{(1/12)}$, which implies a real interest rate of 4 percent. I set the elasticity of substitution across firms to be four ($\varepsilon = 4$) which matches the firms' average markup of 33% in the survey data.[42] Moreover, I assume the elasticity of substitution between goods is the same as that across firms ($\gamma = 4$). However, the value of γ plays little role since there are no common good-specific shocks in the model.

I calibrate the standard deviation of the log difference in nominal demand, σ_m, to match the standard deviation of the growth rate of nominal GDP in New Zealand, 0.0065.[43] There are three key model parameters which should be calibrated: the size of menu costs (θ), the size of marginal costs of information processing (ψ), and the size of idiosyncratic good-specific shocks (σ_a). I assume that the marginal cost of processing a bit of information in both one-good and two-good versions of the model are the same.[44] I calibrate those three parameters to match the median frequency of price changes (once a year), the median size of absolute price changes (6.5%), and slope of backcast errors about the growth rate of aggregate nominal GDP on the number of products (-0.026) observed in the survey data.[45]

The latter is obtained by regressing the firms' backcast errors about the growth rate of nominal GDP on firms' number of products.[46] The estimate shows that controlling for firm and manager characteristics as well as industry controls, the backcast errors decrease by 0.026 percentage points when firms produce one more good.[47] The model counterpart measure is calculated by taking difference between average backcast errors about the growth rate of nominal demand in single- and two-product models. The three moments exactly identify the three key model parameters, and as the Table 5 shows, all the targeted moments are well matched.

Table 6 shows the calibrated and assigned parameters in both single-product and two-product

[42] This value is in the middle of 3 and 7, the elasticity of substitution parameters in Midrigan (2011) and Golosov and Lucas (2007), respectively. It directly affects the slope of profit curves, and thus the estimates of menu costs and the standard deviation of good-specific shocks, without altering main findings.

[43] I restrict the sample to post-1991 New Zealand data since New Zealand has explicitly conducted monetary policy by targeting inflation in that time period.

[44] As shown in Pasten and Schoenle (2016), an alternative assumption for cost of information processing, such as a constant loss per good from imperfect information, does not change the main findings in this paper.

[45] Parker (2017) also finds the median frequency of price changes is once a year in New Zealand using *2010 Business Operations Survey* data carried out by Statistics New Zealand while the data does not provide a quantitative measure of the size of price changes.

[46] The backcast errors about nominal GDP growth rate are from Wave #4 of the survey. In the survey, firms' managers are asked about the current inflation and the real GDP growth rate in New Zealand. I take a summation of both measures to obtain firms' perceived growth rate of nominal GDP in the economy.

[47] See Appendix Table A.5 in for the regressions results with and without controls.

models. The baseline parameterization implies a menu cost of 0.93 percent of steady-state (per good) revenue in the single-product model. Given the average frequency of price changes, the overall cost of price adjustment in the single-product model is around 0.078 percent of steady-state revenue. Similarly, the overall cost of price adjustment in the two-product model is around 0.071 percent of steady-state revenue. These values are smaller than estimates in the previous literature, which often used U.S. data, since the average size of absolute price changes in New Zealand is small.[48] The calibrated standard deviations of the idiosyncratic good-specific shocks are around 2 percent per month in both models, which are about three times bigger than the standard deviation of the monetary policy shock. The calibrated marginal cost of information processing is 0.7 percent of steady-state (per good) revenue. This value implies about 0.2 of an average Kalman gain on the signal about idiosyncratic shocks in both model (see Table 7). This is equivalent to a quarterly gain of 0.38, which is close to the estimate of 0.45 in Coibion and Gorodnichenko (2015), which uses the U.S. Survey of Professional Forecasters data.

4.4 Simulation

In this section, I show the simulation results for two models with a large number of firms facing both idiosyncratic good-specific shocks and the monetary policy shock.[49] I show that the two-product model has a more realistic distribution of price changes compared to the single-product model by generating both large and small price changes. Also, I emphasize two distributional characteristics that will be important to understand how big are the real effects of the monetary shocks in both models, which I extensively investigate in the next section. First, there is a selection in information processing: price adjusters have better information about both idiosyncratic and aggregate shocks than non-adjusters. In particular, I show that selection in information processing about idiosyncratic shocks endogenizes a leptokurtic distribution of desired price changes, which acts as a force to weaken selection effects of price changes. Second, multi-product firms value more information about the monetary policy shock than the single-product firms.

Distribution of Price Changes. Figure 7 shows a distribution of price changes in the single-product model and that in a two-product model. As a comparison, I also plot the distribution

[48] For example, Levy et al. (1997) find menu costs of 0.7 percent of revenue, while Zbaracki et al. (2004) find price adjustment costs as large as 1.2 percent. Stella (2018) estimates the total cost of changing prices between 0.3% and 1.3% of revenues. Moreover, The baseline calibration in Midrigan (2011) implies a menu cost of 0.34 percent of revenue while it is 0.15 percent of revenue in Karadi and Reiff (2019).

[49] A simulation algorithm for two-product model is presented in Appendix D.

of price changes in the menu cost only model with single-product firms (yellow bar). All three models are calibrated to match the same frequency and size of price changes. In the baseline single-product model (blue bar), there are no small price changes since price changes occur when firms believe that their price is outside of its inaction bands. However, the Kurtosis of the distribution in the baseline single-product model is higher than that in the menu cost only model and there is a relatively small fraction of firms around the inaction bands in the baseline model compared to the only menu cost model.[50] Notice that firms in the baseline model have different inaction bands depending on their subjective uncertainty while firms in the only menu cost model have the same inaction bands (black vertical line). The heterogeneity in firms' subjective uncertainty makes the distribution of price changes in the baseline model more dispersed than that in the menu cost only model.

In contrast to the single-product model, the baseline two-product model generates both small and large price changes. This is because of economies of scope in menu cost technology: when a two-product firm believes that one of its price is far away from its perceived optimal price, the firm pays a fixed menu cost to change its price. Since the additional price changes are free after paying this menu cost, the firm also changes the price of the other product even if it is still very close to the perceived optimal price. Thus the economy with two-product firms can have a large fraction of small price changes and a higher kurtosis of price changes.[51] This economies of scope in menu cost technology weakens selection effects of price changes, which act as a strong force to reduce monetary non-neutrality in a standard menu cost model such as Golosov and Lucas (2007).

Selection in Information Processing and Endogenous Leptokurtic Distribution of Price Gaps. Table 7 shows another important characteristics about firms' optimal information choices. The second and third row compare the average Kalman gains of firms that adjust their prices to those of firms that do not adjust their prices. The Kalman gains represent how much firms put their weight on the new information relative to their prior estimates. When firms' signals are perfectly telling about the true underlying shocks, the Kalman gain is one; while in this model firms optimally

[50] The red vertical lines are the average of inaction bands across all firms in the baseline single-product model. Since I calibrate both models to have the same frequency and size of price changes, the average inaction bands in both models are very similar.

[51] Standard menu cost models with two-product firms also can generate small price changes through the same economies of scope motive (e.g. Midrigan, 2011; Bhattarai and Schoenle, 2014; Alvarez and Lippi, 2014). However, the baseline two-product model has a more dispersed distribution of price changes than the menu cost only models with two-product firms since, again, firms' optimal inaction bands are a function of their subjective uncertainty. Figure A.4 shows a comparison of the distribution of price changes in the baseline two-product model with that in the menu cost only model with two-product firms.

choose not to be perfectly informed about the shocks due to the cost of information, the Kalman gain is less than one. Thus, the average Kalman gains can be interpreted as the average degree of firms' attentiveness to the underlying shocks. I find that there is a selection in information processing; price adjusters are more informed about both idiosyncratic and aggregate shocks than price non-adjusters. The price adjusters' average Kalman gains in the signals about idiosyncratic shocks are about three times bigger than those of price non-adjusters. As shown in Appendix Figure A.3, the distributions of price adjusters' subjective uncertainty about both shocks are more concentrated than that of price non-adjusters' subjective uncertainty. The price adjusters also put more weight on new information about the aggregate shock than price non-adjusters. These findings are true in both the single- and the two-product models, implying that selection in information processing operates regardless of the number of products in the model.

This selection in information processing is due to the interaction between firms optimal information and pricing decisions. As shown in Section 3.1, firms' optimal information acquisition policy is affected by their belief about their price gaps. If a firm believes that its price is far away from its optimal level and close to the inaction bands, the potential losses from mistakes in pricing decisions would be very large. This makes the firm process more information about the shocks to reduce the losses. After the realization of shocks, this firm is more likely to be a price adjuster since its prior price gap is close to the inaction bands. For this reason, on average, the price adjusters in the economy are better informed about the underlying shocks than the price non-adjusters.

This new selection mechanism in information processing about idiosyncratic shocks has an important implication about the distribution of desired price changes: it *endogenously* generates a leptokurtic distribution of firms' perceived desired price changes.[52] Figure 8 shows the distributions of the perceived and true price gaps in the single-product model. First, the blue line is a prior distribution about firms' perceived price gap. At the beginning of period, all firms believe that their prices are within their inaction bands and there is a lot of zero perceived price gaps, which were adjusted at the previous period. This implies that the *prior* distribution of firms' perceived price gaps is very concentrated around zero and has a high Kurtosis. After being hit by Gaussian idiosyncratic shocks, the distribution of true price gaps in the economy is Gaussian (red dashed line). If firms have perfect information about their true optimal prices, their pricing decisions would be based on their true price gaps. Since the distribution of the true price gaps

[52] If a firm has a price gap of x % and it is free to change its price, then it would change by $-x$%. I use "price gaps" and "desired price changes" interchangeably.

is Gaussian, as in the standard single-product menu cost model, there would be large selection effects of price changes: an expansionary monetary shock triggers a lot of large price increases, it offset a mass of large price decreases.

However, in the model with both menu costs and informational costs, firms are rationally inattentive about their true optimal prices. Firms all choose their optimal Gaussian signals and update their estimates of price gaps, but they do not do so in the same way. Firms which think that their price gap is well-within their inaction bands and who think it is unlikely that they will need to change prices have little incentive to collect much new information: they choose to remain quite uninformed and update the estimates of their price gaps with a large weight on their (imprecise) priors. In contrast, firms that think they are close to the boundaries of their inaction regions have a high incentive to collect information and therefore choose to become more informed. Since the distribution of priors of firms' perceived price gaps is very concentrated around zero, this selection in information processing makes the distribution of posteriors of the perceived price gaps also *leptokurtic*. This implies a small selection effects of price changes in my baseline model because the rationally inattentive firms' pricing decisions are based on their posterior of perceived price gaps. Thus, the endogenous leptokurtic distribution will act as a strong force to amplify monetary non-neutrality in the general equilibrium model that I show in the following subsection. Previous studies of standard menu cost models often assume exogenously the leptokurtic distribution of idiosyncratic shocks to weaken selection effects of price change (e.g. Gertler and Leahy, 2008; Midrigan, 2011; Vavra, 2013; Karadi and Reiff, 2019; Baley and Blanco, 2019). Unlike these studies, due to selection in information processing about the idiosyncratic shocks, my baseline model with both rational inattention and menu costs can generate the leptokurtic distribution endogenously even if the distribution of underlying shocks is Gaussian.[53]

Value of Information about the Aggregate Shock. Given both idiosyncratic shocks and aggregate shocks, which shocks do firms pay more attention to? Maćkowiak and Wiederholt (2009) show that rationally inattentive firms process more information about an idiosyncratic shock rather than an aggregate shock since the former is more volatile than the latter. Optimal attention allocation in the rational inattention model implies that firms have a large incentive to allocation their attention in more volatile shocks. This is also true in the the current models with single- and two-product firms. The first row of Table 7 shows in both single- and two-product models,

[53]This endogenous leptokurtic distribution of perceived price gaps is also present in the two-product model as shown in Appendix Figure A.5.

the average Kalman gains across all firms for the idiosyncratic shocks are lager than those for the aggregate shock since the idiosyncratic shocks are more volatile than the aggregate shock.

However, the amount of information processing about the aggregate shock is different in the single- and the two-produce model. As I document in Section 3.1, the value of information about the aggregate shock is higher for the two-product firms than the single-product firms since the firms' frictionless optimal prices for all goods are affected by the aggregate shock. Table 7 shows that the two-product firms are more informed about the aggregate shocks than the single-product firms. The Kalman gains for aggregates shocks are about twice in the two-product model than in the single-product model. This implies that firms in the two-product economy will be more responsive to the monetary policy shock than firms in the single-product economy as they are more informed about it.

4.5 Real Effects of Monetary Policy Shocks

In this subsection, I show that monetary non-neutrality in the one-good version of the model is nearly as large as that in the Calvo sticky price model, while it decreases in the two-good version of the model. To show this, I take the calibrated models and hit them with one standard deviation shock to monetary policy. Figure 9 shows the impulse responses of output in the one-good and the two-good versions of the model. I also show the impulse responses in the standard menu cost model with single-product firms and that in the Calvo sticky price model.

The output response to an one standard deviation monetary policy shock in the standard menu cost model is small and short-lived. The half-life of output response is only 2 months. This is a well-known fact in this model: there are large selection effects of price changes, which act as a strong force to reduce monetary non-neutrality (e.g. Golosov and Lucas, 2007). In my one-good version of the baseline model with both menu costs and rational inattention however, the real effects of monetary policy shocks are large and persistent. The impact response increases by 60% and the cumulative output responses, which are defined as an area under the impulse responses of output, are about seven times bigger than those in the standard menu cost model. In fact, this large real effect is comparable to that in the Calvo sticky price model.

However, the large real effects are reduced in the two-good version of the baseline model. The cumulative output responses in the two-good version of the model is 20% smaller than those in the one-good version of the model, and the half-life of output responses also decreases from 7 months in the single-product model to 6 months in the two-product model. Interestingly, as shown in

Figure 7, the implied kurtosis of price changes is higher in the two-good version of the model than in the one-good version of the model. This suggests that in this model with both rational inattention and menu costs, the ratio of kurtosis to the frequency of price changes might not be a sufficient statistic for the output response to a monetary shock, which is derived by Alvarez et al. (2016).[54]

How do the real effects change when I shut down each friction at a time? Consider the economy without informational costs, which is coincides to the standard menu cost models (e.g. Golosov and Lucas, 2007, Midrigan, 2011, Alvarez and Lippi, 2014). Appendix Figure A.6 shows the output responses in the pure menu cost models with single- and two-product firms. The real effects of monetary policy shocks are larger in the two-good version of the menu cost model than in the one-good version of the menu cost model. Since economies of scope of menu cost technology generate many small price changes, selection effects of price changes are small in the two-good version of the model. The weak selection effects lead to the larger real effects in the two-good version of the menu cost model than the one-good version of the model.

Next, I assume that firms are rationally inattentive, but free to adjust their prices without menu costs. This model coincides with the pure rational inattention models (e.g. Maćkowiak and Wiederholt, 2009, Pasten and Schoenle, 2016). Appendix Figure A.7 shows that the output responses in the pure rational inattention models with single- and two-product firms. The output effects are larger in the one-good version of the rational inattention model than in the two-good version of the model. As I show in Appendix B.1, in the pure rational inattention model with multi-product firms, firms' subjective uncertainty about monetary shocks decrease in their number of products. Moreover, I show that the cumulative response of output to a monetary shock is only function of their subjective uncertainty about it. Since firms with a greater number of products have better information and smaller uncertainty about the monetary shocks, they respond more strongly to the monetary shocks by learning about them more rapidly and therefore changing their prices more rapidly. Thus, the real output effects of monetary policy shocks decrease in

[54]The reason the rational inattention models are outside the class of models studied by Alvarez et al. (2016) is imperfect information about monetary policy shocks. In their model, the real effects are calculated by the output responses to a once and for all unexpected monetary shock, which is perfectly observed by firms. Also, after the monetary shock, firms use the same decision rule used in the steady state. In the rational inattention model, however, firms do not have perfect information about the monetary policy shocks, and their optimal policy rule depends on their uncertainty about the monetary shocks. More importantly, firms' optimal information acquisition decision is affected by their product scope. As I show in the following subsection, this makes two-product firms more informed about the monetary shock than single-product firms. Thus, monetary non-neutrality in the two-product model is smaller than that in the single-product model even if kurtosis of the distribution of price changes is higher in the two-product model. This imperfect information about the monetary shocks breaks the application of the sufficient statistics derived by Alvarez et al. (2016).

the number of products.

4.6 Inspecting the Mechanisms

In this subsection, I investigate the key mechanisms behind the results of monetary non-neutrality in the baseline model. To this end, I start from the standard menu cost model with single-product firms, such as Golosov and Lucas (2007), and consider counterfactual models by adding core elements of the baseline model. I discuss five main mechanisms; three of them have been studied in the previous literature while two of them are new in this paper. Figure 10 shows how each counterfactual model is related to the underlying mechanisms which I discuss here in detail.

Endogenous Leptokurtic Distribution. The first model (1A) is the standard menu cost model with single-product firms such as, Golosov and Lucas (2007). In this model, firms have perfect information about both idiosyncratic and monetary shocks. As I discussed above, this model implies small and short-lived real effects of monetary shocks due to large selection effects of price changes (see the black solid line in Figure 11). For comparison with other counterfactual models, I normalize the impact response of output and the cumulative output responses in this model as one.

In the next model (1B), I assume that the single-product firms have perfect information about the monetary shock, but are rationally inattentive to their idiosyncratic good-specific shock. Since firms choose their optimal signals about the idiosyncratic shock, selection in information processing about the idiosyncratic shock, which I discussed in the previous section, makes endogenously the distribution of firms' desired prices be leptokurtic. This implies that price selection effects are small because there are only small fraction of firms around the inaction bands. Thus, the real effects of monetary policy shocks in this counterfactual model are larger than those in the menu cost only model with single-product firms. As shown in Figure 11, the impact output effect in the model (1B) increases by 31% compared to that in the menu cost only model with single product firms.[55] This mechanism is new in the literature; the interaction between menu costs and rational inattention generates the endogenous leptokurtic distribution of desired price changes, which amplifies the real effects of monetary shocks in a non-trivial way.

[55]Since firms have perfect information about monetary policy shocks, and the aggregate demand follows a random walk process, they immediately observe and respond to the shocks if their prices are around the adjustment margins. It makes the real effects of monetary shocks in this counterfactual model also be short-lived.

Imperfect Information about Monetary Policy Shocks. Next, I assume that the single-product firms are not only rationally inattentive to the good-specific shock and choose their optimal signals about it, but informationally constrained about monetary policy shocks. I assume that all single-product firms are *exogenously* given a signal about the monetary policy shocks. The signal has the same precision with the steady-state average precision of signals in the one-good version of the baseline model (1D). In other words, firms in this counterfactual economy have the same degree of attentiveness to the monetary policy shocks with the average firms in the baseline one-good version of the model where all information choices are endogenous. This counterfactual model (1C) captures the role of imperfect information about monetary policy shocks for monetary non-neutrality. This mechanism has been widely studied in the literature (e.g. Lucas, 1972; Woodford, 2003; Maćkowiak and Wiederholt, 2009). Figure 11 shows that this channel has the most important role for amplifying the real effects of monetary policy shocks. The cumulative output effects are seven times bigger in this model than those in the standard menu cost model.

Selection in Information Processing about Monetary Policy Shocks. Now, I assume that the single-product firms choose their optimal signal about the monetary policy shocks rather than receive an exogenous signal. This model (1D) is the baseline one-good version of the model that I studied in the previous section. The comparison of this model to the model (1C) captures the role of selection in information processing about the monetary policy shocks. Notice that firms in both models (1C) and (1D), *on average*, acquire and process the same amount of information about the monetary policy shocks. While all firms have the same amount of information about the monetary shocks in the counterfactual model (1C), price adjusters to have better information about the monetary shocks than non-adjusters in the baseline one good version of the model (1D). This implies that firms that change their prices following the monetary shocks in the baseline model will adjust more strongly and learn quickly about the shocks compared to the price adjusting firms in the model (1C). Thus, the real effects of monetary shocks are smaller in this baseline one-good version of the model than those in the model (1C) with exogenous information about the monetary shocks. Figure 11 shows that the cumulative output responses in the baseline one-good version of the model are 20 % smaller than those in the model (1C). This mechanism is also new in the literature.

Economies of Scope in Price Setting and Information Processing. Lastly, I consider the baseline two-good version of the model. The two-good version of the model entails economies of scope motives in both price setting and information processing that I discussed in the previous section. Notice that both economies of scope motives work in the opposite directions for monetary non-neutrality. Economies of scope in price setting make selection effects of price changes weak in the standard menu costs model and amplify the real effects of monetary policy shocks. On the other hand, in the scope economy in information processing, firms with greater number of products have better information about the monetary policy shocks, and thus the real effects of monetary policy shocks decrease in the number of products firms produce. Due to these opposite forces, it is not clear the implications of multi-product pricing for monetary non-neutrality in the model with both rational inattention and menu costs. Figure 11 shows that the cumulative output effects of monetary shocks decrease by 20% in the two-product model than in the single-product model. This implies that in the calibrated model, economies of scope in information processing act a strong force to reduce monetary non-neutrality.

5 Extension and Additional Evidence

In this section, I study an extension of the baseline model to study the robustness of the results about monetary non-neutrality. I also provide two kinds of empirical evidence that support the key predictions of the baseline model.

5.1 Models with a Large Number of Products

In the Section 4.5, I show that the real effects of monetary policy shocks decrease in the two-good version of the model compared to the one-good version of the model. In this section, I show that this implication of multi-product pricing for monetary non-neutrality can be extended to the models with arbitrary many number of products. The main computational challenge for solving the model with more than two products is that the number of state variables increases linearly in the number of products. To simplify the analysis, I make two assumptions. First, I assume that firms choose how much process information about the underlying shocks *as if* they do not have menu costs. Second, given menu costs, firms choose their prices based on that information, but they are *myopic* in a sense that they do not care about the continuation value of their current pricing decisions. One limitation of taking these assumptions is that it eliminates the interesting interaction between rational inattention and menu costs; under these assumptions, all firms have the same information set about the underlying shocks. However, since it simplifies the model analysis by eliminating the state variables, but keeps the core of the baseline model, I analyze the implications of the multi-product pricing for monetary non-neutrality under these assumptions.[56]

Figure 12 shows that the cumulative responses of output to a monetary policy shock in the simplified models with various number of products. I calibrate each model with different number of products to have the same size and frequency of price changes. I normalize the cumulative output response in the one-good version of the menu cost only model as one. In the menu cost only models (red line with circles), the cumulative output effects increase in firms' number of products. In the models with both rational inattention and menu costs (blue line with diamonds), the real effects decrease in the number of products, but converge to the menu cost only models with large number of products. As the number of products increases in the model, firms' subjective uncertainty about monetary policy shocks decreases and converges to zero, implying firms have almost perfect information about the monetary policy shocks.[57] Again, this implies that the ratio of kurtosis to the frequency of price changes might not be the sufficient statistic for monetary non-neutrality in the models with both rational inattention and menu costs. I also consider the model that exactly matches the distribution of number of products across firms in the New Zealand survey data. The black dashed line shows that the cumulative output effect of that model is larger than that in the one-good version of the menu cost only model, but smaller than that in the Calvo sticky price model.[58] This implies that my main conclusion about the relationship between monetary non-neutrality and firms' product scope can be extended to the model with a large number of products.

5.2 Additional Evidence

In this subsection, I show two kinds of additional evidence that support the key predictions of the baseline model. First, I show that the empirical distribution of desired price changes has a fat-tail

[56]For example, as shown in Appendix Figure A.8, the backcast errors about the growth rate of nominal GDP decreases in the number of products. This stems from the economies of scope in information processing in rational inattention models with multi-product firms. Moreover, kurtosis of the distribution of price changes increases in the number of products and converges to the value of three, which is consistent with the implications of menu cost models with multi-product firms.

[57]In Appendix B.1, I show this negative relationship between the number of products and firms' subjective uncertainty about monetary policy shocks.

[58]The median number of products in the New Zealand survey data is 7 excluding retail and wholesale trade firms. Interestingly, the cumulative output responses in this model are similar to the model with firms that produce the median number of products.

using the New Zealand survey data. Second, I provide evidence of the wait-and-see rule in firms' price setting decision. In the survey data, firms with greater uncertainty are more likely to delay their price changes.

Evidence on Leptokurtic Distribution of Desired Price Changes. One of the main results of this paper is to show that selection in information processing about idiosyncratic good-specific shocks endogenously leads to a fat-tail distribution of desired price changes. I find that this result is empirically consistent with what we observe in the survey data. In the second wave of the survey data, firms' managers were asked how much they would like to change the price of their main product if it was free to change its price in three months. The answer gives firms' desired price changes in three months. To construct a model consistent measure of desired price changes, I define an inflation-adjusted desired price changes as the gap between the desired price changes and their inflation expectations in three months. Appendix Figure A.9 shows that the distribution of desired price changes has much mass near zero, while some desired price changes are very far away from zero. The distribution of desired price changes exhibits excess kurtosis around 5, implying that the survey supports the fat-tail distribution of desired price changes. The baseline model with both rational inattention and menu costs endogenously captures this distributional characteristic without an assumption that the distribution of idiosyncratic good-specific shocks is leptokurtic.[59]

Subjective Uncertainty and the Duration of Price Changes. Another main characteristic of firms' pricing rule is that firms' optimal inaction bands depend on their subjective uncertainty about the underlying shocks. When firms are more uncertain, the inaction bands are wider, implying that the wait-and-see effects are present in firms' optimal price setting decision. I directly test this implication using the second wave of the survey data. Firms in this wave were asked to assign probabilities (from 0-100) to the different outcomes for growth rates of unit sales of their main product over the next 12 months. I calculate the standard deviation, which is a measure of firms' subjective uncertainty, surrounding firms' sales forecast using the implied probability distribution. I regress the duration of firms' expected next price changes on their subjective uncertainty about future sales growth. Appendix Table A.6 shows that firms that have greater uncertainty

[59]Previous literature on menu costs often assumes that this fat-tail distribution of idiosyncratic shocks to weaken selection effects of price changes. Midrigan (2011) supports this assumption by providing evidence of excess kurtosis in the distribution of markup gaps in U.S. retail data.

expect longer duration of next price changes. This finding is consistent with the prediction of the baseline model. When firms are more uncertain about their fundamentals, they are reluctant to change their prices. Instead, firms want to wait and see until they acquire more information to resolve their own uncertainty about their fundamentals.

6 Conclusion

Understanding the nature of firms' expectations formation and price setting behavior has been an active area of research in monetary economics. The first part of this paper uses a firm-level survey from New Zealand to show how firms' product scope is related to their expectations formation and price setting behavior. I find two stylized facts: firms with a greater number of products have both 1) better information about aggregate inflation, and 2) more frequent but smaller price changes.

The second part of the paper then builds a dynamic general equilibrium menu cost model with rationally inattentive multi-product firms to study the aggregate implications for monetary non-neutrality. In this model, the interaction between nominal and informational rigidities gives rise to a novel selection effect of information processing: price adjusters have better information about the underlying shocks than non-adjusters since firms with higher subjective uncertainty are less likely to change prices and want to wait and see until acquiring more information and resolving their uncertainty. I show that this new selection effect leads to a fat-tail distribution of firms' desired price changes, and thus weakens selection effects of price changes. As a result, the real effects of monetary policy shocks in the one-good version of the model are nearly as large as those in the Calvo model. Finally, I show that in the two-good version of the model, the cumulative output effects decrease by 20% than the one-good version of the model due to the strong economies of scope motive in information processing.

References

Abel, Andrew B. and Janice C. Eberly, "The effects of irreversibility and uncertainty on capital accumulation," *Journal of Monetary Economics*, 1999, *44* (3), 339 – 377.

Afrouzi, Hassan, "Sticky Prices, Rational Inattention and Monetary Non-Neutrality," mimeo 2019.

_ , "Strategic Inattention, Inflation Dynamics and the Non-Neutrality of Money," mimeo 2019.

_ **and Choongryul Yang**, "Dynamic Rational Inattention and the Phillips Curve," mimeo 2019.

Alvarez, Fernando and Francesco Lippi, "Price Setting With Menu Cost for Multiproduct Firms," *Econometrica*, 2014, *82* (1), 89–135.

Alvarez, Fernando E., Francesco Lippi, and Luigi Paciello, "Monetary Shocks in Models with Inattentive Producers," *The Review of Economic Studies*, 10 2015, *83* (2), 421–459.

Alvarez, Fernando, Francesco Lippi, and Luigi Paciello, " Optimal Price Setting With Observation and Menu Costs," *The Quarterly Journal of Economics*, 11 2011, *126* (4), 1909–1960.

_ , _ , and _ , "Monetary Shocks in Models with Observation and Menu Costs," *Journal of the European Economic Association*, 05 2017, *16* (2), 353–382.

_ , Hervé Le Bihan, and Francesco Lippi, "The Real Effects of Monetary Shocks in Sticky Price Models: A Sufficient Statistic Approach," *American Economic Review*, October 2016, *106* (10), 2817–51.

Baley, Isaac and Andrés Blanco, "Firm Uncertainty Cycles and the Propagation of Nominal Shocks," *American Economic Journal: Macroeconomics*, January 2019, *11* (1), 276–337.

Barro, Robert J., "A Theory of Monopolistic Price Adjustment," *The Review of Economic Studies*, 01 1972, *39* (1), 17–26.

Bhattarai, Saroj and Raphael Schoenle, "Multiproduct Firms and Price-setting: Theory and Evidence from U.S. Producer Prices," *Journal of Monetary Economics*, 2014, *66*, 178 – 192.

Bloom, Nicholas, "The Impact of Uncertainty Shocks," *Econometrica*, 2009, *77* (3), 623–685.

_ , Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen J. Terry, "Really Uncertain Business Cycles," *Econometrica*, 2018, *86* (3), 1031–1065.

Boneva, Lena, James Cloyne, Martin Weale, and Tomasz Wieladek, "Firms' Price, Cost and Activity Expectations: Evidence from Micro Data," Discussion Papers 1905, Centre for Macroeconomics (CFM) February 2019.

Bonomo, Marco, Carlos Carvalho, Oleksiy Kryvtsov, Sigal Ribon, and Rodolfo Rigato, "Multi-Product Pricing: Theory and Evidence From Large Retailers in Israel," mimeo 2019.

_ , _ , René Garcia, and Vivian Malta, "Persistent Monetary Non-Neutrality in an Estimated Model with Menu Costs and Partially Costly Information," mimeo 2019.

Caplin, Andrew S. and Daniel F. Spulber, "Menu Costs and the Neutrality of Money," *The Quarterly Journal of Economics*, 11 1987, *102* (4), 703–725.

Carvalho, Carlos and Oleksiy Kryvtsov, "Price Selection," Technical Report 2018.

Coibion, Olivier and Yuriy Gorodnichenko, "Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts," *American Economic Review*, August 2015, *105* (8), 2644–78.

_ , _ , and Saten Kumar, "How Do Firms Form Their Expectations? New Survey Evidence," *American Economic Review*, 2018, *108* (9), 2671–2713.

_ , _ , and Tiziano Ropele, "Inflation Expectations and Firm Decisions: New Causal Evidence*," *The Quarterly Journal of Economics*, 09 2019. qjz029.

__ , __ , **Saten Kumar, and Jane Ryngaert**, "Do You Know That I Know That You Know...? Higher-Order Beliefs in Survey Data," Working Paper 24987, National Bureau of Economic Research September 2018.

Denti, Tommaso, "Games with Unrestricted Information Acquisition," Technical Report 2015. Mimeo.

Dixit, Avinash K and Robert S Pindyck, *Investment under uncertainty*, Princeton university press, 1994.

Fisher, Timothy C.G and Jerzy D Konieczny, "Synchronization of price changes by multiproduct firms: evidence from Canadian newspaper prices," *Economics Letters*, 2000, *68* (3), 271 – 277.

Frache, Serafin and Rodrigo Lluberas, "New information and inflation expectations among firms," BIS Working Papers 781, Bank for International Settlements April 2019.

Gagnon, Etienne, David López-Salido, and Nicolas Vincent, "Individual Price Adjustment along the Extensive Margin," *NBER Macroeconomics Annual*, 2013, *27* (1), 235–281.

Gertler, Mark and John Leahy, "A Phillips Curve with an Ss Foundation," *Journal of Political Economy*, 2008, *116* (3), 533–572.

Gilchrist, Simon, Jae W Sim, and Egon Zakrajšek, "Uncertainty, financial frictions, and investment dynamics," Technical Report, National Bureau of Economic Research 2014.

Golosov, Mikhail and Robert E. Lucas, "Menu Costs and Phillips Curves," *Journal of Political Economy*, 2007, *115*, 171–199.

Gorodnichenko, Yuriy, "Endogenous Information, Menu Costs and Inflation Persistence," Working Paper 14184, National Bureau of Economic Research July 2008.

Grasso, Adriana and Tiziano Ropele, "Firms' inflation expectations and investment plans," Temi di discussione (Economic working papers) 1203, Bank of Italy, Economic Research and International Relations Area December 2018.

Hellwig, Christian and Venky Venkateswaran, "Dispersed Information, Sticky Prices and Monetary Business Cycles: A Hayekian Perspective," Technical Report 2015.

Jung, Junehyuk, Jeong Ho (John) Kim, Filip Matějka, and Christopher A Sims, "Discrete Actions in Information-Constrained Decision Problems," *The Review of Economic Studies*, 03 2019. rdz011.

Kaihatsu, Sohei and Noriyuki Shiraki, "Firms' Inflation Expectations and Wage-setting Behaviors," Bank of Japan Working Paper Series 16-E-10, Bank of Japan July 2016.

Karadi, Peter and Adam Reiff, "Menu Costs, Aggregate Fluctuations, and Large Shocks," *American Economic Journal: Macroeconomics*, July 2019, *11* (3), 111–46.

Klenow, Peter J. and Benjamin A. Malin, "Microeconomic Evidence on Price-Setting.," in Benjamin M. Friedman and Michael Woodford, eds., *Handbook of Monetary Economics*, Vol. 3, Elsevier, 2010, pp. 231–284.

— **and Jonathan L. Willis**, "Sticky Information and Sticky Prices," *Journal of Monetary Economics*, 2007, *54*, 79 – 99.

Knotek, Edward S., "A Tale of Two Rigidities: Sticky Prices in a Sticky-Information Environment," *Journal of Money, Credit and Banking*, 2010, *42* (8), 1543–1564.

Kumar, Saten, Hassan Afrouzi, Olivier Coibion, and Yuriy Gorodnichenko, "Inflation Targeting Does Not Anchor Inflation Expectations: Evidence from Firms in New Zealand," *Brookings Papers on Economic Activity*, 2015, *2015* (2), 151–225.

Lach, Saul and Daniel Tsiddon, "Staggering and Synchronization in Price-Setting: Evidence from Multiproduct Firms," *The American Economic Review*, 1996, *86* (5), 1175–1196.

— **and** — , "Small price changes and menu costs," *Managerial and Decision Economics*, 2007, *28* (7), 649–656.

Letterie, Wilko and Øivind Anti Nilsen, "Price Changes - Stickiness and Internal Coordination in Multiproduct Firms," Technical Report 2016.

Levy, Daniel, Mark Bergen, Shantanu Dutta, and Robert Venable, "The Magnitude of Menu Costs: Direct Evidence From Large U. S. Supermarket Chains," *The Quarterly Journal of Economics*, 1997, *112* (3), 791–825.

Lucas, Robert E., "Expectations and the neutrality of money," *Journal of Economic Theory*, April 1972, *4* (2), 103–124.

Maćkowiak, Bartosz and Mirko Wiederholt, "Optimal Sticky Prices under Rational Inattention," *American Economic Review*, June 2009, *99* (3), 769–803.

— **and** — , "Business Cycle Dynamics under Rational Inattention," *The Review of Economic Studies*, 08 2015, *82* (4), 1502–1532.

Matějka, Filip, "Rationally Inattentive Seller: Sales and Discrete Pricing," *The Review of Economic Studies*, 10 2015, *83* (3), 1125–1155.

Midrigan, Virgiliu, "Menu Costs, Multiproduct Firms, and Aggregate Fluctuations," *Econometrica*, 2011, *79* (4), 1139–1180.

Mongey, Simon et al., "Market Structure and Monetary Non-Neutrality," in "2017 Meeting Papers" number 184 Society for Economic Dynamics 2017.

Nakamura, Emi and Jón Steinsson, "Five Facts about Prices: A Reevaluation of Menu Cost Models*," *The Quarterly Journal of Economics*, 11 2008, *123* (4), 1415–1464.

— **and** — , "Monetary Non-neutrality in a Multisector Menu Cost Model," *The Quarterly Journal of Economics*, 08 2010, *125* (3), 961–1013.

— **and** — , "Price Rigidity: Microeconomic Evidence and Macroeconomic Implications," *Annual Review of Economics*, 2013, *5* (1), 133–163.

Paciello, Luigi and Mirko Wiederholt, "Exogenous Information, Endogenous Information, and

Optimal Monetary Policy," *The Review of Economic Studies*, 09 2013, *81* (1), 356–388.

Parker, Miles, "Price-setting Behaviour in New Zealand," *New Zealand Economic Papers*, 2017, *51* (3), 217–236.

Pasten, Ernesto and Raphael Schoenle, "Rational Inattention, Multi-product Firms and the Neutrality of Money," *Journal of Monetary Economics*, 2016, *80*, 1–16.

Sims, Christopher A., "Implications of Rational Inattention," *Journal of Monetary Economics*, 2003, *50* (3), 665 – 690. Swiss National Bank/Study Center Gerzensee Conference on Monetary Policy under Incomplete Information.

_ , "Rational inattention and monetary economics," in "Handbook of monetary economics," Vol. 3, Elsevier, 2010, pp. 155–181.

Steiner, Jakub, Colin Stewart, and Filip Matějka, "Rational Inattention Dynamics: Inertia and Delay in Decision-Making," *Econometrica*, 2017, *85* (2), 521–553.

Stella, Andrea, "Economies of scope in price setting: A moment inequalities estimation," *Journal of Monetary Economics*, 2018.

Stevens, Luminita, "Coarse Pricing Policies," *The Review of Economic Studies*, 07 2019. rdz036.

Tanaka, Mari, Nicholas Bloom, Joel M. David, and Maiko Koga, "Firm performance and macro forecast accuracy," *Journal of Monetary Economics*, 2019.

Tauchen, George, "Finite state markov-chain approximations to univariate and vector autoregressions," *Economics Letters*, 1986, *20* (2), 177 – 181.

Tsiddon, Daniel, "The (Mis)Behaviour of the Aggregate Price Level," *The Review of Economic Studies*, 1993, *60* (4), 889–902.

Vavra, Joseph, " Inflation Dynamics and Time-Varying Volatility: New Evidence and an Ss Interpretation," *The Quarterly Journal of Economics*, 09 2013, *129* (1), 215–258.

Woodford, Michael, "Imperfect Common Knowledge and the Effects of Monetary Policy," in Philippe Aghion, Roman Frydman, Joseph Stiglitz, and Michael Woodford, eds., *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, Princeton University Press, 2003, pp. 25–58.

_ , "Information-Constrained State-Dependent Pricing," *Journal of Monetary Economics*, 2009, *56*, S100–S124.

Zbaracki, Mark J., Mark Ritson, Daniel Levy, Shantanu Dutta, and Mark Bergen, "Managerial and Customer Costs of Price Adjustment: Direct Evidence from Industrial Markets," *The Review of Economics and Statistics*, 2004, *86* (2), 514–533.

Tables and Figures

Table 1: Number of Products and Knowledge about Aggregate Inflation

	(1)	(2)	(3)	(4)
Panel A. Dependent variable: Absolute value of actual minus firm-reported inflation in prior 12 months				
log(number of products)	-0.326**	-0.214***	-0.588***	-0.251***
	(0.145)	(0.060)	(0.149)	(0.060)
Observations	591	580	446	439
R-squared	0.339	0.800	0.345	0.899
Panel B. Dependent variable: Willingness to pay for professional inflation forecasts				
log(number of products)	7.664***	3.503***	7.207**	3.850**
	(2.931)	(1.210)	(2.949)	(1.661)
Observations	381	367	326	317
R-squared	0.202	0.657	0.273	0.705
Firm-level controls	Yes	Yes	Yes	Yes
Industry fixed effects		Yes		Yes
Manager controls			Yes	Yes

Notes: This table reports results for the Huber robust regression. Dependent variables are the absolute value of firm errors about past 12 month inflation from Wave #1 survey (Panel A) and firms' willingness to payment for professional forecaster's forecasts about future inflation from Wave #4 (Panel B). Firm-level controls include log of firms' age, log of firms' employment, foreign trade share, number of competitors, firms' beliefs about price difference from competitors, and the slope of the profit function. Industry fixed effects include dummies for 14 sub-industries excluding retail and wholesale trade sectors. Manager controls include the age of the respondent (each firm's manger), education, income, and tenure at the firm. Sample weights are applied to all specifications. Robust standard errors (clustered at the 3-digit Australian and New Zealand Standard Industrial Classification (ANZ SIC) level) are reported in parentheses. ***, **, * denotes statistical significance at 1%, 5%, and 10% levels respectively. See Section 2.1 for details.

Table 2: Number of Products and Information Updates

	(1)	(2)	(3)	(4)	(5)
Dependent variable: Posterior belief about aggregate inflation (p_i)					
Prior (μ_i)	0.357***	0.197***	0.203**	0.335**	0.542***
	(0.025)	(0.068)	(0.077)	(0.125)	(0.054)
$N_i^{\text{product}} \times \mu_i$		0.186**	0.068*	0.063*	0.475***
		(0.075)	(0.037)	(0.035)	(0.124)
N_i^{product}		-0.578*	-0.332**	-0.313**	-1.145*
		(0.315)	(0.130)	(0.136)	(0.641)
N_i^{product}		$\mathbf{1}_{\{N \geq 4\}}$	$\log(N)$	$\log(N)$	1-output share of main product
Firm size control				Yes	Yes
Observations	130	130	130	130	411
R-squared	0.367	0.410	0.401	0.395	0.512

Notes: This table reports Huber robust regressions of the posterior point prediction of the 12-month-ahead forecasts of inflation on the prior, i.e. the point prediction implied by the reported probability distribution for the future inflation. The prior is the belief of a firm before the firm is presented with additional information. The posterior is the belief of a firm after the firm is presented with additional information. Fixed effects for source of information are included but not reported. Column (2) includes a dummy variable which is 1 if firms' number of products is greater than or equal to 4, and its interaction with the prior. Column (3) and (4) include log of firms' number of products and its interaction with the prior. Column (5) includes 1-the share of total production value for main product and its interaction with the prior. In Column (4) and (5), I add log of firms' employment and its interaction with the prior as controls. Robust standard errors (clustered at the 3-digit ANZ SIC level) are reported in parentheses. ***, **, * denotes statistical significance at 1%, 5%, and 10% levels respectively. See Section 2.1.2 for details.

Table 3: The Treatment Effects of Information about RBNZ Inflation Target on Firms' Choice

	No controls for firm characteristics		Controls for firm characteristics	
	Number of products		Number of products	
	4 or less	More than 4	4 or less	More than 4
	(1)	(2)	(3)	(4)
Change in expectations immediately after information treatment				
One-year ahead inflation	-0.612**	0.345	-0.672*	0.281
	(0.275)	(0.367)	(0.373)	(0.251)
Forecast error in firm-level outcome				
Wage growth	0.227	-0.058	0.053	-0.120
	(0.204)	(0.059)	(0.228)	(0.078)
Investment growth	-3.196**	-0.252	-4.969*	-0.932
	(1.597)	(0.882)	(2.782)	(0.991)
Employment growth	-2.467*	0.394	0.148	-0.415
	(1.411)	(1.395)	(2.187)	(1.572)
Price change	-1.267***	-0.206	-1.299***	-0.123
	(0.258)	(0.131)	(0.228)	(0.190)

Notes: The table shows estimates of the treatment effect of providing information about the inflation target of the Reserve Bank of New Zealand on firms with a smaller number of products (columns (1) and (3)) and on firms with a greater number of products (column (2) and (4)). N stands for the number of products that firms produce. Firm characteristics, such as log of firms' age, log of firms' employment, number of competitors, and the slope of the profit function, are included in column (3) and (4). Influential observations are identified as observations that move the estimation by more than 0.5 of the standard error. These observations are excluded. Robust standard errors (clustered at the 3-digit ANZ SIC level) are reported in parentheses. ***, **, * denotes statistical significance at 1%, 5%, and 10% levels respectively. See Section 2.1.2 for details.

Table 4: Number of Products and Duration and Size of Price Changes

	(1)	(2)	(3)	(4)
Panel A. Dependent variable: Duration of expected next price changes				
log(number of products)	-0.266*	-0.253**	-0.435***	-0.597***
	(0.159)	(0.121)	(0.156)	(0.143)
Observations	587	578	445	442
R-squared	0.419	0.557	0.445	0.511
Panel B. Dependent variable: Size of expected next price changes				
log(number of products)	-0.048	-0.260***	-0.058	-0.281***
	(0.094)	(0.070)	(0.099)	(0.088)
Observations	576	575	431	426
R-squared	0.021	0.599	0.055	0.499
Firm-level controls	Yes	Yes	Yes	Yes
Industry fixed effects		Yes		Yes
Manager controls			Yes	Yes

Notes: This table reports results for the Huber robust regression. Dependent variables are the duration of expected next price changes from Wave #1 (Panel A) and the (absolute) size of expected next price changes from Wave #1 survey (Panel B). Firm-level controls include log of firms' age, log of firms' employment, foreign trade share, number of competitors, and firms' beliefs about price difference from competitors. Industry fixed effects include dummies for 14 sub-industries excluding retail and wholesale trade sectors. Manager controls include the age of the respondent (each firm's manger), education, income, and tenure at the firm. Sample weights are applied to all specifications. Robust standard errors (clustered at the 3-digit ANZ SIC level) are reported in parentheses. ***, **, * denotes statistical significance at 1%, 5%, and 10% levels respectively. See Section 2.2 for details.

Table 5: Data and Model Moments

	Data	Single-product model	Two-product model
Median (absolute) size of price changes	0.0656	0.0656	0.0656
Median frequency of price changes	0.0833	0.0833	0.0833
Slope of the backcast error curve	-0.026	-0.027	

Notes: The table presents moments of the data and simulated series from the single- and two-product models parameterized at the baseline values in Table 6. To get the slope of the backcast error curve, I regress the absolute value of firm errors about past 12 month nominal GDP growth rate from Wave #4 survey on the number of products each firm produces. Regression results are reported in Table A.5 in appendix. See Section 4.3 for details.

Table 6: Calibration and Assigned Parameters

	Single-product model	Two-product model
Panel A. Calibrated parameters		
Menu cost (θ)	0.0093	0.0342
Information cost (ψ)	0.0070	0.0070
Standard deviation of idiosyncratic shocks (σ_a)	0.0189	0.0212
Standard deviation of monetary policy shocks (σ_m)	0.0065	0.0065
Panel B. Assigned parameters		
Time discount factor (β)	0.9966	0.9966
Elasticity of substitution across firms (ε)	4.0	4.0
Elasticity of substitution between goods (γ)		4.0

Notes: The table presents the baseline parameters for the general equilibrium models with single- and two-product firms. Panel A shows the calibrated parameters which match the three key moments shown in Table 5. Panel B shows the assigned parameters. See Section 4.3 for details.

Table 7: Average Kalman Gains in Models

	Single-product model		Two-product model	
Average Kalman gains	Signal about good-specific shocks ($\mathcal{K}^a$)	Signal about monetary shocks ($\mathcal{K}^m$)	Signal about good-specific shocks ($\mathcal{K}^a$)	Signal about monetary shocks ($\mathcal{K}^m$)
All firms	0.219	0.094	0.231	0.124
- Price adjusters	0.556	0.276	0.618	0.411
- Price non-adjusters	0.188	0.078	0.196	0.098

Notes: The table presents average Kalman gains across firms in the baseline single- and two-product models. Column (1) and (3) show the average Kalman gains on the signal about the idiosyncratic good-specific shocks in the single-product model and those in the two-product model, respectively. Column (2) and (4) show the average Kalman gains on the signal about the monetary policy shock in the single-product model and those in the two-product model, respectively.

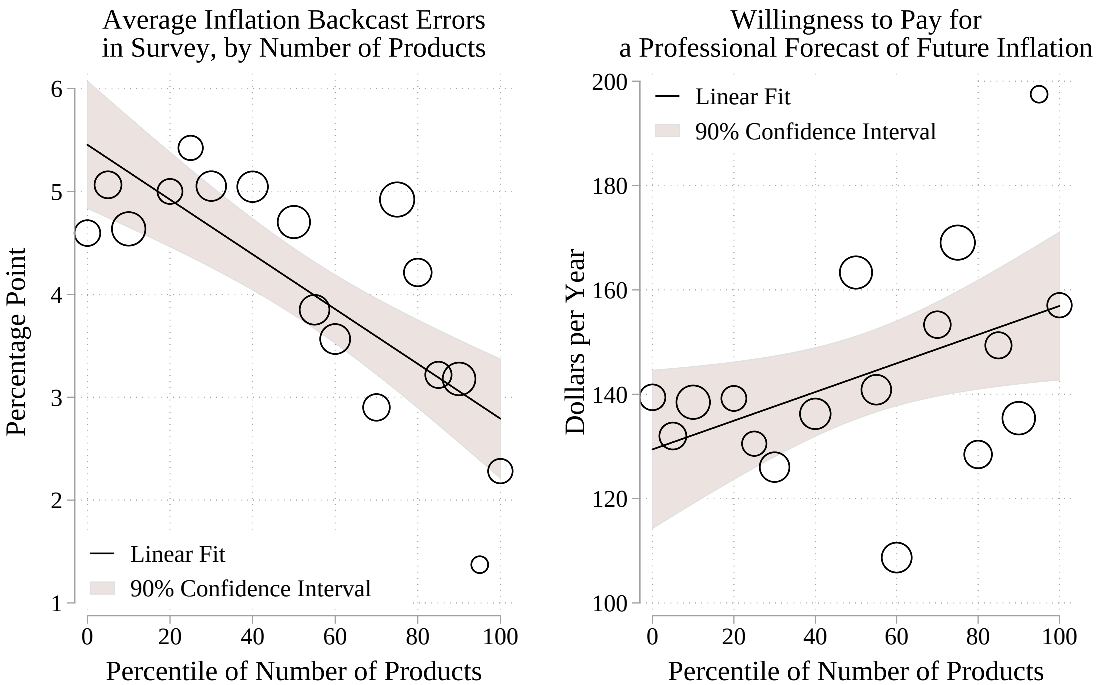


Figure 1: Number of Products and Attentiveness to Aggregate Inflation

Notes: The left panel plots percentile of firms' number of products versus the average of firm backcast errors about past 12 month inflation within each percentile. The right panel plots the percentile of firms' number of products versus the average of willingness to pay for a professional forecast of future inflation. The willingness to pay is measured from answers to the following question in Wave #4 survey: *"How much would you pay per year to have access to a monthly magazine of professional forecasts of future inflation?"* Black lines are linear fitted lines and shaded areas are 90% confidence intervals. The size of bins represents average size of employment of firms in each percentile.

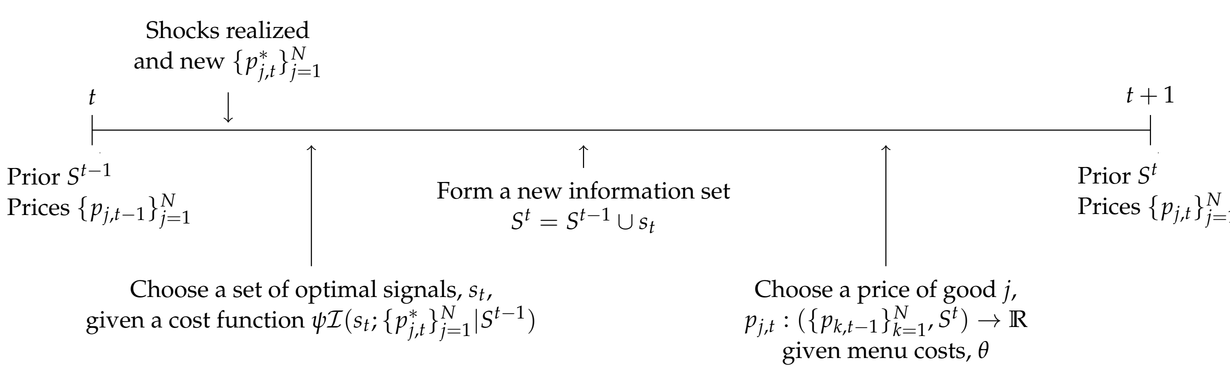


Figure 2: Timing of Events for a Firm's Problem

Notes: This figure shows a sequence of events in each period of the model. See Section 3.1 for details.

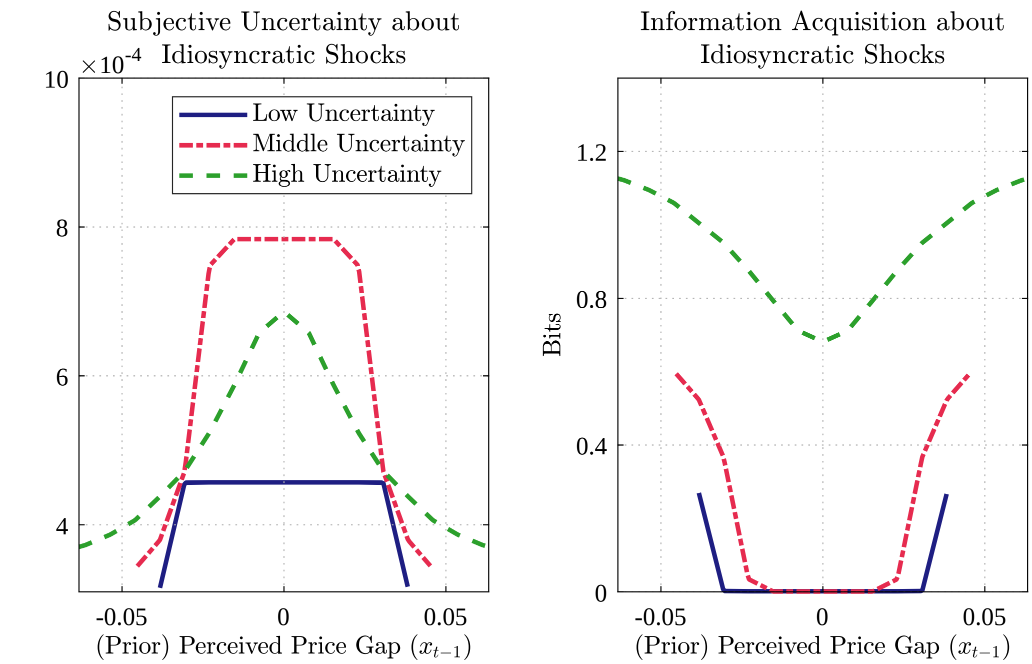


Figure 3: Subjective Uncertainty and Information Acquisition about an Idiosyncratic Shock

Notes: The left panel plots a single-product firm's optimal decision rule for subjective uncertainty about a good-specific shock, $z_t^a(x_{t-1}, z_{t-1}^a, z_{t-1}^m)$, when $z_{t-1}^m = 0.00016$. The right panel plots the implied amounts of information acquisition from the optimal choice of subjective uncertainty about the good-specific shock.

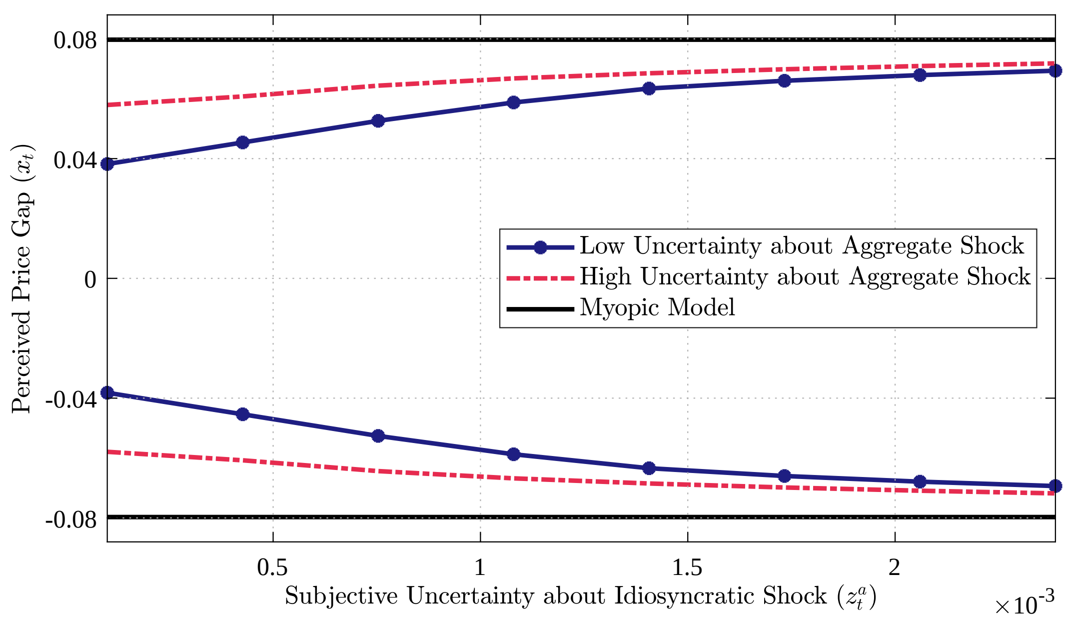


Figure 4: Inaction Bands of a Single-Product Firm by Subjective Uncertainty

Notes: This figure shows inaction bands of a single-product firm as a function of its subjective uncertainty. Different lines represent the inaction bands with different levels of subjective uncertainty about the aggregate shock (z_t^m). Black lines are the inaction bands of a myopic firm whose discount factor is zero. Since this myopic firm does not care about a continuation value of information, the subjective uncertainty is not its state variable, which leads the inaction bands of this firm to be constant.

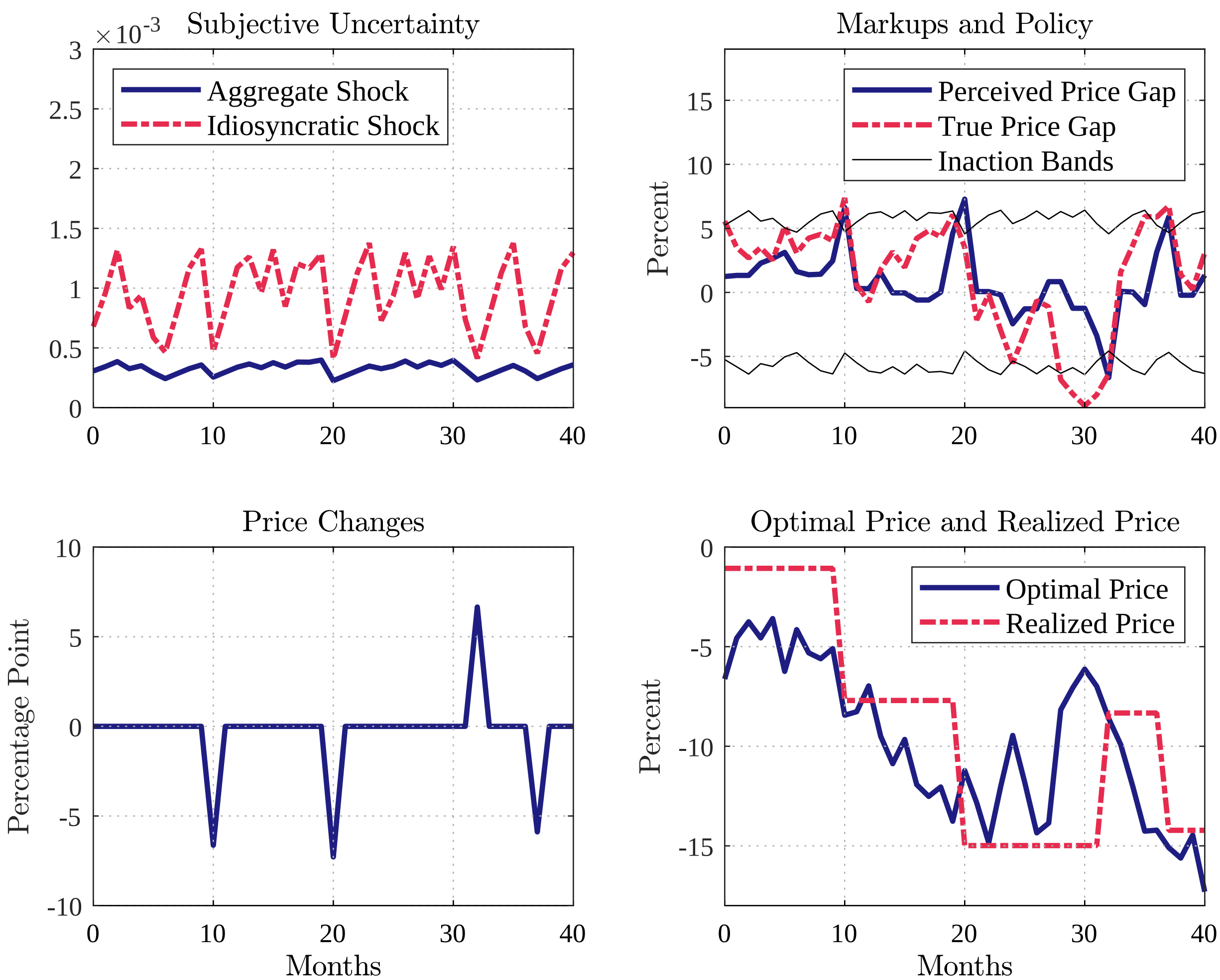


Figure 5: A Single-Product Firm in Model Simulation

Notes: The upper left panel plots a single-product firm's subjective uncertainty about both a good-specific shock (red dash-dot line) and an aggregate shock (blue solid line). The upper right panel plots the firm's perceived price gap (x_t), its true price gap under perfect information, and inaction bands. The firm changes its price when the perceived price gap is out of the inaction bands. The lower left panel plots these price changes. The lower right panel plots realized actual prices and optimal prices under perfect information.

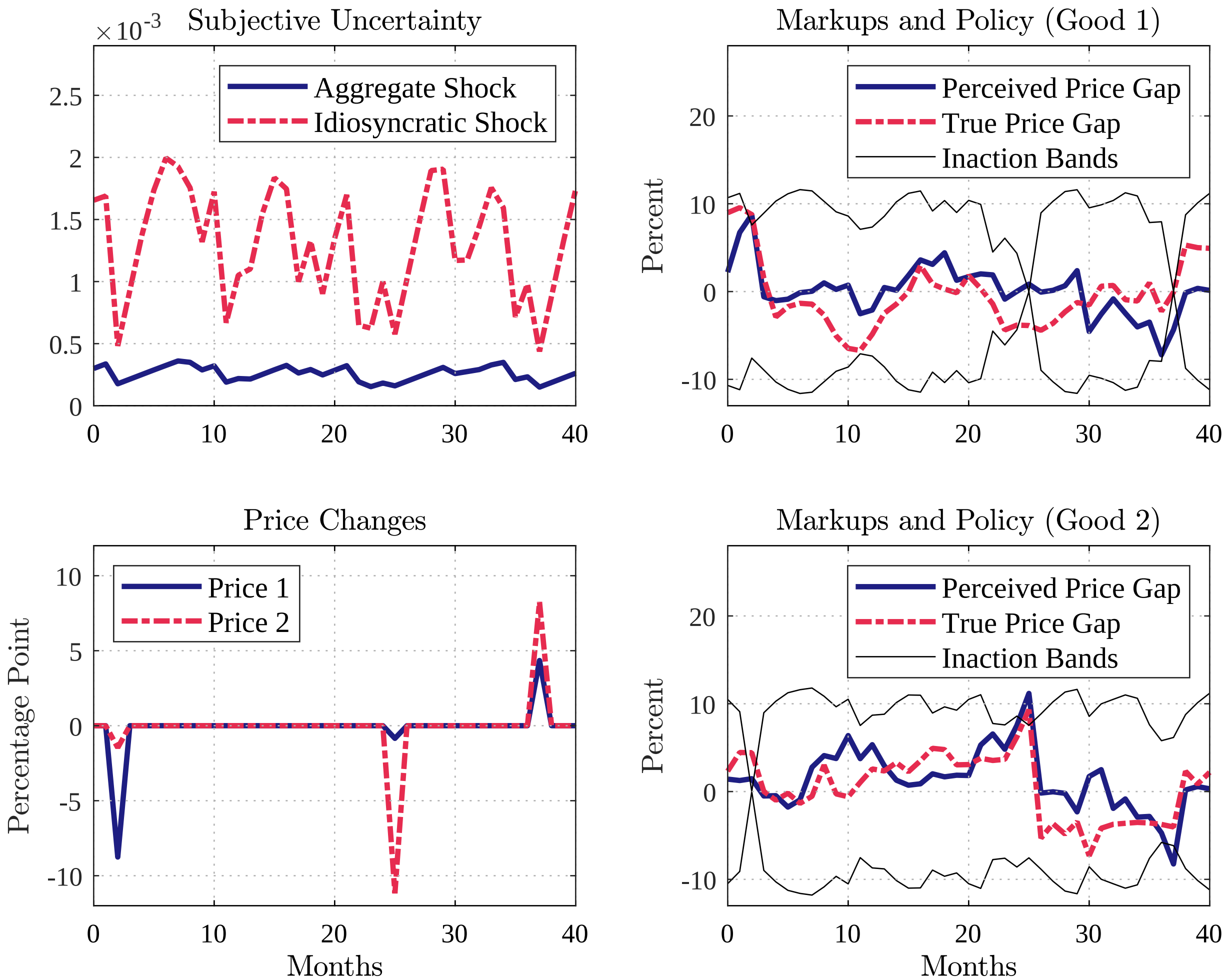


Figure 6: A Two-Product Firm in Model Simulation

Notes: The upper left panel plots a two-product firm's subjective uncertainty about both a good-specific shock (red dash-dot line) and an aggregate shock (blue solid line). The upper right panel plots the firm's perceived price gap (x_t) for good 1, its true price gap for good 1 under perfect information, and inaction bands for good 1. The lower right panel plots the firm's perceived price gap (x_t) for good 2, its true price gap for good 2 under perfect information, and inaction bands for good 2. The firm changes its price when the perceived price gaps are out of the inaction bands. The lower left panel plots these price changes for both goods.

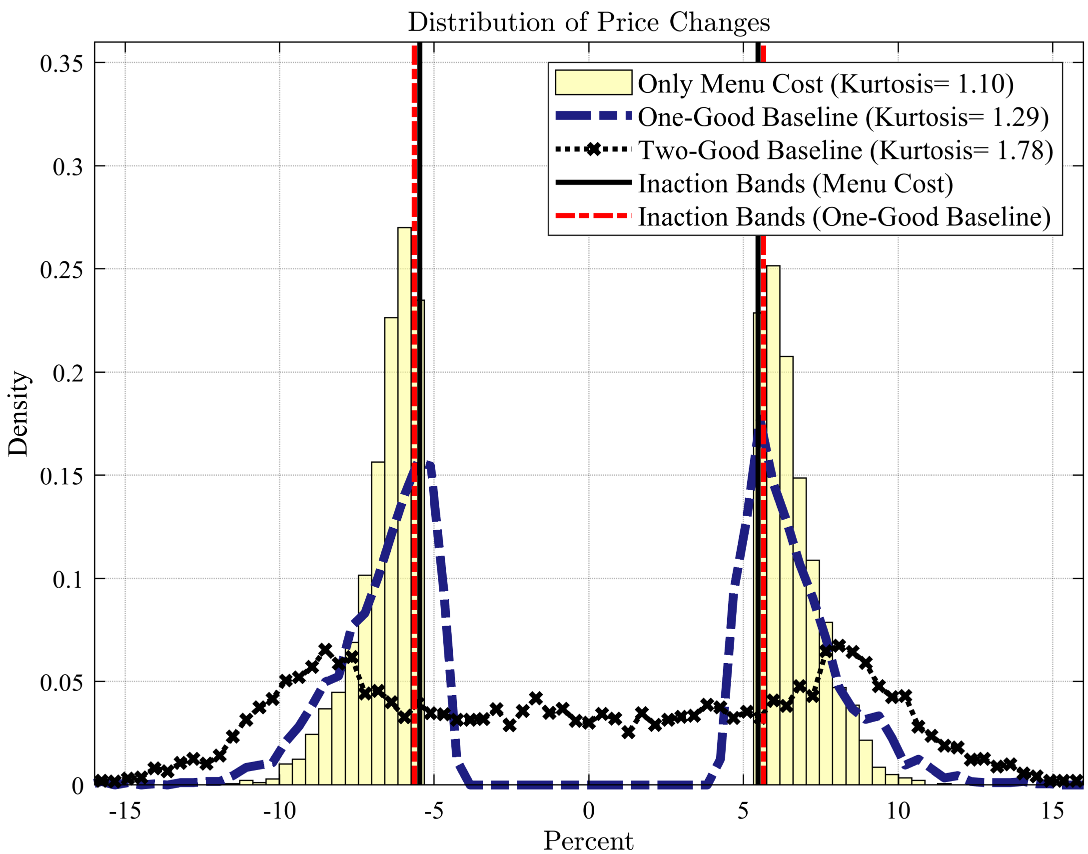


Figure 7: Distributions of Price Changes

Notes: This figure plots the distribution of price changes in the single-product only menu cost model (yellow bar), that in the baseline single-product model (blue dashed line), and that in the baseline two-product model (black line with cross markers). Black vertical lines are the inaction bands for firms in the only menu cost model. In this model, every firms have the same inaction bands. Red vertical dash-dot lines are the average of inaction bands across firms in the baseline single-product model. Notice that in this model, the inaction bands vary with firms' subjective uncertainty, as shown in Figure 4.

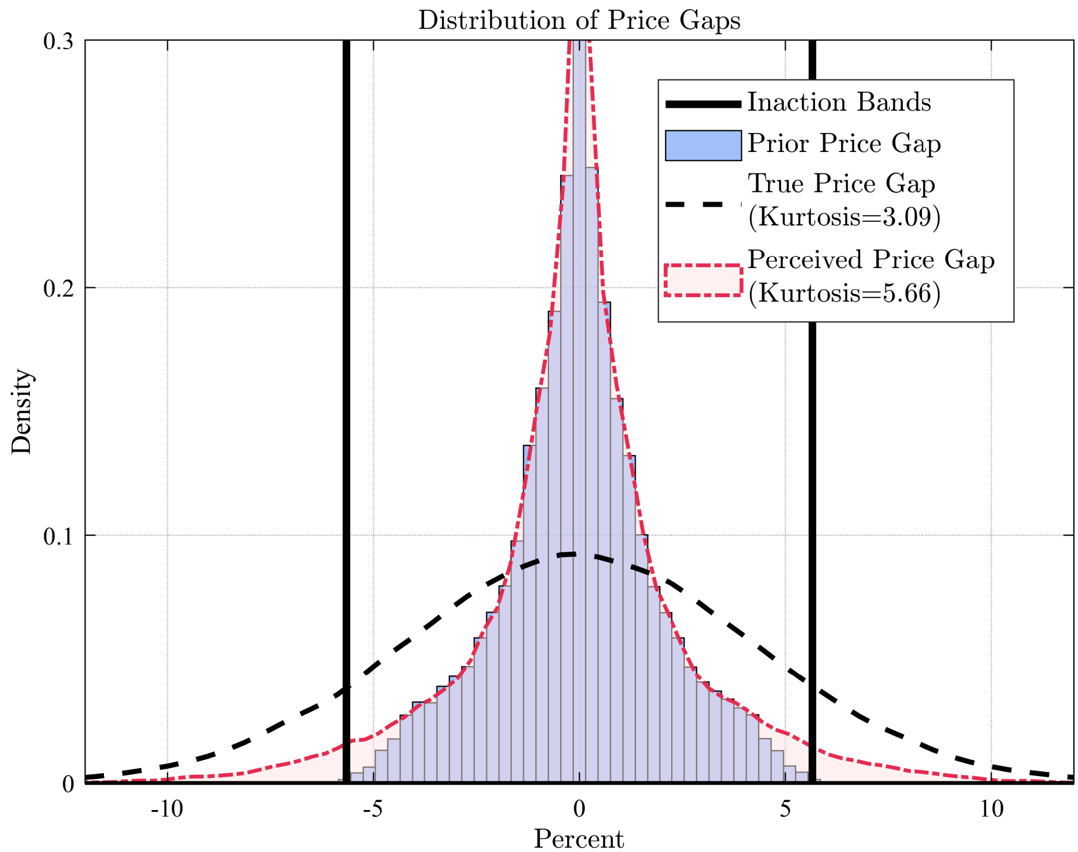


Figure 8: Distributions of True and Perceived Price Gaps in the Single-Product Model

Notes: This figure plots distributions of price gaps in the single-product baseline model. Black lines are the average of inaction bands across firms in the model. At the beginning of period, before the realization of shocks, all firms believe that their price is within the inaction bands. Blue bar graph shows the distribution of firms' *prior* about their price gaps ($p_{i,j,t-1} - \mathbb{E}_{t-1}[p^*_{i,j,t}|S^{t-1}]$) at the beginning of period. After the Gaussian shocks realized, firms' marginal costs change, and thus their true price gap ($p_{i,j,t-1} - p^*_{i,j,t}$) also changes. Black dashed line shows the distribution of these true price gaps. Firms choose their optimal signals about the shocks and form a new posterior about their (frictionless) optimal price. Then, the *posterior* of perceived price gap is $p_{i,j,t-1} - \mathbb{E}_t[p^*_{i,j,t}|S^t]$. Red dash-dot line shows the distribution of these perceived price gaps.

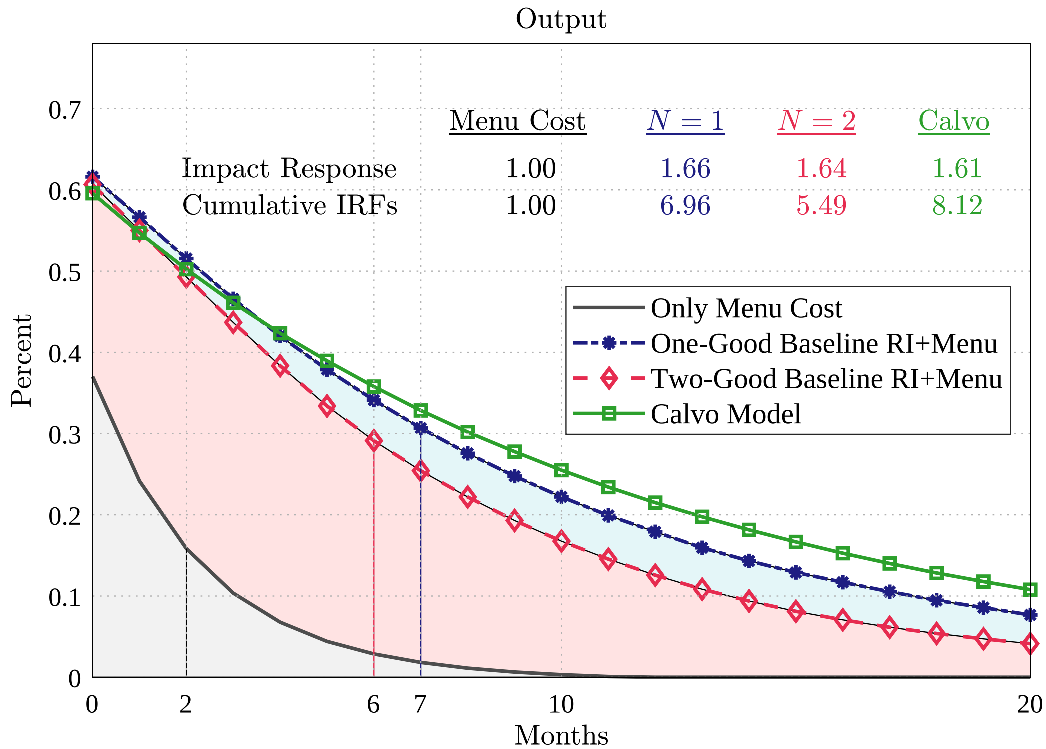


Figure 9: Impulse Response of Output to a One S.D. Monetary Shock

Notes: This figure plots impulse responses of output to a one standard deviation monetary shock. Cumulative IRFs refers to area under the responses of output. I normalize both impact response and cumulative output response in the only menu cost model with single-product firms as 1.

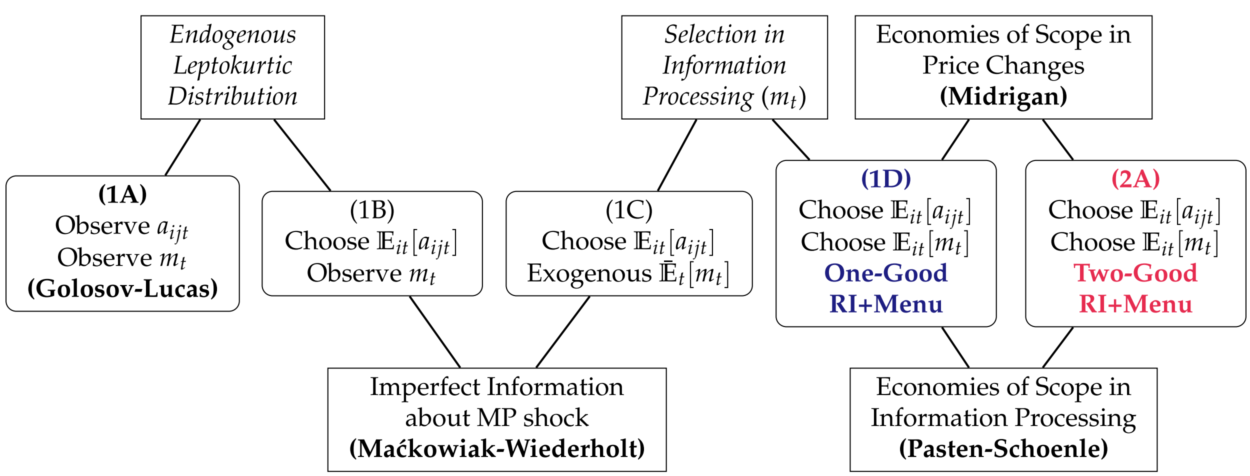
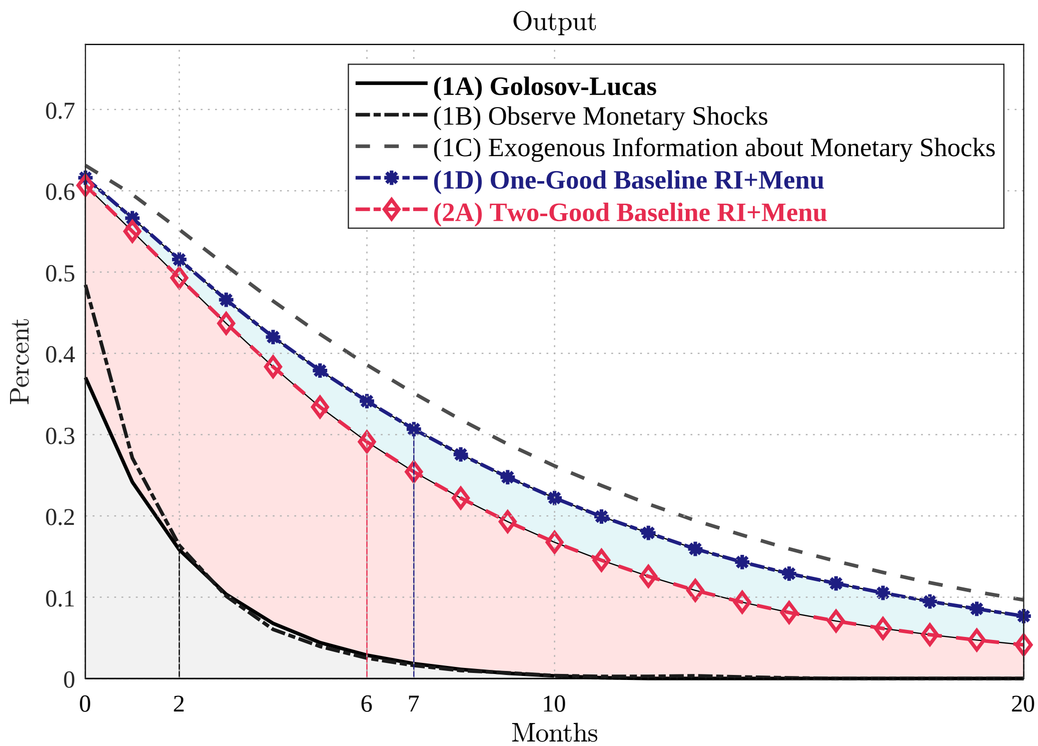


Figure 10: Counterfactuals and Model Mechanism

Notes: This figure shows counterfactual models and the implied model mechanisms. Model (1A), (1B), (1C), and (1D) are single-product menu cost models with different assumptions about firms' information set. In model (1A), firms have full-information about both idiosyncratic and monetary shocks. Firms in model (1B) have perfect information about the monetary shock, but choose their optimal signal about the idiosyncratic shock. All firms in model (1C) are given the same exogenous signal about the monetary shock, while they choose their optimal signal about the idiosyncratic shock. Model (1D) is the baseline single-product model where all firms choose their optimal signals about both shocks. Model (2D) is the baseline two-product model. See Section 4.6 for details.



	(1A)	(1B)	(1C)	**(1D)**	**(2A)**
Impact Response	**1**	1.31	1.70	**1.66**	**1.64**
Cumulative IRFs	**1**	1.08	8.18	**6.96**	**5.49**

Figure 11: Impulse Responses of Output in Counterfactual Models

Notes: This figure plots impulse responses of output to a one standard deviation monetary shock in counterfactual models described in Figure 10. Cumulative IRFs refers to area under the responses of output. I normalize both impact response and cumulative output response in the only menu cost model with single-product firms as 1. See Section 4.6 for details.

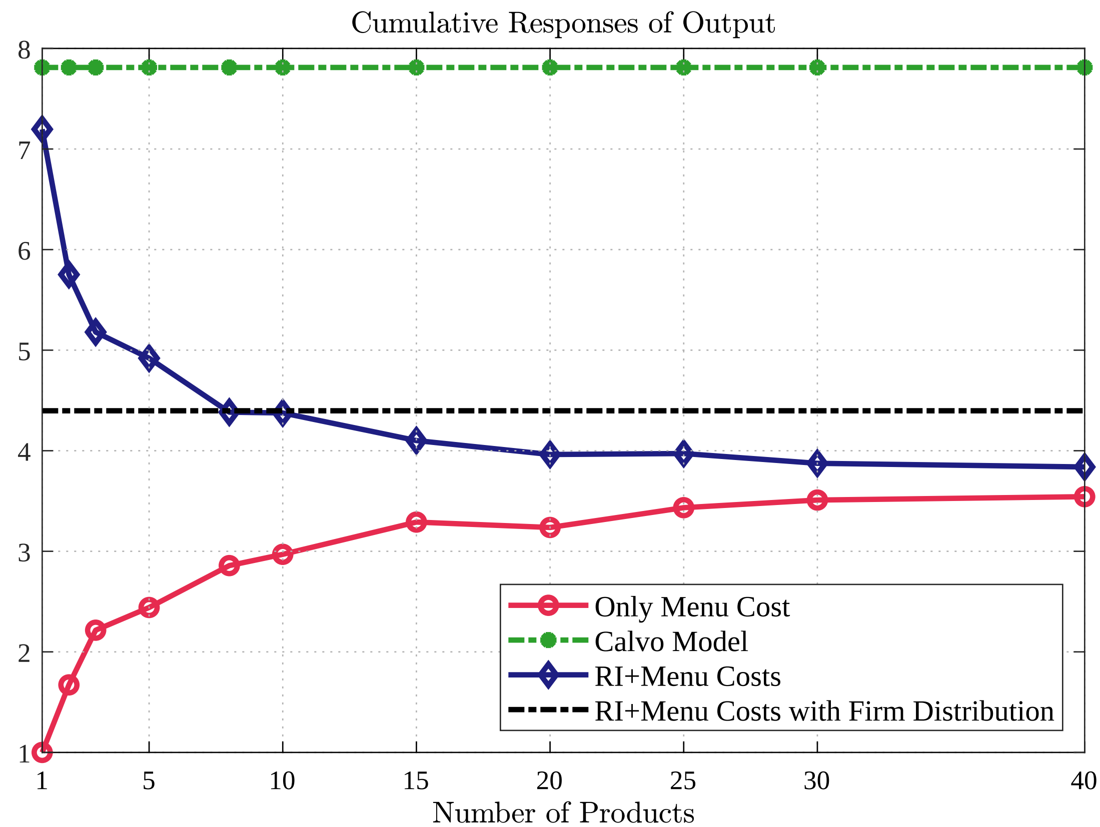


Figure 12: Cumulative Output Responses and Number of Products in the Simplified Models

Notes: This figure plots cumulative output responses in the simplified models with different number of products. RI+Menu refers to the model with both menu costs and rational inattention. Red line shows the cumulative output responses in the only menu cost models with different number of products and blue line shows those in the models with both menu costs and rational inattention with different number of products. Black line shows the cumulative response of output in the RI+Menu model with the same firm distribution of number of products observed in the New Zealand data. See Section 5.1 for details.

APPENDIX FOR ONLINE PUBLICATION

A Computing the Equilibrium

In this appendix, I describe a second order approximation to a firm's profit function. Also, I give a recursive formulation of two-product firms' problem that I studied in Section 4.

A.1 Quadratic Approximation to a Firm's Profit Function

Firm i produces N products indexed by j in monopolistic competitive markets. Its demand for good j is given by

$$Y_{i,j,t} = A_{i,j,t}^{\varepsilon-1}\left(\frac{P_{i,j,t}}{P_{j,t}}\right)^{-\varepsilon}\left(\frac{P_{j,t}}{P_t}\right)^{-\gamma} Y_t,$$

where P_t is the price of aggregate output Y_t, $P_{j,t}$ is the price of good j, γ is the constant elasticity of substitution across different firms that produce the same good, and ε is the constant elasticity of substitution across different goods. Then, the firm's profit function is

$$\begin{aligned}\Pi_{i,t} &= \sum_{j=1}^{N}\left(P_{i,j,t} - W_t A_{i,j,t}\right) Y_{i,j,t} \\ &= \sum_{j=1}^{N}\left(P_{i,j,t} - W_t A_{i,j,t}\right) A_{i,j,t}^{\varepsilon-1}\left(\frac{P_{i,j,t}}{P_{j,t}}\right)^{-\varepsilon}\left(\frac{P_{j,t}}{P_t}\right)^{-\gamma} Y_t,\end{aligned}$$

where

$$P_t = \left(\frac{1}{N}\sum_{j=1}^{N} P_{j,t}^{1-\gamma}\right)^{\frac{1}{1-\gamma}}, \quad P_{j,t} = \left(\int_0^1 \left(P_{i,j,t}\right)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}.$$

Define firm i's markup for good j, $\mu_{i,j,t} = \frac{P_{i,j,t}}{W_t A_{i,j,t}}$. Then, the profit function can be written as a function of the firm's markups for each good:

$$\begin{aligned}\Pi_{i,t} &= \sum_{j=1}^{N}\left(P_{i,j,t} - W_t A_{i,j,t}\right) A_{i,j,t}^{\varepsilon-1}\left(\frac{P_{i,j,t}}{P_{j,t}}\right)^{-\varepsilon}\left(\frac{P_{j,t}}{P_t}\right)^{-\gamma} Y_t \\ &= \sum_{j=1}^{N}\left(\mu_{i,j,t} - 1\right)\left(\mu_{i,j,t}\right)^{-\varepsilon}\left(W_t\right)^{1-\varepsilon}\left(P_{j,t}\right)^{\varepsilon-\gamma}\left(P_t\right)^{\gamma} Y_t.\end{aligned}$$

Let $R_{i,j,t}$ be the revenue from good j:

$$\begin{aligned}R_{i,j,t} &= P_{i,j,t} A_{i,j,t}^{\varepsilon-1}\left(\frac{P_{i,j,t}}{P_{j,t}}\right)^{-\varepsilon}\left(\frac{P_{j,t}}{P_t}\right)^{-\gamma} Y_t \\ &= \mu_{i,j,t}\left(\mu_{i,j,t}\right)^{-\varepsilon}\left(W_t\right)^{1-\varepsilon}\left(P_{j,t}\right)^{\varepsilon-\gamma}\left(P_t\right)^{\gamma} Y_t.\end{aligned}$$

A second order approximation to the profit function around the optimal frictionless markup,

$\mu_j^* = \frac{\varepsilon}{\varepsilon-1}$, yields

$$\Pi\left(\{\mu_{j,t}\}_{j=1}^N\right) = \Pi\left(\left\{\mu_j^*\right\}_{j=1}^N\right) + \frac{1}{2}\sum_{j=1}^N \frac{\partial^2 \Pi_t}{\partial \mu_{j,t}^2}\bigg|_{\left\{\mu_{j,t}=\mu_j^*\right\}_{j=1}^N} \left(\frac{\mu_{j,t}-\mu_j^*}{\mu_j^*}\right)^2 \left(\mu_j^*\right)^2$$

$$= \Pi\left(\left\{\mu_j^*\right\}_{j=1}^N\right) + \frac{1}{2}\sum_{j=1}^N \frac{\partial^2 \Pi_t}{\partial \mu_{j,t}^2}\bigg|_{\left\{\mu_{j,t}=\mu_j^*\right\}_{j=1}^N} \left(\hat{\mu}_{j,t}\right)^2 \left(\mu_j^*\right)^2,$$

where $\hat{\mu}_{j,t} = \log\left(\mu_{j,t}\right) - \log\left(\mu_j^*\right)$ is the realized markup-gap. Then, given the CES demand and the constant returns to scale technology, we can express the expected losses that arise from frictions (both nominal and informational) relative to the frictionless case, expressed as a fraction of per-good revenue:

$$\mathcal{L} \equiv \mathbb{E}\left[\frac{\Pi\left(\{\mu_{j,t}\}_{j=1}^N\right) - \Pi\left(\left\{\mu_j^*\right\}_{j=1}^N\right) - \tilde{\theta}\mathbf{1}_{\left\{\text{for any } j,\ p_{i,j,t}\neq p_{i,j,t-1}\right\}} - \tilde{\psi}\mathcal{I}(s_{i,t};\{\{A_{i,j,t}\}_{j=1}^N, W_t\}|S_i^{t-1})}{R\left(\mu_j^*\right)}\Bigg| S^{t-1}\right]$$

$$= \mathbb{E}\Bigg[\frac{1}{2}\frac{1}{R\left(\mu_j^*\right)}\sum_{j=1}^N \frac{\partial^2 \Pi_t}{\partial \mu_{j,t}^2}\bigg|_{\left\{\mu_{j,t}=\mu_j^*\right\}_{j=1}^N} \left(\hat{\mu}_{j,t}\right)^2 \left(\mu_j^*\right)^2$$

$$- \theta\mathbf{1}_{\left\{\text{for any } j,\ p_{i,j,t}\neq p_{i,j,t-1}\right\}} - \psi\mathcal{I}(s_{i,t};\{\{A_{i,j,t}\}_{j=1}^N, W_t\}|S_i^{t-1})\Bigg| S^{t-1}\Bigg]$$

$$= \mathbb{E}\Bigg[\frac{1}{2}\frac{\Pi\left(\left\{\mu_j^*\right\}_{j=1}^N\right)}{R\left(\mu_j^*\right)}\sum_{j=1}^N \frac{\left(\mu_j^*\right)^2 \frac{\partial^2 \Pi_t}{\partial \mu_{j,t}^2}\Big|_{\left\{\mu_{j,t}=\mu_j^*\right\}_{j=1}^N}}{\Pi\left(\left\{\mu_j^*\right\}_{j=1}^N\right)} \left(\hat{\mu}_{j,t}\right)^2$$

$$- \theta\mathbf{1}_{\left\{\text{for any } j,\ p_{i,j,t}\neq p_{i,j,t-1}\right\}} - \psi\mathcal{I}(s_{i,t};\{\{A_{i,j,t}\}_{j=1}^N, W_t\}|S_i^{t-1})\Bigg| S^{t-1}\Bigg],$$

where

$$\frac{\partial^2 \Pi_t}{\partial \mu_{j,t}^2}\bigg|_{\left\{\mu_{j,t}=\mu_j^*\right\}_{j=1}^N} = \varepsilon\left(\mu_j^*\right)^{-\varepsilon-2}\left[(\varepsilon+1)\left(\mu_j^*-1\right) - 2\mu_j^*\right]\left(\bar{W}\right)^{1-\varepsilon}\left(\bar{P}_j\right)^{\varepsilon-\gamma}\left(\bar{P}\right)^{\gamma}\bar{Y}$$

$$= -\varepsilon\left(\mu_j^*\right)^{-\varepsilon-2}\left(\bar{W}\right)^{1-\varepsilon}\left(\bar{P}_j\right)^{\varepsilon-\gamma}\left(\bar{P}\right)^{\gamma}\bar{Y},$$

$$\Pi\left(\left\{\mu_j^*\right\}_{j=1}^N\right) = \sum_{j=1}^N \left(\mu_j^*-1\right)\left(\mu_j^*\right)^{-\varepsilon}\left(\bar{W}\right)^{1-\varepsilon}\left(\bar{P}_j\right)^{\varepsilon-\gamma}\left(\bar{P}\right)^{\gamma}\bar{Y},$$

and

$$R\left(\mu_j^*\right) = \left(\mu_j^*\right)^{1-\varepsilon} \left(\bar{W}\right)^{1-\varepsilon} \left(\bar{P}_j\right)^{\varepsilon-\gamma} \left(\bar{P}\right)^{\gamma} \bar{Y}.$$

Notice that I express the cost of change price, $\tilde{\theta}$, as a fraction θ of the steady state frictionless revenue from selling one of N products, that is $\tilde{\theta} = \theta R(\mu_j^*)$. Similarly, the marginal cost of information processing is $\tilde{\psi} = \psi R(\mu_j^*)$.

Then, the loss function is

$$\begin{aligned}
\mathcal{L} = \mathbb{E}\Bigg[&\frac{1}{2}\frac{\Pi\left(\left\{\mu_j^*\right\}_{j=1}^N\right)}{R\left(\mu_j^*\right)} \sum_{j=1}^N \frac{\left(\mu_j^*\right)^2 \left.\frac{\partial^2 \Pi_t}{\partial \mu_{j,t}^2}\right|_{\left\{\mu_{j,t}=\mu_j^*\right\}_{j=1}^N}}{\Pi\left(\left\{\mu_j^*\right\}_{j=1}^N\right)} \left(\hat{\mu}_{j,t}\right)^2 \\
&- \theta \mathbf{1}_{\left\{\text{for any } j,\ p_{i,j,t} \neq p_{i,j,t-1}\right\}} - \psi \mathcal{I}(s_{i,t}; \{\{A_{i,j,t}\}_{j=1}^N, W_t\} | S_i^{t-1}) \Bigg| S^{t-1}\Bigg] \\
= -\mathbb{E}\Bigg[&\varepsilon\frac{1}{2}\left(\frac{\sum_{j=1}^N \left(\mu_j^*-1\right)\left(\mu_j^*\right)^{-\varepsilon}\left(\bar{W}\right)^{1-\varepsilon}\left(\bar{P}_j\right)^{\varepsilon-\gamma}\left(\bar{P}\right)^{\gamma}\bar{Y}}{\left(\mu_j^*\right)^{1-\varepsilon}\left(\bar{W}\right)^{1-\varepsilon}\left(\bar{P}_j\right)^{\varepsilon-\gamma}\left(\bar{P}\right)^{\gamma}\bar{Y}}\right) \\
&\times \frac{\sum_{j=1}^N \left(\mu_j^*\right)^{-\varepsilon}\left(\bar{W}\right)^{1-\varepsilon}\left(\bar{P}_j\right)^{\varepsilon-\gamma}\left(\bar{P}\right)^{\gamma}\bar{Y}}{\sum_{j=1}^N \left(\mu_j^*-1\right)\left(\mu_j^*\right)^{-\varepsilon}\left(\bar{W}\right)^{1-\varepsilon}\left(\bar{P}_j\right)^{\varepsilon-\gamma}\left(\bar{P}\right)^{\gamma}\bar{Y}} \left(\hat{\mu}_{j,t}\right)^2 \\
&+ \theta \mathbf{1}_{\left\{\text{for any } j,\ p_{i,j,t} \neq p_{i,j,t-1}\right\}} + \psi \mathcal{I}(s_{i,t}; \{\{A_{i,j,t}\}_{j=1}^N, W_t\} | S_i^{t-1}) \Bigg| S^{t-1}\Bigg].
\end{aligned}$$

Now, assume $\varepsilon = \gamma$ (or by symmetry across product industry due to there are no common industry specific shocks). Then, the second order approximation of the firm's profit function is

$$\mathcal{L} = \mathbb{E}\left[-\varepsilon\frac{1}{2}\left(\frac{1}{\mu_j^*}\right)\sum_{j=1}^N \left(\hat{\mu}_{i,j,t}\right)^2 + \theta \mathbf{1}_{\left\{\text{for any } j,\ p_{i,j,t} \neq p_{i,j,t-1}\right\}} + \psi \mathcal{I}(s_{i,t}; \{\{A_{i,j,t}\}_{j=1}^N, W_t\} | S_i^{t-1}) \middle| S^{t-1}\right].$$

Let $p_{i,j,t}^* = \log(W_t) + \log(A_{i,j,t})$ be the log deviation of (frictionless) optimal price of good j from its non-stochastic steady state. Then, we define firm i's true price gap of good j as

$$\hat{\mu}_{i,j,t} = p_{i,j,t} - p_{i,j,t}^*,$$

where $p_{i,j,t}$ is the log deviation of the price of good j from its non-stochastic steady state.[60] Then, I

[60]The true price gap $\hat{\mu}_{i,j,t}$ can be also written as a markuk gap, $\hat{\mu}_{i,j,t} = \log(\mu_{i,j,t}/\mu_j^*)$, which is a the log deviation of the current markup to the non-stochastic steady state markup $\mu_j^* = \frac{\varepsilon}{\varepsilon-1}$.

derive a second order approximation of firms' loss function:

$$\begin{aligned}\mathcal{L} &= \mathbb{E}\left[-\varepsilon\frac{1}{2}\left(\frac{1}{\mu_j^*}\right)\sum_{j=1}^{N}\left(p_{i,j,t}-p_{i,j,t}^*\right)^2 + \theta\mathbf{1}_{\{\text{for any } j,\ p_{i,j,t}\neq p_{i,j,t-1}\}} + \psi\mathcal{I}(s_{i,t};\{\{A_{i,j,t}\}_{j=1}^N, W_t\}|S_i^{t-1})\middle| S^{t-1}\right] \\ &= -\mathbb{E}\left[B\sum_{j=1}^{N}\left(p_{i,j,t}-p_{i,j,t}^*\right)^2 + \theta\mathbf{1}_{\{\text{for any } j,\ p_{i,j,t}\neq p_{i,j,t-1}\}} + \psi\mathcal{I}(s_{i,t};\{\{A_{i,j,t}\}_{j=1}^N, W_t\}|S_i^{t-1})\middle| S^{t-1}\right],\end{aligned}$$

where $B = \frac{\varepsilon-1}{2}$ is a slope of profit curve.

A.2 A Recursive Formulation

At the beginning of period t, firm i takes its initial information set, S_i^{t-1}, as given and chooses a set of optimal signals, $s_{i,t}$, subject to a cost of information processing, $\psi\mathcal{I}(s_{i,t};\{p_{i,j,t}^*\}_{j=1}^N|S_i^{t-1}|S_i^{t-1})$, where ψ is a marginal cost of a bit of information and $\mathcal{I}(\cdot)$ is Shannon's mutual information function. Notice that I replace the idiosyncratic shocks and the nominal wage in Shannon's mutual information function with the firm's frictionless optimal price since all the firms need to know is its frictionless optimal price after deriving the firms' loss function. It then forms a new information set, $S_i^t = S_i^{t-1}\cup s_{i,t}$, and sets its new prices, $\{p_{i,j,t}\}_{j=1}^N$, based on that. The firm pays a menu cost, θ, if it decides to change any prices. Otherwise, the firm waits for the next period.

Formally, after taking a quadratic approximation of firm i's profit function around non-stochastic steady state and deriving a loss function from the suboptimal prices, the firm's problem can be written as:

$$\begin{aligned}\min_{\{\{p_{i,j,t}\}_{j=1}^N, s_{i,t}\}_{t\geq 0}} \quad & \mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\left\{B\sum_{j=1}^{N}(p_{i,j,t}-p_{i,j,t}^*)^2 + \theta\mathbf{1}_{\{\text{for any } j,\ p_{i,j,t}\neq p_{i,j,t-1}\}} \right.\right. \\ & \left.\left. + \psi\mathcal{I}(s_{i,t};\{p_{i,j,t}^*\}_{j=1}^N|S_i^{t-1})\right\}\middle| S_i^{-1}\right] \\ \text{s.t.}\quad & S_i^t = S_i^{t-1}\cup s_{i,t}.\end{aligned}$$

I assume that the set of available signals satisfies that 1) the firm chooses $N+1$ independent signals for each shock, 2) each signal is Gaussian, and 3) all noise in signals is idiosyncratic and independent. Then, the set of optimal signals about both the idiosyncratic good-specific shocks and the monetary shock can be written as:

$$\begin{aligned}\mathcal{S}_{i,j,t}^a &= \{a_{i,j,t} + \eta_{i,j,t}^a\xi_{i,j,t}^a : \eta_{i,j,t}^a \geq 0,\ \xi_{i,j,t}^a \sim N(0,1)\},\ \text{ for } j = 1,2,\cdots,N, \\ \mathcal{S}_{i,t}^m &= \{m_t + \eta_{i,t}^m\xi_{i,t}^m : \eta_{i,t}^m \geq 0,\ \xi_{i,t}^m \sim N(0,1)\}.\end{aligned}$$

Here, rational inattention errors, $\xi_{i,j,t}^a$ and $\xi_{i,t}^m$, are independent across firms. At each time t, given S^{t-1}, firm i chooses its optimal signals $s_{i,j,t}^a \in \mathcal{S}_{i,j,t}^a$ for $j = 1,2,\cdots,N$, and $s_{i,t}^m \in \mathcal{S}_{i,t}^m$ subject to the cost of information processing. Then, the firm's new information set evolves as $S_i^t = S_i^{t-1}\cup s_{i,t}$

where $s_{i,t} = \{\{s^a_{i,j,t}\}_{j=1}^N, s^m_{i,t}\}$. Now, the firms' problem is virtually identical to the problem I studied in Section 3.

Let $x_{j,t} = p_{j,t} - \mathbb{E}[p^*_{j,t}|S^t]$ be firm's *perceived* price gap about product j and

$$z^a_{j,t} = \mathbb{E}\left[\left(a_{j,t} - \mathbb{E}_t[a_{j,t}|S^t]\right)^2 \middle| S^t\right], \;\; z^m_t = \mathbb{E}\left[\left(m_t - \mathbb{E}_t[m_t|S^t]\right)^2 \middle| S^t\right]$$

be subjective uncertainty about the j-good specific shock and the aggregate shock, respectively. I rewrite the recursive formation of the firms' problem as:

$$\begin{aligned} V\left(\{x_{j,-1}\}_{j=1}^N, \{z^a_{j,-1}\}_{j=1}^N, z^m_{-1}\right) = \max_{\{\{z^a_j\}_{j=1}^N, z^m\}} \mathbb{E}\Bigg[& \max\left\{V^I\left(\{x_j\}_{j=1}^N, \{z^a_j\}_{j=1}^N, z^m\right), V^C\left(\{z^a_j\}_{j=1}^N, z^m\right)\right\} \\ & - \frac{\psi}{2}\left(\sum_{j=1}^N \log_2\left(\frac{z^a_{j,-1}+\sigma^2_a}{z^a_j}\right) + \log_2\left(\frac{z^m_{-1}+\sigma^2_m}{z^m}\right)\right)\Bigg| S^{-1}\Bigg] \\ \text{s.t.} \qquad & 0 \le z^a_j \le z^a_{j,-1} + \sigma^2_a, \quad \forall j = 1, 2, \cdots, N, \\ & 0 \le z^m \le z^m_{-1} + \sigma^2_m, \end{aligned}$$

where

$$V^I\left(\{x_j\}_{j=1}^N, \{z^a_j\}_{j=1}^N, z^m\right) = -B\sum_{j=1}^N\left(x_j^2 + z^a_j + z^m\right) + \beta V\left(\{x_j\}_{j=1}^N, \{z^a_j\}_{j=1}^N, z^m\right)$$

$$\text{with } \mathbf{x} \sim N\left(\mathbf{x}_{-1}, \Sigma\right), \text{ and}$$

$$V^C\left(\{z^a_j\}_{j=1}^N, z^m\right) = \max_{\{y_j\}_{j=1}^N} -B\sum_{j=1}^N\left(y_j^2 + z^a_j + z^m\right) - \theta + \beta V\left(\{y_j\}_{j=1}^N, \{z^a_j\}_{j=1}^N, z^m\right).$$

Here $\mathbf{x} = \{x_1, x_2, \cdots, x_N\}'$ and

$$\Sigma_t(j,k) = \begin{cases} z^m_{-1} + \sigma^2_m - z^m & \text{if } j \ne k \\ z^a_{j,-1} + \sigma^2_a - z^a_j + z^m_{-1} + \sigma^2_m - z^m & \text{if } j = k. \end{cases}$$

B A Rationally Inattentive Firm's Problem without Menu Costs ($\theta = 0$)

In this appendix, I solve a rationally inattentive firm's problem without menu costs. This problem is similar to the one studied in Pasten and Schoenle (2016) with one main difference. They solve the problem by assuming that the cost of information is not discounted and optimizing at the long-run steady-state for information structure. Here, I assume that the firm discounts future costs of information at the same discount rate as their payoffs and solve the dynamic information acquisition problem. This setup is also similar to Afrouzi and Yang (2019) that study the dynamic multivariate rational inattention problem. One difference is that I assume that the set of available signals are partitioned into two subsets, one for signals about idiosyncratic shocks and the other for signals about aggregate shocks, that are independent each other.

Without menu costs, the firm can change its prices and collapses the price gaps to zero whenever it wants, i.e. there is no micro rigidity in price setting. In this case, the firm's prior price gaps are no longer its state variables, and thus the firm's problem is deterministic. Then, in a recursive formation, the firm's problem is:

$$\begin{aligned} V\left(\{z^a_{j,-1}\}_{j=1}^N, z^m_{-1}\right) = & \max_{\{\{z^a_j\}_{j=1}^N, z^m\}} - B\sum_{j=1}^N \left(z^a_j + z^m\right) + \beta V\left(\{z^a_j\}_{j=1}^N, z^m\right) \\ & - \frac{\psi}{2}\left(\sum_{j=1}^N \log_2\left(\frac{z^a_{j,-1}+\sigma_a^2}{z^a_j}\right) + \log_2\left(\frac{z^m_{-1}+\sigma_m^2}{z^m}\right)\right) \\ \text{s.t.} \quad & 0 \le z^a_j \le z^a_{j,-1} + \sigma_a^2, \quad \forall j = 1,2,\cdots,N, \\ & 0 \le z^m \le z^m_{-1} + \sigma_m^2, \end{aligned}$$

Notice that with $\psi > 0$, the constraints $z^a_j \ge 0$ and $z^m_j \ge 0$ will not bind. The first order necessary conditions are:

$$\begin{aligned} \partial z^a_j: \quad & -B + \frac{\psi}{2\log 2}\frac{1}{z^a_j} + \beta V_{z^a_j}\left(\{z^a_j\}_{j=1}^N, z^m\right) - \phi_j = 0, \ \forall j \in \{1,2,\cdots,N\}, \\ \partial z^m: \quad & -BN + \frac{\psi}{2\log 2}\frac{1}{z^m} + \beta V_{z^m}\left(\{z^a_j\}_{j=1}^N, z^m\right) - \phi_m = 0, \\ & V_{z^a_{j,-1}}\left(\{z^a_{j,-1}\}_{j=1}^N, z^m_{-1}\right) = -\frac{\psi}{2\log 2}\frac{1}{z^a_{j,-1}+\sigma_a^2} \ \ \forall j \in \{1,2,\cdots,N\}, \\ & V_{z^m_{-1}}\left(\{z^a_{j,-1}\}_{j=1}^N, z^m_{-1}\right) = -\frac{\psi}{2\log 2}\frac{1}{z^m_{-1}+\sigma_m^2}, \end{aligned}$$

and complementarity slackness conditions where $\{\phi_j\}_{j=1}^N$ and ϕ_m are Lagrangian multipliers for no-forgetting constraints. The no-forgetting constraints will bind when the marginal cost of information processing is high enough. Here, I focus on interior solutions where the constraints do not bind.

When the standard deviation of idiosyncratic good-specific shocks is the same, then subjective uncertainty about idiosyncratic shocks is also the same across all goods. Then, the optimal subjective uncertainty about both shocks satisfies:

$$\begin{aligned} B = & \frac{\psi}{2\log 2}\left(\frac{1}{z^a_j} - \beta\frac{1}{z^a_j+\sigma_a^2}\right), \ \forall j \in \{1,2,\cdots,N\} \\ B\cdot N = & \frac{\psi}{2\log 2}\left(\frac{1}{z^m} - \beta\frac{1}{z^m+\sigma_m^2}\right). \end{aligned} \tag{13}$$

Several interesting characteristics emerge. First, the firm's optimal subjective uncertainty is constant while it is time-varying with menu costs. Second, subjective uncertainty increases in the size of marginal cost of information processing, ψ, and the size of shocks, σ_a^2, while it decreases in the slope of profit function , B, and the time preference parameter, β. Third, optimal subjective uncertainty about idiosyncratic shocks is independent of the number of products that the firm produces. Fourth, optimal subjective uncertainty about aggregate shocks is *decreasing* in the number of products ($\frac{\partial z^m(N)}{\partial N} < 0$) and $\lim_{N\to\infty} z^m(N) = 0$.

B.1 Real Effects of Monetary Policy Shocks

Let $\tilde{z}^m = \frac{z^m}{\sigma_m^2}$ be firms' subjective uncertainty relative to the variance of monetary policy shocks. Then I can rewrite Equation (13) as:

$$\frac{1}{\tilde{z}^m} - \beta \frac{1}{\tilde{z}^m + 1} = \sigma_m^2 \frac{BN}{\psi} \left(2 \log 2\right) \tag{14}$$

Given parameters, firms' subjective uncertainty about monetary shocks decreases in their number of products.

Now, the size of price changes for good j is given by:

$$\begin{aligned}\Delta p_{i,j,t} =& \mathcal{K}^a(N) \left(a_{i,j,t-1} - a_{i,j,t-1|t-1} + \varepsilon_{i,j,t} + \eta_{i,j,t}\right) \\ &+ \mathcal{K}^m(N) \left(m_{t-1} - m_{i,t-1|t-1} + \varepsilon_{m,t} + \eta_{i,m,t}\right),\end{aligned}$$

where

$$\begin{aligned}a_{i,jt} - a_{i,j,t|t} &= (1 - \mathcal{K}^a(N)) \left(a_{i,j,t-1} - a_{i,j,t-1|t-1} + \varepsilon_{i,j,t}\right) - \mathcal{K}^a(N)\eta_{i,j,t}, \\ m_t - m_{i,t|t} &= (1 - \mathcal{K}^m(N)) \left(m_{t-1} - m_{i,t-1|t-1} + \varepsilon_{m,t}\right) - \mathcal{K}^m(N)\eta_{i,m,t},\end{aligned}$$

and Kalman gains are:

$$\mathcal{K}^a(N) = \frac{1}{1 + \tilde{z}_j^a(N)}, \ \ \mathcal{K}^m(N) = \frac{1}{1 + \tilde{z}^m(N)}.$$

Let $p_{j,t} = \int p_{i,j,t} di$. Let $p_{j,t}^{\mathrm{NoMP}}$ and $p_{j,t}^{\mathrm{MP}}$ be price level at time t without and with monetary shocks, respectively. Then, since all noise in signals, $\eta_{i,j,t}$, is idiosyncratic and independent, we have

$$\begin{aligned}p_{j,t}^{\mathrm{NoMP}} &= p_{j,t-1}^{\mathrm{NoMP}} + \Delta p_{j,t}^{\mathrm{NoMP}} \\ &= p_{j,t-1}^{\mathrm{NoMP}} + \left(\mathcal{K}^a(N) \int \left(a_{i,j,t-1} - a_{i,j,t-1|t-1}\right) di + \mathcal{K}^m(N) \int \left(m_{t-1} - m_{i,t-1|t-1}\right) di\right)\end{aligned}$$

and

$$\begin{aligned}p_{j,t}^{\mathrm{MP}} &= p_{j,t-1}^{\mathrm{MP}} + \Delta p_{j,t}^{\mathrm{MP}} \\ &= p_{j,t-1}^{\mathrm{MP}} + \left(\mathcal{K}^a(N) \int \left(a_{i,j,t-1} - a_{i,j,t-1|t-1}\right) di + \mathcal{K}^m(N) \int \left(m_{t-1} - m_{i,t-1|t-1}\right) di + \varepsilon_{m,t}\right)\end{aligned}$$

Then, Notice that by symmetry across goods, we have $p_t = p_{j,t}$ for all j. Define an impulse response of aggregate price to a monetary shock as the gap between the prices with and without the monetary shock, that is,

$$IRF_t^P = p_t^{\text{MP}} - p_t^{\text{NoMP}}.$$

Assume at time 0, there is a monetary shock, $\varepsilon_{m,0}$. Then,

$$\begin{aligned}
IRF_0^P &= \mathcal{K}^m(N)\varepsilon_{m,0} \\
IRF_1^P &= \mathcal{K}^m(N)\varepsilon_{m,0} + \mathcal{K}^m(N)\left(1-\mathcal{K}^m(N)\right)\varepsilon_{m,0} \\
IRF_2^P &= \mathcal{K}^m(N)\varepsilon_{m,0} + \mathcal{K}^m(N)\left(1-\mathcal{K}^m(N)\right)\varepsilon_{m,0} + \mathcal{K}^m(N)\left(1-\mathcal{K}^m(N)\right)^2\varepsilon_{m,0} \\
&\vdots \\
IRF_t^P &= \mathcal{K}^m(N)\left(\frac{1-\left(1-\mathcal{K}^m(N)\right)^{t+1}}{1-\left(1-\mathcal{K}^m(N)\right)}\right)\varepsilon_{m,0} \\
&= \left(1-\left(1-\mathcal{K}^m(N)\right)^{t+1}\right)\varepsilon_{m,0}
\end{aligned}$$

and output responses are given by

$$\begin{aligned}
IRF_t^Y &= \varepsilon_{m,0} - IRF_t^P \\
&= \left(1-\mathcal{K}^m(N)\right)^{t+1}\varepsilon_{m,0}.
\end{aligned}$$

Let a cumulative impulse response of output as a function of the number of product, $M(N)$ be the area under the impulse response function of output. Then,

$$\begin{aligned}
\mathcal{M}(N) &= \int_0^\infty IRF_t^Y dt = \int_0^\infty \left(1-\mathcal{K}^m(N)\right)^{t+1}\varepsilon_{m,0}dt \\
&= -\frac{\left(1-\mathcal{K}^m(N)\right)}{\log\left(1-\mathcal{K}^m(N)\right)}\varepsilon_{m,0} \\
&= -\frac{\left(\frac{\tilde{z}_m(N)}{1+\tilde{z}_m(N)}\right)}{\log\left(\frac{\tilde{z}_m(N)}{1+\tilde{z}_m(N)}\right)}\varepsilon_{m,0}.
\end{aligned}$$

where $\tilde{z}_m(N)$ is a solution of Equation (14) as a function of N. Notice that

$$\frac{\partial\mathcal{M}(N)}{\partial N} = -\underbrace{\frac{\frac{\partial\tilde{z}^m(N)}{\partial N}}{\left(1+\tilde{z}^m(N)\right)^2}}_{<0} \times \underbrace{\frac{1}{\log\left(\frac{\tilde{z}_m(N)}{1+\tilde{z}_m(N)}\right)}}_{<0} \times \underbrace{\left(1-\frac{1}{\log\left(\frac{\tilde{z}_m(N)}{1+\tilde{z}_m(N)}\right)}\right)}_{>0} < 0.$$

Although N is an arbitrary integer number, here I assume $\mathcal{M}(N)$ is continuously differentiable with respect to N. Appendix Figure A.2 shows that both subjective uncertainty about monetary shocks and cumulative responses of output to monetary shocks decreases in number of products that firms produce.

C Computational Procedures for the Two-Product Model

I use the method of value function iteration to solve the two-product firms' optimization problem. There are 5 state variables for the problem: prior perceived price gap for two products, subjective uncertainty about each good-specific shock, and prior subjective uncertainty about monetary policy shocks. Since this problem is non-convex optimization problem with occasionally binding constraints, it should b e solved numerically.

The most computationally burdensome part is to compute firms' expected future values since 1) tomorrow's perceived price gaps are stochastic variables when firms do not change prices today, and 2) the distribution of these price gaps has a mean vector $(x_{1,-1}, x_{2,-1})'$, which is firms' state variable, and a covariance matrix, Σ, which is firms' choice variable. Standard approximation methods for the transition probability of states, such as Tauchen approximation method, are not applicable since the approximation errors are quite large. I compute expected value of the firms' value functions using Gauss-Legendre quadrature which is an explicit numerical integration technique.

I solve the firms' problem and the value function and the optimal policy functions using the following procedure:

1. Construct grids for individual state variables, such as prior of perceived price gaps for each product, $x_{1,-1}$, $x_{2,-1}$, prior subjective uncertainty about two good-specific shocks, $z^a_{1,-1}$, $z^a_{2,-1}$, and prior subjective uncertainty about monetary shocks, z^m_{-1}. I use 21 grids for $x_{1,-1}$, $x_{2,-1}$, and 16 grids for $z^a_{1,-1}$, $z^a_{2,-1}$, and z^m_{-1}. The ranges of $x_{1,-1}$ and $x_{2,-1}$, are $[-1.5\sqrt{\theta/B}, 1.5\sqrt{\theta/B}]$ where θ is the size of menu costs and B is the slope of firms' profit curve.[61] More grid points are assigned around inaction bands. $z^a_{1,-1}$, $z^a_{2,-1}$, and z^m_{-1} are equally spaced in the range of [0 0.004].

2. Construct the abscissas, $\{\tilde{x}_i\}_{i=1}^{N_q}$, and weights, $\{\tilde{w}_i\}_{i=1}^{N_q}$, of the Gauss-Legendre quadrature with $N_q = 500$ points.

3. Solve the individual value functions at each grid point. In this step, I obtain the optimal decision rules for subjective uncertainty about both good-specific shocks and monetary shocks,

   $$g^a_1(x_{1,-1}, x_{2,-1}, z^a_{1,-1}, z^a_{2,-1}, z^m_{-1}),\ g^a_2(x_{1,-1}, x_{2,-1}, z^a_{1,-1}, z^a_{2,-1}, z^m_{-1}),\ g^m(x_{1,-1}, x_{2,-1}, z^a_{1,-1}, z^a_{2,-1}, z^m_{-1}),$$

   and the value function, $V(x_{1,-1}, x_{2,-1}, z^a_{1,-1}, z^a_{2,-1}, z^m_{-1})$. The detailed steps are as follows:

   (a) Make an initial guess for the value functions, V_0, for all grid points.

   (b) Solve firms' optimization problem and compute V_1. Notice that the problem can be

[61] In fact $[-\sqrt{\frac{\theta}{B}}, \sqrt{\frac{\theta}{B}}]$ is the inaction bands for myopic firms ($\beta = 0$). In this regard, I have more conservative ranges of grids for prior price gaps.

written:

$$
\begin{aligned}
&V_1\Big(x_{1,-1}, x_{2,-1}, z^a_{1,-1}, z^a_{2,-1}, z^m_{,-1}\Big) = \\
&\qquad \max_{\{z^a_1, z^a_2, z^m\}} - B\left(z^a_1 + z^a_2 + 2z^m\right) - \frac{\psi}{2}\left(\sum_{j=1}^{2} \log_2\left(\frac{z^a_{j,-1} + \sigma^2_a}{z^a_j}\right) + \log_2\left(\frac{z^m_{-1} + \sigma^2_m}{z^m}\right)\right) \\
&\qquad\qquad + \int_{(x_1, x_2)} \max\Big\{ - B(x_1^2 + x_2^2) + \beta V_0\left(x_1, x_2, z^a_1, z^a_2, z^m\right), \\
&\qquad\qquad\qquad - \theta + \beta V_0\left(0, 0, z^a_1, z^a_2, z^m\right) \Big\} dF((x_1, x_2); (x_{1,-1}, x_{2,-1}), \Sigma) \\
&\qquad \text{s.t.} \qquad 0 \le z^a_j \le z^a_{j,-1} + \sigma^2_a, \quad \forall j = 1, 2 \\
&\qquad\qquad\qquad 0 \le z^m \le z^m_{-1} + \sigma^2_m \\
&\qquad\qquad\qquad \Sigma_t(j,k) = \begin{cases} z^m_{-1} + \sigma^2_m - z^m & \text{if } j \ne k \\ z^a_{j,-1} + \sigma^2_a - z^a_j + z^m_{-1} + \sigma^2_m - z^m & \text{if } j = k. \end{cases}
\end{aligned}
$$

where $F((x_1, x_2); (x_{1,-1}, x_{2,-1}), \Sigma)$ is a joint normal distribution with mean $(x_{1,-1}, x_{2,-1})$ and covariance matrix Σ.

(c) If V_0 and V_1 are close enough for each grid point, and go to the next step. Otherwise, update the value functions ($V_0 = V_1$), and go back to (a).

(d) Simulate the model with a large number of firms to obtain a stationary distribution, $G(x_1, x_2, z^a_1, z^a_2, z^m)$, over firm states $(x_1, x_2, z^a_1, z^a_2, z^m)$. Simulation algorithm is described in Appendix D.

(e) Compute aggregate variables.

D Simulation Algorithm for the Two-Product Model

I simulate the two-good version of the baseline model with 100,000 firms for 5,000 periods.

1. Set initial $x_{i,1,t-1}$, $x_{i,2,t-1}$, $z^a_{i,1,t-1}$, $z^a_{i,2,t-1}$, and $z^m_{i,t-1}$.

2. Generate random numbers for the shocks $\varepsilon^m_t \sim N(0, \sigma^2_m)$, $\varepsilon^a_{i,1,t} \sim N(0, \sigma^2_a)$, and $\varepsilon^a_{i,2,t} \sim N(0, \sigma^2_a)$

3. Find $z^m_{i,t}$, $z^a_{i,1,t}$, and $z^a_{i,2,t}$, given policy functions,

$$
\begin{aligned}
g^a_1\left(x_{i,1,t-1}, x_{i,2,t-1}, z^a_{i,1,t-1}, z^a_{i,2,t-1}, z^m_{i,t-1}\right) \\
g^a_2\left(x_{i,1,t-1}, x_{i,2,t-1}, z^a_{i,1,t-1}, z^a_{i,2,t-1}, z^m_{i,t-1}\right) \\
g^m\left(x_{i,1,t-1}, x_{i,2,t-1}, z^a_{i,1,t-1}, z^a_{i,2,t-1}, z^m_{i,t-1}\right).
\end{aligned}
$$

4. Calculate standard deviations of signal noises and Kalman gains from

$$
\begin{aligned}
z^m_{i,t} &= \left(1 - \mathcal{K}^m_{i,t}\right)\left(z^m_{i,t-1} + \sigma^2_m\right) \\
z^a_{i,1,t} &= \left(1 - \mathcal{K}^a_{i,1,t}\right)\left(z^a_{i,1,t-1} + \sigma^2_1\right) \\
z^a_{i,2,t} &= \left(1 - \mathcal{K}^a_{i,2,t}\right)\left(z^a_{i,2,t-1} + \sigma^2_1\right)
\end{aligned}
$$

and

$$
\mathcal{K}^m_{i,t} = \frac{z^m_{i,t-1} + \sigma^2_m}{z^m_{i,t-1} + \sigma^2_m + \eta^2_{i,m,t}},\ \mathcal{K}^a_{i,1,t} = \frac{z^a_{i,1,t-1} + \sigma^2_1}{z^a_{i,1,t-1} + \sigma^2_1 + \eta^2_{i,1,t}},\ \mathcal{K}^a_{i,2,t} = \frac{z^a_{i,2,t-1} + \sigma^2_2}{z^a_{i,2,t-1} + \sigma^2_2 + \eta^2_{i,2,t}}.
$$

Then

$$
\begin{aligned}
\eta^2_{i,m,t} &= \frac{z^m_{i,t}\left(z^m_{i,t-1} + \sigma^2_m\right)}{z^m_{i,t-1} + \sigma^2_m - z^m_{i,t}} \\
\eta^2_{i,1,t} &= \frac{z^a_{i,1,t}\left(z^a_{i,1,t-1} + \sigma^2_1\right)}{z^a_{i,1,t-1} + \sigma^2_1 - z^a_{i,1,t}} \\
\eta^2_{i,2,t} &= \frac{z^a_{i,2,t}\left(z^a_{i,2,t-1} + \sigma^2_1\right)}{z^a_{i,2,t-1} + \sigma^2_1 - z^a_{i,2,t}}
\end{aligned}
$$

5. Generate random numbers for signal noises $\xi_{i,m,t} \sim \mathcal{N}\left(0, \eta^2_{i,m,t}\right)$, $\xi_{i,1,t} \sim \mathcal{N}\left(0, \eta^2_{i,1,t}\right)$, $\xi_{i,2,t} \sim \mathcal{N}\left(0, \eta^2_{i,2,t}\right)$.

6. Calculate the perceived gap(markup) **after observing their signals** at t

$$
\begin{aligned}
x_{i,1,t} &= x_{i,1,t-1} - \left[\mathcal{K}^m_{i,t}\left(s^m_{i,t} - m_{i,t-1|t-1}\right) + \mathcal{K}^a_{i,1,t}\left(s^a_{i,1,t} - a^a_{i,1,t-1|t-1}\right)\right] \\
&= x_{i,1,t-1} - \Big[\mathcal{K}^m_{i,t}\left(m_{t-1} - m_{i,t-1|t-1} + \varepsilon^m_t + \xi_{i,m,t}\right) \\
&\qquad + \mathcal{K}^a_{i,1,t}\left(a^a_{i,1,t-1} - a^a_{i,1,t-1|t-1} + \varepsilon^a_{i,1,t} + \xi_{i,1,t}\right)\Big]
\end{aligned}
$$

$$
\begin{aligned}
x_{i,2,t} &= x_{i,2,t-1} - \left[\mathcal{K}^m_{i,t}\left(s^m_{i,t} - m_{i,t-1|t-1}\right) + \mathcal{K}^a_{i,2,t}\left(s^a_{i,2,t} - a^a_{i,2,t-1|t-1}\right)\right] \\
&= x_{i,2,t-1} - \Big[\mathcal{K}^m_{i,t}\left(m_{t-1} - m_{i,t-1|t-1} + \varepsilon^m_t + \xi_{i,m,t}\right) \\
&\qquad + \mathcal{K}^a_{i,2,t}\left(a^a_{i,2,t-1} - a^a_{i,2,t-1|t-1} + \varepsilon^a_{i,2,t} + \xi_{i,2,t}\right)\Big]
\end{aligned}
$$

where

$$\begin{aligned}
a_{i,1,t} - a_{i,1,t|t} &= \left(1 - \mathcal{K}^a_{i,1,t}\right)\left(a_{i,1,t-1} - a_{i,1,t-1|t-1}\right) + \varepsilon^a_{i,1,t} - \mathcal{K}^a_{i,1,t}\left(\varepsilon^a_{i,1,t} + \xi_{i,1,t}\right) \\
a_{i,2,t} - a_{i,2,t|t} &= \left(1 - \mathcal{K}^a_{i,2,t}\right)\left(a_{i,2,t-1} - a_{i,2,t-1|t-1}\right) + \varepsilon^a_{i,2,t} - \mathcal{K}^a_{i,2,t}\left(\varepsilon^a_{i,2,t} + \xi_{i,2,t}\right) \\
m_t - m_{i,t|t} &= \left(1 - \mathcal{K}^m_{i,t}\right)\left(m_{t-1} - m_{i,t-1|t-1}\right) + \varepsilon^m_t - \mathcal{K}^m_{i,t}\left(\varepsilon^m_t + \xi_{i,m,t}\right)
\end{aligned}$$

with given $a_{i,1,-1} - a_{i,1,-1|-1} = 0$, $a_{i,2,1} - a_{i,2,-1|-1} = 0$, and $m_{-1} - m_{i,-1|-1} = 0$.

7. Price changes: for $j = \{a, B\}$,

$$\Delta p_{i,j,t} = \begin{cases} 0 & \text{if } -\theta + \beta V\left(0, 0, z^a_{i,1,t}, z^a_{i,2,t}, z^m_{i,t}\right) \\ & \qquad \leq -\left[\left(x_{i,1,t}\right)^2 + \left(x_{i,2,t}\right)^2\right] + \beta V\left(x_{i,1,t}, x_{i,2,t}, z^a_{i,1,t}, z^a_{i,2,t}, z^m_{i,t}\right) \\ -x_{i,j,t} & \text{if } -\theta + \beta V\left(0, 0, z^a_{i,1,t}, z^a_{i,2,t}, z^m_{i,t}\right) \\ & \qquad > -\left[\left(x_{i,1,t}\right)^2 + \left(x_{i,2,t}\right)^2\right] + \beta V\left(x_{i,1,t}, x_{i,2,t}, z^a_{i,1,t}, z^a_{i,2,t}, z^m_{i,t}\right) \end{cases}$$

8. True markup: for $j = \{a, B\}$,

$$\begin{aligned}
\mu_{i,j,t} &= p_{i,j,t} - m_t - a_{i,j,t} \\
&= \Delta p_{i,j,t} + x_{i,j,t-1} - \left(m_{t-1} - m_{i,t-1|t-1}\right) - \left(a_{i,j,t-1} - a_{i,j,t-1|t-1}\right) - \varepsilon^m_t - \varepsilon^a_{i,j,t}
\end{aligned}$$

where $a_{i,j,-1} - a_{i,j,-1|-1} = 0$ and $m_{-1} - m_{i,-1|-1} = 0$.

E Discussion on Model Assumptions

In this appendix, I discuss two key assumptions in my baseline model with both menu costs and rational inattention. First, I provide some evidence of firm-specific menu costs from the survey data. Second, I discuss implications and limitations of the assumptions about the set of available information.

Firm-Specific Menu Costs. In the baseline two-good version of the model, economies of scope in price setting emerge from the existence of firm-specific menu costs. Previous literature found ample evidence of the firm-specific menu costs for multi-product firm. For example, recent work by Stella (2018) and Letterie and Nilsen (2016) directly estimate various types of adjustment costs for the multi-product firms and find that there are sizable component of costs from firm-specific menu costs.[62] In the New Zealand survey data, I also find some evidence of firm-specific menu costs. Managers were asked about how typical it is to synchronize price reviews and price changes across

[62] Bhattarai and Schoenle (2014) and Lach and Tsiddon (2007) test the implications of menu cost models with a single fixed menu cost and find that micro price data support the existence of economies of scope for multi-product firms. Moreover, Lach and Tsiddon (1996) Fisher and Konieczny (2000), and Midrigan (2011) find that price changes within multi-product firms are highly synchronized, suggesting the existence of firm-specific menu costs.

multiple products sold by their firms. They report that on average 75% of their price changes and price review decisions are synchronized within their firms.

While the firm-specific fixed cost implies perfect within-firm synchronization of price changes, the data show that firms synchronize their price changes partially. Bonomo et al. (2019a) also find partial synchronization using Israel retail price data and show that even small departures from full synchronization in menu costs models substantially weaken monetary non-neutrality. This implies that introducing a product-specific menu cost in my baseline model would weaken economies of scope in price setting. In this case, the real effects of monetary shocks in the two-good version of the model will be much smaller than those in the single-product model.

Set of Available Information. In my main model analysis, I assume that the set of available signals has three properties. First, the firm chooses $N + 1$ independent signals for each shock, implying that paying attention to aggregate conditions and paying attention to good-specific idiosyncratic conditions are separate activities. Although this assumption is often made in the rational inattention literature, such as Maćkowiak and Wiederholt (2009) and Pasten and Schoenle (2016), it might be suboptimal for firms to choose to observe independent signals. In fact, Afrouzi and Yang (2019) show that in LQG setting rational inattention models (without menu costs), the number of signals that firms choose to observe are no more than the number of actions. In my model, the number of actions is N as firms choose N prices for their goods, implying that firms might waste their resources to observe additional signals.

Second, I assume that every signal is Gaussian. Gaussian signals are optimal when the underlying shocks are Gaussian and firms' objective function is quadratic. However, in my model, firms' objective is not quadratic due to menu costs. Recent rational inattention literature considers models with general objective functions with some assumptions of a simple stochastic process, a static setup or finite actions and states (Matějka, 2015; Jung et al., 2019; Steiner et al., 2017). Solving fully non-linear dynamic problems under rational inattention is computationally demanding as firms' state variable is an infinitely dimensional object if the shocks are continuously distributed. The assumption of Gaussian signals is for tractability.

Third, I assume that all noise in signals is idiosyncratic and independent. This assumption is without loss of generality since I consider Shannon's mutual information as the cost of information (Denti, 2015; Afrouzi, 2019b).

F Additional Tables and Figures

Table A.1: Summary Statistics for Number of Products

Industries	Obs.	Mean	Median	Std. Dev.	Max.
Total	712	67.4	9	234.2	2115
Total without Retail/Wholesale Trade	627	9.55	7	8.47	48
– Manufacturing	278	9.57	8	7.75	39
– Professional and Financial Services	276	7.95	7	6.09	35
– Other Services	37	14.49	13	11.63	48
– Construction and Transportation	36	8.42	5	8.92	40

Notes: This table reports summary statistics for firms' number of products by sectors. The number of products of each firm is measured from answers to the following question in the second wave of New Zealand Firms' Expectation Survey: "*In addition to your main product or product line, how many other products do you sell?*" See Coibion et al. (2018a) for details about the survey data. Moments are calculated using sampling weights.

Table A.2: Summary Statistics for (Absolute) Backcast Errors about Inflation by Industries

Industries	Quartile 1		Quartile 2		Quartile 3		Quartile 4	
	N	Mean (S.D.)	*N*	Mean (S.D.)	*N*	Mean (S.D.)	*N*	Mean (S.D.)
Total	1-4	5.01 (4.56)	5-9	5.83 (4.93)	10-15	3.89 (5.20)	>15	2.47 (2.99)
Total without Retail/Wholesale	1-3	5.23 (4.10)	4-7	6.30 (5.03)	8-14	4.40 (5.57)	>14	3.53 (3.62)
– Manufacturing	1-4	1.52 (2.19)	5-8	1.70 (1.86)	9-14	2.25 (2.74)	>14	2.46 (2.56)
– Professional and Financial Services	1-3	6.42 (3.17)	4-8	6.16 (4.65)	9-12	7.00 (3.57)	>12	5.51 (3.71)
– Other Services	1-5	2.29 (1.97)	6-13	0.72 (0.46)	14-21	0.76 (0.52)	>21	0.90 (0.51)
– Construction and Transportation	1-3	7.46 (5.06)	4-5	7.38 (4.55)	6-9	10.82 (5.34)	>9	7.36 (7.59)

Notes: This table reports summary statistics for firms' (absolute) backcast errors about aggregate inflation by quartiles of the distribution of the number of products in each industry. The backcast errors are the absolute value of firm errors about past 12 month inflation from Wave #1 survey. Moments are calculated using sampling weights.

Table A.3: Number of Products and Knowledge about Aggregate Inflation (All Industries)

	(1)	(2)	(3)	(4)
Panel A. Dependent variable: Inflation backcast errors				
log(number of products)	-0.401***	-0.103***	-0.210***	-0.0565*
	(0.069)	(0.030)	(0.063)	(0.033)
Observations	670	656	504	492
R-squared	0.225	0.835	0.269	0.897
Panel B. Dependent variable: Willingness to pay for professional inflation forecasts				
log(number of products)	-2.391	2.924**	-3.548**	3.884***
	(1.852)	(1.224)	(1.607)	(1.185)
Observations	436	429	375	373
R-squared	0.101	0.624	0.190	0.640
Firm-level controls	Yes	Yes	Yes	Yes
Industry fixed effects		Yes		Yes
Manager controls			Yes	Yes

Notes: This table reports results for the Huber robust regression. Dependent variables are the absolute value of firm errors about past 12 month inflation from Wave #1 survey (Panel A) and firms' willingness to payment for professional forecaster's forecasts about future inflation from Wave #4 (Panel B). Firm-level controls include log of firms' age, log of firms' employment, foreign trade share, number of competitors, firms' beliefs about price difference from competitors, and the slope of the profit function. Industry fixed effects include dummies for 17 sub-industries. Manager controls include the age of the respondent (each firm's manger), education, income, and tenure at the firm. Sample weights are applied to all specifications. Robust standard errors (clustered at the 3-digit ANZ SIC level) are reported in parentheses. ***, **, * denotes statistical significance at 1%, 5%, and 10% levels respectively.

Table A.4: Model Predictions and Monetary Non-Neutrality

	Menu Costs	Calvo/Taylor	Rational Inattention	Sticky/Noisy Info/ Observational Costs
1) Attentiveness to Aggregate Conditions	Full information	Full information	Increase in N	Independent of N
2) Size/Duration of Price Changes	Decrease in N	Independent of N	Flexible micro price	Independent of N
Monetary Non-Neutrality	Increase in N	Independent of N	Decrease in N	Independent of N

Notes: This table shows the predictions from different models about 1) the relationship between firms' number of products and their attentiveness to aggregate condition and 2) the relationship between firms' number of products and the duration and size of price changes. N stands for the number of products that firms sell in each model. See Section 2.3 for details.

Table A.5: Number of Products and Knowledge about Nominal GDP Growth

	(1)	(2)	(3)	(4)
Dependent variable: Backcast errors about nominal GDP growth rate				
Number of products	-0.033**	-0.025***	-0.032**	-0.026**
	(0.015)	(0.009)	(0.014)	(0.011)
Observations	387	380	332	328
R-squared	0.354	0.569	0.364	0.577
Firm-level controls	Yes	Yes	Yes	Yes
Industry fixed effects		Yes		Yes
Manager controls			Yes	Yes

Notes: This table reports results for the Huber robust regression. Dependent variable is the absolute value of firm errors about the growth rate of nominal GDP from Wave #4 survey. Firms' perceived growth rate of nominal GDP are calculated by taking the summation of firms' belief about current inflation and the real GDP growth rate in New Zealand. Firm-level controls include log of firms' age, log of firms' employment, foreign trade share, number of competitors, firms' beliefs about price difference from competitors, and the slope of the profit function. Industry fixed effects include dummies for 14 sub-industries excluding retail and wholesale trade sectors. Manager controls include the age of the respondent (each firm's manger), education, income, and tenure at the firm. Sample weights are applied to all specifications. Robust standard errors (clustered at the 3-digit ANZ SIC level) are reported in parentheses. ***, **, * denotes statistical significance at 1%, 5%, and 10% levels respectively.

Table A.6: Subjective Uncertainty and Expected Duration of Price Changes

	(1)	(2)	(3)	(4)
Dependent variable: Duration of expected next price changes				
Standard deviation of the growth rate	0.128***	0.094**	0.152***	0.161**
of sales over the next 12 months	(0.040)	(0.043)	(0.056)	(0.067)
Observations	581	589	442	441
R-squared	0.322	0.700	0.337	0.428
Firm-level controls	Yes	Yes	Yes	Yes
Industry fixed effects		Yes		Yes
Manager controls			Yes	Yes

Notes: This table reports results for the Huber robust regression. Dependent variable is the duration of expected next price changes from Wave #2. The regressor is the standard deviation implied by the reported probability distribution for the growth rates of unit sales of firms' main product over the next 12 months. Firm-level controls include log of firms' age, log of firms' employment, the number of competitors, and log of firms' number of products. Industry fixed effects include dummies for 14 sub-industries excluding retail and wholesale trade sectors. Manager controls include the age of the respondent (each firm's manger), education, income, and tenure at the firm. Sample weights are applied to all specifications. Robust standard errors (clustered at the 3-digit ANZ SIC level) are reported in parentheses. ***, **, * denotes statistical significance at 1%, 5%, and 10% levels respectively.

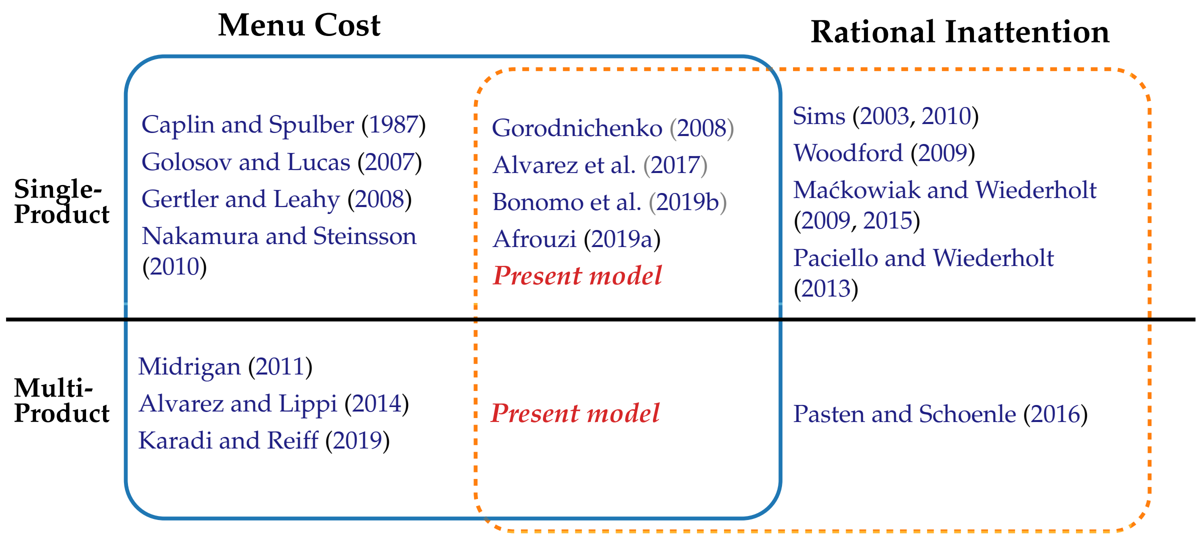


Figure A.1: Classification of Models

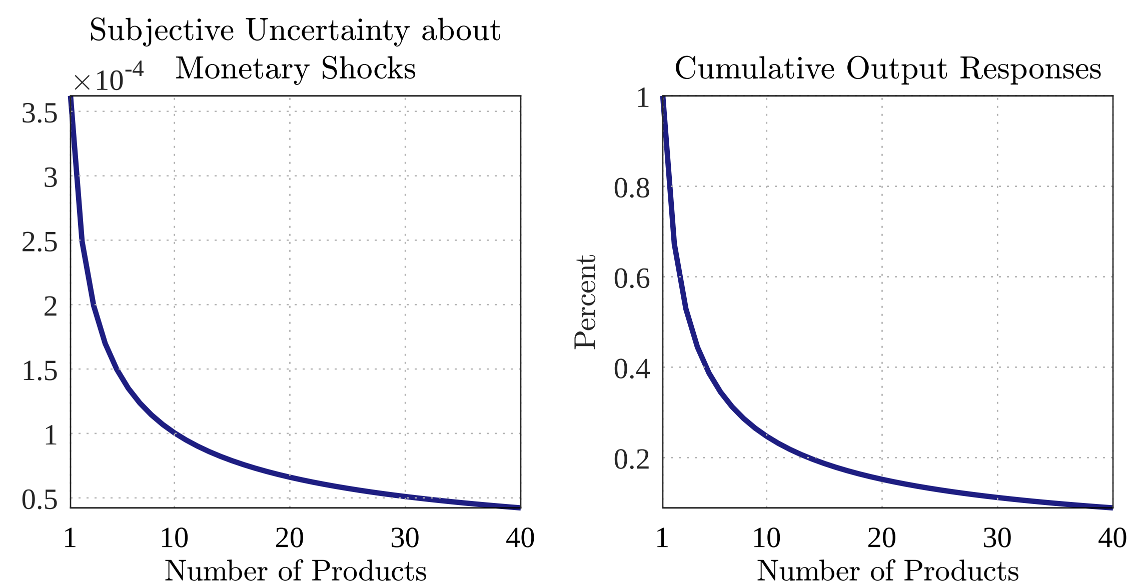


Figure A.2: Subjective Uncertainty and Cumulative Output Responses in a Rational Inattention Only Model

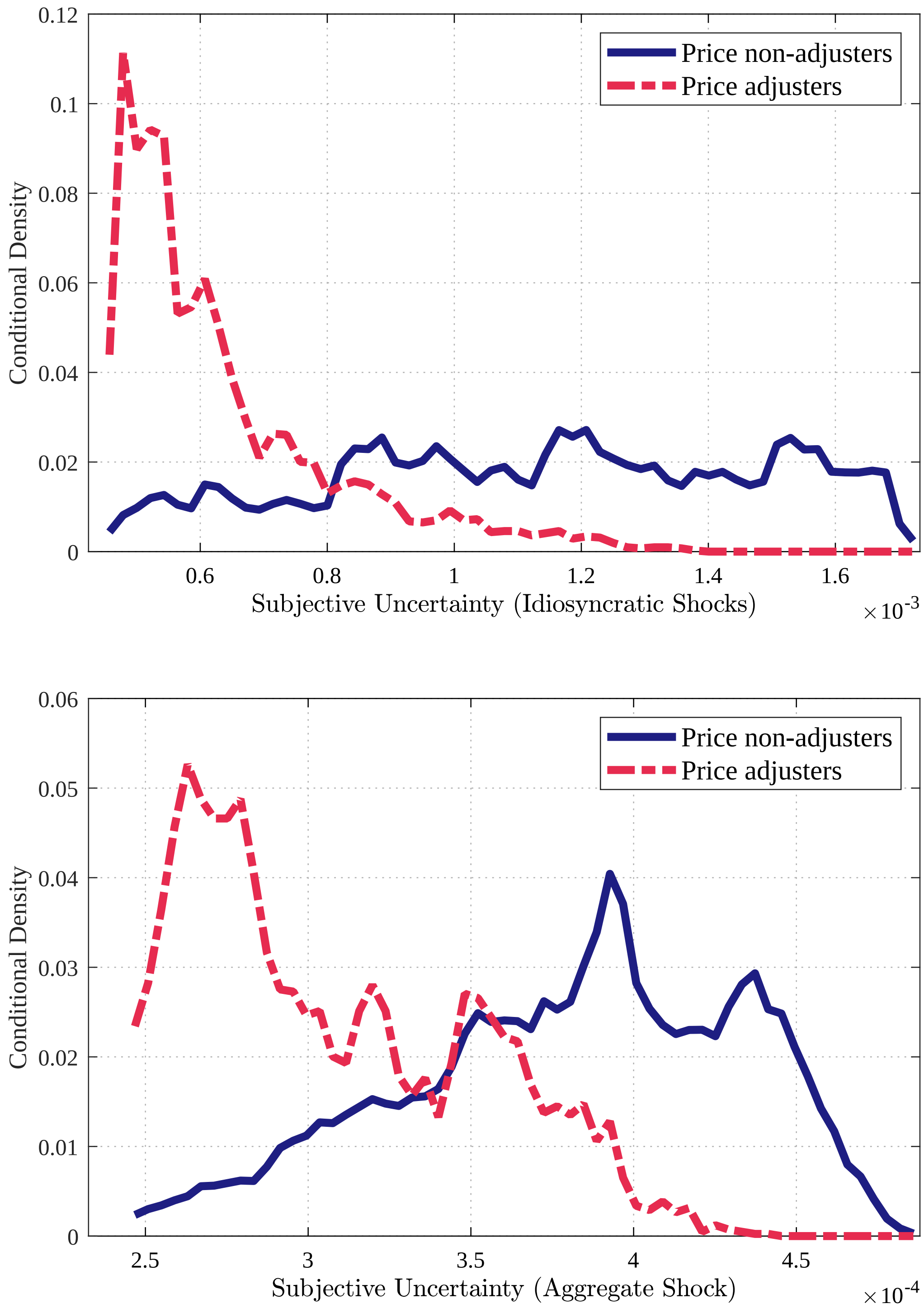


Figure A.3: Distribution of Subjective Uncertainty about the Idiosyncratic and Aggregate Shocks: Single-Product Baseline Model

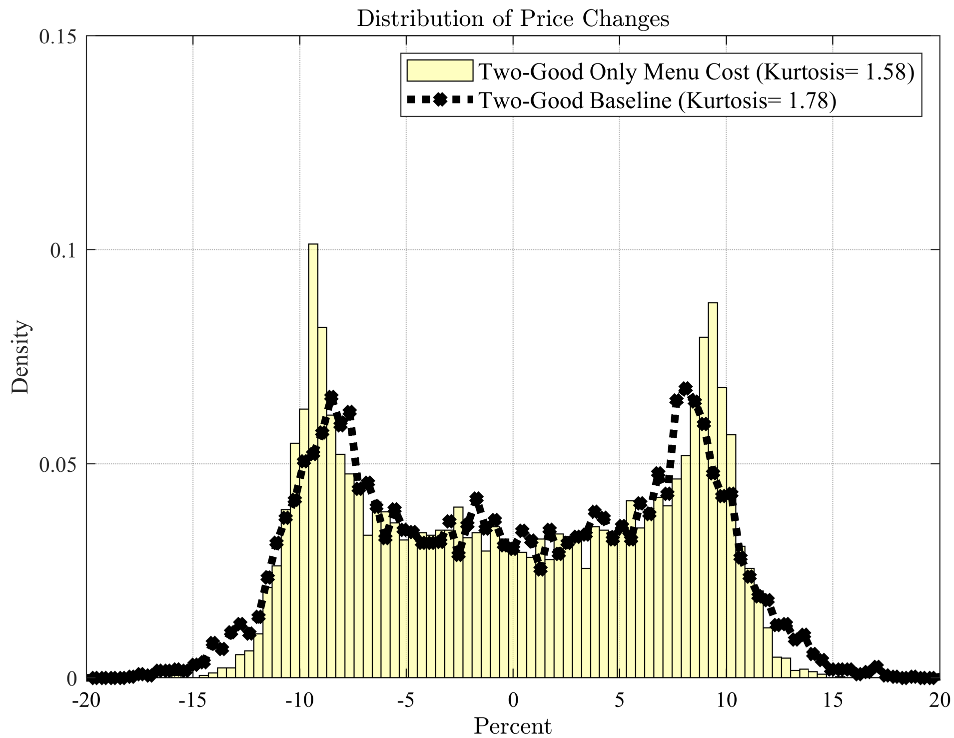


Figure A.4: Distribution of Price Changes in the Two-Product Models

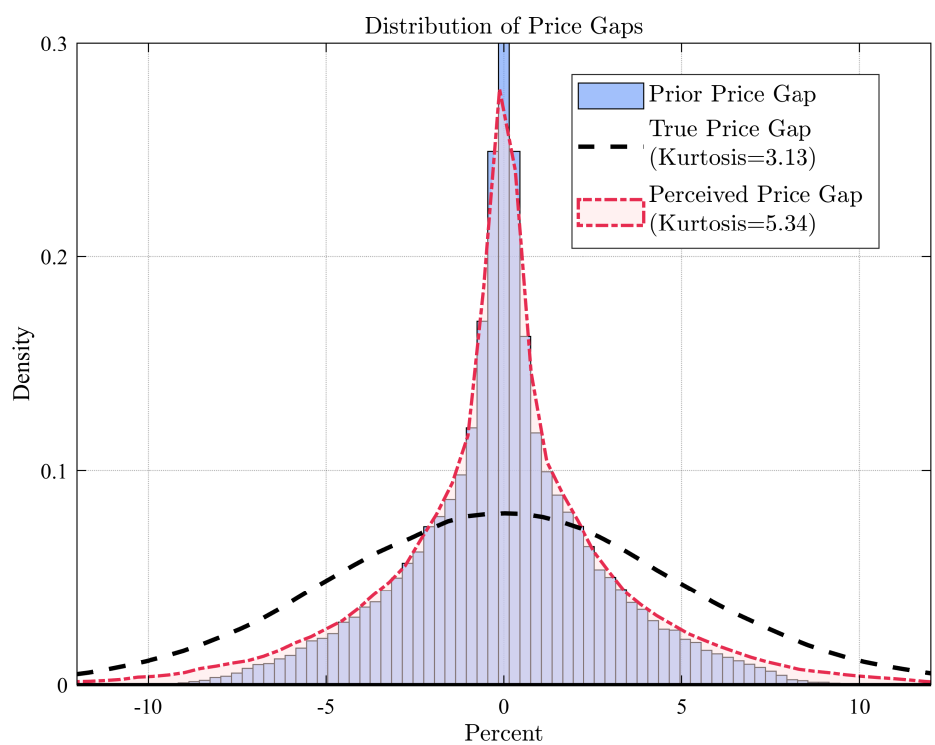


Figure A.5: Distribution of True and Perceived Price Gaps in the Two-Product Baseline Model

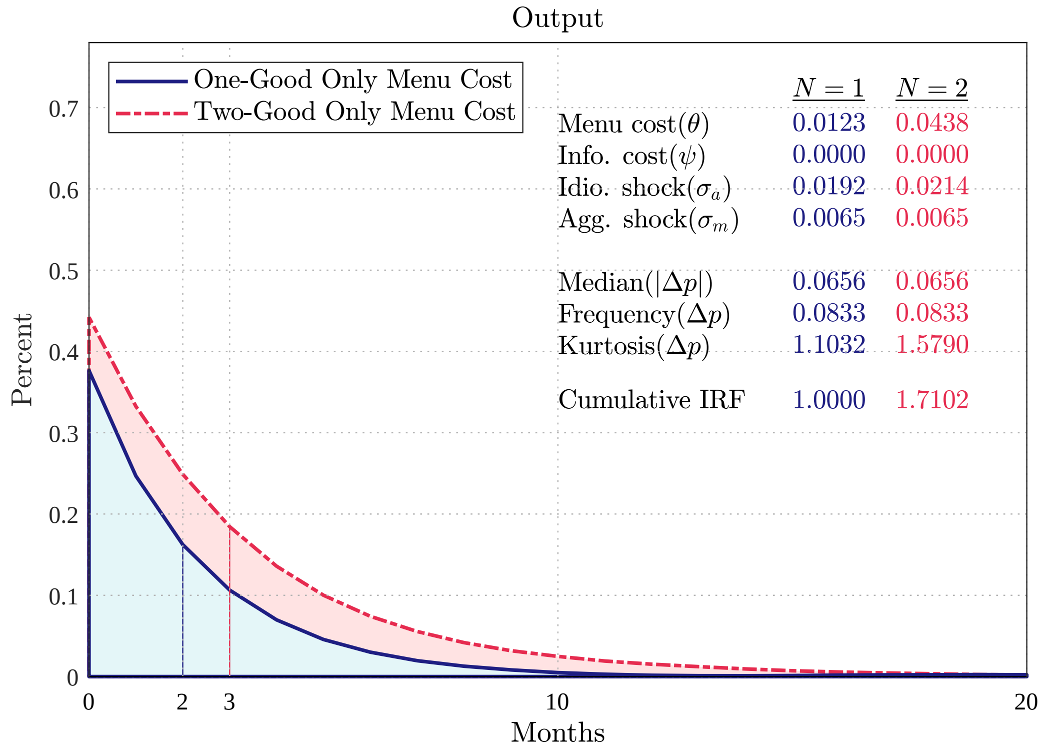


Figure A.6: Impulse Response of Output to a One S.D. Monetary Policy Shock in the Only Menu Costs Models

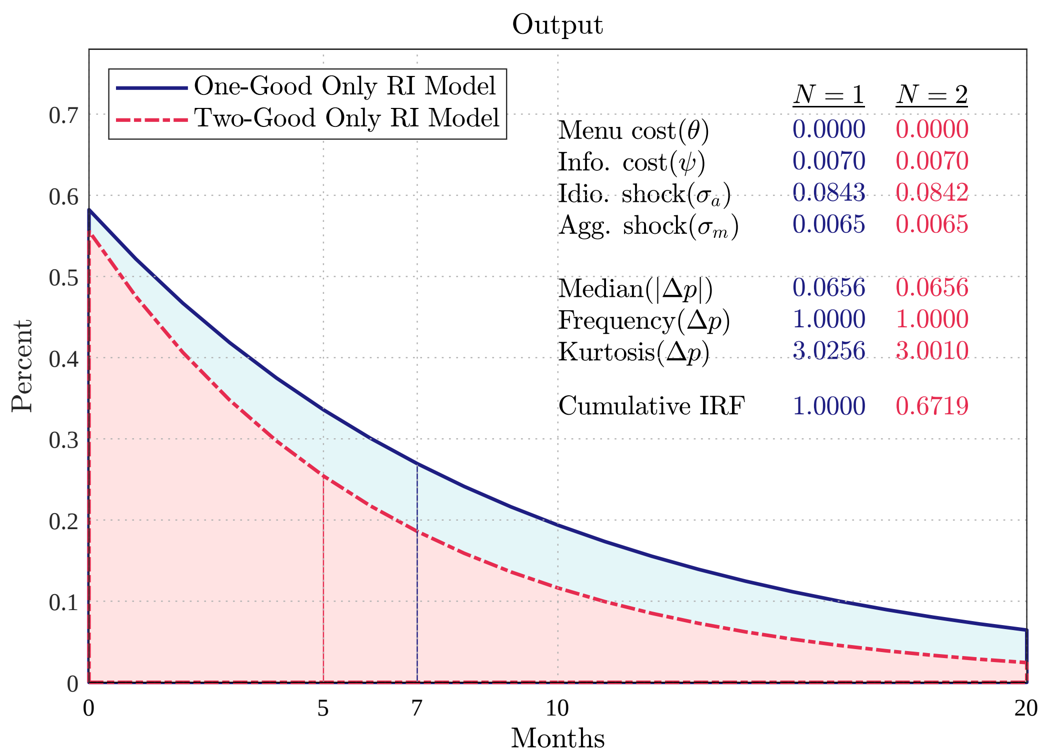


Figure A.7: Impulse Response of Output to a One S.D. Monetary Policy Shock in the Only Rational Inattention Models

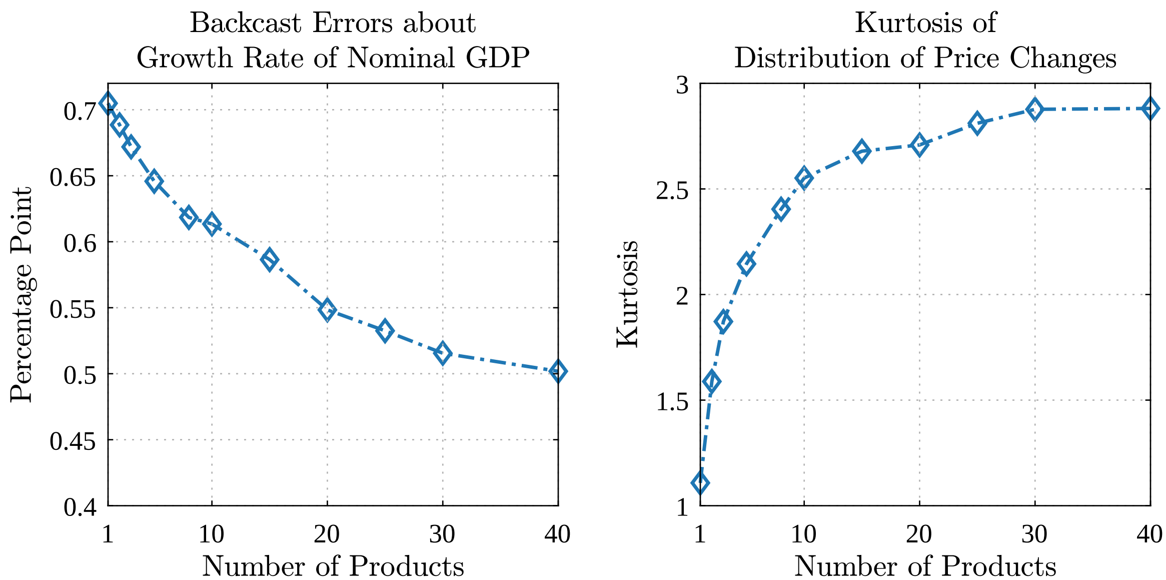


Figure A.8: Backcast Errors about Growth Rate of Nominal GDP and Kurtosis of Price Change Distribution in the Simplified Models

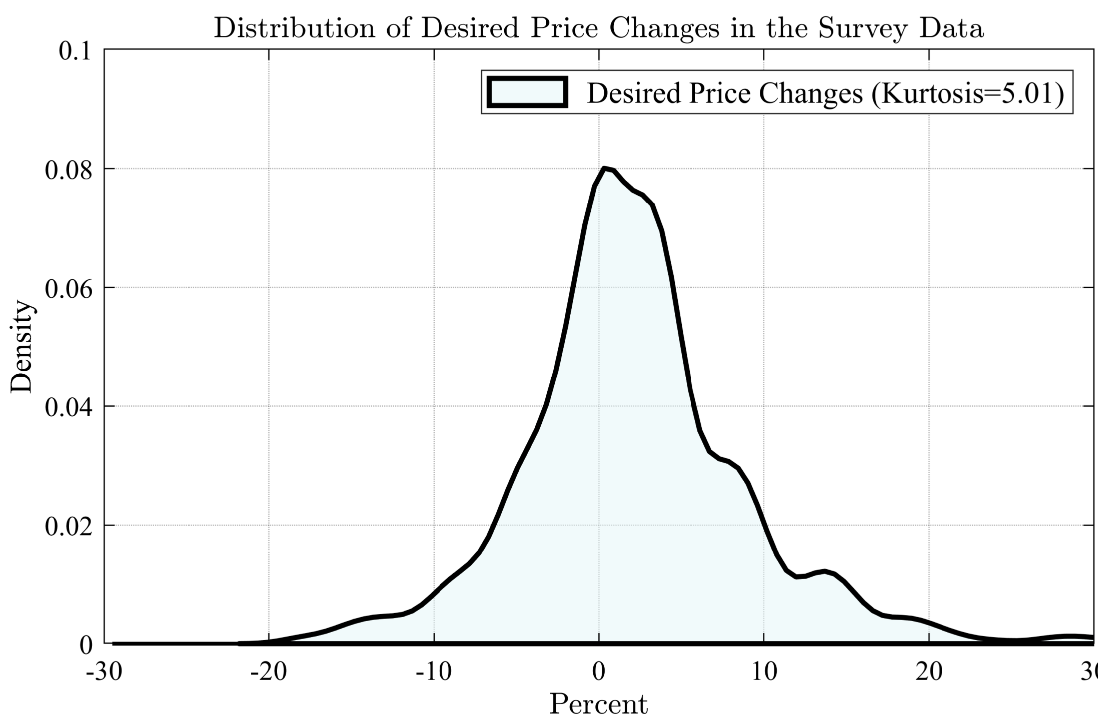


Figure A.9: Distribution of Desired Price Changes in the Survey Data