Macroeconomic Effects of Capital Tax Rate Changes*

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Abstract

We study aggregate, distributional, and welfare effects of a permanent reduction in the capital tax rate in a quantitative equilibrium model with capital-skill complementarity. Such a tax reform leads to expansionary long-run aggregate effects, but is coupled with an increase in wage and income inequality. Moreover, the expansionary aggregate effects are smaller when distortionary labor or consumption tax rates have to increase to finance the capital tax rate cut, driven by effects on labor supply decisions. An extension to a model with heterogeneous households shows that consumption inequality also increases in the long-run, which leads to a further rise in wage inequality. We study transition dynamics and show that joint modeling of monetary and fiscal policy response is important for analyzing short-run effects. Finally, we contrast the long-term aggregate welfare gains with short-term losses, regardless of how the tax cut is financed. In the model with heterogeneous households, we additionally show that welfare gains for the skilled go together with welfare losses for the unskilled.

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1 Introduction

The macroeconomic effects of permanent capital tax cuts have recently become a subject of widespread discussion, spurred by the recent U.S. tax reform that reduced the corporate tax rate from 35% to 21%. Several questions have been raised. What are the long-run and the short-run effects on output, investment, and consumption? What are the distributional consequences in terms of wage, income, and consumption inequality? What are the welfare implications? Will such a large tax cut be self-financing? If not, what are the positive and normative implications of alternative ways to finance the reform? How does the monetary policy response matter for the short-run effects of a capital tax cut?

Given the nature of these questions, it is useful to pursue an analysis through the lens of a quantitative dynamic model. Moreover, since the tax reform is large-scale, it is imperative to consider general equilibrium effects. This paper therefore addresses these questions using a quantitative, dynamic equilibrium model. Compared to existing studies on the effects of capital tax changes, our analysis is integrative in terms of the questions and the model used to address them. We consider several features, such as (equipment) capital-skill complementarity, household heterogeneity, and different types of long-run fiscal adjustments as well as monetary and fiscal policy interactions that matter for the short-run, in a unified framework. These, along with other model elements we include for realistic dynamics, while common in various macroeconomic studies, have not been analyzed simultaneously for our main research questions. We present the long-run equilibrium, as well as full (nonlinear) transition dynamics, and along the way highlight the role of these various features by comparing to a neoclassical benchmark.

The paper starts with a long-run analysis. We show analytically in a simplified model and numerically in the quantitative model with capital-skill complementarity that capital tax cuts, as expected, have expansionary long-run aggregate effects on the economy. In particular, with a permanent reduction of the capital tax rate from 35% to 21%, output in the new steady state, compared to the initial steady state, is greater by 3.8%, structure investment by 19.7%, equipment investment by 6.8%, and consumption by 2.1%, in our baseline calibration. Moreover, skilled wages increase by 3.4%, unskilled wages by 2.7%, and both skilled and unskilled hours increase.

The mechanism for aggregate effects is well understood. A reduction in the capital tax rate leads to a decrease in the rental rate of capital, raising demand for capital by firms. This stimulates investment and capital accumulation. A larger amount of capital stock, in turn, makes workers more productive, raising wages and hours. Finally, given the increase in the factors of production, output expands, which also increases consumption in the long-run.

The way the government finances the capital tax cuts matters through its effect on labor supply decisions. The estimates above are obtained in a scenario in which the government has the ability to finance the capital tax cuts in a completely non-distorting way by cutting back lump-sum transfers. Such a financing leads to increased hours in the long-run. When the government has to rely on distortionary labor or consumption taxes, the effectiveness of capital tax cuts is smaller. We show this result again both analytically in a simplified model and numerically in the quantitative model.

For instance, in our baseline calibration, a permanent reduction of the capital tax rate from 35% to 21% requires an increase in the labor tax rate by 2.7% points.[1] Then, output in the new steady state, compared to the initial steady state, is greater by 1.96%, equipment investment by 4.96%, structures investment by 17.61%, and consumption by 0.31%. The reason for the smaller boost in aggregate variables is a decline of the after-tax wages, by 0.24% for skilled workers and 0.90% for unskilled workers. This in turn leads to a decrease in labor hours in the long-run, for both skilled and unskilled workers.

While skill heterogeneity and capital-skill complementarity have relatively small aggregate effects, they generate important distributional implications. As mentioned above, skilled wages rise relatively more, which leads to a rise in the skill premium of 1.06% points under lump-sum transfer adjustment. This long-run rise in wage inequality is driven by the rise in equipment capital, which raises skilled wages as there is equipment capital-skill complementarity. The capital tax cut thus favors those workers whose skill is not easily substituted by capital. In addition, a measure of income inequality, the ratio of after-tax capital income to labor income, increases.

Furthermore, in an extended model with heterogeneous households where unskilled households cannot smooth consumption over time, consumption inequality also increases in the long-run. In fact, unskilled consumption decreases in the long-run as a result of a decrease in either transfers or after-tax labor income in this extended model. The rise in consumption inequality in turn generates a further increase in wage inequality through wealth effects on labor supply. The two sources of heterogeneity we introduce thus interact in economically meaningful ways to generate new distributional effects.

When it comes to aggregate implications, however, skill and household heterogeneity play a relatively insignificant role. The key reason is again endogenous labor supply decisions. Introducing such heterogeneity moves skilled and unskilled hours in opposite directions relative to where they would be otherwise. These two countervailing forces then contribute to a small differential in aggregate output.[2]

We then move to an analysis of transition dynamics as the economy evolves from the initial steady-state to the new steady-state. During the transition, the economy experiences a decline in not only consumption, as a result of the need for financing greater capital accumulation, but also output. This holds even if lump-sum transfers finance the capital tax rate cut.[3] Consumption and output falls are more severe under distortionary tax rate adjustment, although tax rate smoothing can moderate the contraction. The short-run contraction may be viewed as another side effect of

[1] We keep debt-GDP ratio the same between the initial and the new steady-state. Debt-GDP ratio, however, is allowed to deviate from the steady-state level along the transition path, when we study short-run effects.

[2] We do find a small increase in aggregate output as we sequentially add capital-skill complementarity and household heterogeneity to the model. For instance, under lump-sum transfer adjustment, output increases by 4.2% in our extended model with heterogeneous households (compared to 3.8% in the baseline model and 3.6% in a simplified model which abstracts from capital-skill complementarity).

[3] The short-run fall in output is a result of investment adjustment cost, nominal rigidities, and our empirical monetary policy rule.

a permanent capital tax cut besides the increase in inequality in the long-run. We revisit the issue of these side effects more formally by looking at welfare implications.

Another important aspect of transition dynamics that we highlight is on the need to analyze monetary and fiscal policy adjustments jointly. This is because the short-run effects depend critically on the monetary policy response. In particular, when the government has access only to distortionary labor taxes, we consider the central bank directly accommodating inflation to facilitate government debt stabilization along the transition. In this interesting scenario, the government does not raise labor tax rates as much, and the rise of inflation in the short-run completely negates any short-run contraction in output as well as consumption.

Finally, while our paper does not study optimal policy, we analyze welfare consequences of the permanent capital tax rate cut, given the various financing possibilities we consider. In the baseline quantitative model, we show that long-term aggregate welfare gains contrast with short-term (but, still prolonged) welfare losses, regardless of how the capital tax rate cut is financed. In the extended model with heterogeneous households, we show that the skilled gain at the expense of the unskilled. Unskilled workers suffer from welfare losses both in the short- and long-run as they consume less (due to either reduced transfers or a lower after-tax wage rate,) which in turn forces them to work more through wealth effects on labor supply.

On welfare implications of alternate financing possibilities, we emphasize two results. First, the inflating-away-debt policy described above, while intriguing as it prevents output losses and in fact also increases consumption in the short-run, is still inadvisable. The main reason is the inflation-driven inefficiency that creates a resource misallocation in our model with nominal rigidities. This in turn leads to an increase in the required labor hours to produce a given amount of final output. Such a policy hence does not generate a welfare improvement, even in the short-run. Second, financing a capital tax cut by lump-sum transfer adjustment, while leading to a higher level of aggregate output, is not necessarily better than financing it by labor tax adjustment. Intuitively, when some agents' income relies relatively more on transfers (the low-skilled households in our extended model), a reduction in transfer to offset tax revenue losses from capital tax cuts can decrease their welfare. Moreover, even aggregate social welfare can decrease in such a case. We use a version of our model with heterogeneous households to illustrate this possibility.[4]

Related literature Given the integrative nature of our analysis, this paper is related to several strands of the literature, some of which have been developed without much interaction with each other. While we focus mostly on a positive analysis, analytically and quantitatively assessing the macroeconomic effects of a given reduction in the capital tax rate, our paper is related to classic normative analysis of Chamley (1986) and Judd (1985), which was re-addressed recently in Straub

[4]Regarding the distribution of unearned income, in our baseline case, high-skilled households receive profits from firms and low-skilled households receive transfers from the government. We however, parameterize our model in a way such that other distribution possibilities can be easily explored.

and Werning (2020).[5] We do not analyze optimal policy issues, but do compute welfare implications given the capital tax rate cut and various financing rules we consider.

Our analysis of the central bank allowing inflation to directly facilitate debt stabilization, through passive monetary policy, when the government has access to only distortionary labor taxes relates this paper to the literature on monetary and fiscal policy interactions – in particular, the normative analysis in Sims (2001). We implement this scenario using a rules-based positive description of interest rate policy, as in Leeper (1991), Sims (1994), and Woodford (1994) for instance.[6] Relatedly, our work is also motivated by the study of effects of government spending and how that depends on the monetary policy response, as highlighted recently by Christiano, Eichenbaum and Rebelo (2011), Woodford (2011), and Leeper, Traum and Walker (2017).

In terms of analyzing the long-run effects of changes in the capital tax rate in an equilibrium macroeconomic model, our paper is close to Trabandt and Uhlig (2011) and the more recent work of Barro and Furman (2018) that analyzes the U.S. tax reform. Compared to this literature, one key difference is that our baseline model features capital-skill complementarity, following Krusell et al. (2000), such that wage inequality issues can be analyzed. Trabandt and Uhlig's (2011) focus is on the important issue of the presence of Laffer curves for labor and capital tax, under either transfer or government spending adjustment. We show, both analytically and numerically, how the macroeconomic effects of a given capital tax rate change are different depending on whether non-distortionary or distortionary sources of government financing are available. In addition, we study transition dynamics in detail as well, highlighting that it is imperative to model monetary and fiscal policy adjustments jointly for determining short-run effects, and explore aggregate, distributional, and welfare implications taking dynamics fully into account.

Barro and Furman's (2018) recent important contribution studies macroeconomic implications of a given capital tax rate change, like we do, in a model with more details of the tax code and five types of capital. Our baseline model is simpler in that respect, but features endogenous labor supply such that distortionary sources of capital tax reform financing can be explored carefully. Moreover, as mentioned above, we also study wage inequality (and in an extension, consumption inequality) implications, transition dynamics, and welfare properties.

Another closely related paper is Domeij and Heathcote (2004). Similar to our study, they explicitly take into account both transition dynamics and steady-state change after a tax reform in the welfare analysis, and show a capital tax rate reduction is not Pareto improving. We also find such a result in this paper. Domeij and Heathcote's (2004) model, however, abstracts from capital-skill complementarity. The more recent work of Slavík and Yazici (2019) uses a model

[5]The literature on optimal capital taxation in a dynamic setting is extensive. Earlier work typically finds significant welfare gains from eliminating capital income taxes in representative-household infinite-horizon frameworks. More recent studies move away from the standard setup, featuring heterogeneous households and overlapping generations, and find optimality of non-zero capital taxes (e.g. Aiyagari 1995, Erosa and Gervais 2002, and Conesa, Kitao and Krueger 2009.)

[6]In this case, the central bank does not follow the Taylor principle. Bhattarai, Lee and Park (2014) analytically characterize the effects of such a case in a model with sticky prices. Canzoneri, Cumby and Diba (2010), Leeper and Leith (2016), and Cochrane (2019) provide an excellent survey of the literature.

with capital-skill complementarity and analyzes the effects of a tax reform that eliminates tax differentials between equipment and structure capital.

Besides the different research questions we aim to address, our paper is different from these two contributions in modeling choices. Their models feature a richer form of household heterogeneity, as in Aiyagari (1994), while our model is more stylized in that dimension, focusing on a particular type of heterogeneity as in the tradition of the Two Agent New Keynesian (TANK) literature. Our analysis thus misses potentially important implications of a realistic wealth distribution. It however, allows us to include a richer set of model elements that enable us to conduct a more realistic analysis of transition dynamics. Furthermore, our empirically motivated specifications of monetary and fiscal policy, coupled with model elements that matter for transition, allow us to consider positive and welfare implications of non-trivial monetary and fiscal policy interactions.

As mentioned above, the way we introduce household heterogeneity in the extended version of our model connects our paper to the growing TANK literature. This literature has analyzed extensively various issues on monetary policy (e.g. Bilbiie 2008, Bhattarai, Lee and Park 2015, Cúrdia and Woodford 2016, and Debortoli and Galí 2017.) On the fiscal side, Galí, López-Salido and Vallés (2007), Bilbiie, Monacelli and Perotti (2013) and Eggertsson and Krugman (2012) have considered the effects of government spending and (lump-sum) transfers. Much of the literature, however, abstracts from capital accumulation (with the exception of Galí, López-Salido and Vallés 2007), thereby precluding an analysis of capital income taxes. Moreover, our model importantly also features capital-skill complementarity, which allows us to analyze the effect of a policy change on the wage distribution.[7]

There is by now a fairly large dynamic stochastic general equilibrium modeling literature that assesses the effects of distortionary tax rate changes and of fiscal policy generally. For instance, among others, Forni, Monteforte and Sessa (2009) study transmission of various fiscal policies, including government spending and transfer changes in a quantitative model. Sims and Wolff (2018) additionally study state-dependent effects of tax rate changes. These papers often study effects of transitory and small changes in the tax rate while our main focus is on the long-run effects of a permanent reduction in the capital tax rate under various sources of financing, and then on an analysis of full (nonlinear) transition dynamics following a fairly large reduction. Additionally, we provide several analytical results that help illustrate the key mechanisms on the long-run effects, while in the quantitative part, we use a model that can assess distributional consequences.

While we are motivated by the particular recent U.S. episode of a permanent tax rate change, generally, our paper is influenced also by a large literature that empirically assesses the macroeconomic effects of tax policy. In particular, various identification strategies, such as narrative (Romer and Romer 2010) and statistical (Blanchard and Perotti 2002, Mountford and Uhlig 2009) have been used to assess equilibrium effects of tax changes. Relatedly, House and Shapiro (2008) study a particular case of change in investment tax incentive. The effects on aggregate variables that

[7]On a technical side, the existing literature typically focuses on linear dynamics around a steady state. We present exact, nonlinear transition dynamics in a TANK model with capital-skill complementarity.

we find using a calibrated equilibrium model is consistent with this work, although these papers have generally focused either explicitly on temporary tax policies or do not explicitly separate out permanent changes from transitory ones. We also use our model to assess several distributional and welfare effects following a permanent capital tax rate cut.

2 Model

We now present the baseline model, which is a standard quantitative equilibrium framework augmented with two types of workers (skilled and unskilled) and two types of capital (structures and equipment). We introduce equipment capital-skill complementarity following Krusell et al. (2000), and a skill premium arises endogenously in the model. This framework allows us to study both aggregate and wage inequality implications of a capital tax rate change in a unified way.[8] The model also features adjustment costs in investment, variable capacity utilization, and nominal pricing frictions to enable a realistic study of transition dynamics. Pricing frictions additionally enable an analysis of the role of monetary policy for the transition dynamics.

2.1 Private sector

We start by describing the maximization problems of the private sector.

2.1.1 Households

There are two types of households who supply skilled labor (type s) and unskilled labor (type u), respectively. The measure of type-i household for $i \in \{s, u\}$ is denoted by N^i. The type-i household's problem is to

$$\max_{\left\{C_t^i, H_t^i, B_t^i, I_{b,t}^i, I_{e,t}^i, \hat{K}_{b,t+1}^i, \hat{K}_{e,t+1}^i, u_{e,t}^i, u_{b,t}^i, V_{t+1}^i\right\}} \quad E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U\left(C_t^i, H_t^i\right) \right\}$$

[8] While the baseline model does feature two types of workers, the growth rate of the marginal utility of consumption is equalized between the two types. The model is thus equivalent to a model with a representative family whose members are either skilled or unskilled workers. In an extension, heterogeneous households are introduced to study consumption inequality.

subject to a sequence of flow budget constraints

$$
\begin{aligned}
\left(1+\tau_t^C\right) P_t C_t^i + P_t I_{b,t}^i + P_t I_{e,t}^i + B_t^i + E_t Q_{t,t+1} V_{t+1}^i & \\
= \left(1-\lambda_{\tau^H}^i \tau_t^H\right) W_t^i H_t^i + R_{t-1} B_{t-1}^i + V_t^i & \\
+ \left(1-\tau_t^K\right) R_t^{K,b} u_{b,t}^i \hat{K}_{b,t}^i + \left(1-\tau_t^K\right) R_t^{K,e} u_{e,t}^i \hat{K}_{e,t}^i & \\
+ \lambda_b \tau_t^K P_t I_{b,t}^i + \lambda_e \tau_t^K P_t I_{e,t}^i & \\
- P_t \left(1-\lambda_b \tau_t^K\right) \mathcal{A}_b\left(u_{b,t}^i\right) \hat{K}_{b,t}^i - \frac{P_t}{q_t}\left(1-\lambda_e \tau_t^K\right) \mathcal{A}_e\left(u_{e,t}^i\right) \hat{K}_{e,t}^i & \\
+ P_t \frac{\chi_\Phi^i}{N^i} \Phi_t + P_t \frac{\chi_S^i}{N^i} S_t, &
\end{aligned}
$$

where E_t is the mathematical expectation operator, C_t^i is consumption, H_t^i is hours, and $I_{b,t}^i$ and $I_{e,t}^i$ are investment in the capital stock of structures and equipment denoted by $\hat{K}_{b,t}^i$ and $\hat{K}_{e,t}^i$, respectively. Similarly, $K_{b,t}^i \equiv u_{b,t}^i \hat{K}_{b,t}^i$ and $K_{e,t}^i \equiv u_{e,t}^i \hat{K}_{e,t}^i$ are the effective structure and equipment capital and $u_{b,t}^i$ and $u_{e,t}^i$ are the respective variable capacity utilization rates. $\mathcal{A}_b(u_{b,t}^i)$ and $\mathcal{A}_e(u_{e,t}^i)$ are the costs of variable capital utilization.

Households trade one-period state-contingent nominal securities V_{t+1}^i at price $Q_{t,t+1}$ in period t so as to fully insure against idiosyncratic risks. Thus, there is complete consumption insurance in the model. They trade nominal risk-less one-period government bonds B_t^i as well. Type-i households are paid a fraction χ_Φ^i of the aggregate profits Φ_t from the firms and a fraction χ_S^i of the aggregate lump-sum transfers S_t from the government.[9] The aggregate price level is P_t, W_t^i is the nominal wage for type-i households, R_t is the nominal one-period interest rate, and $R_t^{K,b}$ and $R_t^{K,e}$ are the rental rate of capital structures and equipment, respectively.

The government levies taxes on consumption, labor income, and capital income with tax rates τ_t^C, τ_t^H, and τ_t^K, respectively. The parameter $\lambda_{\tau^H}^i$ is introduced to allow for differential effective labor tax rates on the two types of households and λ_b and λ_e are the rates of expensing of capital investment in structures and equipment, respectively. The discount factor is β.

The evolutions of the two types of capital stock are described by

$$
\begin{aligned}
\hat{K}_{b,t+1}^i &= (1-d_b)\, \hat{K}_{b,t}^i + \left(1-\mathcal{S}\left(\frac{I_{b,t}^i}{I_{b,t-1}^i}\right)\right) I_{b,t}^i, \\
\hat{K}_{e,t+1}^i &= (1-d_e)\, \hat{K}_{e,t}^i + \left(1-\mathcal{S}\left(\frac{I_{e,t}^i}{I_{e,t-1}^i}\right)\right) I_{e,t}^i q_t,
\end{aligned}
$$

where q_t is the relative price between investment in capital structures and equipment and d_b and d_e are the rates of depreciation of the capital stock invested in structures and equipment, respectively.[10]

The period utility $U(C_t, H_t)$, investment adjustment cost $\mathcal{S}\left(\frac{I_t}{I_{t-1}}\right)$, and variable capacity uti-

[9] Due to complete markets (or equivalently, a large family who supplies both types of labor) in the baseline model, the share of profits or fraction of transfers allocated to a particular type of household does not matter regardless of how the capital tax rate cuts are financed.

[10] As we describe in detail later, this relative price is exogenous to ensure balanced growth in the model.

lization cost $\mathcal{A}(u_t)$ have standard properties, which are detailed later.

2.1.2 Firms

The model has final goods firms and intermediate goods firms. Perfectly competitive final goods firms produce aggregate output Y_t by combining a continuum of differentiated intermediate goods, indexed by $i \in [0,1]$, using the CES aggregator given by $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}$, where $\theta > 1$ is the elasticity of substitution between intermediate goods. The corresponding optimal price index P_t for the final good is $P_t = \left(\int_0^1 P_t(i)^{1-\theta} di\right)^{\frac{1}{1-\theta}}$, where $P_t(i)$ is the price of intermediate good i and the optimal demand for good i is $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t$. The final good is used for private and government consumption as well as investment in capital structures and equipment.

Monopolistically competitive intermediate goods firms, indexed by i, produce output using a CRS production function $F(.)$

$$Y_t(i) = F\left(A_t, K_{b,t}(i), K_{e,t}(i), L_{s,t}(i), L_{u,t}(i)\right), \tag{1}$$

where A_t is an exogenous stochastic process that represents technological progress, with its gross growth rate given by $a_t \equiv \frac{A_t}{A_{t-1}} = \bar{a}$.[11] As we describe later, we follow Krusell et al. (2000) in functional form assumptions on $F(.)$, a nested CES formulation, and parameterizations of the elasticities of substitution across factors such that it features (equipment) capital-skill complementarity. Firms rent two types of capital and hire two types of labor in perfectly competitive factor markets.

Intermediate goods firms face nominal rigidity. As in Calvo (1983), a firm resets its price optimally with probability $1-\alpha_P$ every period. Firms that do not optimize adjust their price according to the indexation rule $P_t(i) = P_{t-1}(i)\,\pi_{t-1}^{\gamma_P}\bar{\pi}^{1-\gamma_P}$, where γ_P measures the extent of dynamic indexation and $\bar{\pi}$ is the steady-state value of the gross inflation rate $\pi_t \equiv P_t/P_{t-1}$.

Optimizing firms choose a common price P_t^* to solve their problem

$$\max_{\{P_t^*, Y_{t+k}(i), H_{t+k}(i), K_{t+k}(i)\}} E_t\left\{\sum_{k=0}^{\infty}(\alpha_p\beta)^k \frac{\Lambda_{t+k}}{\Lambda_t} P_{t+k}\Phi_{t+k}(i)\right\},$$

subject to (1), where Λ_t is the marginal utility of nominal income and flow profit $\Phi_t(i)$ is given by

$$\Phi_{t+k}(i) = \frac{P_t^*}{P_{t+k}} X_{P,t,k} Y_{t,t+k}(i) - \frac{W_{t+k}^u}{P_{t+k}} L_{u,t+k}(i) - \frac{W_{t+k}^s}{P_{t+k}} L_{s,t+k}(i) - \frac{R_{t+k}^{K,b}}{P_{t+k}} K_{b,t+k}(i) - \frac{R_{t+k}^{K,e}}{P_{t+k}} K_{e,t+k}(i),$$

where

$$X_{P,t,k} = \begin{cases} \left(\pi_t \pi_{t+1}\cdots\pi_{t+k-1}\right)^{\gamma_P} \bar{\pi}^{(1-\gamma_P)k}, & k \geq 1 \\ 1 & k = 0 \end{cases}$$

[11] Steady-state of a variable x is denoted by $\overline{x}$ throughout. As we discuss later, we restrict preferences and technology such that the model is consistent with balanced growth.

and

$$Y_{t,t+k}\left(i\right)=\left(\frac{P_t^* X_{P,t,k}}{P_{t+k}}\right)^{-\theta}Y_{t+k}.$$

Note that there is a skill premium in the model, which we define as the wage of skilled labor relative to that of unskilled labor, $\frac{W_t^s}{W_t^u}$. Given the CRS production function and the assumption of perfectly competitive factor markets, the factor prices are equal to marginal products of each factor multiplied by firms' marginal costs. Moreover, as we show in detail later, if capital-skill complementarity exists, the skill premium increases in the amount of equipment capital when the quantities of the two types of labor inputs are held fixed. It is also increasing in the ratio of unskilled to skilled labor.

2.2 Government

We now describe the constraint on the government and how it determines monetary and fiscal policy.

2.2.1 Government budget constraint

The government flow budget constraint, written by expressing fiscal variables as ratio of output, is given by

$$\frac{B_t}{P_tY_t}+\tau_t^C\frac{C_t}{Y_t}+\tau_t^H\left(\lambda_{\tau^H}^s\frac{W_t^s}{P_tY_t}L_{s,t}+\lambda_{\tau^H}^u\frac{W_t^u}{P_tY_t}L_{u,t}\right)+\tau_t^K\left(\frac{R_t^{K,b}}{P_tY_t}K_{b,t}-\lambda_b\left(I_{b,t}+\mathcal{AC}_{b,t}\right)+\frac{R_t^{K,e}}{P_tY_t}K_{e,t}-\lambda_e\left(I_{e,t}+\mathcal{AC}_{e,t}\right)\right)$$
$$=R_{t-1}\frac{B_{t-1}}{P_{t-1}Y_{t-1}}\frac{1}{\pi_t}\frac{Y_{t-1}}{Y_t}+\frac{G_t}{Y_t}+\frac{S_t}{Y_t},$$

where $B_t=\sum_{i\in\{s,u\}}N^iB_t^i$, $S_t=\sum_{i\in\{s,u\}}N^iS_t^i$, $S_t^i=\frac{\chi_S^i}{N^i}S_t$, $\mathcal{AC}_{b,t}=\sum_{i\in\{s,u\}}N^i\mathcal{A}_b(u_{b,t}^i)\hat{K}_{b,t}^i$, $\mathcal{AC}_{e,t}=\sum_{i\in\{s,u\}}N^i\mathcal{A}_e(u_{e,t}^i)\hat{K}_{e,t}^i$, and G_t is government spending on the final good.[12]

2.2.2 Monetary policy

Monetary policy is given by a interest-rate feedback rule as in Coibion and Gorodnichenko (2011),

$$\frac{R_t}{\bar{R}}=\left[\frac{R_{t-1}}{\bar{R}}\right]^{\rho_1^R}\left[\frac{R_{t-2}}{\bar{R}}\right]^{\rho_2^R}\left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{Y_{t-1}}\right)^{\phi_{\Delta y}}\left(\frac{Y_t}{Y_t^n}\right)^{\phi_x}\right]^{\left(1-\rho_1^R-\rho_2^R\right)}, \tag{2}$$

where ρ_1^R and ρ_2^R govern interest rate smoothing, $\phi_\pi\geq 0$ is the feedback parameter on inflation, Y_t^n is the natural (that is, flexible price) level of output, $\phi_{\Delta y}$ is the feedback parameter on output growth, ϕ_x is the feedback parameter on output gap, and $\bar{R}$ is the steady-state value of R_t. For large enough feedback coefficients (a combination of ϕ_π, $\phi_{\Delta y}$, and ϕ_x), the Taylor principle is

[12] We introduce government spending in the model for a realistic calibration. As we discuss later, government spending-to-GDP ratio is held fixed throughout in the model.

satisfied.[13] We will also consider a case, described in more detail next, where the Taylor principle is not satisfied, and inflation response will play a direct role in government debt stabilization along the transition.

2.2.3 Fiscal policy

We consider a one-time permanent change in the capital tax rate τ_t^K in period 0, when the economy is in the initial steady-state. In order to isolate the effects of the capital tax rate cut, $\frac{G_t}{Y_t}$ is kept unchanged from its initial steady-state value in all periods. The debt-to-GDP ratio, $\frac{B_t}{P_t Y_t}$, may deviate from the initial steady-state in the short-run but will converge back to the initial steady-state in the long-run, through appropriate changes in fiscal instruments. We consider the following four policy adjustments. First, only lump-sum transfers adjust to maintain $\frac{B_t}{P_t Y_t}$ constant at each point in time.[14]

Second, only labor tax rates τ_t^H adjust following a feedback rule similar to the monetary policy rule specification in Coibion and Gorodnichenko (2011),

$$\begin{aligned}\tau_t^H - \bar{\tau}_{new}^H =& \rho_1^H \left(\tau_{t-1}^H - \bar{\tau}_{new}^H\right) + \rho_2^H \left(\tau_{t-2}^H - \bar{\tau}_{new}^H\right) \\ &+ \left(1 - \rho_1^H - \rho_2^H\right) \left\{ \psi_B^H \left(\frac{B_{t-1}}{P_{t-1} Y_{t-1}} - \overline{\frac{B}{PY}} \right) + \psi_{\Delta y}^H \left(\frac{Y_t}{Y_{t-1}} \right) + \psi_x^H \left(\frac{Y_t}{Y_t^n} \right) \right\},\end{aligned} \tag{3}$$

where $0 \leq \rho_1^H + \rho_2^H < 1$ governs labor tax rate smoothing, $\psi_B^H \geq 0$ is the feedback parameter on outstanding debt, $\psi_{\Delta y}^H$ is the feedback parameter on output growth, ψ_x^H is the feedback parameter on output gap, $\bar{\tau}_{new}^H$ is the new steady-state value of τ_t^H, and $\overline{\frac{B}{PY}}$ is the (initial and new) steady-state value of $\frac{B_t}{P_t Y_t}$.[15] A large enough feedback coefficient on debt (high ψ_B^H) ensures that fiscal policy leads to stationary debt dynamics.[16] This will be our baseline labor tax policy formulation.

Third, only consumption tax rates τ_t^C adjust following the simple feedback rule

$$\begin{aligned}\tau_t^C - \bar{\tau}_{new}^C =& \rho_1^C \left(\tau_{t-1}^C - \bar{\tau}_{new}^C\right) + \rho_2^C \left(\tau_{t-2}^C - \bar{\tau}_{new}^C\right) \\ &+ \left(1 - \rho_1^C - \rho_2^C\right) \left\{ \psi_B^C \left(\frac{B_{t-1}}{P_{t-1} Y_{t-1}} - \overline{\frac{B}{PY}} \right) + \psi_{\Delta y}^C \left(\frac{Y_t}{Y_{t-1}} \right) + \psi_x^C \left(\frac{Y_t}{Y_t^n} \right) \right\},\end{aligned} \tag{4}$$

[13] In the textbook linearized sticky price model, this condition is $\phi_\pi > 1$. In the model here, as there are several endogenous propagation mechanisms and an empirically grounded interest rate rule, such a condition has to be determined numerically, although $\phi_\pi > 1$ is often a good benchmark.

[14] Since transfers are lump-sum and there is complete risk-sharing in the baseline model, the time-path of transfers does not matter, and so we just use a simple formulation.

[15] Feedback rules for fiscal policy were estimated in an early contribution by Bohn (1998). Bhattarai, Lee and Park (2016) estimate slightly simpler versions than above in a general equilibrium model with lump-sum taxes. Note importantly that in (3), distortionary tax rates adjust smoothly during the transition. This is motivated by the theoretical analysis of Barro (1979), but also, by our empirical estimates of tax rules that take this form. For completeness and comparison, we will also consider a case where labor tax rates adjust as necessary to ensure a constant debt-to-GDP ratio throughout the transition.

[16] In the textbook linearized model with one source of taxes, this condition is $\psi_B^H > \beta^{-1} - 1$. In the model here, we solve for non-linear dynamics and additionally, as there are several sources of taxes and an empirically grounded tax rule, such a condition has to be determined numerically.

where $0 \leq \rho_1^C + \rho_2^C < 1$ governs consumption tax rate smoothing, $\psi_B^C \geq 0$ is the feedback parameter on outstanding debt, $\psi_{\Delta y}^C$ is the feedback parameter on output growth, ψ_x^C is the feedback parameter on output gap, and $\bar{\tau}_{new}^C$ is the new steady-state value of τ_t^C. A large enough feedback coefficient on debt (high ψ_B^C) ensures that fiscal policy leads to stationary debt dynamics. This will be our baseline consumption tax policy formulation.

For transition dynamics, the behavior of the monetary authority generally matters. In the three fiscal policies described above, the monetary policy rule (2) satisfies the Taylor principle, which thereby, implies that inflation plays no direct role in government debt stabilization. Moreover, given our restrictions that ψ_B^H and ψ_B^C are high enough, taxes respond strongly enough to ensure that debt dynamics are mean-reverting.

We consider a fourth case to highlight the role of monetary policy response to inflation for transition dynamics. In this case, labor taxes adjust, but not sufficiently, as the tax rule response coefficients are not large enough, and inflation partly plays a direct role in government debt stabilization, as the monetary rule response coefficients are not large enough. The monetary and labor tax rules are still given by (2) and (3), but now with these appropriate restrictions on the feedback parameters. Thus, in this fourth case, we allow debt stabilization, (only) along the transition, to occur partly through distortionary labor taxes and partly through inflation.[17]

2.3 Equilibrium and functional forms

The equilibrium definition is standard, given the maximization problems of the private sector and the monetary and fiscal policy described above. We also have perfect risk sharing across households. Goods, asset, and factor markets clear in equilibrium.

The economy features balanced growth. As we describe below, we use standard assumptions on preferences that ensure balanced growth. Moreover, since our production function features two types of capital and capital-skill complementarity, we impose an additional assumption on the growth rate of q_t, the exogenous relative price between investment in capital structures and equipment. Generally, we normalize variables growing along the balanced growth path by the level of technology. Fiscal variables, as mentioned above, are normalized by output. We use the notation, for instance, $\tilde{Y}_t \equiv \frac{Y_t}{\gamma^t}$ and $\tilde{b}_t \equiv \frac{B_t}{P_t Y_t}$ to denote these stationary variables where γ is the growth rate of output. We also use the notation T_t^C, T_t^H, and T_t^K to denote (real) consumption, labor, and capital tax revenues. Nominal variables are denoted in real terms in small case letters, for instance, $w_t = \frac{W_t}{P_t}$. All the equilibrium conditions are derived and given in detail in the Appendix A.6.

We use the following functional forms for preferences and technology

$$U(C_t^i, H_t^i) \equiv \log C_t^i - \bar{\varpi}^i \frac{\left(H_t^i\right)^{1+\varphi}}{1+\varphi},$$

[17]An analogous consumption tax rule, with low enough response to debt, generates similar results and is thus omitted here. Moreover, while we consider these various fiscal/monetary adjustment scenarios to investigate how both positive and normative results depend on policy choices, our analysis is not in the Ramsey policy tradition.

$$F\left(A_t, K_{b,t}, K_{e,t}, L_{s,t}, L_{u,t}\right) \equiv A_t\left(K_{b,t}\right)^{\alpha}\left[\mu L_{u,t}^{\sigma}+(1-\mu)\left(\lambda\left(K_{e,t}\right)^{\rho}+(1-\lambda)\left(L_{s,t}\right)^{\rho}\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\alpha}{\sigma}},$$

and standard functional forms for the investment adjustment cost and the variable capacity utilization cost

$$\mathcal{S}\left(\frac{I_t}{I_{t-1}}\right) \equiv \frac{\xi}{2}\left(\frac{I_t}{I_{t-1}}-\gamma\right)^2, \ \mathcal{A}\left(u_t\right) \equiv \chi_1\left(u_t-1\right)+\frac{\chi_2^2}{2}\left(u_t-1\right)^2.$$

The utility function is a standard one and consistent with balanced growth. The production function $F(\cdot)$ is a nested CES structure used in Krusell et al. (2000). This implies that equipment capital and skilled labor have the same elasticity of substitution against unskilled labor, given by $1/(1-\sigma)$. The elasticity of substitution between equipment capital and skilled labor is $1/(1-\rho)$. Capital-skill complementarity exists when $\sigma>\rho$. The parameters μ and λ govern income shares. Note that when $\rho \to 0$, the production function reduces to a standard Cobb-Douglas formulation, which we will use for analytical results. Suppose that the gross growth rate of A_t is $\bar{a} = \gamma^{1-\alpha}$. We assume that the exogenous relative price between consumption (structures) and equipment investment, q_t, grows at rate $\gamma_q = 1/\gamma$, which leads to balanced growth of the model. It follows that all growing variables except A_t grow at rate γ.[18]

3 Long-Run Results

This section presents our results on long-run effects of permanent changes in the capital tax rate. We consider three versions of the model. A simplified model, a special case in which we shut down the role of skill heterogeneity and thus capital-skill complementarity, comes first and permits various analytical solutions. It is then followed by the baseline model presented above. Finally, an extended model in which we introduce imperfect risk-sharing between skilled and unskilled households is presented. We present the models in this order to highlight sequentially the role of capital-skill complementarity and household heterogeneity.

In each case, we consider three different fiscal policy adjustments to ensure that the government debt-to-GDP ratio is at the same level in the long-run, as stated above in Section 2.2.3. The first is by (non-distortionary) transfer adjustment, which we take as the starting point. We then look at how a distortionary adjustment of labor tax rate and consumption tax rate alters results.

3.1 Analytical results of a simplified model

We now present several analytical results that help clarify the mechanisms regarding long-run aggregate effects. For this, we simplify the model presented above such that it converges to a textbook business cycle model. In particular, we first assume $\rho \to 0$ to get a nested version of the

[18] King, Plosser and Rebelo (2002) describes the required restrictions on preferences and technology in the standard neoclassical model. Balanced growth with capital-skill complementarity in the production function was shown in Maliar and Maliar (2011), who pointed out the need to have an exogenous path for relative price between consumption (structures) and equipment and restrictions on the growth rate.

model with a Cobb-Douglas production function. It is also assumed the two share parameters to be zero, $\mu = \alpha = 0$, and the fraction of skilled households to be 1, $N^S = 1$. In this case, we now have one type of capital $K_{e,t}$ and one type of labor $L_{s,t}$ and a standard Cobb-Douglas production function that implies a unit elasticity of substitution between capital and labor. In the analytical results below, we then drop subscripts e and s for variables. We also for simplicity do not have expensing of the tax rate.

While there is no skill premium in this simple model, these analytical results on aggregate effects are relevant as not only do they show the mechanisms and provide intuition, but also because as we show later, our baseline model with capital-skill complementarity has similar predictions for aggregate effects, even quantitatively, as the simpler case presented here.

3.1.1 Lump-sum transfer adjustment

We start with the case where lump-sum transfers adjust to finance the capital tax rate cut. Capital tax cuts, as expected, have expansionary long-run effects on the economy. It is useful to state a mild restriction on government spending in steady-state as an assumption.[19]

Assumption 1. $\bar{\tilde{G}} < 1 - \frac{\theta-1}{\theta}\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\left(1-\bar{\tau}^K\right) = 1 - \frac{1}{\lambda}\left(\frac{\bar{\tilde{I}}}{\bar{\tilde{Y}}}\right)$ *in the initial steady-state.*

Then, we can show that a permanent capital tax rate cut leads to an increase in output, consumption, investment, and wages, and a decline in the rental rate of capital in the model. We state this formally below in Lemma 1.

Lemma 1. *Fix $\bar{\tau}^H$ and $\bar{\tilde{b}}$. With lump-sum transfer adjustment,*

1. *Rental rate of capital is increasing, while capital to hours ratio, wage, hours, capital, investment, and output are decreasing in $\bar{\tau}^K$.*
2. *Under Assumption 1, consumption is also decreasing in $\bar{\tau}^K$.*

Proof. See Appendix C.2. □

Intuition for this result is well-understood. A reduction in the capital tax rate leads to a decrease in the rental rate of capital, raising firms' demand for capital. This stimulates investment and capital accumulation. The capital-to-labor ratio increases as a result. A larger amount of capital stock, in turn, makes workers more productive, raising wages and hours. Given the increase in the factors of production, output increases, which also raises consumption unless the steady-state ratio of government spending-to-GDP is unrealistically very high, as ruled out by Assumption 1.[20]

Additionally, we can also derive an exact solution for the change in macroeconomic quantities and factor prices, as well as an approximate solution for small changes in the capital tax rates that are intuitive to understand and sign. We state this formally below in Proposition 1. Note that the results below are in terms of changes from the original steady-state.

[19] This restriction is very mild, and is just to ensure that government spending in steady-state is not very high. For instance, except for a case of an unrealistically high markup, this holds for any reasonable parameterization of government spending in steady-state.

[20] In such a case, government consumption or investment crowds out private consumption.

Proposition 1. *Let $\bar{\tau}^K_{new} = \bar{\tau}^K + \Delta\left(\bar{\tau}^K\right)$. With lump-sum transfer adjustment, relative changes of various variables from their initial steady-states are:*

$$\frac{\bar{r}^K_{new}}{\bar{r}^K} = \left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{-1}, \qquad \frac{\bar{\tilde{w}}_{new}}{\bar{\tilde{w}}} = \left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{\frac{\lambda}{1-\lambda}},$$

$$\left(\frac{\bar{\tilde{K}}_{new}/\bar{H}_{new}}{\bar{\tilde{K}}/\bar{H}}\right) = \left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{\frac{1}{1-\lambda}}, \qquad \frac{\bar{H}_{new}}{\bar{H}} = \left(1 + \Omega\Delta\left(\bar{\tau}^K\right)\right)^{-\frac{1}{1+\varphi}},$$

$$\frac{\bar{\tilde{K}}_{new}}{\bar{\tilde{K}}} = \frac{\bar{\tilde{I}}_{new}}{\bar{\tilde{I}}} = \left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{\frac{1}{1-\lambda}} \frac{\bar{H}_{new}}{\bar{H}}, \qquad \frac{\bar{\tilde{Y}}_{new}}{\bar{\tilde{Y}}} = \left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{\frac{\lambda}{1-\lambda}} \frac{\bar{H}_{new}}{\bar{H}},$$

and

$$\frac{\bar{\tilde{C}}_{new}}{\bar{\tilde{C}}} = \left(1 + \frac{\bar{\tilde{I}}}{\bar{H}}\left(\frac{\bar{\tilde{C}}}{\bar{H}}\left(1-\bar{\tau}^K\right)\right)^{-1}\right)\frac{\bar{\tilde{Y}}_{new}}{\bar{\tilde{Y}}},$$

where $\Omega = \bar{\omega}\bar{H}^{1+\varphi}\frac{\lambda}{1-\lambda}\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H} > 0$. Moreover, for small changes in the capital tax rate $\Delta\left(\bar{\tau}^K\right)$, the percent changes of these variables from their initial steady-states are:

$$\ln\left(\frac{\bar{r}^K_{new}}{\bar{r}^K}\right) = \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}, \qquad \ln\left(\frac{\bar{\tilde{w}}_{new}}{\bar{\tilde{w}}}\right) = -\left(\frac{\lambda}{1-\lambda}\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right),$$

$$\ln\left(\frac{\bar{\tilde{K}}_{new}/\bar{H}_{new}}{\bar{\tilde{K}}/\bar{H}}\right) = -\left(\frac{1}{1-\lambda}\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right), \qquad \ln\left(\frac{\bar{H}_{new}}{\bar{H}}\right) = -\mathcal{M}_H\Delta\left(\bar{\tau}^K\right),$$

$$\ln\left(\frac{\bar{\tilde{K}}_{new}}{\bar{\tilde{K}}}\right) = \ln\left(\frac{\bar{\tilde{I}}_{new}}{\bar{\tilde{I}}}\right) = -\mathcal{M}_K\Delta\left(\bar{\tau}^K\right), \qquad \ln\left(\frac{\bar{\tilde{Y}}_{new}}{\bar{\tilde{Y}}}\right) = -\mathcal{M}_Y\Delta\left(\bar{\tau}^K\right),$$

and

$$\ln\left(\frac{\bar{\tilde{C}}_{new}}{\bar{\tilde{C}}}\right) = -\mathcal{M}_C\Delta\left(\bar{\tau}^K\right),$$

where $\mathcal{M}_H = \frac{\Omega}{1+\varphi}$, $\mathcal{M}_K = \frac{1}{(1-\lambda)(1-\bar{\tau}^K)} + \mathcal{M}_H > 0$, $\mathcal{M}_Y = \frac{\lambda}{(1-\lambda)(1-\bar{\tau}^K)} + \mathcal{M}_H > 0$ and $\mathcal{M}_C = \mathcal{M}_Y - \frac{\bar{\tilde{I}}}{\bar{H}}\left(\frac{\bar{\tilde{C}}}{\bar{H}}\left(1-\bar{\tau}^K\right)\right)^{-1}$. Under Assumption 1, $\mathcal{M}_C > 0$.

Proof. See Appendix C.3. □

Proposition 1 provides a simple representation of the model solution that helps us understand the mechanism for aggregate variables even further. As is standard, the effects on factor prices and capital to labor ratio depend only on the production side parameters. For the level of aggregate quantities (output, consumption and investment), however, the proposition shows that the key step, in the aforementioned channel, is in fact how labor hours respond, $\frac{H_{new}}{H}$.[21] This implies that

[21] Output for example, increases by the same amount (in percentage from the initial steady-state) as pre-tax labor

preference parameters, in particular, the Frisch elasticity of labor supply, generally matter for the effectiveness of a capital tax cut. In fact, it is clear that since hours in the initial steady-state is less than 1, the capital tax elasticity of hours, $\mathcal{M}_H$, is decreasing in φ, and thus hours increase more with higher Frisch elasticity. Moreover, given the importance of hours response, the proposition naturally leads us to a conjecture that a capital tax cut would have a smaller effect if the labor tax rate needed to adjust, which we prove formally in the next subsection. Finally, the solution also reveals that the effectiveness of a tax reform depends on the economy's current tax rates. When the economy is initially farther away from the non-distortionary case (i.e. when $\bar{\tau}^K$, $\bar{\tau}^H$, and $\bar{\tau}^C$ are currently high), a given capital tax cut will have a stronger long-run effect.

3.1.2 Labor tax rate and consumption tax rate adjustment

We next discuss the case where distortionary tax rates increase to finance the capital tax rate cut. We first derive results where labor tax rate increases in the long-run to finance the permanent capital tax rate cuts. Overall, compared to the previous case of lump-sum transfer adjustment, the model predicts qualitatively similar long-run effects on most of the variables – except for labor hours and for after-tax wages. Quantitatively, however, the macroeconomic effects are expected to be smaller because of distortions created by the labor tax rate increase. In fact, for small changes in the capital tax rate, we have analytical results below on exactly how small these effects are and what parameters determine the differences.

Once again, a mild restriction on steady-state government spending is assumed as given below.

Assumption 2. $\bar{\tilde{G}} < 1 - \frac{\theta-1}{\theta}\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)} = 1 - \frac{1}{\lambda(1-\bar{\tau}^K)}\left(\frac{\bar{\tilde{I}}}{\bar{\tilde{Y}}}\right)$ *in the initial steady-state.*

Then, we can show that a permanent capital tax rate cut, financed by an increase in the labor tax rate, leads to an increase in the capital-to-hours ratio and in (pre-tax) wages and a decrease in the rental rate of capital, as before.[22] In contrast to the lump-sum transfer case, however, hours now decline in the new steady-state.

Lemma 2. *Fix $\bar{\tilde{S}}$ and $\bar{\tilde{b}}$. With labor tax rate adjustment,*

1. Rental rate of capital is increasing, while capital to hours ratio and wage are decreasing in $\bar{\tau}^K$.

2. Under Assumption 2, hours are increasing in $\bar{\tau}^K$.

Proof. See Appendix C.4. □

We next show analytically the required adjustment in labor tax rate in the new steady-state as well as the approximate solution for small changes in the capital tax rates that are intuitive to understand and sign. We state the results formally below in Proposition 2.

income.

[22]In fact the entire response of capital-to-hours, rental rate of capital, and (pre-tax) wages are the same between transfer and labor tax rate adjustment.

Proposition 2. *Let $\bar{\tau}^K_{new} = \bar{\tau}^K + \Delta\left(\bar{\tau}^K\right)$. With labor tax rate adjustment,*

1. *New steady-state labor tax rate is given by $\bar{\tau}^H_{new} = \bar{\tau}^H + \Delta\left(\bar{\tau}^H\right)$ where*

$$\Delta\left(\bar{\tau}^H\right) = -\frac{\lambda}{1-\lambda}\left(1+\bar{\tau}^C\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\right)\Delta\left(\bar{\tau}^K\right).$$

2. *For small changes in the capital tax rate $\Delta\left(\bar{\tau}^K\right)$, relative changes of rental rate, wage, after-tax wage, capital to hours ratio and hours from their initial steady-states are:*

$$\ln\left(\frac{\bar{r}^K_{new}}{\bar{r}^K}\right) = \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K},\ \ln\left(\frac{\bar{\tilde{K}}_{new}/\bar{H}_{new}}{\bar{\tilde{K}}/\bar{H}}\right) = -\frac{1}{1-\lambda}\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K},$$
$$\ln\left(\frac{\bar{\tilde{w}}_{new}}{\bar{\tilde{w}}}\right) = -\frac{\lambda}{1-\lambda}\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K},\ \ln\left(\frac{\left(1-\bar{\tau}^H_{new}\right)\bar{\tilde{w}}_{new}}{\left(1-\bar{\tau}^H\right)\bar{\tilde{w}}}\right) = \mathcal{M}_W\Delta\left(\bar{\tau}^K\right),$$

and

$$\ln\left(\frac{\bar{H}_{new}}{\bar{H}}\right) = \mathcal{M}_{H,\tau^H}\Delta\left(\bar{\tau}^K\right),$$

where $\mathcal{M}_{H,\tau^H} = \frac{1-\bar{\tilde{G}}+\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\left(\bar{\tilde{T}}^C+\bar{\tilde{T}}^H+\bar{\tilde{T}}^K-\frac{\theta-1}{\theta}\right)}{(1+\varphi)\frac{1-\lambda}{\lambda}(1-\bar{\tau}^H)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}$ *and* $\mathcal{M}_W = \frac{\lambda\left(1+\bar{\tau}^C\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}-\frac{1-\bar{\tau}^H}{1-\bar{\tau}^K}\right)}{(1-\lambda)(1-\bar{\tau}^H)}$. *Under Assumption 2, $\mathcal{M}_{H,\tau^H} > 0$. Moreover, $\mathcal{M}_W > 0$ if and only if $1+\bar{\tau}^C\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right) > \frac{1-\bar{\tau}^H}{1-\bar{\tau}^K}$.*

Proof. See Appendix C.5. □

The required adjustment in the labor tax rate is approximately given by the ratio of the capital to labor input in the production function, as the government is keeping debt-to-GDP constant and hence has to compensate the loss of capital tax revenue-to-GDP with gains in labor tax revenue. One interesting result on the approximate solution is that the effects on *after-tax* wage rate depends on initial level of labor tax rate relative to the other tax rates. Intuitively, a further increase in labor tax rate (to finance a capital tax cut), when it is sufficiently high already, lowers *after-tax* wage rate. Moreover, again, hours fall, which is the result we highlight given that it is qualitatively different.[23] Additionally, note that the (absolute) capital tax elasticity of hours, $\mathcal{M}_{H,\tau^H}$ in Proposition 2, decreases in φ, and thus hours fall more with a higher Frisch elasticity.

We next discuss the case where consumption tax rate increases in the long-run to finance the permanent capital tax rate cuts. Overall, the results are very similar to the labor tax rate adjustment case as both these distortionary source of taxes affect the consumption-leisure choice in a similar way. Thus, first, we can show that a permanent capital tax rate cut, financed by an increase in the consumption tax rate rate, leads to an increase in the capital-to-hours ratio and wages and a decrease in the rental rate of capital, as before for both transfer and labor tax rate adjustment, as well as a decrease in hours, as before for labor tax rate adjustment.

[23]For this case, because of opposite movement of hours and capital to hours ratio, it is not possible to provide intuitive results on the levels of variables such as output and consumption.

Lemma 3. *Fix $\bar{\tilde{S}}$ and $\bar{\tilde{b}}$. With consumption tax rate adjustment,*

1. *Rental rate of capital is increasing, while capital to hours ratio and wage are decreasing in $\bar{\tau}^K$.*
2. *Hours are increasing in $\bar{\tau}^K$.*

Proof. See Appendix C.6. □

Then, we can also show analytically the required adjustment in consumption tax rate in the new steady-state as well as the approximate solution for small changes in the capital tax rates that are intuitive to understand and sign. We state the results formally below in Proposition 3. The economic mechanisms are very similar to the labor tax rate change scenario that we described in detail above, where here as well, hours decline.

Proposition 3. *Let $\bar{\tau}^K_{new} = \bar{\tau}^K + \Delta\left(\bar{\tau}^K\right)$. With consumption tax rate adjustment,*

1. *New steady-state consumption tax rate is given by $\bar{\tau}^C_{new} = \bar{\tau}^C + \Delta\left(\bar{\tau}^C\right)$ where*

$$\Delta\left(\bar{\tau}^C\right) = -\left(1 + \frac{\bar{a} - (1-d)}{\frac{\bar{a}}{\beta} - (1-d)}\bar{\tau}^C\right)\frac{\Theta_C \Delta\left(\bar{\tau}^K\right)}{1 + \left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\Theta_C \Delta\left(\bar{\tau}^K\right)}.$$

with $\Theta_C = \frac{\lambda \bar{mc}}{\left(1-\bar{\tilde{G}}\right) - \frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda \bar{mc}(1-\bar{\tau}^K)} > 0.$

2. *For small changes in the capital tax rate $\Delta\left(\bar{\tau}^K\right)$, relative changes of rental rate, wage, after-tax wage, capital to hours ratio and hours from their initial steady-states are:*

$$\ln\left(\frac{\bar{r}^K_{new}}{\bar{r}^K}\right) = \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K},\ \ln\left(\frac{\bar{\tilde{K}}_{new}/\bar{H}_{new}}{\bar{\tilde{K}}/\bar{H}}\right) = -\frac{1}{1-\lambda}\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K},\ \ln\left(\frac{\bar{\tilde{w}}_{new}}{\bar{\tilde{w}}}\right) = -\frac{\lambda}{1-\lambda}\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}$$

and

$$\ln\left(\frac{\bar{H}_{new}}{\bar{H}}\right) = \mathcal{M}_{H,\tau^C}\Delta\bar{\tau}^K,$$

where $\mathcal{M}_{H,\tau^C} = \frac{1}{1+\varphi}\frac{\lambda \bar{mc}}{(1+\bar{\tau}^C)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\left(1 - \frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right) > 0.$

Proof. See Appendix C.7. □

Finally, we are also able to compare analytically the change in macroeconomic quantities as a result of the capital tax rate cut for the three fiscal adjustment cases. We do this for the small capital tax rate adjustment approximation and prove in Proposition 4 that the increase in output, capital, investment, consumption, and hours increase by more under adjustment in lump-sum transfers compared to labor tax rate adjustment.[24] Moreover, the differences in these changes

[24]This result does not require Assumption 2. That is, it holds regardless of whether hours increase or decrease following a capital tax rate cut. Additionally, as seen above wages and rental rates are the same across the two fiscal adjustments, as shown in Proposition 1 and 2, and so we do not present these obvious results in Proposition 4.

for output, investment, consumption, and hours are given by the same amount. This constant difference depends intuitively and precisely on the labor supply parameter for a given change in the tax rates. A higher Frisch elasticity ($\frac{1}{\varphi}$) makes workers more responsive to labor tax rates, thereby generating greater distortions, which in turn, magnifies the difference. The two fiscal adjustments produce the same outcomes only if labor supply is completely inelastic ($\frac{1}{\varphi} = 0$). Moreover, as is intuitive, higher is the initial level of the labor tax rate, bigger is the difference. Thus, for the same change in the labor tax rate, if the initial labor tax rate is higher, the increase in output, investment, consumption, and hours will be relatively smaller.

Proposition 4. *Let $\bar{\tau}^K_{new} = \bar{\tau}^K + \Delta\left(\bar{\tau}^K\right)$, $\bar{\tau}^H_{new} = \bar{\tau}^H + \Delta\left(\bar{\tau}^H\right)$. Denote $\bar{X}^T_{new}$ and $\bar{X}^L_{new}$ as the new steady-state variables in transfer adjustment case and in labor tax rate adjustment case, respectively. For small changes in the capital tax rate $\Delta\left(\bar{\tau}^K\right)$, for $X \in \left\{\tilde{C}, \tilde{K}, \tilde{I}, \tilde{Y}, H\right\}$*

$$\ln\left(\frac{\bar{X}^T_{new}}{\bar{X}^L_{new}}\right) = -\Theta\Delta\left(\bar{\tau}^K\right) = \frac{1}{1+\varphi}\left(\frac{1}{1-\bar{\tau}^H}\right)\Delta\left(\bar{\tau}^H\right)$$

where $\Theta = \frac{1}{1+\varphi}\left(\frac{1}{1-\bar{\tau}^H}\right)\frac{\lambda}{1-\lambda}\left(1+\bar{\tau}^C\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right) > 0$. In other words, generally, output, capital, investment, consumption and hours increase by more in the transfer adjustment case than in the labor tax rate adjustment case when capital tax rate is cut.

Proof. See Appendix C.8. □

For completeness, we also show similar comparisons for the consumption tax rate adjustment case in Propositions 5 and 6, which is relegated to the Appendix C.9 to conserve space.

3.2 Numerical results of baseline model

We now present numerical results of the baseline model with capital-skill complementarity, as presented in Section 2.

3.2.1 Parameterization

The frequency of the model is a quarter. Table 1 contains numerical values we use for the parameters that are relevant for long-run effects. The parameterization is standard, and we provide detailed justification and references in Table 1. As given above, we use separable preferences that imply log utility in consumption and then calibrate a modest, unit Frisch elasticity of labor supply ($\frac{1}{\varphi}$= 1) based on Smets and Wouters (2007). For the production function elasticity of substitution parameters, we use the estimates in Krusell et al. (2000) ($\sigma = 0.401, \rho = -0.495$). This parameterization implies (equipment) capital-skill complementarity. We also follow Krusell et al. (2000) in matching the income share of structure ($\alpha = 0.117$) as well as the depreciation rates of the two types of capital. Finally, for the income share of equipment and unskilled labor, we pick parameter values to get a steady-state labor share of 0.56 (Elsby, Hobijn and Şahin 2013) and steady-state skill premium of 60% (Krusell et al. 2000).

Additionally, across various fiscal adjustment scenarios and preference and technology functions specifications, we normalize hours for skilled labor to be 0.330 and hours for unskilled labor to be 0.307 in the initial steady-state by appropriately adjusting the scaling parameters $\bar{\omega}^s$ and $\bar{\omega}^u$. We follow the calibration of Lindquist (2004) for this choice of steady-state hours as well as the fraction of skilled labor (N^s=0.5).

The steady state of the fiscal variables such as the debt-to-GDP ratio, the government spending-to-GDP ratio, and the taxes-to-GDP ratio, is matched to their respective long-run values in the data. The Appendix D describes this data in detail. We then calibrate the steady-state markup to obtain a 35% capital tax rate initially. The implied initial levels of labor tax rate and consumption tax rate are 12.8% and 0.9% respectively. For the effective expensing rates of the two types of capital, we use the estimates in Barro and Furman (2018), which imply lower expensing of structure investment. For the parameters governing the incidence of labor tax rate on the two types of workers, we set equal weights in the baseline ($\lambda^s_{\tau^H} = \lambda^u_{\tau^H}$=1).

We present detailed sensitivity analysis of our baseline parameterization in Section 6.

3.2.2 Results

The first set of numerical results are summarized in Figure 1, where we highlight the role of capital-skill complementarity in comparison to the simplified, analytical model presented above.[25] One main finding is that worker and capital heterogeneity in the model, while certainly generating new distributional implications, have little aggregate effects. The analytical results in the previous section thus serve as a useful benchmark for economic intuition for aggregate variables, as they are relevant both quantitatively and qualitatively. While our focus is on a reduction of the capital tax rate from 35% to 21%, which are clearly shown with colored dots in the Figure, we show the entire range of tax rate changes for completeness.

Let us now discuss the results in more detail, where we start with the case of transfer adjustment. For a reduction of the capital tax rate from 35% to 21%, output increases by 3.8% relative to the initial steady state, structure investment by 19.7%, equipment investment by 6.8%, and consumption by 2.1%.[26] Moreover, skilled wages increase by 3.4%, unskilled wages by 2.7%, skilled hours increase to 0.334 from 0.330, and unskilled hours to 0.310 from 0.307. In terms of financing, as shown in Figure 1, a decrease in the capital tax rate reduces total (tax) revenues-to-GDP ratio (driven by decrease in capital tax revenue-to-GDP ratio), which is financed by a decline in transfers-to-GDP ratio from 1.0% to -0.5%.[27]

[25] We show population weighted aggregates, and note that due to perfect insurance between the two types within a representative family, marginal utilities are equalized across the two types of households.

[26] For comparison, Barro and Furman (2018) predict that the long-run increase in output will be 3.1% for a permanent capital tax rate cut from 38% to 26%, assuming that the employment-to-population ratio is fixed. Unlike their analysis, we model an endogenous labor supply decision, which is especially important when considering distortionary financing.

[27] Note that this result is obtained not only because output (i.e. the denominator) increases. In fact, the total tax

The mechanisms behind aggregate effects are the same as described in Section 3.1.1 for the simple model. In fact, to make this transparent, in Figure 1, we explicitly show the comparison with a nested model where there is a Cobb-Douglas production function, everything else the same. This nested model, other than allowing for expensing to make it closer to the tax code, is the same as our analytical model. As is clear, the aggregate effects are extremely similar between the baseline and this nested model, with output increase of 3.6% and consumption increase of 2.0%. Moreover, this comparison also illustrates the usefulness of the analytical results based on the simplified model we presented in Section 3.1.1, even for quantitative predictions of aggregate effects. For instance, accounting for expensing, results as derived in Proposition 1 would have implied an output increase of 3.9%.[28] In Figure 1, we also show results based on another nested model, where there is a general CES production function, but not capital-skill complementarity. Again, the aggregate effects are very similar. One way to obtain intuition for these similar aggregate output results across various model variations is to look at how hours respond, as suggested by Proposition 1. As shown in Figure 1, skilled hours increase by more, but unskilled hours increase by less, than they would in the absence of capital-skill complementarity. These two countervailing forces contribute to a small differential in aggregate output.

We now turn to distributional implications. First, why does structure investment increase more than equipment investment? Quantitatively, the major reason is the lower expensing rate on structure investment in our calibration. Qualitatively, a role is also played by the fact that in the production function, the elasticity of substitution between equipment investment and skilled hours make them complements.

Second, and more interestingly, as mentioned above, skilled wages increase by more compared to unskilled, and thus, the skill premium or wage inequality increases following a capital tax rate cut. In particular, the skill premium goes up by 1.06% points.[29] To understand the mechanism, in our model, we can express the skill premium as

$$\frac{W_t^s}{W_t^u} = \frac{(1-\mu)(1-\lambda)}{\mu} \left(\lambda \left(\frac{K_{e,t}}{L_{s,t}} \right)^{\rho} + (1-\lambda) \right)^{\frac{\sigma-\rho}{\rho}} \left(\frac{L_{u,t}}{L_{s,t}} \right)^{1-\sigma}.$$

Thus, if a capital-skill complementarity ($\sigma > \rho$) exists, as in our model calibration, the skill premium increases in the amount of equipment capital when the quantities of the two types of labor inputs are held fixed. This mechanism drives our result on the skill premium. Also, note that the skill premium is increasing in the ratio of unskilled-to-skilled labor. This however declines in our experiment. Thus, the main force behind the increase in the skill premium is the increase in equipment capital,

revenues also decline. In particular, there is a significant decrease in capital tax revenues (about 42% decline relative to the initial steady state), which is only partially offset by an increase in consumption and labor tax revenues. The government therefore finances such a deficit by taking resources away from the household: transfers decline by roughly 151% of the initial steady-state. There is a "Laffer curve" for capital tax revenues but the capital tax revenue starts to decline at very high and empirically irrelevant range, such as above 90% in our baseline calibration.

[28]The derivations with expensing would give, $\log\left(\frac{\bar{\tilde{Y}}_{new}}{\bar{\tilde{Y}}}\right) \simeq -\left[\frac{\lambda}{1-\lambda}\frac{1}{1-\bar{\tau}^K} + \frac{\Omega}{1+\varphi}\left(\frac{1}{1-\lambda_e\bar{\tau}^K_{new}}\right)\right]\left(\frac{1-\lambda_e}{1-\lambda_e\bar{\tau}^K}\right)\Delta\bar{\tau}^K$.

[29]In the Appendix C.11, we show an analytical result on how the skill premium increases with the capital tax rate cut in our model.

and in particular, the increase in the equipment-to-skilled labor ratio. Finally, income inequality, measured by the the ratio of after-tax capital-to-labor income, unambiguously increases – although both types of income increase. The increase in wage and income inequality can be considered as caveats to the effectiveness of the capital tax rate cut in our model, even when lump-sum transfers are allowed to finance the tax cut.

Next, we contrast the results in our baseline model with capital-skill complementarity when labor tax rate, instead of transfers, adjusts to finance the capital tax rate cut. Figure 2 shows that in the long-run, to finance the reduction of the capital tax rate from 35% to 21%, labor tax rates have to increase from 22.8% to 25.5%. The same mechanism for aggregate variables as we described for the transfer adjustment case works, and moreover, the capital-to-hours ratio, (pre-tax) wages on both the skilled and unskilled, and rental rate on both types of capital change by the same amount as before. There continues to be an expansion in output, investment, and consumption as a result of the capital tax rate cut.

The increase in output, investment, and consumption is however, less under labor tax rate adjustment – as is consistent with what we proved in Proposition 4 above for small changes in the simplified model. In particular, for the baseline experiment of a reduction of the capital tax rate from 35% to 21%, output increases by 1.96%, equipment investment by 4.96%, structures investment by 17.61%, and consumption by 0.31%. The reason for the smaller boost in aggregate variables is a decline of the after-tax wages, by 0.24% for skilled workers and 0.90% for unskilled workers. This in turn leads to a decrease in labor hours in the long-run, from 0.330 to 0.3282 for skilled workers and from 0.307 to 0.3047 for unskilled workers. These are the first major qualitative differences from the lump-sum transfer adjustment case, as also highlighted by Proposition 2. The decrease in hours dampens the expansionary effect of capital tax cuts on output, consumption, and investment.

We also note that analytical results given by Proposition 4 of the simple model are quantitatively useful here as well in predicting the smaller boost in output and investment when distortionary labor tax rates have to adjust. In fact, Proposition 4 holds not only with expensing in the simple model, but also in the baseline quantitative model here with capital-skill complementarity and expensing. Thus, here as well, we can show

$$\log\left(\frac{\bar{\tilde{I}}^{T}_{new}}{\bar{\tilde{I}}^{L}_{new}}\right) = \log\left(\frac{\bar{\tilde{Y}}^{T}_{new}}{\bar{\tilde{Y}}^{L}_{new}}\right) = \frac{1}{1+\varphi}\frac{\Delta\bar{\tau}^{H}}{1-\bar{\tau}^{H}}.$$

Given this, the implied differences in output change between transfer adjustment and labor tax rate adjustment from Proposition 4 would be 1.84% points, compared to the 1.82% points here, while the implied difference in equipment investment change from Proposition 4 would be 1.90% points, compared to 1.88% points here.

Furthermore, in addition to the smaller expansionary effects, with labor tax adjustment, there continues to be a cost in terms of inequality. First, the skill premium (which is the same regardless of pre- or post-tax measures as the labor tax rate is the same on the two types of labor) increases as before. Our measure of income inequality continues to increases, but in fact by more here compared to transfer adjustment, as after-tax labor income now decreases.

We also analyze the case when consumption tax rate increases in the long-run to finance the capital tax rate cut. Figure 2 shows that in the long-run, to finance the reduction of the capital tax rate from 35% to 21%, consumption tax rates have to increase from 1.3% to 3.5%. Generally, as we emphasized before in analytical results for the simple model, the effects are qualitatively similar to labor tax rate adjustment, with the main distortion again coming in labor supply decisions (here more concentrated on the unskilled), which leads to a smaller expansionary effect.

3.3 Numerical results with heterogeneous households

We now consider an extension to heterogeneous households, a model with a hand-to-mouth household. In particular, the unskilled household is hand-to-mouth and consumes wage income plus government transfers every period. The skilled workers still own capital, have access to government bond markets and make dynamic, optimal consumption and savings decisions.[30] The extended model is detailed in Appendix B. Compared to the baseline model above, we need to parameterize two new parameters.[31] In our baseline, we assume that the profit shares for skilled labor (χ^s_Φ) is 1 and the transfers share for unskilled labor (χ^u_S) is 1.[32]

The results for long-run effects under transfer adjustment for this model are in Figure 3. For comparison, we show the results from the baseline model above as well. The key result is that the tax reform generates consumption inequality, which in turn leads to a further increase in wage inequality compared to the baseline model. Since transfers decline to finance the capital tax rate cut, and transfers are all distributed to the unskilled, consumption of the unskilled falls. Consumption of the skilled continues to rise, as in the baseline.[33] This implies that now consumption inequality, as measured by relative consumption of the skilled vs. the unskilled increases in the long-run. Additionally, these consumption responses cause strong wealth effects on labor supply now, unlike the baseline case. Thus, hours of the unskilled households increase while those of the skilled decline slightly. The increase in labor supply from the unskilled then leads to a much more muted increase in their wages, compared to the baseline. On the flip side, wages of the skilled increase by more now. The wealth effect on labor supply thus produces a further increase in wage inequality or the skill premium.

On the aggregate side, the effects on output and investment are similar to the baseline model, echoing our finding in the previous subsection. For a reduction of the capital tax rate from 35% to 21%, output increases by 4.2% relative to the initial steady state (compared to 3.8% in the baseline model), structure investment by 20.2% (compared to 19.7% in the baseline model), and equipment

[30] This extension, while maintaining tractability, allows us to consider the type of heterogeneity that are being extensively studied in the current business cycle literature (e.g. Bilbiie 2018 and Debortoli and Galí 2017).

[31] Appendix Table A.1 contains numerical values we use for all parameters of the model with heterogeneous households.

[32] We discuss how results might change with alternate assumptions later in a sensitivity analysis in Section 6.

[33] For the baseline model, as we have a representative family, we show the same consumption change for both skill types.

investment by 6.3% (compared to 6.8% in the baseline model). The slightly higher output effects are driven by the increased labor supply of the unskilled worker, while the slightly smaller increase in equipment investment is a result of reduced labor supply of the skilled worker combined with equipment capital-skill complementarity.

The results for long-run effects under labor tax rate adjustment for this model are in Figure 4. Again, for comparison, we show the results from the baseline model above as well. The key result, in comparison to the baseline model, remains the same qualitatively: introducing household heterogeneity generates consumption inequality and more pronounced wage inequality. In comparison to the transfer adjustment case above (i.e. comparing Figure 4 to Figure 3) however, the distributional effects are smaller. The main reason is that the burden from the labor tax increase is shared by both types of households, whereas transfer reduction only affects the unskilled. Therefore, consumption of the unskilled does not fall as much while consumption of the skilled does not increase as much. The reduced consumption inequality in turn diminishes the role of the wealth effect on labor supply, leading to a smaller increase in the skill premium. In addition, unlike in Figure 3, hours of both types now decrease due to lower after-tax wage rates.

On the aggregate side, the effects on output and investment are very similar to the baseline model, again echoing our finding in the previous subsection. For a reduction of the capital tax rate from 35% to 21%, output increases by 2.08% relative to the initial steady state (compared to 1.96% in the baseline model), structure investment by 17.74% (compared to 17.61% in the baseline model), and equipment investment by 4.80% (compared to 4.96% in the baseline model). The differential effects on labor supply of the two types, compared to the baseline model, once again produce similar results.

4 Transition Dynamics

We now discuss transition dynamics associated with a permanent capital tax rate cut, from 35% to 21%. Thus, we trace out the evolution of the economy as it transitions from the initial steady-state to the new steady-state. Studying transition dynamics is important as we find that it typically takes a quite long time, around 150 quarters, for consumption to converge to a new steady-state following a permanent reduction in the capital tax rate. This allows us in particular to analyze short-run effects, which are the focus here.

As in the long-run analysis in the previous section, we present the baseline and extended model in turn, and in each case consider different policy adjustments.[34] Compared to the long-run analysis, we pay a special attention to the role of monetary policy, which can be potentially important due to nominal rigidities in the short-run. An overall theme we highlight thus in this section is how a joint analysis of monetary and fiscal policy is important to understand the short-run effects to a permanent capital tax rate change.

[34] The simplified analytical model is omitted here for brevity.

4.1 Baseline model

We now discuss transition dynamics in our baseline model.

4.1.1 Parameterization

For transition dynamics, the parameterization of policy rules, investment adjustment costs, nominal rigidities, and capacity utilization costs matters. The parameterization is in Table 2. We use estimates from Smets and Wouters (2007) for investment adjustment costs, capacity utilization costs, the probability of resetting prices, and degree of inflation indexation.

We now discuss how we parameterize the policy rules, which govern the associated fiscal and monetary adjustments along the transition. We use estimates from Coibion and Gorodnichenko (2011) for monetary policy rule parameters from the post-Volcker period (1983-2002), where the Taylor principle is satisfied. For the tax rule parameters, we use our estimates of the policy rule (3) using US data. The details of the estimation is presented in Appendix E and Table A.2 shows the estimation results.

We point out here that the tax rule parameters we use for the case where taxes respond sufficiently to debt are based on estimates obtained using post-Volcker period data for the same period as in Coibion and Gorodnichenko (2011) (1983-2002). This combination, using data and estimates for the exact time period, then describes the regime where labor tax rates adjust to ensure stationary debt dynamics while monetary policy stabilizes inflation. Next, the tax rule parameters we use for the case where taxes do not respond sufficiently to debt, while inflation plays a role in debt stabilization in the short-run, are based on estimates obtained using data from a later period (2001-2019). In this latter case, the monetary policy rule does not satisfy the Taylor principle and we parameterize it accordingly.[35] In this regime, inflation plays a partial, but direct role, in debt stabilization.

4.1.2 Four different fiscal/monetary adjustments

We now consider four different fiscal/monetary policy adjustments in our baseline model, as described in Section 2.2.3. In particular, a policy response that we consider here is one where inflation plays a partial, but direct, role in debt stabilization.

Lump-sum transfer adjustment Once again, the starting point is the case of non-distortionary transfer adjustment, shown in Figure 5. What makes the short-run distinct from the long-run is that in principle, capital tax cuts can now generate a contractionary effect during the transition periods.

[35] In particular, we use $\phi_\pi < 1$, $\phi_{\Delta y} = 0$, and $\phi_x = 0$. We do not formally estimate the monetary policy rule for this sub-period due to the binding ZLB for much of the sample and the lack of enough observations. Passive monetary policy arguably characterizes this period well. Our results are robust to the precise parameterization of ϕ_π as long as it is below 1, as we show in Section 6.

The model dynamics can be best understood as depicting transition dynamics when the capital stock is initially below the new steady-state. As mentioned before, a reduction in the capital tax rate leads to a decrease in the rental rate of capital, thereby facilitating capital accumulation via more investment. In the short-run, to finance this increase of investment, consumption in fact declines for many periods. Given this postponement of consumption, combined with sticky prices, output also falls temporarily, before rising towards the high new steady-state. The temporary contraction in output is a result of sticky prices and investment adjustment costs, which renders output (partially) demand-determined and markups countercyclical in the model.[36] The temporary fall in output (which is coupled with increased capital stock), in turn leads to fall in hours of both skilled and unskilled workers.

Inflation also declines. It is determined by forward looking behavior of firms, and thus depends on current and future real marginal costs. As real marginal costs are a function of wages and capital rental rate, the path of inflation roughly follows that of these factor prices. The decrease in wages is driven by both supply and demand forces. The drop in consumption and the rise in marginal utility of consumption raise the supply of hours for a given wage rate. On the other hand, demand declines as firms produce a smaller amount of output as discussed above.

In terms of inequality, the skill premium increases in the short-run and slowly converges to the new steady state. The capital to labor income ratio also increases in the short-run, above the new long-run level.

Moreover, the long-run positive effects of capital tax cuts come at the expense of short-run decline of labor tax revenue– even under lump-sum transfer adjustments. Furthermore, the decrease in labor income requires a larger adjustment of transfers. Transfers fall sharply and in fact, have to go negative immediately. This need to engage in lump-sum taxes is arguably unrealistic, and motivates our study of distortionary financing next.

Labor tax rate adjustment Next, we analyze the case of labor tax rate increase, which is also shown in Figure 5. Here, labor tax rate evolves according to the tax rate rule, (3), given in Section 2.2.3. Overall, model dynamics are qualitatively similar to those in the transfer adjustment case. We still see capital accumulation, achieved by increased investment and postponement of consumption, which in turn also causes output to fall with sticky prices.

Quantitatively, however, the drop in consumption and output is larger in this case compared to the lump-sum transfer adjustment case. As in the lump-sum transfer adjustment case, delayed consumption decreases hours by lowering firms' labor demand. In addition, increased labor tax rate decreases hours even further by discouraging workers from supplying labor. Consequently, hours in equilibrium fall more, of both the skilled and the unskilled. This in turn amplifies the short-run contraction in consumption and output.

Note that the dynamics associated with labor tax rate adjustment are fairly close, especially in the very short-run, compared to lump-sum transfer adjustment. This is driven by our use of an

[36] The monetary policy rule specification also plays a role quantitatively.

empirically driven tax rule where tax rates adjust smoothly, allowing higher-than-normal debt-to-GDP ratio. To highlight this, in Figure 5 we additionally consider a case where labor tax rates adjust as necessary to maintain a constant debt-to-GDP ratio throughout the transition, as in the case of lump-sum transfer adjustment. In that case, the short-run contraction is quantitatively more severe compared to lump-sum transfer adjustment, driven by more rapid increases in the labor tax rates.

Labor tax rate and inflation adjustment The results are quite different, even qualitatively, in the case where labor tax rates increase, but not by enough, and inflation partly plays a role in government debt stabilization, as described in Section 2.2.3.[37] Figure 6 shows these results, where for comparison we also show the pure labor tax rate adjustment case discussed above.

The main difference now compared to the pure labor tax adjustment analysis is that there is a short-run burst of inflation to help stabilize debt. This increase in inflation, as the model has nominal rigidities, helps counteract the short-run contractionary effects. Output, consumption, investment, hours, and wages, in fact, all increase in the short-run. After 8 quarters or so, the transition dynamics are very similar to the labor tax rate increase only case. Interestingly, debt-to-GDP ratio, in sharp contrast to other fiscal adjustment cases, decreases for extended periods due to the rise in output and the price level.

Consumption tax rate adjustment For completeness, we finally study transition dynamics for the case of consumption tax rate adjustment. We show the results in Figure A.8 in Appendix F, where we use the same policy rule parameters as for the labor tax rate adjustment. The transition dynamics associated with the labor tax rate and consumption tax rate adjustment are very similar.

4.2 Model with heterogeneous households

We finally present transition dynamics in the model with heterogeneous households. Figure 7 shows that for the transfer adjustment case, consumption inequality increases throughout the transition, with a large decline along the transition in consumption of the unskilled. This is because of the large dynamic decline in transfers.[38] Because of the effects on marginal utility of the unskilled, they work more, unlike the baseline case. On the other hand, introducing such heterogeneity has little effect on the transition dynamics of aggregate output, mirroring the long-run results in Section 3.3.

Figure 8 shows that for the labor adjustment case, consumption inequality is less pronounced compared to the transfer adjustment case. This fiscal adjustment is relatively more beneficial for

[37] Note that in this case, the monetary policy rule (2) does not satisfy the Taylor principle, which is coupled with a low response of the tax rate in the tax rule (3). Clearly, we can analyze a similar fiscal adjustment case where inflation plays a role in debt stabilization even with lump-sum transfer adjustment. When non distortionary sources of revenue is possible, allowing inflation to play a role in debt stabilization might not be a very insightful experiment and so we do not emphasize this.

[38] In this extended model, the time path of transfers matters due to heterogeneity. We use a smooth adjustment path, using the same smoothing parameter as for labor tax rate adjustment in the baseline model. Appendix Table A.1 contains numerical values we use for all parameters of the model with heterogeneous households.

the unskilled as it does not feature a decline in transfers.[39] The labor tax increase burden is shared by both types, whereas transfer reduction only affects the unskilled. As in the baseline model, the transition dynamics under the consumption tax rate adjustment is similar, and thus omitted here.

5 Welfare Implications

While the focus of this paper is not on normative policy issues, we can nevertheless evaluate welfare implications of the permanent capital tax rate cut from 35% to 21%. Our results in the previous sections suggest that a reduction of the capital tax rate has different welfare implications depending on time horizon, household types, and policy adjustments. In this section, we formally calculate a measure of welfare gain that can be achieved through a permanent capital tax cut, taking into account transition dynamics as well as the long-run effect.

5.1 Welfare measure

Our measure of welfare gain for type-i agent, $\mu^i_{k,t}$, is implicitly defined by

$$\sum_{j=0}^{t} \beta^j U\left(\tilde{C}^i_j, H^i_j\right) = \sum_{j=0}^{t} \beta^j U\left(\left(1+\mu^i_{k,t}\right)\bar{\tilde{C}}^i_j, \bar{H}^i_j\right),$$

where $\left\{\tilde{C}^i_j, H^i_j\right\}$ and $\left\{\bar{\tilde{C}}^i_j, \bar{H}^i_j\right\}$ are respectively the time path of type-i agent's normalized consumption and hours with and without a capital tax cut under the various fiscal/monetary adjustments (indexed by k) we have considered above. We denote by $\mu^i_{T,t}$ the case of transfer adjustment, by $\mu^i_{H,t}$ the case of labor tax rate adjustment, and by $\mu^i_{H\pi,t}$ the case of labor tax rate and inflation adjustment. In this way, $\mu^i_{k,t}$ measures welfare gains from period 0, when the tax reform initiates, till (arbitrary) period t, in units of a percentage of the level of normalized initial consumption.[40] The lifetime (total) welfare gain is then measured by $\lim_{t\to\infty}\mu^i_{k,t}$, which is often of interest in the business cycle literature (Lucas 1987).

5.2 Baseline model

We start with the baseline model with consumption insurance across household types, where we present results on aggregate welfare. Figure 9, in panel (a), presents the evolution of aggregate welfare gain measure over time, $\mu_{k,t}$, along with its limit, $\mu_{k,\infty}$, when the government finances the capital tax cut through lump-sum transfer adjustment and labor tax rate adjustment, (under the smooth adjustment in tax rates following our tax rule).[41] Moreover, while for a realistic calibration and comparison, we had assumed that government spending-to-GDP ratio $\frac{G_t}{Y_t}$ remains unchanged

[39] Labor tax rates change smoothly here, as in the baseline model as well.

[40] It thus measures welfare gains at the point when the agents are t quarters old.

[41] We show the case where labor tax rates adjust to ensure constant debt-GDP ratio throughout the transition later in sensitivity analysis in Section 6.

from an initial positive level after the tax reform, to ensure that our welfare results are not driven by this assumption, we also show results when throughout, both before and after the capital tax reform, government spending-to-GDP ratio $\frac{G_t}{Y_t}$ is zero.[42]

Panel (a) of Figure 9 shows that a reduction in the capital tax rate from 35% to 21% increases discounted aggregate lifetime utility. The lifetime welfare gains amounts to 1.2% of the initial consumption level under transfer adjustment ($\mu_{T,\infty}$) and 0.55% under labor tax rate adjustment ($\mu_{H,\infty}$). Thus, as expected, gains in welfare are lower under distortionary tax adjustment. In both cases, however, there are welfare losses in the short-run: $\mu_{T,t}$ and $\mu_{H,t}$ become positive only 82 quarters and 143 quarters after the onset of the tax reform.[43] Results are very similar when the government spending-to-GDP ratio $\frac{G_t}{Y_t}$ is zero throughout, with higher welfare gains in the long-run and quicker transitions to positive welfare gain, due to no crowding out effect of government spending on consumption.

We now discuss welfare implications when, together with labor tax rate adjustment, inflation also plays a partial, but direct role in debt stabilization. This was the other fiscal adjustment considered above in Figure 6. This regime where inflation adjusts is an intriguing policy response as we pointed out before because consumption increases in the very short-run. Panel (b) of Figure 9 shows that welfare gains in this case ($\mu_{H\pi,t}$) are greater only for the first few quarters and then become smaller for extended periods, compared to when inflation plays no such role ($\mu_{H,t}$). The reason is that hours increase significantly not only to produce more output, but also because high inflation generates an inefficiency that requires more hours to produce even the same amount of output.

Transition dynamics in Figure 6 in fact suggests that such inflation-driven inefficiency may be significant. First, notice that the response of output under this inflating-away-debt regime (displayed by the dotted line) roughly coincides with that under pure labor tax rate adjustment (solid line) after 5 quarters. In producing roughly the same amount of output however, the economy is using more labor hours (of both types) for prolonged periods, as inflation is persistently higher than normal during the transition. Moreover, this is the case even though the economy also has a greater amount of structure and equipment capital, as implied by the investment responses. Our result thus suggests that inflating away debt in the short-run, while potentially intriguing, is still inadvisable from a welfare perspective.

5.3 Model with heterogeneous households

More interesting for welfare implications is arguably the case with heterogeneous households where unskilled workers are hand-to-mouth. Panel (a) of Figure 10 shows this case for a reduction in the capital tax rate from 35% to 21% and where government spending-to-GDP ratio $\frac{G_t}{Y_t}$ is at the

[42] We do this extension to clarify that crowding out of private consumption by higher government spending does not drive our welfare results. It also allows us to assess whether not including a utility yielding role for government spending has any meaningful consequences. For the quantitative positive results presented earlier, this issue has negligible effects.

[43] The black vertical line for reference denotes when the aggregate consumption converges to the new steady-state.

baseline, positive level throughout. It is clear that in this case, the tax reform does not leads to a Pareto improvement even with transfer adjustment: the skilled gain at the expense of the unskilled because the latter type, as we pointed out earlier, consumes less, which in turn also forces the agent to work more through wealth effects on labor supply. Specifically, the lifetime welfare gains under transfer adjustment amounts to 4.59% of the initial consumption level for the skilled and *negative* 8.52% for the unskilled. Turning to the short-run, compared to the baseline case above in panel (a) of Figure 9, the skilled worker's welfare never drops while in contrast, for the unskilled, $\mu^u_{T,t}$ never becomes positive.

The tax reform not leading to a Pareto improvement is also true under labor tax rate adjustment. Moreover, when labor tax rates adjust, compared to transfer adjustment, while welfare gains are smaller for the skilled in the long-run, welfare losses are in fact smaller for the unskilled. Labor tax adjustment works better for the unskilled as the labor tax increase burden is shared by both types, whereas transfer reduction only affects the unskilled. We discussed the same mechanism for the consumption effects in the long-run in Section 3.3 above. This finding implies that lump-sum transfer adjustment, while leading to a higher level of aggregate output, is not necessarily a better policy response than labor tax rate adjustment in this model.

Panel (b) of Figure 10 shows the case for this model where government spending-to-GDP ratio $\frac{G_t}{Y_t}$ is at zero throughout. The results are similar qualitatively. Focusing on the transfer adjustment case, due to the fact that there is no increased government spending in the new steady-state, the cut in transfers is lower. Then, the long-run welfare loss for the unskilled is lower as now their consumption falls by less in the long-run. Note however, that the long-run welfare gain for the skilled is also lower. This is because in fact consumption in the long-run increases by less for the skilled than before, driven by the much stronger effect on consumption for the unskilled.[44] We again also see that labor tax adjustment works better for the unskilled, with in fact welfare gains in the long-run.

6 Sensitivity Analysis

Before concluding the paper, we present some additional results on sensitivity analysis and extensions. As in the rest of the paper, we consider long-run, short-run, and welfare implications in both the baseline and the extended model. All the results from this section are in Appendix F.

6.1 Baseline model

We start with long-run effects. First, we present comparative statics result with respect to Frisch elasticity of labor supply. This is an important parameter, given that different source of financing imply different labor supply response, as we highlighted in the analytical results based on the simple

[44] Thus, while in this case there is certainly no crowding out effect of government spending on aggregate/total consumption, this effect is concentrated on the unskilled consumption and in fact, that ends up crowding out skilled consumption.

model. Given the baseline parameterization of a unit Frisch elasticity, we now show results for a higher and lower Frisch elasticity. Figure A.1 shows the results under transfer adjustment, where consistent with Proposition 1, we find that output effects are higher with higher Frisch elasticity due to a stronger hours response. Figure A.2 shows the results under labor tax rate adjustment, where consistent with Proposition 2, we find that output effects are lower with higher Frisch elasticity due to a stronger negative hours response. In the range for the values we consider here, the Figures show that the results overall are quantitatively not different for aggregate output.

In Figures A.3–A.4, we compare across the two fiscal adjustment for a given Frisch elasticity. Consistent with Proposition 4, the difference between the two cases is bigger for a higher Frisch elasticity. In our baseline calibration of a unit Frisch elasticity, we pointed out above that output increases by 3.8% under lump-sum transfer adjustment and by 1.96% under labor tax rate adjustment. Here, with a Frisch elasticity of 4, output increases by 4.4% under lump-sum transfer adjustment and by 1.5% under labor tax rate adjustment while with a Frisch elasticity of 0.5, output increases by 3.4% under lump-sum transfer adjustment and by 2.2% under labor tax rate adjustment.

Second, for the case of labor tax rate adjustment in the baseline model, we consider a different tax schedule across worker types. Note that in the baseline, for the parameter governing the incidence of labor tax rate on the two types of workers, we set equal weights ($\lambda^s_{\tau^H} = \lambda^u_{\tau^H}$=1), which is arguably a realistic starting-point. In Figure A.5 we show results when we vary these parameters. In particular, consider the case where the skilled workers only pay the labor tax ($\lambda^s_{\tau^H} = 1$, $\lambda^u_{\tau^H} = 0$). This can be considered as a progressive tax regime. In such a case, we see that the after-tax skill premium declines, while the boom in output is also reduced. Thus, wage inequality decrease comes at a cost of lower expansion, with consumption in fact falling in the long-run.

We note however, that letting the tax incidence fall more on the unskilled workers does not lead to a bigger aggregate effect. In fact, Figure A.5 shows that while that case ($\lambda^s_{\tau^H} = 0.1$, $\lambda^u_{\tau^H} = 1$) certainly leads to an increase in wage inequality, it actually goes together with a lower aggregate effect. The aggregate output and consumption effects depend on labor supply responses of the two types of workers, which get distorted to a varying degree with the changing incidence of the labor tax rates and affected in equilibrium from consumption response (as it affects the marginal utility of consumption).

Then finally, we present two comparative statics result that are useful to interpret the long-run effects, especially when it comes to the differential increase in structure and equipment investment following a capital tax rate decrease in our baseline model. First, we show how results depend on different rates of expensing in Figure A.6. Note that in the baseline calibration, structure investment is expensed at a lower rate in our calibration, in line with the data. Figure A.6 shows that if the expensing rates were to be the same, then the long-run increases in investment of the two types of capital would also be more similar. For instance, if both are expensed at the rate of 0.338, then equipment investment increases in the long-run by 16.7% (compared to baseline of 6.8%), while structure investment increases by 22.0% (compared to baseline of 19.7%). It also follows that in such a case, as equipment investment increases by more, the skill premium increases more than the baseline. In light of this result, in terms of wage inequality implications, our baseline calibration can be regarded as conservative.

Second, we show how results depend on changing the elasticity of substitution between equipment capital and skilled labor in Figure A.7. Note that in the production function, the elasticity of substitution between equipment capital and skilled labor is given as $1/(1-\rho)$. As to be expected, a lower elasticity of substitution, making equipment and skilled labor even stronger complements, reduces the long-run increase in equipment investment. This is another reason why in our baseline case, equipment investment increases less than structure investment in the long-run, following a permanent capital tax rate decrease.

Now we move to transition dynamics and welfare implications. Figure A.9 compares the transition dynamics in the baseline model under the labor tax and inflation adjustment finance scheme for different inflation feedback parameters in the Taylor rule (all have to be below 1 in this regime). Our results are very robust. In fact, as can be seen, our baseline parameterization of $\phi_\pi = 0.95$, which is very close to satisfying the Taylor principle, is conservative in terms of the short-run boom in consumption and output. The differences across the parameterizations show up more clearly in inflation and debt dynamics, with a stronger Taylor rule coefficient in fact leading to a bigger effect on inflation dynamically. This is consistent with the analytical results for the simple sticky price model in Bhattarai, Lee and Park (2014).

Figure A.10 shows welfare gains in the baseline model under labor tax adjustment with constant debt and under consumption tax adjustment, the other two fiscal adjustments we have discussed before but did not show explicitly in Section 6. Figure A.11 shows the same results, zooming on the short-run so that differences are visible. The labor tax rate adjustment that ensures a constant debt-GDP ratio throughout has very similar welfare implication as the one where labor tax rates adjust smoothly that we presented in Figure 9, and which is shown for direct comparison here as well. This is because of off-setting effects of lower consumption and hours on welfare. The welfare implications of consumption tax rate adjustment are similar to that of labor tax rate adjustment overall, but the welfare losses are more transient.

6.2 Model with heterogeneous households

We now present additional results in the model with heterogeneous households. We want to first point out that in this model, clearly the assumptions made on how profits and transfers are distributed across the two types of households makes a non-trivial difference for distributional variables. While the assumptions we made in the baseline case are arguably the most realistic, where the skilled workers get the profits stream while the unskilled/hand-to-mouth workers get government transfers, in Figure A.12, we show the long-run results under various other combinations of these distributions. For instance, if the skilled workers get both the profits and (cut in) transfers, which might also be considered reasonable, then it leads to a decline in consumption inequality, in sharp contrast to the baseline case. The results also show that aggregate effects however, are relatively similar across the various possibilities for profits and transfer distributions.

Figure A.13 shows the transition dynamics under transfers adjustment with two different rules for profit and transfer distributions, one the baseline and the other where the skilled workers get both the profits and (cut in) transfers. Again, like with the long-run, the differences show up prominently in distributional variables as now consumption of unskilled falls for a short-period only, whereas skilled consumption falls persistently, as in the baseline model. Moreover, in terms of hours, the unskilled does not increase labor supply unlike the baseline case as there are no longer strong wealth effects due to the fall in transfer.[45] This leads to a slightly stronger contraction in output in the short-run.

7 Conclusion

We study aggregate, distributional, and welfare effects of a permanent reduction in the capital tax rate in a quantitative equilibrium model with capital-skill complementarity. Such a tax reform leads to expansionary long-run aggregate effects, but is coupled with an increase in wage and income inequality. Moreover, the expansionary aggregate effects are smaller when distortionary labor or consumption tax rates have to increase to finance the capital tax rate cut, driven by their effects on labor supply decisions. An extension to a model with heterogeneous households, where the unskilled cannot smooth consumption over time, shows that consumption inequality also increases in the long-run, which leads to a further rise in wage inequality. The two sources of heterogeneity we introduce thus interact in economically meaningful ways for new distributional implications, even though they have relatively small aggregate effects.

We study transition dynamics and show that there are contractionary effects in the short-run, with a fall not just in consumption, but also output, and which is in turn coupled with increases in wage inequality. Importantly, we show that joint modeling of monetary and fiscal policy response is critical for analyzing short-run effects. In particular, when the government has access only to distortionary labor taxes, we consider the central bank directly accommodating inflation to facilitate government debt stabilization along the transition. In this interesting scenario, the government does not raise labor tax rates as much, and the rise of inflation in the short-run completely negates any short-run contraction in output as well as consumption. Finally, while we do not study optimal policy, we analyze welfare consequences of the permanent capital tax rate cut. We contrast the long-term aggregate welfare gains with short-term (but, still prolonged) losses, regardless of how the capital tax rate cut is financed. In the model with heterogeneous households, we additionally show that the skilled gain at the expense of the unskilled.

Introducing some additional forms of heterogeneity is a potentially important extension. Our analysis of the short-run and the long-run suggests that the proposed tax reform will have heterogeneous effects on different generations. Thus, exploring generational heterogeneity is a particularly

[45] In fact, due to log-utility, the income and substitution effect of wage change exactly cancel, and as the unskilled faces a purely static optimization problem on hours, hours do not change at all.

interesting avenue for future research. Introducing firm heterogeneity and financing constraints, similar to the household heterogeneity extension, might also be an interesting avenue for research.

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8 Tables and figures

Table 1: Calibration for Long-Run Analysis

	Value	Description	References
Households			
β	0.9975	Time preference	Smets and Wouters (2007)
φ	1.0	Inverse of Frisch elasticity of labor supply	Smets and Wouters (2007)
$\bar{\omega}^s$	7.14	Labor supply disutility parameter	Steady-state $\bar{H}^s = 0.33$
$\bar{\omega}^u$	4.75		Steady-state $\bar{H}^u = 0.31$
N^s	0.5	Fraction of skilled labor	Lindquist (2004)
d_e	0.031	Equipment capital depreciation	Krusell et al. (2000)
d_b	0.014	Structures capital depreciation	Krusell et al. (2000)
Firms			
σ	0.401	Elasticity of substitution between unskilled labor and equipment	Krusell et al. (2000)
ρ	-0.495	Elasticity of substitution between skilled labor and equipment	Krusell et al. (2000)
α	0.117	Structures capital Income share	Krusell et al. (2000)
λ	0.35	Equipment capital income share	Steady-state labor share: 56% (Elsby, Hobijn and Şahin 2013)
μ	0.345	Unskilled labor income share	Steady-state skill premium: 60% (Krusell et al. 2000)
γ	1.0054	Long-run growth rate of output	Smets and Wouters (2007)
$\bar{\pi}$	1.0078	Steady-state inflation rate	Smets and Wouters (2007)
θ	4.0	Elasticity of substitution between goods	Steady-state markup: 33%
q_0	0.95	Relative price of structure to equipment capital	Maliar and Maliar (2011)
Government			
$\bar{\bar{b}}$	0.363	Steady-state debt to GDP ratio	Data (See Appendix D)
$\bar{\bar{G}}$	0.161	Steady-state government spending to GDP ratio	Data (See Appendix D)
$\bar{\bar{T}}^C$	0.009	Steady-state consumption tax revenue to GDP ratio	Data (See Appendix D)
$\bar{\bar{T}}^H$	0.128	Steady-state labor tax revenue to GDP ratio	Data (See Appendix D)
$\lambda^s_{\tau^H}$	1.0	Effective rate of labor tax on skilled workers	Assigned
$\lambda^u_{\tau^H}$	1.0	Effective rate of labor tax on unskilled workers	Assigned
λ_b	0.338	Effective expensing rate of structure investment	Barro and Furman (2018)
λ_e	0.812	Effective expensing rate of equipment investment	Barro and Furman (2018)

Table 2: Calibration for Transition Dynamics

	Value	Description	References
Households			
ξ	4.0	Investment adjustment cost	Smets and Wouters (2007)
$\frac{\mathcal{A}''}{\mathcal{A}'}$	0.85	Elasticity of cost of capital utilization	Smets and Wouters (2007)
Firms			
α_P	0.65	Calvo sticky price parameter	Smets and Wouters (2007)
γ_P	0.22	Degree of price indexation	Smets and Wouters (2007)
Government(Fiscal/Monetary Policy): Transfer or Labor Tax Rate Adjustment			
ρ_1^R	1.12	Interest rate smoothing parameter lag 1	Coibion and Gorodnichenko (2011)
ρ_2^R	-0.18	Interest rate smoothing parameter lag 2	Coibion and Gorodnichenko (2011)
ϕ_π	1.58	Inflation feedback parameter under Taylor rule	Coibion and Gorodnichenko (2011)
ϕ_x	0.11	Output gap feedback parameter under Taylor rule	Coibion and Gorodnichenko (2011)
$\phi_{\Delta y}$	2.21	Output growth feedback parameter under Taylor rule	Coibion and Gorodnichenko (2011)
ρ_1^H	0.869	Labor tax rate smoothing parameter lag 1	Estimated (See Appendix E)
ρ_2^H	0.0	Labor tax rate smoothing parameter lag 2	Estimated (See Appendix E)
ψ_B^H	0.111	Labor tax rate response to debt	Estimated (See Appendix E)
$\psi_{\Delta y}^H$	0.831	Labor tax rate response to output growth	Estimated (See Appendix E)
ψ_x^H	0.0	Labor tax rate response to output gap	Estimated (See Appendix E)
Government(Fiscal/Monetary Policy): Labor Tax Rate and Inflation Adjustment			
ρ_1^R	1.12	Interest rate smoothing parameter lag 1	Coibion and Gorodnichenko (2011)
ρ_2^R	-0.18	Interest rate smoothing parameter lag 2	Coibion and Gorodnichenko (2011)
ϕ_π	0.95	Inflation feedback parameter under Taylor rule	Assigned
ϕ_x	0.0	Output gap feedback parameter under Taylor rule	Assigned
$\phi_{\Delta y}$	0.0	Output growth feedback parameter under Taylor rule	Assigned
ρ_1^H	0.785	Labor tax rate smoothing parameter lag 1	Estimated (See Appendix E)
ρ_2^H	0.107	Labor tax rate smoothing parameter lag 2	Estimated (See Appendix E)
ψ_B^H	0.007	Labor tax rate response to debt	Estimated (See Appendix E)
$\psi_{\Delta y}^H$	1.821	Labor tax rate response to output growth	Estimated (See Appendix E)
ψ_x^H	0.0	Labor tax rate response to output gap	Estimated (See Appendix E)

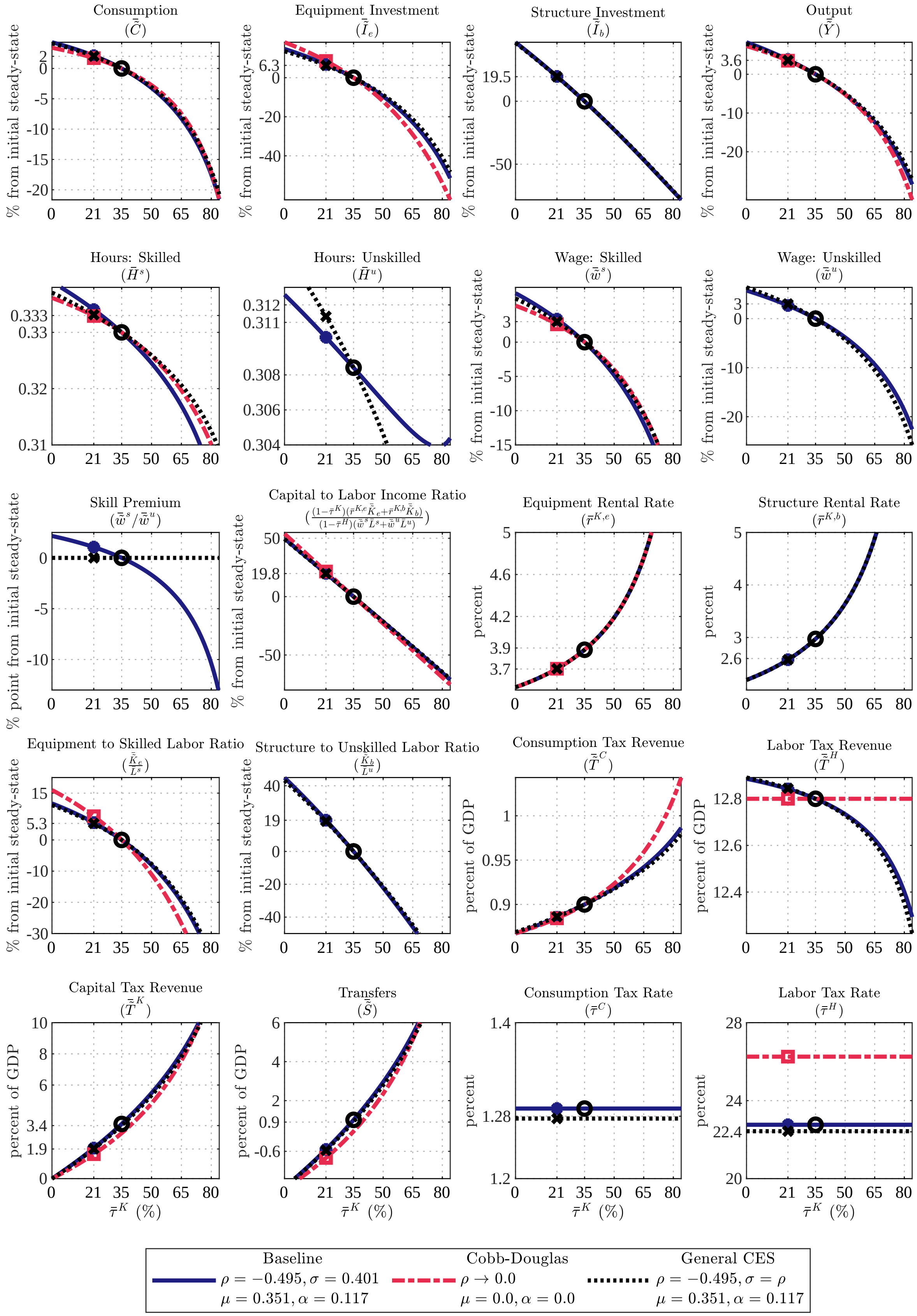


Figure 1: Long-run Effects of Permanent Capital Tax Rate Changes (Transfer Adjustment and Comparison with Nested Models)

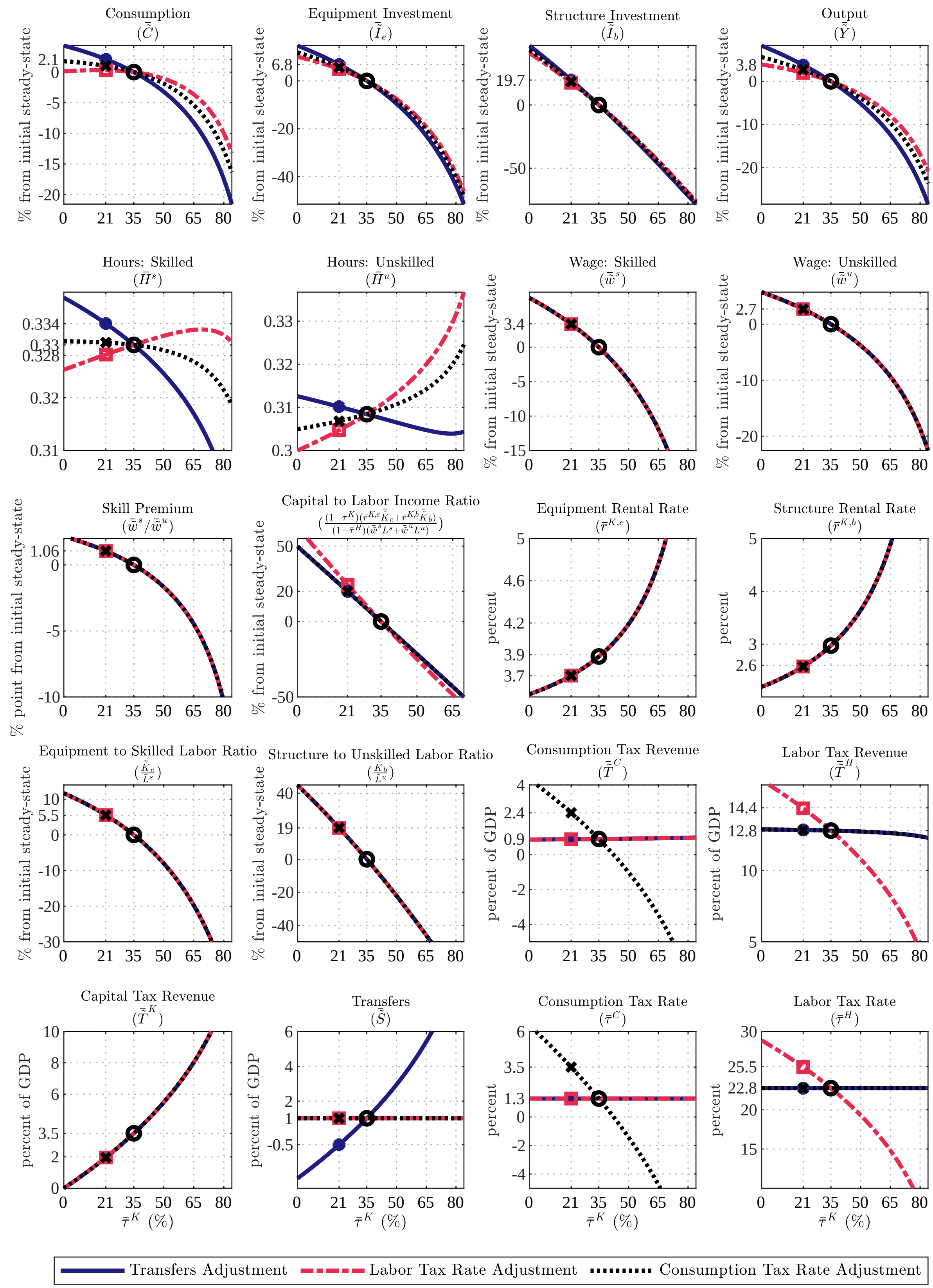


Figure 2: Long-run Effects of Permanent Capital Tax Rate Changes

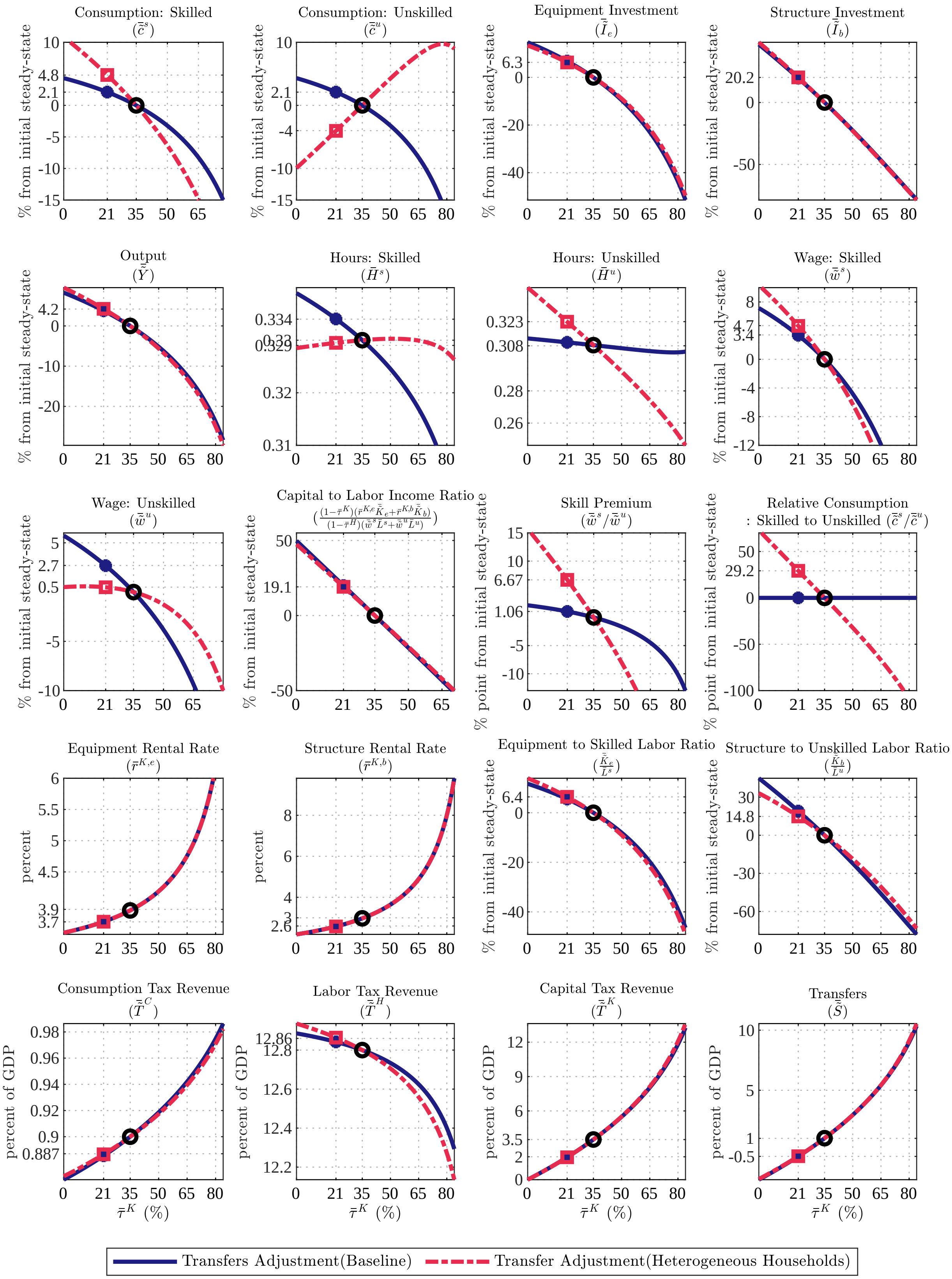


Figure 3: Long-run Effects of Permanent Capital Tax Rate Changes (Comparison with Heterogeneous Households Model)

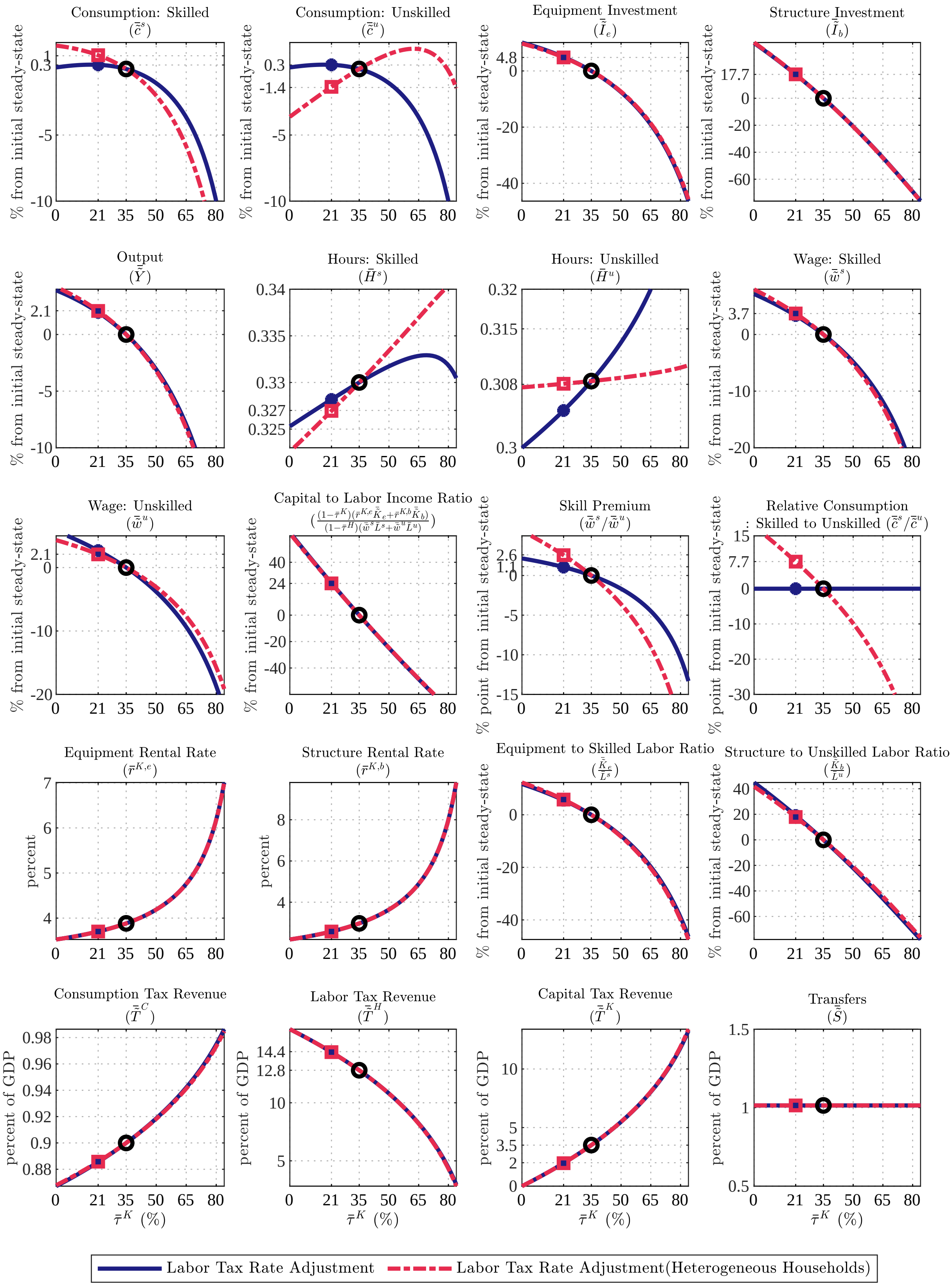


Figure 4: Long-run Effects of Permanent Capital Tax Rate Changes with Labor Tax Adjustment (Comparison with Heterogeneous Households Model)

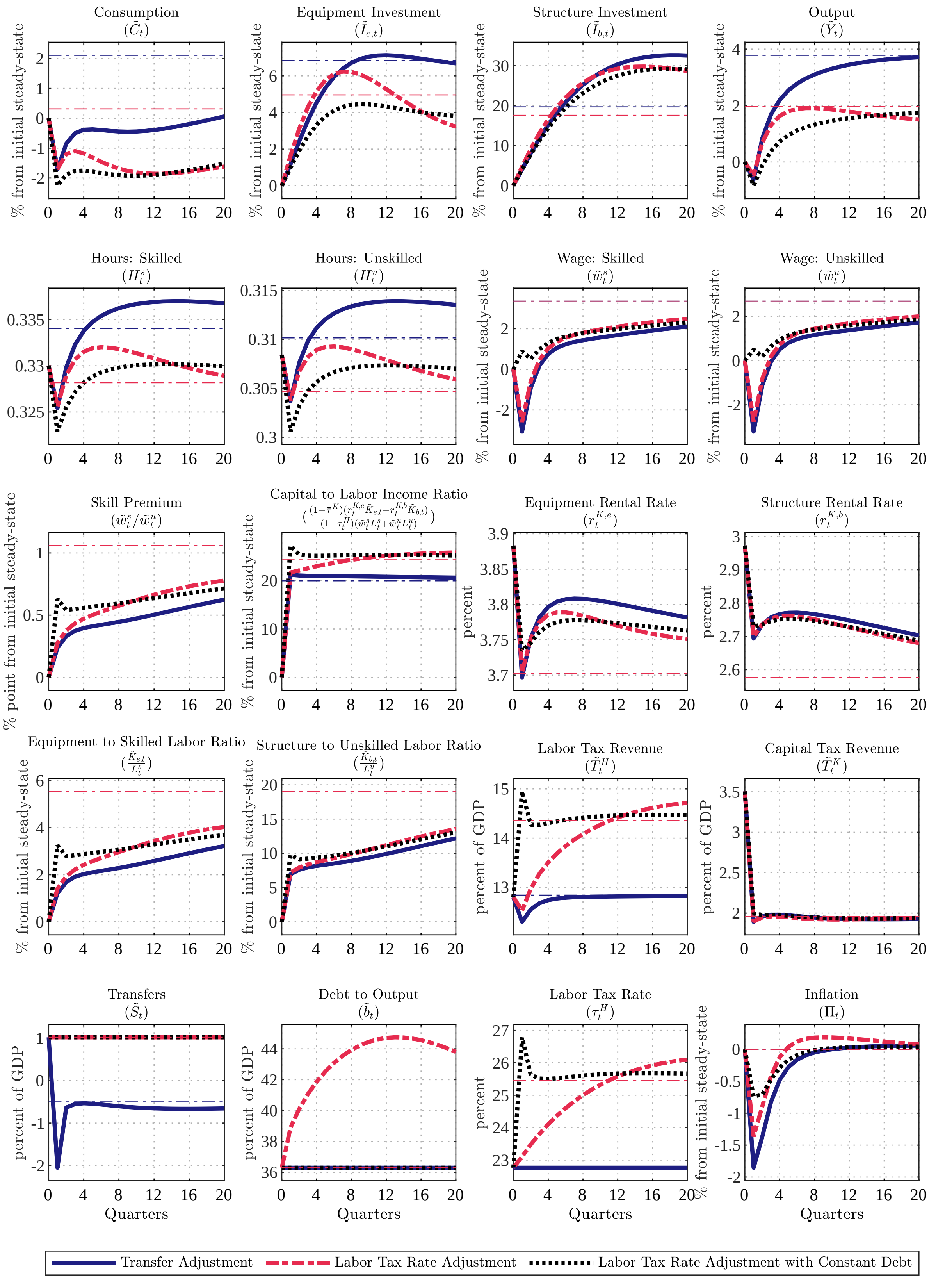


Figure 5: Transition Dynamics of a Permanent Capital Tax Rate Decrease Under Alternate Financing

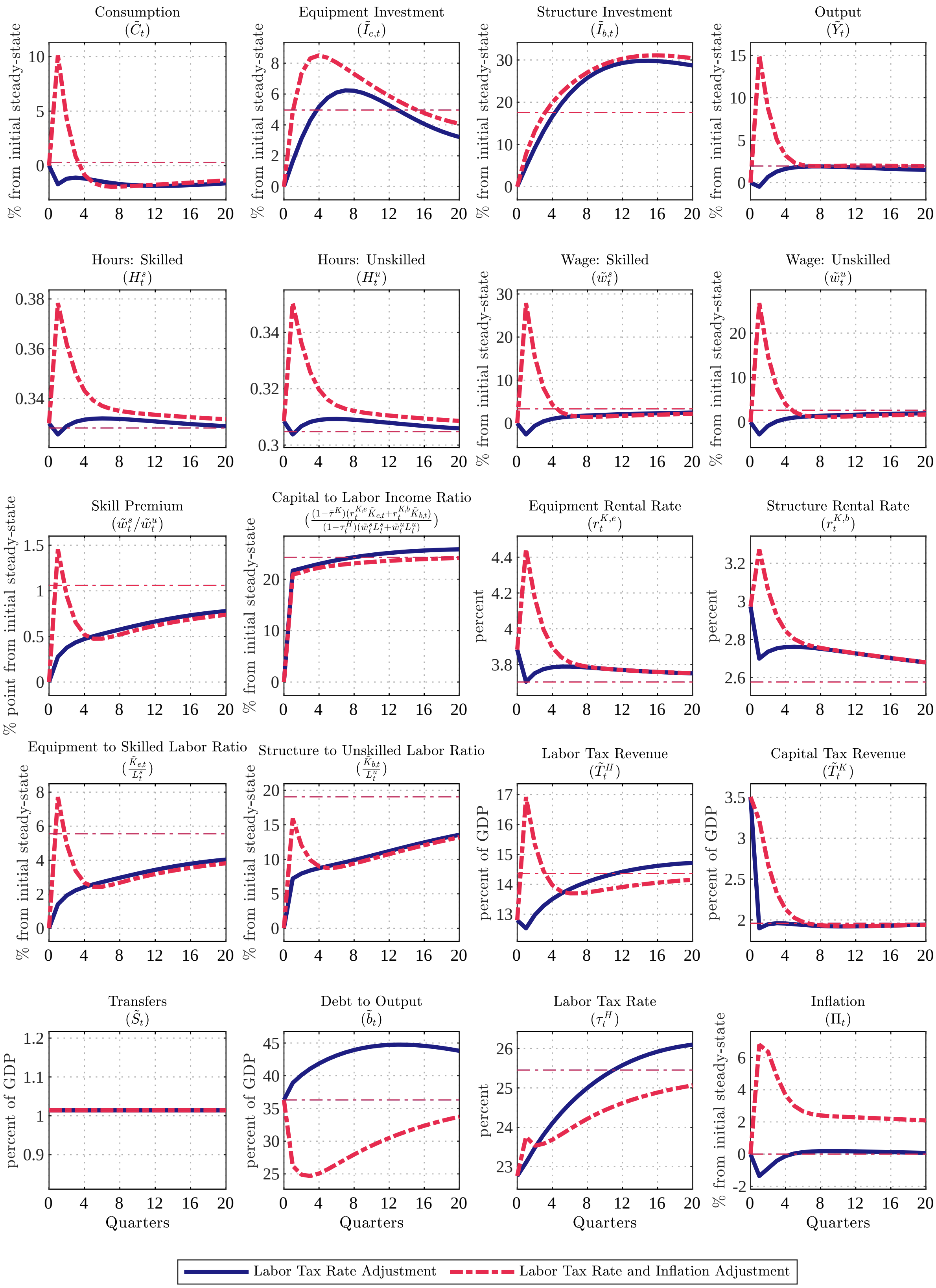


Figure 6: Transition Dynamics of a Permanent Capital Tax Rate Decrease Under Labor Tax Rate and Inflation Adjustment

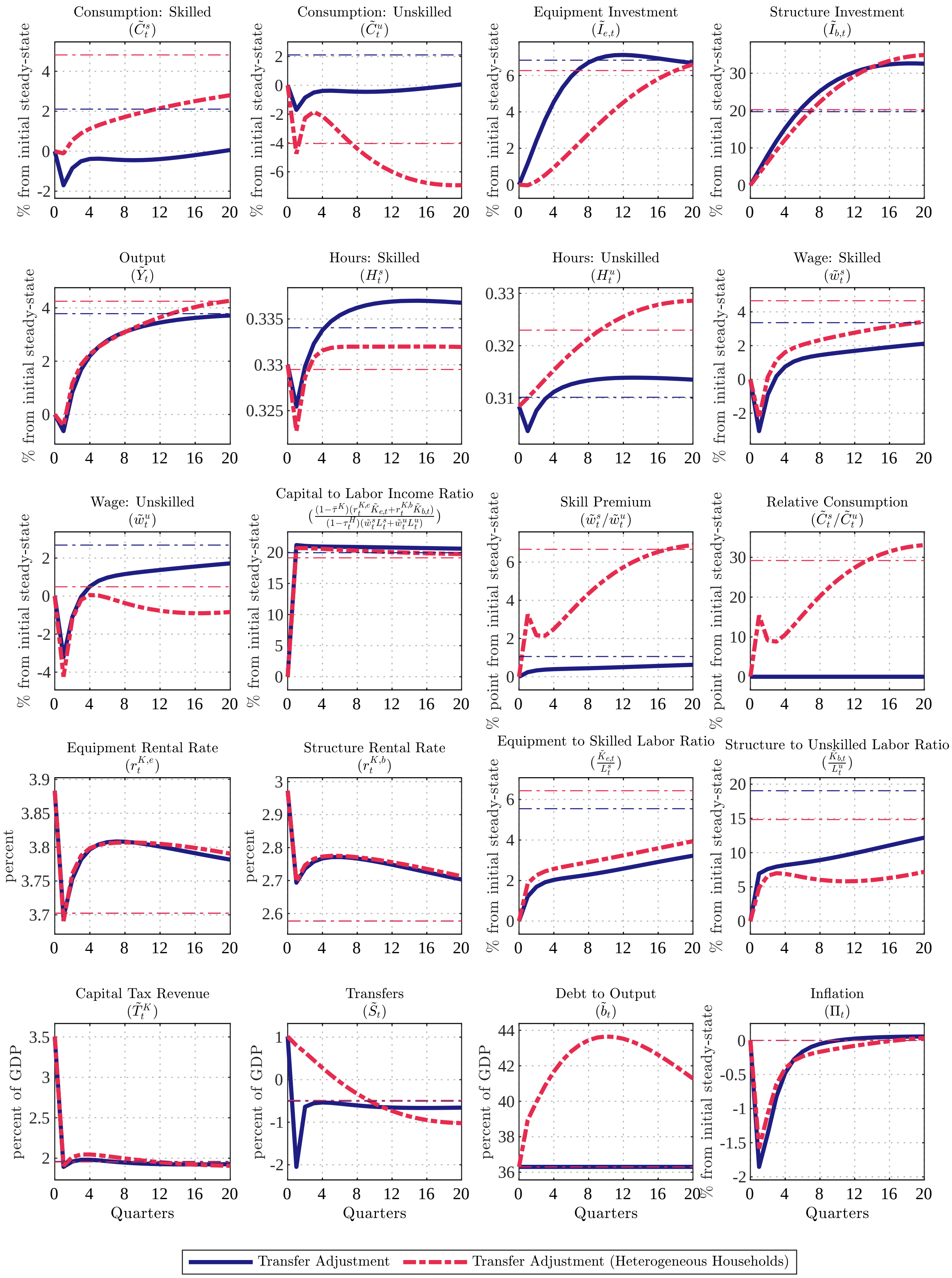


Figure 7: Transition Dynamics of Permanent Capital Tax Rate Changes Under Transfer Adjustment with Heterogeneous Households

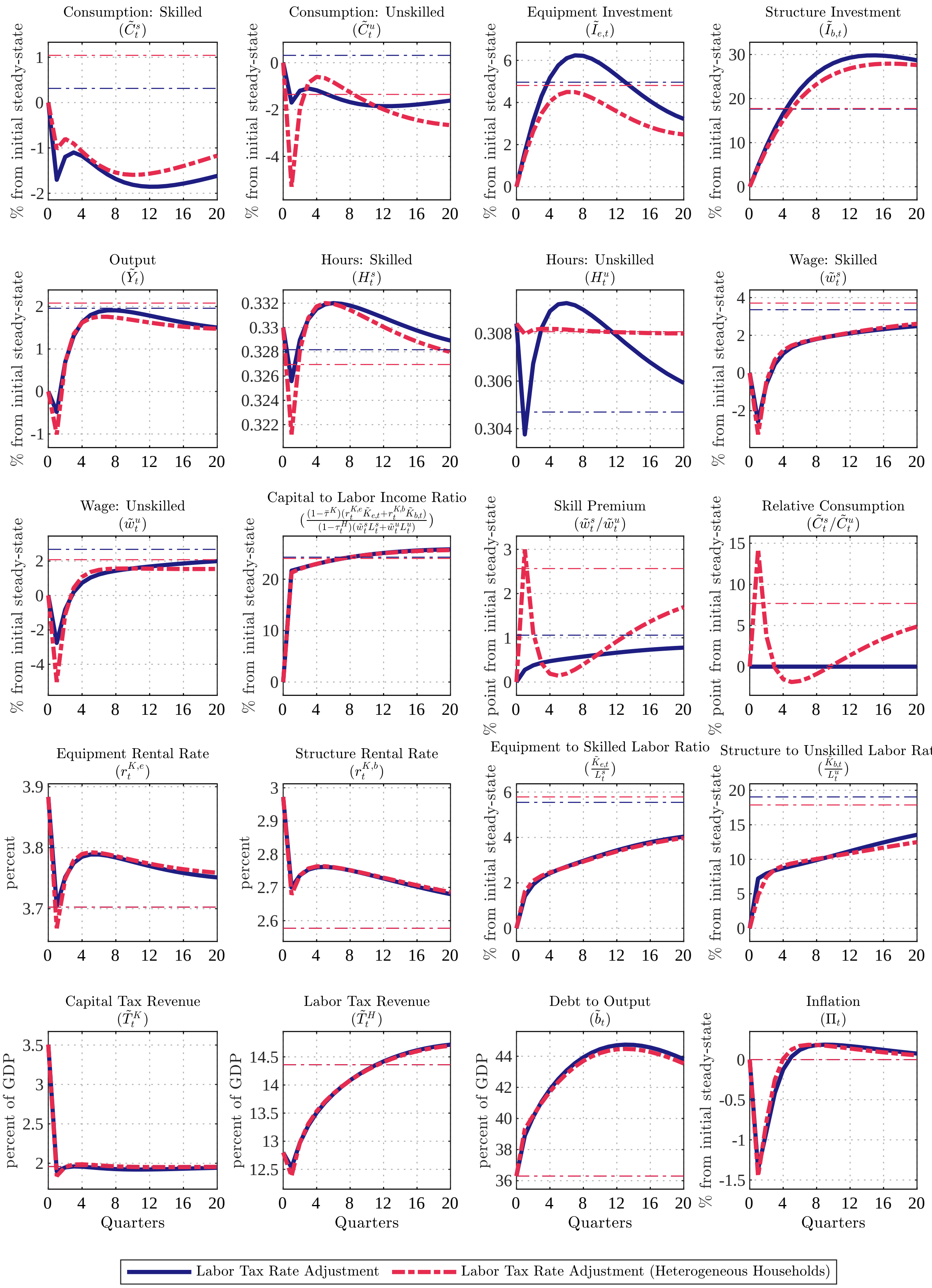


Figure 8: Transition Dynamics of Permanent Capital Tax Rate Changes Under Labor Tax Rate Adjustment with Heterogeneous Households

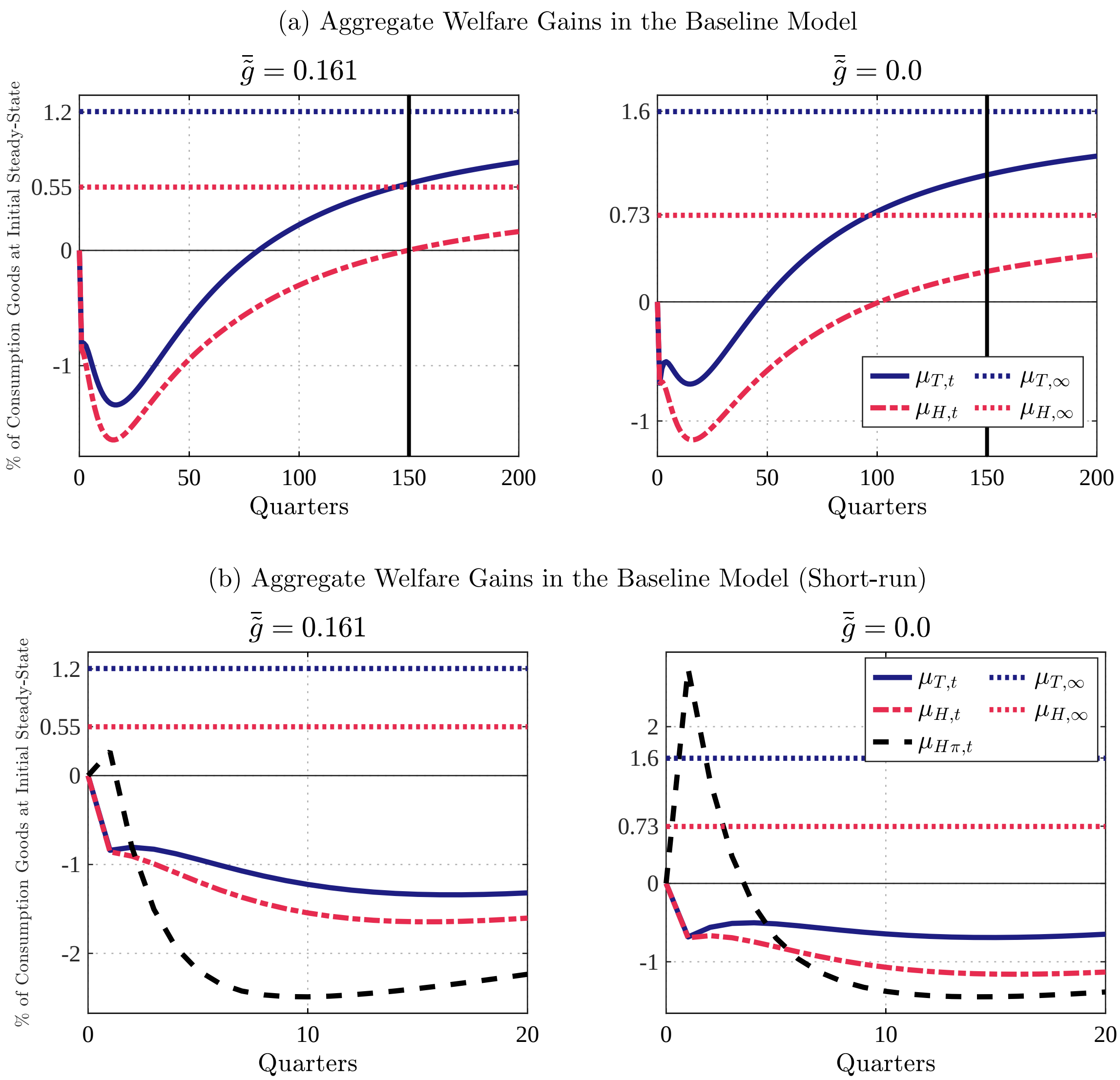
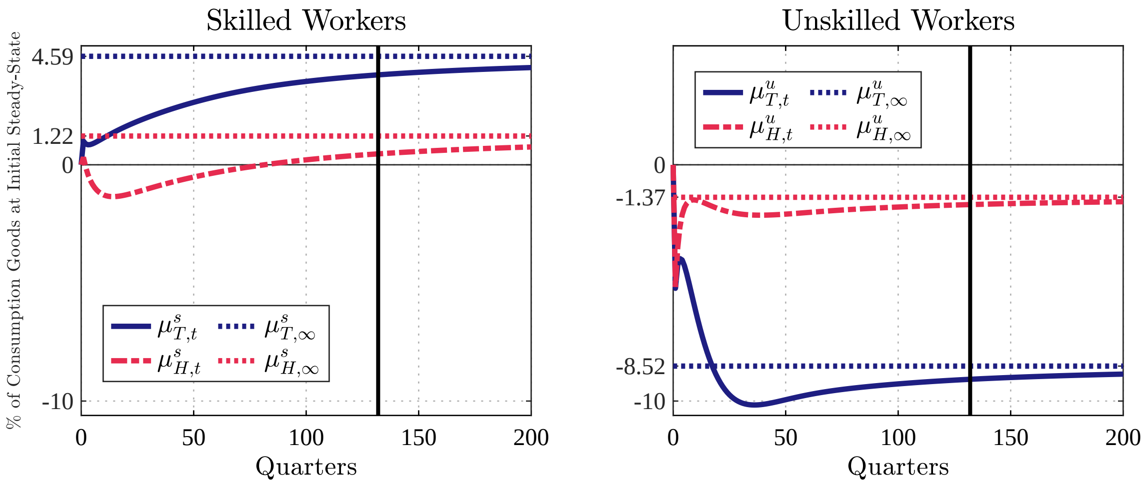


Figure 9: Welfare Implications of a Permanent Capital Tax Rate Decrease

(a) Heterogeneous Households Model: Welfare Gains by Household Type ($\bar{\tilde{g}} = 0.161$)



(b) Heterogeneous Households Model: Welfare Gains by Household Type ($\bar{\tilde{g}} = 0.0$)

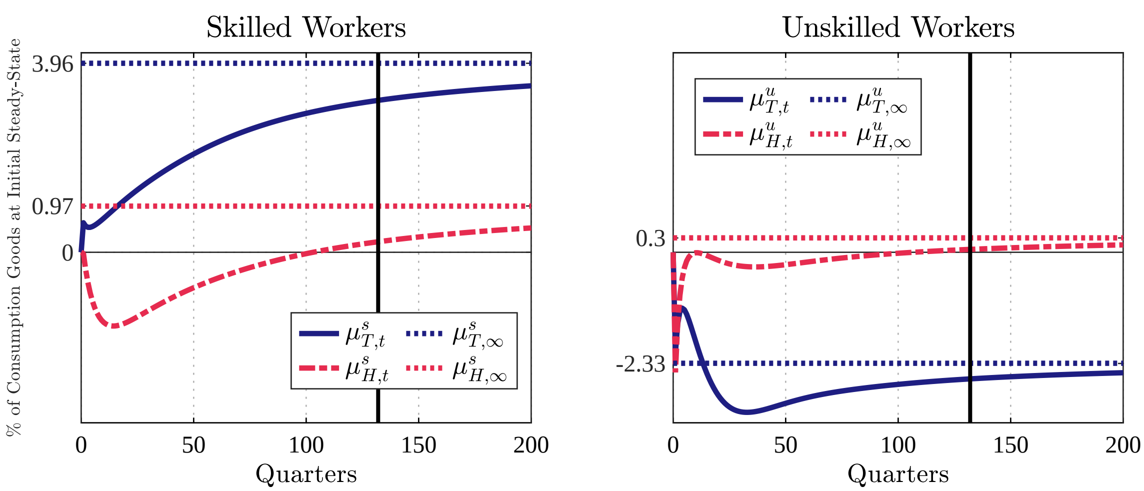


Figure 10: Welfare Implications of a Permanent Capital Tax Rate Decrease

Appendix (not for publication)

A Model

A.1 Households

There are two types of agents: skilled(s) and unskilled(u). We assume there is no population growth. We denote N^i by the mass of i-type agents for $i \in \{s, u\}$. There are two types of capital: capital structures ($K_{b,t}$) and capital equipment ($K_{e,t}$). The economy has two sectors: one sector produces consumption goods and capital structures, and the other sector produces capital equipment. Both sectors use the same technology; however, there is a technology factor specific to the capital equipment sector. We aggregate the production of the two sectors by introducing an exogenous relative price between consumption (structures) and equipment, q_t. We assume that q_t grows a constant rate γ_q, that is $q_t = q_0 \gamma_q^t$. Households' maximization problem is as follows:

$$
\begin{aligned}
\max_{\left\{C_t^i, H_t^i, B_t^i, I_{b,t}^i, I_{e,t}^i, \bar{K}_{b,t+1}^i, \bar{K}_{e,t+1}^i, u_{e,t}^i, u_{b,t}^i, V_{t+1}^i\right\}} \quad & E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\log\left(C_t^i\right) - \bar{\omega}^i \frac{\left(H_t^i\right)^{1+\varphi}}{1+\varphi} \right] \right\} \\
\text{s.t.} \quad & \left(1 + \tau_t^C\right) P_t C_t^i + P_t I_{b,t}^i + P_t I_{e,t}^i + B_t^i + E_t Q_{t,t+1} V_{t+1}^i \\
& = \left(1 - \lambda_{\tau^H}^i \tau_t^H\right) W_t^i H_t^i + R_{t-1} B_{t-1}^i + V_t^i \\
& + \left(1 - \tau_t^K\right) R_t^{K,b} u_{b,t}^i \hat{K}_{b,t}^i + \left(1 - \tau_t^K\right) R_t^{K,e} u_{e,t}^i \hat{K}_{e,t}^i \\
& + \lambda_b \tau_t^K P_t I_{b,t}^i + \lambda_e \tau_t^K P_t I_{e,t}^i \\
& - P_t \left(1 - \lambda_b \tau_t^K\right) \mathcal{A}_b\left(u_{b,t}^i\right) \hat{K}_{b,t}^i - \frac{P_t}{q_t} \left(1 - \lambda_e \tau_t^K\right) \mathcal{A}_e\left(u_{e,t}^i\right) \hat{K}_{e,t}^i \\
& + P_t \frac{\chi_\Phi^i}{N^i} \Phi_t + P_t \frac{\chi_S^i}{N^i} S_t, \\
\hat{K}_{b,t+1}^i = & \left(1 - d_b\right) \hat{K}_{b,t}^i + \left(1 - \mathcal{S}\left(\frac{I_{b,t}^i}{I_{b,t-1}^i}\right)\right) I_{b,t}^i \\
\hat{K}_{e,t+1}^i = & \left(1 - d_e\right) \hat{K}_{e,t}^i + \left(1 - \mathcal{S}\left(\frac{I_{e,t}^i}{I_{e,t-1}^i}\right)\right) I_{e,t}^i q_t
\end{aligned}
$$

where E is the expectation operator, C_t^i is consumption, H_t^i is hours, and $I_{b,t}^i$ and $I_{e,t}^i$ are capital structure investment and capital equipment investment, respectively. Similarly, $K_{b,t}^i \equiv u_{b,t}^i \hat{K}_{b,t}^i$ and $K_{e,t}^i = u_{e,t}^i \hat{K}_{e,t}^i$ are the effective capital stock of structures and equipment, respectively. Here $u_{b,t}^i$ and $u_{e,t}^i$ are the variable capacity utilization rates for structures and equipment capital and $\mathcal{A}_b(u_{b,t}^i)$ and $\mathcal{A}_e(u_{e,t}^i)$ are the costs of capital utilization. In steady-state, $\bar{u}_b^i = \bar{u}_e^i = 1$ and $\mathcal{A}_e(1) = \mathcal{A}_b(1) = 0$. Households trade at time t one-period state-contingent nominal securities V_{t+1}^i at price $Q_{t,t+1}$, and hence fully insure against idiosyncratic risk. Thus, there is complete consumption insurance in the model. B_t^i is nominal risk-less one-period government bonds, Φ_t is profits from firms, and χ_Φ^i is the share of profits for i-type households.

P_t is the aggregate price level, W_t^i is nominal wage for type-i agent, and R_t is the nominal one-period interest rate. Moreover, $R_{b,t}^K$ and $R_{e,t}^K$ are the rental rate of capital invested in structures and equipment, respectively, while q_t is the relative price between consumption (structures) and equipment. S_t^i is lump-sum transfers from the government and χ_Φ^i is the fraction of the transfers for i-type households. τ_t^C is the tax rate on consumption, τ_t^H is the tax rate on wage income, and τ_t^K is the tax rate on capital income. The parameter $\lambda_{\tau^H}^i$ governs the (relative) effective labor tax rate on the two types of agents. β is the discount factor, and d_b and d_e are the rates of depreciation of the capital stock invested in structures and equipment, respectively. Moreover, λ_b and λ_e are the rates of expensing of the capital stock invested in structures and equipment, respectively.

A.2 Firms

A.2.1 Final goods firms

Competitive final goods firms produce aggregate output Y_t by combining a continuum of differentiated intermediate goods using a CES production function $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}$, where θ is the elasticity of substitution between intermediate goods indexed by i. The corresponding optimal price index P_t for the final good is $P_t = \left(\int_0^1 P_t(i)^{1-\theta} di\right)^{\frac{1}{1-\theta}}$, where $P_t(i)$ is the price of intermediate goods and the optimal demand for $Y_t(i)$ is

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t. \tag{5}$$

A.2.2 Intermediate goods firms

Intermediate goods firms indexed by i produce output using a CRS production function

$$Y_t(i) = A_t K_{b,t}^{\alpha}(i) \left[\mu L_{u,t}^{\sigma}(i) + (1-\mu)\left(\lambda K_{e,t}^{\rho}(i) + (1-\lambda) L_{s,t}^{\rho}(i)\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\alpha}{\sigma}} \tag{6}$$

where A_t represents exogenous economy-wide technological progress. The gross growth rate of technology is given by $a_t \equiv \frac{A_t}{A_{t-1}}=\bar{a}$. Firms rent capital and hire labor in economy wide competitive factor markets.

As in Calvo (1983), a firm resets its price optimally with probability $1-\alpha_P$ every period. Firms that do not optimize adjust their price according to the simple partial dynamic indexation rule:

$$P_t(i) = P_{t-1}(i)\, \pi_{t-1}^{\gamma_P} \bar{\pi}^{1-\gamma_P},$$

where γ_P measures the extent of indexation and $\bar{\pi}$ is the steady-state value of the gross inflation rate $\pi_t \equiv P_t/P_{t-1}$. All optimizing firms choose a common price P_t^* to maximize the present discounted value of future profits:

$$\max_{\left\{P_t^*, Y_{t+k}(i), H_{t+k}(i), K_{t+k}(i)\right\}} E_t \left\{\sum_{k=0}^{\infty} (\alpha_p \beta)^k \frac{\Lambda_{t+k}}{\Lambda_t} P_{t+k} \Phi_{t+k}(i)\right\}$$

subject to (6), where Λ_t is the marginal utility of nominal income. Here flow profit $\Phi_{t+k}(i)$ is given by

$$\Phi_{t+k}(i) = \frac{P_t^*}{P_{t+k}} X_{P,t,k} Y_{t,t+k}(i) - \frac{W_{t+k}^u}{P_{t+k}} L_{u,t+k}(i) - \frac{W_{t+k}^s}{P_{t+k}} L_{s,t+k}(i) - \frac{R_{t+k}^{K,b}}{P_{t+k}} K_{b,t+k}(i) - \frac{R_{t+k}^{K,e}}{P_{t+k}} K_{e,t+k}(i),$$

where

$$X_{P,t,k} = \begin{cases} (\pi_t \pi_{t+1} \cdots \pi_{t+k-1})^{\gamma_P} \, \bar{\pi}^{(1-\gamma_P)k}, & k \geq 1 \\ 1 & k = 0 \end{cases}$$

and

$$Y_{t,t+k}(i) = \left(\frac{P_t^* X_{P,t,k}}{P_{t+k}} \right)^{-\theta} Y_{t+k}.$$

A.3 Government

A.3.1 Government budget constraint

The government flow budget constraint, written by expressing fiscal variables as ratio of output, is given by

$$\frac{B_t}{P_t Y_t} + \left(\frac{T_t^C}{Y_t} + \frac{T_t^H}{Y_t} + \frac{T_t^{K,b}}{Y_t} + \frac{T_t^{K,e}}{Y_t} \right) = R_{t-1} \frac{B_{t-1}}{P_{t-1} Y_{t-1}} \frac{1}{\pi_t} \frac{Y_{t-1}}{Y_t} + \frac{G_t}{Y_t} + \frac{S_t}{Y_t}$$

where

$$\begin{aligned}
B_t &= \sum_{i \in \{s,u\}} N^i B_t^i \\
T_t^C &= \sum_{i \in \{s,u\}} N^i \tau_t^C C_t^i \\
T_t^H &= \tau_t^H \sum_{i \in \{s,u\}} \frac{W_t^i}{P_t} \lambda_{\tau^H}^i N^i H_t^i \\
T_t^{K,b} &= \tau_t^K \sum_{i \in \{s,u\}} N^i \left(\frac{R_t^{K,b}}{P_t} \hat{K}_{b,t}^i u_{b,t}^i - \lambda_b \left(I_{b,t}^i + \mathcal{A}_b \left(u_{b,t}^i \right) \hat{K}_{b,t}^i \right) \right) \\
T_t^{K,e} &= \tau_t^K \sum_{i \in \{s,u\}} N^i \left(\frac{R_t^{K,e}}{P_t} \hat{K}_{e,t}^i u_{e,t}^i - \lambda_e \left(I_{e,t}^i + \frac{1}{q_t} \mathcal{A}_e \left(u_{e,t}^i \right) \hat{K}_{e,t}^i \right) \right) \\
S_t &= \sum_{i \in \{s,u\}} N^i S_t^i \\
N^s S_t^s &= \chi_S^s S_t \\
N^u S_t^u &= \chi_S^u S_t = (1 - \chi_S^s) S_t
\end{aligned}$$

and G_t is government spending on the final good.

A.3.2 Monetary policy

Monetary policy is given by a simple interest-rate feedback rule

$$\frac{R_t}{\bar{R}} = \left[\frac{R_{t-1}}{\bar{R}} \right]^{\rho_1^R} \left[\frac{R_{t-2}}{\bar{R}} \right]^{\rho_2^R} \left[\left(\frac{\pi_t}{\bar{\pi}} \right)^{\phi_\pi} \left(\frac{Y_t}{Y_{t-1}} \right)^{\phi_{\Delta y}} \left(\frac{Y_t}{Y_t^n} \right)^{\phi_x} \right]^{(1-\rho_1^R-\rho_2^R)}$$

where ρ_1^R and ρ_2^R govern interest rate smoothing, $\phi_\pi \geq 0$ is the feedback parameter on inflation $\left(\pi_t = \frac{P_t}{P_{t-1}} \right)$, Y_t^n is the natural level of output, ϕ_x is the feedback parameter on output gap, $\phi_{\Delta y}$ is the feedback parameter on output growth, ϕ_x is the feedback parameter on output gap, $\bar{R}$ is the steady-state value of R_t, and $\bar{\pi}$ is the steady-state value of π_t. For large enough feedback response coefficients (a combination of ϕ_x , $\phi_{\Delta y}$, and ϕ_x) , the Taylor principle is satisfied. We will also consider a case, described in more detailed next, where the Taylor principle is not satisfied, and inflation response will play a direct role in government debt stabilization along the transition.

A.3.3 Fiscal policy

We consider a one-time permanent change in the capital tax rate τ_t^K in period 0, when the economy is in the initial steady-state. In order to isolate the effects of the capital tax rate cut, $\frac{G_t}{Y_t}$ is kept unchanged from its initial steady-state value in all periods. The debt-to-GDP ratio, $\frac{B_t}{P_t Y_t}$, may deviate from the initial steady-state in the short run but will converge back to the initial steady-state in the long-run, through appropriate changes in fiscal instruments.

We will study both long-run effects of such permanent changes in the capital tax rate, as well as in extensions, full transition dynamics as the economy evolves towards the new steady-state. We consider the following fiscal policy adjustments so that in the long-run, debt-to-GDP stays at the same level as the initial level through appropriate adjustment of fiscal instruments. First, only lump-sum transfers adjust to maintain $\frac{B_t}{P_t Y_t}$ constant at each point in time.[46]

Second, only labor tax rates τ_t^H adjust following a feedback rule similar to the monetary policy rule specification in Coibion and Gorodnichenko (2011)

$$\begin{aligned}\tau_t^H - \bar{\tau}_{new}^H =& \rho_1^H \left(\tau_{t-1}^H - \bar{\tau}_{new}^H\right) + \rho_2^H \left(\tau_{t-2}^H - \bar{\tau}_{new}^H\right) \\ &+ \left(1 - \rho_1^H - \rho_2^H\right) \left\{\psi_B^H \left(\frac{B_{t-1}}{P_{t-1}Y_{t-1}} - \overline{\frac{B}{PY}}\right) + \psi_{\Delta y}^H \left(\frac{Y_t}{Y_{t-1}}\right) + \psi_x^H \left(\frac{Y_t}{Y_t^n}\right)\right\},\end{aligned}$$

where $0 \leq \rho_1^H + \rho_2^H < 1$ governs labor tax rate smoothing, $\psi_B^H \geq 1-\beta$ is the feedback parameter on outstanding debt, $\psi_{\Delta y}^H$ is the feedback parameter on output growth, ψ_x^H is the feedback parameter on output gap, $\bar{\tau}_{new}^H$ is the new steady-state value of τ_t^H, and $\overline{\frac{B}{PY}}$ is the (initial and new) steady-state value of $\frac{B_t}{P_t Y_t}$.

Third, only consumption tax rates τ_t^C adjust following the simple feedback rule ,

$$\begin{aligned}\tau_t^C - \bar{\tau}_{new}^C =& \rho_1^C \left(\tau_{t-1}^C - \bar{\tau}_{new}^C\right) + \rho_2^C \left(\tau_{t-2}^C - \bar{\tau}_{new}^C\right) = \\ &+ \left(1 - \rho_1^C - \rho_2^C\right) \left\{\psi_B^C \left(\frac{B_{t-1}}{P_{t-1}Y_{t-1}} - \overline{\frac{B}{PY}}\right) + \psi_{\Delta y}^C \left(\frac{Y_t}{Y_{t-1}}\right) + \psi_x^C \left(\frac{Y_t}{Y_t^n}\right)\right\},\end{aligned}$$

where $0 \leq \rho_1^C + \rho_2^C < 1$ governs consumption tax rate smoothing, $\psi_B^C \geq 1 - \beta$ is the feedback parameter on outstanding debt, $\psi_{\Delta y}^C$ is the feedback parameter on output growth, ψ_x^C is the feedback parameter on output gap, and $\bar{\tau}_{new}^C$ is the new steady-state value of τ_t^C.

[46]Since transfers are lump-sum and there is complete risk-sharing, the time-path of transfers does not matter, and so we just use a simple formulation.

A.4 Market Clearing

$$\int_0^1 L_{u,t}(i)\,di = L_{u,t} = N^u H_t^u$$
$$\int_0^1 L_{s,t}(i)\,di = L_{s,t} = N^s H_t^s$$
$$\int_0^1 K_{b,t}(i)\,di = K_{b,t} = N^s K_{b,t}^s + N^u K_{b,t}^u$$
$$\int_0^1 K_{e,t}(i)\,di = K_{e,t} = N^s K_{e,t}^s + N^u K_{e,t}^u$$

where for $i \in \{s, u\}$,

$$K_{b,t}^i = u_{b,t}^i \hat{K}_{b,t}^i$$
$$K_{e,t}^i = u_{e,t}^i \hat{K}_{e,t}^i$$

Aggregate resource constraint:

$$\begin{aligned} Y_t = N^s &\left(C_t^s + I_{b,t}^s + I_{e,t}^s + \mathcal{A}_b\left(u_{b,t}^s\right)\hat{K}_{b,t}^s + \frac{1}{q_t}\mathcal{A}_e\left(u_{e,t}^s\right)\hat{K}_{e,t}^s \right) \\ + N^u &\left(C_t^u + I_{b,t}^u + I_{e,t}^u + \mathcal{A}_b\left(u_{b,t}^s\right)\hat{K}_{b,t}^s + \frac{1}{q_t}\mathcal{A}_e\left(u_{e,t}^s\right)\hat{K}_{e,t}^s \right) + G_t. \end{aligned}$$

Capital accumulation: for $i \in \{s, u\}$,

$$\hat{K}_{b,t+1}^i = (1-d_b)\hat{K}_{b,t}^i + \left(1 - \mathcal{S}\left(\frac{I_{b,t}^i}{I_{b,t-1}^i}\right)\right) I_{b,t}^i$$
$$\hat{K}_{e,t+1}^i = (1-d_e)\hat{K}_{e,t}^i + \left(1 - \mathcal{S}\left(\frac{I_{e,t}^i}{I_{e,t-1}^i}\right)\right) I_{e,t}^i q_t$$

A.5 Nonlinear Equilibrium Conditions

In this section, we derive the equilibrium conditions that are necessary to solve the model.

A.5.1 Firms

- Production function

$$Y_t(i) = A_t K_{b,t}^{\alpha}(i)\left[\mu L_{u,t}^{\sigma}(i) + (1-\mu)\left(\lambda K_{e,t}^{\rho}(i) + (1-\lambda)L_{s,t}^{\rho}(i)\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\alpha}{\sigma}}$$

- Cost minimization: capital-labor ratio

$$R_t^{K,b} = \alpha MC_t(i) A_t K_{b,t}^{\alpha-1}(i)\left[\mu L_{u,t}^{\sigma}(i) + (1-\mu)\left(\lambda K_{e,t}^{\rho}(i) + (1-\lambda)L_{s,t}^{\rho}(i)\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\alpha}{\sigma}} = \alpha MC_t(i)\frac{Y_t(i)}{K_{b,t}(i)}$$

$$R_t^{K,e} = (1-\alpha) MC_t(i)\frac{Y_t(i)}{K_{e,t}(i)}\left(\frac{(1-\mu)\left(\lambda K_{e,t}^{\rho}(i) + (1-\lambda)L_{s,t}^{\rho}(i)\right)^{\frac{\sigma}{\rho}}}{\mu L_{u,t}^{\sigma}(i) + (1-\mu)\left(\lambda K_{e,t}^{\rho}(i) + (1-\lambda)L_{s,t}^{\rho}(i)\right)^{\frac{\sigma}{\rho}}}\right)\left(\frac{\lambda K_{e,t}^{\rho}(i)}{\lambda K_{e,t}^{\rho}(i) + (1-\lambda)L_{s,t}^{\rho}(i)}\right)$$

$$W_t^u = (1-\alpha) MC_t(i)\frac{Y_t(i)}{L_{u,t}(i)}\left(\frac{\mu L_{u,t}^{\sigma}(i)}{\mu L_{u,t}^{\sigma}(i) + (1-\mu)\left(\lambda K_{e,t}^{\rho}(i) + (1-\lambda)L_{s,t}^{\rho}(i)\right)^{\frac{\sigma}{\rho}}}\right)$$

$$W_t^s = (1-\alpha) MC_t(i) \frac{Y_t(i)}{L_{s,t}(i)} \left(\frac{(1-\mu)\left(\lambda K_{e,t}^{\rho}(i) + (1-\lambda) L_{s,t}^{\rho}(i)\right)^{\frac{\sigma}{\rho}}}{\mu L_{u,t}^{\sigma}(i) + (1-\mu)\left(\lambda K_{e,t}^{\rho}(i) + (1-\lambda) L_{s,t}^{\rho}(i)\right)^{\frac{\sigma}{\rho}}} \right) \left(\frac{(1-\lambda) L_{s,t}^{\rho}(i)}{\lambda K_{e,t}^{\rho}(i) + (1-\lambda) L_{s,t}^{\rho}(i)} \right)$$

- Skill-premium

$$\frac{W_t^s}{W_t^u} = \frac{(1-\mu)(1-\lambda)}{\mu} \left(\lambda \left(\frac{K_{e,t}(i)}{L_{s,t}(i)} \right)^{\rho} + (1-\lambda) \right)^{\frac{\sigma-\rho}{\rho}} \left(\frac{L_{u,t}(i)}{L_{s,t}(i)} \right)^{1-\sigma}$$

$$\frac{R_t^{K,e}}{R_t^{K,b}} = \frac{1-\alpha}{\alpha} \left(\frac{(1-\mu)\left(\lambda K_{e,t}^{\rho}(i) + (1-\lambda) L_{s,t}^{\rho}(i)\right)^{\frac{\sigma}{\rho}}}{\mu L_{u,t}^{\sigma}(i) + (1-\mu)\left(\lambda K_{e,t}^{\rho}(i) + (1-\lambda) L_{s,t}^{\rho}(i)\right)^{\frac{\sigma}{\rho}}} \right) \left(\frac{\lambda K_{e,t}^{\rho}(i)}{\lambda K_{e,t}^{\rho}(i) + (1-\lambda) L_{s,t}^{\rho}(i)} \right) \frac{K_{b,t}(i)}{K_{e,t}(i)}$$

$$\frac{W_t^s}{R_t^{K,e}} = \frac{1-\lambda}{\lambda} \left(\frac{K_{e,t}(i)}{L_{s,t}(i)} \right)^{1-\rho}$$

- Profit maximization: Notice that from above, all firms have the same marginal costs, $MC_t(i) = MC_t$. Then,

$$\max_{\{P_t^*\}} E_t \left\{ \sum_{k=0}^{\infty} (\alpha_p \beta)^k \frac{\Lambda_{t+k}}{\Lambda_t} P_{t+k} \left(\frac{P_t^*}{P_{t+k}} X_{P,t,k} - \frac{MC_{t+k}}{P_{t+k}} \right) Y_{t,t+k}(i) \right\}$$

where

$$X_{P,t,k} = \begin{cases} (\pi_t \pi_{t+1} \cdots \pi_{t+k-1})^{\gamma_P} \, \bar{\pi}^{(1-\gamma_P)k}, & k \geq 1 \\ 1 & k = 0 \end{cases}$$

and

$$Y_{t,t+k}(i) = \left(\frac{P_t^* X_{P,t,k}}{P_{t+k}} \right)^{-\theta} Y_{t+k}.$$

The first-order condition is:

$$P_t^* = \frac{\theta}{\theta-1} \frac{E_t \left\{ \sum_{k=0}^{\infty} (\alpha_p \beta)^k \frac{\Lambda_{t+k}}{\Lambda_t} \left(MC_{t+k} \left(\frac{X_{P,t,k}}{P_{t+k}} \right)^{-\theta} Y_{t+k} \right) \right\}}{E_t \left\{ \sum_{k=0}^{\infty} (\alpha_p \beta)^k \frac{\Lambda_{t+k}}{\Lambda_t} \left(X_{P,t,k} \left(\frac{X_{P,t,k}}{P_{t+k}} \right)^{-\theta} Y_{t+k} \right) \right\}}$$

Now write the price-setting condition recursively as:

$$P_t^* = \frac{\theta}{\theta-1} \frac{Z_{1,t}}{Z_{2,t}}$$

where

$$\begin{aligned} Z_{1,t} &= E_t \left\{ \sum_{k=0}^{\infty} (\alpha_p \beta)^k \frac{\Lambda_{t+k}}{\Lambda_t} \left(MC_{t+k} \left(\frac{X_{P,t,k}}{P_{t+k}} \right)^{-\theta} Y_{t+k} \right) \right\} \\ &= MC_t (P_t)^{\theta} Y_t + \alpha_p \beta \left((\pi_t)^{\gamma_P} \bar{\pi}^{(1-\gamma_P)} \right)^{-\theta} E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} Z_{1,t+1} \right\} \end{aligned}$$

$$\begin{aligned} Z_{2,t} &= E_t \left\{ \sum_{k=0}^{\infty} (\alpha_p \beta)^k \frac{\Lambda_{t+k}}{\Lambda_t} \left(X_{P,t,k} \left(\frac{X_{P,t,k}}{P_{t+k}} \right)^{-\theta} Y_{t+k} \right) \right\} \\ &= (P_t)^{\theta} Y_t + \alpha_p \beta \left((\pi_t)^{\gamma_P} \bar{\pi}^{(1-\gamma_P)} \right)^{1-\theta} E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} Z_{2,t+1} \right\}. \end{aligned}$$

- Profit

$$\Phi_t(i) = \frac{P_t(i)}{P_t} Y_t(i) - \frac{W_t^u}{P_t} L_{u,t}(i) - \frac{W_t^s}{P_t} L_{s,t}(i) - \frac{R_t^{K,b}}{P_t} K_{b,t}(i) - \frac{R_t^{K,e}}{P_t} K_{e,t}(i).$$

A.5.2 Households

- Maximization Problem:

$$
\begin{aligned}
& E_0\left\{\sum_{t=0}^{\infty}\beta^t\left[\log\left(C_t^i\right)-\bar{\omega}^i\frac{\left(H_t^i\right)^{1+\varphi}}{1+\varphi}\right]\right\} \\
& -E_0\left\{\sum_{t=0}^{\infty}\beta^t\Lambda_t^i\left\{\left(1+\tau_t^C\right)P_tC_t^i+P_tI_{b,t}^i+Q_{t,t+1}V_{t+1}^i+P_tI_{e,t}^i+B_t^i\right\}\right\} \\
& +E_0\left\{\sum_{t=0}^{\infty}\beta^t\Lambda_t^i\left\{\left(1-\lambda_{\tau^H}^i\tau_t^H\right)W_t^iH_t^i+V_t^i+R_{t-1}B_{t-1}^i+P_t\Phi_t^i+P_tS_t^i\right\}\right\} \\
& +E_0\left\{\sum_{t=0}^{\infty}\beta^t\Lambda_t^i\left\{\left(1-\tau_t^K\right)R_t^{K,b}u_{b,t}^i\hat{K}_{b,t}^i+\left(1-\tau_t^K\right)R_t^{K,e}u_{e,t}^i\hat{K}_{e,t}^i+\lambda_b\tau_t^KP_tI_{b,t}^i+\lambda_e\tau_t^KP_tI_{e,t}^i\right\}\right\} \\
& -E_0\left\{\sum_{t=0}^{\infty}\beta^t\Lambda_t^i\left\{P_t\left(1-\lambda_b\tau_t^K\right)\mathcal{A}_b\left(u_{b,t}^i\right)\hat{K}_{b,t}^i+\frac{P_t}{q_t}\left(1-\lambda_e\tau_t^K\right)\mathcal{A}_e\left(u_{e,t}^i\right)\hat{K}_{e,t}^i\right\}\right\} \\
& +E_0\left\{\sum_{t=0}^{\infty}\beta^t\Psi_{b,t}^i\left\{\left(1-d_b\right)\hat{K}_{b,t}^i+\left(1-\mathcal{S}\left(\frac{I_{b,t}^i}{I_{b,t-1}^i}\right)\right)I_{b,t}^i-\hat{K}_{b,t+1}^i\right\}\right\} \\
& +E_0\left\{\sum_{t=0}^{\infty}\beta^t\Psi_{e,t}^i\left\{\left(1-d_e\right)\hat{K}_{e,t}^i+\left(1-\mathcal{S}\left(\frac{I_{e,t}^i}{I_{e,t-1}^i}\right)\right)I_{e,t}^iq_t-\hat{K}_{e,t+1}^i\right\}\right\}
\end{aligned}
$$

- FOCs:

$$
\begin{aligned}
C_t^i:&\quad P_t\Lambda_t^i\left(1+\tau_t^C\right)=\frac{1}{C_t^i} \\
H_t^i:&\quad \Lambda_t^i\left(1-\lambda_{\tau^H}^i\tau_t^H\right)W_t^i=\bar{\omega}\left(H_t^i\right)^{\varphi} \\
B_t^i:&\quad \Lambda_t^i=\beta R_tE_t\left\{\Lambda_{t+1}^i\right\} \\
V_{t+1}^i:&\quad Q_{t,t+1}=\beta\left\{\frac{\Lambda_{t+1}^i}{\Lambda_t^i}\right\} \\
\hat{K}_{b,t+1}^i:&\quad \Psi_{b,t}^i=\beta E_t\left\{\left(1-d_b\right)\Psi_{b,t+1}^i+\left[\left(1-\tau_{t+1}^K\right)R_{t+1}^{K,b}u_{b,t+1}^i-P_{t+1}\left(1-\lambda_b\tau_{t+1}^K\right)\mathcal{A}_b\left(u_{b,t+1}^i\right)\right]\Lambda_{t+1}^i\right\} \\
I_{b,t}^i:&\quad \left(1-\lambda_b\tau_t^K\right)P_t\Lambda_t^i=\Psi_{b,t}^i\left(1-\mathcal{S}\left(\frac{I_{b,t}^i}{I_{b,t-1}^i}\right)-\mathcal{S}'\left(\frac{I_{b,t}^i}{I_{b,t-1}^i}\right)\frac{I_{b,t}^i}{I_{b,t-1}^i}\right) \\
&\quad +\beta E_t\left\{\Psi_{b,t+1}^i\left(\frac{I_{b,t+1}^i}{I_{b,t}^i}\right)^2\mathcal{S}'\left(\frac{I_{b,t+1}^i}{I_{b,t}^i}\right)\right\} \\
\hat{K}_{e,t+1}^i:&\quad \Psi_{e,t}^i=\beta E_t\left\{\left(1-d_e\right)\Psi_{e,t+1}^i+\left[\left(1-\tau_{t+1}^K\right)R_{t+1}^{K,e}u_{e,t+1}^i-\frac{P_{t+1}}{q_{t+1}}\left(1-\lambda_e\tau_{t+1}^K\right)\mathcal{A}_e\left(u_{e,t+1}^i\right)\right]\Lambda_{t+1}^i\right\} \\
I_{e,t}^i:&\quad \left(1-\lambda_e\tau_t^K\right)P_t\Lambda_t^i=q_t\Psi_{e,t}^i\left(1-\mathcal{S}\left(\frac{I_{e,t}^i}{I_{e,t-1}^i}\right)-\mathcal{S}'\left(\frac{I_{e,t}^i}{I_{e,t-1}^i}\right)\frac{I_{e,t}^i}{I_{e,t-1}^i}\right) \\
&\quad +\beta E_t\left\{q_{t+1}\Psi_{e,t+1}^i\left(\frac{I_{e,t+1}^i}{I_{e,t}^i}\right)^2\mathcal{S}'\left(\frac{I_{e,t+1}^i}{I_{e,t}^i}\right)\right\} \\
u_{b,t}^i:&\quad \left(1-\tau_t^K\right)R_t^{K,b}=P_t\left(1-\lambda_b\tau_t^K\right)\mathcal{A}_b^{'}\left(u_{b,t}^i\right) \\
u_{e,t}^i:&\quad \left(1-\tau_t^K\right)R_t^{K,e}=\frac{P_t}{q_t}\left(1-\lambda_e\tau_t^K\right)\mathcal{A}_e^{'}\left(u_{e,t}^i\right)
\end{aligned}
$$

Notice that from above $u_{b,t}^s=u_{b,t}^u=u_{b,t}$ and $u_{e,t}^s=u_{e,t}^u=u_{e,t}$.

- Definition of effective capital: for $i \in \{s, u\}$

$$
\begin{aligned}
K_{b,t}^i &= u_{b,t}\hat{K}_{b,t}^i \\
K_{e,t}^i &= u_{e,t}\hat{K}_{e,t}^i
\end{aligned}
$$

- Capital accumulation: for $i \in \{s, u\}$

$$
\begin{aligned}
\hat{K}_{b,t+1}^i &= (1-d_b)\,\hat{K}_{b,t}^i + \left(1 - \mathcal{S}\left(\frac{I_{b,t}^i}{I_{b,t-1}^i}\right)\right) I_{b,t}^i \\
\hat{K}_{e,t+1}^i &= (1-d_e)\,\hat{K}_{e,t}^i + \left(1 - \mathcal{S}\left(\frac{I_{e,t}^i}{I_{e,t-1}^i}\right)\right) I_{e,t}^i q_t
\end{aligned}
$$

A.5.3 Government and Market Clearing

- Government budget constraint

$$
\frac{B_t}{P_t Y_t} + \left(\frac{T_t^C + T_t^H + T_t^{K,b} + T_t^{K,e}}{Y_t}\right) = R_{t-1}\frac{B_{t-1}}{P_{t-1}Y_{t-1}}\frac{1}{\pi_t}\frac{Y_{t-1}}{Y_t} + \frac{G_t}{Y_t} + \frac{S_t}{Y_t}
$$

where

$$
\begin{aligned}
B_t &= \sum_{i\in\{s,u\}} N^i B_t^i \\
T_t^C &= \sum_{i\in\{s,u\}} N^i \tau_t^C C_t^i \\
T_t^H &= \tau_t^H \sum_{i\in\{s,u\}} \frac{W_t^i}{P_t}\lambda_{\tau^H}^i N^i H_t^i \\
T_t^{K,b} &= \tau_t^K \sum_{i\in\{s,u\}} N^i \left(\frac{R_t^{K,b}}{P_t}\hat{K}_{b,t}^i u_{b,t}^i - \lambda_b\left(I_{b,t}^i + \mathcal{A}_b\left(u_{b,t}\right)\hat{K}_{b,t}^i\right)\right) \\
T_t^{K,e} &= \tau_t^K \sum_{i\in\{s,u\}} N^i \left(\frac{R_t^{K,e}}{P_t}\hat{K}_{e,t}^i u_{e,t}^i - \lambda_e\left(I_{e,t}^i + \frac{1}{q_t}\mathcal{A}_e\left(u_{e,t}\right)\hat{K}_{e,t}^i\right)\right) \\
S_t &= \sum_{i\in\{s,u\}} N^i S_t^i \\
N^s S_t^s &= \chi_S^s S_t \\
N^u S_t^u &= \chi_S^u S_t = (1-\chi_S^s)\,S_t
\end{aligned}
$$

- Profit distribution

$$
\begin{aligned}
\Phi_t &= \sum_{i\in\{s,u\}} N^i \Phi_t^i \\
N^s \Phi_t^s &= \chi_\Phi^s \Phi_t \\
N^u \Phi_t^u &= \chi_\Phi^u \Phi_t = (1-\chi_\Phi^s)\,\Phi_t
\end{aligned}
$$

- Total profit:

$$
\begin{aligned}
\Phi_t &= \int_0^1 \left(\frac{P_t\,(i)}{P_t}\right)^{1-\theta} Y_t di - \frac{W_t^u}{P_t}\int_0^1 L_{u,t}\,(i)\,di - \frac{W_t^s}{P_t}\int_0^1 L_{s,t}\,(i)\,di - \frac{R_t^{K,b}}{P_t}\int_0^1 K_{b,t}\,(i)\,di - \frac{R_t^{K,e}}{P_t}\int_0^1 K_{e,t}\,(i)\,di \\
&= Y_t - N^u\left(\frac{W_t^u}{P_t}H_t^u + \frac{R_t^{K,b}}{P_t}K_{b,t}^u + \frac{R_t^{K,e}}{P_t}K_{e,t}^u\right) - N^s\left(\frac{W_t^s}{P_t}H_t^s + \frac{R_t^{K,b}}{P_t}K_{b,t}^s + \frac{R_t^{K,e}}{P_t}K_{e,t}^s\right)
\end{aligned}
$$

- Aggregate resource constraint:

$$\begin{aligned}Y_t = {} & N^s\left(C_t^s + I_{b,t}^s + I_{e,t}^s + \mathcal{A}_b\left(u_{b,t}\right)\hat{K}_{b,t}^s + \frac{1}{q_t}\mathcal{A}_e\left(u_{e,t}\right)\hat{K}_{e,t}^s\right) \\ & + N^u\left(C_t^u + I_{b,t}^u + I_{e,t}^u + \mathcal{A}_b\left(u_{b,t}\right)\hat{K}_{b,t}^s + \frac{1}{q_t}\mathcal{A}_e\left(u_{e,t}\right)\hat{K}_{e,t}^s\right) + G_t\end{aligned}$$

- Aggregate production function:

$$\int_0^1 A_t K_{b,t}^{\alpha}(i)\left[\mu L_{u,t}^{\sigma}(i) + (1-\mu)\left(\lambda K_{e,t}^{\rho}(i) + (1-\lambda)L_{s,t}^{\rho}(i)\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\alpha}{\sigma}} di = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t di$$

  From equilibrium conditions, all firms hire (equipment/structure) capital and (skilled and unskilled) labor in the same ratio:

$$\int_0^1 A_t\left(\frac{K_{b,t}}{L_{s,t}}\right)^{\alpha} L_{s,t}(i)\left[\mu\left(\frac{L_{u,t}}{L_{s,t}}\right)^{\sigma} + (1-\mu)\left(\lambda\left(\frac{K_{e,t}}{L_{s,t}}\right)^{\rho} + (1-\lambda)\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\alpha}{\sigma}} di = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t di$$

  Then,

$$A_t K_{b,t}^{\alpha}\left[\mu L_{u,t}^{\sigma} + (1-\mu)\left(\lambda K_{e,t}^{\rho} + (1-\lambda)L_{s,t}^{\rho}\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\alpha}{\sigma}} = Y_t \Xi_t$$

  where

$$\begin{aligned}\Xi_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta} di &= \int_0^{1-\alpha_P}\left(\frac{P_t^*}{P_t}\right)^{-\theta} di + \int_{1-\alpha_P}^1 \left(\frac{P_{t-1}(i)\,\pi_{t-1}^{\gamma_P}\bar{\pi}^{1-\gamma_P}}{P_t}\right)^{-\theta} di \\ &= (1-\alpha_P)\left(\frac{P_t^*}{P_t}\right)^{-\theta} + \alpha_P \pi_t^{\theta}\left(\pi_{t-1}^{\gamma_P}\bar{\pi}^{1-\gamma_P}\right)^{-\theta}\Xi_{t-1}.\end{aligned}$$

- Aggregate price index

$$\begin{aligned}P_t^{1-\theta} &= (1-\alpha_P)\left(P_t^*\right)^{1-\theta} + \alpha_P\left(\pi_{t-1}^{\gamma_P}\bar{\pi}^{1-\gamma_P}\right)^{1-\theta}\int_0^1 \left(P_{t-1}(i)\right)^{1-\theta} di \\ \pi_t^{1-\theta} &= (1-\alpha_P)\left(\pi_t\frac{P_t^*}{P_t}\right)^{1-\theta} + \alpha_P\left(\pi_{t-1}^{\gamma_P}\bar{\pi}^{1-\gamma_P}\right)^{1-\theta}\end{aligned}$$

A.6 Stationary Equilibrium

We consider a symmetric equilibrium across firms, where all firms set the same price and produce the same amount of output.

A.6.1 Notations

A balanced growth path can be achieved if $\frac{A_t}{A_{t-1}} = \bar{a} = \gamma^{1-\alpha}$and $\frac{q_t}{q_{t-1}} = \gamma_q = \frac{1}{\gamma}$, i.e. $\gamma_q\gamma = 1$. In this case, the growth rate of output $\frac{Y_t}{Y_{t-1}} = \gamma$.

$$\text{Quantities :}\tilde{C}_t^i = \frac{C_t^i}{\gamma^t},\ \tilde{I}_{b,t}^i = \frac{I_{b,t}^i}{\gamma^t},\ \tilde{I}_{e,t}^i = \frac{I_{e,t}^i}{\gamma^t},\ \tilde{K}_{b,t}^i = \frac{K_{b,t}^i}{\gamma^t},\ \tilde{K}_{e,t}^i = \frac{K_{e,t}^i}{(\gamma_q\gamma)^t} \text{ for } i \in \{s,u\}$$

$$\tilde{Y}_t = \frac{Y_t}{\gamma^t},\ \tilde{K}_{b,t} = \frac{K_{b,t}}{\gamma^t},\ \tilde{K}_{e,t} = \frac{K_{e,t}}{(\gamma_q\gamma)^t},\ \tilde{\hat{K}}_{b,t} = \frac{\hat{K}_{b,t}}{\gamma^t},\ \tilde{\hat{K}}_{e,t} = \frac{\hat{K}_{e,t}}{(\gamma_q\gamma)^t}$$

$$\text{Prices : } \tilde{w}_t^s = \frac{W_t^s}{P_t\gamma^t},\ \tilde{w}_t^u = \frac{W_t^u}{P_t\gamma^t},\ r_t^{K,b} = \frac{R_t^{K,b}}{P_t},\ r_t^{K,e} = \frac{R_t^{K,e}}{P_t\gamma^t}$$

$$\tilde{\Phi}_t = \frac{\Phi_t}{\gamma^t},\ \tilde{\Phi}_t^i = \frac{\Phi_t^i}{\gamma^t},\ \pi_t = \frac{P_t}{P_{t-1}},\ \tilde{p}_t^* = \frac{P_t^*}{P_t},\ mc_t = \frac{MC_t}{P_t}$$

$$\tilde{Z}_{1,t} = \frac{Z_{1,t}}{(P_t)^{\theta+1}\gamma^t},\ \tilde{Z}_{2,t} = \frac{Z_{2,t}}{(P_t)^{\theta}\gamma^t}$$

$$\text{Fiscal variables: } \tilde{b}_t = \frac{B_t}{P_tY_t},\ \tilde{G}_t = \frac{G_t}{Y_t},\ \tilde{T}_t^C = \frac{T_t^C}{Y_t},\ \tilde{T}_t^H = \frac{T_t^H}{Y_t}$$

$$\tilde{T}_t^{K,b} = \frac{T_t^{K,b}}{Y_t},\ \tilde{T}_t^{K,e} = \frac{T_t^{K,e}}{Y_t},\ \tilde{S}_t = \frac{S_t}{Y_t},\ \tilde{S}_t^i = \frac{S_t^i}{Y_t}$$

$$\text{Multipliers: } \tilde{\Lambda}_t^i = \gamma^t P_t\Lambda_t^i,\ \tilde{\Psi}_{b,t}^i = \gamma^t\Psi_{b,t}^i,\ \tilde{\Psi}_{e,t}^i = \Psi_{e,t}^i$$

A.6.2 Stationary Equilibrium Conditions

- Production function (Let $A_0 = 1$)

$$\tilde{Y}_t^A = \tilde{K}_{b,t}^{\alpha}\left[\mu L_{u,t}^{\sigma} + (1-\mu)\left(\lambda\tilde{K}_{e,t}^{\rho} + (1-\lambda)L_{s,t}^{\rho}\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\alpha}{\sigma}} \tag{7}$$

- Aggregate output

$$\tilde{Y}_t^A = \tilde{Y}_t\Xi_t \tag{8}$$

- Cost minimization

$$r_t^{K,b} = \alpha mc_t\frac{\tilde{Y}_t^A}{\tilde{K}_{b,t}} \tag{9}$$

$$r_t^{K,e} = (1-\alpha)mc_t\frac{\tilde{Y}_t^A}{\tilde{K}_{e,t}}\left(\frac{(1-\mu)\left(\lambda\tilde{K}_{e,t}^{\rho} + (1-\lambda)L_{s,t}^{\rho}\right)^{\frac{\sigma}{\rho}}}{\mu L_{u,t}^{\sigma} + (1-\mu)\left(\lambda\tilde{K}_{e,t}^{\rho} + (1-\lambda)L_{s,t}^{\rho}\right)^{\frac{\sigma}{\rho}}}\right)\left(\frac{\lambda\tilde{K}_{e,t}^{\rho}}{\lambda\tilde{K}_{e,t}^{\rho} + (1-\lambda)L_{s,t}^{\rho}}\right) \tag{10}$$

$$\tilde{w}_t^u = (1-\alpha)mc_t\frac{\tilde{Y}_t^A}{L_{u,t}}\left(\frac{\mu L_{u,t}^{\sigma}}{\mu L_{u,t}^{\sigma} + (1-\mu)\left(\lambda\tilde{K}_{e,t}^{\rho} + (1-\lambda)L_{s,t}^{\rho}\right)^{\frac{\sigma}{\rho}}}\right) \tag{11}$$

$$\tilde{w}_t^s = (1-\alpha)mc_t\frac{\tilde{Y}_t^A}{L_{s,t}}\left(\frac{(1-\mu)\left(\lambda\tilde{K}_{e,t}^{\rho} + (1-\lambda)L_{s,t}^{\rho}\right)^{\frac{\sigma}{\rho}}}{\mu L_{u,t}^{\sigma} + (1-\mu)\left(\lambda\tilde{K}_{e,t}^{\rho} + (1-\lambda)L_{s,t}^{\rho}\right)^{\frac{\sigma}{\rho}}}\right)\left(\frac{(1-\lambda)L_{s,t}^{\rho}}{\lambda\tilde{K}_{e,t}^{\rho} + (1-\lambda)L_{s,t}^{\rho}}\right) \tag{12}$$

- Skill-premium

$$\frac{\tilde{w}_t^s}{\tilde{w}_t^u} = \frac{(1-\mu)(1-\lambda)}{\mu}\left(\lambda\left(\frac{\tilde{K}_{e,t}}{L_{s,t}}\right)^{\rho} + (1-\lambda)\right)^{\frac{\sigma-\rho}{\rho}}\left(\frac{L_{u,t}}{L_{s,t}}\right)^{1-\sigma}$$

- Firms' maximization:

$$\tilde{Z}_{1,t} = mc_t \tilde{Y}_t + \alpha_p \beta \left((\pi_t)^{\gamma_P} \bar{\pi}^{(1-\gamma_P)} \right)^{-\theta} E_t \left\{ \frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t} \tilde{Z}_{1,t+1} (\pi_{t+1})^{\theta} \right\} \tag{13}$$

$$\tilde{Z}_{2,t} = \tilde{Y}_t + \alpha_p \beta \left((\pi_t)^{\gamma_P} \bar{\pi}^{(1-\gamma_P)} \right)^{1-\theta} E_t \left\{ \frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t} \tilde{Z}_{2,t+1} (\pi_{t+1})^{\theta-1} \right\} \tag{14}$$

- Price dispersion

$$\Xi_t = (1-\alpha_P) (\tilde{p}_t^*)^{-\theta} + \alpha_P \pi_t^{\theta} \left(\pi_{t-1}^{\gamma_P} \bar{\pi}^{1-\gamma_P} \right)^{-\theta} \Xi_{t-1} \tag{15}$$

where

$$\tilde{p}_t^* = \frac{\theta}{\theta-1} \frac{\tilde{Z}_{1,t}}{\tilde{Z}_{2,t}}. \tag{16}$$

- Aggregate price index

$$\pi_t^{1-\theta} = (1-\alpha_P) (\pi_t \tilde{p}_t^*)^{1-\theta} + \alpha_P \left(\pi_{t-1}^{\gamma_P} \bar{\pi}^{1-\gamma_P} \right)^{1-\theta} \tag{17}$$

- Profit

$$\tilde{\Phi}_t = \tilde{Y}_t - \tilde{w}_t^u L_{u,t} - \tilde{w}_t^s L_{s,t} - r_t^{K,b} \tilde{K}_{b,t} - r_t^{K,e} \tilde{K}_{e,t} \tag{18}$$

- Households
  - Marginal utilities:

$$\tilde{\Lambda}_t^i \left(1 + \tau_t^C \right) = \frac{1}{\tilde{C}_t^i} \tag{19}$$

$$\tilde{\Lambda}_t^i \left(1 - \lambda_{\tau^H}^i \tau_t^H \right) \tilde{w}_t^i = \bar{\omega}^i \left(H_t^i \right)^{\varphi}$$

  - Capacity utilization costs

$$\mathcal{A}_b (u_{b,t}) = \chi_{b,1} (u_{b,t} - 1) + \frac{\chi_{b,2}^2}{2} (u_{b,t} - 1)^2 \tag{20}$$

$$\mathcal{A}_e (u_{e,t}) = \chi_{e,1} (u_{e,t} - 1) + \frac{\chi_{e,2}^2}{2} (u_{e,t} - 1)^2 \tag{21}$$

  - Budget constraint

$$\begin{aligned}
\left(1 + \tau_t^C \right) \tilde{C}_t^i + \tilde{I}_{b,t}^i + \tilde{I}_{e,t}^i + \tilde{b}_t^i \tilde{Y}_t = & \left(1 - \lambda_{\tau^H}^i \tau_t^H \right) \tilde{w}_t^i H_t^i + R_{t-1} \tilde{b}_{t-1}^i \tilde{Y}_{t-1} \frac{1}{\gamma \pi_t} \\
& + \left(1 - \tau_t^K \right) r_t^{K,b} u_{b,t} \tilde{\hat{K}}_{b,t}^s + \left(1 - \tau_t^K \right) r_t^{K,e} u_{e,t} \tilde{\hat{K}}_{e,t}^i \\
& - \left(1 - \lambda_b \tau_t^K \right) \mathcal{A}_b (u_{b,t}) \tilde{\hat{K}}_{b,t}^i - \frac{1}{q_0} \left(1 - \lambda_e \tau_t^K \right) \mathcal{A}_e (u_{e,t}) \tilde{\hat{K}}_{e,t}^i \\
& + \lambda_b \tau_t^K \tilde{I}_{b,t}^i + \lambda_e \tau_t^K \tilde{I}_{e,t}^i \\
& + \frac{\chi_{\Phi}^i}{N^s} \tilde{\Phi}_t + \frac{\chi_S^i}{N^s} \tilde{S}_t \tilde{Y}_t
\end{aligned}$$

– FOCs and capital accumulations

$$\tilde{w}_t^i \frac{1-\lambda_{\tau^H}^i \tau_t^H}{1+\tau_t^C} = \bar{\omega}^i \tilde{C}_t^i \left(H_t^i\right)^{\varphi} \tag{22}$$

$$\tilde{\Lambda}_t^i = \frac{\beta}{\gamma} R_t E_t \left\{ \tilde{\Lambda}_{t+1}^i \frac{1}{\pi_{t+1}} \right\} \tag{23}$$

$$Q_{t,t+1} = \frac{\beta}{\gamma} \frac{\tilde{\Lambda}_{t+1}^i}{\tilde{\Lambda}_t^i} \frac{1}{\pi_{t+1}} \tag{24}$$

$$\tilde{\Psi}_{b,t}^i = \frac{\beta}{\gamma} E_t \left\{ (1-d_b)\, \tilde{\Psi}_{b,t+1}^i + \left[\left(1-\tau_{t+1}^K\right) r_{t+1}^{K,b} u_{b,t+1} - \left(1-\lambda_b \tau_{t+1}^K\right) \mathcal{A}_b\left(u_{b,t+1}\right) \right] \tilde{\Lambda}_{t+1}^i \right\} \tag{25}$$

$$\begin{aligned}\left(1-\lambda_b \tau_t^K\right) \tilde{\Lambda}_t^i = {} & \tilde{\Psi}_{b,t}^i \left(1 - \mathcal{S}\left(\frac{\tilde{I}_{b,t}^i}{\tilde{I}_{b,t-1}^i}\gamma \right) - \mathcal{S}'\left(\frac{\tilde{I}_{b,t}^i}{\tilde{I}_{b,t-1}^i}\gamma \right) \frac{\tilde{I}_{b,t}^i}{\tilde{I}_{b,t-1}^i}\gamma \right) \\ & + \frac{\beta}{\gamma} E_t \left\{ \tilde{\Psi}_{b,t+1}^i \left(\frac{\tilde{I}_{b,t+1}^i}{\tilde{I}_{b,t}^i}\gamma \right)^2 \mathcal{S}'\left(\frac{\tilde{I}_{b,t+1}^i}{\tilde{I}_{b,t}^i}\gamma \right) \right\}\end{aligned} \tag{26}$$

$$\tilde{\Psi}_{e,t}^i = \beta E_t \left\{ (1-d_e)\, \tilde{\Psi}_{e,t+1}^i + \left[\left(1-\tau_{t+1}^K\right) r_{t+1}^{K,e} u_{e,t+1} - \frac{1}{q_0} \left(1-\lambda_e \tau_{t+1}^K\right) \mathcal{A}_e\left(u_{e,t+1}\right) \right] \tilde{\Lambda}_{t+1}^i \right\} \tag{27}$$

$$\begin{aligned}\left(1-\lambda_e \tau_t^K\right) \frac{1}{q_0} \tilde{\Lambda}_t^i = {} & \tilde{\Psi}_{e,t}^i \left(1 - \mathcal{S}\left(\frac{\tilde{I}_{e,t}^i}{\tilde{I}_{e,t-1}^i}\gamma \right) - \mathcal{S}'\left(\frac{\tilde{I}_{e,t}^i}{\tilde{I}_{e,t-1}^i}\gamma \right) \frac{\tilde{I}_{e,t}^i}{\tilde{I}_{e,t-1}^i}\gamma \right) \\ & + \frac{\beta}{\gamma} E_t \left\{ \tilde{\Psi}_{e,t+1}^i \left(\frac{\tilde{I}_{e,t+1}^i}{\tilde{I}_{e,t}^i}\gamma \right)^2 \mathcal{S}'\left(\frac{\tilde{I}_{e,t+1}^i}{\tilde{I}_{e,t}^i}\gamma \right) \right\}\end{aligned} \tag{28}$$

$$\gamma \tilde{\hat{K}}_{b,t+1}^i = (1-d_b)\, \tilde{\hat{K}}_{b,t}^i + \left(1 - \mathcal{S}\left(\frac{\tilde{I}_{b,t}^i}{\tilde{I}_{b,t-1}^i}\gamma \right) \right) \tilde{I}_{b,t}^i \tag{29}$$

$$\tilde{\hat{K}}_{e,t+1}^i = (1-d_e)\, \tilde{\hat{K}}_{e,t}^i + \left(1 - \mathcal{S}\left(\frac{\tilde{I}_{e,t}^i}{\tilde{I}_{e,t-1}^i}\gamma \right) \right) \tilde{I}_{e,t}^i q_0 \tag{30}$$

$$\left(1-\tau_t^K\right) r_t^{K,b} = \left(1-\lambda_b \tau_t^K\right) \mathcal{A}_b'\left(u_{b,t}\right) \tag{31}$$

$$\left(1-\tau_t^K\right) r_t^{K,e} = \frac{1}{q_0} \left(1-\lambda_e \tau_t^K\right) \mathcal{A}_e'\left(u_{e,t}\right) \tag{32}$$

- Resource constraint

$$\left(1-\tilde{G}_t\right) \tilde{Y}_t = \tilde{C}_t + \tilde{I}_{b,t} + \tilde{I}_{e,t} + \mathcal{A}_b\left(u_{b,t}\right) \tilde{\hat{K}}_{b,t} + \frac{1}{q_0} \mathcal{A}_e\left(u_{e,t}\right) \tilde{\hat{K}}_{e,t} \tag{33}$$

- Government budget constraint

$$\tilde{b}_t + \tilde{T}_t^C + \tilde{T}_t^H + \tilde{T}_t^{K,b} + \tilde{T}_t^{K,e} = R_{t-1} \tilde{b}_{t-1} \frac{1}{\pi_t \gamma} \frac{\tilde{Y}_{t-1}}{\tilde{Y}_t} + \tilde{G}_t + \tilde{S}_t \tag{34}$$

where

$$\tilde{T}_t^C = \tau^C \frac{\tilde{C}_t}{\tilde{Y}_t},\ \tilde{T}_t^H = \tau_t^H \sum_{i \in s,u} \left(\lambda_{\tau^H}^i \tilde{w}_t^i \frac{L_{i,t}}{\tilde{Y}_t} \right),$$

$$\tilde{T}_t^{K,b} = \tau_t^K \left(r_t^{K,b} \frac{\tilde{\hat{K}}_{b,t}}{\tilde{Y}_t} u_{b,t} - \lambda_b \left(\frac{\tilde{I}_{b,t}}{\tilde{Y}_t} + \mathcal{A}_b\left(u_{b,t}\right) \frac{\tilde{\hat{K}}_{b,t}}{\tilde{Y}_t} \right) \right),$$

$$\tilde{T}_t^{K,e} = \tau_t^K \left(r_t^{K,e} \frac{\tilde{\hat{K}}_{e,t}}{\tilde{Y}_t} u_{e,t} - \lambda_e \left(\frac{\tilde{I}_{e,t}}{\tilde{Y}_t} + \frac{1}{q_0} \mathcal{A}_e\left(u_{e,t}\right) \frac{\tilde{\hat{K}}_{e,t}}{\tilde{Y}_t} \right) \right)$$

- Market clearing

$$\tilde{C}_t = N^s \tilde{C}_t^s + N^u \tilde{C}_t^u \tag{35}$$
$$L_{u,t} = N^u H_t^u \tag{36}$$
$$L_{s,t} = N^s H_t^s \tag{37}$$
$$\tilde{K}_{b,t} = N^s \tilde{K}_{b,t}^s + N^u \tilde{K}_{b,t}^u \tag{38}$$
$$\tilde{K}_{e,t} = N^s \tilde{K}_{e,t}^s + N^u \tilde{K}_{e,t}^u \tag{39}$$
$$\tilde{I}_{b,t} = N^s \tilde{I}_{b,t}^s + N^u \tilde{I}_{b,t}^u \tag{40}$$
$$\tilde{I}_{e,t} = N^s \tilde{I}_{e,t}^s + N^u \tilde{I}_{e,t}^u \tag{41}$$
$$\tilde{K}_{b,t}^s = u_{b,t}^s \tilde{\hat{K}}_{b,t}^s \tag{42}$$
$$\tilde{K}_{e,t}^s = u_{e,t}^s \tilde{\hat{K}}_{e,t}^s \tag{43}$$
$$\tilde{K}_{b,t}^u = u_{b,t}^u \tilde{\hat{K}}_{b,t}^u \tag{44}$$
$$\tilde{K}_{e,t}^u = u_{e,t}^u \tilde{\hat{K}}_{e,t}^u \tag{45}$$

- Fiscal Policy Rules

$$\begin{aligned}
\tau_t^H - \bar{\tau}_{new}^H =& \rho_1^H \left(\tau_{t-1}^H - \bar{\tau}_{new}^H\right) + \rho_2^H \left(\tau_{t-2}^H - \bar{\tau}_{new}^H\right) \\
&+ \left(1 - \rho_1^H - \rho_2^H\right) \left\{\psi_B^H \left(\frac{B_{t-1}}{P_{t-1}Y_{t-1}} - \overline{\frac{B}{PY}}\right) + \psi_{\Delta y}^H \left(\frac{Y_t}{Y_{t-1}}\right) + \psi_x^H \left(\frac{Y_t}{Y_t^n}\right)\right\},
\end{aligned} \tag{46}$$

$$\begin{aligned}
\tau_t^C - \bar{\tau}_{new}^C =& \rho_1^C \left(\tau_{t-1}^C - \bar{\tau}_{new}^C\right) + \rho_2^C \left(\tau_{t-2}^C - \bar{\tau}_{new}^C\right) = \\
&+ \left(1 - \rho_1^C - \rho_2^C\right) \left\{\psi_B^C \left(\frac{B_{t-1}}{P_{t-1}Y_{t-1}} - \overline{\frac{B}{PY}}\right) + \psi_{\Delta y}^C \left(\frac{Y_t}{Y_{t-1}}\right) + \psi_x^C \left(\frac{Y_t}{Y_t^n}\right)\right\}
\end{aligned} \tag{47}$$

$$\tau_t^K = \begin{cases} \bar{\tau}^K & \text{if } t = 0 \\ \bar{\tau}_{New}^K & \text{if } t > 0 \end{cases} \tag{48}$$

$$\tilde{G}_t = \bar{\tilde{G}} \tag{49}$$

- Monetary policy

$$\frac{R_t}{\bar{R}} = \left[\frac{R_{t-1}}{\bar{R}}\right]^{\rho_1^R} \left[\frac{R_{t-2}}{\bar{R}}\right]^{\rho_2^R} \left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_{t-1}}\right)^{\phi_{\Delta y}} \left(\frac{Y_t}{Y_t^n}\right)^{\phi_x}\right]^{\left(1-\rho_1^R-\rho_2^R\right)} \tag{50}$$

- Government debt to GDP (or transfers):

$$\begin{cases} \tilde{b}_t = \bar{\tilde{b}} & \text{if adjusting transfers} \\ \tilde{S}_t = \bar{\tilde{S}} & \text{if adjusting labor tax rate} \end{cases} \tag{51}$$

A.7 Steady State

Recall that in steady-state, $\mathcal{S}(\gamma) = \mathcal{S}'(\gamma) = 0$:

From (13)–(16), we get

$$\bar{mc} = \frac{\theta - 1}{\theta}.$$

From (25) and (26), we get

$$\bar{r}^{K,b} = \left(\frac{\gamma}{\beta} - (1-d_b)\right)\frac{1-\lambda_b\bar{\tau}^K}{1-\bar{\tau}^K} \tag{52}$$

$$\bar{r}^{K,e} = \frac{1}{q_0}\left(\frac{1}{\beta} - (1-d_e)\right)\frac{1-\lambda_e\bar{\tau}^K}{1-\bar{\tau}^K} \tag{53}$$

From the production function, we get

$$\frac{\bar{\tilde{Y}}^A}{\bar{L}_s} = \frac{\bar{\tilde{Y}}}{\bar{L}_s} = \left(\frac{\bar{\tilde{K}}_b}{\bar{L}_s}\right)^{\alpha}\left[\mu\left(\frac{\bar{L}_u}{\bar{L}_s}\right)^{\sigma} + (1-\mu)\left(\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho} + (1-\lambda)\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\alpha}{\sigma}} \tag{54}$$

and

$$\bar{\Xi} = 1.$$

From firms' optimality conditions, we get

$$\bar{\tilde{w}}^s = (1-\lambda)(1-\mu)(1-\alpha)\bar{m}c\left(\frac{\bar{r}^{K,b}}{\alpha\bar{m}c}\right)^{1-\frac{\sigma}{1-\alpha}}\left(\frac{\bar{\tilde{K}}_b}{\bar{L}_s}\right)^{1-\sigma}\left(\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho} + (1-\lambda)\right)^{\frac{\sigma}{\rho}-1} \tag{55}$$

$$\bar{\tilde{w}}^u\left(\frac{\bar{L}_u}{\bar{L}_s}\right)^{1-\sigma} = \mu(1-\alpha)\bar{m}c\left(\frac{\bar{r}^{K,b}}{\alpha\bar{m}c}\right)^{\frac{1-\alpha-\sigma}{1-\alpha}}\left(\frac{\bar{\tilde{K}}_b}{\bar{L}_s}\right)^{1-\sigma} \tag{56}$$

$$\bar{r}^{K,e}\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{1-\rho} = \lambda(1-\mu)(1-\alpha)\bar{m}c\left(\frac{\bar{r}^{K,b}}{\alpha\bar{m}c}\right)^{1-\frac{\sigma}{1-\alpha}}\left(\frac{\bar{\tilde{K}}_b}{\bar{L}_s}\right)^{1-\sigma}\left(\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho} + (1-\lambda)\right)^{\frac{\sigma}{\rho}-1} \tag{57}$$

$$\left(\frac{\bar{r}^{K,b}}{\alpha\bar{m}c}\right) = \left(\frac{\tilde{K}_b}{\bar{L}_s}\right)^{\alpha-1}\left[\mu\left(\frac{\bar{L}_u}{\bar{L}_s}\right)^{\sigma} + (1-\mu)\left(\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho} + (1-\lambda)\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\alpha}{\sigma}} \tag{58}$$

Also, from skilled-household's budget constraint, we have budget constraint

$$\begin{aligned}\left(1+\bar{\tau}^C\right)\frac{\bar{\tilde{C}}^s}{\bar{\tilde{Y}}} &= \left(1-\lambda^s_{\tau^H}\bar{\tau}^H\right)\bar{\tilde{w}}^s\frac{\bar{H}^s}{\bar{\tilde{Y}}} \\ &\quad + \left(\left(1-\bar{\tau}^K\right)\bar{r}^{K,b} - \left(1-\lambda_b\bar{\tau}^K\right)(\gamma-(1-d_b))\right)\frac{\bar{\tilde{\hat{K}}}{}^s_b}{\bar{\tilde{Y}}} \\ &\quad + \left(\left(1-\bar{\tau}^K\right)\bar{r}^{K,e} - \left(1-\lambda_e\bar{\tau}^K\right)\frac{d_e}{q_0}\right)\frac{\bar{\tilde{\hat{K}}}{}^s_e}{\bar{\tilde{Y}}} \\ &\quad + \left(\frac{1}{\beta}-1\right)\bar{\tilde{b}}^s + \frac{\chi^s_\Phi}{N^s}\frac{\tilde{\Phi}_t}{\bar{\tilde{Y}}} + \frac{\chi^s_S}{N^s}\tilde{S}_t\end{aligned}$$

where the profit is

$$\frac{\bar{\tilde{\Phi}}}{\bar{\tilde{Y}}} = 1 - \bar{\tilde{w}}^u\frac{\bar{L}_u}{\bar{\tilde{Y}}} - \bar{\tilde{w}}^s\frac{\bar{L}_s}{\bar{\tilde{Y}}} - \bar{r}^{K,b}\frac{\bar{\tilde{K}}_b}{\bar{\tilde{Y}}} - \bar{r}^{K,e}\frac{\bar{\tilde{K}}_e}{\bar{\tilde{Y}}}$$

and the transfer is

$$\bar{\tilde{S}} = \bar{\tau}^C\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}} + \bar{\tau}^H\left(\sum_{i\in\{s,u\}}\lambda^i_{\tau^H}\bar{\tilde{w}}^i\frac{\bar{L}_i}{\bar{\tilde{Y}}}\right) + \bar{\tau}^K\bar{r}^{K,b}\left(\frac{\bar{\tilde{\hat{K}}}_b}{\bar{\tilde{Y}}} - \lambda_b\frac{\bar{\tilde{I}}_b}{\bar{\tilde{Y}}}\right) + \bar{\tau}^K\bar{r}^{K,e}\left(\frac{\bar{\tilde{\hat{K}}}_e}{\bar{\tilde{Y}}} - \lambda_e\frac{\bar{\tilde{I}}_e}{\bar{\tilde{Y}}}\right) - \left(\left(\frac{1}{\beta}-1\right)\bar{\tilde{b}} + \bar{\tilde{G}}\right).$$

From (29) and (30) for both types of households, we get

$$(\gamma - (1 - d_b)) \frac{\bar{\tilde{\hat{K}}}_b^s}{\bar{H}^s} = \frac{\bar{\tilde{I}}_b^s}{\bar{H}^s} \tag{59}$$

$$d_e \frac{\bar{\tilde{\hat{K}}}_e^s}{\bar{H}^s} = \frac{\bar{\tilde{I}}_e^s}{\bar{H}^s} q_0 \tag{60}$$

$$(\gamma - (1 - d_b)) \frac{\bar{\tilde{\hat{K}}}_b^u}{\bar{H}^u} = \frac{\bar{\tilde{I}}_b^u}{\bar{H}^u} \tag{61}$$

$$d_e \frac{\bar{\tilde{\hat{K}}}_e^u}{\bar{H}^u} = \frac{\bar{\tilde{I}}_e^u}{\bar{H}^u} q_0 \tag{62}$$

From household's intra-temporal Euler equations, we have

$$\frac{\tilde{\Lambda}^s \left(1 - \lambda^s_{\tau^H} \bar{\tau}^H\right) \bar{\tilde{w}}^s}{\tilde{\Lambda}^u \left(1 - \lambda^u_{\tau^H} \bar{\tau}^H\right) \bar{\tilde{w}}^u} = \frac{\bar{\omega}^s}{\bar{\omega}^u} \left(\frac{\bar{H}^s}{\bar{H}^u}\right)^{\varphi} \tag{63}$$

$$\tilde{\bar{w}}^s \left(\frac{1 - \lambda^s_{\tau^H} \bar{\tau}^H}{1 + \bar{\tau}^C}\right) = \bar{\omega}^s \bar{\tilde{C}}^s \left(\bar{H}^s\right)^{\varphi} \tag{64}$$

$$\Lambda_{s,u} = \frac{\bar{\tilde{\Lambda}}^s}{\bar{\tilde{\Lambda}}^u} = \frac{\bar{\tilde{C}}^u}{\bar{\tilde{C}}^s} \tag{65}$$

$$\bar{\tilde{C}}^s = \left(\frac{1}{N^u \Lambda_{s,u} + N^s}\right) \bar{\tilde{C}}$$

$$\bar{\tilde{C}}^u = \left(\frac{\Lambda_{s,u}}{N^u \Lambda_{s,u} + N^s}\right) \bar{\tilde{C}}$$

From the market clearing conditions, we have

$$\frac{\bar{\tilde{K}}_b}{\bar{L}_s} = \frac{\bar{\tilde{K}}_b^s}{\bar{H}^s} + \frac{\bar{L}_u}{\bar{L}_s} \frac{\bar{\tilde{K}}_b^u}{\bar{H}^u} \tag{66}$$

$$\frac{\bar{\tilde{K}}_e}{\bar{L}_s} = \frac{\bar{\tilde{K}}_e^s}{\bar{H}^s} + \frac{\bar{L}_u}{\bar{L}_s} \frac{\bar{\tilde{K}}_e^u}{\bar{H}^u} \tag{67}$$

From (33), we get

$$\frac{\bar{\tilde{C}}}{\bar{L}_s} + (\gamma - (1 - d_b)) \frac{\bar{\tilde{K}}_b}{\bar{L}_s} + \frac{d_e}{q_0} \frac{\bar{\tilde{K}}_e}{\bar{L}_s} = \left(1 - \tilde{G}\right) \frac{\bar{\tilde{Y}}}{L_s} \tag{68}$$

The nominal interest rate is obtained from Euler equation (23)

$$\bar{R} = \frac{\gamma \bar{\pi}}{\beta}.$$

We fix steady-state hours for skilled labor $\bar{H}^s = 0.33$ by assuming the skilled works 40 hours per week and $\bar{H}^u = 0.93 * \bar{H}^s$(Skilled workers work 7% more than low-skilled worker).

B Model with Incomplete Markets

B.1 Households and Firms

- Skilled households make saving/investment decisions and own the entire capital in the economy

$$
\begin{aligned}
\max_{\left\{C_t^s, H_t^s, B_t^s, I_{b,t}^s, I_{e,t}^s, \bar{K}_{b,t+1}^s, \bar{K}_{e,t+1}^s, u_{e,t}^s, u_{b,t}^s, V_{t+1}^s\right\}} \quad & E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\log\left(C_t^s\right) - \bar{\omega}^s \frac{\left(H_t^s\right)^{1+\varphi}}{1+\varphi} \right] \right\} \\
\text{s.t.} \quad & \left(1+\tau_t^C\right) P_t C_t^s + P_t I_{b,t}^s + P_t I_{e,t}^s + B_t^s + E_t Q_{t,t+1} V_{t+1}^s \\
& = \left(1 - \lambda_{\tau^H}^s \tau_t^H\right) W_t^s H_t^s + R_{t-1} B_{t-1}^s + V_t^s \\
& + \left(1-\tau_t^K\right) R_t^{K,b} u_{b,t}^s \hat{K}_{b,t}^s + \left(1-\tau_t^K\right) R_t^{K,e} u_{e,t}^s \hat{K}_{e,t}^s \\
& + \lambda_b \tau_t^K P_t I_{b,t}^s + \lambda_e \tau_t^K P_t I_{e,t}^s \\
& - P_t \left(1 - \lambda_b \tau_t^K\right) \mathcal{A}_b \left(u_{b,t}^s\right) \hat{K}_{b,t}^i - \frac{P_t}{q_t} \left(1 - \lambda_e \tau_t^K\right) \mathcal{A}_e \left(u_{e,t}^s\right) \hat{K}_{e,t}^s \\
& + P_t \Phi_t^s + P_t S_t^s, \\
\hat{K}_{b,t+1}^s = (1-d_b)\, \hat{K}_{b,t}^s + & \left(1 - \mathcal{S}\left(\frac{I_{b,t}^s}{I_{b,t-1}^s}\right)\right) I_{b,t}^s \\
\hat{K}_{e,t+1}^s = (1-d_e)\, \hat{K}_{e,t}^s + & \left(1 - \mathcal{S}\left(\frac{I_{e,t}^s}{I_{e,t-1}^s}\right)\right) I_{e,t}^s q_t
\end{aligned}
$$

- Unskilled households are hand-to-mouth households, that is, they consume their disposable income every period.

$$
\begin{aligned}
\max_{\left\{C_t^u, H_t^u\right\}} \quad & \log C_t^u - \bar{\omega}^u \frac{\left(H_t^u\right)^{1+\varphi}}{1+\varphi} \\
\text{s.t.} \quad & \left(1+\tau_t^C\right) P_t C_t^u = \left(1 - \lambda_{\tau^H}^u \tau_t^H\right) W_t^u H_t^u + P_t \Phi_t^u + P_t S_t^u
\end{aligned}
$$

- Firms' problem are identical to that of the baseline model with complete markets.

B.2 Government Budget Constraint, Monetary Policy, and Fiscal Policy

- Monetary policy and fiscal policy rules are identical to those under the model with complete markets.

- The government flow budget constraint, written by expressing fiscal variables as ratio of output, is given by

$$
\frac{B_t}{P_t Y_t} + \left(\frac{T_t^C}{Y_t} + \frac{T_t^H}{Y_t} + \frac{T_t^{K,b}}{Y_t} + \frac{T_t^{K,e}}{Y_t} \right) = R_{t-1} \frac{B_{t-1}}{P_{t-1} Y_{t-1}} \frac{1}{\pi_t} \frac{Y_{t-1}}{Y_t} + \frac{G_t}{Y_t} + \frac{S_t}{Y_t}
$$

where

$$
\begin{aligned}
B_t &= N^s B_t^s \\
T_t^C &= \sum_{i \in \{s,u\}} N^i \tau_t^C C_t^i \\
T_t^H &= \tau_t^H \sum_{i \in \{s,u\}} \frac{W_t^i}{P_t} \lambda_{\tau^H}^i N^i H_t^i \\
T_t^{K,b} &= \tau_t^K N^s \left(\frac{R_t^{K,b}}{P_t} \hat{K}_{b,t}^s u_{b,t}^s - \lambda_b \left(I_{b,t}^s + \mathcal{A}_b \left(u_{b,t}^s \right) \hat{K}_{b,t}^s \right) \right) \\
T_t^{K,e} &= \tau_t^K N^s \left(\frac{R_t^{K,e}}{P_t} \hat{K}_{e,t}^s u_{e,t}^s - \lambda_e \left(I_{e,t}^s + \frac{1}{q_t} \mathcal{A}_e \left(u_{e,t}^s \right) \hat{K}_{e,t}^s \right) \right) \\
S_t &= \sum_{i \in \{s,u\}} N^i S_t^i
\end{aligned}
$$

$$
N^s S_t^s = \chi_S^s S_t, \; N^u S_t^u = \chi_S^u S_t = \left(1 - \chi_S^s\right) S_t
$$

- Profit distribution

$$
\begin{aligned}
\Phi_t &= \sum_{i \in \{s,u\}} N^i \Phi_t^i \\
N^s \Phi_t^s &= \chi_\Phi^s \Phi_t \\
N^u \Phi_t^u &= \chi_\Phi^u \Phi_t = \left(1 - \chi_\Phi^s\right) \Phi_t
\end{aligned}
$$

B.3 Market Clearing

$$
\begin{aligned}
\int_0^1 L_{u,t}(i)\, di &= L_{u,t} = N^u H_t^u \\
\int_0^1 L_{s,t}(i)\, di &= L_{s,t} = N^s H_t^s \\
\int_0^1 K_{b,t}(i)\, di &= K_{b,t} = N^s K_{b,t}^s \\
\int_0^1 K_{e,t}(i)\, di &= K_{e,t} = N^s K_{e,t}^s \\
\int_0^1 \Phi_t(i)\, di &= N^s \Phi_t^s + N^u \Phi_t^u
\end{aligned}
$$

where

$$
K_{b,t}^s = u_{b,t}^s \hat{K}_{b,t}^s, \; K_{e,t}^s = u_{e,t}^s \hat{K}_{e,t}^s
$$

Aggregate resource constraint:

$$
Y_t = N^s \left(C_t^s + I_{b,t}^s + I_{e,t}^s + \mathcal{A}_b \left(u_{b,t}^s \right) \hat{K}_{b,t}^s + \frac{1}{q_t} \mathcal{A}_e \left(u_{e,t}^s \right) \hat{K}_{e,t}^s \right) + N^u C_t^u + G_t
$$

Capital accumulation:

$$\hat{K}^s_{b,t+1} = (1-d_b)\,\hat{K}^s_{b,t} + \left(1 - \mathcal{S}\left(\frac{I^s_{b,t}}{I^s_{b,t-1}}\right)\right) I^s_{b,t}$$
$$\hat{K}^s_{e,t+1} = (1-d_e)\,\hat{K}^s_{e,t} + \left(1 - \mathcal{S}\left(\frac{I^s_{e,t}}{I^s_{e,t-1}}\right)\right) I^s_{e,t} q_t.$$

B.4 Stationary Equilibrium

We consider a symmetric equilibrium across firms, where all firms set the same price and produce the same amount of output. Given nonlinear equilibrium conditions, we detrended variables to specify stationary equilibrium conditions. All the notations are the same as before. We now state all the stationary equilibrium equations under incomplete markets.

- Production function:(Let $A_0 = 1$)

$$\tilde{Y}^A_t = \tilde{K}^{\alpha}_{b,t}\left[\mu L^{\sigma}_{u,t} + (1-\mu)\left(\lambda \tilde{K}^{\rho}_{e,t} + (1-\lambda)\,L^{\rho}_{s,t}\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\alpha}{\sigma}} \tag{69}$$

- Aggregate output

$$\tilde{Y}^A_t = \tilde{Y}_t \Xi_t \tag{70}$$

- Cost minimization

$$r^{K,b}_t = \alpha mc_t \frac{\tilde{Y}^A_t}{\tilde{K}_{b,t}} \tag{71}$$

$$r^{K,e}_t = (1-\alpha)\,mc_t \frac{\tilde{Y}^A_t}{\tilde{K}_{e,t}} \left(\frac{(1-\mu)\left(\lambda\tilde{K}^{\rho}_{e,t} + (1-\lambda)\,L^{\rho}_{s,t}\right)^{\frac{\sigma}{\rho}}}{\mu L^{\sigma}_{u,t} + (1-\mu)\left(\lambda\tilde{K}^{\rho}_{e,t} + (1-\lambda)\,L^{\rho}_{s,t}\right)^{\frac{\sigma}{\rho}}}\right)\left(\frac{\lambda\tilde{K}^{\rho}_{e,t}}{\lambda\tilde{K}^{\rho}_{e,t} + (1-\lambda)\,L^{\rho}_{s,t}}\right) \tag{72}$$

$$\tilde{w}^u_t = (1-\alpha)\,mc_t \frac{\tilde{Y}^A_t}{L_{u,t}} \left(\frac{\mu L^{\sigma}_{u,t}}{\mu L^{\sigma}_{u,t} + (1-\mu)\left(\lambda\tilde{K}^{\rho}_{e,t} + (1-\lambda)\,L^{\rho}_{s,t}\right)^{\frac{\sigma}{\rho}}}\right) \tag{73}$$

$$\tilde{w}^s_t = (1-\alpha)\,mc_t \frac{\tilde{Y}^A_t}{L_{s,t}} \left(\frac{(1-\mu)\left(\lambda\tilde{K}^{\rho}_{e,t} + (1-\lambda)\,L^{\rho}_{s,t}\right)^{\frac{\sigma}{\rho}}}{\mu L^{\sigma}_{u,t} + (1-\mu)\left(\lambda\tilde{K}^{\rho}_{e,t} + (1-\lambda)\,L^{\rho}_{s,t}\right)^{\frac{\sigma}{\rho}}}\right)\left(\frac{(1-\lambda)\,L^{\rho}_{s,t}}{\lambda\tilde{K}^{\rho}_{e,t} + (1-\lambda)\,L^{\rho}_{s,t}}\right) \tag{74}$$

- Skill-premium

$$\frac{\tilde{w}^s_t}{\tilde{w}^u_t} = \frac{(1-\mu)\,(1-\lambda)}{\mu}\left(\lambda\left(\frac{\tilde{K}_{e,t}}{L_{s,t}}\right)^{\rho} + (1-\lambda)\right)^{\frac{\sigma-\rho}{\rho}}\left(\frac{L_{u,t}}{L_{s,t}}\right)^{1-\sigma}$$

- Firms' maximization:

$$\tilde{Z}_{1,t} = mc_t\tilde{Y}_t + \alpha_p\beta\left((\pi_t)^{\gamma_P}\,\bar{\pi}^{(1-\gamma_P)}\right)^{-\theta} E_t\left\{\frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}\tilde{Z}_{1,t+1}\,(\pi_{t+1})^{\theta}\right\},$$

$$\tilde{Z}_{2,t} = \tilde{Y}_t + \alpha_p\beta\left((\pi_t)^{\gamma_P}\,\bar{\pi}^{(1-\gamma_P)}\right)^{1-\theta} E_t\left\{\frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}\tilde{Z}_{2,t+1}\,(\pi_{t+1})^{\theta-1}\right\}.$$

- Price dispersion

$$\Xi_t = (1-\alpha_P)\left(\tilde{p}_t^*\right)^{-\theta} + \alpha_P \pi_t^{\theta}\left(\pi_{t-1}^{\gamma_P}\bar{\pi}^{1-\gamma_P}\right)^{-\theta}\Xi_{t-1},$$

where

$$\tilde{p}_t^* = \frac{\theta}{\theta-1}\frac{\tilde{Z}_{1,t}}{\tilde{Z}_{2,t}}.$$

- Aggregate price index

$$\pi_t^{1-\theta} = (1-\alpha_P)\left(\pi_t\tilde{p}_t^*\right)^{1-\theta} + \alpha_P\left(\pi_{t-1}^{\gamma_P}\bar{\pi}^{1-\gamma_P}\right)^{1-\theta}$$

- Profit

$$\tilde{\Phi}_t = \tilde{Y}_t - \tilde{w}_t^u L_{u,t} - \tilde{w}_t^s L_{s,t} - r_t^{K,b}\tilde{K}_{b,t} - r_t^{K,e}\tilde{K}_{e,t}$$

- Hand-to-mouth households

$$\tilde{w}_t^u \frac{1-\lambda_{\tau^H}^u \tau_t^H}{1+\tau_t^C} = \bar{\omega}^u\left(\tilde{C}_t^u\right)\left(H_t^u\right)^{\varphi}$$
$$\left(1+\tau_t^C\right)\tilde{C}_t^u = \left(1-\lambda_{\tau^H}^u\tau_t^H\right)\tilde{w}_t^u H_t^u + \tilde{\Phi}_t^u + \tilde{S}_t^u\tilde{Y}_t$$

- Skilled households

  – Marginal utilities:

$$\tilde{\Lambda}_t^s\left(1+\tau_t^C\right) = \frac{1}{\tilde{C}_t^s} \tag{75}$$
$$\tilde{\Lambda}_t^i\left(1-\lambda_{\tau^H}^s\tau_t^H\right)\tilde{w}_t^s = \bar{\omega}^s\left(H_t^s\right)^{\varphi}$$

  – Capacity utilization costs

$$\mathcal{A}_b\left(u_{e,t}\right) = \chi_{b,1}\left(u_{b,t}-1\right) + \frac{\chi_{b,2}^2}{2}\left(u_{b,t}-1\right)^2$$

$$\mathcal{A}_e\left(u_{e,t}\right) = \chi_{e,1}\left(u_{e,t}-1\right) + \frac{\chi_{e,2}^2}{2}\left(u_{e,t}-1\right)^2$$

  – FOCs and Capital Accumulation

$$\begin{aligned}
\tilde{w}_t^s\frac{1-\lambda_{\tau^H}^s\tau_t^H}{1+\tau_t^C} &= \bar{\omega}^s\tilde{C}_t^s\left(H_t^s\right)^{\varphi} \\
\tilde{\Lambda}_t^s &= \frac{\beta}{\gamma}R_t E_t\left\{\tilde{\Lambda}_{t+1}^s\frac{1}{\pi_{t+1}}\right\} \\
Q_{t,t+1} &= \frac{\beta}{\gamma}\frac{\tilde{\Lambda}_{t+1}^s}{\tilde{\Lambda}_t^s}\frac{1}{\pi_{t+1}} \\
\tilde{\Psi}_{b,t}^s &= \frac{\beta}{\gamma}E_t\left\{(1-d_b)\tilde{\Psi}_{b,t+1}^s + \left[\left(1-\tau_{t+1}^K\right)r_{t+1}^{K,b}u_{b,t+1} - \left(1-\lambda_b\tau_{t+1}^K\right)\mathcal{A}_b\left(u_{b,t+1}\right)\right]\tilde{\Lambda}_{t+1}^s\right\} \\
\left(1-\lambda_b\tau_t^K\right)\tilde{\Lambda}_t^s &= \tilde{\Psi}_{b,t}^s\left(1-\mathcal{S}\left(\frac{\tilde{I}_{b,t}^s}{\tilde{I}_{b,t-1}^s}\gamma\right) - \mathcal{S}'\left(\frac{\tilde{I}_{b,t}^s}{\tilde{I}_{b,t-1}^s}\gamma\right)\frac{\tilde{I}_{b,t}^s}{\tilde{I}_{b,t-1}^s}\gamma\right) \\
&\quad + \frac{\beta}{\gamma}E_t\left\{\tilde{\Psi}_{b,t+1}^s\left(\frac{\tilde{I}_{b,t+1}^s}{\tilde{I}_{b,t}^s}\gamma\right)^2\mathcal{S}'\left(\frac{\tilde{I}_{b,t+1}^s}{\tilde{I}_{b,t}^s}\gamma\right)\right\}
\end{aligned}$$

$$
\begin{aligned}
\tilde{\Psi}^s_{e,t} &= \beta E_t \left\{ (1-d_e)\, \tilde{\Psi}^s_{e,t+1} + \left[\left(1-\tau^K_{t+1}\right) r^{K,e}_{t+1} u_{e,t+1} - \frac{1}{q_0} \left(1-\lambda_e \tau^K_{t+1}\right) \mathcal{A}_e\left(u_{e,t+1}\right) \right] \tilde{\Lambda}^s_{t+1} \right\} \\
\left(1-\lambda_e \tau^K_t\right) \frac{1}{q_0} \tilde{\Lambda}^s_t &= \tilde{\Psi}^s_{e,t} \left(1 - \mathcal{S}\left(\frac{\tilde{I}^s_{e,t}}{\tilde{I}^s_{e,t-1}} \gamma \right) - \mathcal{S}'\left(\frac{\tilde{I}^s_{e,t}}{\tilde{I}^s_{e,t-1}} \gamma \right) \frac{\tilde{I}^s_{e,t}}{\tilde{I}^s_{e,t-1}} \gamma \right) \\
&\quad + \frac{\beta}{\gamma} E_t \left\{ \tilde{\Psi}^s_{e,t+1} \left(\frac{\tilde{I}^s_{e,t+1}}{\tilde{I}^s_{e,t}} \gamma \right)^2 \mathcal{S}'\left(\frac{\tilde{I}^s_{e,t+1}}{\tilde{I}^s_{e,t}} \gamma \right) \right\} \\
\gamma \tilde{\hat{K}}^s_{b,t+1} &= (1-d_b)\, \tilde{\hat{K}}^s_{b,t} + \left(1 - \mathcal{S}\left(\frac{\tilde{I}^s_{b,t}}{\tilde{I}^s_{b,t-1}} \gamma \right) \right) \tilde{I}^s_{b,t} \\
\tilde{\hat{K}}^s_{e,t+1} &= (1-d_e)\, \tilde{\hat{K}}^s_{e,t} + \left(1 - \mathcal{S}\left(\frac{\tilde{I}^s_{e,t}}{\tilde{I}^s_{e,t-1}} \gamma \right) \right) \tilde{I}^s_{e,t} q_0 \\
\left(1-\tau^K_t\right) r^{K,b}_t &= \left(1-\lambda_b \tau^K_t\right) \mathcal{A}_b'\left(u_{b,t}\right) \\
\left(1-\tau^K_t\right) r^{K,e}_t &= \frac{1}{q_0} \left(1-\lambda_e \tau^K_t\right) \mathcal{A}_e'\left(u_{e,t}\right)
\end{aligned}
$$

- Resource constraint

$$
\begin{aligned}
\left(1-\tilde{G}_t\right) \tilde{Y}_t &= N^s \left(\tilde{C}^s_t + \tilde{I}^s_{b,t} + \tilde{I}^s_{e,t} + \mathcal{A}_b\left(u_{b,t}\right) \tilde{\hat{K}}^s_{b,t} + \frac{1}{q_0} \mathcal{A}_e\left(u_{e,t}\right) \tilde{\hat{K}}^s_{e,t} \right) + N^u \tilde{C}^u_t \\
&= \tilde{C}_t + \tilde{I}_{b,t} + \tilde{I}_{e,t} + \mathcal{A}_b\left(u_{b,t}\right) \tilde{\hat{K}}_{b,t} + \frac{1}{q_0} \mathcal{A}_e\left(u_{e,t}\right) \tilde{\hat{K}}_{e,t}
\end{aligned}
$$

- Market clearing

$$
\tilde{C}_t = N^s \tilde{C}^s_t + N^u \tilde{C}^u_t
$$

$$
\begin{gathered}
L_{u,t} = N^u H^u_t,\; L_{s,t} = N^s H^s_t \\
\tilde{K}_{b,t} = N^s \tilde{K}^s_{b,t},\; \tilde{K}_{e,t} = N^s \tilde{K}^s_{e,t} \\
\tilde{I}_{b,t} = N^s \tilde{I}^s_{b,t},\; \tilde{I}_{e,t} = N^s \tilde{I}^s_{e,t} \\
\tilde{K}^s_{b,t} = u^s_{b,t} \tilde{\hat{K}}^s_{b,t},\; \tilde{K}^s_{e,t} = u^s_{e,t} \tilde{\hat{K}}^s_{e,t}
\end{gathered}
$$

- Government budget constraint

$$
\tilde{b}_t + \tilde{T}^C_t + \tilde{T}^H_t + \tilde{T}^{K,b}_t + \tilde{T}^{K,e}_t = R_{t-1} \tilde{b}_{t-1} \frac{1}{\pi_t \gamma} \frac{\tilde{Y}_{t-1}}{\tilde{Y}_t} + \tilde{G}_t + \tilde{S}_t
$$

where

$$
\begin{aligned}
\tilde{T}^C_t &= \tau^C \frac{\tilde{C}_t}{\tilde{Y}_t},\; \tilde{T}^H_t = \tau^H_t \sum_{i \in s,u} \left(\lambda^i_{\tau^H} \tilde{w}^i_t \frac{L_{i,t}}{\tilde{Y}_t} \right), \\
\tilde{T}^{K,b}_t &= \tau^K_t \left(r^{K,b}_t \frac{\tilde{\hat{K}}_{b,t}}{\tilde{Y}_t} u_{b,t} - \lambda_b \left(\frac{\tilde{I}_{b,t}}{\tilde{Y}_t} + \mathcal{A}_b\left(u_{b,t}\right) \frac{\tilde{\hat{K}}_{b,t}}{\tilde{Y}_t} \right) \right), \\
\tilde{T}^{K,e}_t &= \tau^K_t \left(r^{K,e}_t \frac{\tilde{\hat{K}}_{e,t}}{\tilde{Y}_t} u_{e,t} - \lambda_e \left(\frac{\tilde{I}_{e,t}}{\tilde{Y}_t} + \frac{1}{q_0} \mathcal{A}_e\left(u_{e,t}\right) \frac{\tilde{\hat{K}}_{e,t}}{\tilde{Y}_t} \right) \right)
\end{aligned}
$$

- Fiscal Policy Rules: for $i \in \{H, C\}$,

$$\begin{aligned}
\tau_t^i - \bar{\tau}_{new}^i =& \rho_1^i \left(\tau_{t-1}^i - \bar{\tau}_{new}^i\right) + \rho_2^i \left(\tau_{t-2}^i - \bar{\tau}_{new}^i\right) \\
&+ \left(1 - \rho_1^i - \rho_2^i\right) \left\{ \psi_B^i \left(\frac{B_{t-1}}{P_{t-1} Y_{t-1}} - \overline{\frac{B}{PY}} \right) + \psi_{\Delta y}^i \left(\frac{Y_t}{Y_{t-1}} \right) + \psi_x^i \left(\frac{Y_t}{Y_t^n} \right) \right\},
\end{aligned}$$

$$\tau_t^K = \begin{cases} \bar{\tau}^K & \text{if } t = 0 \\ \bar{\tau}_{New}^K & \text{if } t > 0 \end{cases}$$

$$\tilde{G}_t = \bar{\tilde{G}}$$

- Monetary policy

$$\frac{R_t}{\bar{R}} = \left[\frac{R_{t-1}}{\bar{R}}\right]^{\rho_1^R} \left[\frac{R_{t-2}}{\bar{R}}\right]^{\rho_2^R} \left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_{t-1}}\right)^{\phi_{\Delta y}} \left(\frac{Y_t}{Y_t^n}\right)^{\phi_x} \right]^{\left(1-\rho_1^R-\rho_2^R\right)}$$

- Government debt to GDP (or transfers):

$$\begin{cases} \tilde{b}_t = \bar{\tilde{b}} & \text{if adjusting transfers} \\ \tilde{S}_t = \bar{\tilde{S}} & \text{if adjusting labor tax rate} \end{cases}$$

C Proofs of Propositions

C.1 Steady-state equilibrium equations for a nested version of the model

We assume $\mu = 0$, $\alpha = 0$, and $\rho \to 0$ to get a nested version of the model with a Cobb-Douglas production function. In this case, to have balanced growth, the growth rate of output is the same with the growth rate of technology,$\gamma = \bar{a}$, and the growth rate of relative price (γ_q) is 1. Let $q_0 = 1$. Also, let the fraction of skilled workers $N^S = 1$ and set $\chi_\Phi^s = 1$ and $\chi_S^s = 1$ for profit and transfers distributions. Then, in this economy, we have one type of capital $K_{e,t}$ and one type of labor $L_{s,t}$. We derive steady-state equilibrium equations for this nested version of the model and drop subscripts e and s.

- Marginal cost

$$\bar{mc} = \frac{\theta - 1}{\theta}.$$

- Capital rental rate

$$\bar{r}^K = \frac{\frac{\bar{a}}{\beta} - (1 - d)}{1 - \bar{\tau}^K} \tag{76}$$

- Production function

$$\frac{\bar{\tilde{Y}}}{\bar{H}} = \left(\frac{\bar{\tilde{K}}}{\bar{H}}\right)^\lambda \tag{77}$$

- Wages and capital-to-labor ratio

$$\frac{\bar{\tilde{K}}}{\bar{H}} = \left(\frac{\bar{r}^K}{\lambda \bar{mc}}\right)^{\frac{1}{\lambda-1}} \tag{78}$$

$$\begin{aligned} \bar{\tilde{w}} &= (1-\lambda)\, \bar{mc} \left(\frac{\bar{\tilde{K}}}{\bar{H}}\right)^{\lambda} \\ &= (1-\lambda)\,(\lambda)^{\frac{\lambda}{1-\lambda}} \,(\bar{mc})^{\frac{1}{1-\lambda}} \left(\bar{r}^K\right)^{\frac{\lambda}{\lambda-1}} \end{aligned} \tag{79}$$

- Resource constraint

$$\frac{\bar{\tilde{C}}}{\bar{H}} = \left(1 - \bar{\tilde{G}}\right) \frac{\bar{\tilde{Y}}}{\bar{H}} - \frac{\bar{\tilde{I}}}{\bar{H}}. \tag{80}$$

- Profit

$$\frac{\bar{\tilde{\Phi}}}{\bar{\tilde{Y}}} = 1 - \bar{\tilde{w}} \frac{\bar{H}}{\bar{\tilde{Y}}} - \bar{r}^K \frac{\bar{\tilde{K}}}{\bar{\tilde{Y}}}$$

- Transfer

$$\bar{\tilde{S}} = \left(1 - \frac{\bar{R}}{\bar{\pi}\bar{a}}\right) \bar{\tilde{b}} - \bar{\tilde{G}} + \bar{\tilde{T}}^C + \bar{\tilde{T}}^H + \bar{\tilde{T}}^K. \tag{81}$$

- The consumption, labor income and capital income tax rates are respectively given as:

$$\bar{\tau}^C = \frac{\bar{\tilde{T}}^C}{\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}},\ \bar{\tau}^H = \frac{1}{\bar{\tilde{w}}} \frac{\bar{\tilde{T}}^H}{\frac{\bar{H}}{\bar{\tilde{Y}}}},\ \bar{\tau}^K = \frac{1}{\bar{r}^K} \frac{\bar{\tilde{T}}^K}{\frac{\bar{\tilde{K}}}{\bar{\tilde{Y}}}}.$$

- Intra-temporal Euler equation

$$\bar{H} = \left(\frac{1}{\bar{\omega}\left(\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H} \frac{\bar{\tilde{C}}}{\bar{H}} \frac{1}{\bar{\tilde{w}}}\right)}\right)^{\frac{1}{1+\varphi}}. \tag{82}$$

- Investment

$$\frac{\bar{\tilde{I}}}{\bar{H}} = \frac{\bar{\tilde{K}}}{\bar{H}} \left(\bar{a} - (1-d)\right). \tag{83}$$

- Nominal interest rate

$$\bar{R} = \frac{\bar{a}\bar{\pi}}{\beta}.$$

C.2 Proof of Lemma 1

Proof. From (76) and (79), we get

$$\frac{\partial \bar{r}^K}{\partial \bar{\tau}^K} = \frac{\bar{r}^K}{1-\bar{\tau}^K} > 0$$
$$\frac{\partial \bar{\bar{w}}}{\partial \bar{\tau}^K} = -\left(\frac{\bar{\bar{w}}}{\bar{r}^K}\right)\left(\frac{\lambda}{1-\lambda}\right)\frac{\partial \bar{r}^K}{\partial \bar{\tau}^K} < 0.$$

Let $\bar{\bar{k}} = \frac{\bar{\bar{K}}}{\bar{H}}$ and $\bar{\bar{y}} = \frac{\bar{\bar{Y}}}{\bar{H}}$. From (78) and (77), we get

$$\frac{\partial \bar{\bar{k}}}{\partial \bar{\tau}^K} = -\frac{\bar{\bar{k}}}{\bar{r}^K}\frac{1}{1-\lambda}\frac{\partial \bar{r}^K}{\partial \bar{\tau}^K} < 0$$
$$\frac{\partial \bar{\bar{y}}}{\partial \bar{\tau}^K} = \lambda\left(\frac{\bar{\bar{y}}}{\bar{\bar{k}}}\right)^{\frac{1}{\varepsilon}}\frac{\partial \bar{\bar{k}}}{\partial \bar{\tau}^K} < 0$$

Combining (79) and (80) with (82), we rewrite the steady-state hours as

$$\bar{H} = \left(\bar{\omega}\left(\frac{1}{1-\lambda}\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H}\left(\frac{1-\bar{\bar{G}}}{\bar{mc}} - \lambda\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right)\right)\right)^{-\frac{1}{1+\varphi}}.$$

Then, the partial derivative with respect to capital tax rate is

$$\frac{\partial \bar{H}}{\partial \bar{\tau}^K} = -\frac{\bar{H}^{2+\varphi}}{1+\varphi}\left(\bar{\omega}\frac{\lambda}{1-\lambda}\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H}\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right) < 0.$$

Now, we find the partial derivatives of levels of variables. For capital, investment and output, we can easily verify that

$$\frac{\partial \bar{\bar{K}}}{\partial \bar{\tau}^K} = \bar{H}\frac{\partial \bar{\bar{k}}}{\partial \bar{\tau}^K} + \bar{\bar{k}}\frac{\partial \bar{H}}{\partial \bar{\tau}^K} < 0$$
$$\frac{\partial \bar{\bar{I}}}{\partial \bar{\tau}^K} = \frac{\partial \bar{\bar{K}}}{\partial \bar{\tau}^K}\left(\bar{a}-(1-d)\right) < 0$$
$$\frac{\partial \bar{\bar{Y}}}{\partial \bar{\tau}^K} = \bar{H}\frac{\partial \bar{\bar{y}}}{\partial \bar{\tau}^K} + \bar{\bar{y}}\frac{\partial \bar{H}}{\partial \bar{\tau}^K} < 0$$

For consumption, combining (77) and (83) with (80), we get

$$\begin{aligned}\bar{\bar{C}} &= \left(\left(1-\bar{\bar{G}}\right)\frac{\bar{\bar{Y}}}{\bar{H}} - \frac{\bar{\bar{I}}}{\bar{H}}\right)\bar{H}\\ &= \left(\bar{mc}\frac{\lambda\left(1-\bar{\tau}^K\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)^{\frac{\lambda}{1-\lambda}}\left[\left(1-\bar{\bar{G}}\right) - \lambda\bar{mc}\frac{\left(\bar{a}-(1-d)\right)}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right]\bar{H}.\end{aligned}$$

Then, the partial derivative of consumption with respect to capital tax rate is

$$\frac{\partial \bar{\tilde{C}}}{\partial \bar{\tau}^K} = -\left(\frac{\lambda}{1-\lambda}\frac{1}{1-\bar{\tau}^K}\right)\left(\frac{\lambda \bar{mc}\left(1-\bar{\tau}^K\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)^{\frac{\lambda}{1-\lambda}}\left[\left(1-\bar{\tilde{G}}\right)-\bar{mc}\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right]\bar{H}+\frac{\bar{\tilde{C}}}{\bar{H}}\frac{\partial \bar{H}}{\partial \bar{\tau}^K}.$$

Under Assumption 1, we find $\frac{\partial \bar{\tilde{C}}}{\partial \bar{\tau}^K} < 0$. $\square$

C.3 Proof of Proposition 1

Proof. Let $\bar{\tau}^K_{new} = \bar{\tau}^K + \Delta\left(\bar{\tau}^K\right)$. From (76), we get

$$\frac{\bar{r}^K_{new}}{\bar{r}^K} = \left(\frac{1-\bar{\tau}^K}{1-\bar{\tau}^K_{new}}\right) = \left(1-\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{-1}$$

From (79), (78) and (77), we get

$$\begin{aligned}
\frac{\bar{\tilde{w}}_{new}}{\bar{\tilde{w}}} &= \left(1-\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{\frac{\lambda}{1-\lambda}} \\
\frac{\bar{\tilde{k}}_{new}}{\bar{\tilde{k}}} &= \left(1-\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{\frac{1}{1-\lambda}} \\
\frac{\bar{\tilde{y}}_{new}}{\bar{\tilde{y}}} &= \left(1-\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{\frac{\lambda}{1-\lambda}}.
\end{aligned}$$

Combining (79) and (80) with (82), we get

$$\begin{aligned}
\frac{\bar{H}_{new}}{\bar{H}} &= \left(\frac{\frac{1}{1-\lambda}\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H}\left(\frac{1-\bar{\tilde{G}}}{\bar{mc}}-\lambda\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)\right)}{\frac{1}{1-\lambda}\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H}\left(\frac{1-\bar{\tilde{G}}}{\bar{mc}}-\lambda\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right)}\right)^{-\frac{1}{1+\varphi}} \\
&= \left(1+\bar{\omega}\bar{H}^{1+\varphi}\frac{\lambda}{1-\lambda}\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H}\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\Delta\left(\bar{\tau}^K\right)\right)^{-\frac{1}{1+\varphi}} \\
&= \left(1+\Omega\Delta\left(\bar{\tau}^K\right)\right)^{-\frac{1}{1+\varphi}}
\end{aligned}$$

where $\Omega = \bar{\omega}\bar{H}^{1+\varphi}\frac{\lambda}{1-\lambda}\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H} > 0$.

Now, we find changes of levels of variables. For capital, investment and output, we can easily verify that

$$\frac{\bar{\tilde{K}}_{new}}{\bar{\tilde{K}}} = \frac{\bar{\tilde{k}}_{new}}{\bar{\tilde{k}}}\frac{\bar{H}_{new}}{\bar{H}} = \left(1-\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{\frac{1}{1-\lambda}}\left(1+\Omega\Delta\left(\bar{\tau}^K\right)\right)^{-\frac{1}{1+\varphi}}$$

$$\frac{\bar{\tilde{I}}_{new}}{\bar{\tilde{I}}} = \frac{\bar{\tilde{K}}_{new}}{\bar{\tilde{K}}} = \left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{\frac{1}{1-\lambda}} \left(1+\Omega\Delta\left(\bar{\tau}^K\right)\right)^{-\frac{1}{1+\varphi}}$$

and

$$\frac{\bar{\tilde{Y}}_{new}}{\bar{\tilde{Y}}} = \left(\frac{\bar{\tilde{k}}_{new}}{\bar{\tilde{k}}}\right)^{\lambda} \frac{\bar{H}_{new}}{\bar{H}} = \left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{\frac{\lambda}{1-\lambda}} \left(1+\Omega\Delta\left(\bar{\tau}^K\right)\right)^{-\frac{1}{1+\varphi}}.$$

For consumption, combining (77) and (83) with (80), we get

$$\begin{aligned}
\frac{\bar{\tilde{C}}_{new}}{\bar{\tilde{C}}} &= \frac{\left(\bar{mc}\frac{\lambda\left(1-\bar{\tau}^K_{new}\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)^{\frac{\lambda}{1-\lambda}} \left[\left(1-\bar{\tilde{G}}\right) - \lambda\bar{mc}\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)\right]\bar{H}_{new}}{\left(\bar{mc}\frac{\lambda\left(1-\bar{\tau}^K\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)^{\frac{\lambda}{1-\lambda}} \left[\left(1-\bar{\tilde{G}}\right) - \lambda\bar{mc}\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right]\bar{H}} \\
&= \left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{\frac{\lambda}{1-\lambda}} \left(1 + \frac{\lambda\bar{mc}\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}}{\left(1-\bar{\tilde{G}}\right) - \lambda\bar{mc}\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)}\Delta\left(\bar{\tau}^K\right)\right)\left(1+\Omega\Delta\left(\bar{\tau}^K\right)\right)^{-\frac{1}{1+\varphi}} \\
&= \left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{\frac{\lambda}{1-\lambda}} \left(1 + \frac{\bar{\tilde{I}}}{\bar{H}}\left(\frac{\bar{\tilde{C}}}{\bar{H}}\left(1-\bar{\tau}^K\right)\right)^{-1}\Delta\left(\bar{\tau}^K\right)\right)\left(1+\Omega\Delta\left(\bar{\tau}^K\right)\right)^{-\frac{1}{1+\varphi}}.
\end{aligned}$$

Now for small changes in the capital tax rate $\Delta\left(\bar{\tau}^K\right)$, the percent changes of rental rate, wages, capital to hours ratio, output to hours ratio from their initial steady-states are:

$$\ln\left(\frac{\bar{r}^K_{new}}{\bar{r}^K}\right) = -\ln\left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right) \simeq \left(\frac{1}{1-\bar{\tau}^K}\right)\Delta\left(\bar{\tau}^K\right).$$

$$\ln\left(\frac{\bar{\tilde{w}}_{new}}{\bar{\tilde{w}}}\right) = \frac{\lambda}{1-\lambda}\ln\left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right) \simeq -\left(\frac{\lambda}{1-\lambda}\frac{1}{1-\bar{\tau}^K}\right)\Delta\left(\bar{\tau}^K\right)$$

$$\ln\left(\frac{\bar{\tilde{k}}_{new}}{\bar{\tilde{k}}}\right) = \frac{1}{1-\lambda}\ln\left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right) \simeq -\left(\frac{1}{1-\lambda}\frac{1}{1-\bar{\tau}^K}\right)\Delta\left(\bar{\tau}^K\right)$$

$$\ln\left(\frac{\bar{\tilde{y}}_{new}}{\bar{\tilde{y}}}\right) = \frac{\lambda}{1-\lambda}\ln\left(1 - \frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right) \simeq -\left(\frac{\lambda}{1-\lambda}\frac{1}{1-\bar{\tau}^K}\right)\Delta\left(\bar{\tau}^K\right)$$

and

$$\ln\left(\frac{\bar{H}_{new}}{\bar{H}}\right) = -\frac{1}{1+\varphi}\ln\left(1+\Omega\Delta\left(\bar{\tau}^K\right)\right) \simeq -\frac{1}{1+\varphi}(1+\Omega)\Delta\left(\bar{\tau}^K\right).$$

The percent changes of levels of capital and investment from their initial steady-states are:

$$\begin{aligned}
\ln\left(\frac{\bar{\tilde{K}}_{new}}{\bar{\tilde{K}}}\right) = \ln\left(\frac{\bar{\tilde{I}}_{new}}{\bar{\tilde{I}}}\right) &\simeq -\left(\frac{1}{(1-\lambda)\left(1-\bar{\tau}^K\right)} + \frac{\Omega}{1+\varphi}\right)\Delta\left(\bar{\tau}^K\right) \\
&= -\mathcal{M}_K\Delta\left(\bar{\tau}^K\right)
\end{aligned}$$

where $\mathcal{M}_K = \frac{1}{(1-\lambda)(1-\bar{\tau}^K)} + \frac{\Omega}{1+\varphi} > 0$. Also, the percent change of output from the initial steady-state is:

$$\ln\left(\frac{\bar{\tilde{Y}}_{new}}{\bar{\tilde{Y}}}\right) \simeq -\left(\frac{\lambda}{(1-\lambda)\left(1-\bar{\tau}^K\right)}+\frac{\Omega}{1+\varphi}\right)\Delta\left(\bar{\tau}^K\right)$$
$$= -\mathcal{M}_Y \Delta\left(\bar{\tau}^K\right)$$

where $\mathcal{M}_Y = \frac{\lambda}{(1-\lambda)(1-\bar{\tau}^K)}+\frac{\Omega}{1+\varphi} > 0$.

Finally, the percent change of consumption from its initial steady-state is:

$$\ln\left(\frac{\bar{\tilde{C}}_{new}}{\bar{\tilde{C}}}\right) \simeq -\left[\frac{\lambda}{(1-\lambda)\left(1-\bar{\tau}^K\right)}+\frac{\Omega}{1+\varphi}-\frac{\lambda \bar{m}c\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}}{\left(1-\bar{\tilde{G}}\right)-\lambda \bar{m}c\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)}\right]\Delta\left(\bar{\tau}^K\right)$$
$$= -\frac{\lambda}{1-\lambda}\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}^{\eta}}{\beta}-(1-d)}\right)\times$$
$$\left(\frac{\bar{\omega}\bar{H}^{1+\varphi}\left(\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H}\right)}{1+\varphi}+\frac{\left(1-\bar{\tilde{G}}\right)-\bar{m}c\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\left(1-\bar{\tau}^K\right)}{\frac{(\bar{a}-(1-d))\left(1-\bar{\tau}^K\right)}{\frac{\bar{a}}{\beta}-(1-d)}\left(\left(1-\bar{\tilde{G}}\right)-\frac{\lambda\bar{m}c(\bar{a}-(1-d))\left(1-\bar{\tau}^K\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)}\right)\Delta\left(\bar{\tau}^K\right)$$
$$= -\mathcal{M}_C\Delta\left(\bar{\tau}^K\right).$$

Notice that under Assumption 1, the numerator of the second term in the large bracket is greater than zero. Thus, we have $\mathcal{M}_C = \mathcal{M}_Y - \frac{\bar{\tilde{I}}}{\bar{H}}\left(\frac{\bar{\tilde{C}}}{\bar{H}}\left(1-\bar{\tau}^K\right)\right)^{-1} > 0$. □

C.4 Proof of Lemma 2

Proof. Notice that rental rate of capital, wage, capital to hours ratio, and output to hours ratio are the same with the lump-sum transfers adjustment case in C.2.

To show hours are increasing in $\bar{\tau}^K$, we rewrite (81) as the following:

$$\bar{\tilde{S}} = \left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}}-\bar{\tilde{G}}+\bar{\tau}^C\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}+\bar{\tau}^H\bar{\tilde{w}}\frac{\bar{H}}{\bar{\tilde{Y}}}+\bar{\tau}^K\bar{r}^K\frac{\tilde{K}}{\tilde{Y}}$$
$$= \left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}}-\bar{\tilde{G}}+\bar{\tau}^C\left[\left(1-\bar{\tilde{G}}\right)-\frac{\lambda\bar{m}c\,(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right]+\bar{\tau}^H(1-\lambda)\,\bar{m}c+\bar{\tau}^K\lambda\bar{m}c$$
$$1-\bar{\tau}^H = \frac{(1-\lambda)\,\bar{m}c-\bar{\tilde{S}}+\left[\left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}}-\bar{\tilde{G}}+\bar{\tau}^C\left(\left(1-\bar{\tilde{G}}\right)-\frac{\lambda\bar{m}c(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right)+\bar{\tau}^K\lambda\bar{m}c\right]}{(1-\lambda)\,\bar{m}c}$$

Then, from (82), we get

$$\bar{H} = \left(\frac{\bar{\omega}\left(1+\bar{\tau}^C\right)\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{m}c(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right)}{(1-\lambda)\,\bar{m}c-\bar{\tilde{S}}+\left[\left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}}-\bar{\tilde{G}}+\bar{\tau}^C\left(\left(1-\bar{\tilde{G}}\right)-\frac{\lambda\bar{m}c(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right)+\bar{\tau}^K\lambda\bar{m}c\right]}\right)^{-\frac{1}{1+\varphi}} \tag{84}$$

Taking a partial derivative with respect to capital tax rate gives:

$$
\begin{aligned}
\frac{\partial \bar{H}}{\partial \bar{\tau}^K} &= \frac{\bar{H}^{-\varphi}}{1+\varphi}\left[\frac{\bar{\tau}^C \frac{\lambda \bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)} + \lambda \bar{mc}}{\bar{\omega}\left(1+\bar{\tau}^C\right)\left(1-\bar{\tilde{G}}-\frac{\lambda \bar{mc}(\bar{a}-(1-d))\left(1-\bar{\tau}^K\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)} - \frac{\bar{\omega}\left(1+\bar{\tau}^C\right)\frac{\lambda \bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\lambda\right)\bar{mc}\left(1-\bar{\tau}^H\right)}{\left(\bar{\omega}\left(1+\bar{\tau}^C\right)\left(1-\bar{\tilde{G}}-\frac{\lambda \bar{mc}(\bar{a}-(1-d))\left(1-\bar{\tau}^K\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\right)^2}\right] \\
&= \frac{\frac{1}{1+\varphi}\bar{H}^{-\varphi}}{\bar{\omega}\left(1+\bar{\tau}^C\right)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\left[\bar{\tau}^C \frac{\lambda \bar{mc}\left(\bar{a}-(1-d)\right)}{\frac{\bar{a}}{\beta}-(1-d)} + \lambda \bar{mc} - \frac{(1-\lambda)\bar{mc}\left(1-\bar{\tau}^H\right)\frac{\lambda \bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}}{\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\right] \\
&= \frac{\frac{1}{1+\varphi}\bar{H}^{-\varphi}\frac{\lambda \bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}}{\bar{\omega}\left(1+\bar{\tau}^C\right)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\left[\frac{\bar{\tau}^C \frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}} + \frac{\frac{\bar{a}}{\beta}-(1-d)}{(\bar{a}-(1-d))}\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}} - (1-\lambda)\bar{mc}\left(1-\bar{\tau}^H\right)}{\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\right] \\
&= \frac{\frac{1}{1+\varphi}\bar{H}^{-\varphi}\frac{\lambda \bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}}{\bar{\omega}\left(1+\bar{\tau}^C\right)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\left[\frac{\bar{\tau}^C \frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}} + \bar{\tau}^H \bar{\tilde{w}}\frac{\bar{H}}{\bar{\tilde{Y}}} + \frac{\frac{\bar{a}}{\beta}-(1-d)}{(\bar{a}-(1-d))}\left(\left(1-\bar{\tilde{G}}\right) - \frac{\bar{\tilde{K}}}{\bar{\tilde{Y}}}\left(\bar{a}-(1-d)\right)\right) - (1-\lambda)\bar{mc}}{\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\right] \\
&= \frac{\frac{1}{1+\varphi}\bar{H}^{-\varphi}\frac{\lambda \bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}}{\bar{\omega}\left(1+\bar{\tau}^C\right)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\left[\frac{\bar{\tau}^C \frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}} + \bar{\tau}^H \bar{\tilde{w}}\frac{\bar{H}}{\bar{\tilde{Y}}} + \bar{\tau}^K\frac{\bar{\tilde{K}}}{\bar{\tilde{Y}}} + \frac{\frac{\bar{a}}{\beta}-(1-d)}{(\bar{a}-(1-d))}\left(1-\bar{\tilde{G}}\right) - \bar{mc}}{\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\right] \\
&= \frac{\frac{1}{1+\varphi}\bar{H}^{-\varphi}\frac{1}{\left(1-\bar{\tau}^K\right)}\frac{\bar{\tilde{I}}}{\bar{\tilde{C}}}}{\bar{\omega}\left(1+\bar{\tau}^C\right)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\left[\bar{\tilde{T}}^C + \bar{\tilde{T}}^K + \bar{\tilde{T}}^H + \frac{\frac{\bar{a}}{\beta}-(1-d)}{(\bar{a}-(1-d))}\left(\left(1-\bar{\tilde{G}}\right) - \frac{\bar{mc}\left(\bar{a}-(1-d)\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\right]
\end{aligned}
$$

Under Assumption 2, $\frac{\partial \bar{H}}{\partial \bar{\tau}^K} > 0$. □

C.5 Proof of Proposition 2

Proof. Let $\bar{\tau}^K_{new} = \bar{\tau}^K + \Delta\left(\bar{\tau}^K\right)$. Under the labor tax adjustment case, the steady-state labor tax rate is:

$$
\bar{\tau}^H = \frac{\bar{\tilde{S}} - \left[\left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}} - \bar{\tilde{G}} + \bar{\tilde{T}}^C + \bar{\tilde{T}}^K\right]}{\bar{\tilde{w}}\frac{\bar{H}}{\bar{\tilde{Y}}}}
$$

Then, after capital tax rate changes, the new steady-state labor tax rate is given by:

$$
\begin{aligned}
\bar{\tau}^H_{new} &= \frac{\bar{\tilde{S}} - \left[\left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}} - \bar{\tilde{G}} + \bar{\tau}^C\left(\left(1-\bar{\tilde{G}}\right) - \frac{\lambda \bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)\right) + \bar{\tau}^K_{new}\lambda \bar{mc}\right]}{(1-\lambda)\bar{mc}} \\
&= \bar{\tau}^H - \frac{\lambda}{1-\lambda}\left(1+\bar{\tau}^C\frac{\left(\bar{a}-(1-d)\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\Delta\left(\bar{\tau}^K\right) \qquad (85) \\
&= \bar{\tau}^H + \Delta\left(\bar{\tau}^H\right).
\end{aligned}
$$

Notice that the relative changes of rental rate, wage and capital to hours ratio from their initial steady-states are the same with the lump-sum transfers adjustment case. For after-tax wage, from

(79) and (85), we get

$$\ln\left(\frac{\left(1-\bar{\tau}^H_{new}\right)\bar{\tilde{w}}_{new}}{\left(1-\bar{\tau}^H\right)\bar{\tilde{w}}}\right) = \ln\left(\left(\left(1+\frac{\lambda}{1-\lambda}\frac{\Delta\left(\bar{\tau}^K\right)}{\left(1-\bar{\tau}^H\right)}\left(1+\bar{\tau}^C\frac{\left(\bar{a}-(1-d)\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\right)\left(1-\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}\right)^{\frac{\lambda}{1-\lambda}}\right).$$

$$\simeq \frac{\lambda}{1-\lambda}\left(\frac{1}{1-\bar{\tau}^H}\right)\left(\left(1+\bar{\tau}^C\frac{\left(\bar{a}-(1-d)\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)-\frac{1-\bar{\tau}^H}{1-\bar{\tau}^K}\right)\Delta\left(\bar{\tau}^K\right)$$

$$=\mathcal{M}_W\Delta\left(\bar{\tau}^K\right)$$

where $\mathcal{M}_W>0$ if $\left(1+\bar{\tau}^C\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)>\frac{1-\bar{\tau}^H}{1-\bar{\tau}^K}$. For hours, from (84), we get

$$\frac{\bar{H}_{new}}{\bar{H}} = \left(\frac{\dfrac{(1-\lambda)\bar{mc}-\bar{\tilde{S}}+\left[\left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}}-\bar{\tilde{G}}+\bar{\tau}^C\left(\left(1-\bar{\tilde{G}}\right)-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K-\Delta\bar{\tau}^K\right)\right)+\left(\bar{\tau}^K+\Delta\left(\bar{\tau}^K\right)\right)\lambda\bar{mc}\right]}{\bar{\omega}\left(1+\bar{\tau}^C\right)\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K-\Delta\left(\bar{\tau}^K\right)\right)\right)}}{\dfrac{(1-\lambda)\bar{mc}-\bar{\tilde{S}}+\left[\left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}}-\bar{\tilde{G}}+\bar{\tau}^C\left(\left(1-\bar{\tilde{G}}\right)-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right)+\bar{\tau}^K\lambda\bar{mc}\right]}{\bar{\omega}\left(1+\bar{\tau}^C\right)\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right)}}\right)^{\frac{1}{1+\varphi}}$$

$$=\left(\frac{1+\left(\frac{\lambda\bar{mc}+\frac{\bar{\tau}^C}{1-\bar{\tau}^K}\frac{\bar{\tilde{I}}}{\bar{\tilde{Y}}}}{\left(1-\bar{\tau}^H\right)(1-\lambda)\bar{mc}}\right)\Delta\left(\bar{\tau}^K\right)}{1+\left(\frac{\frac{1}{1-\bar{\tau}^K}\frac{\bar{\tilde{I}}}{\bar{\tilde{Y}}}}{1-\bar{\tilde{G}}-\frac{\bar{\tilde{I}}}{\bar{\tilde{Y}}}}\right)\Delta\left(\bar{\tau}^K\right)}\right)^{\frac{1}{1+\varphi}}$$

Then, for small changes of capital tax rate $\Delta\left(\bar{\tau}^K\right)$, we get:

$$\ln\left(\frac{\bar{H}_{new}}{\bar{H}}\right)$$

$$=\frac{\frac{1}{1+\varphi}\left(\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)}{1-\bar{\tilde{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))\left(1-\bar{\tau}^K\right)}{\frac{\bar{a}}{\beta}-(1-d)}}\left[\frac{\bar{\tau}^C\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}+\frac{\frac{\bar{a}}{\beta}-(1-d)}{\bar{a}-(1-d)}\left(1-\bar{\tilde{G}}\right)-\left(1-\bar{\tau}^K\right)\frac{\bar{\tilde{K}}}{\bar{\tilde{Y}}}\bar{r}^K-(1-\lambda)\bar{mc}\left(1-\bar{\tau}^H\right)}{(1-\lambda)\bar{mc}\left(1-\bar{\tau}^H\right)}\right]\Delta\left(\bar{\tau}^K\right)$$

$$=\frac{\frac{1}{1+\varphi}\left(\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)}{1-\bar{\tilde{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))\left(1-\bar{\tau}^K\right)}{\frac{\bar{a}}{\beta}-(1-d)}}\left[\frac{\bar{\tau}^C\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}+\bar{\tau}^K\bar{r}^K\frac{\bar{\tilde{K}}}{\bar{\tilde{Y}}}+\bar{\tau}^H\bar{\tilde{w}}\frac{\bar{H}}{\bar{\tilde{Y}}}+\frac{\frac{\bar{a}}{\beta}-(1-d)}{\bar{a}-(1-d)}\left(\left(1-\bar{\tilde{G}}\right)-\frac{\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)}{(1-\lambda)\bar{mc}\left(1-\bar{\tau}^H\right)}\right]\Delta\left(\bar{\tau}^K\right)$$

$$=\frac{\frac{1}{1+\varphi}\frac{\lambda}{1-\lambda}\left(\bar{a}-(1-d)\right)}{\left(1-\bar{\tau}^H\right)\left(\frac{\bar{a}}{\beta}-(1-d)\right)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\left[\bar{\tilde{T}}^C+\bar{\tilde{T}}^H+\bar{\tilde{T}}^K+\frac{\frac{\bar{a}}{\beta}-(1-d)}{\bar{a}-(1-d)}\left(\left(1-\bar{\tilde{G}}\right)-\frac{\bar{mc}\left(\bar{a}-(1-d)\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\right]\Delta\left(\bar{\tau}^K\right)$$

$$=\frac{\frac{1}{1+\varphi}\frac{\lambda}{1-\lambda}\left(\bar{a}-(1-d)\right)}{\left(1-\bar{\tau}^H\right)\left(\frac{\bar{a}}{\beta}-(1-d)\right)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\left[\bar{\tilde{T}}+\frac{\frac{\bar{a}}{\beta}-(1-d)}{\bar{a}-(1-d)}\left(\left(1-\bar{\tilde{G}}\right)-\frac{\bar{mc}\left(\bar{a}-(1-d)\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\right]\Delta\left(\bar{\tau}^K\right)$$

$$=\frac{1}{1+\varphi}\frac{\lambda}{1-\lambda}\left[\frac{\left(1-\bar{\tilde{G}}\right)+\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\left(\bar{\tilde{T}}-\bar{mc}\right)}{\left(1-\bar{\tau}^H\right)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\right]\Delta\left(\bar{\tau}^K\right)$$

$$=\mathcal{M}_H\Delta\left(\bar{\tau}^K\right)$$

where $\mathcal{M}_H>0$ under Assumption 2. □

C.6 Proof of Lemma 3

Proof. Notice that rental rate of capital, wage, capital to hours ratio, and output to hours ratio are the same with the lump-sum transfers adjustment case in C.2.

To show hours are increasing in $\bar{\tau}^K$, we rewrite (81) as the following:

$$\bar{\tilde{S}} = \left(1 - \frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}} - \bar{\tilde{G}} + \bar{\tau}^C \frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}} + \bar{\tau}^H \bar{\tilde{w}} \frac{\bar{H}}{\bar{\tilde{Y}}} + \bar{\tau}^K \bar{r}^K \frac{\tilde{K}}{\tilde{Y}}$$
$$= \left(1 - \frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}} - \bar{\tilde{G}} + \bar{\tau}^C \left[\left(1 - \bar{\tilde{G}}\right) - \frac{\lambda \bar{mc}\left(\bar{a} - (1-d)\right)}{\frac{\bar{a}}{\beta} - (1-d)}\left(1 - \bar{\tau}^K\right)\right] + \bar{\tau}^H (1-\lambda)\bar{mc} + \bar{\tau}^K \lambda \bar{mc}$$

$$1 + \bar{\tau}^C = \frac{\left(1 - \bar{\tilde{G}}\right) - \frac{\lambda \bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1 - \bar{\tau}^K\right) + \bar{\tilde{S}} - \left[\left(1 - \frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}} - \bar{\tilde{G}} + \bar{\tau}^H (1-\lambda)\bar{mc} + \bar{\tau}^K \lambda \bar{mc}\right]}{\left(1 - \bar{\tilde{G}}\right) - \frac{\lambda \bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1 - \bar{\tau}^K\right)}$$

Then, from (82), we get

$$\bar{H} = \left(\bar{\omega}\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H}\frac{\bar{\tilde{C}}}{\bar{H}}\frac{1}{\bar{\tilde{w}}}\right)^{-\frac{1}{1+\varphi}}$$
$$= \left(\frac{\bar{\omega}\left(\left(1 - \bar{\tilde{G}}\right) - \frac{\lambda \bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1 - \bar{\tau}^K\right) + \bar{\tilde{S}} - \left[\left(1 - \frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}} - \bar{\tilde{G}} + \bar{\tau}^H (1-\lambda)\bar{mc} + \bar{\tau}^K \lambda \bar{mc}\right]\right)}{(1-\lambda)\bar{mc}\left(1 - \bar{\tau}^H\right)}\right)^{-\frac{1}{1+\varphi}}$$

Taking a partial derivative with respect to capital tax rate gives:

$$\frac{\partial \bar{H}}{\partial \bar{\tau}^K} = \frac{\bar{\omega}\bar{H}^{2+\varphi}}{1+\varphi}\frac{\lambda}{(1-\lambda)\left(1-\bar{\tau}^H\right)}\left(1 - \frac{\left(\bar{a} - (1-d)\right)}{\frac{\bar{a}}{\beta} - (1-d)}\right) > 0.$$

□

C.7 Proof of Proposition 3

Proof. Let $\bar{\tau}^K_{new} = \bar{\tau}^K + \Delta\left(\bar{\tau}^K\right)$. Under the consumption tax adjustment case, the steady-state consumption tax rate is:

$$\bar{\tau}^C = \frac{\bar{\tilde{S}} - \left[\left(1 - \frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}} - \bar{\tilde{G}} + \bar{\tilde{T}}^H + \bar{\tilde{T}}^K\right]}{\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}$$

Then, after capital tax rate changes, the new steady-state consumption tax rate is given by:

$$\bar{\tau}^C_{new} = \frac{\bar{\tilde{S}} - \left[\left(1 - \frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}} - \bar{\tilde{G}} + \bar{\tau}^H (1-\lambda)\bar{mc} + \bar{\tau}^K \lambda \bar{mc} + \Delta\left(\bar{\tau}^K\right)\lambda \bar{mc}\right]}{\left(1 - \bar{\tilde{G}}\right) - \frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\lambda \bar{mc}\left(1 - \bar{\tau}^K_{new}\right)}$$
$$= \frac{\bar{\tau}^C - \frac{\lambda \bar{mc}}{\left(1-\bar{\tilde{G}}\right) - \frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda \bar{mc}\left(1-\bar{\tau}^K\right)}\Delta\left(\bar{\tau}^K\right)}{1 + \frac{\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda \bar{mc}}{\left(1-\bar{\tilde{G}}\right) - \frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda \bar{mc}\left(1-\bar{\tau}^K\right)}\Delta\left(\bar{\tau}^K\right)}$$
$$= \bar{\tau}^C + \Delta\left(\bar{\tau}^C\right)$$

Then,

$$\Delta\left(\bar{\tau}^{C}\right)=-\left(1+\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\bar{\tau}^{C}\right)\frac{\Theta_{C}\Delta\left(\bar{\tau}^{K}\right)}{1+\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\Theta_{C}\Delta\left(\bar{\tau}^{K}\right)}$$

where $\Theta_C=\frac{\lambda\bar{m}c}{\left(1-\bar{\tilde{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{m}c\left(1-\bar{\tau}^{K}\right)}=\bar{r}^{K}\frac{\bar{\tilde{K}}}{\bar{\tilde{C}}}>0.$

Notice that the relative changes of rental rate, wage and capital to hours ratio from their initial steady-states are the same with the lump-sum transfers adjustment case. For hours, we get

$$\begin{aligned}
\frac{\bar{H}_{new}}{\bar{H}}&=\left(\frac{\left(1-\bar{\tilde{G}}\right)-\frac{\lambda\bar{m}c(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^{K}_{new}\right)+\bar{\tilde{S}}-\left[\left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}}-\bar{\tilde{G}}+\bar{\tau}^{H}\left(1-\lambda\right)\bar{m}c+\bar{\tau}^{K}_{new}\lambda\bar{m}c\right]}{\left(1-\bar{\tilde{G}}\right)-\frac{\lambda\bar{m}c(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^{K}\right)+\bar{\tilde{S}}-\left[\left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}}-\bar{\tilde{G}}+\bar{\tau}^{H}\left(1-\lambda\right)\bar{m}c+\bar{\tau}^{K}\lambda\bar{m}c\right]}\right)^{-\frac{1}{1+\varphi}}\\
&=\left(1-\frac{\left(1-\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)\lambda\bar{m}c\Delta\left(\bar{\tau}^{K}\right)}{\left(1-\bar{\tilde{G}}\right)-\frac{\lambda\bar{m}c(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^{K}\right)+\bar{\tilde{S}}-\left[\left(1-\frac{\bar{R}}{\bar{\pi}\bar{a}}\right)\bar{\tilde{b}}-\bar{\tilde{G}}+\bar{\tau}^{H}\left(1-\lambda\right)\bar{m}c+\bar{\tau}^{K}\lambda\bar{m}c\right]}\right)^{-\frac{1}{1+\varphi}}\\
&=\left(1-\frac{\left(1-\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)\lambda\bar{m}c}{\left(1+\bar{\tau}^{C}\right)\left(\left(1-\bar{\tilde{G}}\right)-\frac{\lambda\bar{m}c(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^{K}\right)\right)}\Delta\left(\bar{\tau}^{K}\right)\right)^{-\frac{1}{1+\varphi}}\\
&=\left(1-\frac{\left(1-\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)\lambda\bar{m}c}{\left(1+\bar{\tau}^{C}\right)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\Delta\left(\bar{\tau}^{K}\right)\right)^{-\frac{1}{1+\varphi}}
\end{aligned}$$

Then, for small changes of capital tax rate $\Delta\left(\bar{\tau}^{K}\right)$, we get:

$$\begin{aligned}
\ln\left(\frac{\bar{H}_{new}}{\bar{H}}\right)&=\frac{1}{1+\varphi}\frac{\lambda\bar{m}c}{\left(1+\bar{\tau}^{C}\right)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\left(1-\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)\Delta\left(\bar{\tau}^{K}\right)\\
&=\mathcal{M}_{H,\tau^{C}}\Delta\left(\bar{\tau}^{K}\right)
\end{aligned}$$

where $\mathcal{M}_{H,\tau^C}=\frac{1}{1+\varphi}\frac{\lambda\bar{m}c}{\left(1+\bar{\tau}^{C}\right)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\left(1-\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)>0.$

Now, we find changes of levels of variables. For capital, investment and output, we can easily verify that

$$\frac{\bar{\tilde{K}}_{new}}{\bar{\tilde{K}}}=\frac{\bar{\tilde{k}}_{new}}{\bar{\tilde{k}}}\frac{\bar{H}_{new}}{\bar{H}}=\left(1-\frac{\Delta\left(\bar{\tau}^{K}\right)}{1-\bar{\tau}^{K}}\right)^{\frac{1}{1-\lambda}}\left(1-\frac{\left(1-\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)\lambda\bar{m}c}{\left(1+\bar{\tau}^{C}\right)\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}}\Delta\left(\bar{\tau}^{K}\right)\right)^{-\frac{1}{1+\varphi}}$$

Now for small changes in the capital tax rate $\Delta\left(\bar{\tau}^{K}\right)$, the percent changes of rental rate, wages, capital to hours ratio, output to hours ratio from their initial steady-states are:

$$
\begin{aligned}
\ln\left(\frac{\bar{\tilde{K}}_{new}}{\bar{\tilde{K}}}\right) = \ln\left(\frac{\bar{\tilde{I}}_{new}}{\bar{\tilde{I}}}\right) &\simeq -\frac{1}{1-\lambda}\left(1-\left(1-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\frac{1}{1+\varphi}\frac{\left(1-\bar{\tau}^K\right)\bar{\tilde{w}}\bar{H}}{\left(1+\bar{\tau}^C\right)\bar{\tilde{C}}}\right)\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K} \\
&= -\frac{1}{1-\lambda}\left(1-\left(1-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\left(\frac{1-\bar{\tau}^K}{1-\tau_t^H}\right)\frac{\bar{\omega}H_t^{1+\varphi}}{1+\varphi}\right)\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K} \\
&= -\frac{1}{1-\lambda}\left(1-\frac{1}{1+\varphi}\frac{1-\lambda}{\lambda}\frac{1}{1+\bar{\tau}^C}\left(\frac{\left(1-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\lambda\bar{mc}\left(1-\bar{\tau}^K\right)}{\left(1-\bar{\tilde{G}}\right)-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)}\right)\right)\frac{\Delta\left(\bar{\tau}^K\right)}{1-\bar{\tau}^K}
\end{aligned}
$$

□

C.8 Proof of Proposition 4

Proof. From (76), (79) and (78), we get

$$
\begin{aligned}
\frac{\bar{\tilde{K}}}{\bar{H}} &= \frac{\bar{\tilde{w}}}{\bar{r}^K}\frac{\lambda}{1-\lambda} \\
&= \left(\frac{\lambda\bar{mc}}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right)^{\frac{1}{1-\lambda}}.
\end{aligned}
$$

The amount of changes in capital to hours ratio to the capital tax cut is the same in both lump-sum transfers adjustment case and labor tax rate case. In a similar way, we know that output to hours ratio, investment to hours ratio, and consumption to hours ratio change by the same amount in both cases. Thus, all the magnitudes of changes in macro quantities to capital tax cuts are determined by the hours responses. Now, we compare the changes in hours to capital tax rate changes under the transfers adjustment case with the changes under the labor tax rate adjustment case. Notice that the initial steady-states are the same in both cases. Let $\bar{H}^T_{new}$ and $\bar{H}^L_{new}$ denote the steady-state hours after the capital tax changes in the transfers adjustment case and in the labor tax rate adjustment case, respectively. Then, from (82) and (85), we get

$$
\begin{aligned}
\frac{\bar{H}^T_{new}}{\bar{H}^L_{new}} &= \left(\frac{\frac{\bar{\omega}}{(1-\lambda)\bar{mc}}\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H}\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)\right)}{\frac{\bar{\omega}}{(1-\lambda)\bar{mc}}\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H_{new}}\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)\right)}\right)^{-\frac{1}{1+\varphi}} \\
&= \left(1-\frac{\Delta\left(\bar{\tau}^H\right)}{1-\bar{\tau}^H}\right)^{-\frac{1}{1+\varphi}} \\
&= \left(1+\frac{\lambda}{1-\lambda}\left(\frac{1}{1-\bar{\tau}^H}\right)\left(1+\bar{\tau}^C\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)\Delta\left(\bar{\tau}^K\right)\right)^{-\frac{1}{1+\varphi}}.
\end{aligned}
$$

For small changes in capital tax rate $\Delta\left(\bar{\tau}^K\right)$, we get

$$
\begin{aligned}
\ln\left(\frac{\bar{H}^T_{new}}{\bar{H}^L_{new}}\right) &= -\frac{1}{1+\varphi}\frac{\lambda}{1-\lambda}\left(\frac{1}{1-\bar{\tau}^H}\right)\left(1+\bar{\tau}^C\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)\Delta\left(\bar{\tau}^K\right) \\
&= -\Theta\Delta\left(\bar{\tau}^K\right)
\end{aligned}
$$

where $\Theta = \frac{1}{1+\varphi}\frac{\lambda}{1-\lambda}\left(\frac{1}{1-\bar{\tau}^H}\right)\left(1+\bar{\tau}^C\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right) > 0$. Then, for the levels of output, consumption, capital and investment, the differences are the same: that is,

$$\ln\left(\frac{\bar{\tilde{Y}}^T_{new}}{\bar{\tilde{Y}}^L_{new}}\right) = \ln\left(\frac{\bar{\tilde{C}}^T_{new}}{\bar{\tilde{C}}^L_{new}}\right) = \ln\left(\frac{\bar{\tilde{K}}^T_{new}}{\bar{\tilde{K}}^L_{new}}\right) = \ln\left(\frac{\bar{\tilde{I}}^T_{new}}{\bar{\tilde{I}}^L_{new}}\right) = \ln\left(\frac{\bar{H}^T_{new}}{\bar{H}^L_{new}}\right) = -\Theta\Delta\left(\bar{\tau}^K\right).$$

□

C.9 Relative Changes of Variables in the Consumption Tax Rate Adjustment Case

Proposition 5. *Let $\bar{\tau}^K_{new} = \bar{\tau}^K + \Delta\left(\bar{\tau}^K\right)$. Denote $\bar{X}^T_{new}$ and $\bar{X}^C_{new}$ as the new steady-state variables in transfer adjustment case and in consumption tax rate adjustment case, respectively. For small changes in the capital tax rate $\Delta\left(\bar{\tau}^K\right)$, for $X \in \left\{\tilde{C}, \tilde{K}, \tilde{I}, \tilde{Y}, H\right\}$, we get*

$$\ln\left(\frac{\bar{X}^T_{new}}{\bar{X}^C_{new}}\right) = -\Theta^T_C\Delta\left(\bar{\tau}^K\right)$$

where $\Theta^T_C > 0$ if and only if $\bar{\tilde{G}} < 1 - \lambda\frac{\theta-1}{\theta}\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)$.

Proof. Let $\bar{H}^T_{new}$ and $\bar{H}^C_{new}$ denote the steady-state hours after the capital tax changes in the transfers adjustment case and in the consumption tax rate adjustment case, respectively. Then, we get

$$\begin{aligned}
\frac{\bar{H}^T_{new}}{\bar{H}^C_{new}} &= \left(\frac{\frac{\bar{\omega}}{(1-\lambda)\bar{m}c}\frac{1+\bar{\tau}^C}{1-\bar{\tau}^H}\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{m}c(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)\right)}{\frac{\bar{\omega}}{(1-\lambda)\bar{m}c}\frac{1+\bar{\tau}^C_{new}}{1-\bar{\tau}^H}\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{m}c(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K_{new}\right)\right)}\right)^{-\frac{1}{1+\varphi}} \\
&= \left(1+\frac{\Delta\left(\bar{\tau}^C\right)}{1+\bar{\tau}^C}\right)^{\frac{1}{1+\varphi}} \\
&= \left(1-\left(\frac{1+\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\bar{\tau}^C}{1+\bar{\tau}^C}\right)\frac{\tilde{\Delta}\left(\bar{\tau}^K\right)}{1+\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\tilde{\Delta}\left(\bar{\tau}^K\right)}\right)^{\frac{1}{1+\varphi}}
\end{aligned}$$

For small changes in capital tax rate $\Delta\left(\bar{\tau}^K\right)$, we get

$$\ln\left(\frac{\bar{H}^T_{new}}{\bar{H}^C_{new}}\right) = -\frac{1}{1+\varphi}\left(\frac{1+\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\bar{\tau}^C}{1+\bar{\tau}^C}\right)\left(\frac{\lambda\bar{m}c}{\left(1-\bar{\tilde{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{m}c\left(1-\bar{\tau}^K_{new}\right)}\right)\Delta\left(\bar{\tau}^K\right)$$

Then, for the levels of output, consumption, capital and investment, the differences are the same:

that is,

$$
\begin{aligned}
\ln\left(\frac{\bar{\tilde{Y}}^{T}_{new}}{\bar{\tilde{Y}}^{C}_{new}}\right) = \ln\left(\frac{\bar{\tilde{C}}^{T}_{new}}{\bar{\tilde{C}}^{C}_{new}}\right) = \ln\left(\frac{\bar{\tilde{K}}^{T}_{new}}{\bar{\tilde{K}}^{C}_{new}}\right) = \ln\left(\frac{\bar{\tilde{I}}^{T}_{new}}{\bar{\tilde{I}}^{C}_{new}}\right) &= \ln\left(\frac{\bar{H}^{T}_{new}}{\bar{H}^{C}_{new}}\right) \\
&= \frac{1}{1+\varphi}\frac{\Delta\left(\bar{\tau}^{C}\right)}{1+\bar{\tau}^{C}} \\
&= -\frac{1}{1+\varphi}\left(\frac{1+\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\bar{\tau}^{C}}{1+\bar{\tau}^{C}}\right)\left(\frac{\lambda\bar{mc}}{\left(1-\bar{\tilde{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^{K}_{new}\right)}\right)\Delta\left(\bar{\tau}^{K}\right) \\
&= -\mathcal{M}^{T}_{C}\Delta\left(\bar{\tau}^{K}\right).
\end{aligned}
$$

Then, $\mathcal{M}^{T}_{C} = \frac{1}{1+\varphi}\left(\frac{1+\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\bar{\tau}^{C}}{1+\bar{\tau}^{C}}\right)\left(\frac{\lambda\bar{mc}}{\left(1-\bar{\tilde{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^{K}_{new}\right)}\right) > 0$ if

$$
\begin{aligned}
\bar{\tilde{G}} &< 1-\lambda\frac{\theta-1}{\theta}\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^{K}_{new}\right) \\
&\approx 1-0.15 = 0.85
\end{aligned}
$$

in our calibration. In this case, capital tax cut is more expansionary in the transfer adjustment case than in the consumption tax rate adjustment case. □

Proposition 6. *Let $\bar{\tau}^{K}_{new} = \bar{\tau}^{K} + \Delta\left(\bar{\tau}^{K}\right)$. Denote $\bar{X}^{L}_{new}$ and $\bar{X}^{C}_{new}$ as the new steady-state variables in labor tax rate adjustment case and in consumption tax rate adjustment case, respectively. For small changes in the capital tax rate $\Delta\left(\bar{\tau}^{K}\right)$, for $X \in \left\{\tilde{C}, \tilde{K}, \tilde{I}, \tilde{Y}, H\right\}$, we get*

$$
\ln\left(\frac{\bar{X}^{L}_{new}}{\bar{X}^{C}_{new}}\right) = \Theta^{L}_{C}\Delta\left(\bar{\tau}^{K}\right)
$$

where $\Theta^{L}_{C} > 0$ if and only if $\bar{\tilde{G}} < 1-\lambda\frac{\theta-1}{\theta}\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^{K}_{new}\right)-(1-\lambda)\frac{\theta-1}{\theta}\frac{1-\bar{\tau}^{H}}{1+\bar{\tau}^{C}}$.

Proof. Let $\bar{H}^{L}_{new}$ and $\bar{H}^{C}_{new}$ denote the steady-state hours after the capital tax changes in the labor tax adjustment case and in the consumption tax adjustment case, respectively. Then, we get

$$
\begin{aligned}
\frac{\bar{H}^{L}_{new}}{\bar{H}^{C}_{new}} &= \left(\frac{\frac{\bar{\omega}}{(1-\lambda)\bar{mc}}\frac{1+\bar{\tau}^{C}}{1-\bar{\tau}^{H}_{new}}\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^{K}_{new}\right)\right)}{\frac{\bar{\omega}}{(1-\lambda)\bar{mc}}\frac{1+\bar{\tau}^{C}_{new}}{1-\bar{\tau}^{H}}\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{mc}(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^{K}_{new}\right)\right)}\right)^{-\frac{1}{1+\varphi}} \\
&= \left(\frac{1+\bar{\tau}^{C}_{new}}{1+\bar{\tau}^{C}}\frac{1-\bar{\tau}^{H}_{new}}{1-\bar{\tau}^{H}}\right)^{\frac{1}{1+\varphi}} \\
&= \left(\left(1+\frac{\Delta\left(\bar{\tau}^{C}\right)}{1+\bar{\tau}^{C}}\right)\left(1-\frac{\Delta\left(\bar{\tau}^{H}\right)}{1-\bar{\tau}^{H}}\right)\right)^{\frac{1}{1+\varphi}} \\
&= \left(\left(1-\frac{1+\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\bar{\tau}^{C}}{1+\bar{\tau}^{C}}\frac{\lambda\bar{mc}}{\left(1-\bar{\tilde{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^{K}_{new}\right)}\Delta\left(\bar{\tau}^{K}\right)\right)\left(1+\frac{1+\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\bar{\tau}^{C}}{1-\bar{\tau}^{H}}\frac{\lambda}{1-\lambda}\Delta\left(\bar{\tau}^{K}\right)\right)\right)^{\frac{1}{1+\varphi}}
\end{aligned}
$$

For small changes in capital tax rate $\Delta\left(\bar{\tau}^{K}\right)$, we get

$$
\begin{aligned}
\ln\left(\frac{\bar{H}^{L}_{new}}{\bar{H}^{C}_{new}}\right) &= \frac{1}{1+\varphi}\left(\frac{\Delta\left(\bar{\tau}^{C}\right)}{1+\bar{\tau}^{C}} - \frac{\Delta\left(\bar{\tau}^{H}\right)}{1-\bar{\tau}^{H}}\right) \\
&= \frac{1}{1+\varphi}\left(1+\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\bar{\tau}^{C}\right)\left(\frac{1}{1-\bar{\tau}^{H}}\frac{\lambda}{1-\lambda} - \frac{1}{1+\bar{\tau}^{C}}\frac{\lambda\bar{mc}}{\left(1-\bar{\tilde{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^{K}_{new}\right)}\right)\Delta\left(\bar{\tau}^{K}\right) \\
&= \frac{1}{1+\varphi}\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{1+\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\bar{\tau}^{C}}{1-\bar{\tau}^{H}}\right)\left(1-\frac{1-\bar{\tau}^{H}}{1+\bar{\tau}^{C}}\frac{(1-\lambda)\,\bar{mc}}{\left(1-\bar{\tilde{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^{K}_{new}\right)}\right)\Delta\left(\bar{\tau}^{K}\right)
\end{aligned}
$$

Then, for the levels of output, consumption, capital and investment, the differences are the same: that is,

$$
\begin{aligned}
\ln\left(\frac{\bar{\tilde{Y}}^{L}_{new}}{\bar{\tilde{Y}}^{C}_{new}}\right) = \ln\left(\frac{\bar{\tilde{C}}^{L}_{new}}{\bar{\tilde{C}}^{C}_{new}}\right) = \ln\left(\frac{\bar{\tilde{K}}^{L}_{new}}{\bar{\tilde{K}}^{C}_{new}}\right) = \ln\left(\frac{\bar{\tilde{I}}^{L}_{new}}{\bar{\tilde{I}}^{C}_{new}}\right) = \ln\left(\frac{\bar{H}^{L}_{new}}{\bar{H}^{C}_{new}}\right) &= \frac{1}{1+\varphi}\left(\frac{\Delta\left(\bar{\tau}^{C}\right)}{1+\bar{\tau}^{C}} - \frac{\Delta\left(\bar{\tau}^{H}\right)}{1-\bar{\tau}^{H}}\right) \\
&= \frac{1}{1+\varphi}\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{1+\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\bar{\tau}^{C}}{1-\bar{\tau}^{H}}\right)\left(1-\frac{1-\bar{\tau}^{H}}{1+\bar{\tau}^{C}}\frac{(1-\lambda)\,\bar{mc}}{\left(1-\bar{\tilde{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}\left(1-\bar{\tau}^{K}_{new}\right)}\right)\Delta\left(\bar{\tau}^{K}\right) \\
&= \mathcal{M}^{L}_{C}\Delta\left(\bar{\tau}^{K}\right).
\end{aligned}
$$

Notice that $\mathcal{M}^{L}_{C} = \frac{1}{1+\varphi}\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{1+\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\bar{\tau}^{C}}{1-\bar{\tau}^{H}}\right)\left(1-\frac{1-\bar{\tau}^{H}}{1+\bar{\tau}^{C}}\frac{(1-\lambda)\bar{mc}}{\left(1-\bar{\tilde{G}}\right)-\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\lambda\bar{mc}(1-\bar{\tau}^{K}_{new})}\right) > 0$ if

$$
\begin{aligned}
\bar{\tilde{G}} &< 1-\lambda\frac{\theta-1}{\theta}\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^{K}_{new}\right)-(1-\lambda)\frac{\theta-1}{\theta}\frac{1-\bar{\tau}^{H}}{1+\bar{\tau}^{C}} \\
&\approx 1-0.4976 = 0.5023
\end{aligned}
$$

in our baseline calibration. In this case, capital tax cut is more expansionary in the consumption tax rate adjustment case than in the labor tax rate adjustment case. □

C.10 Changes in Output with Infinite Frisch Elasticity

How do the changes in output to the capital tax rate vary with the different Frisch elasticity parameters under labor tax rate adjustment case? Notice that from (84), we get

$$
\begin{aligned}
\ln\left(\frac{\bar{\tilde{Y}}_{new}}{\bar{\tilde{Y}}}\right) &= \ln\left(\frac{\bar{\tilde{y}}_{new}}{\bar{\tilde{y}}}\right) + \ln\left(\frac{\bar{H}_{new}}{\bar{H}}\right). \\
&= -\left[\left(\frac{\lambda}{1-\lambda}\frac{1}{1-\bar{\tau}^{K}}\right) - \mathcal{M}_{H}\right]\Delta\left(\bar{\tau}^{K}\right)
\end{aligned}
$$

where $\mathcal{M}_H = \dfrac{\frac{1}{1+\varphi}\left(\frac{\lambda}{(1-\lambda)}\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)\left[\bar{\tilde{T}}+\frac{\frac{\bar{a}^{\eta}}{\beta}-(1-d)}{(\bar{a}-(1-d))}\left(1-\bar{\tilde{G}}-\frac{\bar{m}c(\bar{a}-(1-d))}{\frac{\bar{a}^{\eta}}{\beta}-(1-d)}\right)\right]}{(1-\bar{\tau}^H)\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{m}c(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}(1-\bar{\tau}^K)\right)}$ and $\bar{\tilde{T}} = \bar{\tilde{T}}^C + \bar{\tilde{T}}^H + \bar{\tilde{T}}^K$. Then, we can rewrite $\ln\left(\frac{\bar{\tilde{Y}}_{new}}{\bar{\tilde{Y}}}\right)$ as

$$-\frac{\ln\left(\frac{\bar{\tilde{Y}}_{new}}{\bar{\tilde{Y}}}\right)}{\Delta\left(\bar{\tau}^K\right)} = \frac{\lambda}{1-\lambda}\left(\frac{1}{1-\bar{\tau}^K} - \frac{\frac{1}{1+\varphi}\left(\frac{\lambda}{(1-\lambda)}\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)\left[\bar{\tilde{T}}+\frac{\frac{\bar{a}}{\beta}-(1-d)}{(\bar{a}-(1-d))}\left(1-\bar{\tilde{G}}-\frac{\bar{m}c(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\right)\right]}{\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{m}c(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right)\left(1-\bar{\tau}^H\right)}\right).$$

Notice that under Assumption 2, the second term in the RHS is positive. Thus, $-\frac{\ln\left(\frac{\bar{\tilde{Y}}_{new}}{\bar{\tilde{Y}}}\right)}{\Delta(\bar{\tau}^K)}$ is increasing in φ and it has a lower bound at $\frac{1}{\varphi}=\infty$. That is, for $\Delta\left(\bar{\tau}^K\right)<0$,

$$\frac{\lambda}{1-\lambda}\left[\frac{1}{1-\bar{\tau}^K} - \frac{\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(\bar{\tilde{T}}^C+\bar{\tilde{T}}^H+\bar{\tilde{T}}^K-\bar{m}c\right)+\left(1-\bar{\tilde{G}}\right)}{\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{m}c(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^K\right)\right)\left(1-\bar{\tau}^H\right)}\right] \leq -\frac{\ln\left(\frac{\bar{\tilde{Y}}_{new}}{\bar{\tilde{Y}}}\right)}{\Delta\left(\bar{\tau}^K\right)} < \frac{\lambda}{1-\lambda}\left(\frac{1}{1-\bar{\tau}^K}\right).$$

The lower bound is:

$$\begin{aligned}
&\frac{\lambda}{1-\lambda}\left(\frac{1}{1-\bar{\tau}^K} - \frac{\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(\bar{\tilde{T}}^C+\bar{\tilde{T}}^H+\bar{\tilde{T}}^K-\bar{m}c\right)+\left(1-\bar{\tilde{G}}\right)}{\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{m}c(\bar{a}-(1-d))(1-\bar{\tau}^K)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\left(1-\bar{\tau}^H\right)}\right)\\
=&\frac{\lambda}{1-\lambda}\left(\frac{1}{1-\bar{\tau}^K} - \frac{\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\left(\bar{\tau}^C\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}+\bar{\tau}^H\left(1-\lambda\right)\bar{m}c+\bar{\tau}^K\lambda\bar{m}c-\bar{m}c\right)+\left(1-\bar{\tilde{G}}\right)}{\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{m}c(\bar{a}-(1-d))(1-\bar{\tau}^K)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\left(1-\bar{\tau}^H\right)}\right)\\
=&\frac{\left(\frac{1-\bar{\tau}^H}{1-\bar{\tau}^K}-\left(1+\frac{(\bar{a}-(1-d))\bar{\tau}^C}{\frac{\bar{a}}{\beta}-(1-d)}\right)\right)\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{m}c(\bar{a}-(1-d))\left(1-\bar{\tau}^K\right)}{\frac{\bar{a}}{\beta}-(1-d)}\right)+\frac{(\bar{a}-(1-d))(1-\lambda)\bar{m}c}{\frac{\bar{a}}{\beta}-(1-d)}\left(1-\bar{\tau}^H\right)}{\frac{1-\lambda}{\lambda}\left(1-\bar{\tilde{G}}-\frac{\lambda\bar{m}c(\bar{a}-(1-d))(1-\bar{\tau}^K)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\left(1-\bar{\tau}^H\right)}\\
=&\frac{\lambda}{1-\lambda}\left[\frac{1}{1-\bar{\tau}^K}-\frac{1+\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\bar{\tau}^C}{1-\bar{\tau}^H}+\left(\frac{\bar{a}-(1-d)}{\frac{\bar{a}}{\beta}-(1-d)}\right)\frac{(1-\lambda)\bar{m}c}{\left(\frac{\bar{\tilde{C}}}{\bar{\tilde{Y}}}\right)}\right]\\
=&\frac{\lambda}{1-\lambda}\left(\frac{1}{1-\bar{\tau}^H}\right)\left(\left(1+\frac{1-\lambda}{\lambda}\frac{\bar{\tilde{I}}}{\bar{\tilde{C}}}\right)\frac{1-\bar{\tau}^H}{1-\bar{\tau}^K}-\left(1+\frac{(\bar{a}-(1-d))}{\frac{\bar{a}}{\beta}-(1-d)}\bar{\tau}^C\right)\right)
\end{aligned}$$

Thus, the change in output is positive even under the infinite Frisch elasticity if $\frac{1-\bar{\tau}^H}{1-\bar{\tau}^K} > 1+\bar{\tau}^C$.

C.11 Analytical Results with Capital-Skill Complementarity

Equipment and Structure Capital Rental Rates

From equations (52) and (53), we have

$$\frac{\partial\bar{r}^{K,e}}{\partial\bar{\tau}^K} = \frac{\bar{r}^{K,e}}{1-\bar{\tau}^K} > 0,\ \frac{\partial\bar{r}^{K,b}}{\partial\bar{\tau}^K} = \frac{\bar{r}^{K,b}}{1-\bar{\tau}^K} > 0.$$

Equipment Capital to Skilled Labor Ratio

We can combine equations (57) and (58) to get

$$\left(\bar{r}^{K,e}\right)^{\frac{1}{1-\alpha}}\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{1-\rho}=\frac{\lambda\left(1-\mu\right)\left(\frac{1-\alpha}{\alpha}\right)\left(\alpha\bar{mc}\right)^{\frac{1}{1-\alpha}}}{\left(q_0\frac{\frac{\gamma}{\beta}-(1-d_b)}{\frac{1}{\beta}-(1-d_e)}\right)^{\frac{\alpha}{1-\alpha}}}\left(\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho}+(1-\lambda)\right)^{\frac{\sigma}{\rho}-1}$$
$$\times\left[\mu\left(\frac{\bar{L}_u}{\bar{L}_s}\right)^{\sigma}+(1-\mu)\left(\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho}+(1-\lambda)\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\sigma}{\sigma}}.$$

Combining equations (55), (56) and (63), we derive unskilled to skilled labor ratio ($\bar{L}_u/\bar{L}_s$),

$$\left(\frac{\bar{L}_u}{\bar{L}_s}\right)=\left(\frac{(1-\lambda)(1-\mu)}{\mu}\frac{\tilde{\Lambda}_{s,u}}{\frac{\bar{\omega}^s}{\bar{\omega}^u}\left(\frac{N^u}{N^s}\right)^{\varphi}}\frac{\left(1-\lambda^s_{\tau^H}\bar{\tau}^H\right)}{\left(1-\lambda^u_{\tau^H}\bar{\tau}^H\right)}\left(\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho}+(1-\lambda)\right)^{\frac{\sigma}{\rho}-1}\right)^{\frac{1}{\sigma-1-\varphi}}$$
$$=\left(\frac{(1-\lambda)(1-\mu)}{\mu}\frac{\tilde{\Lambda}_{s,u}}{\frac{\bar{\omega}^s}{\bar{\omega}^u}\left(\frac{N^u}{N^s}\right)^{\varphi}}\frac{\left(1-\lambda^s_{\tau^H}\bar{\tau}^H\right)}{\left(1-\lambda^u_{\tau^H}\bar{\tau}^H\right)}\right)^{\frac{1}{\sigma-\varphi-1}}\left(\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho}+(1-\lambda)\right)^{\frac{(\sigma-\rho)}{\rho(\sigma-\varphi-1)}}.$$

Let's combine the above two equations to get a nonlinear equation which determines steady-state equipment capital to skilled labor ratio ($\bar{\tilde{K}}_e/\bar{L}_s$) which is a function of equipment capital rental rate ($\bar{r}^{K,e}$),

$$\left(\bar{r}^{K,e}\right)^{\frac{1}{1-\alpha}}=\chi_A\left[\mu\chi_C\left(F\right)^{\frac{\sigma(\sigma-\rho)}{\rho(\sigma-\varphi-1)}}+(1-\mu)\left(F\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1-\sigma}{\sigma}}\left(F\right)^{\frac{\sigma}{\rho}-1}\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho-1} \tag{86}$$
$$=\chi_A\left[\mu\chi_C\left(F\right)^{\frac{\sigma}{\rho}\left(\frac{\varphi(\sigma-\rho)}{(1-\sigma)(1-\sigma+\varphi)}\right)}+(1-\mu)\left(F\right)^{\frac{\sigma}{\rho}\left(\frac{1-\rho}{1-\sigma}\right)}\right]^{\frac{1-\sigma}{\sigma}}\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho-1}$$

where

$$\chi_A=\frac{\lambda\left(1-\mu\right)\left(\frac{1-\alpha}{\alpha}\right)\left(\alpha\bar{mc}\right)^{\frac{1}{1-\alpha}}}{\left(q_0\frac{\frac{\gamma}{\beta}-(1-d_b)}{\frac{1}{\beta}-(1-d_e)}\right)^{\frac{\alpha}{1-\alpha}}}>0$$
$$\chi_C=\left(\frac{(1-\lambda)(1-\mu)}{\mu}\frac{\tilde{\Lambda}_{s,u}}{\frac{\bar{\omega}^s}{\bar{\omega}^u}\left(\frac{N^u}{N^s}\right)^{\varphi}}\frac{\left(1-\lambda^s_{\tau^H}\bar{\tau}^H\right)}{\left(1-\lambda^u_{\tau^H}\bar{\tau}^H\right)}\right)^{\frac{\sigma}{\sigma-\varphi-1}}>0$$

and

$$F\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)=\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho}+(1-\lambda)>0.$$

Taking a partial derivative of (86) with respect to capital tax rate $\bar{\tau}^K$, we have

$$
\begin{aligned}
\frac{1}{1-\alpha}\left(\bar{r}^{K,e}\right)^{\frac{\alpha}{1-\alpha}}\frac{\partial\left(\bar{r}^{K,e}\right)}{\partial\bar{\tau}^K} =& \frac{(1-\sigma)\left(\bar{r}^{K,e}\right)^{\frac{1}{1-\alpha}}}{F}\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho-1}\frac{\partial}{\partial\bar{\tau}^K}\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)\\
&\times\left(\frac{\left(\frac{\varphi(\sigma-\rho)}{(1-\sigma)(1-\sigma+\varphi)}\right)\mu\chi_C\left(F\right)^{\frac{\sigma}{\rho}\left(\frac{\varphi(\sigma-\rho)}{(1-\sigma)(1-\sigma+\varphi)}\right)}+\left(\frac{1-\rho}{1-\sigma}\right)(1-\mu)\left(F\right)^{\frac{\sigma}{\rho}\left(\frac{1-\rho}{1-\sigma}\right)}}{\mu\chi_C\left(F\right)^{\frac{\sigma}{\rho}\left(\frac{\varphi(\sigma-\rho)}{(1-\sigma)(1-\sigma+\varphi)}\right)}+(1-\mu)\left(F\right)^{\frac{\sigma}{\rho}\left(\frac{1-\rho}{1-\sigma}\right)}}\right)\\
&+(\rho-1)\left(\bar{r}^{K,e}\right)^{\frac{1}{1-\alpha}}\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{-1}\frac{\partial}{\partial\bar{\tau}^K}\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)\\
\frac{1}{1-\alpha}\frac{\partial\left(\bar{r}^{K,e}\right)}{\partial\bar{\tau}^K} =& -\frac{\bar{r}^{K,e}}{F}\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{-1}\frac{\partial}{\partial\bar{\tau}^K}\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)\\
&\times\left[\left(\frac{(1-\sigma)\frac{(1-\rho+\varphi)}{(1-\sigma+\varphi)}\mu\chi_C\left(F\right)^{\frac{\sigma}{\rho}\left(\frac{\varphi(\sigma-\rho)}{(1-\sigma)(1-\sigma+\varphi)}\right)}}{\mu\chi_C\left(F\right)^{\frac{\sigma}{\rho}\left(\frac{\varphi(\sigma-\rho)}{(1-\sigma)(1-\sigma+\varphi)}\right)}+(1-\mu)\left(F\right)^{\frac{\sigma}{\rho}\left(\frac{1-\rho}{1-\sigma}\right)}}\right)\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho}+(1-\rho)(1-\lambda)\right]
\end{aligned}
$$

Thus

$$
\begin{aligned}
\frac{\partial}{\partial\bar{\tau}^K}\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right) =& -\frac{F}{1-\alpha}\frac{1}{\bar{r}^{K,e}}\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\left[(1-\sigma)\frac{(1-\rho+\varphi)}{(1-\sigma+\varphi)}\left(\frac{\mu\chi_C}{\mu\chi_C+(1-\mu)F^{\frac{\sigma}{\rho}\left(\frac{1-\rho+\varphi}{1-\sigma+\varphi}\right)}}\right)\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho}+(1-\rho)(1-\lambda)\right]^{-1}\frac{\partial\left(\bar{r}^{K,e}\right)}{\partial\bar{\tau}^K}\\
<& \ 0.
\end{aligned}
$$

Unskilled to skilled labor

Combining equations (55), (56) and (63), we derive unskilled to skilled labor ratio ($\bar{L}_u/\bar{L}_s$),

$$
\begin{aligned}
\left(\frac{\bar{L}_u}{\bar{L}_s}\right) &= \left(\frac{(1-\lambda)(1-\mu)}{\mu}\frac{\tilde{\Lambda}_{s,u}}{\frac{\bar{\omega}^s}{\bar{\omega}^u}\left(\frac{N^u}{N^s}\right)^{\varphi}}\frac{\left(1-\lambda^s_{\tau^H}\bar{\tau}^H\right)}{\left(1-\lambda^u_{\tau^H}\bar{\tau}^H\right)}\right)^{\frac{1}{\sigma-\varphi-1}}\left(\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho}+(1-\lambda)\right)^{\frac{(\sigma-\rho)}{\rho(\sigma-\varphi-1)}}\\
&= \chi_c^{\frac{1}{\sigma}}\left(\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho}+(1-\lambda)\right)^{\frac{(\sigma-\rho)}{\rho(\sigma-\varphi-1)}}
\end{aligned}
\tag{87}
$$

where $\chi_c = \left(\frac{(1-\lambda)(1-\mu)}{\mu}\frac{\tilde{\Lambda}_{s,u}}{\frac{\bar{\omega}^s}{\bar{\omega}^u}\left(\frac{N^u}{N^s}\right)^{\varphi}}\frac{\left(1-\lambda^s_{\tau^H}\bar{\tau}^H\right)}{\left(1-\lambda^u_{\tau^H}\bar{\tau}^H\right)}\right)^{\frac{\sigma}{\sigma-\varphi-1}} > 0$. Then,

$$
\begin{aligned}
\frac{\partial}{\partial\bar{\tau}^K}\left(\frac{\bar{L}_u}{\bar{L}_s}\right) &= -\left(\frac{\sigma-\rho}{1-\sigma+\varphi}\right)\frac{\bar{L}_u}{\bar{L}_s}\left(\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho}+(1-\lambda)\right)^{-1}\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho-1}\frac{\partial}{\partial\bar{\tau}^K}\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)\\
&> 0
\end{aligned}
$$

Skill premium

Combining equations (55), (56) and (87), we derive the skill premium

$$
\begin{aligned}
\frac{\bar{\tilde{w}}^s}{\bar{\tilde{w}}^u} &= \frac{(1-\lambda)(1-\mu)}{\mu}\frac{\left(\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho}+(1-\lambda)\right)^{\frac{\sigma}{\rho}-1}}{\left(\frac{\bar{L}_u}{\bar{L}_s}\right)^{\sigma-1}}\\
&= \frac{(1-\lambda)(1-\mu)}{\mu}\chi_c^{\frac{1-\sigma}{\sigma}}\left(F\right)^{\frac{\sigma-\rho}{\rho}\left(\frac{\varphi}{1-\sigma+\varphi}\right)}
\end{aligned}
$$

where $\chi_c = \left(\frac{(1-\lambda)(1-\mu)}{\mu} \frac{\tilde{\Lambda}_{s,u}}{\frac{\bar{\omega}^s}{\bar{\omega}^u}\left(\frac{N^u}{N^s}\right)^{\varphi}} \frac{\left(1-\lambda^s_{\tau^H}\bar{\tau}^H\right)}{\left(1-\lambda^u_{\tau^H}\bar{\tau}^H\right)} \right)^{\frac{\sigma}{\sigma-\varphi-1}} > 0$ and $F\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right) = \lambda \left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho} + (1-\lambda) > 0.$
Then,

$$\begin{aligned} \frac{\partial}{\partial \bar{\tau}^K}\left(\frac{\bar{\tilde{w}}^s}{\bar{\tilde{w}}^u}\right) &= (\sigma-\rho)\left(\frac{\varphi}{1-\sigma+\varphi}\right)\frac{\bar{\tilde{w}}^s}{\bar{\tilde{w}}^u}\frac{1}{F}\lambda\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right)^{\rho-1}\frac{\partial}{\partial \bar{\tau}^K}\left(\frac{\bar{\tilde{K}}_e}{\bar{L}_s}\right) \\ &< 0. \end{aligned}$$

D Data Appendix

We calibrate the steady-state fiscal variables using US quarterly data for the post-Volcker period from 1982:Q4 to 2008:Q2.

D.1 Debt and spending data

We use the following definitions for our debt and spending variables:

- Government debt = market value of privately held gross federal debt;
- Government expenditures = government consumption;

Note that we use a single price level, GDP deflator, for both variables.

The market value of privately held gross federal debt series was obtained from Federal Reserve Bank of Dallas and the government consumption data series was taken from National Income and Product Accounts (NIPA) tables.

D.2 Tax data

We follow a method originally based on Jones (2002). Additionally, we use the tax revenues of the federal government and local property taxes.

We use federal taxes on production and imports (lines 4 of NIPA Table 3.2) for consumption tax revenues. Let this be T^C.

The average personal income tax rate is computed to get both capital tax revenues and labor tax revenues. We first compute the average personal income tax rate as

$$\tau^P = \frac{IT}{W + PRI/2 + CI}$$

where IT is the personal current tax revenues (line 3 of NIPA Table 3.2), W is wage and salary accruals (line 3 of NIPA Table 1.12), PRI is proprietor's income (line 9 of NIPA Table 1.12), and CI is capital income, which is the sum of rental income (line 12 of NIPA Table 1.12), corporate profits (line 13 of NIPA Table 1.12), interest income (line 18 of NIPA Table 1.12), and $PRI/2$. We here regard half of proprietor's income as wage labor income and the other half as capital income.

Then the capital tax revenue is

$$T^K = \tau^P CI + CT + PT$$

where CT is taxes on corporate income (line 7 of NIPA Table 3.2), and PT is property taxes (line 8 of NIPA Table 3.3). In NIPA, home owners are thought of as renting their houses to themselves and thus property taxes are included as taxes on rental income or capital income. The labor tax revenue is computed

$$T^H = \tau^P \left(W + PRI/2\right) + CSI$$

where CSI is contributions for government social insurance (line 11 of NIPA Table 3.2).

E Estimation of Labor Tax Adjustment Rule

The labor tax rate adjustment rule is specified as the following:

$$\begin{aligned}\tau_t^H - \bar{\tau}_{new}^H &= \rho_1^H \left(\tau_{t-1}^H - \bar{\tau}_{new}^H\right) + \rho_2^H \left(\tau_{t-2}^H - \bar{\tau}_{new}^H\right) \\ &\quad + \left(1 - \rho_1^H - \rho_2^H\right) \left\{ \psi_B^H \left(\frac{B_{t-1}}{P_{t-1} Y_{t-1}} - \overline{\frac{B}{PY}} \right) + \psi_{\Delta Y}^H \left(\frac{Y_t}{Y_{t-1}} \right) + \psi_x^H \left(\frac{Y_t}{Y_t^n} \right) \right\} \end{aligned} \tag{88}$$

where Y_t^n is the natural level of output. We estimate an empirical version of this rule by OLS. We first estimate the composite coefficients $\left(1 - \rho_1^H - \rho_2^H\right) \psi_B^H$, $\left(1 - \rho_1^H - \rho_2^H\right) \psi_{\Delta Y}^H$, and $\left(1 - \rho_1^H - \rho_2^H\right) \psi_x^H$ and then recover ψ_B^H, $\psi_{\Delta Y}^H$, and ψ_x^H using the estimate of ρ_1^H and ρ_2^H. Quarterly US data is used for estimation: tax revenues-to-output ratio, market value of government debt-to-output ratio, output growth and the gap between actual output and potential output. The rule is estimated on two sub-periods. The first sub-sample covers the period from 1983Q1 through 2002Q4 as in Coibion and Gorodnichenko (2011) and the second sub-sample covers the period from 2001Q1 through 2019Q3. For the first sub-sample, we drop the second lag of the tax revenues to ensure stationarity of the tax rule. The data is taken from FRED of the Federal Reserve Bank of St. Louis. Potential output is real potential gross domestic product estimated by US Congressional Budget Office. Table A.2 shows the estimation results.

F Appendix Table and Figures

Table A.1: Calibration for a Heterogeneous Households Model

	Value	Description	References
Households			
β	0.9975	Time preference	Smets and Wouters (2007)
φ	1.0	Inverse of Frisch elasticity of labor supply	Smets and Wouters (2007)
$\bar{\omega}^s$	4.70	Labor supply disutility parameter	Steady-state $\bar{H}^s = 0.33$
$\bar{\omega}^u$	9.89		Steady-state $\bar{H}^u = 0.31$
N^s	0.5	Fraction of skilled labor	Lindquist (2004)
d_e	0.031	Equipment capital depreciation	Krusell et al. (2000)
d_b	0.014	Structures capital depreciation	Krusell et al. (2000)
ξ	4.0	Investment adjustment cost	Smets and Wouters (2007)
$\frac{\mathcal{A}''}{\mathcal{A}'}$	0.85	Elasticity of cost of capital utilization	Smets and Wouters (2007)
Firms			
σ	0.401	Elasticity of substitution between unskilled labor and equipment	Krusell et al. (2000)
ρ	-0.495	Elasticity of substitution between skilled labor and equipment	Krusell et al. (2000)
α	0.117	Structures capital Income share	Krusell et al. (2000)
λ	0.35	Equipment capital income share	Steady-state labor share: 56% (Elsby, Hobijn and Şahin 2013)
μ	0.345	Unskilled labor income share	Steady-state skill premium: 60% (Krusell et al. 2000)
γ	1.0054	Long-run growth rate of output	Smets and Wouters (2007)
$\bar{\pi}$	1.0078	Steady-state inflation rate	Smets and Wouters (2007)
θ	4.0	Elasticity of substitution between goods	Steady-state markup: 33%
q_0	0.95	Relative price of structure to equipment capital	Maliar and Maliar (2011)
α_P	0.65	Calvo sticky price parameter	Smets and Wouters (2007)
γ_P	0.22	Degree of price indexation	Smets and Wouters (2007)
Government(Fiscal/Monetary Policy): Transfer or Labor Tax Rate Adjustment			
$\bar{\bar{b}}$	0.363	Steady-state debt to GDP ratio	Data (See Appendix D)
$\bar{\bar{G}}$	0.161	Steady-state government spending to GDP ratio	Data (See Appendix D)
$\bar{\bar{T}}^C$	0.009	Steady-state consumption tax revenue to GDP ratio	Data (See Appendix D)
$\bar{\bar{T}}^H$	0.128	Steady-state labor tax revenue to GDP ratio	Data (See Appendix D)
$\lambda^s_{\tau^H}$	1.0	Effective rate of labor tax on skilled workers	Assigned
$\lambda^u_{\tau^H}$	1.0	Effective rate of labor tax on unskilled workers	Assigned
λ_b	0.338	Effective expensing rate of structure investment	Barro and Furman (2018)
λ_e	0.812	Effective expensing rate of equipment investment	Barro and Furman (2018)
χ^s_Φ	1	Fraction of profit distribution to skilled worker	Assigned
χ^s_S	0	Fraction of transfer to skilled worker	Assigned
ρ^R_1	1.12	Interest rate smoothing parameter lag 1	Coibion and Gorodnichenko (2011)
ρ^R_2	-0.18	Interest rate smoothing parameter lag 2	Coibion and Gorodnichenko (2011)
ϕ_π	1.58	Inflation feedback parameter under Taylor rule	Coibion and Gorodnichenko (2011)
ϕ_x	0.11	Output gap feedback parameter under Taylor rule	Coibion and Gorodnichenko (2011)
$\phi_{\Delta y}$	2.21	Output growth feedback parameter under Taylor rule	Coibion and Gorodnichenko (2011)
ρ^S_1	0.869	Transfers smoothing parameter lag 1 (Transfer Adjustment)	Assigned
ρ^S_1	0.0	Transfers smoothing parameter lag 2 (Transfer Adjustment)	Assigned
ψ^S_B	-0.111	Transfers response to debt	Assigned
$\psi^S_{\Delta y}$	0.831	Transfers response to output growth	Assigned
ρ^H_1	0.869	Labor tax rate smoothing parameter lag 1	Estimated (See Appendix E)
ρ^H_2	0.0	Labor tax rate smoothing parameter lag 2	Estimated (See Appendix E)
ψ^H_B	0.111	Labor tax rate response to debt	Estimated (See Appendix E)
$\psi^H_{\Delta y}$	0.831	Labor tax rate response to output growth	Estimated (See Appendix E)
ψ^H_x	0.0	Labor tax rate response to output gap	Estimated (See Appendix E)

Table A.2: Estimation Results for Labor Tax Rate Adjustment Rules

	(1) Sample (1983Q1-2002Q4)	(2) Sample (2001Q1-2019Q3)
ρ_1^H	0.869	0.785
	(0.075)	(0.171)
ρ_2^H		0.107
		(0.168)
ψ_B^H	0.111	0.007
	(0.107)	(0.069)
$\psi_{\Delta Y}^H$	0.831	1.821
	(0.633)	(1.473)
ψ_x^H	0.032	0.040
	(0.035)	(0.099)
constant	0.001	-0.002
	(0.003)	(0.001)
R^2	0.832	0.853
Observations	79	75

Notes: The table shows OLS estimates of the labor tax rate adjustment rule (88). Quarterly US data is used for estimation: tax revenues-to-output ratio, market value of government debt-to-output ratio, output growth and the gap between actual output and potential output. Column (1) shows the estimation results using the sample from from 1983Q1 through 2002Q4 and column (2) shows the estimation results using the sample from 2001Q1 through 2019Q3. For the column (1), we drop the second lag of the tax revenues to ensure stationarity of the tax rule. See Appendix E for details.

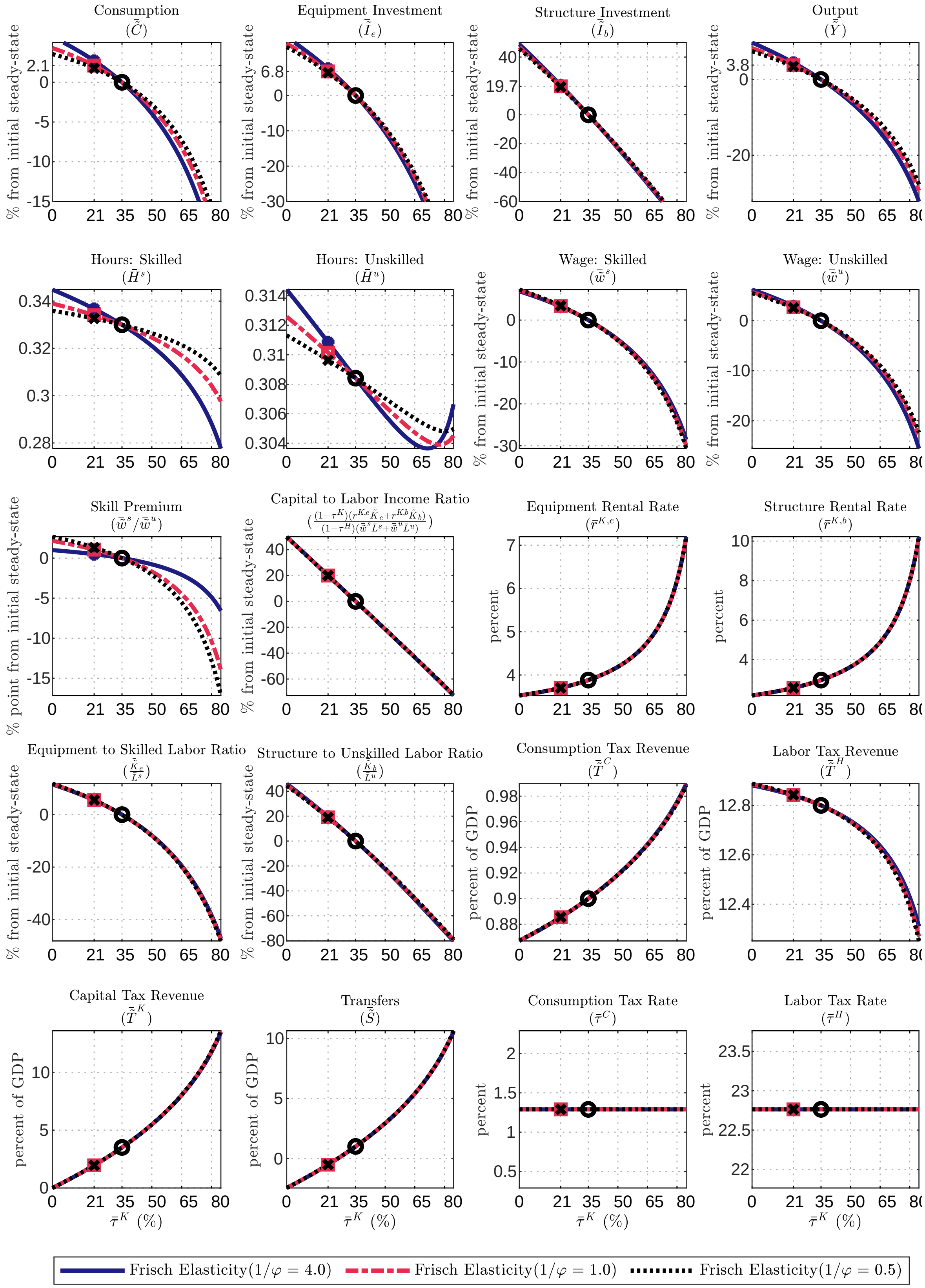


Figure A.1: Long-run Effects of Permanent Capital Tax Rate Changes under Transfer Adjustment with Different Frisch Elasticity

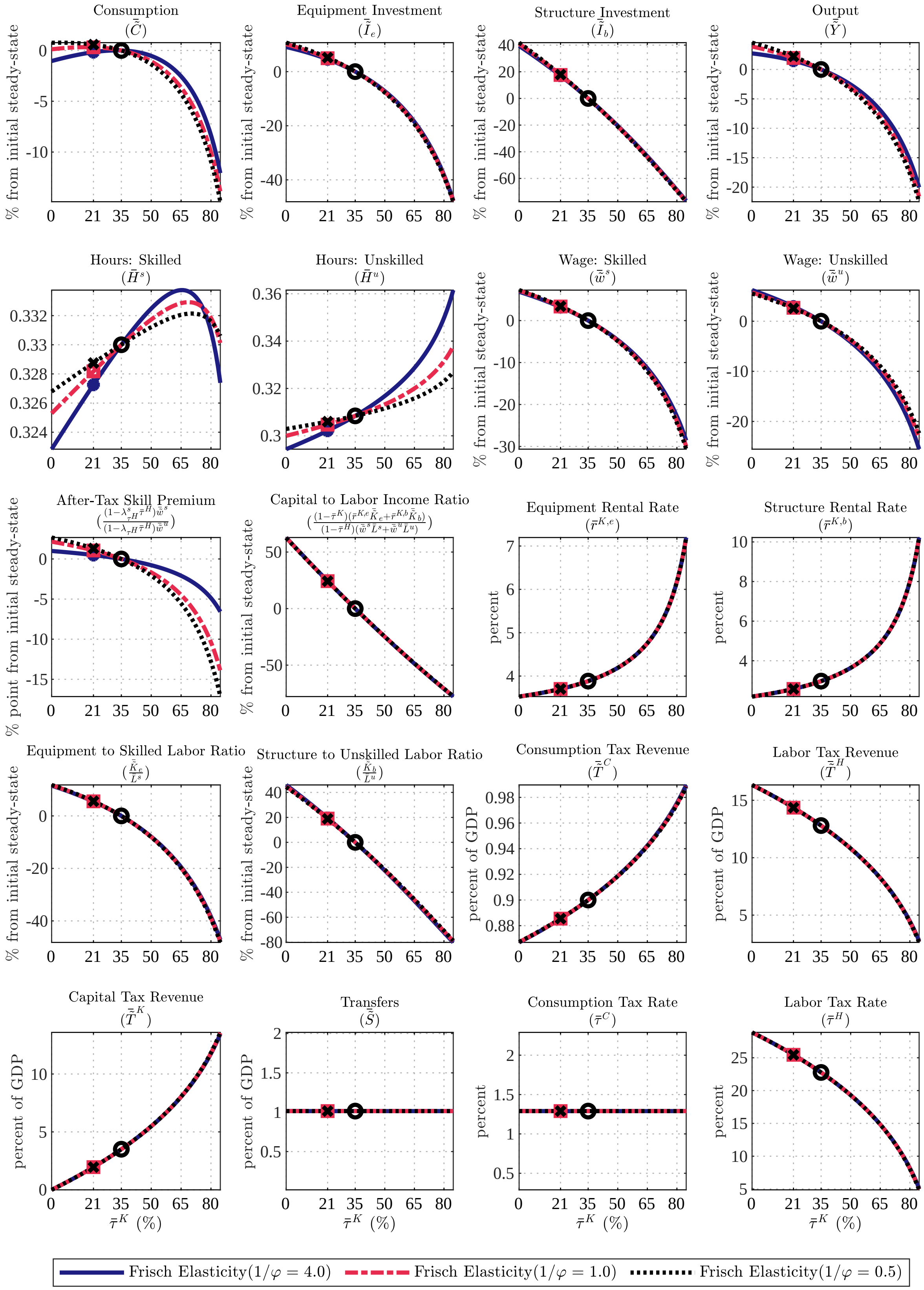


Figure A.2: Long-run Effects of Permanent Capital Tax Rate Changes under Labor Tax Rate Adjustment with Different Frisch Elasticity

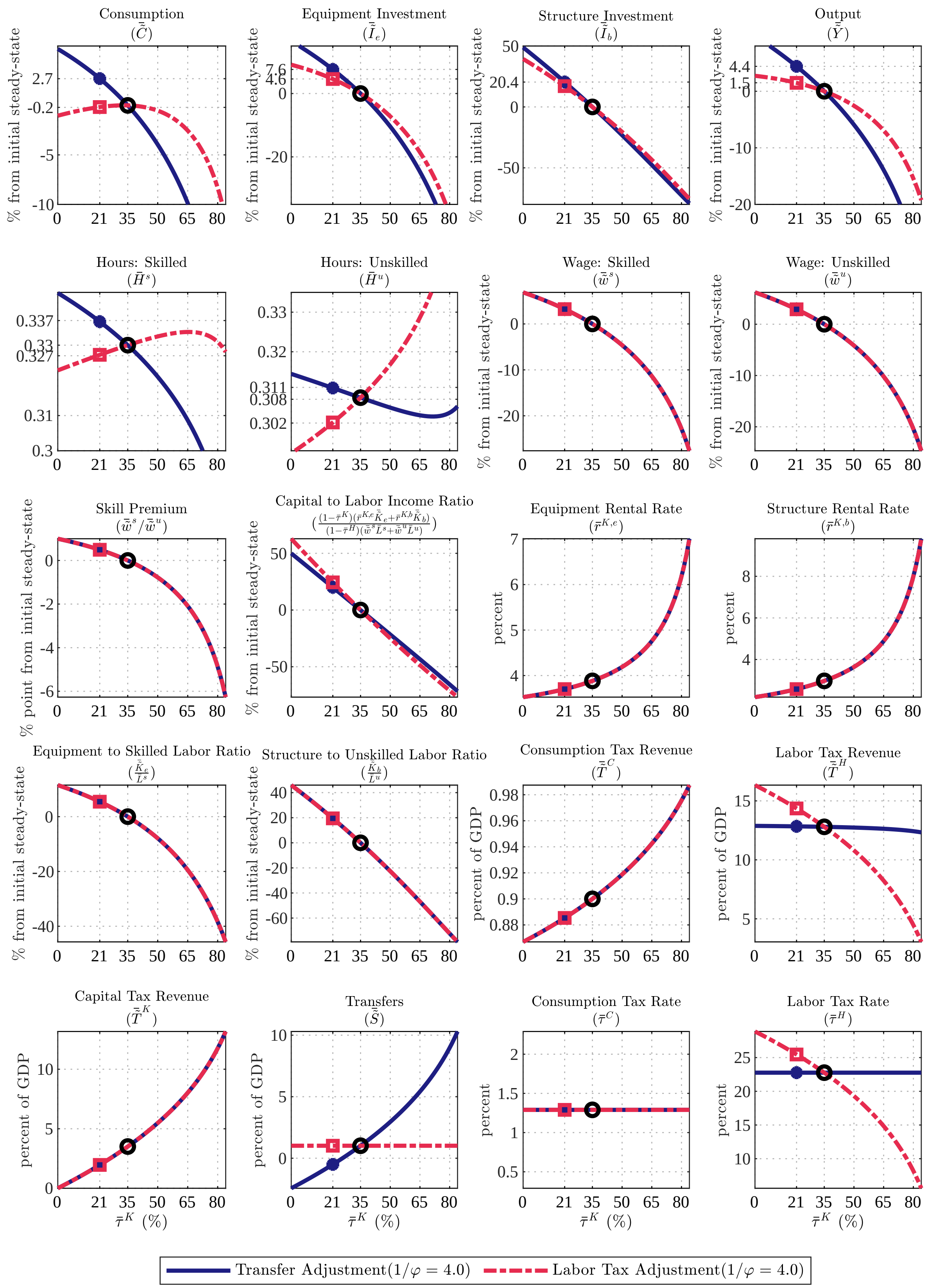


Figure A.3: Long-run Effects of Permanent Capital Tax Rate Changes under Transfer Adjustment and Labor Tax Rate Adjustment

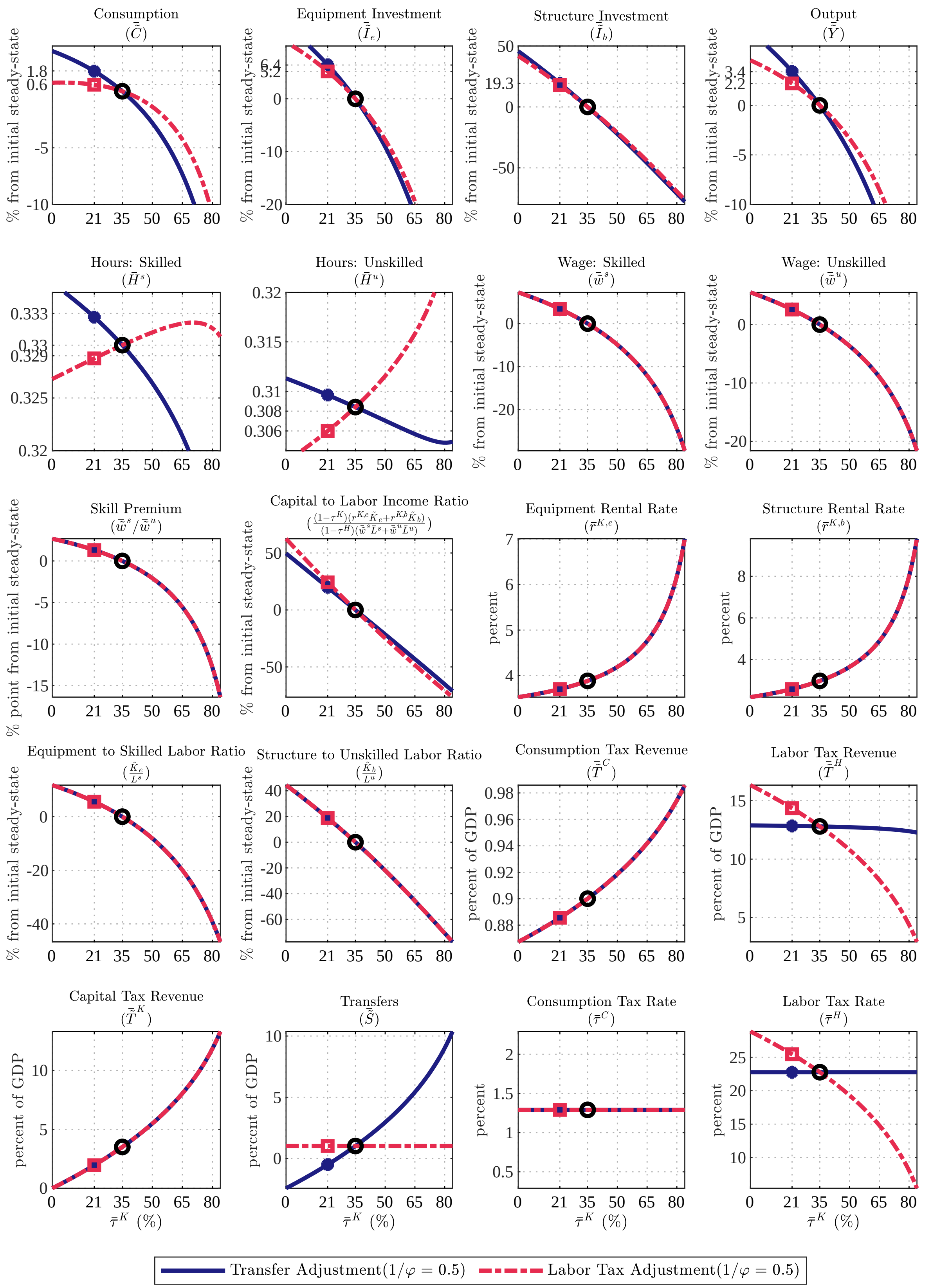


Figure A.4: Long-run Effects of Permanent Capital Tax Rate Changes under Transfer Adjustment and Labor Tax Rate Adjustment

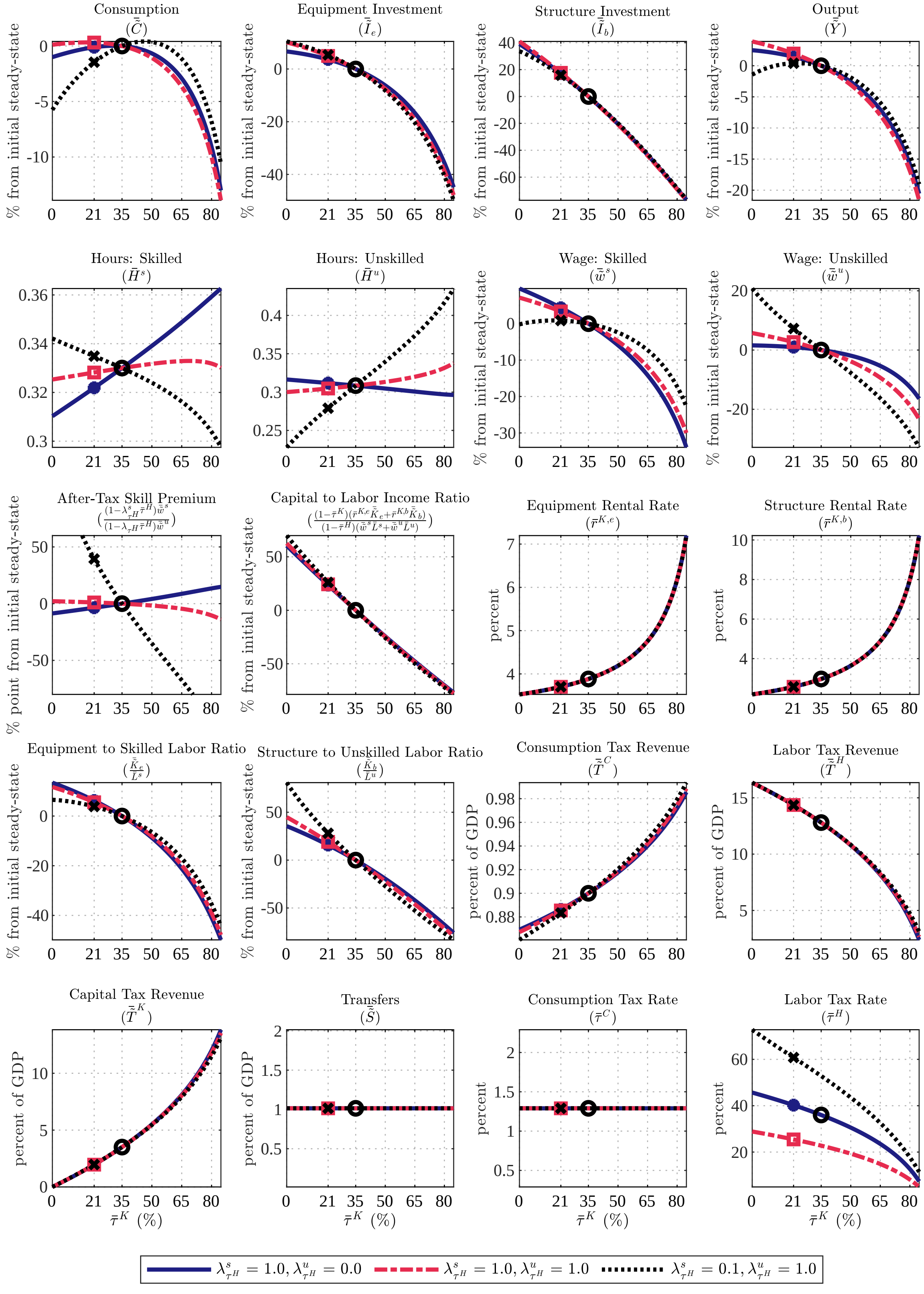


Figure A.5: Long-run Effects of Permanent Capital Tax Rate Changes (Progressiveness of Labor Taxes)

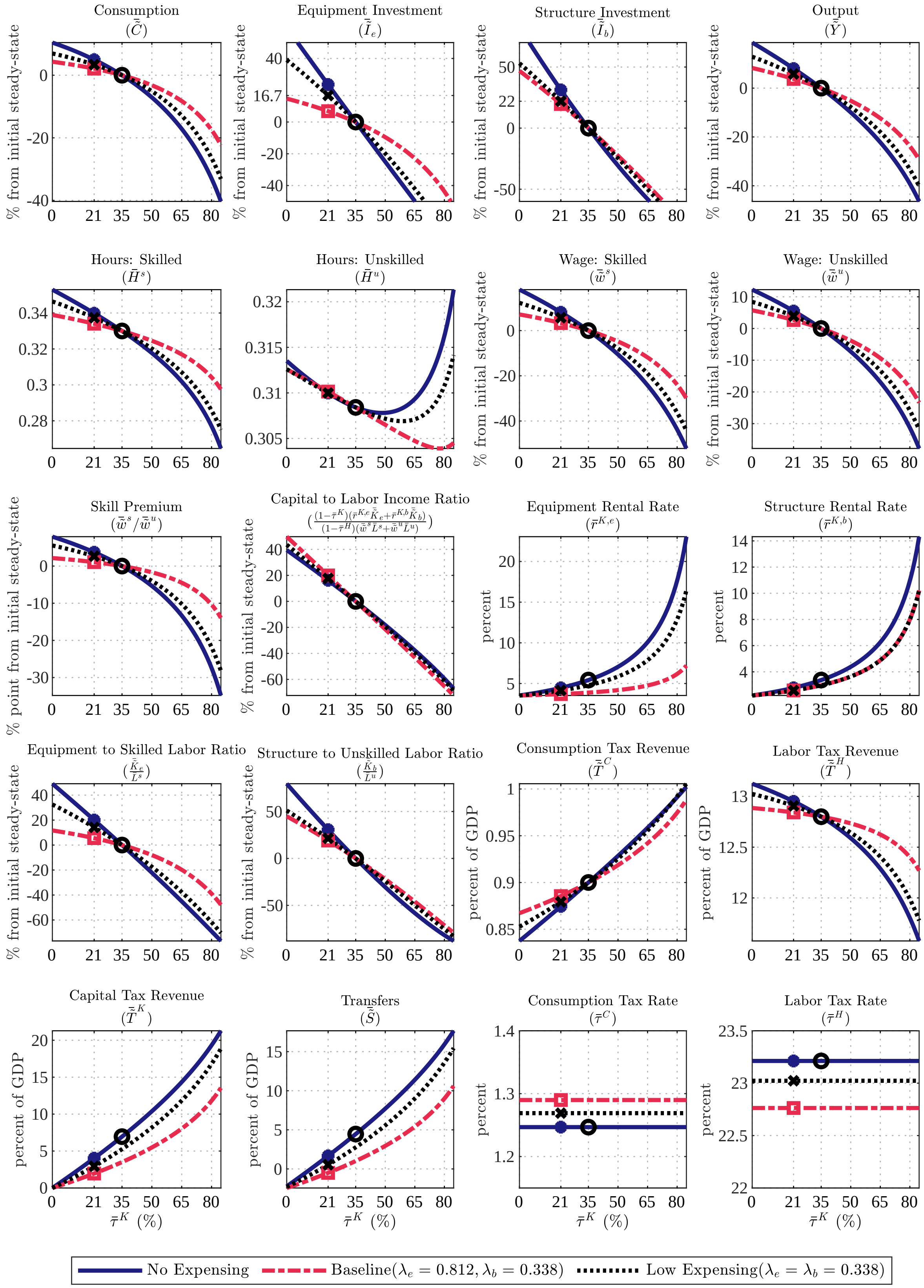


Figure A.6: Long-run Effects of Permanent Capital Tax Rate Changes Under Transfer Adjustment with Different Expensing Rules

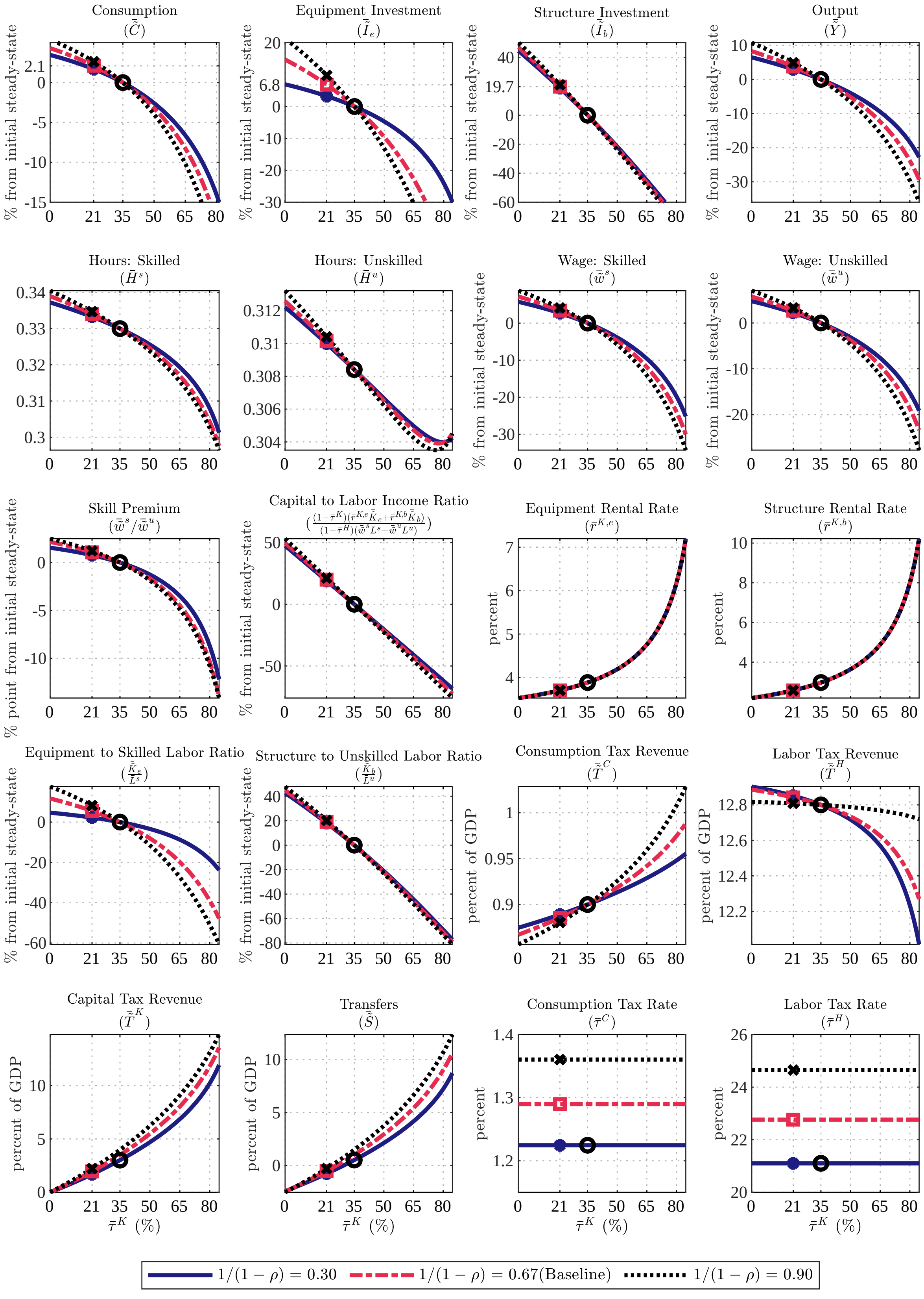


Figure A.7: Long-run Effects of Permanent Capital Tax Rate Changes under Transfer Adjustment with Different Elasticity of Substitution between Skilled and Equipment Capital

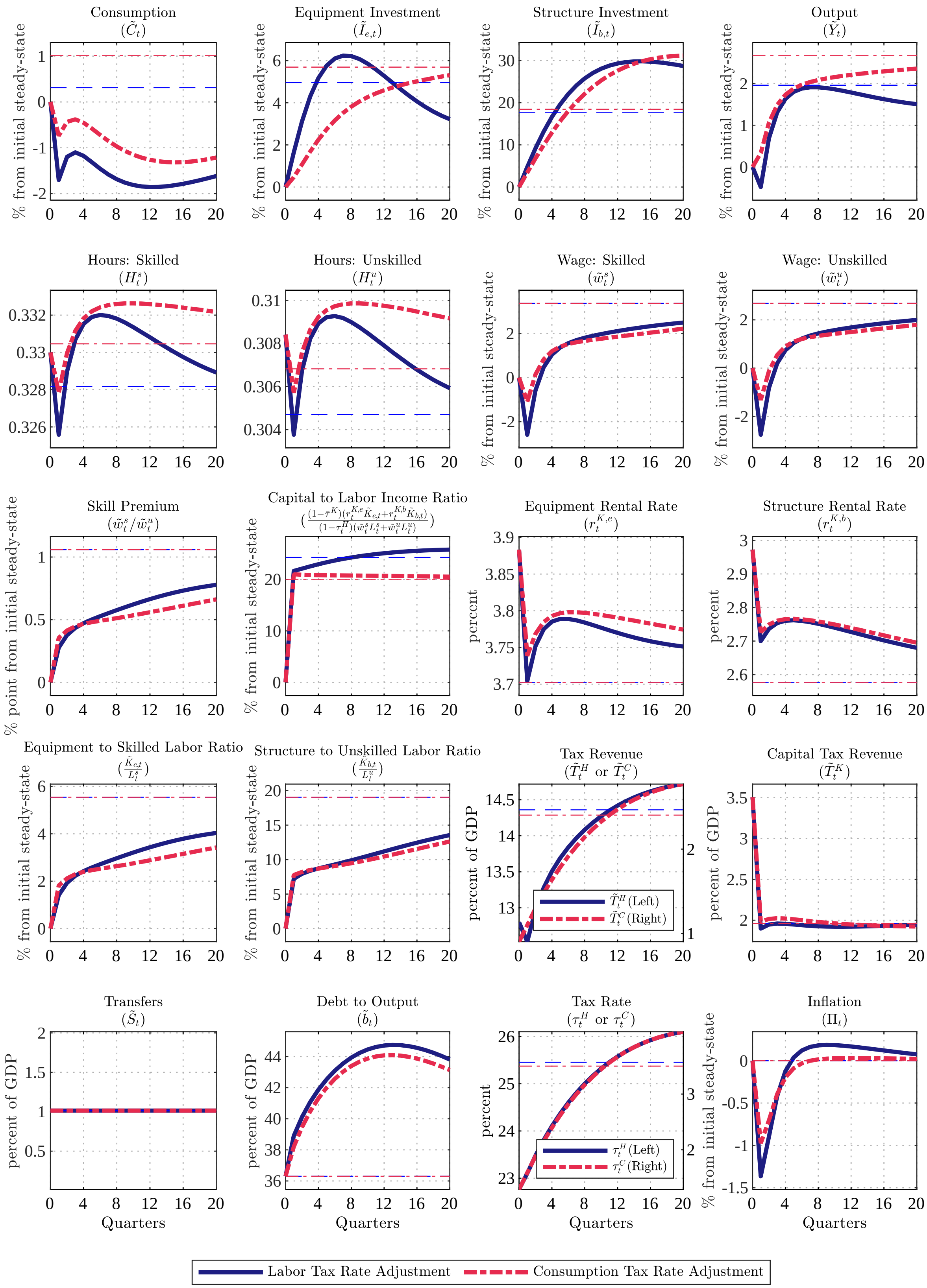


Figure A.8: Transition Dynamics Under Labor Tax Rate and Consumption Tax Rate Adjustment

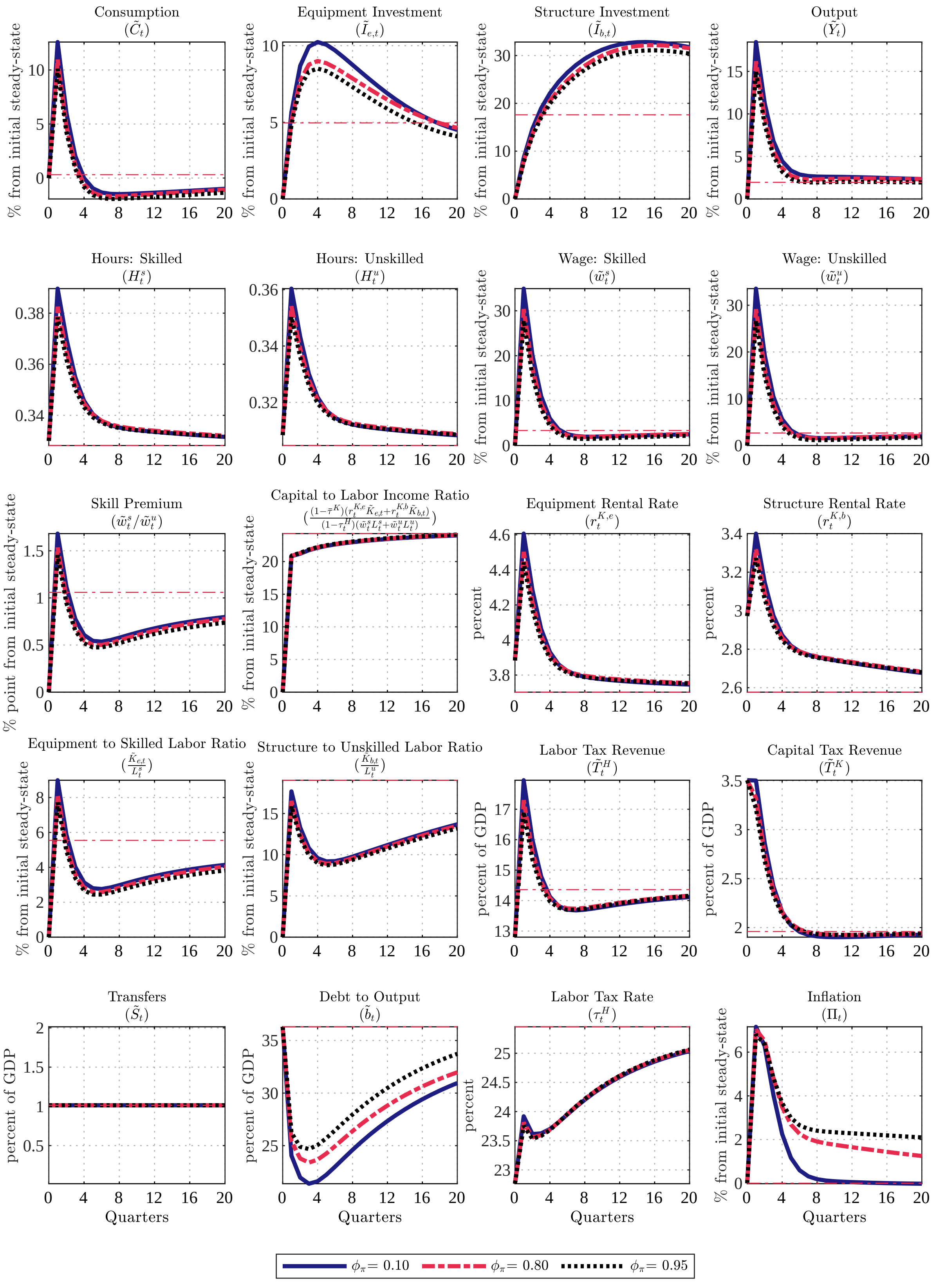
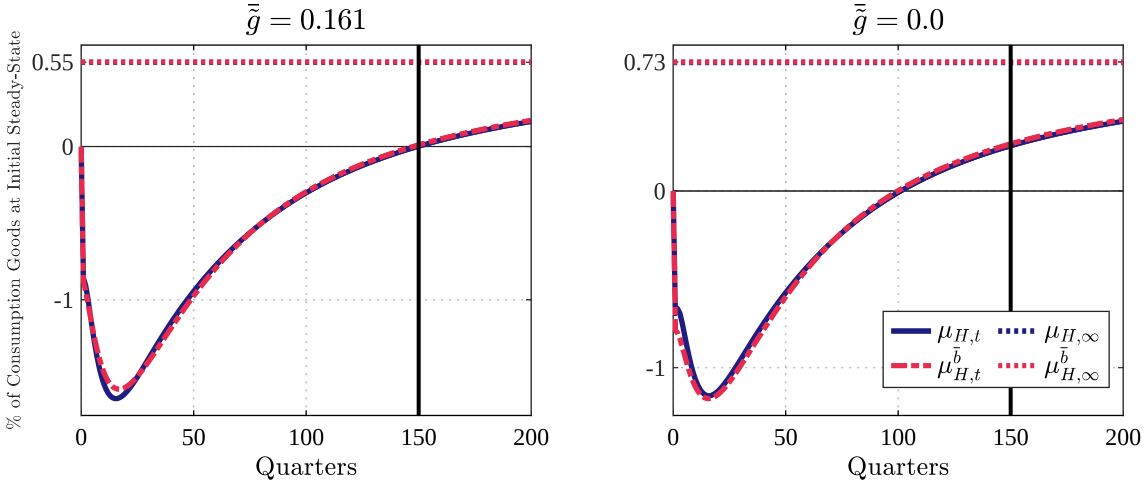


Figure A.9: Transition Dynamics Under Labor Tax Rate Adjustment with Different Inflation Feedback Parameters

(a) Aggregate Welfare Gains in the Baseline Model with Labor Tax Rate Adjustment



(b) Aggregate Welfare Gains in the Baseline Model with Labor and Consumption Tax Rate Adjustment

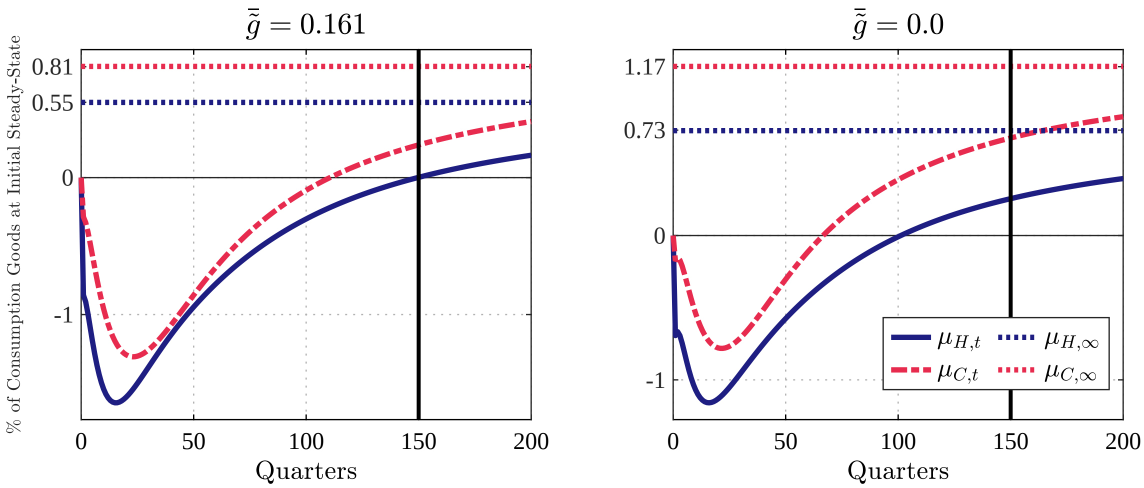
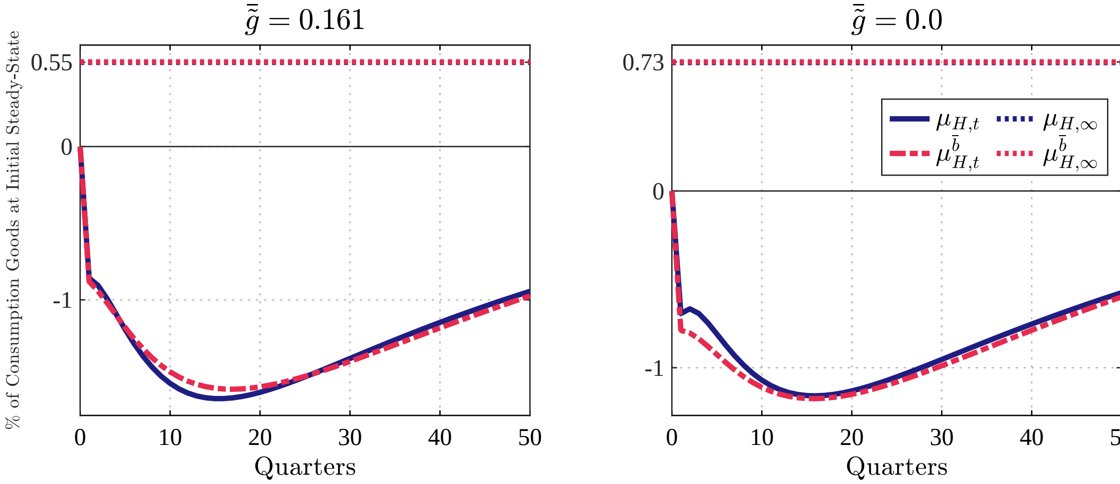


Figure A.10: Welfare Implications of a Permanent Capital Tax Rate Decrease

(a) Aggregate Welfare Gains in the Baseline Model with Labor Tax Rate Adjustment



(b) Aggregate Welfare Gains in the Baseline Model with Labor and Consumption Tax Rate Adjustment

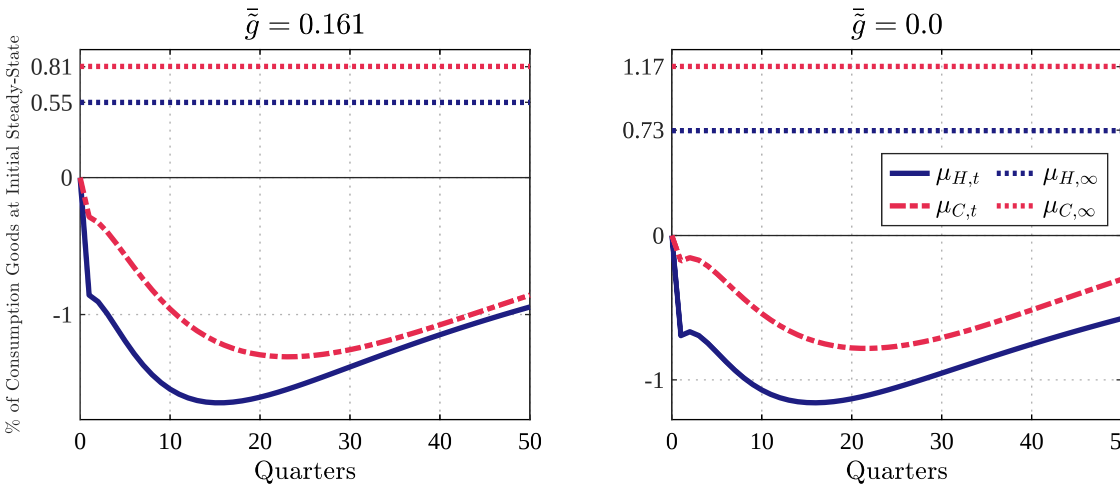


Figure A.11: Welfare Implications of a Permanent Capital Tax Rate Decrease

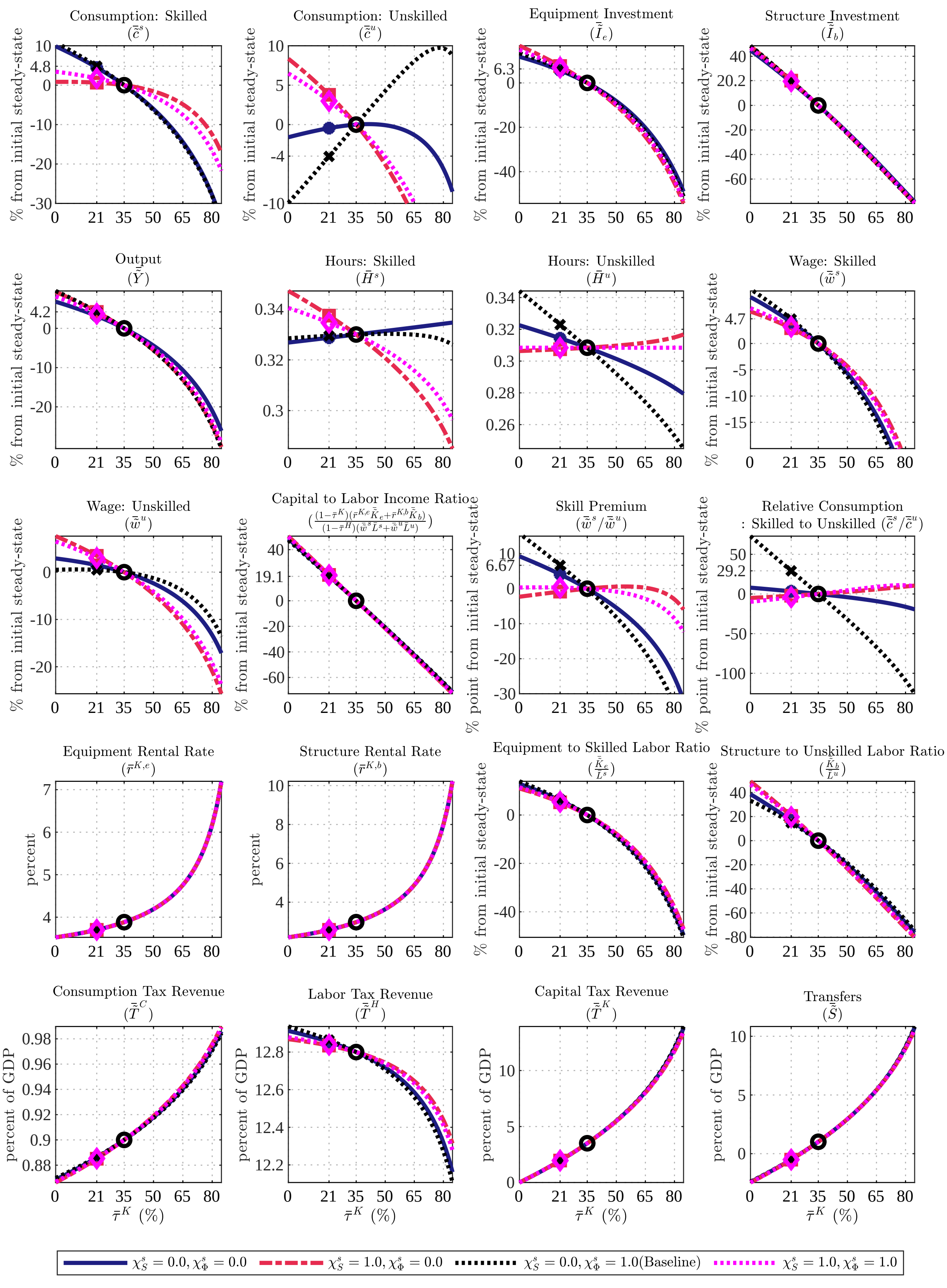


Figure A.12: Long-run Effects of Permanent Capital Tax Rate Changes (Incomplete Market Model with Different Profits and Transfer Distribution Rules)

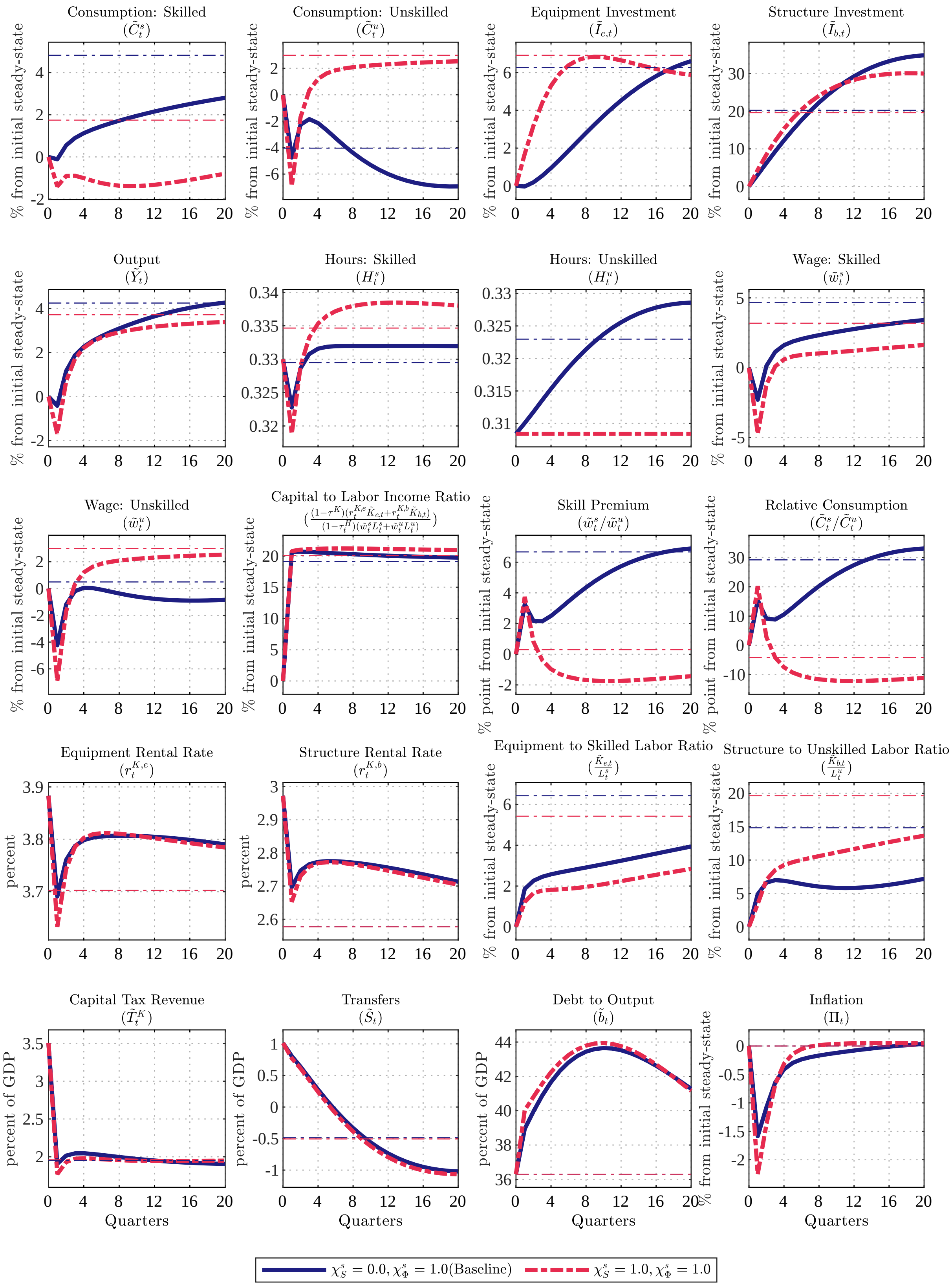


Figure A.13: Transition Dynamics Under Transfer Adjustment with Different Profit and Transfers Distributions