Rational Inattention, Menu Costs, and Multi-Product Firms: Micro Evidence and Aggregate Implications*

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Abstract

How does a firm's product scope affect its decisions regarding price setting and information acquisition? Using a firm-level survey from New Zealand, I show that firms that produce more goods have both better information about aggregate inflation and more frequent but smaller price changes. To characterize the aggregate implications of these empirical findings, I develop a general equilibrium menu cost model with rationally inattentive multi-product firms. I show that the interaction of the menu cost and rational inattention frictions leads to a new selection effect: price adjusters choose to be better informed than non-adjusters. This selection effect endogenously generates a leptokurtic distribution of desired price changes, which amplifies monetary non-neutrality and leads to large real effects of monetary shocks comparable to a Calvo model. Compared to a one-product baseline, the effects are 12% smaller in a two-product model, as firms with more products will optimally choose to have better information.

JEL Classification: E31, E32, E37, E52, E70

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1 Introduction

The way firms set their prices and form their expectations has important implications for the transmission of monetary policy shocks. The extensive empirical literature studying detailed microdata on firms' beliefs and pricing behavior has found that many firms are not fully aware of macroeconomic conditions and change their prices infrequently.[1] Economic models have embedded realistic features of price setting and firms' expectations formation, such as costly price adjustments and a lack of awareness of economic conditions, to study the ability of monetary policy to stimulate the economy. Most of those models, however, assume firms produce only one product.[2] This assumption can be justified if multi-product firms act approximately like a collection of single-product firms, a view that has been difficult to challenge due to a dearth of empirical evidence regarding multi-product firms' economic decisions. A second justification is that there are significant computational challenges in integrating multi-product pricing into menu cost and rational inattention models. This paper challenges both justifications by first providing new empirical evidence that multi-product firms price their products and form their expectations differently and then developing a new theoretical model in which multi-product pricing under both nominal and informational rigidities has important implications for monetary policy transmission.

How are firms' price-setting and information acquisition decisions related to their product scope? Using a detailed survey of firms' pricing and information acquisition decisions, I first show that multi-product firms change their prices more frequently (and by less) and are more informed about the economy on average. These results imply that it is necessary to jointly consider the scope of the products sold by firms when formalizing their pricing and information acquisition decisions. The second part of this paper then investigates how firms' product scopes affect the transmission of monetary policy shocks when both adjusting prices and acquiring information are costly. To do so, I develop a general equilibrium model disciplined by micro-evidence and seek answers to the following two theoretical questions. First, how do economies of scope in multi-product firms quantitatively affect monetary policy transmission? Second, regardless of firms' product scopes, are there any new insights about how the interaction between nominal and informational rigidities affects monetary non-neutrality?

[1] See, for instance, Klenow and Malin (2010) and Nakamura and Steinsson (2008, 2013) for comprehensive reviews about micro price stickiness. See Kumar et al. (2015), Coibion et al. (2018a), Boneva et al. (2019), and Afrouzi (2019b) for evidence on pervasive inattention on the firm side to macroeconomic variables.

[2] Notable exceptions are Midrigan (2011) and Alvarez and Lippi (2014), who study menu cost models with multi-product firms, and Pasten and Schoenle (2016), who build a rational inattention model with multi-product firms.

This paper builds on a growing literature studying how firms form their expectations and process information. For example, Kumar et al. (2015), Coibion et al. (2018a), and Afrouzi (2019b) study various factors that determine how much attention firms devote to tracking macroeconomic conditions.[3] My new empirical contribution to this literature is to document that the number of products that firms produce is an important determinant of firm-level inattention to macroeconomic conditions: firms with a greater product scope have better information about aggregate economic conditions. Moreover, I investigate how firms' product scopes affect their price-setting decisions and show that firms with a greater product scope have more frequent but smaller price changes. The joint characterization of the relationship between firms' product scopes and their decisions regarding both price setting and information acquisition complements previous literature studying microdata on multi-product pricing (e.g. Lach and Tsiddon, 1996; Bhattarai and Schoenle, 2014; Stella, 2018; Bonomo et al., 2019a).

This paper also contributes to our understanding of monetary non-neutrality by using a new general equilibrium model with both nominal and informational rigidities in a world of multi-product pricing. This model builds on two strands of monetary models with multi-product firms to capture micro-evidence on economies of scope. First, menu cost models with multi-product firms, such as Midrigan (2011) and Alvarez and Lippi (2014), exhibit economies of scope in price setting: firms with a greater number of products have more frequent but smaller price changes, as they can change the prices of all their products by paying a single fixed cost. Second, rational inattention models with multi-product firms, such as Pasten and Schoenle (2016), exhibit economies of scope in information processing: firms with a greater number of products have a large incentive to acquire and process information about aggregate shocks. My theoretical contribution is to combine all three elements—menu costs, rational inattention, and multi-product firms—in a unified framework to study the ability of monetary policy to stimulate the economy.[4] This model, disciplined by micro-evidence, serves to quantitatively study how monetary non-neutrality is affected 1) by firms' product scopes and 2) by the interaction between menu costs and rational inattention regardless of firms' product scopes.

My starting point is to explore the empirical characteristics of multi-product firms in terms of their price-setting and information acquisition decisions. To do so, I use a representative survey of New Zealand firms' macroeconomic beliefs. Inattention among firms to macroeconomic

[3] See also Frache and Lluberas (2019), Grasso and Ropele (2018), and Coibion et al. (2019).

[4] Appendix Figure A.1 shows how my model fits within the literature on menu costs and rational inattention models.

conditions is pervasive. For example, on average, firms overstate aggregate inflation over the previous 12 months by 4.5 percentage points. Moreover, firms in the survey change their prices infrequently, and there is a great deal of heterogeneity in the frequency and size of price changes. For example, the expected duration until subsequent price changes, on average, is 12 months, though this varies substantially across sectors.

My empirical analysis documents that firms' product scopes are systematically related to their inattention to macroeconomic conditions and price-setting decisions. First, I find that firms with a greater number of goods are better informed about aggregate inflation. Firms make systematically smaller errors about recent values of aggregate inflation when they produce a greater number of products. Moreover, firms with a greater product scope are also willing to pay more for information about future inflation. This finding implies that firms with more products have incentives to process more information about macroeconomic conditions. To the best of my knowledge, these results are the first empirical evidence documenting differential information acquisition decisions of firms based on the number of products they sell. Second, I show that firms with a greater number of products have more frequent but smaller price changes. This finding is consistent with the previous empirical literature that used different micro price data.[5] Jointly, these results illustrate that the scope of products sold by firms affects both their information acquisition and price-setting decisions.[6]

What are the aggregate implications of the micro-evidence that I show above for the ability of monetary policy to stimulate the economy? To answer this question, I develop a new model that captures the behavior of firms in the survey. Specifically, I assume that it is costly for firms to observe and process information regarding underlying shocks. Firms optimally choose their information set given the costs of information. This captures the pervasive inattention among firms in the survey. Second, firms have to pay a fixed menu cost to reset their prices, which leads to the infrequent price changes observed in the survey data. I further assume that this fixed cost is independent of how many prices firms change. This assumption introduces economies of scope in price setting: firms with greater product scope change their prices more frequently and by smaller amounts because the average costs of changing prices are lower for them. Finally, I assume that firms face two types of shocks—idiosyncratic good-specific shocks and aggregate

[5] Using the U.S. PPI microdata, Bhattarai and Schoenle (2014) find that multi-product firms change their prices more frequently and by smaller amounts. Parker (2017) reports similar findings using New Zealand data. Stella (2018) estimates common menu costs for multi-product firms and finds substantial economies of scope in price setting.

[6] These results are robust to controlling for firm-level characteristics, such as firms' sizes and ages, and industry fixed effects. See Section 2 for details.

monetary shocks—and the marginal cost of processing information is independent of the firms' number of products. This assumption introduces economies of scope in information processing: firms with a greater product scope want to learn more about aggregate monetary shocks because information about them can be used to price all their goods.[7]

I embed this setup of firm decision making into a full-fledged dynamic general equilibrium model and study the macroeconomic implications for monetary non-neutrality. I use three target moments from the survey data to discipline the model parameters: the frequency and size of price changes and the slope of the backcast error curve on the number of products.[8] The first two help calibrate the menu cost parameter and the size of idiosyncratic shocks, while the third helps calibrate the informational cost parameter. I focus on two key questions with this general equilibrium model. First, I explore how the interaction between rational inattention and menu costs affects firms' optimal decisions and, therefore, how the economy responds to monetary shocks, regardless of firms' product scopes. To do this, I compare the output responses to monetary shocks in the one-good version of my model with those in the standard menu-cost-only model, such as Golosov and Lucas (2007). Second, I show how firms' product scopes affect monetary non-neutrality through economies of scope in multi-product firms by comparing the macroeconomic dynamics to monetary shocks in the one-good vs. two-good versions of my model.

My first theoretical finding is that the baseline one-good version of the model generates large real effects of monetary policy shocks that are seven times larger than those in the standard menu cost model and nearly as large as those in the Calvo sticky price model. Standard menu cost models have small and short-lived real effects of monetary shocks due to the strong selection effects of price changes: an expansionary monetary shock triggers numerous price increases that originate from far below the average level and offset many price decreases.[9] The extent of the selection effects depends on the underlying distribution of firms' desired price changes. When the distri-

[7]This model nests the baseline menu cost and rational inattention models as special cases. For example, if there is no informational cost, this model coincides with the menu-cost-only models with either single-product firms (e.g., Golosov and Lucas, 2007) or multi-product firms (e.g., Midrigan, 2011; Alvarez and Lippi, 2014). If there is no menu cost, this model nests the rational-inattention-only models with either single-product firms (e.g., Maćkowiak and Wiederholt, 2009) or multi-product firms (e.g., Pasten and Schoenle, 2016), with one main difference. While the previous literature solves the rational inattention problem by assuming that the cost of information is not discounted and by optimizing at the long-run steady state for the information structure, firms in my model discount future costs of information at the same discount rate as their payoffs and solve the dynamic rational inattention problem. See Section 3 for details.

[8]The slope of the backcast error curve captures the magnitude of the decrease in firm-level errors regarding recent values of the growth rate of nominal GDP if the number of products that firms produce increases by one unit.

[9]Gagnon et al. (2013) study the effect of large inflationary shocks on the timing of price changes using Mexican CPI data and find direct support for a selection effect. Carvalho and Kryvtsov (2018) find evidence of strong price selection across goods and services using detailed micro-level consumer price data for the UK, the US, and Canada.

bution is Gaussian, many prices are clustered around the adjustment margins, leading to large price selection effects. Previous menu cost models that try to explain the strong non-neutrality of money found in the data have often assumed that the distribution of idiosyncratic shocks has excess kurtosis and a fat tail, so the majority of desired price changes are near zero while some of them are very far from zero (e.g., Gertler and Leahy, 2008; Midrigan, 2011; Vavra, 2013; Karadi and Reiff, 2019; Baley and Blanco, 2019). My new contribution to this literature is to show that the interaction between menu costs and rational inattention frictions can *endogenously* generate a distribution of firms' desired price changes with excess kurtosis. I show that this leptokurtic distribution of desired price changes weakens the selection effects of price changes, amplifying the impact response of output to monetary shocks by 23% in my baseline one-good version of the model compared with standard menu cost models with fully informed firms.

How does the interaction between nominal and informational rigidities generate the leptokurtic distribution of desired price changes? As highlighted in previous models with both information and nominal rigidities, when both acquiring information and adjusting prices are costly, firms' optimal price-setting rules depend on their own subjective uncertainty (e.g. Gorodnichenko, 2008; Afrouzi, 2019a). Firms with large uncertainty regarding underlying shocks have a wider inaction region for pricing decisions, as they want to wait and see until they get more information to resolve their uncertainty.[10] This leads to another kind of selection effect regarding information processing: price adjusters have better information than non-adjusters. A new finding of this paper is that this selection effect in information processing endogenously leads to a leptokurtic distribution of firms' desired price changes, with the majority near zero and some far away from zero.

Consider an economy with a large number of one-product firms.[11] At the beginning of each period, firms' prior beliefs about their price gaps are all within their inaction bands, implying a high kurtosis in the distribution of *prior* price gaps. After being hit by Gaussian idiosyncratic shocks, the distribution of *true* price gaps is Gaussian. Firms then choose their information sets and update their estimates of the price gaps, but they do not do so in the same way. Firms that think it is unlikely that they will need to change prices have little incentive to collect much new

[10]The real option value and the "wait-and-see" rule have been studied in the literature on the effects of steady-state uncertainty (e.g., Dixit and Pindyck, 1994; Abel and Eberly, 1999) or second-moment uncertainty shocks (e.g., Bloom, 2009; Vavra, 2013; Gilchrist et al., 2014; Bloom et al., 2018). Unlike this literature, the interaction between information and nominal rigidities makes firms' optimal pricing rules depend on their *own* subjective uncertainty.

[11]While the mechanism I explain here also operates in the two-good version of the model, the one-good version of the model gives a clear comparison to the standard menu cost models with one-product firms, such as Golosov and Lucas (2007).

information—they choose to remain quite uninformed. In contrast, firms that think they are close to the boundaries of their inaction region have a high incentive to collect information and therefore choose to become more informed. This disparity results in a leptokurtic distribution of *posterior* price gaps, or, equivalently, their desired price changes.[12]

The second theoretical finding is that the real effects of monetary policy shocks decrease by 12% in the two-good version of the model compared with the one-good version of the baseline model. As I discussed above, the two-good version of the model exhibits two types of economies of scope in multi-product firms. First, there are economies of scope in price setting. Because paying the menu cost allows firms to change the prices of all of their goods simultaneously, there are many small and large price changes in the two-good version of the model. This weakens selection effects of price changes, as emphasized by Midrigan (2011) and Alvarez and Lippi (2014), and should tend to amplify the real effects of monetary policy shocks in the two-good version of the model. Second, I also find significant economies of scope in information processing: the value of information about aggregate shocks increases with firms' product scopes. Under my benchmark calibration, firms in the two-good version of the model have better information and lower uncertainty about monetary policy shocks than firms in the one-good version of the model. Because multi-product firms learn about monetary policy shocks rapidly, this force will tend to reduce the real effects of nominal shocks, as highlighted by Pasten and Schoenle (2016). The quantitative analysis shows that cumulative output effects are smaller in the two-good version of the model than the one-good version of the model. This implies that the scope motive in information processing quantitatively dominates its effect on pricing decisions, thereby leading to reduced effects of nominal shocks on economic activity as we move to a multi-product environment.

My results are robust to introducing higher numbers of products in the model. I simplify the baseline model by assuming that firms' information acquisition decisions are independent of their price-setting decisions.[13] I solve the simplified version of the model with an arbitrary number of products to explore whether my main results can be extended to higher numbers of products. Although this assumption eliminates the interesting interaction between nominal and informational rigidities, the simplified version of the model shares the core predictions of the baseline model:

[12] In Section 5.2, I provide evidence on the leptokurtic distribution of desired price changes using the New Zealand survey data. Moreover, I show that firms with greater subjective uncertainty expect a longer duration before their next price changes. This finding is direct evidence of the "wait-and-see" rule in firms' price-setting decisions.

[13] The main computational challenge for solving the baseline model with more than two products is that the number of state variables doubles with each additional good produced by the firms and their optimal information choice problem is subject to occasionally binding constraints. The simplifying assumption eliminates the state variables for the firms' problem. See Section 5.1 for details.

firms with a greater product scope have better information about aggregate shocks, and the kurtosis of the distribution of price changes increases with the number of products that firms produce. I calibrate each model with different numbers of products to match the same frequency and size of price changes and study the impulse response analysis to monetary policy shocks. Consistent with the results from the full baseline model, I find that the cumulative output responses in the simplified version of the model decrease with the number of products sold by firms. This result implies that in the class of rational inattention models, the ratio of kurtosis and the frequency of price changes might not be a sufficient statistic, which is derived by Alvarez et al. (2016), for the real effects of monetary shocks.

The paper is organized as follows. Section 2 empirically evaluates how firms' attentiveness to aggregate inflation and price-setting behavior are related to their number of products. In Section 3, I develop a new menu cost model with rationally inattentive multi-product firms that captures the behavior of firms in the survey, and I study firms' optimal decisions regarding information acquisition and price setting. In Section 4, I extend my model to show how product scope affects firm behavior and monetary policy transmission. In Section 5, I study an extension of the baseline model to show the robustness of my results and provide additional evidence that supports the model's predictions using the survey data. Section 6 concludes the paper.

2 Empirical Evidence

In this section, I empirically explore how firms' product scopes relate to 1) their attentiveness to aggregate economic conditions and 2) the frequency and size of their price changes. To this end, I use a quantitative survey of firms' expectations toward macroeconomic conditions in New Zealand.[14] The survey was conducted in multiple waves among a random sample of firms in New Zealand with broad sectoral coverage. This paper contributes two novel empirical results relative to previous studies that used the same survey.[15] First, I show that firms producing a greater number of products are better informed about current aggregate inflation. Second, I document that both the duration and average size of their price changes decrease with the number of products firms produce.

[14] See Coibion et al. (2018a) and Kumar et al. (2015) for a comprehensive description of the survey.

[15] Several papers use the data to characterize how firms form their expectations. For example, Afrouzi (2019b) shows that strategic complementarity decreases with competition, and documents that firms with more competitors have more certain posteriors about the aggregate inflation. Also, Coibion et al. (2018b) evaluate the relation between first-order and higher-order expectations of firms, including how they adjust their beliefs in response to a variety of information treatments.

2.1 Firms' Product Scopes in the Survey Data

I use the second wave of the survey, which was implemented between February and April 2014, to identify the number of products firms produce. In the survey, firms' managers were asked the following question:

> **"In addition to your main product or product line, how many other products do you sell?"**
>
> *Answer*: .... products

Appendix Table A.1 shows the summary statistics regarding firms' number of products by industry. The median is 9, but it is 7 when firms in retail and wholesale trade sectors are excluded. In the baseline regressions, I exclude these retail and wholesale trade firms because their strategies for pricing and information processing are likely to be different from those of firms in other sectors, such as manufacturing and service industries.[16] In the data, a large fraction of firms (about 18% of all firms) compared with other studies sell only one product or have one product line.[17] There are two reasons behind the large fraction of single-product firms in this survey. First, the firms included in the survey were relatively small, with the average number of employees being about 31 and the largest number being about 600 employees. Second, the survey question is about the number of products or product lines at a firm. As there might be several similar types of products in a product line, this question captures firms' perceptions of the unit of their product scope. In fact, I find that the average of firms' output shares of their main product (or product line) is about 60% excluding single-product firms, implying that firms define their unit of product scope a bit broadly.

2.2 Number of Products and Attentiveness to Aggregate Inflation

I first investigate the relationship between the firms' number of products and their attentiveness to current aggregate economic conditions. Firms' attentiveness to aggregate economic conditions, measured by their backcast error in aggregate inflation, is defined to capture firms' knowledge about the current aggregate economy. Given that recent aggregate economic conditions are largely observable in real time, I define the backcast error as the absolute values of the difference between the actual past 12 months of aggregate inflation and managers' corresponding beliefs from the

[16] Including retail and wholesale trade firms in the sample does not change the baseline results that I show later. See, for example, Appendix Table A.3.

[17] For example, Bhattarai and Schoenle (2014) document that 98% of all prices are set by firms with more than one good in the microdata that underlie the calculation of the U.S. PPI.

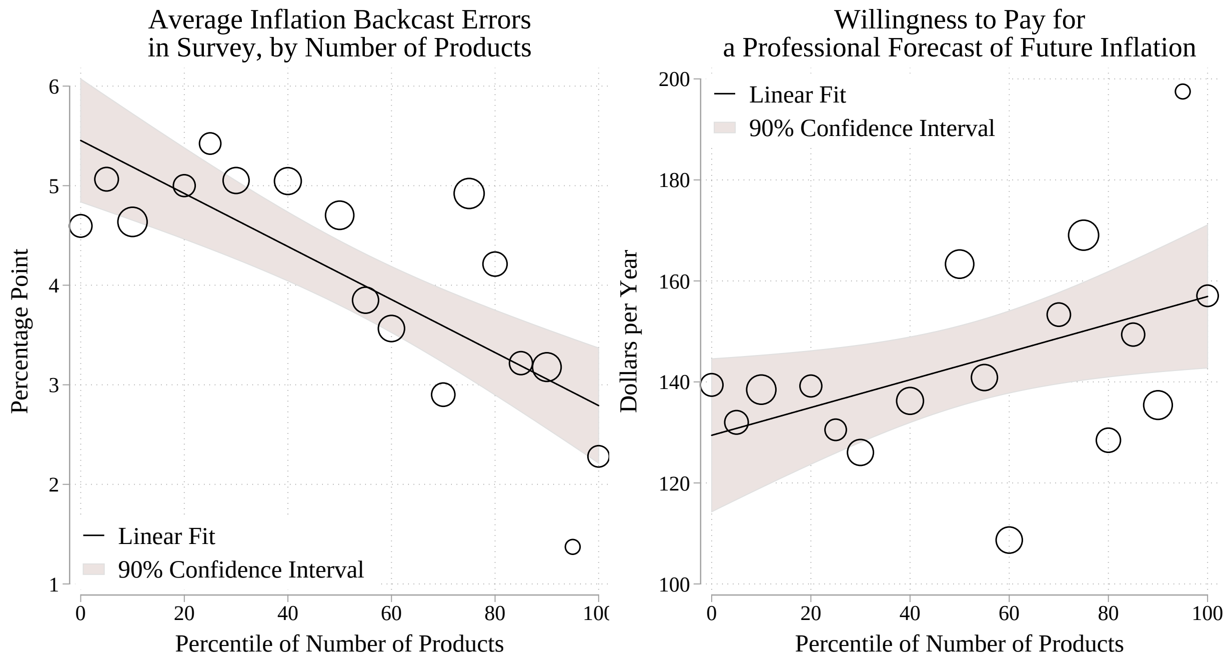


Figure 1: Number of Products and Attentiveness to Aggregate Inflation

Notes: The left panel plots percentile of firms' number of products versus the average of firm backcast errors about past 12 month inflation within each percentile. The right panel plots the percentile of firms' number of products versus the average of willingness to pay for a professional forecast of future inflation. The willingness to pay is measured from answers to the following question in Wave #4 survey: *"How much would you pay per year to have access to a monthly magazine of professional forecasts of future inflation?"* Black lines are linear fitted lines and shaded areas are 90% confidence intervals. The size of bins represents average size of employment of firms in each percentile.

survey.[18] As documented in Coibion et al. (2018a), firms are not well informed about current aggregate inflation, resulting in 4.5% backcast errors on average.[19]

In addition, I find that firms' backcast errors are related to their number of products.[20] Figure 1 shows there is a clear positive relationship between the number of products firms produce and their attentiveness to aggregate inflation. The left panel shows that firms with a smaller product scope produce larger backcast errors on average. In the right panel, I show the relationship between firms' product scopes and their willingness to pay for professional forecasts about future inflation from the fourth wave of the survey. The latter is another measure of firms' incentives

[18] The Consumer Price Index (CPI) is used to calculate the actual past 12 month aggregate inflation. The baseline results are quantitatively similar when I use the GDP deflator or the Produce Price Index to calculate the actual inflation rate.

[19] One might have a concern that firms do not know what the inflation means. However, Kumar et al. (2015) document that 86% of firm managers in the survey could correctly explain what inflation means and they believed that statistical agencies were credible in measuring price changes. Coibion et al. (2018a) also highlight that the large errors are not driven by specific language about the definition of inflation used in the survey.

[20] See Appendix Table A.2 for the summary statistics of the firms' backcast error is aggregate inflation by the quartiles of firms' product scopes within different industries.

Table 1: Number of Products and Knowledge about Aggregate Inflation

	(1)	(2)	(3)	(4)
Panel A. Dependent variable: Absolute value of actual minus firm-reported inflation in prior 12 months				
log(number of products)	-0.314**	-0.207***	-0.580***	-0.252***
	(0.148)	(0.059)	(0.150)	(0.060)
Observations	593	582	448	440
R-squared	0.341	0.801	0.344	0.900
Panel B. Dependent variable: Willingness to pay for professional inflation forecasts				
log(number of products)	5.717**	3.805***	6.965**	3.960**
	(2.565)	(1.186)	(2.635)	(1.643)
Observations	381	368	328	320
R-squared	0.242	0.657	0.289	0.697
Firm-level controls	Yes	Yes	Yes	Yes
Industry fixed effects		Yes		Yes
Manager controls			Yes	Yes

Notes: This table reports results for the Huber robust regression. Dependent variables are the absolute value of firm errors about past 12 month inflation from Wave #1 survey (Panel A) and firms' willingness to payment for professional forecaster's forecasts about future inflation from Wave #4 (Panel B). Firm-level controls include log of firms' age, log of firms' employment, foreign trade share, number of competitors, firms' beliefs about price difference from competitors, and the slope of the profit function. Industry fixed effects include dummies for 14 sub-industries excluding retail and wholesale trade sectors. Manager controls include the age of the respondent (each firm's manger), education, income, and tenure at the firm. Sample weights are applied to all specifications. Robust standard errors (clustered at the 3-digit Australian and New Zealand Standard Industrial Classification (ANZ SIC) level) are reported in parentheses. ***, **, * denotes statistical significance at 1%, 5%, and 10% levels respectively. See Section 2.2 for details.

to be attentive to the aggregate economy. Here, I find a positive correlation between the number of products firms produce and their willingness to pay for professional forecasts regarding future aggregate inflation, which implies that firms with larger product scopes are likely to pay more attention to aggregate conditions.

One potential concern is that this negative correlation between the number of products firms produce and their knowledge of current aggregate inflation is driven by other firm-level characteristics. For example, as big firms are likely to have a larger product scope and a larger capacity to process information, it might seem that the negative correlation stems from the size of the firm rather than its product scope.[21] To address this concern, I regress firms' inattention to inflation, as measured by 1) their absolute errors in regard to recent inflation rates and 2) their willingness to pay for professional future inflation forecasts. I use a log of the firms' number of products, controlling for firm-level characteristics such as a log of firm age, a log of total employment, foreign

[21] Kaihatsu and Shiraki (2016) show that the size of firms significantly affects differences in their inflation expectations. Also, Frache and Lluberas (2019) find that large firms have lower forecast errors in aggregate inflation than small firms.

trade share, the firms' number of competitors, their beliefs about price differences relative to their competitors, and the slope of the profit function.[22] Column (1) of Table 1 shows that firms with a large number of products are likely to make small errors about current aggregate inflation and are more willing to pay more for professional forecasts about future aggregate inflation. Column (2) shows the significant negative correlation after controlling for industry fixed effects.

Another potential concern is that the survey respondents—here the managers of firms—have different abilities or incentives to pay attention to aggregate economic conditions.[23] To address this issue, in Column (3), I report regression results after controlling for managers' characteristics, such as age, education, income level, and number of years at their firms. Again, after controlling for manager characteristics, I find a negative correlation between the firms' number of products and their knowledge of or attentiveness to aggregate inflation.[24]

2.3 Number of Products and Size and Frequency of Price Changes

In this subsection, I document the relationship between the firms' number of products and the frequency and size of their price changes. Firms' managers were asked the following question:

> **"Please report when and by how much you expect to next change the price of your main product and your second main product.** Please provide a numerical answer in months for the durations (e.g. "0" for within the next month, 1 for one month from now, ...) and a percentage answer for the size of the price change (e.g. "+10%" for a 10% increase in price or "-10%" for a 10% decrease)

After controlling for firms' characteristics and their incentives for changing their prices, this question quantifies the frequency and size of firms' price changes.

Table 2 shows the relationship between the firms' number of products and the frequency and size of their price changes. Panel A shows that, after controlling for firm-level characteristics, the duration of price changes is negatively correlated with the number of products. This negative correlation is even stronger when I control for managers' characteristics. In Panel B, I also present

[22] The slope of a firm's profit function is calculated as the ratio of the extent to which a firm could increase its profit (as a percent of revenue) if it could reset its price freely at the time of the survey to the percent price change the firm would implement if it could reset its price freely at the time of the survey.

[23] For example, Tanaka et al. (2019) show that managers' GDP forecasting ability is linked to their management ability and experience.

[24] In Appendix Table A.4, I show that firms' backcast errors in the growth rate of nominal GDP also decrease with their number of products. In the general equilibrium model I study in Section 4, I calibrate the information cost parameter to match the slope coefficient of the regression of the backcast errors in the growth rate of nominal GDP. See Section 4.3 for details.

Table 2: Number of Products and Duration and Size of Price Changes

	(1)	(2)	(3)	(4)
Panel A. Dependent variable: Duration of expected next price changes				
log(number of products)	-0.169	-0.205*	-0.276*	-0.474***
	(0.144)	(0.116)	(0.142)	(0.139)
Observations	588	580	440	445
R-squared	0.495	0.604	0.520	0.548
Panel B. Dependent variable: Size of expected next price changes				
log(number of products)	-0.111	-0.318***	-0.113	-0.239***
	(0.066)	(0.089)	(0.079)	(0.081)
Observations	578	578	432	431
R-squared	0.073	0.612	0.075	0.423
Firm-level controls	Yes	Yes	Yes	Yes
Industry fixed effects		Yes		Yes
Manager controls			Yes	Yes

Notes: This table reports results for the Huber robust regression. Dependent variables are the duration of expected next price changes from Wave #1 (Panel A) and the (absolute) size of expected next price changes from Wave #1 survey (Panel B). Firm-level controls include log of firms' age, log of firms' employment, foreign trade share, number of competitors, firms' beliefs about price difference from competitors, and the slope of the profit function. Industry fixed effects include dummies for 14 sub-industries excluding retail and wholesale trade sectors. Manager controls include the age of the respondent (each firm's manger), education, income, and tenure at the firm. Sample weights are applied to all specifications. Robust standard errors (clustered at the 3-digit ANZ SIC level) are reported in parentheses. ***, **, * denotes statistical significance at 1%, 5%, and 10% levels respectively. See Section 2.3 for details.

that after controlling for industry fixed effects, there is a negative correlation between the number of products firms produce and the size of their price changes. This result shows that, conditional on the price change, firms with a greater product scope change their prices by smaller amounts.

Previous studies have also found negative correlations between the firms' number of products and the duration and size of their price changes. For example, using microdata that underlies the calculation of the U.S. PPI, Bhattarai and Schoenle (2014) show that firms with larger product scopes are more likely to change their prices frequently, and, conditional on price changes, they change by smaller amounts. Parker (2017) also identified this negative correlation between firms' product scopes and the duration and size of their price changes using other survey data on New Zealand firms from Statistics New Zealand in 2010.

2.4 Summary and Relation to Monetary Models

In this section, I find two stylized facts about how firms' information acquisition about aggregate inflation and price-setting decisions are related to their number of products such that

1. firms with a greater number of products have better information about aggregate inflation;
2. firms with a greater number of products have more frequent but smaller price changes.

Can existing monetary models with multi-product firms explain both empirical findings? In fact, empirical finding 1 is consistent with the prediction of rational inattention models with multi-product firms, such as Pasten and Schoenle (2016). In the presence of both good-specific shocks and aggregate shocks, multi-product firms want to be more informed about the aggregate shocks, as they will affect the marginal costs of all their products. Thus, this model implies a negative correlation between the firms' number of products and their inattentiveness to aggregate shocks.

On the other hand, empirical finding 2 is consistent with the prediction of menu cost models with multi-product firms, such as Midrigan (2011) and Alvarez and Lippi (2014). When firms can change all of their prices by paying a firm-specific fixed cost, then firms with a greater number of products are likely to change their prices more frequently and by smaller amounts.

However, those two models are not capable of explaining both findings simultaneously by assumption. Rational inattention models assume flexible prices to focus on the effects of information rigidity while menu cost models assume perfect information to focus on the effects of nominal rigidity. Those two assumptions are inconsistent with the empirical evidence that firms change their prices infrequently and are not fully aware of macroeconomic conditions.[25]

Moreover, the previous literature finds a contradictory implication of firms' product scopes for monetary non-neutrality. In menu cost models, the effects of monetary shocks on output *increase* with the firms' number of products, as selection effects of price changes from menu cost technology decrease with the firms' number of products, as shown in Alvarez and Lippi (2014).[26] In contrast, in rational inattention models, the real effects of monetary shocks *decrease* with the number of products firms produce because firms with a greater product scope have better information about monetary policy shocks.[27]

[25] Appendix Table A.5 shows that other models with nominal or information rigidities also cannot explain those two stylized facts I find in this section. For example, Calvo or Taylor sticky price models imply that the frequency and size of price changes are independent of the number of products, as the frequency of price changes is exogenously given in the model. Similarly, sticky and noisy information models imply that attentiveness to aggregate conditions is independent of the firms' number of products because information acquisition is exogenous in those models. Observational cost models such as Alvarez et al. (2015) also imply the independence results, as it is assumed that firms use steady-state policy rules which are not affected by aggregate shocks in the model.

[26] Alvarez et al. (2016) show that in menu cost models, the cumulative output response to a monetary shock increases in the number of products, N, and is given by $\frac{3N}{N+2}$.

[27] In Appendix B.1, I show that in a rational inattention model, the cumulative response of output to a monetary shock is only a function of firms' average subjective uncertainty about the monetary shock and decreases with the firms' number of products.

In sum, neither model with multi-product firms can account for the empirical relationship between firms' product scopes and their decisions regarding both price setting and information acquisition, and they have contradictory implications regarding firms' product scopes for monetary non-neutrality. This calls for a new model that is disciplined by the empirical findings from microdata in order to study the macroeconomic implications of monetary non-neutrality. This is the goal of the following sections.

3 Price Setting with Menu Costs for a Rationally Inattentive Multi-Product Firm

In this section, I develop a menu cost model for a rationally inattentive multi-product firm. Before constructing a full-fledged dynamic general equilibrium model in the next section, I consider the decision problem of a rationally inattentive firm that faces both idiosyncratic good-specific shocks and an aggregate monetary shock. The firm chooses a set of optimal signals about the underlying shocks given costs of information and pays a single fixed cost (i.e., a menu cost) to reset all of its prices. The goal of this section is to explore how the firm's decisions regarding information acquisition and price setting are affected by 1) the interaction between menu costs and rational inattention and 2) the firm's product scope .

First, to show the interaction between menu costs and rational inattention frictions, I characterize a single-product firm's optimal decision rules. When information is costly, the rationally inattentive firm should decide how much new information to acquire given the cost of information. Moreover, given menu costs, the firm decides whether to change its price based on its information set. I find that the firm's optimal price-setting decisions exhibit the *wait-and-see* property: the more the firm is uncertain about the underlying shocks, the less likely it is to change its price. Because the firm optimally decides its subjective uncertainty, the wait-and-see property implies that the timing of price changes has a selection feature such that it is more likely that the firm changes its price when it has more information and less uncertainty about the underlying shocks. This interaction of nominal and informational rigidities plays a key role in amplifying monetary non-neutrality in the dynamic general equilibrium model that I will discuss in the next section.

Second, given both menu costs and rational inattention frictions, I investigate how a two-product firm is different from a single-product firm in terms of its information and price-setting decisions to highlight movibes based on economies of scope in both price changes and information processing. I show that the two-product firm optimally chooses to be more informed about the aggregate shock than the single-product firm because information about the aggregate shock can be utilized for its pricing decisions for all goods. Moreover, due to the single fixed cost of price changes, the two-product firm changes all of its prices at the same time. Then, the two-product economy will generate small price changes that are absent in the single-product economy. These two motives of economies of scope are also key elements for monetary non-neutrality in the general equilibrium model with multi-product firms.

3.1 A Rationally Inattentive Firm's Problem

Consider a multi-product firm that produces N goods, indexed by $j = 1, 2, \cdots, N$. The firm sets its price of good j, $p_{j,t}$, to match a (frictionless) optimal price, $p^*_{j,t}$.[28] Suppose its optimal price of good j consists of two components, a good-specific shock, $a_{j,t}$, and an aggregate shock, m_t:

$$p^*_{j,t} = a_{j,t} + m_t.$$

I assume that both shocks follow random walk processes:[29]

$$\begin{aligned} a_{j,t} = &\quad a_{j,t-1} + \varepsilon^a_{j,t}, \quad \varepsilon^a_{j,t} \sim N(0, \sigma^2_a) \;\text{ for }\; j = 1, 2, \cdots, N, \\ m_t = &\quad m_{t-1} + \varepsilon^m_t, \quad \varepsilon^m_t \sim N(0, \sigma^2_m), \end{aligned}$$

where $\varepsilon^a_{j,t}$ and ε^m_t are independent and identically distributed. A flow loss of the firm in profits is the sum of the distance between its price of each good and the (frictionless) optimal price:[30]

$$B \sum_{j=1}^{N} \left(p_{j,t} - p^*_{j,t} \right)^2,$$

where B captures the concavity of the firm's profit function with respect to each price.

This firm is rationally inattentive. At the beginning of each period, the firm has to choose how precisely it wants to observe its current set of (frictionless) optimal prices subject to a cost

[28] Small letters denote log deviation from (frictionless) steady state. When we consider the firm as a monopolistically competitive producer, its optimal reset price of good j without any frictions is a constant markup over the marginal cost: $P^*_{j,t} = \mu \times MC_{j,t}$. Then the log deviation of the optimal price is that of marginal cost: $p^*_{j,t} = mc_{j,t}$.

[29] The random walk process of the underlying shock is a common assumption in the menu cost literature, as it simplifies the firm's problem by making it choose its price gaps, which are defined by the difference between the frictionless and the actual prices. See, among others, Barro (1972), Tsiddon (1993), and Alvarez and Lippi (2014).

[30] While I take this characteristic as an assumption, this loss function can also be derived as a second order approximation to a twice-differentiable profit function around the non-stochastic steady state.

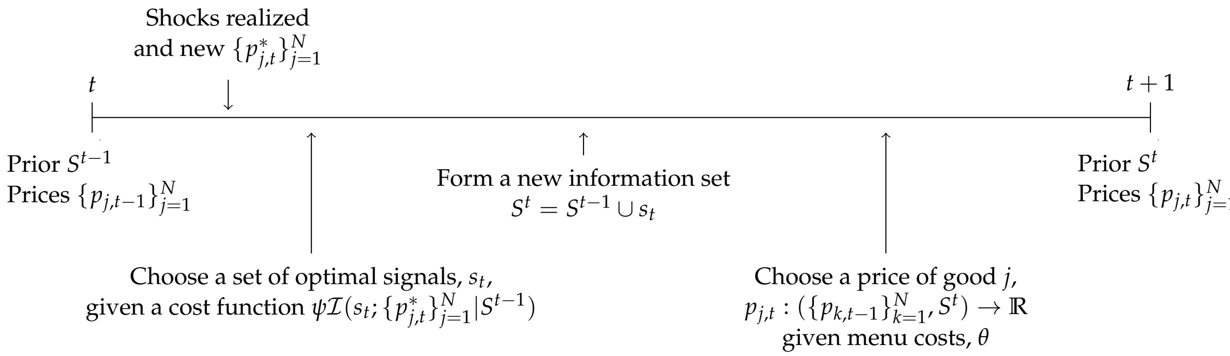


Figure 2: Timing of Events for a Firm's Problem

Notes: This figure shows a sequence of events in each period of the model. See Section 3.1 for details.

of information processing . Formally, at time t, the firm chooses a set of signals about both good-specific and aggregate shocks from a set of available signals, $\mathcal{S}_t = \{\mathcal{S}^a_{j,t}\}_{j=1}^N \cup \mathcal{S}^m_t$, such that

$$
\begin{aligned}
\mathcal{S}^a_{j,t} &= \{a_{j,t} + \eta_{j,t}\xi^a_{j,t} : \eta_{j,t} \geq 0,\ \xi^a_{j,t} \sim N(0,1)\}, \text{ for } j = 1, 2, \cdots, N, \\
\mathcal{S}^m_t &= \{m_t + \eta_{m,t}\xi^m_t : \eta_{m,t} \geq 0,\ \xi^m_t \sim N(0,1)\},
\end{aligned}
$$

where $\{\xi^a_{j,t}\}_{j=1}^N$ and ξ^m_t are the firm's rational inattention errors. Let S^{t-1} be the firm's information set at the beginning of period t before it receives new signals about its frictionless optimal prices. At each time t, given S^{t-1}, the firm chooses a set of its signals $s^a_{j,t} \in \mathcal{S}^a_{j,t}$ for $j = 1, 2, \cdots, N$, and $s^m_t \in \mathcal{S}^m_t$ subject to the cost of information processing. Then, the firm's information set evolves as follows:

$$
S^t = S^{t-1} \cup s_t,
$$

where $s_t = \{\{s^a_{j,t}\}_{j=1}^N, s^m_t\}$. The evolution of information set implies that the firm does not forget information over time. This "no-forgetting constraint" implies that the current information choice has a continuation value and thus the optimal information choice is a solution of a dynamic information acquisition problem.

I assume that the cost of information is linear in Shannon's mutual information function. The firm pays ψ units of its (per-good) revenue for every bit of expected reduction in uncertainty, where uncertainty is measured by entropy. Denote this cost as $\psi\mathcal{I}(s_t; \{p^*_{j,t}\}_{j=1}^N | S^{t-1})$, which will be defined later in this section. At each period, based on its optimal choice of information, the firm chooses whether to change its prices. I assume that the firm can change all of its prices by paying a single fixed cost, θ. This cost is independent of the number of prices the firm changes.[31]

Figure 2 shows the timing of events for the firm's problem: at the beginning of period t, the firm starts with an a priori information set, S^{t-1}, and forms a prior over its optimal prices at that time. Then it chooses a new set of signals, s_t, subject to the cost of information processing and updates its information set, S^t. Given this time t information set, the firm decides whether to change its prices and pay the fixed cost θ or to wait until the next period without changing its prices. If the firm decides to change its prices, it also chooses how much it changes the prices. Thus, the firm optimally chooses a set of signals about the underlying shocks and prices ($p_{j,t}$) over time, contingent on the evolution of its beliefs.

Formally, the firm's problem is as follows:

$$\min_{\{\{p_{j,t}\}_{j=1}^{N}, s_t\}_{t=0}^{\infty}} \mathbb{E}\Bigg[\sum_{t=0}^{\infty} \beta^t \Bigg(\underbrace{B \sum_{j=1}^{N} \left(p_{j,t} - p^*_{j,t}\right)^2}_{\textbf{loss from suboptimal prices}} + \underbrace{\theta \mathbf{1}_{\{\text{for any } j,\ p_{j,t} \neq p_{j,t-1}\}}}_{\textbf{cost of price changes}}$$

$$+ \underbrace{\psi \mathcal{I}\left(s_t; \{p^*_{j,t}\}_{j=1}^{N} | S^{t-1}\right)}_{\textbf{cost of information processing}} \Bigg) \Big| S^{-1} \Bigg] \tag{1}$$

subject to

$$p^*_{j,t} = a_{j,t} + m_t, \quad \forall j = 1, 2, \cdots, N,$$
$$S^t = S^{t-1} \cup s_t, \;\; S^{-1} \text{ is given,}$$

where $\mathbf{1}_{\{\text{for any } j,\ p_{j,t} \neq p_{j,t-1}\}}$ is an indicator function that is 1 if it changes any one of its prices.[32]

Cost of Information Processing. The cost of information processing is linear in Shannon's mutual information function. Let $\mathcal{H}(X|Y)$ be a conditional entropy of a random variable of X given

[31] In Section E, I discuss some evidence of the firm-specific menu costs using the survey data. I also discuss the implications of adding product-specific menu costs in the model.

[32] Besides the existence of menu costs, this problem is different from the previous rational inattention models in LQG settings, such as Maćkowiak and Wiederholt (2015) or Pasten and Schoenle (2016), which solve the problem by assuming that the cost of information is not discounted and optimizing at the long-run steady state for the information structure. In contrast, I assume that the firm discounts future costs of information at the same discount rate as its payoffs and solve the dynamic rational inattention problem. This dynamic nature of firms' information acquisition is the reason that firms are subject to the no-forgetting constraints. See, for instance, Afrouzi and Yang (2019) for adetailed discussion of solutions for the dynamic rational inattention problem in LQG setups.

knowledge of Y. The firm's flow cost of information at time t is $\psi\mathcal{I}(s_t; \{p^*_{j,t}\}_{j=1}^N | S^{t-1})$, where

$$\mathcal{I}(s_t; \{p^*_{j,t}\}_{j=1}^N | S^{t-1}) = \mathcal{H}(\{p^*_{j,t}\}_{j=1}^N | S^{t-1}) - \mathbb{E}[\mathcal{H}(\{p^*_{j,t}\}_{j=1}^N | S^t) | S^{t-1}]$$

is the reduction in uncertainty about its (frictionless) optimal prices that the firm experiences by observing the set of signals, s_t, given its prior information set, S^{t-1}, and ψ is the marginal cost of a bit of information.

Let $z^a_{j,t} \equiv var(a_{j,t}|S^t)$ and $z^m_t \equiv var(m_t|S^t)$ be the firm's subjective uncertainty about the j-good-specific shock and about the aggregate shock, respectively. Then, I can rewrite the cost of information processing at time t in terms of $\{z^a_{j,t}\}_{j=1}^N$ and $z^m_t \equiv var(m_t|S^t)$:

$$\begin{aligned}
\mathcal{I}\left(s_t; \{p^*_{j,t}\}_{j=1}^N | S^{t-1}\right) &= \sum_{j=1}^N \mathcal{I}\left(s^a_{j,t}; a_{j,t} | S^{t-1}\right) + \mathcal{I}\left(s^m_t; m_t | S^{t-1}\right) \\
&= \frac{1}{2}\left(\sum_{j=1}^N \log_2\left(\frac{z^a_{j,t-1} + \sigma^2_a}{z^a_{j,t}}\right) + \log_2\left(\frac{z^m_{t-1} + \sigma^2_m}{z^m_t}\right)\right),
\end{aligned} \tag{2}$$

where $\{z^a_{j,-1}\}_{j=1}^N$ and z^m_{-1} are given. The first equality follows from the fact that the underlying shocks are independent and the firm observes independent signals about them. The second equality holds because the firm observes Gaussian signals about the underlying shocks, which are also Gaussian. Moreover, in this setup, I can rewrite the no-forgetting constraint, $S^t = S^{t-1} \cup s_t$, in terms of the firm's subjective uncertainty:

$$\begin{aligned}
&0 \le z^a_{j,t} \le z^a_{j,t-1} + \sigma^2_a \ \text{ for } \ j = 1, 2, \cdots, N, \\
&0 \le z^m_t \le z^m_{t-1} + \sigma^2_m.
\end{aligned}$$

This reformulation shows that the cost of information processing is directly related to how much each firm reduces its subjective uncertainty about the good-specific shocks and the aggregate shock, given their prior uncertainty about those shocks. If the marginal cost of information processing, ψ, is zero, the firm would like to choose zero subjective uncertainty about both underlying shocks. As it is costly for the firm to reduce a large amount of uncertainty about the underlying shocks when $\psi > 0$, it optimally chooses to observe less precise signals and to be optimally uncertain about the underlying shocks.

Recursive Formulation of the Firm's Problem. I reformulate the firm's problem (1) in a recursive form to characterize its optimal decision rules and simulate the model numerically. Let $x_{j,t} = p_{j,t} - \mathbb{E}[p^*_{j,t}|S^t]$ be the firm's *perceived* price gap of product j. Then, the loss from suboptimal prices can be decomposed into two components:

$$\mathbb{E}\left[\left(p_{j,t} - p^*_{j,t}\right)^2 \middle| S^t\right] = \underbrace{z^a_{j,t} + z^m_t}_{\substack{\text{contemporaneous loss from} \\ \textbf{imperfect information}}} + \underbrace{x^2_{j,t}}_{\substack{\text{contemporaneous loss from} \\ \textbf{nominal rigidities}}}, \tag{3}$$

where

$$z^a_{j,t} = \mathbb{E}\left[\left(a_{j,t} - \mathbb{E}_t[a_{j,t}|S^t]\right)^2 \middle| S^t\right], \;\; z^m_t = \mathbb{E}\left[\left(m_t - \mathbb{E}_t[m_t|S^t]\right)^2 \middle| S^t\right]$$

are the subjective uncertainty about the j-good-specific shock and about the aggregate shock, respectively.

On the one hand, if there is no informational cost, $\psi = 0$, then the firm chooses zero subjective uncertainty and thus there is no contemporaneous loss from imperfect information. In this case, the firm's problem is identical to the problem in a standard menu cost model with multi-product firms. On the other hand, if there is no menu cost, $\theta = 0$, then the firm can always adjust its prices freely and thus will choose zero *perceived* price gaps, $x_{j,t} = 0$ for all $j = 1, 2, \cdots, N$. In this case, the firm's problem is identical to the problem in a standard rational inattention model with multi-product firms.

Notice that the perceived price gaps, $\{x_{j,t}\}$, are the firm's choice variables when it decides to change its prices. However, if the firm does not want to change its prices, then the perceived price gaps are stochastic variables that evolve according to

$$\mathbf{x}_t \sim N(\mathbf{x}_{t-1}, \Sigma_t),$$

where $\mathbf{x}_t = \{x_{1,t}, x_{2,t}, \cdots, x_{N,t}\}'$ and

$$\Sigma_t(j,k) = \begin{cases} z^m_{t-1} + \sigma^2_m - z^m_t & \text{if } j \neq k \\ z^a_{j,t-1} + \sigma^2_a - z^a_{j,t} + z^m_{t-1} + \sigma^2_m - z^m_t & \text{if } j = k. \end{cases} \tag{4}$$

Given (2), (3), and (4), I reformulate the firm's problem (1) in a recursive form with $2N + 1$

state variables and occasionally binding constraints:

$$
\begin{aligned}
V\left(\{x_{j,-1}\}_{j=1}^{N},\{z_{j,-1}^{a}\}_{j=1}^{N},z_{-1}^{m}\right)=\max_{\{\{z_j^a\}_{j=1}^N,z^m\}}\mathbb{E}\Bigg[&\max\left\{V^{I}\left(\{x_j\}_{j=1}^{N},\{z_j^a\}_{j=1}^{N},z^m\right),V^{C}\left(\{z_j^a\}_{j=1}^{N},z^m\right)\right\}\\
&-\frac{\psi}{2}\left(\sum_{j=1}^{N}\log_2\left(\frac{z_{j,-1}^{a}+\sigma_a^2}{z_j^a}\right)+\log_2\left(\frac{z_{-1}^{m}+\sigma_m^2}{z^m}\right)\right)\Bigg|S^{-1}\Bigg],\\
\text{s.t.}\qquad &0\le z_j^a\le z_{j,-1}^{a}+\sigma_a^2,\quad \forall j=1,2,\cdots,N,\\
&0\le z^m\le z_{-1}^{m}+\sigma_m^2,
\end{aligned}
$$

where

$$
V^{I}\left(\{x_j\}_{j=1}^{N},\{z_j^a\}_{j=1}^{N},z^m\right)=-B\sum_{j=1}^{N}\left(x_j^2+z_j^a+z^m\right)+\beta V\left(\{x_j\}_{j=1}^{N},\{z_j^a\}_{j=1}^{N},z^m\right)
$$

$$
\text{with } \mathbf{x}\sim N\left(\mathbf{x}_{-1},\Sigma\right),\ \text{and}
$$

$$
V^{C}\left(\{z_j^a\}_{j=1}^{N},z^m\right)=\max_{\{y_j\}_{j=1}^{N}}-B\sum_{j=1}^{N}\left(y_j^2+z_j^a+z^m\right)-\theta+\beta V\left(\{y_j\}_{j=1}^{N},\{z_j^a\}_{j=1}^{N},z^m\right).
$$

Here $V^I(\{x_j\}_{j=1}^N,\{z_j^a\}_{j=1}^N,z^m)$ represents the firm's value of not changing its prices. Similarly, $V^C(\{z_j^a\}_{j=1}^N,z^m)$ is the firm's value of changing its prices.

3.2 Decision Rules

In this section, I describe key properties of the firm's optimal decision rules. First, because of the quadratic objective function and the symmetry of the normal distribution, the value function is also symmetric around the null vector for the perceived price gaps. Second, given the optimal choices of subjective uncertainty about the good-specific shocks ($\{z_{j,t}^a\}$) and about the aggregate shock (z_t^m), the value function is decreasing in the absolute values of perceived price gaps. These two properties imply that, given optimal choices of subjective uncertainty, the firm chooses to have zero perceived price gaps for all their goods whenever it decides to change its prices by paying the menu cost, θ. Then, the value function of the firm that changes its prices can be written as

$$
V^{C}\left(\{z_j^a\}_{j=1}^{N},z^m\right)=-B\sum_{j=1}^{N}\left(z_j^a+z^m\right)-\theta+\beta V\left(\{0\},\{z_j^a\}_{j=1}^{N},z^m\right).
$$

Because the firm's problem is a non-convex optimization problem in its price-setting decision and there are occasionally binding no-forgetting constraints for its choices of subjective uncer-

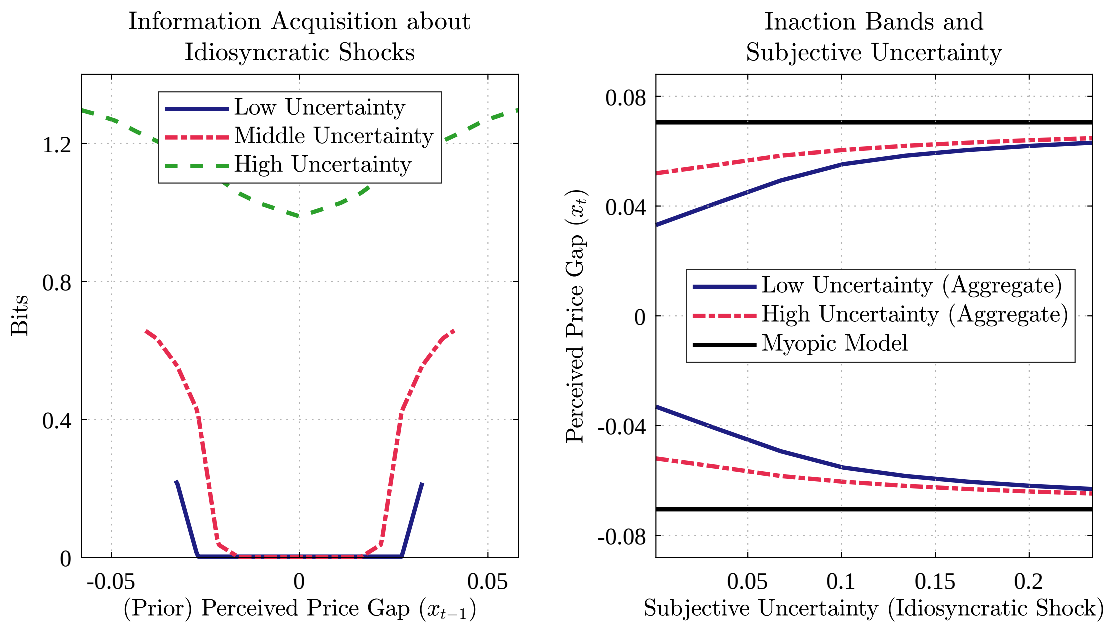


Figure 3: Information Acquisition and Inaction Bands of a Single-Product Firm

Notes: The left panel plots the amounts of information acquisition from a single-product firm's optimal choice of subjective uncertainty about the good-specific shock. The right panel shows inaction bands of a single-product firm as a function of its subjective uncertainty. Different lines represent the inaction bands with different levels of subjective uncertainty about the aggregate shock (z_t^m). Black lines are the inaction bands of a myopic firm whose discount factor is zero. Since this myopic firm does not care about a continuation value of information, the subjective uncertainty is not its state variable, which leads the inaction bands of this firm to be constant

tainty, it needs to be solved numerically.[33] Using the method of value function iteration, I solve the problems of two types of firms: a single-product firm and a two-product firm. I first investigate how the interaction between menu costs and rational inattention frictions affects the single-product firm by characterizing its optimal information acquisition and price-setting decisions. Then, I show how economies of scope in both price setting and information processing affect the two-product firm by comparing with the single-product firm.

3.2.1 A Single-Product Firm

I first consider a single-product firm's optimal decision rules.[34] Here I drop the j-index because the firm produces only one product.

[33]The most computationally burdensome part is to compute firms' expected future values. When the firm does not change prices, its price gaps are stochastic variables that are jointly normally distributed with a mean vector, $\mathbf{x}_{-1}$, which is the firm's state variable, and a covariance matrix, Σ, which is its choice variable. Standard approximation methods for the transition probability of states, such as Tauchen (1986), are not applicable, as approximation errors are quite large. I compute the expected value of the firms' value functions using an explicit numerical integration technique. See Appendix C for the detailed description.

[34]To illustrate this example, I set $\theta = 0.0074$, $\psi = 0.0035$, $\sigma_a = 0.0168$, and $\sigma_m = 0.0044$. These are the parameters that I calibrate when I solve a general equilibrium single-product model in the next section.

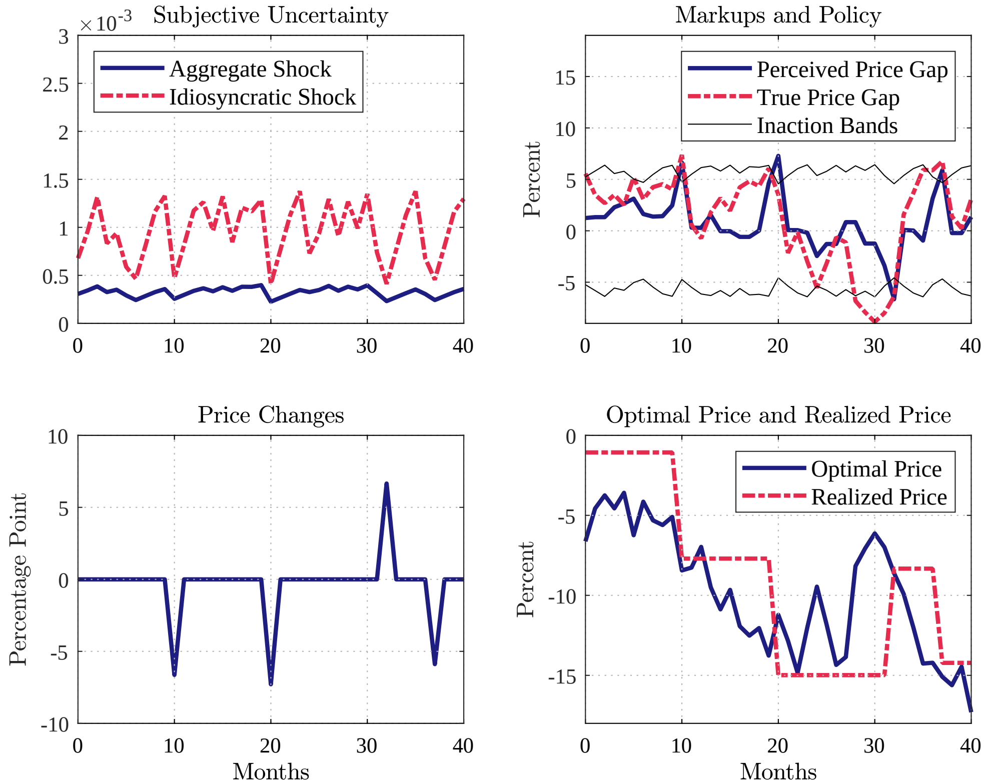


Figure 4: A Single-Product Firm in Model Simulation

Notes: The upper left panel plots a single-product firm's subjective uncertainty about both a good-specific shock (red dash-dot line) and an aggregate shock (blue solid line). The upper right panel plots the firm's perceived price gap (x_t), its true price gap under perfect information, and inaction bands. The firm changes its price when the perceived price gap is out of the inaction bands. The lower left panel plots these price changes. The lower right panel plots realized actual prices and optimal prices under perfect information.

Optimal Information Acquisition. Optimal policy rules for choosing a firm's subjective uncertainty about the good-specific shock are presented in Figure 3. In particular, the left panel of Figure 3 shows that when its prior uncertainty is low enough and its prior price gap is close to zero, the no-forgetting constraint binds and the firm does not acquire new information. The amount of information acquisition increases in both the firm's prior uncertainty (z^a_{t-1}) and the distance between its current price and the frictionless optimal price ($|x_{t-1}|$). In other words, the firm has a large incentive to collect and process information when the firm is quite uncertain about the realization of the underlying shocks and thinks that it is likely that it will need to change prices. This is because potential losses from mistakes in the price-setting decisions are large if the firm thinks that it is likely that it will need to change its price. The firm could make wrong decisions either by paying the menu cost and changing the price when it was not supposed to do so or by choosing not to change its price when it should.

Price-Setting Decision Given the Optimal Information Choices. After choosing optimal signals about the underlying shocks and forming the new information set, the firm decides whether to change its price, based on the new information set. Due to the fixed menu cost of adjusting prices, the firm adopts S-s rules in setting its prices, like in standard menu cost models. There are adjustment thresholds (s, S) such that if the firm's *perceived* price gap is greater than S, it pays the fixed cost and decreases its price to the frictionless optimal level. Similarly, the firm increases its price and collapses the perceived gap to zero when it believes its price gap is less than s. The adjustment thresholds are the firm's inaction bands. As we can see in the upper-right panel of Figure 4, one interesting feature in this model is that the inaction bands are time-varying. Formally, let $\hat{x}_t$ be the firm's posterior belief about its perceived price gap *after* observing the new optimal signals and *before* changing its price at time t. Then,

$$\begin{aligned}\hat{x}_t &= p_{t-1} - \mathbb{E}[a_t + m_t | S^t] \\ &= x_{t-1} - \{\mathcal{K}_t^a (s_t^a - \mathbb{E}[a_t | S^{t-1}]) + \mathcal{K}_t^m (s_t^m - \mathbb{E}[m_t | S^{t-1}])\},\end{aligned}$$

where $\mathcal{K}_t^a$ and $\mathcal{K}_t^m$ are the optimal Kalman gains for the good-specific shock and for the aggregate shock, respectively. A higher value of the Kalman gains implies that the firm chooses to observe more precise signals about the underlying shocks. Given the firm's optimal choice of subjective uncertainty, z_t^a and z_t^m, there exists an adjustment threshold $\tilde{x}(z_t^a, z_t^m) \geq 0$ such that

$$-\tilde{x}_t(z_t^a, z_t^m)^2 + \beta V\left(B\tilde{x}_t(z_t^a, z_t^m), z_t^a, z_t^m\right) = -\theta + \beta V\left(0, z_t^a, z_t^m\right).$$

The firm will change its price if $|\hat{x}_t| > \tilde{x}_t(z_t^a, z_t^m)$. Then, the perceived price gap at the end of period t, x_t, is

$$x_t = \begin{cases} \hat{x}_t & \text{if } |\hat{x}_t| \leq \tilde{x}_t(z_t^a, z_t^m) \\ 0 & \text{if } |\hat{x}_t| > \tilde{x}_t(z_t^a, z_t^m). \end{cases}$$

The right panel of Figure 3 shows the inaction bands, $(-\tilde{x}_t(\cdot, z_t^m), \tilde{x}_t(\cdot, z_t^m))$, for the various values of z_t^m. Notice that the inaction bands in a myopic model ($\beta = 0$) are constant and given by $(-\sqrt{\theta/B}, \sqrt{\theta/B})$ because the firm's subjective uncertainty is no longer a state variable for the firm's problem. With $\beta > 0$, the inaction bands vary with the firm's subjective uncertainty. When the firm is more uncertain about the underlying shocks, the inaction bands are wider. This makes sense because when the firm is uncertain about the underlying shocks, it is optimal to wait until it gets more information about them. As a result, as shown in Figure 4, the firm is likely to change its price when its subjective uncertainty is low and thus the inaction bands are narrow.

The main implication of the interaction between menu costs and rational inattention frictions is that the firm is likely to be more informed about the underlying shocks when it changes its price than when it does not. As will be clear in the next section, in a general equilibrium model with a large number of firms, this interaction leads to a selection effect of information processing such that price adjusters are more informed about the underlying shocks than non-adjusters.[35]

Another interesting characteristic of the firm's pricing rule is that the firm makes mistakes in both the timing and size of price changes. As the firm's pricing decision is based on its *belief* about the price gap ($p_t - \mathbb{E}[p_t^*|S^t]$) rather than the *true* price gap ($p_t - p_t^*$), the firm changes its price by a wrong amount when it decides to change. Moreover, it makes mistakes on the timing of its price changes. The firm changes its price when it is not supposed to, or it does not adjust the price when it should. Figure 4, for example, shows that in period 20, the firm decreases its price when the true price gap (a red dashed line) is well-within the inaction bands while in period 30, the firm does not change its price when the true price gap is outside of the inaction bands.

3.2.2 A Two-Product Firm

Now, I consider the two-product firm's optimal information acquisition and price setting decision. In fact, the two-product firm shares the same characteristics about its optimal decision rules with the single-product firm that I discussed earlier. However, two interesting motives based on economies of scope emerge in the two-product firm's optimal choices—economies of scope in price changes through the menu cost technology and economies of scope in information processing through the rational inattention friction.

Economies of Scope in Price Changes. Figure 5 shows a two-product firm's information acquisition and price setting behavior in the model simulation. Like a single-product firm, the inaction bands of the two-product firm also depend on its subjective uncertainty. The main difference in

[35]Models with both menu costs and observational costs such as Alvarez et al. (2011) and Bonomo et al. (2019b) also have similar implications. In these models, a firm has to pay a fixed cost to acquire full-information about the underlying shocks and can change its price by paying a fixed menu cost. The optimal pricing rule implies that the firm only changes its price when it pays the observational cost, which is an extreme case of the selection effect of information processing in the sense that the firm has full information when it changes its price while it does not acquire new information at all otherwise. Gorodnichenko (2008) also shows that firms have an incentive to buy an additional signal prior to changing prices in a model with menu costs and endogenous information choice.

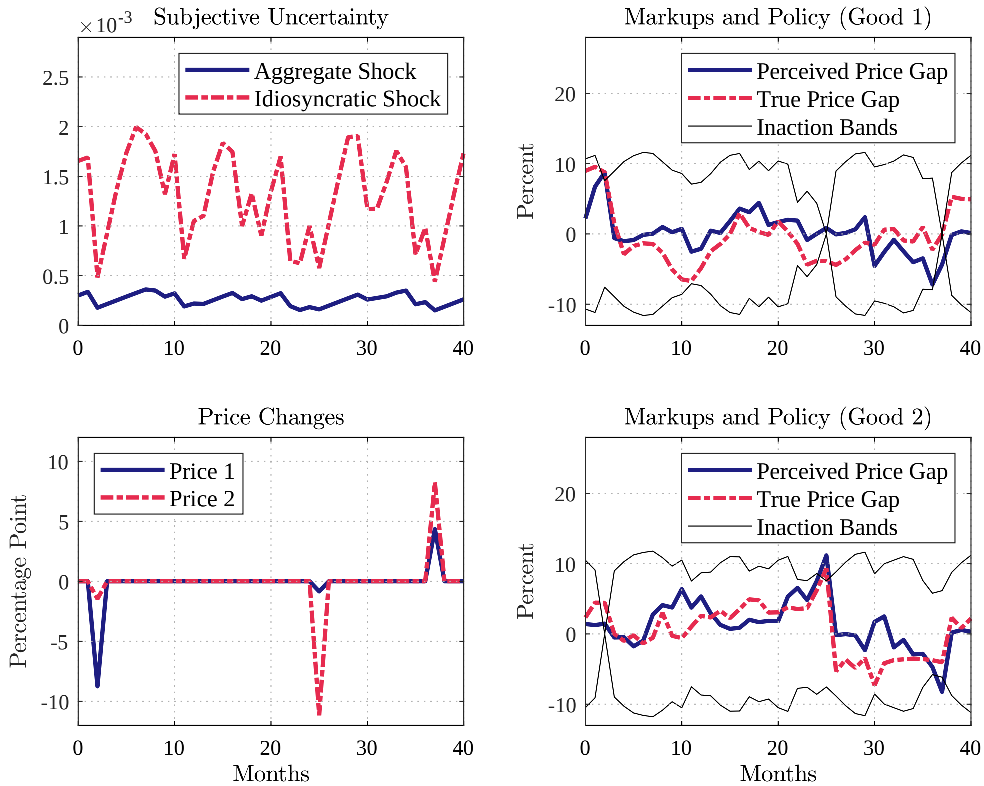


Figure 5: A Two-Product Firm in Model Simulation

Notes: The upper left panel plots a two-product firm's subjective uncertainty about both a good-specific shock (red dash-dot line) and an aggregate shock (blue solid line). The upper right panel plots the firm's perceived price gap (x_t) for good 1, its true price gap for good 1 under perfect information, and inaction bands for good 1. The lower right panel plots the firm's perceived price gap (x_t) for good 2, its true price gap for good 2 under perfect information, and inaction bands for good 2. The firm changes its price when the perceived price gaps are out of the inaction bands. The lower left panel plots these price changes for both goods.

the two-product firm's price setting decision is that the price change of one of its products depends on the perceived price gap of the other product. For example, the right upper and lower panels of Figure 5 show that when the perceived price gap of the second product is large, the inaction bands for the first product are narrow. This relationship implies that the timing of price changes within the firm is synchronized and, more importantly there are both large and small price changes. This is called economies of scope in price changes from the menu cost technology. If the firm decides to pay the menu cost to change one of its prices, then the price change of the other products is free for the firm. As a result, given the single menu cost, the two-product firm is likely to change its prices more frequently than the single-product firm. Moreover, as the additional price change is free and thus there are many small price changes, the two-product firm changes its prices, on average, by a smaller amount than the single-product firm. As I show in Section 2.3, these results are consistent with my second empirical finding that firms producing more goods have more frequent but smaller price changes. As highlighted in the menu cost literature, economies of scope in price changes will weaken the selection effects of price changes by letting some price changes be random (e.g. Midrigan 2011, Alvarez and Lippi 2014). The weak selection effects of price changes in an economy with a large number of multi-product firms will then lead to an amplified real effect of monetary shocks.

Economies of Scope in Information Processing. The two-product firm is also different from the single-product firm in terms of its optimal information acquisition about the aggregate shock. In particular, I find that given the same marginal cost of information processing, the two-product firm is more informed about the aggregate shock than the single-product firm. Because the firm's optimal prices for all goods are affected by the aggregate shock, the value of information about the aggregate shock will be higher if the firm produces more products. This relationship is clearly seen if we consider the firm's problem without menu costs ($\theta = 0$).[36] In this case, the firm's optimal subjective uncertainty about the aggregate shock satisfies the following FOC:

$$B \cdot N = \frac{\psi}{2 \log 2} \left(\frac{1}{z_t^m} - \beta \frac{1}{z_t^m + \sigma_m^2} \right), \tag{5}$$

where z_t^m is decreasing in the number of products, N.

With menu costs ($\theta > 0$), I also find the economies of scope in the information processing of the two-product firm. The average subjective uncertainty about the aggregate shock for the two-product firm is 25% smaller than that for the single-product firm. As I show in Section 2.2, this result is consistent with my first empirical finding that firms that produce more goods have better information about aggregate inflation. As the two-product firm has lower subjective uncertainty about the aggregate shock than the single-product firm, it is more likely that the two-product firm responds more strongly to monetary policy shocks by learning about them more rapidly and therefore changing their prices more rapidly. In the economy with a large number of firms, this scope motive in information processing is likely to act as a strong force to weaken monetary non-neutrality.

In sum, the two-product firm's information acquisition and price-setting decisions show two economies of scope motives that work in opposite directions for monetary non-neutrality. The question is how quantitatively large each scope motive is. To draw the implications of firms' product scopes for monetary non-neutrality in the model with both menu costs and rational inat-

[36] See Appendix B for the optimal solutions for the rationally inattentive firm's problem without menu costs.

tention friction, we need to extend the model in a general equilibrium setup and discipline it with micro-evidence. This is the goal of the next section.

4 A Dynamic General Equilibrium Model

In this section, I extend the menu cost model with rationally inattentive multi-product firms of Section 3 to a dynamic general equilibrium model. The model is disciplined using micro-evidence of Section 2 and then used for quantitative analysis on the transmission of monetary shocks. The focus of this analysis is twofold. First, I investigate how the interaction between rational inattention and menu costs affects the distribution of firms' desired price changes as well as the distribution of subjective uncertainty. I show these two distributions are important determinants of monetary non-neutrality. Second, I compare the two-good and one-good versions of the model to study how multi-product pricing affects the real effects of monetary shocks through economies of scope motives in both information processing and price setting that I studied in the previous section.

4.1 Environment

The economy is populated by a representative household and a unit measure of monopolistically competitive firms. Each firm sells N products. I first discuss the household problem and then present the firm problem and define equilibrium.

Households. The representative household consumes a Dixit-Stiglitz aggregate consumption, C_t, of a basket of multiple goods $j \in \{1, 2, \cdots, N\}$ purchased from firms $i \in [0, 1]$, and supplies labor L_t to maximize the expected lifetime utility with a discount factor $\beta \in (0, 1)$.

The representative household's problem is

$$\max_{\{\{C_{i,j,t}\}_{j=1}^{N}, C_t, L_t, B_t\}_{t\geq 0}} \mathbb{E}_0^f \left[\sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \right], \tag{6}$$

subject to

$$\int \Big(\sum_{j=1}^{N} P_{i,j,t} C_{i,j,t} \Big) di + B_t \leq R_{t-1} B_{t-1} + W_t L_t + \Pi_t, \quad \text{for all } t,$$

where

$$C_t = \left(\frac{1}{N} \sum_{j=1}^{N} C_{j,t}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}, \quad C_{j,t} = \left(\int \left(A_{i,j,t} C_{i,j,t} \right)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Here $\mathbb{E}_t^f[\cdot]$ is the full-information rational expectation operator at time t. As the main purpose of this paper is to study the effects of nominal rigidity and rational inattention among firms, I assume that the household is fully informed about all prices and wages. B_t is the demand for nominal bonds and R_{t-1} is the nominal interest rate.[37] L_t is the labor supply of the household, W_t is the nominal wage, and Π_t is the aggregate profit from the firms. C_t is the aggregator over the consumption for differentiated goods and $C_{j,t}$ is an aggregator over the consumption of good j. $A_{i,j,t}$ is the quality of the good j produced by firm i. Higher $A_{i,j,t}$ increases the marginal utility of consumption for that good while it also increases the production cost for that good, as I describe below.[38] ε is the constant elasticity of substitution across different firms that produce the same good and γ is the constant elasticity of substitution across different goods.

The above formulation implies that demand for good j produced by firm i is

$$C_{i,j,t} = A_{i,j,t}^{\varepsilon-1}\left(\frac{P_{i,j,t}}{P_{j,t}}\right)^{-\varepsilon}\left(\frac{P_{j,t}}{P_t}\right)^{-\gamma} C_t,$$

where P_t is the price of the aggregate consumption bundle C_t, and $P_{j,t}$ is the price index for good j. These prices are given by

$$P_t = \left(\frac{1}{N}\sum_{j=1}^{N} P_{j,t}^{1-\gamma}\right)^{\frac{1}{1-\gamma}}, \quad P_{j,t} = \left(\int_0^1 \left(\frac{P_{i,j,t}}{A_{i,j,t}}\right)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}.$$

Firms. There is a measure one of firms, indexed by i, that operate in monopolistically competitive markets. Each firm produces N goods, indexed by j. Firms take wages and demands for their goods as given, and choose their prices $\{P_{i,j,t}\}_{j=1}^{N}$ based on their information set, S_i^t, at that time. After setting their prices, firms hire labor from a competitive labor market and produce the realized level of demand that their prices induce with a production function for good j,

$$Y_{i,j,t} = \frac{1}{A_{i,j,t}} L_{i,j,t},$$

where $L_{i,j,t}$ is firm i's demand for labor for producing good j. Notice that higher quality products require extra labor input. I assume that shocks to $A_{i,j,t}$ are independently and identically

[37] Natural borrowing limits on the demand for nominal bond are imposed to rule out Ponzi schemes.

[38] The assumption that idiosyncratic shocks affect both the cost at which a good is sold and the household's marginal utility for the good is common in the menu cost literature to reduce the dimensionality of the state space and thus the computational burden. See, for instance, Midrigan (2011), Alvarez and Lippi (2014), Mongey (2017), and Karadi and Reiff (2019).

distributed and the log of the j-good specific shock, $a_{i,j,t} \equiv \log(A_{i,j,t})$, follows a random walk process:

$$\log(A_{i,j,t}) = \log(A_{i,j,t-1}) + \varepsilon^{a}_{i,j,t}, \;\; \varepsilon^{a}_{i,j,t} \sim N(0, \sigma_a^2) \;\text{ for }\; j = 1, 2, \cdots, N.$$

Then, firm i's nominal profit from sales of all goods at prices $\{P_{i,j,t}\}_{j=1}^{N}$ is given by

$$\Pi_{i,t}(\{P_{i,j,t}, A_{i,j,t}, P_{j,t}\}_{j=1}^{N}, W_t, P_t, Y_t) = \sum_{j=1}^{N} \left(P_{i,j,t} - W_t A_{i,j,t}\right) A_{i,j,t}^{\varepsilon-1} \left(\frac{P_{i,j,t}}{P_{j,t}}\right)^{-\varepsilon} \left(\frac{P_{j,t}}{P_t}\right)^{-\gamma} Y_t, \tag{7}$$

where Y_t is the nominal aggregate demand.

At each period, firms optimally decide their prices and signals subject to the costs of changing prices and of processing information. First, changing the price entails a fixed cost, $\tilde{\theta}$. I express this cost as a fraction θ of the steady-state frictionless revenue from selling one of N products. This cost is incurred when at least one price is changed and is independent of the total number of prices that the firm adjusts. Second, firms are rationally inattentive in the sense that they choose their optimal information set by taking into account the cost of obtaining and processing information. At the beginning of period t, firm i wakes up with its initial information set, S_i^{t-1}. Then it chooses optimal signals, $s_{i,t}$, from a set of available signals, $\mathcal{S}_{i,t}$, subject to the cost of information, which is linear in Shannon's mutual information function. Denote $\tilde{\psi}$ as the marginal cost of information processing. Again, I express this marginal cost as a fraction ψ of the steady-state frictionless revenue from selling one of N products. Firm i forms a new information set, $S_i^t = S_i^{t-1} \cup s_{i,t}$, and uses it to set its new prices, $\{P_{i,j,t}\}_{j=1}^{N}$.

Firm i chooses a set of signals to observe over time $(s_{i,t} \in \mathcal{S}_{i,t})_{t=0}^{\infty}$ and a pricing strategy that maps the set of its prices at $t-1$ and its information set at t to its optimal price at any given period, $P_{i,j,t} : (\{P_{i,j,t-1}\}_{j=1}^{N}, S_i^t) \to \mathbb{R}$ where $S_i^t = S_i^{t-1} \cup s_{i,t} = S_i^{-1} \cup \{s_{i,\tau}\}_{\tau=0}^{t}$ is the firm's information set at time t. Then, firm i's problem is to maximize the net present value of its life time profits given an initial information set:

$$\begin{aligned}
\max_{\{s_{i,t} \in \mathcal{S}_{i,t}, \{P_{i,j,t}(\{P_{i,k,t-1}\}_{k=1}^{N}, S_i^t)\}_{j=1}^{N}\}_{t\geq 0}} \quad & \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \Lambda_t \left\{ \Pi_{i,t}(\{P_{i,j,t}, A_{i,j,t}, P_{j,t}\}_{j=1}^{N}, W_t, P_t, Y_t) \right.\right. \\
& \left.\left. - \tilde{\theta} \mathbf{1}_{\{\text{for any } j,\; P_{i,j,t} \neq P_{i,j,t-1}\}} - \tilde{\psi} \mathcal{I}(s_{i,t}; \{\{A_{i,j,t}\}_{j=1}^{N}, W_t\} | S_i^{t-1}) \right\} \middle| S_i^{-1} \right] \\
\text{s.t.} \quad & S_i^t = S_i^{t-1} \cup s_{i,t},
\end{aligned} \tag{8}$$

where $\Lambda_t = \frac{U_{c,t}/P_t}{U_{c,0}/P_0}$ and $\mathcal{I}(s_{i,t}; \{\{A_{i,j,t}\}_{j=1}^N, W_t\}|S_i^{t-1})$ is Shannon's mutual information function.[39]

Monetary Policy. For simplicity, I assume that nominal spending must be equal to the money supply:

$$\int \Big(\sum_{j=1}^{N} P_{i,j,t} C_{i,j,t} \Big) di = P_t C_t = M_t, \tag{9}$$

and the log of money supply, $m_t \equiv \log(M_t)$, follows a random walk process:

$$m_t = m_{t-1} + \varepsilon_t^m, \quad \varepsilon_t^m \sim N(0, \sigma_m^2), \tag{10}$$

where ε_t^m is an independently and identically distributed normal disturbance.

Definition of Equilibrium. Given exogenous processes for idiosyncratic good-specific shocks $\{\{\{a_{i,j,t}\}_{j=1}^N\}_{i\in[0,1]}\}_{t\geq 0}$, a general equilibrium of the economy consists of an allocation for the representative household,

$$\Omega^H \equiv \Big\{ C_t, \{C_{i,j,t}\}_{j=1}^N, L_t, B_t \Big\}_{t=0}^{\infty},$$

an allocation for every firm $i \in [0,1]$ given the initial set of signals,

$$\Omega_i^F \equiv \Big\{ s_{i,t} \in \mathcal{S}_{i,t}, \{P_{i,j,t}, L_{i,j,t}, Y_{i,j,t}\}_{j=1}^N \Big\}_{t=0}^{\infty},$$

a set of prices $\Big\{ \{P_{j,t}\}_{j=1}^N, P_t, R_t, W_t \Big\}_{t=0}^{\infty}$, and a stationary distribution over firms' states such that

1. given the set of prices and $\{\Omega_i^F\}_{i\in[0,1]}$, the household's allocation solves its problem as specified in Equation (6) ;

2. given the set of prices and Ω^H, and the implied labor supply and output demand, firms' allocations solve their problem as specified in Equation (8) ;

3. given the set of prices, Ω^H, and $\{\Omega_i^F\}_{i\in[0,1]}$, $\{M_t \equiv P_t C_t\}_{t\geq 0}$ satisfies the monetary policy rule specified in Equation (10) ;

[39] While I implicitly assume that firms' signals are informative only about the idiosyncratic good-specific shocks and aggregate nominal wage, this signal structure is optimal after taking the second-order approximation to the firms' profit function as I do when solving the problem. Up to the second-order approximation, only the aggregate nominal wage and the idiosyncratic shocks matter for the firm's profit function. Thus, this assumption is without loss of generality.

4. all markets clear: for all $t \geq 0$,

$$
\begin{aligned}
Y_{i,j,t} =& C_{i,j,t}, \qquad \text{for all } i \in [0,1] \text{ and } j = 1, 2, \cdots, N, \\
L_t =& \int \left(\sum_{j=1}^{N} L_{i,j,t} \right) di, \\
B_t =& 0\,;
\end{aligned}
$$

5. the stationary distribution is consistent with actions.

4.2 Computing the Equilibrium

I assume the representative household's preferences of the form $U(C, L) = \log(C) - L$. The log-utility of consumption implies that the intertemporal optimal condition relates the nominal interest rate to the law of motion of aggregate demand. This assumption enables me to formulate monetary policy in terms of either the nominal interest rates or the aggregate nominal demand. Moreover, linear disutility in labor ensures that the nominal wage is proportional to the nominal aggregate demand, $W_t = P_t C_t = M_t$. This closely follows Golosov and Lucas (2007) and Midrigan (2011) and makes monetary shocks translate one-for-one into changes in the firms' nominal marginal costs.

Firms' profit functions (7) imply that without any frictions in price setting and in information processing, firm i's frictionless optimal price of good j, $P^*_{i,j,t}$, is a constant markup over its nominal marginal cost:

$$P^*_{i,j,t} = \frac{\varepsilon}{\varepsilon - 1} W_t A_{i,j,t}.$$

Let $\mu_{i,j,t} = \bar{\mu} \frac{P_{i,j,t}}{P^*_{i,j,t}}$ be firm i's true price gap for good j, which is a gap between the actual price and the frictionless optimal price. Here $\bar{\mu} = \frac{\varepsilon}{\varepsilon - 1}$ is a non-stochastic steady-state level of the price gap. Then, firm i's nominal flow profit at time t can be written as a function of the set of these price gaps:

$$\sum_{j=1}^{N} \left(\mu_{i,j,t} - 1\right) \left(\mu_{i,j,t}\right)^{-\varepsilon} \left(W_t\right)^{1-\varepsilon} \left(P_{j,t}\right)^{\varepsilon - \gamma} \left(P_t\right)^{\gamma} Y_t. \tag{11}$$

To solve the firm's problem, I take the second-order approximation to the firms' profit function (11) and derive firms' losses from sub-optimal pricing following the rational inattention literature (e.g., Maćkowiak and Wiederholt 2009, Pasten and Schoenle 2016, Afrouzi 2019b).[40] I assume

[40] See Appendix A for the detailed derivation of the second-order approximation and the recursive representation for

that the set of available signals, $\mathcal{S}_{i,t}$, has the following properties. First, the firm chooses $N+1$ independent signals for each shock, implying that paying attention to aggregate conditions and paying attention to good-specific idiosyncratic conditions are separate activities. Second, each signal is Gaussian. Third, all noise in signals is idiosyncratic and independent.[41] The second-order approximation reduces the state space of the problem from an entire distribution to its covariance matrix. Moreover, because the signals that firms choose are Gaussian and the objective function is quadratic, it enables us to focus on a Gaussian posterior. Under these assumptions, each firm's problem is virtually identical to that studied in Section 3.1.

I compare two economies with $N = 1$ and $N = 2$.[42] I use the method of value function iteration to solve the firms' problem and simulate the economy with a large number of firms for a long period to make sure that the economy reaches a stationary distribution over firms' states. I describe the computational details in Appendix C.

4.3 Calibration and Parameterization

In the numerical exercises, I set the monthly discount factor to $\beta = 0.96^{(1/12)}$, which implies a real interest rate of 4 percent. I set the elasticity of substitution across firms to be four ($\varepsilon = 4$), which matches the firms' average markup of 33% in the survey data.[43] Moreover, I assume the elasticity of substitution between goods is the same as that across firms ($\gamma = 4$). However, the value of γ plays little role, as there are no common good-specific shocks in the model.

I calibrate the standard deviation of the log difference in nominal demand, σ_m, to match the standard deviation of the growth rate of nominal GDP in New Zealand, 0.0044.[44] There are three key model parameters that should be calibrated: the size of menu costs (θ), the size of marginal costs of information processing (ψ), and the size of idiosyncratic good-specific shocks (σ_a). I assume that the marginal cost of processing one bit of information in both one-good and two-good

the firms' problem.

[41] See Appendix E for a discussion on the implications and limitations of these assumptions.

[42] Because the number of state variables is increasing linearly with the number of products as $2N+1$, a model with more than two products is hard to solve. A two-product economy is also considered the baseline in Midrigan (2011) and Karadi and Reiff (2019). Moreover, in the New Zealand survey data, the average of the main product's share of total output value is about 60%, excluding single-product firms. This number implies that a two-product firm is a good benchmark for a multi-product firm. In Section 5.1, I solve models with any arbitrary number of products under some simplifying assumptions.

[43] This value is in the middle of 3 and 7, the elasticity of substitution parameters in Midrigan (2011) and Golosov and Lucas (2007), respectively. It directly affects the slopes of profit curves, and thus the estimates of menu costs and the standard deviation of good-specific shocks, without altering the main findings.

[44] I restrict the sample to post-1991 New Zealand data, as New Zealand has explicitly conducted monetary policy by targeting inflation in that time period.

Table 3: Data and Model Moments

	Data	Single-product model	Two-product model
Median (absolute) size of price changes	0.0576	0.0576	0.0576
Median frequency of price changes	0.0833	0.0833	0.0833
Slope of the backcast error curve	-0.020	-0.020	

Notes: The table presents moments of the data and simulated series from the single- and two-product models parameterized at the baseline values in Table 4. To get the slope of the backcast error curve, I regress the absolute value of firm errors about past 12 month nominal GDP growth rate from Wave #4 survey on the number of products each firm produces. Regression results are reported in Table A.4 in appendix. See Section 4.3 for details.

versions of the model are the same.[45] I calibrate these three parameters to match the median frequency of price changes (once a year), the median size of absolute price changes (5.76%), and slope of backcast errors in the growth rate of aggregate nominal GDP on the number of products (-0.02) observed in the survey data.[46]

The latter is obtained by regressing the firms' backcast errors in the growth rate of nominal GDP on the firms' number of products.[47] The estimate shows that, controlling for firm and manager characteristics as well as industry controls, the backcast errors decrease by 0.02 percentage points when firms produce one more good.[48] The model counterpart measure is calculated by taking the difference between average backcast errors in the growth rate of nominal demand in single- and two-product models. The three moments exactly identify the three key model parameters, and, as Table 3 shows, all the targeted moments are well matched.

Table 4 shows the calibrated and assigned parameters in both single-product and two-product models. The baseline parameterization implies a menu cost of 0.74 percent of steady-state (per good) revenue in the single-product model. Given the average frequency of price changes, the overall cost of price adjustment in the single-product model is around 0.062 percent of steady-state revenue. Similarly, the overall cost of price adjustment in the two-product model is around 0.058 percent of steady-state revenue. These values are smaller than estimates in the previous literature, which often used U.S. data, as the average size of absolute price changes in New Zealand

[45]As shown in Pasten and Schoenle (2016), an alternative assumption for the cost of information processing, such as a constant loss per good from imperfect information, does not change the main findings in this paper.

[46]Parker (2017) also finds that the median frequency of price changes is once a year in New Zealand using *2010 Business Operations Survey* data carried out by Statistics New Zealand, though the data does not provide a quantitative measure of the size of price changes.

[47]The backcast errors in the nominal GDP growth rate are from the fourth wave of the survey. In the survey, firms' managers are asked about current inflation and the real GDP growth rate in New Zealand. I take a summation of both measures to obtain firms' perceived growth rate of nominal GDP in the economy.

[48]See Appendix Table A.4 in for the regressions results with and without controls.

Table 4: Calibration and Assigned Parameters

	Single-product model	Two-product model
Panel A. Calibrated parameters		
Menu cost (θ)	0.0074	0.0281
Information cost (ψ)	0.0035	0.0035
Standard deviation of idiosyncratic shocks (σ_a)	0.0183	0.0188
Standard deviation of monetary policy shocks (σ_m)	0.0044	0.0044
Panel B. Assigned parameters		
Time discount factor (β)	0.9966	0.9966
Elasticity of substitution across firms (ε)	4.0	4.0
Elasticity of substitution between goods (γ)		4.0

Notes: The table presents the baseline parameters for the general equilibrium models with single- and two-product firms. Panel A shows the calibrated parameters which match the three key moments shown in Table 3. Panel B shows the assigned parameters. See Section 4.3 for details.

is small.[49] The calibrated standard deviations of the idiosyncratic good-specific shocks are around 1.7 percent per month in both models, which are about four times bigger than the standard deviation of the monetary policy shock. The calibrated cost of information processing, $\psi\mathcal{I}(\cdot;\cdot)$, is 0.11 percent of steady-state (per good) revenue. The cost is relatively smaller than menu costs and this small cost implies that imperfect information models do not require large information costs to match the data. The implied average Kalman gains on the signal about idiosyncratic shocks are 0.27 in both models (see Table 5). This is equivalent to a quarterly gain of 0.60, which is slightly higher than the estimate of 0.50 in Coibion and Gorodnichenko (2015), which uses the U.S. Survey of Professional Forecasters data.

4.4 Simulation

In this section, I show the simulation results for two models with a large number of firms facing both idiosyncratic good-specific shocks and the monetary policy shock.[50] I show that the two-product model has a more realistic distribution of price changes compared with the single-product model by generating both large and small price changes. Also, I emphasize two distributional characteristics that will be important to understand the sizes of the real effects of the monetary

[49]For example, Levy et al. (1997) find menu costs of 0.7 percent of revenue, while Zbaracki et al. (2004) find price adjustment costs as large as 1.2 percent. Stella (2018) estimates the total cost of changing prices to be between 0.3% and 1.3% of revenues. Moreover, the baseline calibration in Midrigan (2011) implies a menu cost of 0.34 percent of revenue, while it is 0.15 percent of revenue in Karadi and Reiff (2019).

[50]A simulation algorithm for the two-product model is presented in Appendix D.

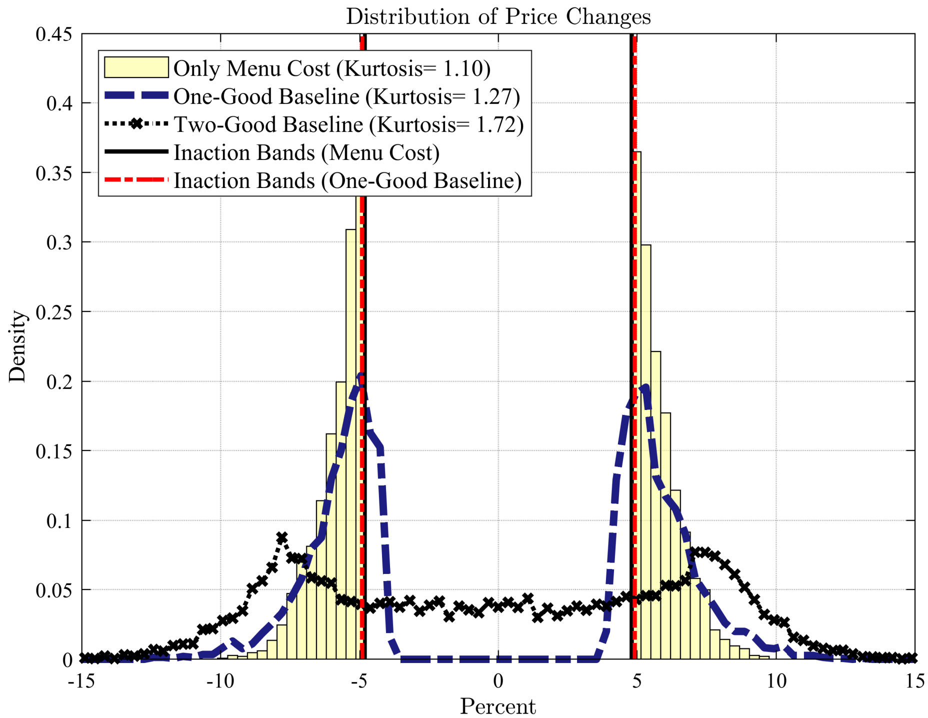


Figure 6: Distributions of Price Changes

Notes: This figure plots the distribution of price changes in the single-product menu cost model with perfect information (yellow bar), that in the one-good version of the baseline model (blue dashed line), and that in the two-good version of the baseline model (black line with cross markers). Black vertical lines are the inaction bands for firms in the menu cost model with perfect information. In this model, every firms have the same inaction bands. Red vertical dash-dot lines are the average of inaction bands across firms in the one-good version of the baseline model. Notice that in this model, the inaction bands vary with firms' subjective uncertainty, as shown in Figure **??**.

shocks in both models, which I extensively investigate in the next section. First, there is a selection in information processing: price adjusters have better information about both idiosyncratic and aggregate shocks than non-adjusters. In particular, I show that selection in information processing about idiosyncratic shocks endogenizes a leptokurtic distribution of desired price changes, which acts as a force to weaken selection effects of price changes. Second, multi-product firms value more information about the monetary policy shock than the single-product firms.

Distribution of Price Changes. Figure 6 shows the distribution of price changes in the single-product model and in a two-product model. As a comparison, I also plot the distribution of price changes in the menu-cost-only model with single-product firms (yellow bar). All three models are calibrated to match the same frequency and size of price changes. In the baseline single-product model (blue bar), there are no small price changes because price changes occur when firms believe that their price is outside of their inaction bands. However, the kurtosis of the distribution in the baseline single-product model is higher than that in the menu-cost-only model, and there is a relatively small fraction of firms around the inaction bands in the baseline model compared with the menu-cost-only model.[51] Notice that firms in the baseline model have different inaction bands depending on their subjective uncertainty, while firms in the menu-cost-only model have the same inaction bands (black vertical line). The heterogeneity in firms' subjective uncertainty makes the distribution of price changes in the baseline model more dispersed than that in the menu-cost-only model.

In contrast to the single-product model, the baseline two-product model generates both small and large price changes because of economies of scope in menu cost technology: when a two-product firm believes that one of its prices is far away from its perceived optimal price, the firm pays a fixed menu cost to change its price. As additional price changes are free after paying this menu cost, the firm also changes the price of the other product, even if it is still very close to the perceived optimal price. Thus, the economy with two-product firms can have a large fraction of small price changes and a higher kurtosis of price changes.[52] This motive based on economies of scope in menu cost technology weakens selection effects of price changes, which act as a strong force to reduce monetary non-neutrality in a standard menu cost model such as Golosov and Lucas (2007).

Selection in Information Processing and Endogenous Leptokurtic Distribution of Price Gaps. Table 5 shows another important characteristic about firms' optimal information choices. The second and third rows compare the average Kalman gain of firms that adjust their prices to those of firms that do not adjust their prices. The Kalman gain represents how much weight firms put on new information relative to their prior estimates. When firms' signals are perfectly telling about the true underlying shocks, the Kalman gain is 1; in this model, when firms optimally choose not to be perfectly informed about the shocks due to the cost of information, the Kalman gain is less than 1. Thus, the average Kalman gains can be interpreted as the average degree of firms'

[51] The red vertical lines are the average inaction bands across all firms in the baseline single-product model. Because I calibrate both models to have the same frequency and size of price changes, the average inaction bands in both models are very similar.

[52] Standard menu cost models with two-product firms can also generate small price changes through the same economies of scope motive (e.g. Midrigan, 2011; Bhattarai and Schoenle, 2014; Alvarez and Lippi, 2014). However, the baseline two-product model has a more dispersed distribution of price changes than the menu cost only models with two-product firms because, again, firms' optimal inaction bands are a function of their subjective uncertainty. Figure A.3 shows a comparison of the distribution of price changes in the baseline two-product model with that in the menu cost only model with two-product firms.

Table 5: Average Kalman Gains in Models

	Single-product model		Two-product model	
Average Kalman gains	Signal about good-specific shocks ($\mathcal{K}^a$)	Signal about monetary shocks ($\mathcal{K}^m$)	Signal about good-specific shocks ($\mathcal{K}^a$)	Signal about monetary shocks ($\mathcal{K}^m$)
All firms	0.267	0.089	0.279	0.131
- Price adjusters	0.611	0.257	0.653	0.259
- Price non-adjusters	0.235	0.074	0.244	0.120

Notes: The table presents average Kalman gains across firms in the baseline single- and two-product models. Column (1) and (3) show the average Kalman gains on the signal about the idiosyncratic good-specific shocks in the single-product model and those in the two-product model, respectively. Column (2) and (4) show the average Kalman gains on the signal about the monetary policy shock in the single-product model and those in the two-product model, respectively.

attentiveness to the underlying shocks. I find that there is a selection in information processing; price adjusters are more informed about both idiosyncratic and aggregate shocks than price non-adjusters. The price adjusters' average Kalman gains in the signals about idiosyncratic shocks are about three times bigger than those of price non-adjusters. The price adjusters also put more weight on new information about the aggregate shock than price non-adjusters. These findings are true in both the single- and the two-product models, implying that selection in information processing operates regardless of the number of products in the model.

This selection in information processing is due to the interaction between firms' optimal information and pricing decisions. As shown in Section 3.1, firms' optimal information acquisition policies are affected by their beliefs about their price gaps. If a firm believes that its price is far away from its optimal level and close to the inaction bands, the potential losses from mistakes in pricing decisions would be very large, which makes the firm process more information about the shocks to reduce losses. After the realization of shocks, this firm is more likely to be a price adjuster because its prior price gap is close to the inaction bands. For this reason, on average, the price adjusters in the economy are better informed about the underlying shocks than the price non-adjusters.

This new selection mechanism in information processing about idiosyncratic shocks has an important implication about the distribution of desired price changes: it *endogenously* generates a leptokurtic distribution of firms' perceived desired price changes.[53] Figure 7 shows the distribu-

[53]If a firm has a price gap of x % and it is free to change its price, then it would change by $-x$%. I use "price gaps" and "desired price changes" interchangeably.

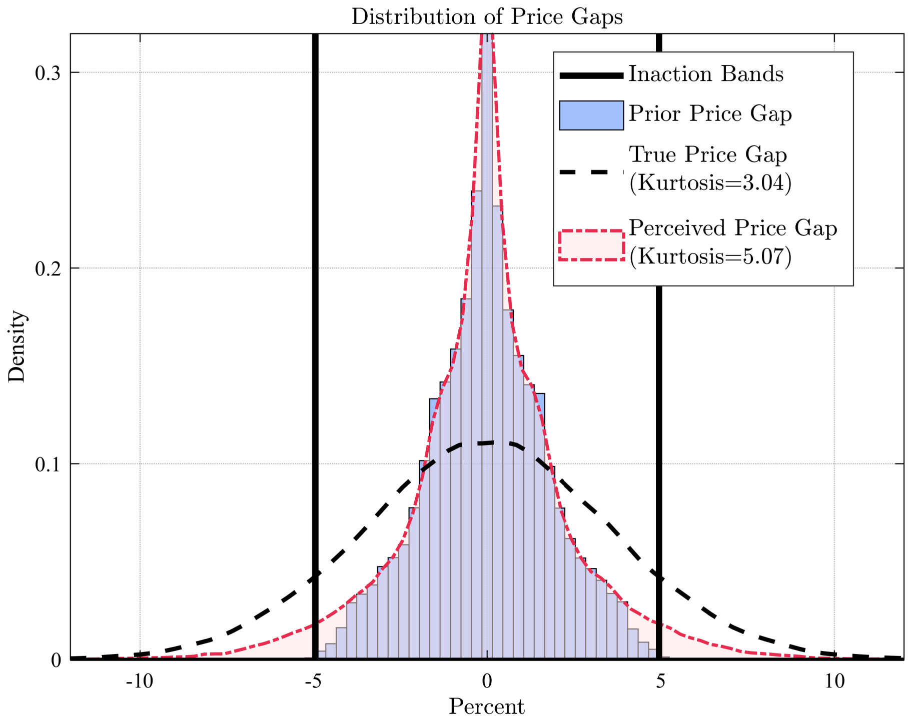


Figure 7: Distributions of True and Perceived Price Gaps in the Single-Product Model

Notes: This figure plots distributions of price gaps in the one-good version of the baseline model. Black lines are the average of inaction bands across firms in the model. At the beginning of period, before the realization of shocks, all firms believe that their price is within the inaction bands. Blue bar graph shows the distribution of firms' *prior* about their price gaps ($p_{i,j,t-1} - \mathbb{E}_{t-1}[p^*_{i,j,t}|S^{t-1}]$) at the beginning of period. After the Gaussian shocks realized, firms' marginal costs change, and thus their true price gap ($p_{i,j,t-1} - p^*_{i,j,t}$) also changes. Black dashed line shows the distribution of these true price gaps. Firms choose their optimal signals about the shocks and form a new posterior about their (frictionless) optimal price. Then, the *posterior* of perceived price gap is $p_{i,j,t-1} - \mathbb{E}_t[p^*_{i,j,t}|S^t]$. Red dash-dot line shows the distribution of these perceived price gaps.

tions of the perceived and true price gaps in the single-product model. First, the blue line is a prior distribution about firms' perceived price gaps. At the beginning of the period, all firms believe that their prices are within their inaction bands and there are many zero perceived price gaps of zero, which were adjusted at the previous period. This implies that the *prior* distribution of firms' perceived price gaps is very concentrated around zero and has a high kurtosis. After being hit by Gaussian idiosyncratic shocks, the distribution of true price gaps in the economy is Gaussian (red dashed line). If firms have perfect information about their true optimal prices, their pricing decisions would be based on their true price gaps. As the distribution of the true price gaps is Gaussian, as in the standard single-product menu cost model, there would be large selection effects of price changes: an expansionary monetary shock triggers many large price increases, but it offsets a mass of large price decreases.

However, in the model with both menu costs and informational costs, firms are rationally inattentive about their true optimal prices. Firms all choose their optimal Gaussian signals and update their estimates of price gaps, but they do not do so in the same way. Firms that think that their price gap is well within their inaction bands and that think it is unlikely that they will need to change prices have little incentive to collect much new information: they choose to remain quite uninformed and update the estimates of their price gaps with a large weight on their (imprecise) priors. In contrast, firms that think they are close to the boundaries of their inaction regions have a high incentive to collect information and therefore choose to become more informed. Because the distribution of priors of firms' perceived price gaps is very concentrated around zero, this selection in information processing makes the distribution of posteriors of the perceived price gaps *leptokurtic*. This leptokurtic distribution implies a small selection effect of price changes in my baseline model because the rationally inattentive firms' pricing decisions are based on their posterior of perceived price gaps. Thus, the endogenous leptokurtic distribution will act as a strong force to amplify monetary non-neutrality in the general equilibrium model that I show in the following subsection. Previous studies of standard menu cost models often assume exogenously a leptokurtic distribution of idiosyncratic shocks to weaken selection effects of price changes (e.g., Gertler and Leahy, 2008; Midrigan, 2011; Vavra, 2013; Karadi and Reiff, 2019; Baley and Blanco, 2019). Unlike these studies, due to selection in information processing about the idiosyncratic shocks, my baseline model with both rational inattention and menu costs can generate the leptokurtic distribution endogenously even if the distribution of underlying shocks is Gaussian.[54]

Value of Information about the Aggregate Shock. Given both idiosyncratic shocks and aggregate shocks, which shocks do firms pay more attention to? Maćkowiak and Wiederholt (2009) show that rationally inattentive firms process more information about an idiosyncratic shock rather than an aggregate shock because the former is more volatile than the latter. Optimal attention allocation in rational inattention models implies that firms have a large incentive to allocate their attention toward more volatile shocks. This finding is also true in my baseline models with single- and two-product firms. The first row of Table 5 shows, in both single- and two-product models, that the average Kalman gains across all firms for the idiosyncratic shocks are larger than those for the aggregate shock because the idiosyncratic shocks are more volatile than the aggregate

[54]This endogenous leptokurtic distribution of perceived price gaps is also present in the two-product model as shown in Appendix Figure A.4.

shock.

However, the amount of information processing about the aggregate shock is different in the single- and the two-product models. As I document in Section 3.1, the value of information about the aggregate shock is higher for the two-product firms than the single-product firms, as the firms' frictionless optimal prices for all goods are affected by the aggregate shock. Table 5 shows that the two-product firms are more informed about the aggregate shocks than the single-product firms. The Kalman gains for aggregates shocks are about twice as large in the two-product model than in the single-product model. This implies that firms in the two-product economy will be more responsive to the monetary policy shock than firms in the single-product economy as they are more informed about it.

4.5 Real Effects of Monetary Policy Shocks

In this subsection, I show that monetary non-neutrality in the one-good version of the model is nearly as large as that in the Calvo sticky price model, while it decreases in the two-good version of the model. To show this, I take the calibrated models and hit them with a one-standard-deviation shock to monetary policy. Figure 8 shows the impulse responses of output in the one-good and the two-good versions of the model. I also show the impulse responses in the standard menu cost model with single-product firms and in the Calvo sticky price model.

The output response to a one-standard-deviation monetary policy shock in the standard menu cost model is small and short-lived. The half-life of output response is only two months. This is a well-known fact in this model: there are large selection effects of price changes, which act as a strong force to reduce monetary non-neutrality (e.g., Golosov and Lucas, 2007). In my one-good version of the baseline model with both menu costs and rational inattention however, the real effects of monetary policy shocks are large and persistent. The impact response increases by 60% and the cumulative output responses, which are defined as the area under the impulse responses of output, are about seven times larger than those in the standard menu cost model. In fact, this large real effect is comparable to that in the Calvo sticky price model.

However, the large real effects are reduced in the two-good version of the baseline model. The cumulative output responses in the two-good version of the model are 12% smaller than those in the one-good version of the model, and the half-life of output responses also decreases from eight months in the single-product model to seven months in the two-product model. Interestingly, as shown in Figure 6, the implied kurtosis of the distribution of price changes is higher in the two-

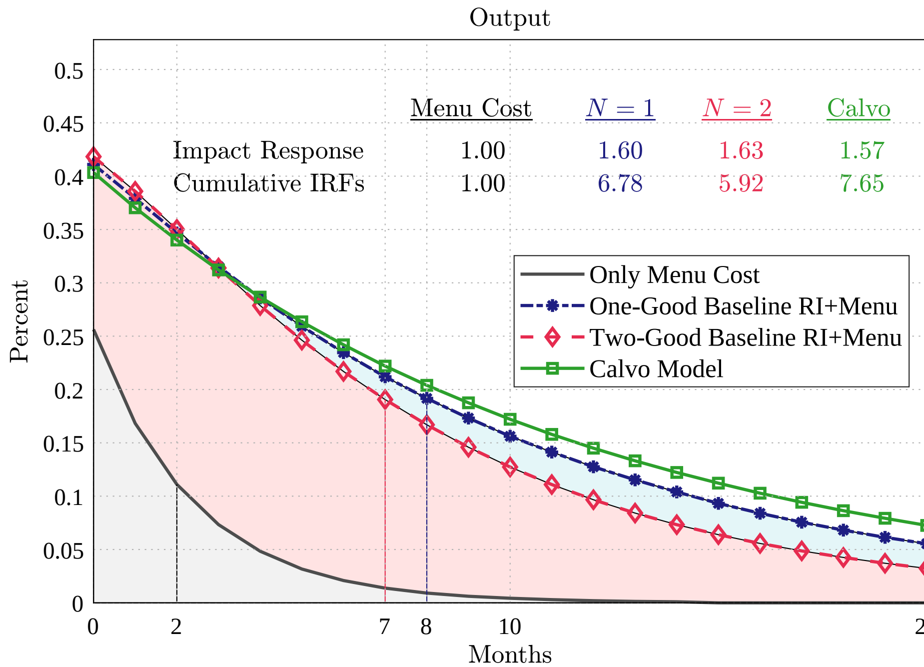


Figure 8: Impulse Response of Output to a One S.D. Monetary Shock

Notes: This figure plots impulse responses of output to a one standard deviation monetary shock. Cumulative IRFs refers to area under the responses of output. I normalize both impact response and cumulative output response in the only menu cost model with single-product firms as one.

good version of the model than in the one-good version of the model. This suggests that in this model with both rational inattention and menu costs, the ratio of kurtosis to the frequency of price changes might not be a sufficient statistic for the output response to a monetary shock, which is derived by Alvarez et al. (2016).[55]

How do the real effects change when I shut down one friction at a time? Consider the economy without informational costs, which coincides with the standard menu cost models (e.g., Golosov and Lucas, 2007, Midrigan, 2011, Alvarez and Lippi, 2014). Appendix Figure A.5 shows the output responses in the pure menu cost models with single- and two-product firms. The real effects of monetary policy shocks are larger in the two-good version of the menu cost model than in the one-good version of the menu cost model. Because the economies of scope of the menu cost technology

[55] The rational inattention models are outside the class of models studied by Alvarez et al. (2016) because of imperfect information about monetary policy shocks. In their model, the real effects are calculated by the output responses to a once and for all unexpected monetary shock that is perfectly observed by firms. In addition, after the monetary shock, firms use the same decision rule used in the steady state. In the rational inattention model, however, firms do not have perfect information about the monetary policy shocks, and their optimal policy rule depends on their uncertainty about the shocks. More importantly, firms' optimal information acquisition decisions are affected by their product scope. As I show in the following subsection, this relationship makes two-product firms more informed about the monetary shock than single-product firms. Thus, monetary non-neutrality in the two-product model is smaller than that in the single-product model even if kurtosis of the distribution of price changes is higher in the two-product model. This imperfect information about the monetary shocks breaks the application of the sufficient statistics derived by Alvarez et al. (2016).

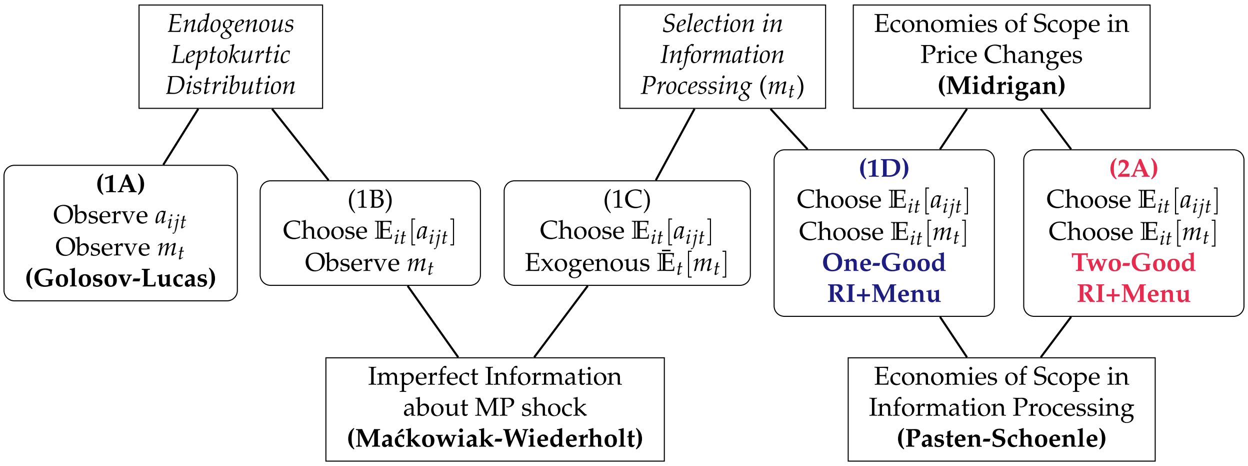
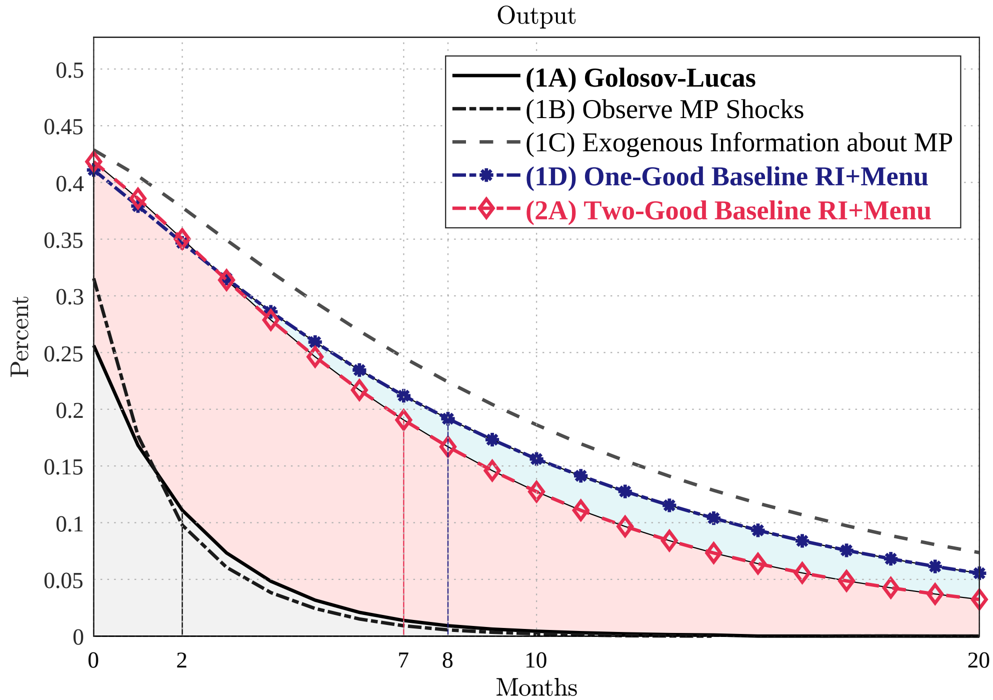


Figure 9: Counterfactuals and Model Mechanism

Notes: This figure shows counterfactual models and the implied model mechanisms. Model (1A), (1B), (1C), and (1D) are single-product menu cost models with different assumptions about firms' information set. In model (1A), firms have full-information about both idiosyncratic and monetary shocks. Firms in model (1B) have perfect information about the monetary shock, but choose their optimal signal about the idiosyncratic shock. All firms in model (1C) are given the same exogenous signal about the monetary shock, while they choose their optimal signal about the idiosyncratic shock. Model (1D) is the baseline single-product model where all firms choose their optimal signals about both shocks. Model (2D) is the baseline two-product model. See Section 4.6 for details.

generate many small price changes, selection effects of price changes are small in the two-good version of the model. The weak selection effects lead to larger real effects in the two-good version of the menu cost model than the one-good version of the model.

Next, I assume that firms are rationally inattentive but free to adjust their prices without menu costs. This model coincides with the pure rational inattention models (e.g., Maćkowiak and Wiederholt, 2009, Pasten and Schoenle, 2016). Appendix Figure A.6 shows the output responses in the pure rational inattention models with single- and two-product firms. The output effects are larger in the one-good version of the rational inattention model than in the two-good version of the model. As I show in Appendix B.1, in the pure rational inattention model with multi-product firms, firms' subjective uncertainty about monetary shocks decrease with their number of products. Moreover, I show that the cumulative response of output to a monetary shock is only a function of their subjective uncertainty about it. Because firms with a greater number of products have better information and smaller uncertainty about monetary shocks, they respond more strongly to the monetary shocks by learning about them more rapidly and therefore changing their prices more rapidly. Thus, the real output effects of monetary policy shocks decrease with the number of products.



	(1A)	(1B)	(1C)	**(1D)**	**(2A)**
Impact Response	**1**	1.23	1.67	**1.60**	**1.63**
Cumulative IRFs	**1**	1.11	7.95	**6.78**	**5.92**

Figure 10: Impulse Responses of Output in Counterfactual Models

Notes: This figure plots impulse responses of output to a one standard deviation monetary shock in counterfactual models described in Figure 9. Cumulative IRFs refers to area under the responses of output. I normalize both impact response and cumulative output response in the only menu cost model with single-product firms as 1. See Section 4.6 for details.

4.6 Inspecting the Mechanisms

In this subsection, I investigate the key mechanisms behind the results of monetary non-neutrality in the baseline model. To this end, I start from the standard menu cost model with single-product firms, such as Golosov and Lucas (2007), and consider counterfactual models by adding core elements of the baseline model. I discuss five main mechanisms; three of them have been studied in the previous literature while two of them are new in this paper. Figure 9 shows how each counterfactual model is related to the underlying mechanisms that I discuss here in detail.

Endogenous Leptokurtic Distribution. The first model (1A) is the standard menu cost model with single-product firms such as in Golosov and Lucas (2007). In this model, firms have perfect information about both idiosyncratic and monetary shocks. As I discussed earlier, this model implies small and short-lived real effects of monetary shocks due to large selection effects of price changes (see the black solid line in Figure 10). For comparison with other counterfactual models, I normalize the impact response of output and the cumulative output responses in this model to 1.

In the next model (1B), I assume that the single-product firms have perfect information about the monetary shock but are rationally inattentive to their idiosyncratic good-specific shock. Because firms choose their optimal signals about the idiosyncratic shock, selection in information processing about the idiosyncratic shock, which I discussed in the previous section, makes the distribution of firms' desired prices endogenously leptokurtic. This implies that price selection effects are small because there is only a small fraction of firms around the inaction bands. Thus, the real effects of monetary policy shocks in this counterfactual model are larger than those in the menu-cost-only model with single-product firms. As shown in Figure 10, the impact output effect in model (1B) increases by 23% compared with that in the menu-cost-only model with single-product firms.[56] This mechanism is new in the literature; the interaction between menu costs and rational inattention generates the endogenous leptokurtic distribution of desired price changes, which amplifies the real effects of monetary shocks in a non-trivial way.

Imperfect Information about Monetary Policy Shocks. Next, I assume that single-product firms are not only rationally inattentive to the good-specific shock and choose their optimal signals about it, but also informationally constrained about monetary policy shocks. I assume that all single-product firms are *exogenously* given a signal about the monetary policy shocks. The signal has the same precision as the steady-state average precision of signals in the one-good version of the baseline model (1D). In other words, firms in this counterfactual economy have the same degree of attentiveness to the monetary policy shocks as do firms in the baseline one-good version of the model where all information choices are endogenous. This counterfactual model (1C) captures the role of imperfect information about monetary policy shocks for monetary non-neutrality. This mechanism has been widely studied in the literature (e.g., Lucas, 1972; Woodford, 2003; Maćkowiak and Wiederholt, 2009). Figure 10 shows that this channel has the most important role for amplifying the real effects of monetary policy shocks. The cumulative output effects are

[56] As firms have perfect information about monetary policy shocks and the aggregate demand follows a random walk process, they immediately observe and respond to the shocks if their prices are around the adjustment margins, which also makes the real effects of monetary shocks in this counterfactual model short-lived.

seven times bigger in this model than those in the standard menu cost model.

Selection in Information Processing about Monetary Policy Shocks. Now, I assume that the single-product firms choose their optimal signal about the monetary policy shocks rather than receive an exogenous signal. This model (1D) is the baseline one-good version of the model that I studied in the previous section. The comparison of this model to model (1C) captures the role of selection in information processing about the monetary policy shocks. Notice that firms in both models (1C) and (1D), *on average*, acquire and process the same amount of information about the monetary policy shocks. While all firms have the same amount of information about the monetary shocks in the counterfactual model (1C), price adjusters to have better information about the monetary shocks than non-adjusters in the baseline one-good version of model (1D). This implies that firms that change their prices following the monetary shocks in the baseline model will adjust more strongly and learn quickly about the shocks compared with the price-adjusting firms in model (1C). Thus, the real effects of monetary shocks are smaller in this baseline one-good version of the model than those in the model (1C) with exogenous information about the monetary shocks. Figure 10 shows that the cumulative output responses in the baseline one-good version of the model are 20 % smaller than those in model (1C). This mechanism is also new in the literature.

Economies of Scope in Price Setting and Information Processing. Lastly, I consider the baseline two-good version of the model. The two-good version of the model entails economies of scope motives in both price setting and information processing that I discussed in the previous section. Notice that both economies of scope motives work in the opposite directions for monetary non-neutrality. On the one hand, economies of scope in price setting make selection effects of price changes weak in the standard menu costs model and amplify the real effects of monetary policy shocks. On the other hand, in the scope economy in information processing, firms with a greater number of products have better information about monetary policy shocks, and thus the real effects of monetary policy shocks decrease with the number of products firms produce. Due to these opposite forces, it is not clear the implications of multi-product pricing for monetary non-neutrality in the model with both rational inattention and menu costs. Figure 10 shows that the cumulative output effects of monetary shocks decrease by 12% in the two-product model compared with the single-product model, which implies that in the calibrated model, economies of scope in information processing act as a strong force to reduce monetary non-neutrality.

5 Extension and Additional Evidence

In this section, I study an extension of the baseline model to study the robustness of the results about monetary non-neutrality. I also provide two kinds of empirical evidence that support the key predictions of the baseline model.

5.1 Models with a Large Number of Products

In the Section 4.5, I show that the real effects of monetary policy shocks decrease in the two-good version of the model compared with the one-good version of the model. In this section, I show that this implication of multi-product pricing for monetary non-neutrality can be extended to the models with an arbitrary large number of products. The main computational challenge for solving the baseline model with more than two products is that the number of state variables double with an additional good produced by the firms and firms' optimal information choice problem is subject to occasionally binding constraints. To simplify the analysis, I make two assumptions. First, I assume that firms choose how much to process information about the underlying shocks *as if* they do not have menu costs. Second, given menu costs, firms choose their prices based on that information, but they are *myopic* in the sense that they do not care about the continuation value of their current pricing decisions. One limitation of making these assumptions is that it eliminates the interesting interaction between rational inattention and menu costs; under these assumptions, all firms have the same information set about the underlying shocks. However, as it simplifies the model analysis by eliminating state variables but keeps the core of the baseline model, I analyze the implications of multi-product pricing for monetary non-neutrality under these assumptions.[57]

Figure 11 shows the cumulative responses of output to a monetary policy shock in the simplified models with various numbers of products. I calibrate each model with a different number of products to have the same size and frequency of price changes. I normalize the cumulative output response in the one-good version of the menu-cost-only model to one. In the menu-cost-only models (red line with circles), the cumulative output effects increase with the firms' number of products. In the models with both rational inattention and menu costs (blue line with diamonds), the real effects decrease with the number of products but converge to the menu-cost-only mod-

[57] For example, as shown in Appendix Figure A.7, the backcast errors in the growth rate of nominal GDP decrease with the number of products. This relationship stems from the economies of scope in information processing in rational inattention models with multi-product firms. Moreover, kurtosis of the distribution of price changes increases with the number of products and converges to a value of three, which is consistent with the implications of menu cost models with multi-product firms.

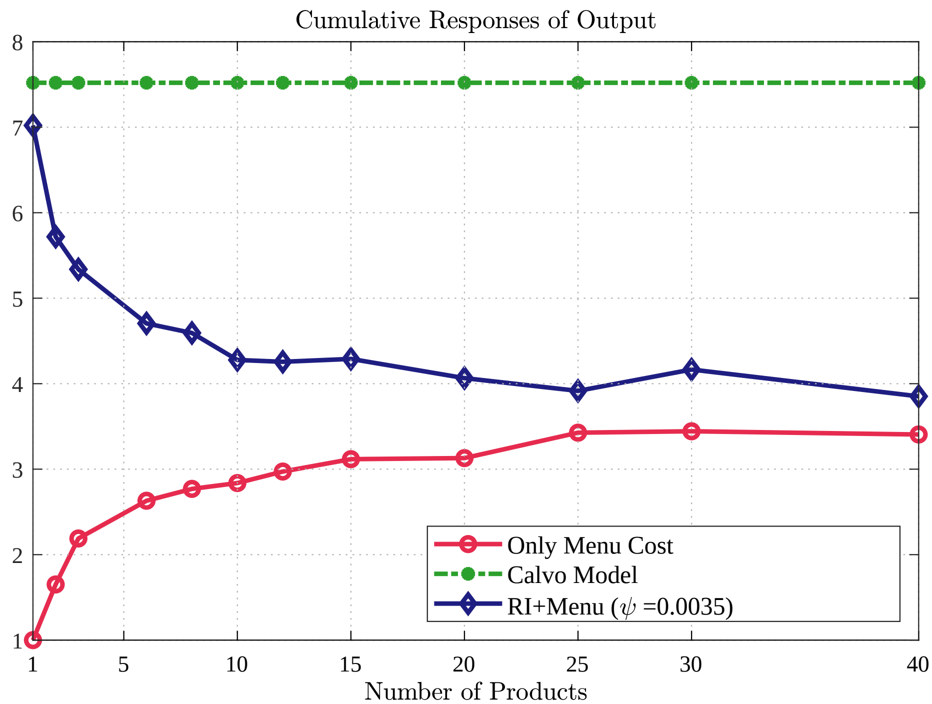


Figure 11: Cumulative Output Responses and Number of Products in the Simplified Models

Notes: This figure plots cumulative output responses in the simplified models with different number of products. "RI+Menu Cost" refers to the model with both menu costs and rational inattention. Red line shows the cumulative output responses in the only menu cost models with different number of products and blue line shows those in the models with both menu costs and rational inattention with different number of products. See Section 5.1 for details.

els with a large number of products. As the number of products increases in the model, firms' subjective uncertainty about monetary policy shocks decreases and converges to zero, implying firms have almost perfect information about the monetary policy shocks.[58] Again, this implies that the ratio of kurtosis to the frequency of price changes might not be a sufficient statistic for monetary non-neutrality in the models with both rational inattention and menu costs. My main conclusion about the relationship between monetary non-neutrality and firms' product scopes can be extended to the model with a large number of products.

5.2 Additional Evidence

In this subsection, I show two additional kinds of evidence that support the key predictions of the baseline model. First, I show using the New Zealand survey data that the empirical distribution of desired price changes has a fat tail. Second, I provide evidence of the wait-and-see rule in firms' price-setting decisions. In the survey data, firms with greater uncertainty are more likely to delay

[58]In Appendix B.1, I show this negative relationship between the number of products and firms' subjective uncertainty about monetary policy shocks.

their price changes.

Evidence on the Leptokurtic Distribution of Desired Price Changes. One of the main results of this paper is to show that selection in information processing about idiosyncratic good-specific shocks endogenously leads to a fat-tail distribution of desired price changes. I find that this result is empirically consistent with what we observe in the survey data. In the second wave of the survey data, firms' managers were asked how much they would like to change the price of their main product if it was free to change its price in three months. The answer gives firms' desired price changes in three months. To construct a model-consistent measure of desired price changes, I define an inflation-adjusted desired price changes as the gap between the desired price changes and their inflation expectations in three months. The left panel of Appendix Figure A.8 shows that the distribution of desired price changes has a cluster near zero, while some desired price changes are very far away from zero. The distribution of desired price changes exhibits kurtosis around 5, implying that the survey supports the fat-tail distribution of desired price changes. The baseline model with both rational inattention and menu costs endogenously captures this distributional characteristic without an assumption that the distribution of idiosyncratic good-specific shocks is leptokurtic.[59]

Subjective Uncertainty and the Duration of Price Changes. Another main characteristic of firms' pricing rule is that firms' optimal inaction bands depend on their subjective uncertainty about the underlying shocks. When firms are more uncertain, the inaction bands are wider, implying that the wait-and-see effects are present in firms' optimal price-setting decision. I directly test this implication using the second wave of the survey data. Firms in this wave were asked to assign probabilities (from 0 to 100) to the different outcomes for growth rates of unit sales of their main product over the next 12 months. I calculate the standard deviation—which is a measure of firms' subjective uncertainty—surrounding firms' sales forecast using the implied probability distribution. The right panel of Appendix Figure A.8 shows that the firms are shorter duration of next price changes when they are less uncertain about their future sales. I regress the duration of firms' expected next price changes on their subjective uncertainty about future sales growth. Appendix Table A.6 shows that firms that have greater uncertainty expect a longer duration be-

[59] The previous literature on menu costs often assumes that this fat-tail distribution of idiosyncratic shocks weakens selection effects of price changes. Midrigan (2011) supports this assumption by providing evidence of excess kurtosis in the distribution of markup gaps in U.S. retail data.

fore their next price changes. This finding is consistent with the prediction of the baseline model. When firms are more uncertain about their fundamentals, they are reluctant to change their prices. Instead, firms want to wait until they acquire more information to resolve their own uncertainty about their fundamentals.

6 Conclusion

Understanding the nature of firms' expectations formation and price setting behavior has been an active area of research in monetary economics. The first part of this paper uses a firm-level survey from New Zealand to show how firms' product scopes are related to their expectations formation and price setting behavior. I find two stylized facts: firms with a greater number of products have both 1) better information about aggregate inflation and 2) more frequent but smaller price changes.

The second part of the paper then builds a dynamic general equilibrium menu cost model with rationally inattentive multi-product firms to study the aggregate implications for monetary non-neutrality. In this model, the interaction between nominal and informational rigidities gives rise to a novel selection effect of information processing: price adjusters have better information about the underlying shocks than non-adjusters because firms with higher subjective uncertainty are less likely to change prices and want to wait until they acquire more information and resolve their uncertainty. I show that this new selection effect leads to a fat-tail distribution of firms' desired price changes and thus weakens selection effects of price changes. As a result, the real effects of monetary policy shocks in the one-good version of the model are nearly as large as those in the Calvo model. Finally, I show that in the two-good version of the model, the cumulative output effects decrease by 12% compared with the one-good version of the model due to the strong economies of scope motive in information processing.

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APPENDIX FOR ONLINE PUBLICATION

A Computing the Equilibrium

In this appendix, I describe a second order approximation to a firm's profit function. Also, I give a recursive formulation of two-product firms' problem that I studied in Section 4.

A.1 Quadratic Approximation to a Firm's Profit Function

Firm i produces N products indexed by j in monopolistic competitive markets. Its demand for good j is given by

$$Y_{i,j,t} = A_{i,j,t}^{\varepsilon-1} \left(\frac{P_{i,j,t}}{P_{j,t}}\right)^{-\varepsilon} \left(\frac{P_{j,t}}{P_t}\right)^{-\gamma} Y_t,$$

where P_t is the price of aggregate output Y_t, $P_{j,t}$ is the price of good j, γ is the constant elasticity of substitution across different firms that produce the same good, and ε is the constant elasticity of substitution across different goods. Then, the firm's profit function is

$$\begin{aligned}
\Pi_{i,t} &= \sum_{j=1}^{N} \left(P_{i,j,t} - W_t A_{i,j,t}\right) Y_{i,j,t} \\
&= \sum_{j=1}^{N} \left(P_{i,j,t} - W_t A_{i,j,t}\right) A_{i,j,t}^{\varepsilon-1} \left(\frac{P_{i,j,t}}{P_{j,t}}\right)^{-\varepsilon} \left(\frac{P_{j,t}}{P_t}\right)^{-\gamma} Y_t,
\end{aligned}$$

where

$$P_t = \left(\frac{1}{N}\sum_{j=1}^{N} P_{j,t}^{1-\gamma}\right)^{\frac{1}{1-\gamma}}, \quad P_{j,t} = \left(\int_0^1 \left(P_{i,j,t}\right)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}.$$

Define firm i's markup for good j, $\mu_{i,j,t} = \frac{P_{i,j,t}}{W_t A_{i,j,t}}$. Then, the profit function can be written as a function of the firm's markups for each good:

$$\begin{aligned}
\Pi_{i,t} &= \sum_{j=1}^{N} \left(P_{i,j,t} - W_t A_{i,j,t}\right) A_{i,j,t}^{\varepsilon-1} \left(\frac{P_{i,j,t}}{P_{j,t}}\right)^{-\varepsilon} \left(\frac{P_{j,t}}{P_t}\right)^{-\gamma} Y_t \\
&= \sum_{j=1}^{N} \left(\mu_{i,j,t} - 1\right) \left(\mu_{i,j,t}\right)^{-\varepsilon} \left(W_t\right)^{1-\varepsilon} \left(P_{j,t}\right)^{\varepsilon-\gamma} \left(P_t\right)^{\gamma} Y_t.
\end{aligned}$$

Let $R_{i,j,t}$ be the revenue from good j:

$$\begin{aligned}
R_{i,j,t} &= P_{i,j,t} A_{i,j,t}^{\varepsilon-1} \left(\frac{P_{i,j,t}}{P_{j,t}}\right)^{-\varepsilon} \left(\frac{P_{j,t}}{P_t}\right)^{-\gamma} Y_t \\
&= \mu_{i,j,t} \left(\mu_{i,j,t}\right)^{-\varepsilon} \left(W_t\right)^{1-\varepsilon} \left(P_{j,t}\right)^{\varepsilon-\gamma} \left(P_t\right)^{\gamma} Y_t.
\end{aligned}$$

A second order approximation to the profit function around the optimal frictionless markup,

$\mu_j^* = \frac{\varepsilon}{\varepsilon - 1}$, yields

$$\Pi\left(\{\mu_{j,t}\}_{j=1}^N\right) = \Pi\left(\left\{\mu_j^*\right\}_{j=1}^N\right) + \frac{1}{2}\sum_{j=1}^N \frac{\partial^2 \Pi_t}{\partial \mu_{j,t}^2}\bigg|_{\left\{\mu_{j,t}=\mu_j^*\right\}_{j=1}^N} \left(\frac{\mu_{j,t}-\mu_j^*}{\mu_j^*}\right)^2 \left(\mu_j^*\right)^2$$

$$= \Pi\left(\left\{\mu_j^*\right\}_{j=1}^N\right) + \frac{1}{2}\sum_{j=1}^N \frac{\partial^2 \Pi_t}{\partial \mu_{j,t}^2}\bigg|_{\left\{\mu_{j,t}=\mu_j^*\right\}_{j=1}^N} \left(\hat{\mu}_{j,t}\right)^2 \left(\mu_j^*\right)^2,$$

where $\hat{\mu}_{j,t} = \log\left(\mu_{j,t}\right) - \log\left(\mu_j^*\right)$ is the realized markup-gap. Then, given the CES demand and the constant returns to scale technology, we can express the expected losses that arise from frictions (both nominal and informational) relative to the frictionless case, expressed as a fraction of per-good revenue:

$$\mathcal{L} \equiv \mathbb{E}\left[\frac{\Pi\left(\{\mu_{j,t}\}_{j=1}^N\right) - \Pi\left(\left\{\mu_j^*\right\}_{j=1}^N\right) - \tilde{\theta}\mathbf{1}_{\{\text{for any } j,\ p_{i,j,t}\neq p_{i,j,t-1}\}} - \tilde{\psi}\mathcal{I}(s_{i,t}; \{\{A_{i,j,t}\}_{j=1}^N, W_t\}|S_i^{t-1})}{R\left(\mu_j^*\right)}\bigg| S^{t-1}\right]$$

$$= \mathbb{E}\Bigg[\frac{1}{2}\frac{1}{R\left(\mu_j^*\right)}\sum_{j=1}^N \frac{\partial^2 \Pi_t}{\partial \mu_{j,t}^2}\bigg|_{\left\{\mu_{j,t}=\mu_j^*\right\}_{j=1}^N} \left(\hat{\mu}_{j,t}\right)^2 \left(\mu_j^*\right)^2$$

$$- \theta\mathbf{1}_{\{\text{for any } j,\ p_{i,j,t}\neq p_{i,j,t-1}\}} - \psi\mathcal{I}(s_{i,t}; \{\{A_{i,j,t}\}_{j=1}^N, W_t\}|S_i^{t-1})\bigg| S^{t-1}\Bigg]$$

$$= \mathbb{E}\Bigg[\frac{1}{2}\frac{\Pi\left(\left\{\mu_j^*\right\}_{j=1}^N\right)}{R\left(\mu_j^*\right)}\sum_{j=1}^N \frac{\left(\mu_j^*\right)^2 \frac{\partial^2 \Pi_t}{\partial \mu_{j,t}^2}\Big|_{\left\{\mu_{j,t}=\mu_j^*\right\}_{j=1}^N}}{\Pi\left(\left\{\mu_j^*\right\}_{j=1}^N\right)} \left(\hat{\mu}_{j,t}\right)^2$$

$$- \theta\mathbf{1}_{\{\text{for any } j,\ p_{i,j,t}\neq p_{i,j,t-1}\}} - \psi\mathcal{I}(s_{i,t}; \{\{A_{i,j,t}\}_{j=1}^N, W_t\}|S_i^{t-1})\bigg| S^{t-1}\Bigg],$$

where

$$\frac{\partial^2 \Pi_t}{\partial \mu_{j,t}^2}\bigg|_{\left\{\mu_{j,t}=\mu_j^*\right\}_{j=1}^N} = \varepsilon\left(\mu_j^*\right)^{-\varepsilon-2}\left[(\varepsilon+1)\left(\mu_j^*-1\right) - 2\mu_j^*\right]\left(\bar{W}\right)^{1-\varepsilon}\left(\bar{P}_j\right)^{\varepsilon-\gamma}\left(\bar{P}\right)^{\gamma}\bar{Y}$$

$$= -\varepsilon\left(\mu_j^*\right)^{-\varepsilon-2}\left(\bar{W}\right)^{1-\varepsilon}\left(\bar{P}_j\right)^{\varepsilon-\gamma}\left(\bar{P}\right)^{\gamma}\bar{Y},$$

$$\Pi\left(\left\{\mu_j^*\right\}_{j=1}^N\right) = \sum_{j=1}^N\left(\mu_j^*-1\right)\left(\mu_j^*\right)^{-\varepsilon}\left(\bar{W}\right)^{1-\varepsilon}\left(\bar{P}_j\right)^{\varepsilon-\gamma}\left(\bar{P}\right)^{\gamma}\bar{Y},$$

and

$$R\left(\mu_j^*\right)=\left(\mu_j^*\right)^{1-\varepsilon}\left(\bar{W}\right)^{1-\varepsilon}\left(\bar{P}_j\right)^{\varepsilon-\gamma}\left(\bar{P}\right)^{\gamma}\bar{Y}.$$

Notice that I express the cost of change price, $\tilde{\theta}$, as a fraction θ of the steady state frictionless revenue from selling one of N products, that is $\tilde{\theta}=\theta R(\mu_j^*)$. Similarly, the marginal cost of information processing is $\tilde{\psi}=\psi R(\mu_j^*)$.

Then, the loss function is

$$\begin{aligned}\mathcal{L}=\mathbb{E}\Bigg[&\frac{1}{2}\frac{\Pi\left(\left\{\mu_j^*\right\}_{j=1}^N\right)}{R\left(\mu_j^*\right)}\sum_{j=1}^N\frac{\left(\mu_j^*\right)^2\left.\frac{\partial^2\Pi_t}{\partial\mu_{j,t}^2}\right|_{\left\{\mu_{j,t}=\mu_j^*\right\}_{j=1}^N}}{\Pi\left(\left\{\mu_j^*\right\}_{j=1}^N\right)}\left(\hat{\mu}_{j,t}\right)^2\\&-\theta\mathbf{1}_{\left\{\text{for any } j,\ p_{i,j,t}\neq p_{i,j,t-1}\right\}}-\psi\mathcal{I}(s_{i,t};\{\{A_{i,j,t}\}_{j=1}^N,W_t\}|S_i^{t-1})\Bigg|S^{t-1}\Bigg]\\=-\mathbb{E}\Bigg[&\varepsilon\frac{1}{2}\left(\frac{\sum_{j=1}^N\left(\mu_j^*-1\right)\left(\mu_j^*\right)^{-\varepsilon}\left(\bar{W}\right)^{1-\varepsilon}\left(\bar{P}_j\right)^{\varepsilon-\gamma}\left(\bar{P}\right)^{\gamma}\bar{Y}}{\left(\mu_j^*\right)^{1-\varepsilon}\left(\bar{W}\right)^{1-\varepsilon}\left(\bar{P}_j\right)^{\varepsilon-\gamma}\left(\bar{P}\right)^{\gamma}\bar{Y}}\right)\\&\times\frac{\sum_{j=1}^N\left(\mu_j^*\right)^{-\varepsilon}\left(\bar{W}\right)^{1-\varepsilon}\left(\bar{P}_j\right)^{\varepsilon-\gamma}\left(\bar{P}\right)^{\gamma}\bar{Y}}{\sum_{j=1}^N\left(\mu_j^*-1\right)\left(\mu_j^*\right)^{-\varepsilon}\left(\bar{W}\right)^{1-\varepsilon}\left(\bar{P}_j\right)^{\varepsilon-\gamma}\left(\bar{P}\right)^{\gamma}\bar{Y}}\left(\hat{\mu}_{j,t}\right)^2\\&+\theta\mathbf{1}_{\left\{\text{for any } j,\ p_{i,j,t}\neq p_{i,j,t-1}\right\}}+\psi\mathcal{I}(s_{i,t};\{\{A_{i,j,t}\}_{j=1}^N,W_t\}|S_i^{t-1})\Bigg|S^{t-1}\Bigg].\end{aligned}$$

Now, assume $\varepsilon=\gamma$ (or by symmetry across product industry due to there are no common industry specific shocks). Then, the second order approximation of the firm's profit function is

$$\mathcal{L}=\mathbb{E}\left[-\varepsilon\frac{1}{2}\left(\frac{1}{\mu_j^*}\right)\sum_{j=1}^N\left(\hat{\mu}_{i,j,t}\right)^2+\theta\mathbf{1}_{\left\{\text{for any } j,\ p_{i,j,t}\neq p_{i,j,t-1}\right\}}+\psi\mathcal{I}(s_{i,t};\{\{A_{i,j,t}\}_{j=1}^N,W_t\}|S_i^{t-1})\middle|S^{t-1}\right].$$

Let $p_{i,j,t}^*=\log(W_t)+\log(A_{i,j,t})$ be the log deviation of (frictionless) optimal price of good j from its non-stochastic steady state. Then, we define firm i's true price gap of good j as

$$\hat{\mu}_{i,j,t}=p_{i,j,t}-p_{i,j,t}^*,$$

where $p_{i,j,t}$ is the log deviation of the price of good j from its non-stochastic steady state.[60] Then, I

[60]The true price gap $\hat{\mu}_{i,j,t}$ can be also written as a markuk gap, $\hat{\mu}_{i,j,t}=\log(\mu_{i,j,t}/\mu_j^*)$, which is a the log deviation of the current markup to the non-stochastic steady state markup $\mu_j^*=\frac{\varepsilon}{\varepsilon-1}$.

derive a second order approximation of firms' loss function:

$$\mathcal{L} = \mathbb{E}\left[-\varepsilon\frac{1}{2}\left(\frac{1}{\mu_j^*}\right)\sum_{j=1}^{N}\left(p_{i,j,t}-p_{i,j,t}^*\right)^2 + \theta\mathbf{1}_{\{\text{for any } j,\ p_{i,j,t}\neq p_{i,j,t-1}\}} + \psi\mathcal{I}(s_{i,t};\{\{A_{i,j,t}\}_{j=1}^{N},W_t\}|S_i^{t-1})\middle| S^{t-1}\right]$$
$$= -\mathbb{E}\left[B\sum_{j=1}^{N}\left(p_{i,j,t}-p_{i,j,t}^*\right)^2 + \theta\mathbf{1}_{\{\text{for any } j,\ p_{i,j,t}\neq p_{i,j,t-1}\}} + \psi\mathcal{I}(s_{i,t};\{\{A_{i,j,t}\}_{j=1}^{N},W_t\}|S_i^{t-1})\middle| S^{t-1}\right],$$

where $B = \frac{\varepsilon-1}{2}$ is a slope of profit curve.

A.2 A Recursive Formulation

At the beginning of period t, firm i takes its initial information set, S_i^{t-1}, as given and chooses a set of optimal signals, $s_{i,t}$, subject to a cost of information processing, $\psi\mathcal{I}(s_{i,t};\{p_{i,j,t}^*\}_{j=1}^{N}|S_i^{t-1}|S_i^{t-1})$, where ψ is a marginal cost of a bit of information and $\mathcal{I}(\cdot)$ is Shannon's mutual information function. Notice that I replace the idiosyncratic shocks and the nominal wage in Shannon's mutual information function with the firm's frictionless optimal price since all the firms need to know is its frictionless optimal price after deriving the firms' loss function. It then forms a new information set, $S_i^t = S_i^{t-1}\cup s_{i,t}$, and sets its new prices, $\{p_{i,j,t}\}_{j=1}^{N}$, based on that. The firm pays a menu cost, θ, if it decides to change any prices. Otherwise, the firm waits for the next period.

Formally, after taking a quadratic approximation of firm i's profit function around non-stochastic steady state and deriving a loss function from the suboptimal prices, the firm's problem can be written as:

$$\min_{\{\{p_{i,j,t}\}_{j=1}^{N},s_{i,t}\}_{t\geq 0}} \mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\left\{B\sum_{j=1}^{N}(p_{i,j,t}-p_{i,j,t}^*)^2 + \theta\mathbf{1}_{\{\text{for any } j,\ p_{i,j,t}\neq p_{i,j,t-1}\}} + \psi\mathcal{I}(s_{i,t};\{p_{i,j,t}^*\}_{j=1}^{N}|S_i^{t-1})\right\}\middle| S_i^{-1}\right]$$
$$\text{s.t.}\quad S_i^t = S_i^{t-1}\cup s_{i,t}.$$

I assume that the set of available signals satisfies that 1) the firm chooses $N+1$ independent signals for each shock, 2) each signal is Gaussian, and 3) all noise in signals is idiosyncratic and independent. Then, the set of optimal signals about both the idiosyncratic good-specific shocks and the monetary shock can be written as:

$$\mathcal{S}_{i,j,t}^a = \{a_{i,j,t} + \eta_{i,j,t}^a\xi_{i,j,t}^a : \eta_{i,j,t}^a \geq 0,\ \xi_{i,j,t}^a \sim N(0,1)\},\ \text{ for } j = 1,2,\cdots,N,$$
$$\mathcal{S}_{i,t}^m = \{m_t + \eta_{i,t}^m\xi_{i,t}^m : \eta_{i,t}^m \geq 0,\ \xi_{i,t}^m \sim N(0,1)\}.$$

Here, rational inattention errors, $\xi_{i,j,t}^a$ and $\xi_{i,t}^m$, are independent across firms. At each time t, given S^{t-1}, firm i chooses its optimal signals $s_{i,j,t}^a \in \mathcal{S}_{i,j,t}^a$ for $j = 1,2,\cdots,N$, and $s_{i,t}^m \in \mathcal{S}_{i,t}^m$ subject to the cost of information processing. Then, the firm's new information set evolves as $S_i^t = S_i^{t-1}\cup s_{i,t}$

where $s_{i,t} = \{\{s^a_{i,j,t}\}_{j=1}^N, s^m_{i,t}\}$. Now, the firms' problem is virtually identical to the problem I studied in Section 3.

Let $x_{j,t} = p_{j,t} - \mathbb{E}[p^*_{j,t}|S^t]$ be firm's *perceived* price gap about product j and

$$z^a_{j,t} = \mathbb{E}\left[\left(a_{j,t} - \mathbb{E}_t[a_{j,t}|S^t]\right)^2 \middle| S^t\right], \quad z^m_t = \mathbb{E}\left[\left(m_t - \mathbb{E}_t[m_t|S^t]\right)^2 \middle| S^t\right]$$

be subjective uncertainty about the j-good specific shock and the aggregate shock, respectively. I rewrite the recursive formation of the firms' problem as:

$$\begin{aligned} V\left(\{x_{j,-1}\}_{j=1}^N, \{z^a_{j,-1}\}_{j=1}^N, z^m_{-1}\right) = \max_{\{\{z^a_j\}_{j=1}^N, z^m\}} &\mathbb{E}\Bigg[\max\left\{V^I\left(\{x_j\}_{j=1}^N, \{z^a_j\}_{j=1}^N, z^m\right), V^C\left(\{z^a_j\}_{j=1}^N, z^m\right)\right\} \\ &- \frac{\psi}{2}\left(\sum_{j=1}^N \log_2\left(\frac{z^a_{j,-1}+\sigma_a^2}{z^a_j}\right) + \log_2\left(\frac{z^m_{-1}+\sigma_m^2}{z^m}\right)\right)\Bigg| S^{-1}\Bigg] \\ \text{s.t.} \qquad & 0 \le z^a_j \le z^a_{j,-1} + \sigma_a^2, \quad \forall j = 1, 2, \cdots, N, \\ & 0 \le z^m \le z^m_{-1} + \sigma_m^2, \end{aligned}$$

where

$$V^I\left(\{x_j\}_{j=1}^N, \{z^a_j\}_{j=1}^N, z^m\right) = -B\sum_{j=1}^N\left(x_j^2 + z^a_j + z^m\right) + \beta V\left(\{x_j\}_{j=1}^N, \{z^a_j\}_{j=1}^N, z^m\right)$$

$$\text{with } \mathbf{x} \sim N\left(\mathbf{x}_{-1}, \Sigma\right), \text{ and}$$

$$V^C\left(\{z^a_j\}_{j=1}^N, z^m\right) = \max_{\{y_j\}_{j=1}^N} -B\sum_{j=1}^N\left(y_j^2 + z^a_j + z^m\right) - \theta + \beta V\left(\{y_j\}_{j=1}^N, \{z^a_j\}_{j=1}^N, z^m\right).$$

Here $\mathbf{x} = \{x_1, x_2, \cdots, x_N\}'$ and

$$\Sigma_t(j,k) = \begin{cases} z^m_{-1} + \sigma_m^2 - z^m & \text{if } j \neq k \\ z^a_{j,-1} + \sigma_a^2 - z^a_j + z^m_{-1} + \sigma_m^2 - z^m & \text{if } j = k. \end{cases}$$

B A Rationally Inattentive Firm's Problem without Menu Costs ($\theta = 0$)

In this appendix, I solve a rationally inattentive firm's problem without menu costs. This problem is similar to the one studied in Pasten and Schoenle (2016) with one main difference. They solve the problem by assuming that the cost of information is not discounted and optimizing at the long-run steady-state for information structure. Here, I assume that the firm discounts future costs of information at the same discount rate as their payoffs and solve the dynamic information acquisition problem. This setup is also similar to Afrouzi and Yang (2019) that study the dynamic multivariate rational inattention problem. One difference is that I assume that the set of available signals are partitioned into two subsets, one for signals about idiosyncratic shocks and the other for signals about aggregate shocks, that are independent each other.

Without menu costs, the firm can change its prices and collapses the price gaps to zero whenever it wants, i.e. there is no micro rigidity in price setting. In this case, the firm's prior price gaps are no longer its state variables, and thus the firm's problem is deterministic. Then, in a recursive formation, the firm's problem is:

$$\begin{aligned} V\left(\{z^a_{j,-1}\}_{j=1}^N, z^m_{-1}\right) = \max_{\{\{z^a_j\}_{j=1}^N, z^m\}} & -B\sum_{j=1}^N\left(z^a_j + z^m\right) + \beta V\left(\{z^a_j\}_{j=1}^N, z^m\right) \\ & -\frac{\psi}{2}\left(\sum_{j=1}^N \log_2\left(\frac{z^a_{j,-1}+\sigma_a^2}{z^a_j}\right) + \log_2\left(\frac{z^m_{-1}+\sigma_m^2}{z^m}\right)\right) \\ \text{s.t.} \qquad & 0 \le z^a_j \le z^a_{j,-1} + \sigma_a^2, \quad \forall j = 1, 2, \cdots, N, \\ & 0 \le z^m \le z^m_{-1} + \sigma_m^2, \end{aligned}$$

Notice that with $\psi > 0$, the constraints $z^a_j \ge 0$ and $z^m_j \ge 0$ will not bind. The first order necessary conditions are:

$$\begin{aligned} \partial z^a_j : \quad & -B + \frac{\psi}{2\log 2}\frac{1}{z^a_j} + \beta V_{z^a_j}\left(\{z^a_j\}_{j=1}^N, z^m\right) - \phi_j = 0, \ \ \forall\, j \in \{1, 2, \cdots, N\}, \\ \partial z^m : \quad & -BN + \frac{\psi}{2\log 2}\frac{1}{z^m} + \beta V_{z^m}\left(\{z^a_j\}_{j=1}^N, z^m\right) - \phi_m = 0, \\ & V_{z^a_{j,-1}}\left(\{z^a_{j,-1}\}_{j=1}^N, z^m_{-1}\right) = -\frac{\psi}{2\log 2}\frac{1}{z^a_{j,-1}+\sigma_a^2} \ \ \forall\, j \in \{1, 2, \cdots, N\}, \\ & V_{z^m_{-1}}\left(\{z^a_{j,-1}\}_{j=1}^N, z^m_{-1}\right) = -\frac{\psi}{2\log 2}\frac{1}{z^m_{-1}+\sigma_m^2}, \end{aligned}$$

and complementarity slackness conditions where $\{\phi_j\}_{j=1}^N$ and ϕ_m are Lagrangian multipliers for no-forgetting constraints. The no-forgetting constraints will bind when the marginal cost of information processing is high enough. Here, I focus on interior solutions where the constraints do not bind.

When the standard deviation of idiosyncratic good-specific shocks is the same, then subjective uncertainty about idiosyncratic shocks is also the same across all goods. Then, the optimal subjective uncertainty about both shocks satisfies:

$$\begin{aligned} B = & \frac{\psi}{2\log 2}\left(\frac{1}{z^a_j} - \beta\frac{1}{z^a_j+\sigma_a^2}\right), \ \ \forall\, j \in \{1, 2, \cdots, N\} \\ B \cdot N = & \frac{\psi}{2\log 2}\left(\frac{1}{z^m} - \beta\frac{1}{z^m+\sigma_m^2}\right). \end{aligned} \tag{12}$$

Several interesting characteristics emerge. First, the firm's optimal subjective uncertainty is constant while it is time-varying with menu costs. Second, subjective uncertainty increases in the size of marginal cost of information processing, ψ, and the size of shocks, σ_a^2, while it decreases in the slope of profit function , B, and the time preference parameter, β. Third, optimal subjective uncertainty about idiosyncratic shocks is independent of the number of products that the firm produces. Fourth, optimal subjective uncertainty about aggregate shocks is *decreasing* in the number of products ($\frac{\partial z^m(N)}{\partial N} < 0$) and $\lim_{N\to\infty} z^m(N) = 0$.

B.1 Real Effects of Monetary Policy Shocks

Let $\tilde{z}^m = \frac{z^m}{\sigma_m^2}$ be firms' subjective uncertainty relative to the variance of monetary policy shocks. Then I can rewrite Equation (12) as:

$$\frac{1}{\tilde{z}^m} - \beta \frac{1}{\tilde{z}^m + 1} = \sigma_m^2 \frac{BN}{\psi} (2 \log 2) \tag{13}$$

Given parameters, firms' subjective uncertainty about monetary shocks decreases in their number of products.

Now, the size of price changes for good j is given by:

$$\begin{aligned} \Delta p_{i,j,t} =& \mathcal{K}^a(N) \left(a_{i,j,t-1} - a_{i,j,t-1|t-1} + \varepsilon_{i,j,t} + \eta_{i,j,t} \right) \\ &+ \mathcal{K}^m(N) \left(m_{t-1} - m_{i,t-1|t-1} + \varepsilon_{m,t} + \eta_{i,m,t} \right), \end{aligned}$$

where

$$\begin{aligned} a_{i,jt} - a_{i,j,t|t} &= (1 - \mathcal{K}^a(N)) \left(a_{i,j,t-1} - a_{i,j,t-1|t-1} + \varepsilon_{i,j,t} \right) - \mathcal{K}^a(N) \eta_{i,j,t}, \\ m_t - m_{i,t|t} &= (1 - \mathcal{K}^m(N)) \left(m_{t-1} - m_{i,t-1|t-1} + \varepsilon_{m,t} \right) - \mathcal{K}^m(N) \eta_{i,m,t}, \end{aligned}$$

and Kalman gains are:

$$\mathcal{K}^a(N) = \frac{1}{1 + \tilde{z}_j^a(N)}, \quad \mathcal{K}^m(N) = \frac{1}{1 + \tilde{z}^m(N)}.$$

Let $p_{j,t} = \int p_{i,j,t} di$. Let $p_{j,t}^{\text{NoMP}}$ and $p_{j,t}^{\text{MP}}$ be price level at time t without and with monetary shocks, respectively. Then, since all noise in signals, $\eta_{i,j,t}$, is idiosyncratic and independent, we have

$$\begin{aligned} p_{j,t}^{\text{NoMP}} &= p_{j,t-1}^{\text{NoMP}} + \Delta p_{j,t}^{\text{NoMP}} \\ &= p_{j,t-1}^{\text{NoMP}} + \left(\mathcal{K}^a(N) \int \left(a_{i,j,t-1} - a_{i,j,t-1|t-1} \right) di + \mathcal{K}^m(N) \int \left(m_{t-1} - m_{i,t-1|t-1} \right) di \right) \end{aligned}$$

and

$$\begin{aligned} p_{j,t}^{\text{MP}} &= p_{j,t-1}^{\text{MP}} + \Delta p_{j,t}^{\text{MP}} \\ &= p_{j,t-1}^{\text{MP}} + \left(\mathcal{K}^a(N) \int \left(a_{i,j,t-1} - a_{i,j,t-1|t-1} \right) di + \mathcal{K}^m(N) \int \left(m_{t-1} - m_{i,t-1|t-1} \right) di + \varepsilon_{m,t} \right) \end{aligned}$$

Then, Notice that by symmetry across goods, we have $p_t = p_{j,t}$ for all j. Define an impulse response of aggregate price to a monetary shock as the gap between the prices with and without the monetary shock, that is,

$$IRF_t^P = p_t^{\text{MP}} - p_t^{\text{NoMP}}.$$

Assume at time 0, there is a monetary shock, $\varepsilon_{m,0}$. Then,

$$\begin{aligned}
IRF_0^P &= \mathcal{K}^m(N)\varepsilon_{m,0} \\
IRF_1^P &= \mathcal{K}^m(N)\varepsilon_{m,0} + \mathcal{K}^m(N)\left(1-\mathcal{K}^m(N)\right)\varepsilon_{m,0} \\
IRF_2^P &= \mathcal{K}^m(N)\varepsilon_{m,0} + \mathcal{K}^m(N)\left(1-\mathcal{K}^m(N)\right)\varepsilon_{m,0} + \mathcal{K}^m(N)\left(1-\mathcal{K}^m(N)\right)^2\varepsilon_{m,0} \\
&\vdots \\
IRF_t^P &= \mathcal{K}^m(N)\left(\frac{1-\left(1-\mathcal{K}^m(N)\right)^{t+1}}{1-\left(1-\mathcal{K}^m(N)\right)}\right)\varepsilon_{m,0} \\
&= \left(1-\left(1-\mathcal{K}^m(N)\right)^{t+1}\right)\varepsilon_{m,0}
\end{aligned}$$

and output responses are given by

$$\begin{aligned}
IRF_t^Y &= \varepsilon_{m,0} - IRF_t^P \\
&= \left(1-\mathcal{K}^m(N)\right)^{t+1}\varepsilon_{m,0}.
\end{aligned}$$

Let a cumulative impulse response of output as a function of the number of product, $M(N)$ be the area under the impulse response function of output. Then,

$$\begin{aligned}
\mathcal{M}(N) &= \int_0^\infty IRF_t^Y dt = \int_0^\infty \left(1-\mathcal{K}^m(N)\right)^{t+1}\varepsilon_{m,0}dt \\
&= -\frac{\left(1-\mathcal{K}^m(N)\right)}{\log\left(1-\mathcal{K}^m(N)\right)}\varepsilon_{m,0} \\
&= -\frac{\left(\frac{\tilde{z}_m(N)}{1+\tilde{z}_m(N)}\right)}{\log\left(\frac{\tilde{z}_m(N)}{1+\tilde{z}_m(N)}\right)}\varepsilon_{m,0}.
\end{aligned}$$

where $\tilde{z}_m(N)$ is a solution of Equation (13) as a function of N. Notice that

$$\frac{\partial\mathcal{M}(N)}{\partial N} = -\underbrace{\frac{\frac{\partial\tilde{z}^m(N)}{\partial N}}{\left(1+\tilde{z}^m(N)\right)^2}}_{<0} \times \underbrace{\frac{1}{\log\left(\frac{\tilde{z}_m(N)}{1+\tilde{z}_m(N)}\right)}}_{<0} \times \underbrace{\left(1-\frac{1}{\log\left(\frac{\tilde{z}_m(N)}{1+\tilde{z}_m(N)}\right)}\right)}_{>0} < 0.$$

Although N is an arbitrary integer number, here I assume $\mathcal{M}(N)$ is continuously differentiable with respect to N. Appendix Figure A.2 shows that both subjective uncertainty about monetary shocks and cumulative responses of output to monetary shocks decreases in number of products that firms produce.

C Computational Procedures for the Two-Product Model

I use the method of value function iteration to solve the two-product firms' optimization problem. There are 5 state variables for the problem: prior perceived price gap for two products, subjective uncertainty about each good-specific shock, and prior subjective uncertainty about monetary policy shocks. Since this problem is non-convex optimization problem with occasionally binding constraints, it should b e solved numerically.

The most computationally burdensome part is to compute firms' expected future values since 1) tomorrow's perceived price gaps are stochastic variables when firms do not change prices today, and 2) the distribution of these price gaps has a mean vector $(x_{1,-1}, x_{2,-1})'$, which is firms' state variable, and a covariance matrix, Σ, which is firms' choice variable. Standard approximation methods for the transition probability of states, such as Tauchen approximation method, are not applicable since the approximation errors are quite large. I compute expected value of the firms' value functions using Gauss-Legendre quadrature which is an explicit numerical integration technique.

I solve the firms' problem and the value function and the optimal policy functions using the following procedure:

1. Construct grids for individual state variables, such as prior of perceived price gaps for each product, $x_{1,-1}$, $x_{2,-1}$, prior subjective uncertainty about two good-specific shocks, $z^a_{1,-1}$, $z^a_{2,-1}$, and prior subjective uncertainty about monetary shocks, z^m_{-1}. I use 21 grids for $x_{1,-1}$, $x_{2,-1}$, and 16 grids for $z^a_{1,-1}$, $z^a_{2,-1}$, and z^m_{-1}. The ranges of $x_{1,-1}$ and $x_{2,-1}$, are $[-1.5\sqrt{\theta/B}, 1.5\sqrt{\theta/B}]$ where θ is the size of menu costs and B is the slope of firms' profit curve.[61] More grid points are assigned around inaction bands. $z^a_{1,-1}$, $z^a_{2,-1}$, and z^m_{-1} are equally spaced in the range of [0 0.004].

2. Construct the abscissas, $\{\tilde{x}_i\}_{i=1}^{N_q}$, and weights, $\{\tilde{w}_i\}_{i=1}^{N_q}$, of the Gauss-Legendre quadrature with $N_q = 500$ points.

3. Solve the individual value functions at each grid point. In this step, I obtain the optimal decision rules for subjective uncertainty about both good-specific shocks and monetary shocks,

   $$g^a_1(x_{1,-1}, x_{2,-1}, z^a_{1,-1}, z^a_{2,-1}, z^m_{-1}),\ g^a_2(x_{1,-1}, x_{2,-1}, z^a_{1,-1}, z^a_{2,-1}, z^m_{-1}),\ g^m(x_{1,-1}, x_{2,-1}, z^a_{1,-1}, z^a_{2,-1}, z^m_{-1}),$$

   and the value function, $V(x_{1,-1}, x_{2,-1}, z^a_{1,-1}, z^a_{2,-1}, z^m_{-1})$. The detailed steps are as follows:

   (a) Make an initial guess for the value functions, V_0, for all grid points.

   (b) Solve firms' optimization problem and compute V_1. Notice that the problem can be

[61] In fact $[-\sqrt{\frac{\theta}{B}}, \sqrt{\frac{\theta}{B}}]$ is the inaction bands for myopic firms ($\beta = 0$). In this regard, I have more conservative ranges of grids for prior price gaps.

written:

$$
\begin{aligned}
&V_1\Big(x_{1,-1}, x_{2,-1}, z^a_{1,-1}, z^a_{2,-1}, z^m_{,-1}\Big) = \\
&\qquad \max_{\{z^a_1, z^a_2, z^m\}} - B\left(z^a_1 + z^a_2 + 2z^m\right) - \frac{\psi}{2}\left(\sum_{j=1}^{2}\log_2\left(\frac{z^a_{j,-1}+\sigma_a^2}{z^a_j}\right) + \log_2\left(\frac{z^m_{-1}+\sigma_m^2}{z^m}\right)\right) \\
&\qquad\qquad + \int_{(x_1,x_2)} \max\Big\{ - B(x_1^2 + x_2^2) + \beta V_0\left(x_1, x_2, z^a_1, z^a_2, z^m\right), \\
&\qquad\qquad\qquad\qquad - \theta + \beta V_0\left(0, 0, z^a_1, z^a_2, z^m\right)\Big\} dF((x_1,x_2);(x_{1,-1},x_{2,-1}),\Sigma) \\
&\qquad \text{s.t.} \qquad 0 \le z^a_j \le z^a_{j,-1} + \sigma_a^2, \quad \forall j = 1,2 \\
&\qquad\qquad\qquad 0 \le z^m \le z^m_{-1} + \sigma_m^2 \\
&\qquad\qquad\qquad \Sigma_t(j,k) = \begin{cases} z^m_{-1} + \sigma_m^2 - z^m & \text{if } j \neq k \\ z^a_{j,-1} + \sigma_a^2 - z^a_j + z^m_{-1} + \sigma_m^2 - z^m & \text{if } j = k. \end{cases}
\end{aligned}
$$

where $F((x_1,x_2);(x_{1,-1},x_{2,-1}),\Sigma)$ is a joint normal distribution with mean $(x_{1,-1}, x_{2,-1})$ and covariance matrix Σ.

(c) If V_0 and V_1 are close enough for each grid point, and go to the next step. Otherwise, update the value functions ($V_0 = V_1$), and go back to (a).

(d) Simulate the model with a large number of firms to obtain a stationary distribution, $G(x_1, x_2, z^a_1, z^a_2, z^m)$, over firm states $(x_1, x_2, z^a_1, z^a_2, z^m)$. Simulation algorithm is described in Appendix D.

(e) Compute aggregate variables.

D Simulation Algorithm for the Two-Product Model

I simulate the two-good version of the baseline model with 100,000 firms for 5,000 periods.

1. Set initial $x_{i,1,t-1}$, $x_{i,2,t-1}$, $z^a_{i,1,t-1}$, $z^a_{i,2,t-1}$, and $z^m_{i,t-1}$.

2. Generate random numbers for the shocks $\varepsilon^m_t \sim N(0,\sigma_m^2)$, $\varepsilon^a_{i,1,t} \sim N(0,\sigma_a^2)$, and $\varepsilon^a_{i,2,t} \sim N(0,\sigma_a^2)$

3. Find $z^m_{i,t}$, $z^a_{i,1,t}$, and $z^a_{i,2,t}$, given policy functions,

$$
\begin{aligned}
g^a_1\left(x_{i,1,t-1}, x_{i,2,t-1}, z^a_{i,1,t-1}, z^a_{i,2,t-1}, z^m_{i,t-1}\right) \\
g^a_2\left(x_{i,1,t-1}, x_{i,2,t-1}, z^a_{i,1,t-1}, z^a_{i,2,t-1}, z^m_{i,t-1}\right) \\
g^m\left(x_{i,1,t-1}, x_{i,2,t-1}, z^a_{i,1,t-1}, z^a_{i,2,t-1}, z^m_{i,t-1}\right).
\end{aligned}
$$

4. Calculate standard deviations of signal noises and Kalman gains from

$$\begin{aligned} z^m_{i,t} &= \left(1-\mathcal{K}^m_{i,t}\right)\left(z^m_{i,t-1}+\sigma^2_m\right) \\ z^a_{i,1,t} &= \left(1-\mathcal{K}^a_{i,1,t}\right)\left(z^a_{i,1,t-1}+\sigma^2_1\right) \\ z^a_{i,2,t} &= \left(1-\mathcal{K}^a_{i,2,t}\right)\left(z^a_{i,2,t-1}+\sigma^2_1\right) \end{aligned}$$

and

$$\mathcal{K}^m_{i,t} = \frac{z^m_{i,t-1}+\sigma^2_m}{z^m_{i,t-1}+\sigma^2_m+\eta^2_{i,m,t}},\ \mathcal{K}^a_{i,1,t} = \frac{z^a_{i,1,t-1}+\sigma^2_1}{z^a_{i,1,t-1}+\sigma^2_1+\eta^2_{i,1,t}},\ \mathcal{K}^a_{i,2,t} = \frac{z^a_{i,2,t-1}+\sigma^2_2}{z^a_{i,2,t-1}+\sigma^2_2+\eta^2_{i,2,t}}.$$

Then

$$\begin{aligned} \eta^2_{i,m,t} &= \frac{z^m_{i,t}\left(z^m_{i,t-1}+\sigma^2_m\right)}{z^m_{i,t-1}+\sigma^2_m-z^m_{i,t}} \\ \eta^2_{i,1,t} &= \frac{z^a_{i,1,t}\left(z^a_{i,1,t-1}+\sigma^2_1\right)}{z^a_{i,1,t-1}+\sigma^2_1-z^a_{i,1,t}} \\ \eta^2_{i,2,t} &= \frac{z^a_{i,2,t}\left(z^a_{i,2,t-1}+\sigma^2_1\right)}{z^a_{i,2,t-1}+\sigma^2_1-z^a_{i,2,t}} \end{aligned}$$

5. Generate random numbers for signal noises $\xi_{i,m,t} \sim \mathcal{N}\left(0,\eta^2_{i,m,t}\right)$, $\xi_{i,1,t} \sim \mathcal{N}\left(0,\eta^2_{i,1,t}\right)$, $\xi_{i,2,t} \sim \mathcal{N}\left(0,\eta^2_{i,2,t}\right)$.

6. Calculate the perceived gap(markup) **after observing their signals** at t

$$\begin{aligned} x_{i,1,t} &= x_{i,1,t-1} - \left[\mathcal{K}^m_{i,t}\left(s^m_{i,t}-m_{i,t-1|t-1}\right)+\mathcal{K}^a_{i,1,t}\left(s^a_{i,1,t}-a^a_{i,1,t-1|t-1}\right)\right] \\ &= x_{i,1,t-1} - \Big[\mathcal{K}^m_{i,t}\left(m_{t-1}-m_{i,t-1|t-1}+\varepsilon^m_t+\xi_{i,m,t}\right) \\ &\qquad + \mathcal{K}^a_{i,1,t}\left(a^a_{i,1,t-1}-a^a_{i,1,t-1|t-1}+\varepsilon^a_{i,1,t}+\xi_{i,1,t}\right)\Big] \end{aligned}$$

$$\begin{aligned} x_{i,2,t} &= x_{i,2,t-1} - \left[\mathcal{K}^m_{i,t}\left(s^m_{i,t}-m_{i,t-1|t-1}\right)+\mathcal{K}^a_{i,2,t}\left(s^a_{i,2,t}-a^a_{i,2,t-1|t-1}\right)\right] \\ &= x_{i,2,t-1} - \Big[\mathcal{K}^m_{i,t}\left(m_{t-1}-m_{i,t-1|t-1}+\varepsilon^m_t+\xi_{i,m,t}\right) \\ &\qquad + \mathcal{K}^a_{i,2,t}\left(a^a_{i,2,t-1}-a^a_{i,2,t-1|t-1}+\varepsilon^a_{i,2,t}+\xi_{i,2,t}\right)\Big] \end{aligned}$$

where

$$\begin{aligned}
a_{i,1,t}-a_{i,1,t|t} &= \left(1-\mathcal{K}^a_{i,1,t}\right)\left(a_{i,1,t-1}-a_{i,1,t-1|t-1}\right)+\varepsilon^a_{i,1,t}-\mathcal{K}^a_{i,1,t}\left(\varepsilon^a_{i,1,t}+\xi_{i,1,t}\right)\\
a_{i,2,t}-a_{i,2,t|t} &= \left(1-\mathcal{K}^a_{i,2,t}\right)\left(a_{i,2,t-1}-a_{i,2,t-1|t-1}\right)+\varepsilon^a_{i,2,t}-\mathcal{K}^a_{i,2,t}\left(\varepsilon^a_{i,2,t}+\xi_{i,2,t}\right)\\
m_t-m_{i,t|t} &= \left(1-\mathcal{K}^m_{i,t}\right)\left(m_{t-1}-m_{i,t-1|t-1}\right)+\varepsilon^m_t-\mathcal{K}^m_{i,t}\left(\varepsilon^m_t+\xi_{i,m,t}\right)
\end{aligned}$$

with given $a_{i,1,-1}-a_{i,1,-1|-1}=0$, $a_{i,2,1}-a_{i,2,-1|-1}=0$, and $m_{-1}-m_{i,-1|-1}=0$.

7. Price changes: for $j\in\{1,2\}$,

$$\Delta p_{i,j,t}=\begin{cases}0 & \text{if } -\theta+\beta V\left(0,0,z^a_{i,1,t},z^a_{i,2,t},z^m_{i,t}\right)\\
& \qquad\leq -\left[\left(x_{i,1,t}\right)^2+\left(x_{i,2,t}\right)^2\right]+\beta V\left(x_{i,1,t},x_{i,2,t},z^a_{i,1,t},z^a_{i,2,t},z^m_{i,t}\right)\\
-x_{i,j,t} & \text{if } -\theta+\beta V\left(0,0,z^a_{i,1,t},z^a_{i,2,t},z^m_{i,t}\right)\\
& \qquad> -\left[\left(x_{i,1,t}\right)^2+\left(x_{i,2,t}\right)^2\right]+\beta V\left(x_{i,1,t},x_{i,2,t},z^a_{i,1,t},z^a_{i,2,t},z^m_{i,t}\right)\end{cases}$$

8. True markup: for $j\in\{1,2\}$,

$$\begin{aligned}
\mu_{i,j,t} &= p_{i,j,t}-m_t-a_{i,j,t}\\
&= \Delta p_{i,j,t}+x_{i,j,t-1}-\left(m_{t-1}-m_{i,t-1|t-1}\right)-\left(a_{i,j,t-1}-a_{i,j,t-1|t-1}\right)-\varepsilon^m_t-\varepsilon^a_{i,j,t}
\end{aligned}$$

where $a_{i,j,-1}-a_{i,j,-1|-1}=0$ and $m_{-1}-m_{i,-1|-1}=0$.

E Discussion on Model Assumptions

In this appendix, I discuss two key assumptions in my baseline model with both menu costs and rational inattention. First, I provide some evidence of firm-specific menu costs from the survey data. Second, I discuss implications and limitations of the assumptions about the set of available information.

Firm-Specific Menu Costs. In the baseline two-good version of the model, economies of scope in price setting emerge from the existence of firm-specific menu costs. Previous literature found ample evidence of the firm-specific menu costs for multi-product firm. For example, recent work by Stella (2018) and Letterie and Nilsen (2016) directly estimate various types of adjustment costs for the multi-product firms and find that there are sizable component of costs from firm-specific menu costs.[62] In the New Zealand survey data, I also find some evidence of firm-specific menu costs. Managers were asked about how typical it is to synchronize price reviews and price changes across

[62] Bhattarai and Schoenle (2014) and Lach and Tsiddon (2007) test the implications of menu cost models with a single fixed menu cost and find that micro price data support the existence of economies of scope for multi-product firms. Moreover, Lach and Tsiddon (1996) Fisher and Konieczny (2000), and Midrigan (2011) find that price changes within multi-product firms are highly synchronized, suggesting the existence of firm-specific menu costs.

multiple products sold by their firms. They report that on average 75% of their price changes and price review decisions are synchronized within their firms.

While the firm-specific fixed cost implies perfect within-firm synchronization of price changes, the data show that firms synchronize their price changes partially. Bonomo et al. (2019a) also find partial synchronization using Israel retail price data and show that even small departures from full synchronization in menu costs models substantially weaken monetary non-neutrality. This implies that introducing a product-specific menu cost in my baseline model would weaken economies of scope in price setting. In this case, the real effects of monetary shocks in the two-good version of the model will be much smaller than those in the single-product model.

Set of Available Information. In my main model analysis, I assume that the set of available signals has three properties. First, the firm chooses $N+1$ independent signals for each shock, implying that paying attention to aggregate conditions and paying attention to good-specific idiosyncratic conditions are separate activities. Although this assumption is often made in the rational inattention literature, such as Maćkowiak and Wiederholt (2009) and Pasten and Schoenle (2016), it might be suboptimal for firms to choose to observe independent signals. In fact, Afrouzi and Yang (2019) show that in LQG setting rational inattention models (without menu costs), the number of signals that firms choose to observe are no more than the number of actions. In my model, the number of actions is N as firms choose N prices for their goods, implying that firms might waste their resources to observe additional signals.

Second, I assume that every signal is Gaussian. Gaussian signals are optimal when the underlying shocks are Gaussian and firms' objective function is quadratic. However, in my model, firms' objective is not quadratic due to menu costs. Recent rational inattention literature considers models with general objective functions with some assumptions of a simple stochastic process, a static setup or finite actions and states (Matějka, 2015; Jung et al., 2019; Steiner et al., 2017). Solving fully non-linear dynamic problems under rational inattention is computationally demanding as firms' state variable is an infinitely dimensional object if the shocks are continuously distributed. The assumption of Gaussian signals is for tractability.

Third, I assume that all noise in signals is idiosyncratic and independent. This assumption is without loss of generality since I consider Shannon's mutual information as the cost of information (Denti, 2015; Afrouzi, 2019b).

F Additional Tables and Figures

Appendix Table A.1: Summary Statistics for Number of Products

Industries	Obs.	Mean	Median	Std. Dev.	Max.
Total	712	67.4	9	234.2	2115
Total without Retail/Wholesale Trade	627	9.55	7	8.47	48
− Manufacturing	278	9.57	8	7.75	39
− Professional and Financial Services	276	7.95	7	6.09	35
− Other Services	37	14.49	13	11.63	48
− Construction and Transportation	36	8.42	5	8.92	40

Notes: This table reports summary statistics for firms' number of products by sectors. The number of products of each firm is measured from answers to the following question in the second wave of New Zealand Firms' Expectation Survey: "*In addition to your main product or product line, how many other products do you sell?*" See Coibion et al. (2018a) for details about the survey data. Moments are calculated using sampling weights.

Appendix Table A.2: Summary Statistics for (Absolute) Backcast Errors about Inflation by Industries

Industries	Quartile 1		Quartile 2		Quartile 3		Quartile 4	
	N	Mean (S.D.)	*N*	Mean (S.D.)	*N*	Mean (S.D.)	*N*	Mean (S.D.)
Total	1-5	5.01 (4.56)	6-9	5.83 (4.93)	10-16	3.89 (5.20)	>16	2.48 (2.99)
Total without Retail/Wholesale	1-4	5.23 (4.10)	5-8	6.30 (5.03)	9-15	4.40 (5.57)	>15	3.54 (3.62)
− Manufacturing	1-5	1.52 (2.19)	6-9	1.70 (1.86)	10-15	2.25 (2.74)	>15	2.46 (2.56)
− Professional and Financial Services	1-4	6.42 (3.17)	5-8	6.16 (4.65)	9-13	7.00 (3.57)	>13	5.51 (3.71)
− Other Services	1-6	2.29 (1.97)	7-15	0.72 (0.46)	16-22	0.76 (0.52)	>22	0.90 (0.51)
− Construction and Transportation	1-3	7.46 (5.06)	4-5	7.38 (4.55)	6-9	10.82 (5.34)	>9	7.36 (7.59)

Notes: This table reports summary statistics for firms' (absolute) backcast errors about aggregate inflation by quartiles of the distribution of the number of products in each industry. The backcast errors are the absolute value of firm errors about past 12 month inflation from Wave #1 survey. Moments are calculated using sampling weights.

Appendix Table A.3: Number of Products and Knowledge about Aggregate Inflation (All Industries)

	(1)	(2)	(3)	(4)
Panel A. Dependent variable: Inflation backcast errors				
log(number of products)	-0.405***	-0.102***	-0.205***	-0.057*
	(0.069)	(0.030)	(0.063)	(0.033)
Observations	673	658	506	495
R-squared	0.229	0.835	0.273	0.896
Panel B. Dependent variable: Willingness to pay for professional inflation forecasts				
log(number of products)	-1.968	2.359*	-2.945*	3.871***
	(1.782)	(1.268)	(1.532)	(1.162)
Observations	438	434	378	372
R-squared	0.102	0.626	0.171	0.662
Firm-level controls	Yes	Yes	Yes	Yes
Industry fixed effects		Yes		Yes
Manager controls			Yes	Yes

Notes: This table reports results for the Huber robust regression. Dependent variables are the absolute value of firm errors about past 12 month inflation from Wave #1 survey (Panel A) and firms' willingness to payment for professional forecaster's forecasts about future inflation from Wave #4 (Panel B). Firm-level controls include log of firms' age, log of firms' employment, foreign trade share, number of competitors, firms' beliefs about price difference from competitors, and the slope of the profit function. Industry fixed effects include dummies for 17 sub-industries. Manager controls include the age of the respondent (each firm's manger), education, income, and tenure at the firm. Sample weights are applied to all specifications. Robust standard errors (clustered at the 3-digit ANZ SIC level) are reported in parentheses. ***, **, * denotes statistical significance at 1%, 5%, and 10% levels respectively.

Appendix Table A.4: Number of Products and Knowledge about Nominal GDP Growth

	(1)	(2)	(3)	(4)
Dependent variable: Backcast errors about nominal GDP growth rate				
Number of products	-0.041***	-0.020***	-0.035***	-0.017*
	(0.012)	(0.007)	(0.011)	(0.009)
Observations	390	378	334	326
R-squared	0.375	0.615	0.412	0.610
Firm-level controls	Yes	Yes	Yes	Yes
Industry fixed effects		Yes		Yes
Manager controls			Yes	Yes

Notes: This table reports results for the Huber robust regression. Dependent variable is the absolute value of firm errors about the growth rate of nominal GDP from Wave #4 survey. Firms' perceived growth rate of nominal GDP are calculated by taking the summation of firms' belief about current inflation and the real GDP growth rate in New Zealand. Firm-level controls include log of firms' age, log of firms' employment, foreign trade share, number of competitors, firms' beliefs about price difference from competitors, and the slope of the profit function. Industry fixed effects include dummies for 14 sub-industries excluding retail and wholesale trade sectors. Manager controls include the age of the respondent (each firm's manger), education, income, and tenure at the firm. Sample weights are applied to all specifications. Robust standard errors (clustered at the 3-digit ANZ SIC level) are reported in parentheses. ***, **, * denotes statistical significance at 1%, 5%, and 10% levels respectively.

Appendix Table A.5: Model Predictions and Monetary Non-Neutrality

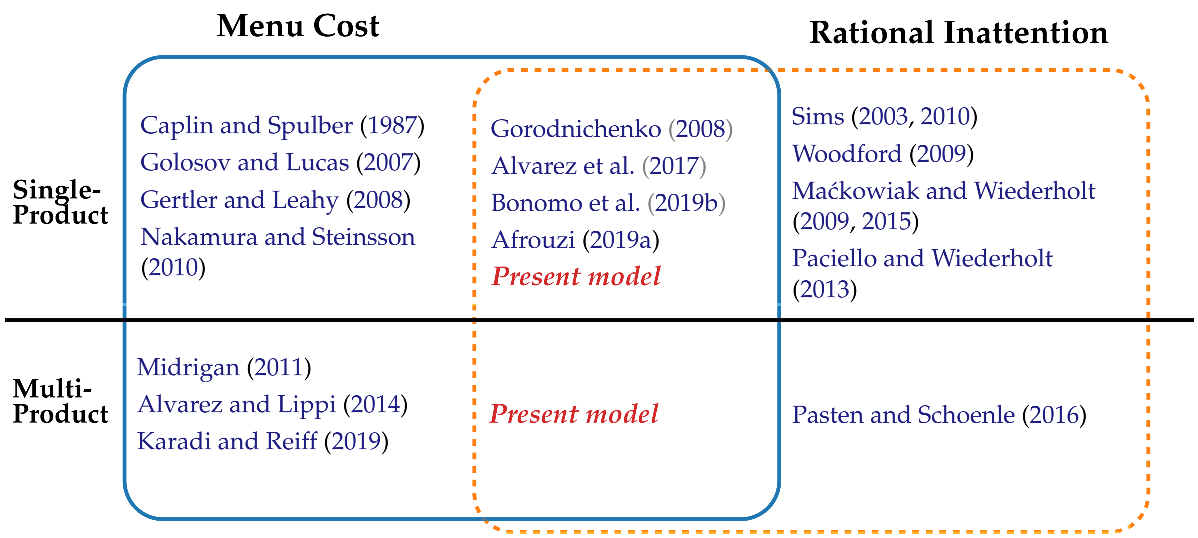
	Menu Costs	Calvo/Taylor	Rational Inattention	Sticky/Noisy Info/ Observational Costs
1) Attentiveness to Aggregate Conditions	Full information	Full information	Increase in N	Independent of N
2) Size/Duration of Price Changes	Decrease in N	Independent of N	Flexible micro price	Independent of N
Monetary Non-Neutrality	Increase in N	Independent of N	Decrease in N	Independent of N

Notes: This table shows the predictions from different models about 1) the relationship between firms' number of products and their attentiveness to aggregate condition and 2) the relationship between firms' number of products and the duration and size of price changes. N stands for the number of products that firms sell in each model. See Section 2.4 for details.

Appendix Table A.6: Subjective Uncertainty and Expected Duration of Price Changes

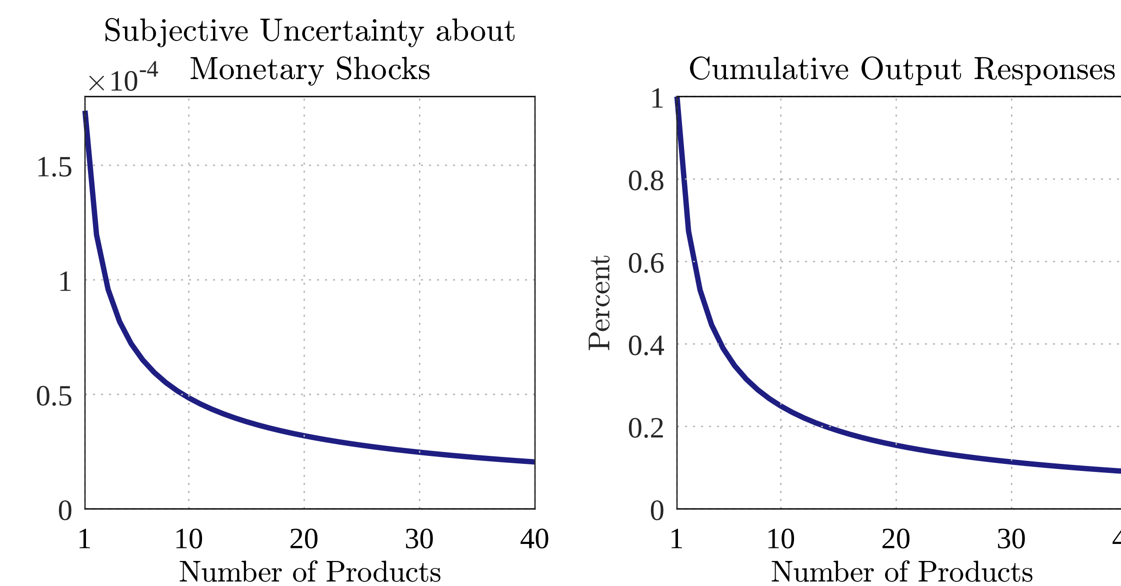
	(1)	(2)	(3)	(4)
Dependent variable: Duration of expected next price changes				
Standard deviation of the growth rate of sales over the next 12 months	0.132***	0.104**	0.153***	0.160**
	(0.039)	(0.042)	(0.054)	(0.065)
Observations	583	591	443	442
R-squared	0.323	0.697	0.342	0.436
Firm-level controls	Yes	Yes	Yes	Yes
Industry fixed effects		Yes		Yes
Manager controls			Yes	Yes

Notes: This table reports results for the Huber robust regression. Dependent variable is the duration of expected next price changes from Wave #2. The regressor is the standard deviation implied by the reported probability distribution for the growth rates of unit sales of firms' main product over the next 12 months. Firm-level controls include log of firms' age, log of firms' employment, the number of competitors, and log of firms' number of products. Industry fixed effects include dummies for 14 sub-industries excluding retail and wholesale trade sectors. Manager controls include the age of the respondent (each firm's manger), education, income, and tenure at the firm. Sample weights are applied to all specifications. Robust standard errors (clustered at the 3-digit ANZ SIC level) are reported in parentheses. ***, **, * denotes statistical significance at 1%, 5%, and 10% levels respectively.



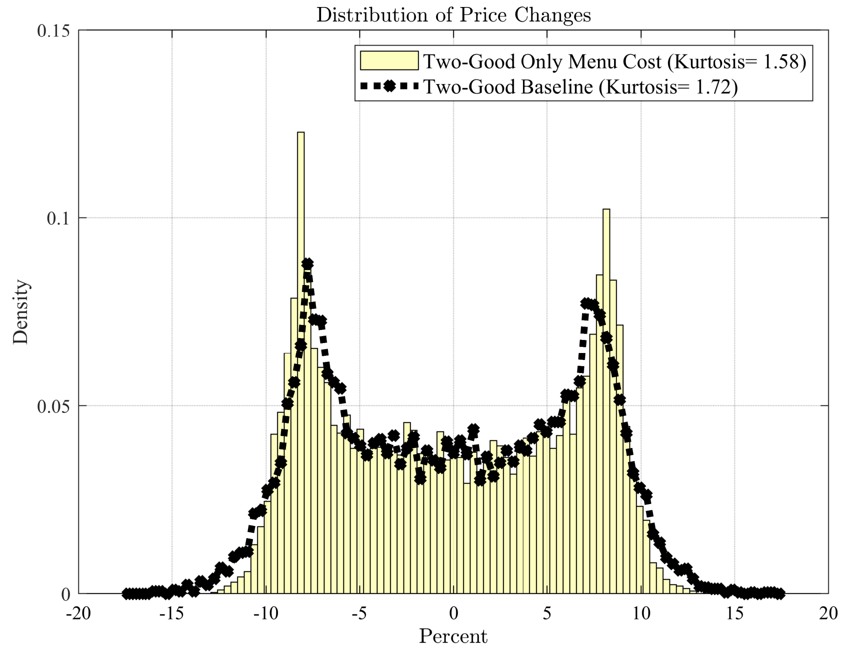
Appendix Figure A.1: Classification of Models

Notes: This figure shows classification of models in the literature on menu costs and rational inattention.



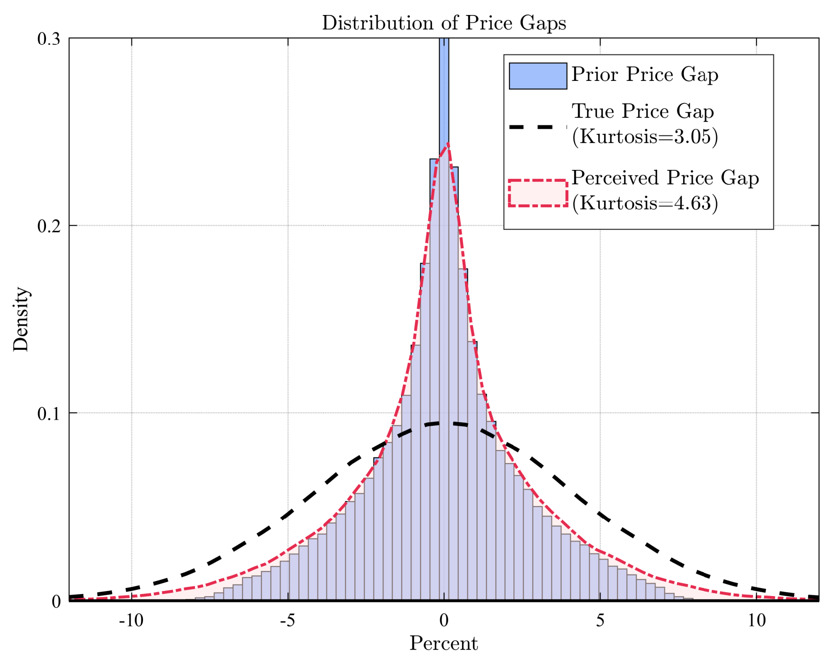
Appendix Figure A.2: Subjective Uncertainty and Cumulative Output Responses in Rational Inattention Only Models with Different Numbers of Products

Notes: The left panel plots firms' subjective uncertainty about monetary shocks in the rational inattention only model with different numbers of products. The right panel plots cumulative output responses to a one standard deviation monetary shock in the rational inattention only models with different numbers of products sold by firms. The cumulative output response in the single-product model is normalized to one.



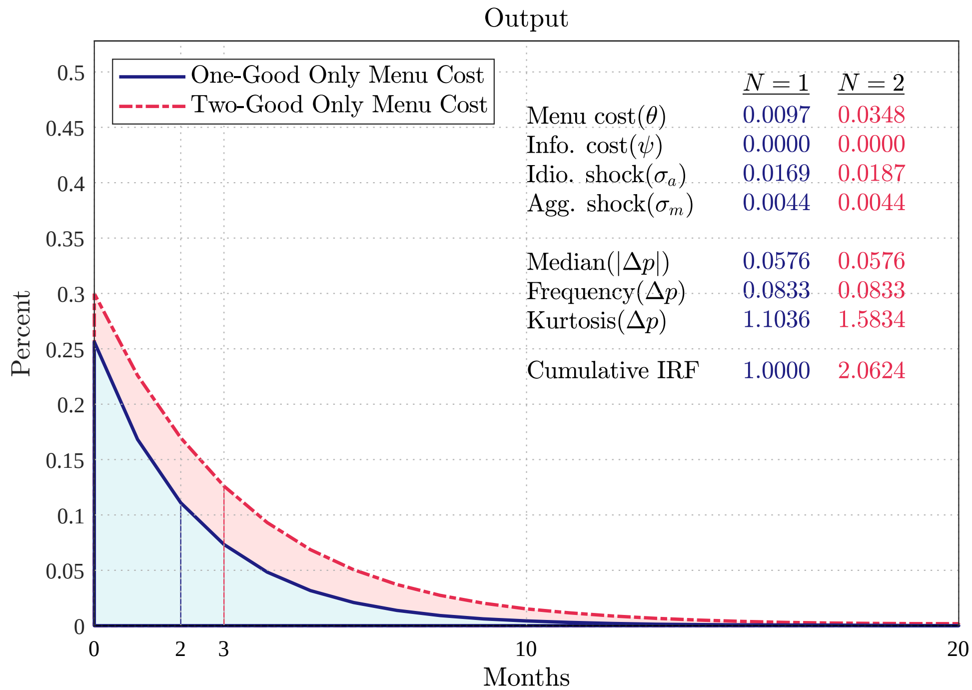
Appendix Figure A.3: Distribution of Price Changes in the Two-Product Models

Notes: This figure plots the distribution of price changes in the two-product menu cost model with perfect information (yellow bar) and that in the two-good version of the baseline model (black line with circle markers).



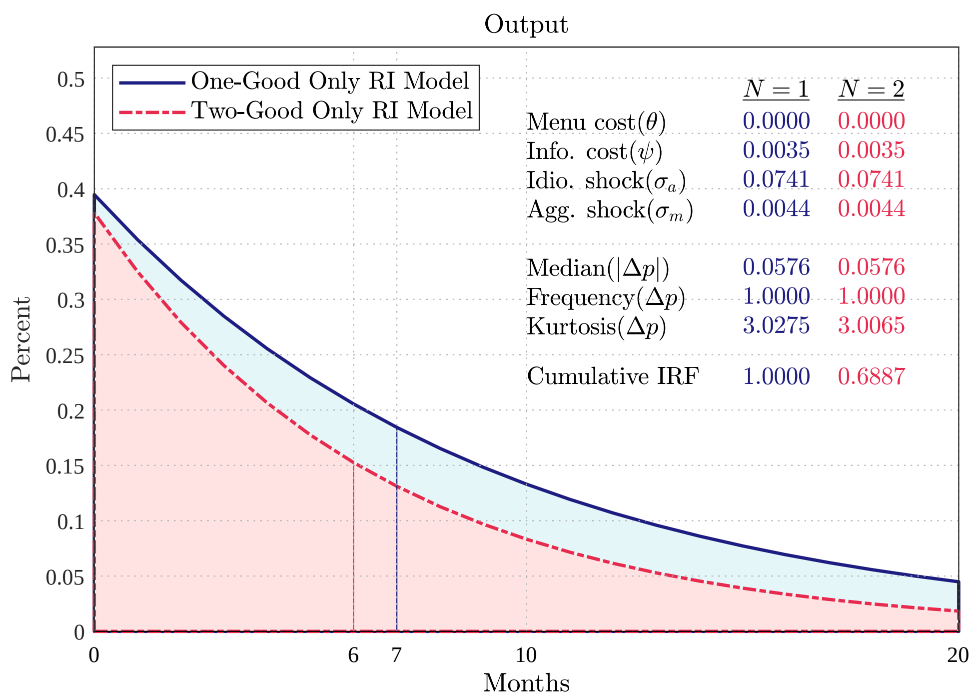
Appendix Figure A.4: Distribution of True and Perceived Price Gaps in the Two-Product Baseline Model

Notes: This figure plots distributions of price gaps in the two-good version of the baseline model. Blue bar graph shows the distribution of firms' *prior* about their price gaps at the beginning of period. Black dashed line shows the distribution of firms' true price gaps after their Gaussian shocks realized. Firms choose their optimal signals about the shocks and form a new posterior about their (frictionless) optimal price. Red dash-dot line shows the distribution of the *posterior* of perceived price gaps.



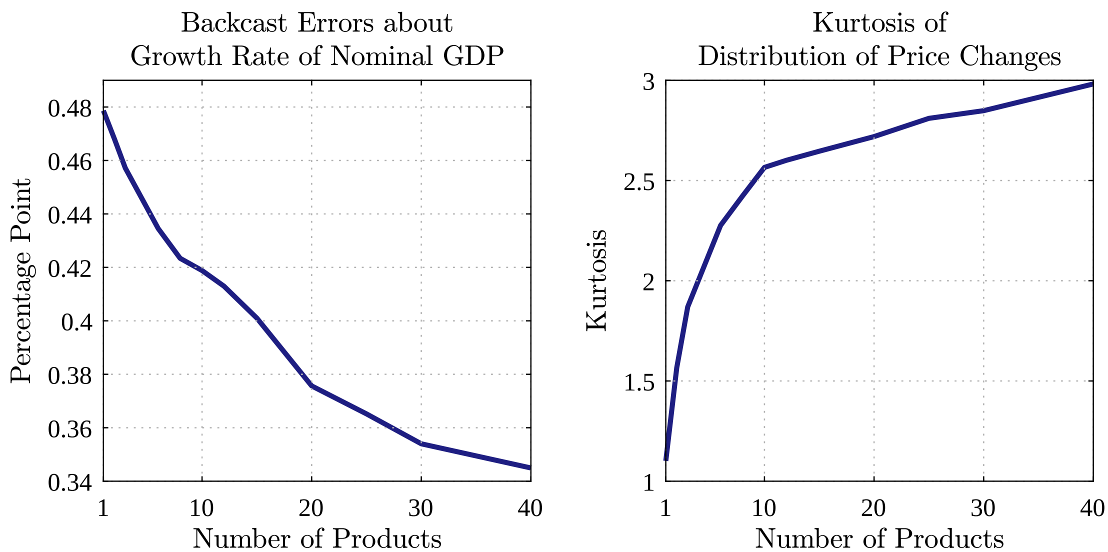
Appendix Figure A.5: Impulse Response of Output in the Menu Costs Models with Perfect Information

Notes: This figure plots impulse responses of output to a one standard deviation monetary shock in the one-good and two-good versions of menu cost models with perfect information. Cumulative IRFs refers to area under the responses of output. I normalize the cumulative output response in the one-good version of the model as one.



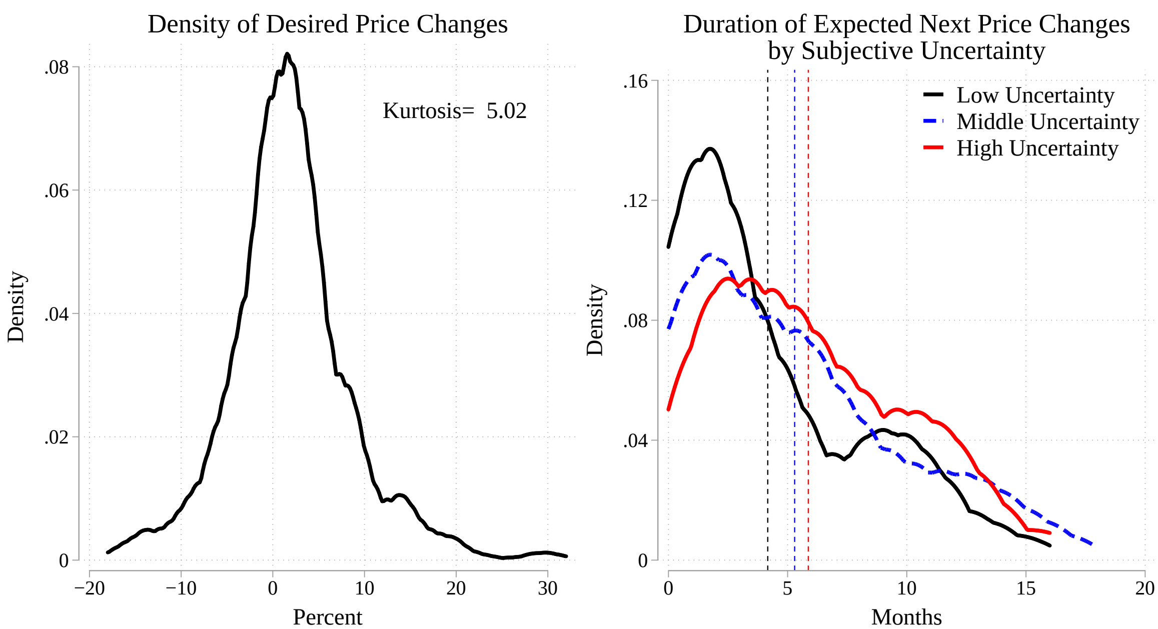
Appendix Figure A.6: Impulse Response of Output the Rational Inattention Models without Menu Costs

Notes: This figure plots impulse responses of output to a one standard deviation monetary shock in the one-good and two-good versions of rational inattention models without menu costs. Cumulative IRFs refers to area under the responses of output. I normalize the cumulative output response in the one-good version of the model as one.



Appendix Figure A.7: Backcast Errors and Kurtosis of Price Changes in the Simplified Models

Notes: This figure shows shows the backcast errors of firms about the growth rate of nominal GDP (left panel) and the kurtosis of distribution of price changes (right panel) in the simplified version of the baseline models with different numbers of products sold by firms. See section 5.1 for details.



Appendix Figure A.8: Distribution of Desired Price Changes and Duration of Expected Price Changes in the Survey Data

Notes: This left panel shows the distribution of desired price changes in the second wave of the survey data. Firms' managers in the survey were asked how much they would like to change the price of their main product if it was free to change its price in three months. The answer gives firms' desired price changes in three months. I define the inflation-adjusted desired price changes as the gap between firms' desired price changes in three months and their three-month ahead inflation expectations. The right panel shows the distribution of firms' expected duration of their next price changes by their degree of subjective uncertainty. The subjective uncertainty is measured by the standard deviation implied by the reported probability distribution for the growth rates of unit sales of firms' main product over the next 12 months. See section 5.2 for details.