We note that the cause of the problem of sample impoverishment is due to the low number of particles being drawn from the tails of the proposal distribution, leading to low numbers of particles with effective weights when $y_k - \mathbf{CA}\mu_{k-1}^{(i)}$ is large

Thus, one method of reducing sample impoverishment in the particle filter would be to simply generate a larger number of particles from the tails of the proposal distribution. In order to preserve the required proposal distribution, we need to then scale the weights of the extra particles generated from the tails appropriately.

This leads to the following algorithm for sampling from the proposal distribution given in **??**:

- Define the hyperparameters ϵ_λ and m. In this example, we select $\epsilon_\lambda = 1 \times 10^{-1}$ and $m = 3$.
- Calculate the number of extra particles (N_{extra}) to draw from the right tail using:

$$threshold = \frac{f_{pareto}(\lambda_k|\alpha/2, 1) - S_{\alpha/2}(\lambda_k|1, 1, 0)}{S_{\alpha/2}(\lambda_k|1, 1, 0)} \tag{1}$$

$$N_{extra} = P(\lambda > threshold) \times Nm \tag{2}$$

- Draw N_{extra} particles from the right tail:

$$\lambda_k^{(i)} \sim f_{bounded\ pareto}(\lambda_k|\alpha = \alpha/2, \beta = 1, L = threshold) \qquad i = 1...N_{extra} \tag{3}$$

- Draw $N - N_{extra}$ particles from the truncated alpha stable distribution using a simple rejection sampler:
  - Draw $\lambda_k^{(i)} \sim S_{\alpha/2}(\lambda_k|1, 1, 0)$
  - Accept if $\lambda_k^{(i)} < threshold$
- Alter the weights of the particles drawn from the tails:

$$w_i = \frac{1}{m} w_i \qquad \text{for: } i \in [1, N_{extra}]$$

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