We take a brief segue to provide some theory into the tail approximation to the α-stable distribution that will be useful in the later sections.

When $\alpha < 2$, the tails of the α-stable distribution are are asymptotically equivalent to a Pareto law. More precisely, if λ is a standardized α-stable variable $\lambda \sim S_\alpha(\sigma, \beta, \mu)$ with $0 < \alpha < 2$, $\sigma = 1$, $\mu = 0$ then as $\lambda \to \infty$ [**samoradnitsky2017stable**]:

$$\lim_{x \to \infty} P(\lambda > x) = C_\alpha (1 + \beta) x^{-\alpha} \tag{1}$$

where:

$$C_\alpha = \left(2 \int_0^\infty x^{-\alpha} \sin x dx \right)^{-1} = \frac{1}{\pi} \Gamma(\alpha) \sin(\frac{\alpha \pi}{2})$$

We can then differentiate Equation 1 formally to obtain the probability distribution function (PDF) of the tail approximation:

$$\begin{aligned} \lim_{x \to \infty} S_\alpha(\lambda = x | \sigma = 1, \beta, \mu = 0) &= f_{pareto}(\lambda = x | \alpha, \beta) \\ &= \alpha C_\alpha (1 + \beta) x^{-\alpha - 1} \end{aligned} \tag{2}$$

Numerical accuracy of the tail approximation

We first define a metric for the absolute relative error in the tail approximation:

$$\epsilon_{rel}(x) = \left| \frac{S_\alpha(\lambda = x | 1, \beta, 0) - f_{pareto}(\lambda = x | \alpha, \beta)}{S_\alpha(\lambda = x | 1, \beta, 0)} \right| \tag{3}$$

We can then investigate the numerical accuracy of the tail approximation for $\beta = 1$ and $\alpha < 1$ using Figure 1. As expected, we note that the relative error in the tail approximation decreases monotically.

Note that for $\alpha < 1$, there is no clear dependence on the value of α and the rate of convergence of the α-stable distribution to the paretian tail approximation.

Sampling from the paretian tail

Using the paretian tail approximation to the α-stable distribution , we note that up to a certain scale parameter($\alpha(1 + \beta)C_\alpha$), sampling from a left-truncated α-stable distribution can be approximated by sampling from a left-truncated pareto distribution.

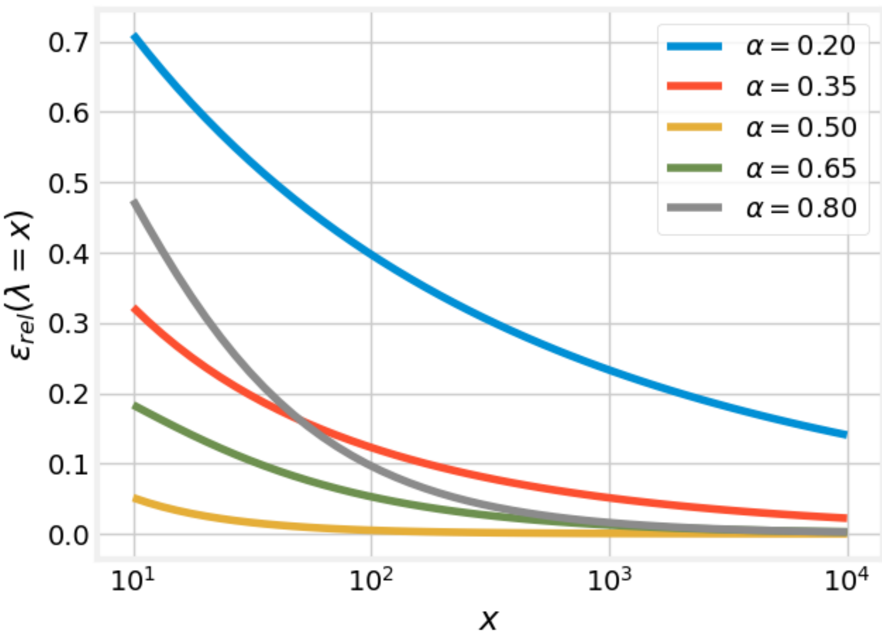

Figure 1: Relative error in the tail approximation $\epsilon_{rel}(x)$ for $\beta = 1$ and $\alpha < 1$

For a left-truncated pareto distribution $f_{pareto}(x|\alpha, \beta)$ lower bounded by $x \geq L$, the corresponding PDF $f_{bounded\ pareto}(x|\alpha, \beta, L)$ is given by Equation 4:

$$f_{bounded\ pareto}(x|\alpha, \beta, L) = \alpha(1+\beta)C_\alpha L^\alpha x^{-\alpha-1} \tag{4}$$

We can then sample from this left-truncated pareto distribution easily using the inverse transform method.

$$x = \left(\frac{1-U}{L^\alpha(1+\beta)C_\alpha}\right)^{-\frac{1}{\alpha}} \qquad \text{where: } U \sim \text{Unif}(0,1) \tag{5}$$

Following Equation 5, x will be left-truncated pareto distributed according to $f_{bounded\ pareto}(x|\alpha, \beta, L)$

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