ANOVA test

Hypothesis testing in ANOVA (Partitioning of Variance in ANOVA)

H_0 : $\mu_1 = \mu_2 = \mu_3 = \cdots = \mu_k$

H_A : Atleast one of the sample mean is not equal

$\mu_1 = \mu_2 = \mu_3 \cdots \neq \mu_k$

* Test statistics = $\frac{\text{Variance between Samples}}{\text{Variance within Samples.}}$

$H_0 : \bar{x}_1 = \bar{x}_2 = \bar{x}_3$

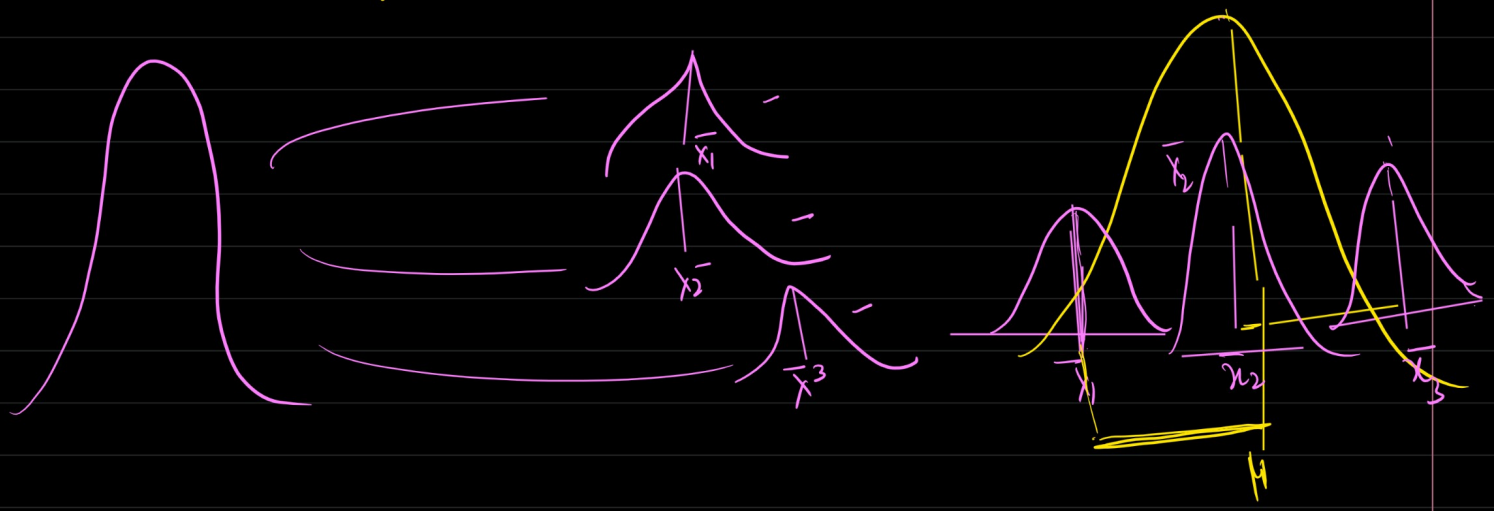
H_A : Atleast one sample mean is not equal.

s^2

variance between sample

x_1	x_2	x_3
1	5	9
2	6	10
3	7	11
4	8	12

Variance within Sample



One way ANOVA

↳ one factor with atleast levels

Q There are three dosage of a medicine given to three sample of the patients. rating if headeal is reduced (1-10) Are there differences in the three condition. $\alpha = 0.05$?

0 mg	10 mg	100 mg
9	7	4
8	6	3
7	6	2
8	7	3
8	8	4
9	7	3
8	6	2

① $H_0 : \mu_{0mg} = \mu_{10mg} = \mu_{100mg}$
H_A : Not all are equal.

② $\alpha = 0.05$, one tail test

③ Calculate F statistics.

$$F\text{statistics} = \frac{\text{Variance b/w Sample}}{\text{Variance within sample.}}$$

	Sum of squares (SS)	df	MS (SS/df)	F
Between the Sample				
Within sample				
total				

0 mg	10 mg	100 mg
9	7	4
8	6	3
7	6	2
8	7	3
8	8	4
9	7	3
8	6	2

* Sum of squares b/w Sample $= \sum \frac{(\sum a_i)^2}{n} - \frac{T^2}{N}$

0mg : $9+8+7+8+8+9+8 = 57$
10mg : $7+6+6+7+8+7+6 = 47$
100mg : $4+3+2+3+4+3+2 = 21$

$$= \frac{57^2+47^2+21}{7} - \frac{(57+47+21)^2}{21} \quad (125) = 98.67.$$

* Sum of Square within the group.

$$SS_{within} = \sum y^2 - \frac{\sum (\sum a_i)^2}{n}$$

0 mg	10 mg	100 mg
9	7	4
8	6	3
7	6	2
8	7	3
8	8	4
9	7	3
8	6	2

$$\sum y^2 = 9^2+8^2+7^2+8^2 \cdots +7^2+6^2+6^2+ \cdots 4^2+3^2+2^2$$
$$= 853$$

$$= 853 - \left(\frac{57^2+47^2+21^2}{7}\right) = 10.29.$$

	Sum of squares (SS)	df	MS (SS/df)	F
Between the sample	98.67	2	49.34	
Within sample	10.29	18	0.54	
total	108.96	20		

dof → no. of samples − 1

3 − 1 = 2

degree of freedom within sample

$df_{within} = N - a$ (no. of groups)

$= 21 - 3 = 18$

$\Rightarrow N - 1$

$\Rightarrow 21 - 1$

$\Rightarrow 20$

$F_{statistic} = \frac{MS_{between}}{MS_{within}}$ (var b/w sample / var within sample)

$F = \frac{49.34}{0.54} = 86.56$

Step-4

Fcritical for $\alpha = 0.05$

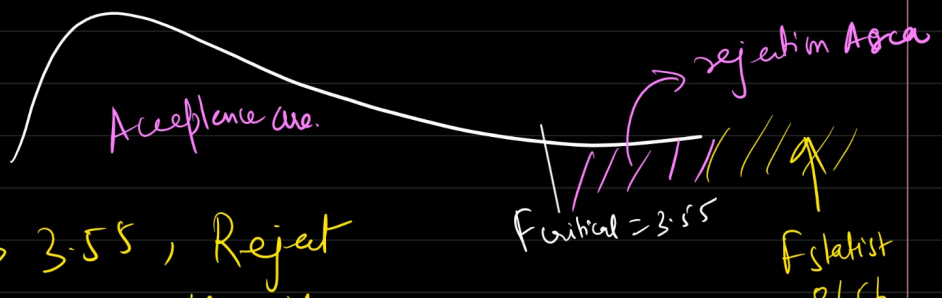
$df_{between} = 3 - 1 = 2$ ← df_1

$df_{within} = 21 - 3 = 18$ ← df_2

F-table

$F_{critical\ \alpha=0.05(2,18)} = 3.55$

Step-5



86.56 > 3.55, Reject the Ho