Confidence interval and Margin of error

* Example → What is the potential score?
→ 80 marks ✗
75 – 85 → interval

80 ± 5

Analysis
→ Sample → Avg ht of Employees of a company → Population

Sample → $\bar{x}$ → Point estimate

* estimate → An estimate of a population parameter is an approximation depending solely on sample information.

* A point estimate is a single no.

eg. 180 cm (ht)

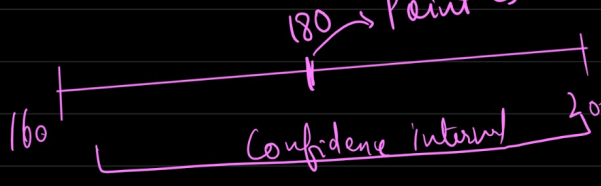
* Confidence interval is an interval.

eg. Point Estimate ± Error

180 ± 20

Confidence Interval = 160 — 200

⇒ Point estimate is located exactly in the middle of Confidence interval



Confidence Interval (CI) = Point Estimate ± Margin of error.

eg: People visiting this restaurant spends 1000 Rs. on an avg.

↳ It is much safer to say that people spend 800 – 1200 Rs

1000 Rs. ← point estimate.

800 – 1200 Rs ← Confidence interval

↓

more accurate representation of reality

⇓

Why? → you can not be 100% confident unless you go through entire population.

$$C.I = \text{point estimate} \pm \text{margin of error.}$$

Z_{test} (two tail test) $\longrightarrow$ $CI = \bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

where σ is population's standard deviation

n – sample size

$\bar{x}$ – sample mean

$Z_{\alpha/2}$ → Z score corresponding to given $\alpha/2$

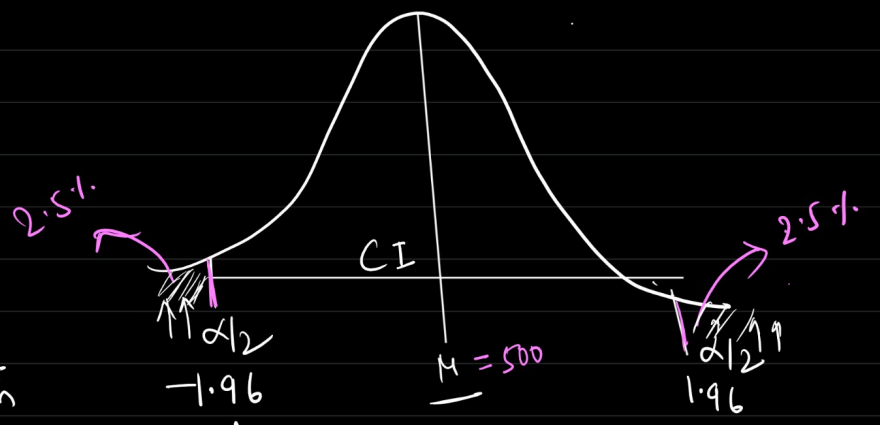
α – level of significance

Z_{test} (one tail test) $\longrightarrow$ $CI = \bar{x} \pm Z_{\alpha} \frac{\sigma}{\sqrt{n}}$ ← margin of error.

t-test $= \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$ ← Sample's standard dev.

* CI → what value the sample statistics will take can be known through confidence interval.

Q In an exam the standard deviation of marks is 100. A sample of 36 students has a mean of 500 marks.
Construct a 95% Confidence Interval about the mean?



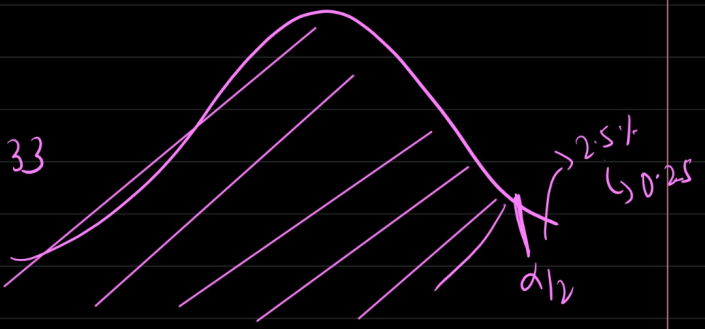
$$C.I = \bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$500 \pm 1.96 \times \frac{100}{\sqrt{36}}$$

Lower C.I

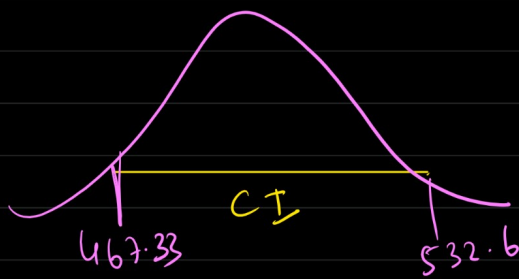
$$\text{Lower CI} = 500 - 1.96 \times \frac{100}{\sqrt{36}} = 467.33$$

$$\text{Upper CI} = 500 + 1.96 \times \frac{100}{\sqrt{36}} = 532.6$$



$\Rightarrow 1 - 0.25$

$\Rightarrow 0.9750$



Z table → Corresponding to 0.9750 what is Z score.

→ The Z score corresponding to 0.9750 is 1.96

→ I am 95% Confident that the mean score in the exam lies between 467.3 and 532.6