Discrete Uniform distribution

→ A uniform distribution refers to a type of prob distribution in which outcomes are equally likely.

→

Uniform distribution

- Discrete Uniform Dist (pmf)
- Continuous Uniform distribution (pdf)

→ In a discrete Uniform distribution, the outcomes are discrete and have the same prob.

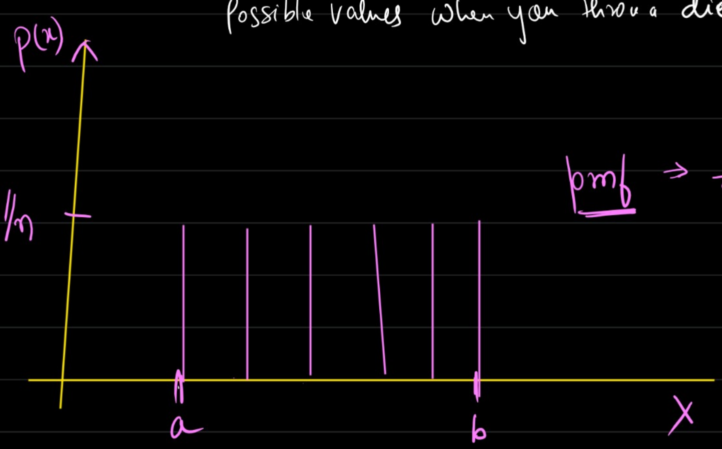
- eg. rolling a dice
- eg. tossing a coin
- eg. picking up a card from well shuffled deck

1, 2, 3, 4, 5, 6

$P(X=1) = 1/6$

Notation of Uniform distribution ⇒ $U(a,b)$ | Unif $\{a,b\}$

Possible values when you throw a die → 1–6

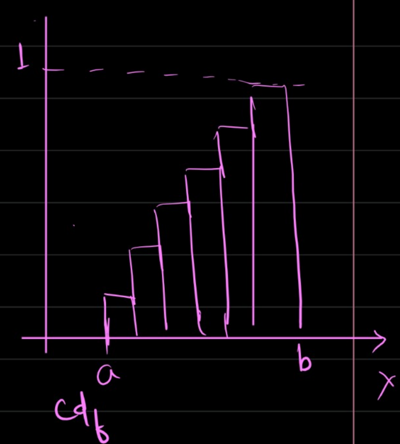


pmf → $\frac{1}{n}$

where $n = b - a + 1$

$= 6 - 1 + 1$

$= 6$



Q what is prob of getting 3 when you throw a dice?

→ $P(X=3) = 1/6$

mean of discrete Uniform distribution ⇒ $\frac{a+b}{2}$

Variance " " " ⇒ $\frac{n^2-1}{12}$

* mean = Sum of nos divided by number of numbers.
* Expected value → the long run avg value of repetitions of experiment it represents.

→ long term avg value of a random value.

dice = 1, 2, 3, 4, 5, 6

$$\text{mean} = \frac{1+2+3+4+5+6}{6} = 3.5$$

$$E.V = \sum_{i=1}^{6} x_i \, P(x_i)$$

x	prob	$x \cdot p$
1	$1/6 \Rightarrow 0.167$	0.17
2	0.167	0.33
3	0.167	0.50
4	0.167	0.67
5	0.167	0.83
6	0.167	1

Sum (px) = 3.50

→ mean is used freq. distribution

→ Expected value is used for prob distribution

$$E(X) \,|\, \mu \Rightarrow \sum_{i=1}^{i} x_i \, p(x_i)$$

$$Var(X) = E(X^2) - (E(X))^2$$

$$E(X) = \sum x \, p(x)$$

$$= \sum_{x=1}^{N} x \cdot \frac{1}{N} \Rightarrow \frac{1}{N}(1+2+\dots N)$$

$$= \frac{1}{\cancel{N}}\left(\frac{\cancel{N}(N+1)}{2}\right)$$

$$\Rightarrow \frac{N+1}{2}$$

Sum of first 'N' no. $= \frac{N(N+1)}{2}$

$1+2+3 \dots 10$

$\Rightarrow \frac{10(10+1)}{2}$

Sum of first N squares $= \sum_{i=1}^{N} x^2 = \frac{N(N+1)(2n+1)}{6}$

$1^2 + 2^2 + \dots 10^2$

$\frac{10(10+1)(2 \times 10+1)}{6}$

$$E(X^2) = \sum x^2 \, p(x)$$

$$\Rightarrow \sum_{i=1}^{N} x^2 \cdot \frac{1}{N}$$

$$\Rightarrow \frac{1}{N}\left(1^2 + 2^2 + 3 + \cdots N^2\right)$$

$$= \frac{1}{\cancel{N}}\left(\frac{\cancel{N}(N+1)(2N+1)}{6}\right)$$

$$= \frac{(N+1)(2N+1)}{6}$$

$$Var(x) = E(X^2) - (E(x))^2$$

$$= \frac{(N+1)(2N+1)}{6} - \left(\frac{N+1}{2}\right)^2$$

$$\Rightarrow \frac{2N^2+3N+1}{6} - \frac{N^2+2N+1}{4}$$

$$Var(x) \Rightarrow \frac{N^2-1}{12}$$

$$\sigma = \sqrt{\frac{N^2-1}{12}}$$