Bernoulli distribution → Binary ← two outcomes

A discrete prob distribution of a random variable which takes only two possible outcomes, typically labelled as Success (coded as 1) and failure (coded as 0) with a fixed prob of Success and failure 1.

* To model any experiment with only two possible outcome.

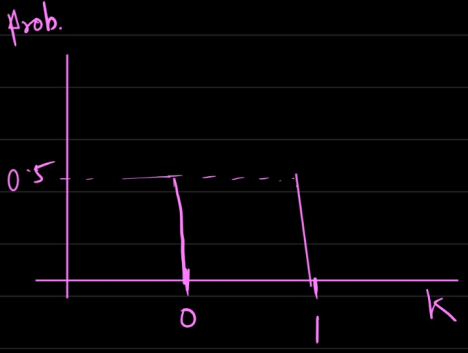
eg: Tossing a coin

Head or Tail $(K \in \{0,1\}$

$P(X=0) = 0.5 = p$

$P(X=1) = 1-0.5 = q$

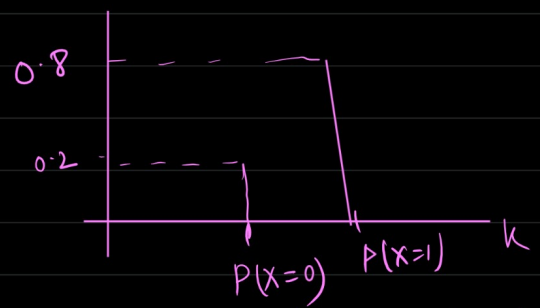
$(1-p)$



Pmf ⇒

$$P(X=k) = \begin{cases} p & \text{if } k=1 \\ 1-p & \text{if } k=0 \end{cases}$$

$$P(X=k) = p^k (1-p)^{1-k}$$



$P(X=1) = 1 - P(X=0)$
$= 1 - 0.2 = 0.8$

① if $k=1$

$P(X=1) = p^1 (1-p)^{1-1} = p^1 (1-p)^0$

$= p$

② if $k=0$

$P(X=0) = p^0 (1-p)^{1-0} \Rightarrow 1-p$

$$p + (1-p) = 1$$

Conditions of Bernoulli dist

1. No of trial should be finite
2. Each trial should be independent — Pass
3. Only two possible outcome — fail
4. Prob of each output should be same in every trial.

1 2 3 4 5

P(H)=0.5 P(H)=0.5 P(H)=0.5 P(H)=0.5 P(H)=0.5

Examples

→ tossing a coin
→ Someone has committed a fraud or not
→ getting a six or not
→ pass / fail
→ win / or not
→ Customer Conversion
→ rain or not

Q Bumrah bowls 6 balls at wicket, with prob of 0.6 at hitting the stump with each ball. What is prob of not hitting a wicket?

→ Bernoulli dist^n.

p(hitting a wicket) = 0.6

p(hitting not a wicket) = 1 − 0.6 = 0.4

* Mean and Variance of Bernoulli dist^n.

↓ p

↓ p(1−p)

① Mean

$$E(X) = \sum_{i=1}^{k} X\, P(X)$$

= X=1 + X=0

= 1 × 0.6 + 0 × 0.4

= 0.6 ⇒ p

E(X) = p

k can take two values 0 & 1

P(X=1) = 0.6 = p

P(X=0) = 0.4 = q

(1−p)

② Var(X) ⇒ $E(X^2) - E(X)^2$ → p

$E(X^2) = \sum x^2 p(x)$

$$= 1^2 \times p + 0 \times (1-p)$$

$$= p$$

$$\text{Var}(x) = p - (p)^2$$

$$= p(1-p)$$