Continuous Uniform distribution

- A uniform distribution refers to a type of prob distribution in which outcomes are equally likely.

→

Uniform distribution

- Discrete Uniform Dist (Pmf)
- Continuous Uniform distribution (pdf)

* A continuous uniform prob distribution is a dist that has an infinite no of values defined in a specified range / bound.

→ r.v is continuous.
→ rectangular dist.

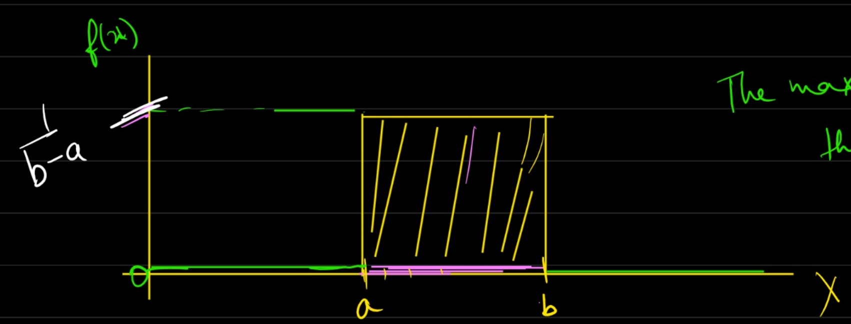
Examples – A perfect Random no generator.)
- Prob of guessing exact time at any moments
→ Waiting time at a bus stop (consistent bus arrival)
→ Temp variation in a day [if a temp fluctuating b/w a min and max value)

Notation: $U(a, b)$

Parameter: $-\infty < a < b < \infty$

$b >> a$ a is min, b is max.

pdf



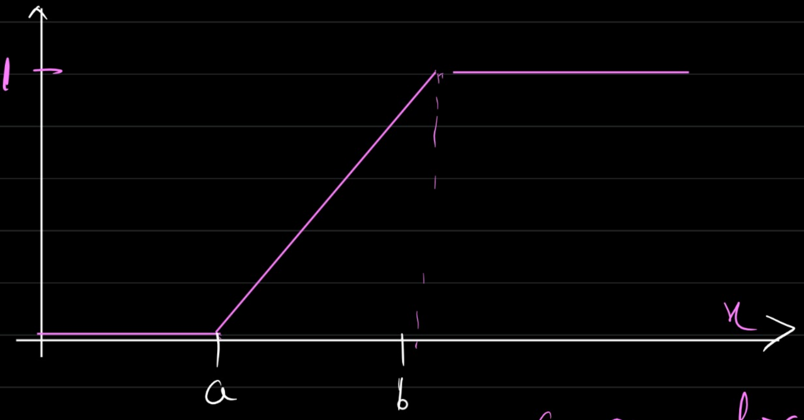
The maximum prob of the variable X is 1.

Area of rectangle = base × height

$1 = b - a \times f(x)$

$f(x) = \frac{1}{b-a}$

cdf

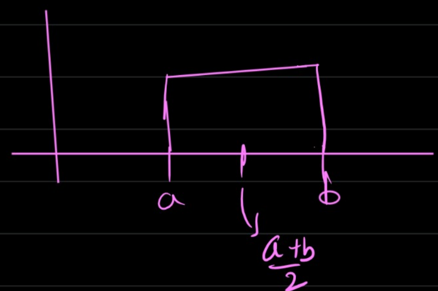


$$\text{cdf} = \begin{cases} 0 & \text{for } x < a \\ \dfrac{x-a}{b-a} & x \in [a, b] \\ 1 & x > 1 \end{cases}$$

mean/median $= \frac{1}{2}(a+b)$

↓

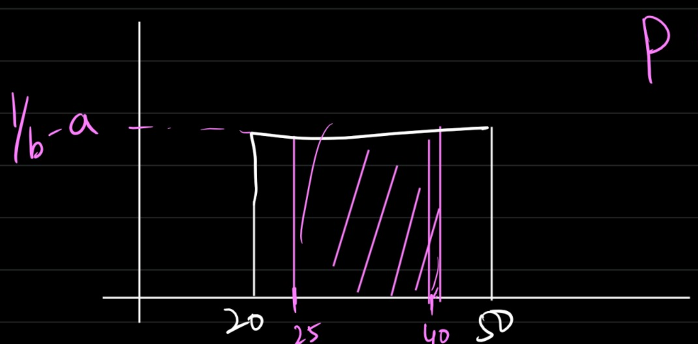
Avg of pdf | Center of distribution.



variance $= \frac{1}{12}(b-a)^2$

Q1 The number of items sold at a shop daily is uniformly distributed with max and min item sold 50 and 20 respectively.

→ Prob of daily sales to fall between 25 & 40



$P(25 \le X \le 40)$

⇓

breadth × ht

$(40-25) \times \frac{1}{50-20}$

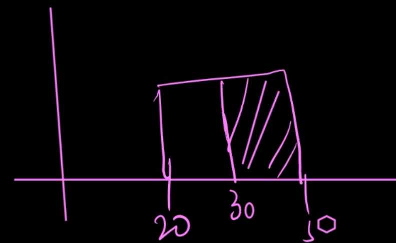
$\Rightarrow 15 \times \frac{1}{30} = \frac{1}{2} = 0.5$

50% chance the no of items sold [25, 40]

→ prob. that sales of ≥ 30

$$P(X \geq 30) = (50-30) \times \frac{1}{50-20}$$

$$\Rightarrow 2\cancel{0} \times \frac{1}{3\cancel{0}} = 66.66$$



Q The amount of time for Pizza delivery is Uniformly distributed b/w 15 and 60 mins. what is standard deviation of the amount of time it takes for a pizza to be delivered?

$\Rightarrow$ $a = 15$ min
$b = 60$ min

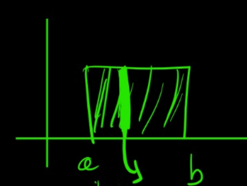
$$\text{var}(X) = \frac{(b-a)^2}{12} = \frac{(60-15)^2}{12} = \frac{45^2}{12} = 168.75 \text{ min}$$

$$\sigma = \sqrt{\text{var}} = \sqrt{168.75} \approx 13 \text{ mins}$$

---

for a cont^n random var with prob density fn $f(x)$

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$$



$\int_a^b dx$

$$E(X) = \int_a^b x\, f(x)\, dx$$

$$\Rightarrow \int_a^b x \cdot \frac{1}{b-a}\, dx$$

$$\Rightarrow \frac{1}{b-a} \int_a^b x\, dx$$

$$\Rightarrow \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b = \frac{1}{2} \cdot \frac{1}{b-a}\left(b^2 - a^2\right)$$

$$= \frac{1}{2}\frac{1}{\cancel{b-a}}\cancel{(b-a)}(b+a)$$

$$\Rightarrow \frac{a+b}{2}$$

if $X \sim U(a,b)$

$\int_a^b x\, dx$
$= \frac{x^2}{2}$
$= \frac{1}{2}\left[x^2\right]_a^b$
$\frac{1}{2}\left(b^2 - a^2\right)$

$b^2 - a^2$
$= (a+b)(a-b)$

$$\text{Var}(X) = E[X^2] - [E[X]]^2$$

$E[X] \to \frac{a+b}{2}$

$E[x^2]$ $\int_a^b x^2 \underline{f(x)}\, dx$ $\hookrightarrow \frac{1}{b-a}$ $\Rightarrow \frac{1}{b-a}\int_a^b x^2\, dx \Rightarrow \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b$

$$\Rightarrow \frac{1}{3(b-a)}\left(b^3-a^3\right)$$

$$= \frac{1}{3\cancel{(b-a)}}\cancel{(b-a)}(b^2+ab+a^2)$$

$$= \frac{b^2+ab+a^2}{3}$$

$$Var(x) = \frac{b^2+ab+a^2}{3} - \left(\frac{a+b}{2}\right)^2$$

$$\Rightarrow \frac{b^2+ab+a^2}{3} - \frac{a^2+b^2+2ab}{4}$$

$$\Rightarrow \frac{4b^2+4ab+4a^2-3a^2-6ab-3b^2}{12} \Rightarrow \frac{b^2-2ab+a^2}{12}$$

$$Var(X) \Rightarrow \frac{(b-a)^2}{12} \qquad \left((a-b)^2 = a^2-2ab+b^2\right)$$

$\sim U(a,b)$