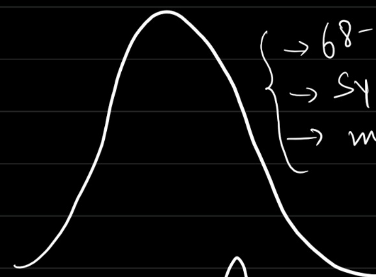
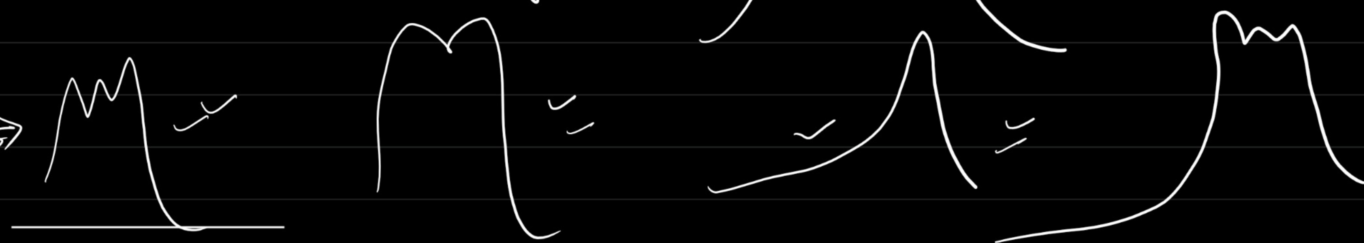
Central limit theorem



- $\rightarrow$ 68 - 95 - 99.7%
- $\rightarrow$ Symmetrical
- $\rightarrow$ mean = median = mode

$\rightarrow$ Distribution will be irregular.

$\Rightarrow$



$\rightarrow$ The CLT states that if you have a population with mean μ and standard deviation σ and take sufficiently large random samples from the population with replacement, then the distribution of sample means will be approximately normally distribution.

$\rightarrow$ Sampling mean of a population(μ,σ) will approximately be a normal distribution $\longrightarrow \mu = \mu,\ \sigma = \sigma/\sqrt{n}$

$S_1(50) \longrightarrow \bar{X}_{S_1}$

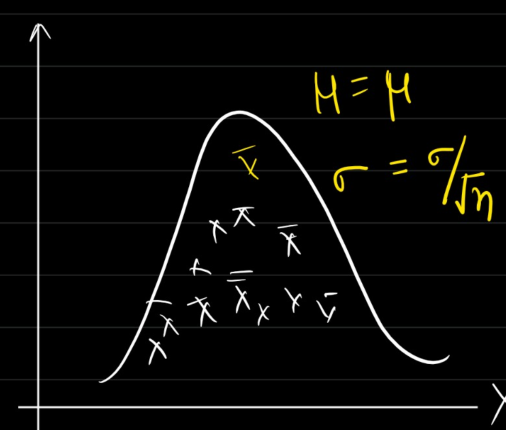
$S_2(50) \longrightarrow \bar{X}_{S_2}$

$S_3(50) \longrightarrow \bar{X}_{S_3}$

$\vdots$

$S_{20k}(50) \longrightarrow \bar{X}_{S_{20k}}$

$\longrightarrow$



$\mu = \mu$

$\sigma = \sigma/\sqrt{n}$

CLT $\rightarrow$ Sampling distribution of the mean $\Longrightarrow$

$\mu = \mu$

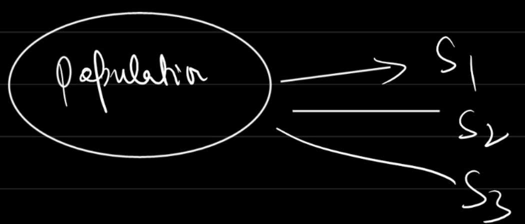
$\sigma = \sigma/\sqrt{n}$

Population (μ,σ) $\longrightarrow$ large No of sample $\longrightarrow$ Sampling mean $\rightarrow$ plot $\hookrightarrow$ N.D $\mu = \mu,\ \sigma = \sigma/\sqrt{n}$

Two conditions of CLT :-

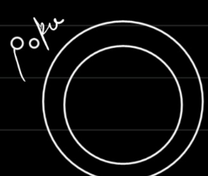
1. The no. of samples should be large.
2. The sample size should be greater than or equals to 30 (except the population distribution which is already a Normal Dist.)

Standard error $= \sigma/\sqrt{n}$



$\dfrac{\sigma}{\sqrt{n}}$

Higher the sample size, SE will be low



$SE \propto \dfrac{1}{n}$ $\quad SE\downarrow \; n\uparrow$

Q You have a population with a $\mu = 100$ and Std dev $(\sigma) = 20$. If you take sample size 50 from this population. What is the prob that sample mean will be less than 105.

→ $\mu = 100, \quad \sigma = 20, \quad n = 50, \quad \bar{X} = 105$

$$Z\text{score} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{105 - 100}{\frac{20}{\sqrt{50}}} = \frac{5 \cdot \sqrt{2}}{4}$$

$$Z\text{score} = \frac{5 \cdot \sqrt{2}}{4} \Rightarrow \frac{5 \times 1.414}{4} = \frac{7.07}{4} = 1.7675$$

$$= 0.8944$$