D7047E - Advanced Deep Learning Exercises

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1 Exercise 1

1.1 Task 1.1 Convolution

Given an image matrix I and a kernel matrix k

$$I = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & -3 & -4 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$k = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

calculate a *same* convolution $I * k$.

Solution: The goal is to perform same convolution and get a resultant matrix R with size 4×4. We start by zero padding the matrix I to get the matrix I'.

$$I' = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 2 & 1 & 0 \\ 0 & 1 & -3 & -4 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{1}$$

And then we want to flip the kernel matrix to get the matrix k'.

$$k' = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix} \tag{2}$$

To get the resultant matrix R, we multiply each element pointwise and sum all the results

$$I' \odot k' = \sum_{i,j} f_{i,j} \times g_{i,j}. \tag{3}$$

$$\begin{aligned}
(f * g)(0,0) \quad &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix} \\
&= (0 \times 0) + (0 \times 1) + (0 \times 0) \\
&+ (0 \times 1) + (1 \times 2) + (1 \times 1) \\
&+ (0 \times 0) + (1 \times 1) + (1 \times 0) \\
&= 4
\end{aligned}$$

$$\begin{aligned}
(f * g)(0,1) \quad &= \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix} \\
&= (0 \times 0) + (0 \times 1) + (0 \times 0) \\
&+ (1 \times 1) + (1 \times 2) + (1 \times 1) \\
&+ (1 \times 0) + (1 \times 1) + (2 \times 0) \\
&= 5
\end{aligned}$$

Repeating the calculation 3 for $(i, j) = 1, 2, 3, 4$ and we get the resultant matrix

$$R = \begin{bmatrix} 4 & 5 & 6 & 4 \\ 5 & 3 & 3 & 6 \\ 1 & -7 & -7 & 0 \\ 4 & 1 & 0 & 4 \end{bmatrix} \tag{4}$$

1.2 Task 1.2 Non Linearity

Apply the ReLU activation to the resultant matrix R.

Solution: The ReLU activation is defined as

$$f(x) = \begin{cases} 1, & \text{if } x \, 0 \\ 0, & \text{otherwise.} \end{cases}$$

Apply to R and we get

$$R = \begin{bmatrix} 4 & 5 & 6 & 4 \\ 5 & 3 & 3 & 6 \\ 1 & 0 & 0 & 0 \\ 4 & 1 & 0 & 4 \end{bmatrix} \tag{5}$$

1.3 Task 1.3 Max Pooling

Apply *valid* Max Pooling with a filter size of (2,2) on the matrix R in (5).

Solution: Applying Max Pooling with filter size (2,2) gives us the resultant matrix

$$R_{\text{maxpool}} = \begin{bmatrix} 5 & 6 \\ 4 & 4 \end{bmatrix} \tag{6}$$

1.4 Task 1.4 Flattening

Flatten the resultant matrix R_{maxpool}.

Solution:

$$R_{\text{flattened}} = [5, 6, 4, 4]^T \tag{7}$$

1.5 Task 1.5 Fully Connected Layer

Calculate the output of a Fully Connected layer. Use the follow W matrix:

$$W = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}. \tag{8}$$

Solution: We perform matrix multiplication $W \times R_{\text{flattened}}$ to get the output

$$\mathbf{z} = \begin{bmatrix} 45 \\ 121 \end{bmatrix}. \tag{9}$$

1.6 Task 1.6 SoftMax

Apply a softmax to the output $\mathbf{z}$ and indicate which is the output class (1st or 2nd). The definition of softmax is

$$\sigma(\mathbf{z})_i = \frac{e^{z_i}}{\sum_{j=1}^{K} e^{z_j}}$$

where K is the number of classifications.

Solution: Applying softmax function to $\mathbf{z}$ gives us

$$\sigma(z) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{10}$$

which indicates output of the 2nd class.