The perimeter of a circle is 2pi times its radius, and its area is pi times the radius squared.

But why?

Well, the perimeter is by definition. The area... is more complicated.

Since ancient times, mathematicians have wondered about the perimeter of a circle.

What did they do? Approach it by a regular polygon, whose perimeter they could calculate. As the sides of the polygon increased, it approached the shape of a circle, bringing the value of the perimeter closer to that of a circle.

Conclusion? The perimeter is the diameter times a constant 3.14159..., etc. We call this constant, the ratio between the circumference and its diameter, “PI”, and the perimeter is pi times the diameter... ...although since the radius is usually used, half the diameter, we leave it as 2pi for the radius.

— Now, what about the area of the circle? The same thing: we approximate it with regular polygons with more and more sides, but now we look for their areas.

Look at this pentagon, for example. It is regular, so its sides measure the same and its angles measure the same. From each vertex, draw a line that divides each angle in half. These 5 lines all meet in the center, dividing the pentagon into 5 triangles. All with the same 3 angles, and the same base: one of the sides of the pentagon. That makes them congruent triangles: they have the same sides, the same height, the same AREA.

The height of each triangle is what in the original pentagon was the distance between one side and the center, which is called “apothem.”

So the area of a triangle is its base (one of the sides L of the pentagon), times its height (the apothem a of the pentagon), divided by 2. L times a divided into 2. And since these 5 triangles of equal area reconstruct the pentagon, the area of this pentagon is 5 times the area of a triangle: 5La/2.

In a regular hexagon, the same: it is divided into 6 congruent triangles, all of them have the same base (one side of the hexagon) and the same height (the apothem), and therefore the same area (La/2), so the area of the hexagon is 6 times La/2.

And in general a regular polygon with n sides is divided into n triangles of area La/2. The general area formula would then be nLa/2, for any regular polygon.

Now the idea is to increase the number of sides to infinity, getting closer to a circle, and that's it! We can find the area of the circle with this formula...

...mmm, moment. Something's wrong. The number n of sides is infinitely large, and each side L is infinitely small.

Let's go back to the beginning, to the pentagon. Its area is 5La / 2. Look at that 5L, the length of one side, times 5. That is precisely the PERIMETER of the pentagon. So we leave the area as PERIMETER by apothem divided into 2.

The same with the hexagon. Its area is 6La/2, but 6L is the perimeter of the hexagon.

In general, for a polygon with n sides whose area is nLa/2, that nL, the measurement L of its side times the number n of sides it has, is its PERIMETER P.

In other words, for all these polygons, its area is Pa /2. It doesn't matter how many sides they have. If we increase n towards infinity, approaching a circle, the formula should remain the same: perimeter times apothem over 2.

BUT: the perimeter of the circle is 2pi times its radius. The apothem ends up being precisely THE RADIO. We just have to replace this in the area formula, and simplify. And this is why the area of the circle is pi times the radius squared.

That's it for the video, I hope you liked it, and have a happy new year.