

Machine Learning

Homework Assignment 3

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ASS-3 mag

②

$$\begin{bmatrix} 0 & 14 \\ 6 & 9 \end{bmatrix}$$

(a) $Au = Kv$

$$\begin{bmatrix} 0-K & 14 \\ 6 & 9-K \end{bmatrix} = \begin{bmatrix} -K & 14 \\ 6 & 9-K \end{bmatrix}$$

$$\begin{aligned} -K(9-K) - (14 \times 6) &= -9K + K^2 - 84 \\ &= \underline{\underline{K^2 - 9K - 84}} \end{aligned}$$

(b) $K^2 - 9K - 84 = 0$

$$K = 14.71 \text{ or } K = -5.71$$

(c) For $K = 14.71$

$$\begin{bmatrix} -14.71 & 14 \\ 6 & 9-14.71 \end{bmatrix} = \begin{bmatrix} -14.71 & 14 \\ 6 & -5.71 \end{bmatrix}$$

$$\begin{aligned} -14.71x + 14y &= 0 & \& \quad \sqrt{x^2 + y^2} = 1 \\ 6x - 5.71y &= 0 \end{aligned}$$

Solving from (i) & (ii) ✓

$$c) \Rightarrow K_1 = 14.71$$

$$K_2 = 5.71$$

for K_1
$$\begin{bmatrix} -14.71 & 14 \\ 6 & -5.71 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

~~for K_1~~ $6x - 5.71y = 0$ $\sqrt{x^2 + y^2} = 1$

$$x = \frac{5.71}{6}y = 0.952y$$

$$\sqrt{0.9063y^2 + y^2} = 1$$

eigen vector

$$y = 0.7243 \text{ \& } x = 0.6895 \quad \boxed{[0.6895, 0.7243]}$$

for K_2

$$\begin{bmatrix} 5.71 & 14 \\ 6 & 14.71 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow 6x + 14.71y = 0, \sqrt{x^2 + y^2} = 1$$

$$x = -\frac{14.71}{6}y$$

~~$\boxed{2}$~~

~~$\rightarrow 2.648$~~ $\boxed{[0.92594, 0.37762]}$
 $K_1 \quad K_2$

d) eigen values = $[-5.71028893, 14.71028893]$

eigen vector = $\begin{bmatrix} -0.9259401 & -0.6894021 \\ 0.37767039 & -0.72437887 \end{bmatrix}$

eigen vector for K_1 : $[-0.9259401 \quad 0.37767039]$

eigen vector for K_2 : $[-0.6894021 \quad -0.72437887]$

(3a)

$$\begin{bmatrix} 5 & 2 & 4 \\ 9 & 6 & 4 \\ 7 & 1 & 0 \\ 2 & 5 & 6 \end{bmatrix}$$

$$S_1 = \frac{5+9+7+2}{4} = 5.75$$

$$S_2 = \frac{2+6+1+5}{4} = \frac{14}{4} = 3.5$$

$$S_3 = \frac{14}{4} = 3.5$$

$$b = \begin{bmatrix} -0.75 & -1.5 & 0.5 \\ 3.25 & 2.5 & 0.5 \\ 1.25 & -2.5 & -3.5 \\ -3.75 & 0.5 & 2.5 \end{bmatrix}$$

$$(b) \quad S_{1,3} = \sum_t (x_i^t - S_i) * (x_j^t - S_j)$$

$$= \frac{(x_1^1 - S_1) * (x_3^1 - S_3) + (x_1^2 - S_1) * (x_3^2 - S_3) + (x_1^3 - S_1) * (x_3^3 - S_3) + (x_1^4 - S_1) * (x_3^4 - S_3)}{N-1}$$

$$= \frac{(-0.75)(0.5) + (3.25)(0.5) + (1.25)(-3.5) + (-3.75)(2.5)}{3}$$

$$= \frac{12.5}{3} = 4.1\bar{6}$$

(C)

Sample Covariance Matrix of B

$$\begin{bmatrix} 8.9166667 & 0.166\bar{6} & -4.166\bar{6} \\ 0.166\bar{6} & 5.66\bar{6} & 4.33\bar{3} \\ -4.166\bar{6} & 4.33\bar{3} & 6.33\bar{3} \end{bmatrix} \quad (3 \times 3)$$

~~(-0.75 - 5.75)(2.5 - 3.5)~~ eigenvalue for this

$$\begin{bmatrix} 0.99703373 & 12.97881887 & 7.44081437 \end{bmatrix}$$

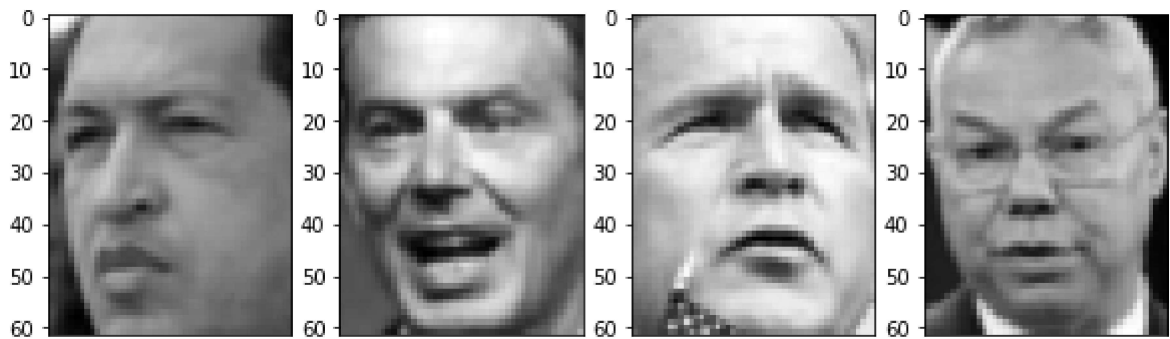
Largest eigenvalue = 12.97881887

d

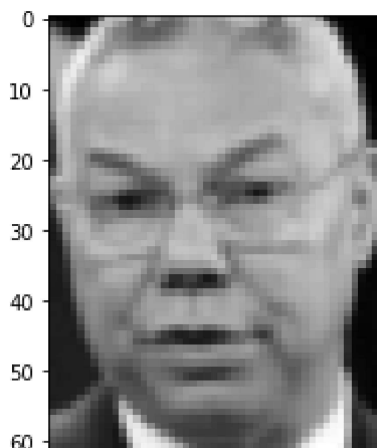
0.26018677	-1.41900435
-0.87353472	4.03721245
-4.04749635	-1.8486773
4.66084433	-0.7695308


```
In [19]: import numpy as np
import matplotlib
import matplotlib.pyplot as plt
import pandas as pd
from sklearn.datasets import fetch_lfw_people
lfw_people = fetch_lfw_people(min_faces_per_person=70)
n_samples, h, w = lfw_people.images.shape
print(lfw_people.images.shape)
npix = h*w
fea = lfw_people.data
def plt_face(x):
    global h,w
    plt.imshow(x.reshape((h, w)), cmap=plt.cm.gray)
    plt.xticks([])
plt.figure(figsize=(10,20))
nplt = 4
for i in range(nplt):
    plt.subplot(1,nplt,i+1)
    plt_face(fea[i])
plt.show()
```

(1288, 62, 47)

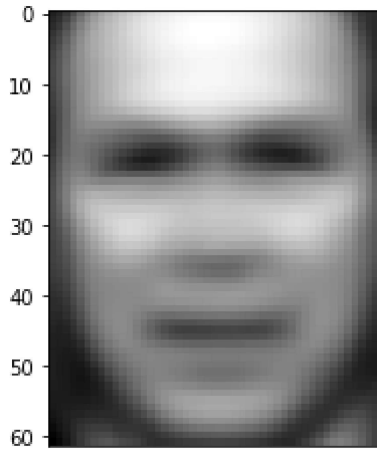


```
In [31]: #Question 5 (a)
plt_face(fea[3])
plt.show()
```




```
In [24]: #Question 5 (b)
y=np.mean(fea,axis=0)
print(y.shape)
plt_face(y)
plt.show()
```

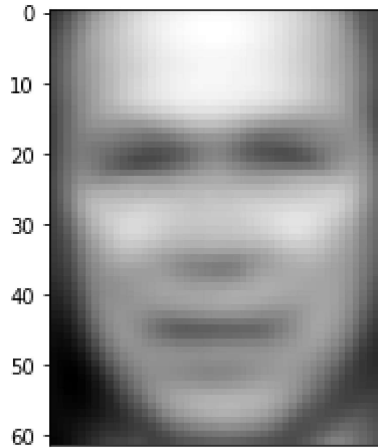
(2914,)



```
In [82]: #Question 5 C
import sklearn.decomposition as skd
import numpy as np
X=fea-y
pca = skd.PCA(n_components = 5)
skd.PCA.fit(pca,X)
W1 = pca.components_
W = W1.transpose()
Z = pca.transform(X)
print(Z[3,])
```

[202.54202 -261.47684 418.97412 -29.39988 39.783478]

```
In [83]: #Question 5 (d)
# for components = 5
XX=np.matmul(Z,W.transpose())
XX+=y
plt_face(XX[3])
plt.show()
```

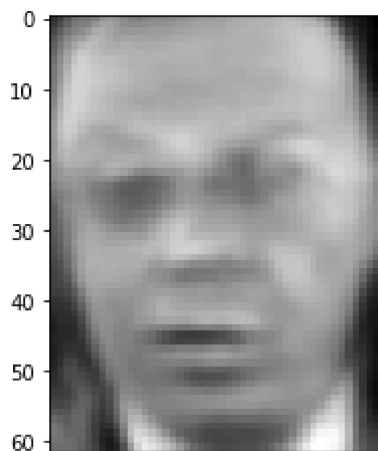


```
In [79]: #Question 5 (d)
# for components = 50
import sklearn.decomposition as skd
import numpy as np
X=fea-y
pca = skd.PCA(n_components = 50)
skd.PCA.fit(pca,X)
W1 = pca.components_
W = W1.transpose()
Z = pca.transform(X)
print(W1.shape)
print(Z.shape)

XX=np.matmul(Z,W.transpose())
XX+=y
plt_face(XX[3])
plt.show()
```

(50, 2914)

(1288, 50)



In []:

In []: