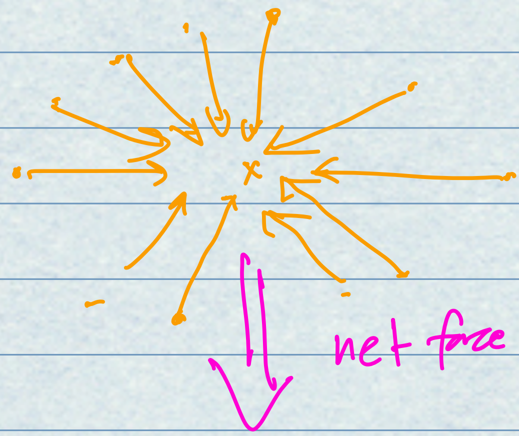
Electrodynamics

* charge, 'Q'; +q and/or -q

$$\boxed{\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}\hat{r}}$$ → source charge

$$\boxed{\vec{F} = q\vec{E}}$$



Suppose several point charges 'q_1', 'q_2' . . . .

$$\vec{F} = \vec{F_1} + \vec{F_2} + \ldots$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1Q}{r_1^2}\hat{r_1} + \frac{q_2Q}{r_2^2}\hat{r_2} + \ldots\right]$$

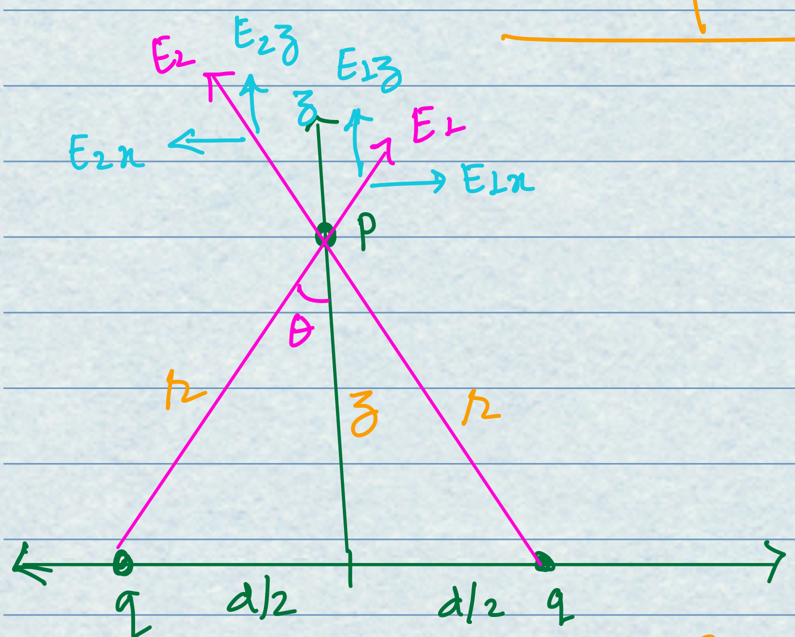
$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1^2}\hat{r}_1 + \frac{q_2}{r_2^2}\hat{r}_2 + \cdots\right]$$

$$\boxed{\vec{F} = Q\vec{E}}$$

$$\Downarrow$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1^2}\hat{r}_1 + \frac{q_2}{r_2^2}\hat{r}_2 + \cdots\right]$$

Find: electric field distance 'z' above midpoint between 'q' a distance 'd' apart.



$E_{1z} = E_1 \cos\theta$

$\cancel{E_{1x} = E_1 \sin\theta}$

$E_{2z} = E_2 \cos\theta$

$\cancel{E_{2x} = -E_1 \sin\theta}$

So, net force:

$$E_z = (E_1 + E_2)\cos\theta$$

or, $$E_z = \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} + \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right)\cos\theta$$

$$E_z = 2 \cdot \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \cos\theta$$

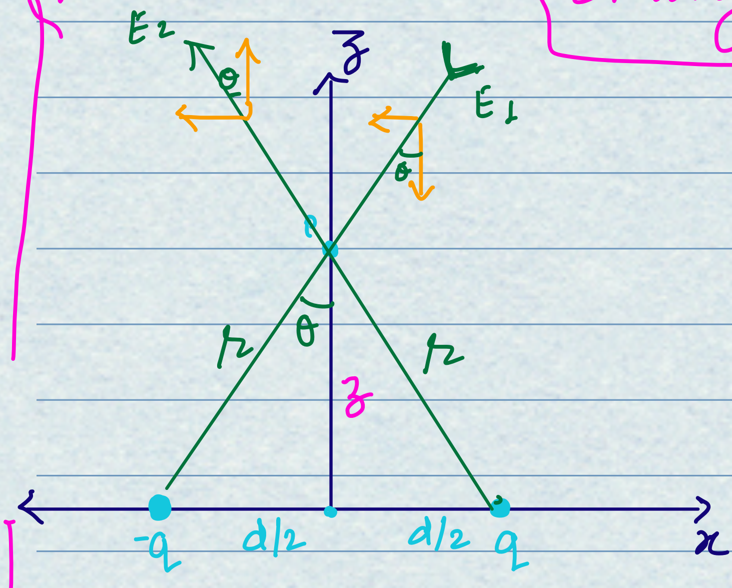
$r^2 = z^2 + d^2/4$

$|r| = \sqrt{z^2 + d^2/4}$

$\cos\theta = \frac{z}{\sqrt{z^2 + d^2/4}}$

$$E_z = 2 \cdot \frac{1}{4\pi\varepsilon_0} \frac{q}{(z^2+d^2/4)} \frac{z}{\sqrt{(z^2+d^2/4)}}$$

P 2.2 # Same problem but first charge is '-q'.



~~$E_{1z} = -E_1 \cos\theta$~~

$E_{1x} = -E_1 \sin\theta$

~~$E_{2z} = E_2 \cos\theta$~~

$E_{2x} = -E_2 \sin\theta$

<u>net field</u>: in x-direction:

$$E_x = E_{1x} + E_{2x}$$

$$= -E_1 \sin\theta - E_2 \sin\theta$$

$$= -2E_1 \sin\theta$$

$$= -2\left[\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right]\sin\theta$$

$$r^2 = z^2 + (d^2/4)$$

$$r = \sqrt{z^2 + (d^2/4)}$$

$$\sin\theta = \frac{d/2}{\sqrt{z^2 + d^2/4}}$$

$$\vec{E} = -2\left[\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right]\sin\theta$$

$$\text{or, } \vec{E} = -\cancel{2}\left[\frac{1}{4\pi\varepsilon_0}\frac{q}{(z^2+d^2/4)}\right]\frac{d/\cancel{2}}{\sqrt{z^2+d^2/4}}$$

$$\text{So, } \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{qd}{(z^2+d^2/4)^{3/2}}$$