Finding Phase Factors for Quantum Signal Processing Using Quantum Machine Learning

Halil İbrahim Olgun

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Abstract

Quantum Signal Processing(QSP)[1][2] applies polynomial transformations through composite rotation sequences. It forms the basis of Quantum Singular Value Transformation (QSVT)[3][4], where these phase sequences are used to transform the singular values of unitary matrices. QSVT is often regarded as a unifying framework for quantum algorithms, with major algorithms like Grover's Search Algorithm[5], and Hamiltonian Simulation[6] and sub-parts of Shor's Factoring Algorithm[7] emerging as specific cases. The challenge of finding optimal phase factors for QSP has persisted for years, with various classical methods proposed for finding and optimizing the phase factors[8][9]. This project aims to leverage quantum machine learning tools to explore and optimize phase factors for QSP.

Theorem 1(Quantum Signal Processing(QSP)): The QSP sequence $U_{\vec{\phi}} = \{\phi_0, \phi_1, ..., \phi_k\}$ produces a matrix which may be expressed as polynomial function of a:

$$e^{i\phi_0\sigma_z}\prod_{k=1}^{d} W(a)e^{i\phi_k\sigma_z} = \begin{pmatrix} P(a) & iQ(a)\sqrt{1-a^2} \\ iQ^*(a)\sqrt{1-a^2} & P^*(a) \end{pmatrix}, \quad W(a) = \begin{pmatrix} a & i\sqrt{1-a^2} \\ i\sqrt{1-a^2} & a \end{pmatrix}$$

for $a \in [-1, 1]$, and a $\vec{\phi}$ exists for any polynomials P, Q in a such that:

1. $\deg(P) \leq d$, $\deg(Q) \leq d-1$,
2. P has parity $d \bmod 2$ and Q has parity $(d-1) \bmod 2$,
3. $|P|^2 + (1-a^2)|Q|^2 = 1$.

For a polynomial satisfying these conditions, a phase sequence $U_{\vec{\phi}}$ exists that applies the transformation. However, finding the phase factors is challenging because of emerging constraints on the achievable polynomials. Current approaches involve approximating the target polynomial to fit the conditions in Theorem 1, then using various classical numerical methods to find and optimize the phase factors. The goal of this project is to explore this problem using quantum machine learning techniques, with the use of variational quantum circuits to determine these phase factors, investigating suitable cost functions and optimizers.

Given a polynomial $Q(x)(or\ P(x))$ satisfying the conditions in Theorem 1, the existence of $P(x)(or\ Q(x))$ may not be unique. In this project, a quantum generative adversarial network (QGAN) will be employed to explore the possibility of generating alternative polynomials by training an appropriate discriminator and generator.

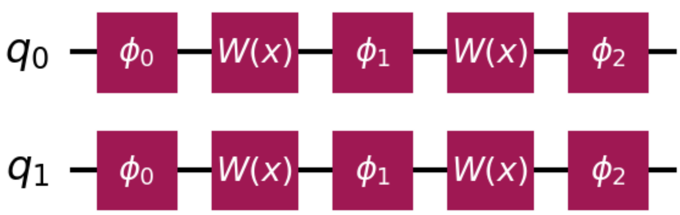

Figure 1: Example Simple Variational Circuit

References

[1] Guang Hao Low, Theodore J. Yoder, and Isaac L. Chuang. Methodology of resonant equiangular composite quantum gates. *Physical Review X*, 6(4), December 2016.

[2] Guang Hao Low and Isaac L. Chuang. Optimal hamiltonian simulation by quantum signal processing. *Physical Review Letters*, 118(1), January 2017.

[3] András Gilyén, Yuan Su, Guang Hao Low, and Nathan Wiebe. Quantum singular value transformation and beyond: exponential improvements for quantum matrix arithmetics. In *Proceedings of the 51st Annual ACM SIGACT Symposium on Theory of Computing*, qsvt. ACM, June 2019.

[4] John M. Martyn, Zane M. Rossi, Andrew K. Tan, and Isaac L. Chuang. Grand unification of quantum algorithms. *PRX Quantum*, 2(4), December 2021.

[5] Lov K. Grover. A fast quantum mechanical algorithm for database search, 1996.

[6] Richard P. Feynman. Simulating physics with computers. *International Journal of Theoretical Physics*, 21(6):467–488, 1982.

[7] Peter W. Shor. Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer. *SIAM Journal on Computing*, 26(5):1484–1509, October 1997.

[8] Yulong Dong, Xiang Meng, K. Birgitta Whaley, and Lin Lin. Efficient phase-factor evaluation in quantum signal processing. *Physical Review A*, 103(4), April 2021.

[9] Hongkang Ni and Lexing Ying. Fast phase factor finding for quantum signal processing, 2024.