MA374 MATHEMATICAL FINANCE LAB 08

Name : Rigzen Chosdon

Roll no : 220123053

Q1)

The **Black-Scholes-Merton (BSM) model** provides a way to price European call and put options. The formulas for the **call price** C and **put price** P are:

$$C = S_0 N(d_1) - K e^{-rT} N(d_2)$$

$$P = K e^{-rT} N(-d_2) - S_0 N(-d_1)$$

where:

$$d_1 = \frac{\ln(S_0/K) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Using S = 1,T= 1,K = 1, r= 0.05,sigma = 0.6

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Price of Call Option:  0.255232056656095
Price of Put Option:  0.20646148115680896
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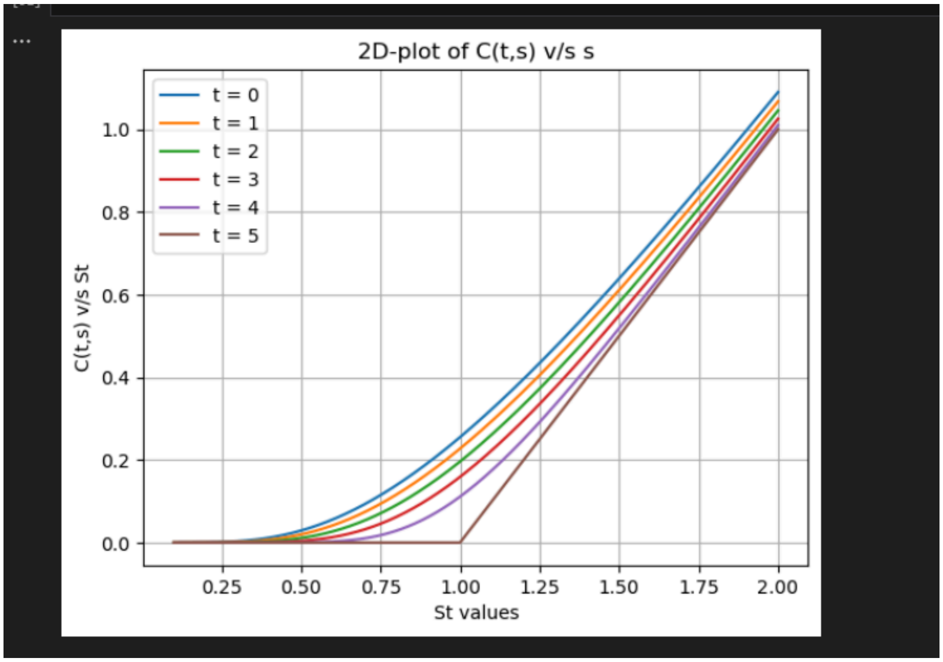
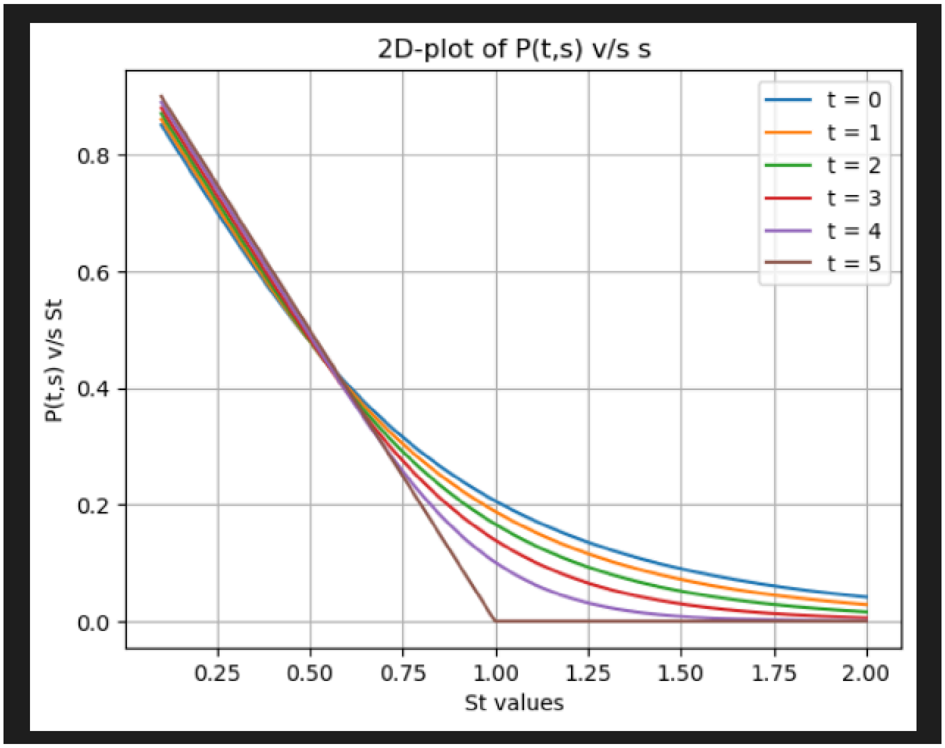
Q2T = 1,K = 1,r = 0.05,σ = 0.6. Plot, in a single graph, C(t,s) as a function of s alone for t =0,0.2,0.4,0.6,0.8,1. Do a similar plot for P(t,s) as a function of s



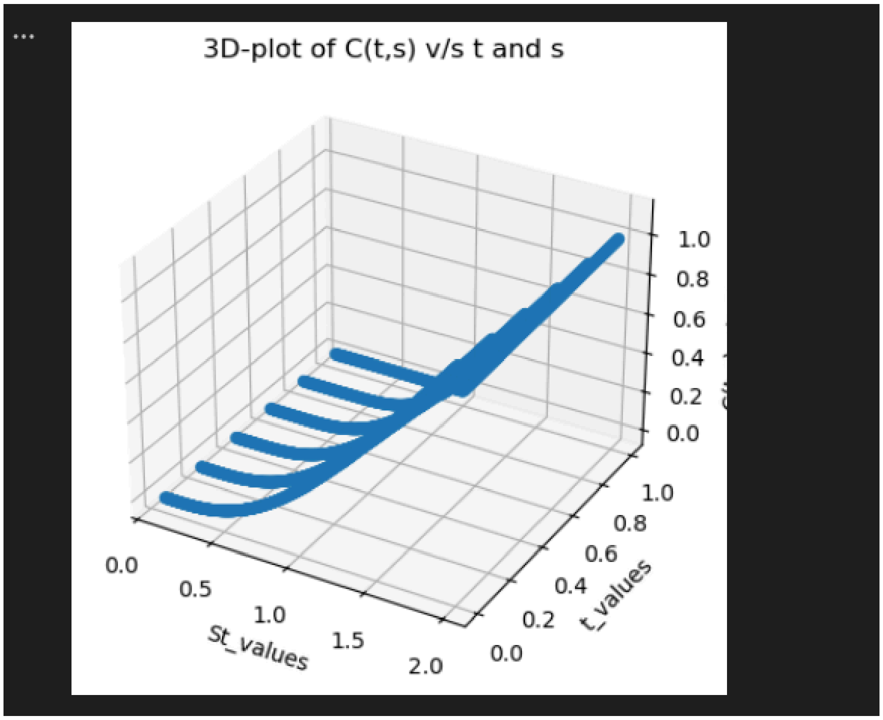
1. Call option value decreases over time as expiration approaches, reducing its relative price.
2. For low StS_t, the ratio C(t,s)/S_t is near zero, meaning the option has little value.
3. For high StS_t, the ratio approaches 1, indicating the option behaves like the stock.
4. Time decay effect is visible, with earlier curves (higher tt) lying lower than the initial ones.

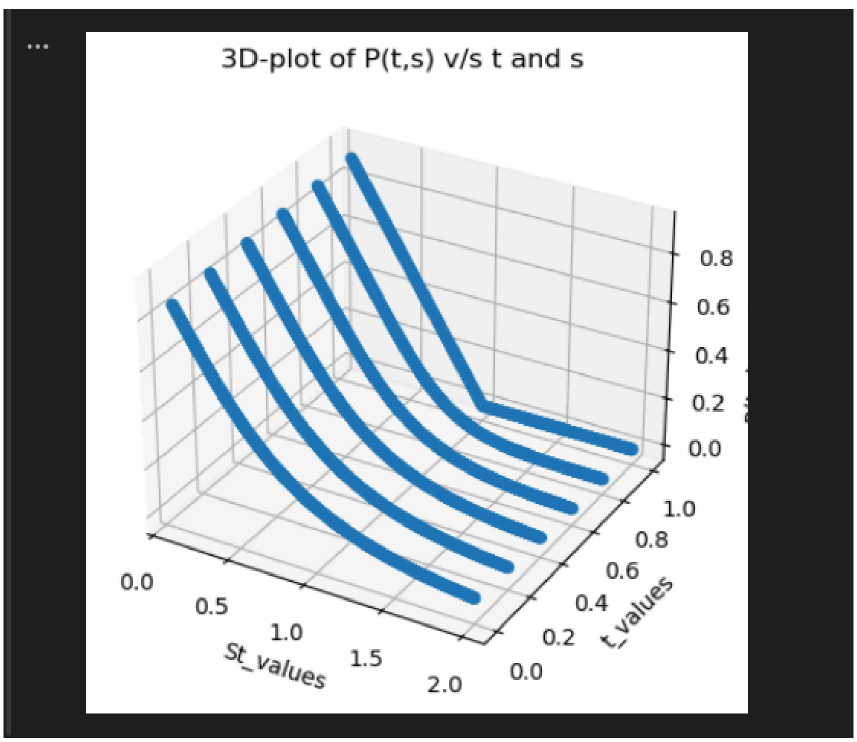


1. Call option value decreases over time as expiration approaches, reducing its relative price.
2. For low StS_t, the ratio C(t,s)/St is near zero, meaning the option has little value.
3. For high St, the ratio approaches 1, indicating the option behaves like the stock.
4. Time decay effect is visible, with earlier curves (higher tt) lying lower than the initial ones.

Q3)

Plot C(t,s) and P(t,s) as a smooth surface above the (t,s)-plane.

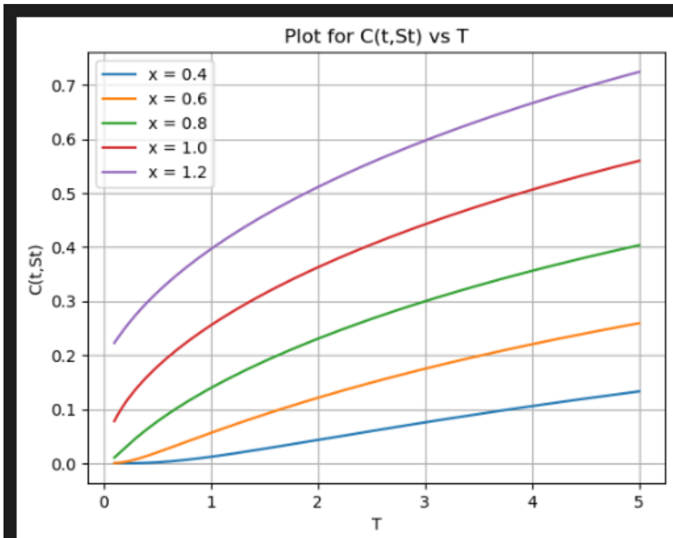

3D-plot of C(t,s) v/s t and s
St_values
t_values

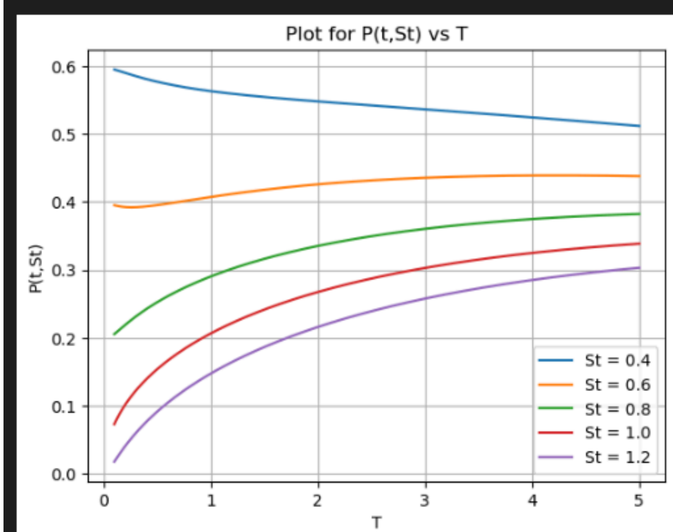

3D-plot of P(t,s) v/s t and s
St_values
t_values

The sensitivity of both the functions C and P as a function of model parameters
Sensivity T:

------------------------ Sensitivity Analysis with respect to T ------------------------

S. No	T	C(t,St)	P(t,St)
1	0.1	0.0104876	0.2055
2	0.590982	0.0897777	0.260661
3	1.08196	0.147911	0.29525
4	1.57295	0.195092	0.319458
5	2.06393	0.235421	0.33737
6	2.55491	0.270874	0.350952
7	3.04589	0.302594	0.361329
8	3.53687	0.331314	0.369225
9	4.02786	0.357547	0.375138
10	4.51884	0.381665	0.379429



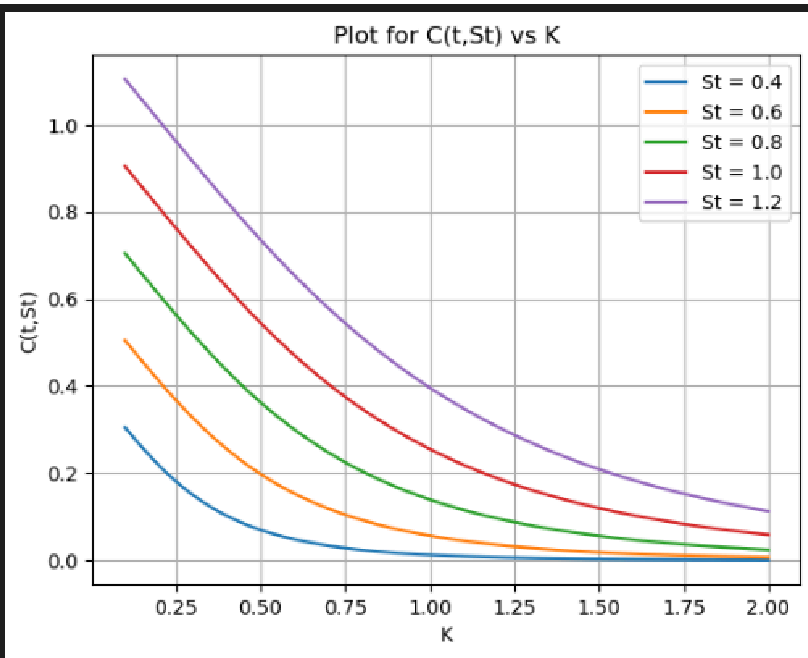


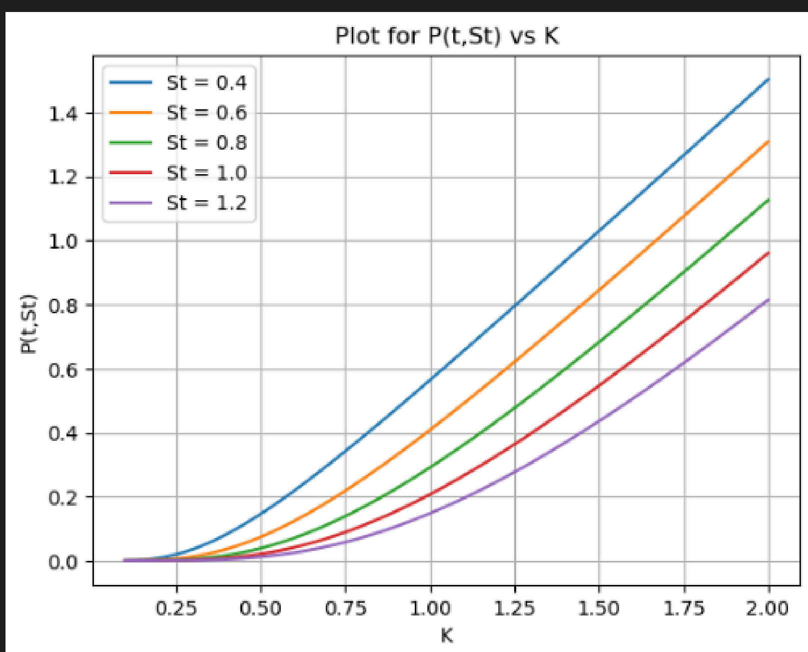
The call option price C(t,St)C(t, S_t) increases with time to maturity TT, showing the impact of time value. The put option price P(t,St)P(t, S_t) generally increases with TT, except for deep in-the-money puts, which slightly decrease. Higher stock prices lead to higher call values and lower put values. Both plots exhibit convexity, indicating diminishing marginal changes over time.

Sensitivity with K:

```
------------------------ Sensitivity Analysis with respect to K ------------------------

  S. No         K    C(t,St)      P(t,St)
-------  --------  ---------  -----------
      1  0.1       0.704885   7.64124e-06
      2  0.290381  0.527951   0.00417013
      3  0.480762  0.376097   0.0334112
      4  0.671142  0.261841   0.100251
      5  0.861523  0.181355   0.200861
      6  1.0519    0.126098   0.3267
      7  1.24228   0.0883989  0.470096
      8  1.43267   0.0625982  0.625392
      9  1.62305   0.0448062  0.788695
     10  1.81343   0.0324182  0.957403
```





1. The **call option price** C(t, S_t) **decreases** as the strike price K increases, showing that higher strike prices reduce the value of call options.
2. The **put option price** P(t, S_t) **increases** with K, reflecting the fact that a higher strike price makes a put option more valuable.
3. The curves are **convex**, showing that the rate of change in option prices slows as K increases.

4. **Sensitivity with r:**

```
------------------------ Sensitivity Analysis with respect to r ------------------------

  S. No    r    C(t,St)    P(t,St)
-------  ---  ---------  ---------
      1  0     0.126249  0.326249
      2  0.1   0.152689  0.257526
      3  0.2   0.181639  0.20037
      4  0.3   0.212714  0.153533
      5  0.4   0.24544   0.11576
      6  0.5   0.279282  0.0858129
      7  0.6   0.313685  0.0624963
      8  0.7   0.348098  0.0446838
      9  0.8   0.382015  0.0313436
     10  0.9   0.414987  0.0215565
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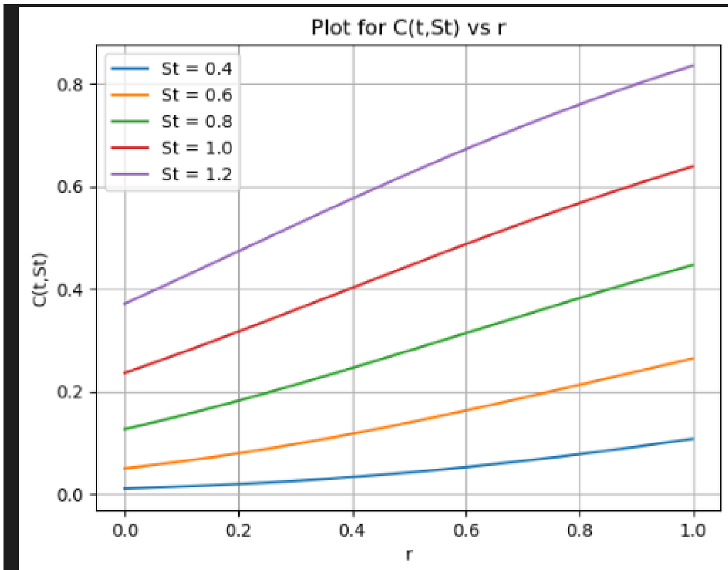
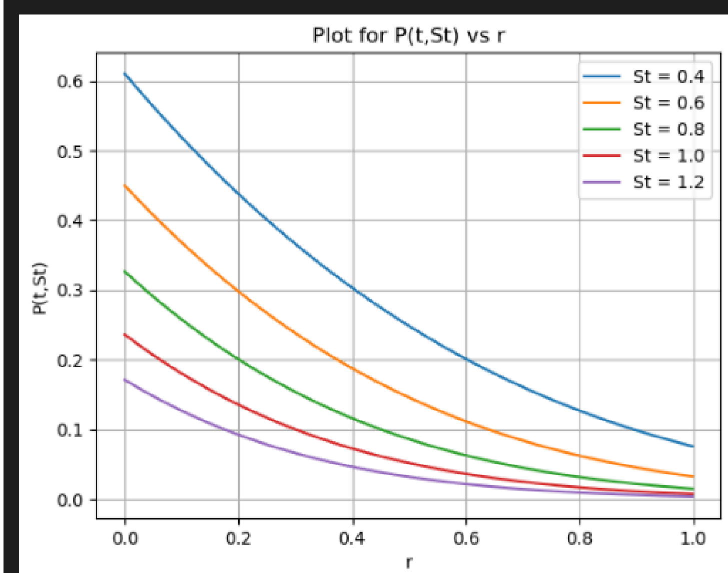
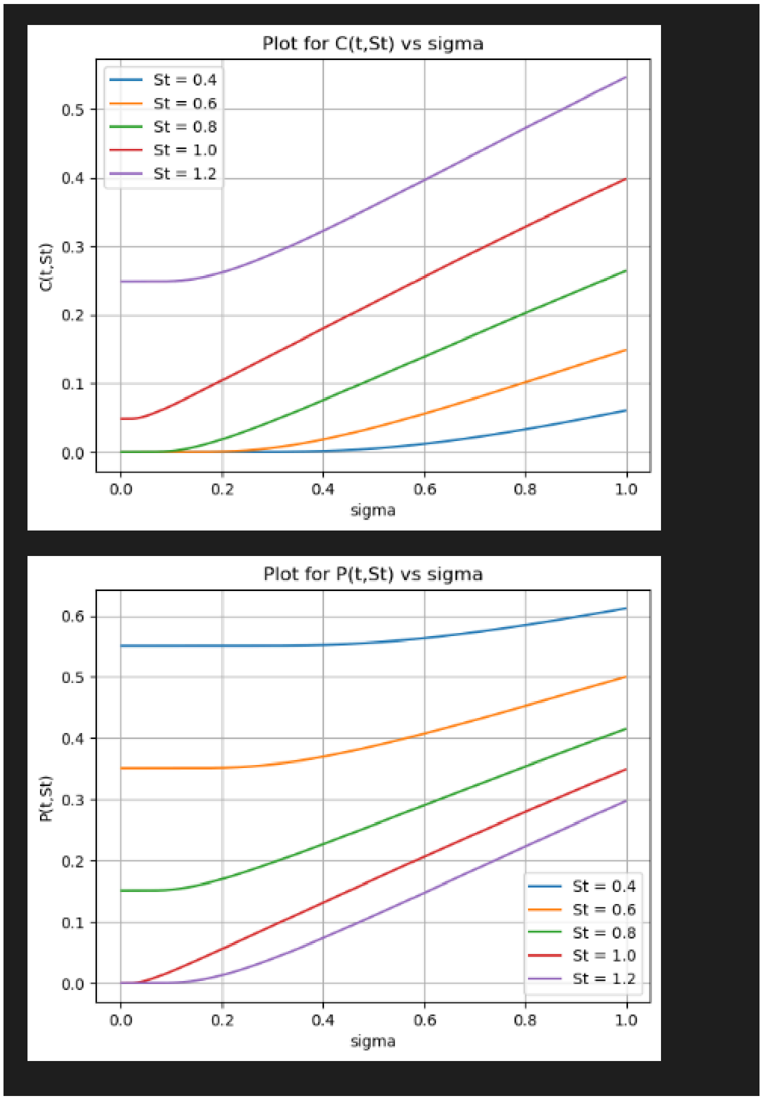




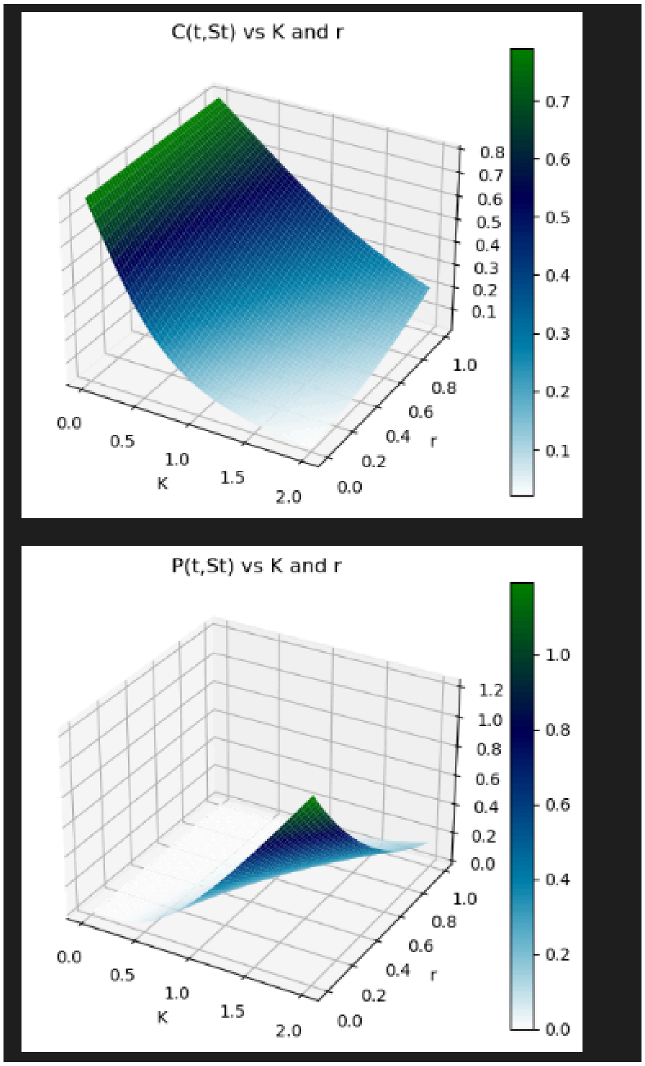
The call option price C(t,St) increases with the risk-free rate rr, as higher rates reduce the present value of the strike price. The put option price P(t,St) decreases with r, as a higher discount factor lowers its value. Higher stock prices lead to higher call values and lower put values. The trends confirm the theoretical relationship between option prices and interest rates.

S. No	sigma	C(t,St)	P(t,St)
1	0.001	0	0.151229
2	0.1009	0.00154652	0.152776
3	0.2008	0.0187849	0.170014
4	0.3007	0.0457362	0.196966
5	0.4006	0.0759687	0.227198
6	0.5005	0.10742	0.258649
7	0.6004	0.139262	0.290492
8	0.7003	0.171081	0.32231
9	0.8002	0.202627	0.353856
10	0.9001	0.233733	0.384963

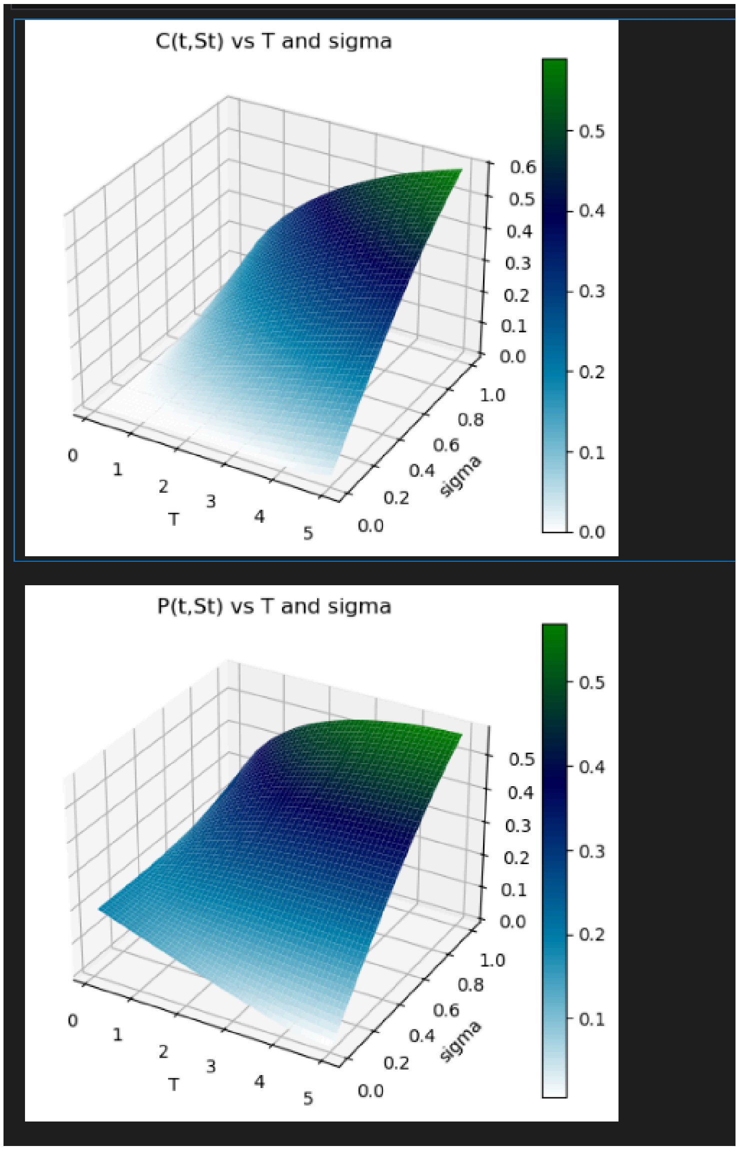


The call option price C(t,St)C(t, S_t) and put option price P(t,St)P(t, S_t) both increase with volatility σ\sigma. Higher volatility raises option values due to greater uncertainty, increasing the probability of favorable price movements. This aligns with the Black-Scholes model, where option prices are positively correlated with volatility.

Sensitivity K and r

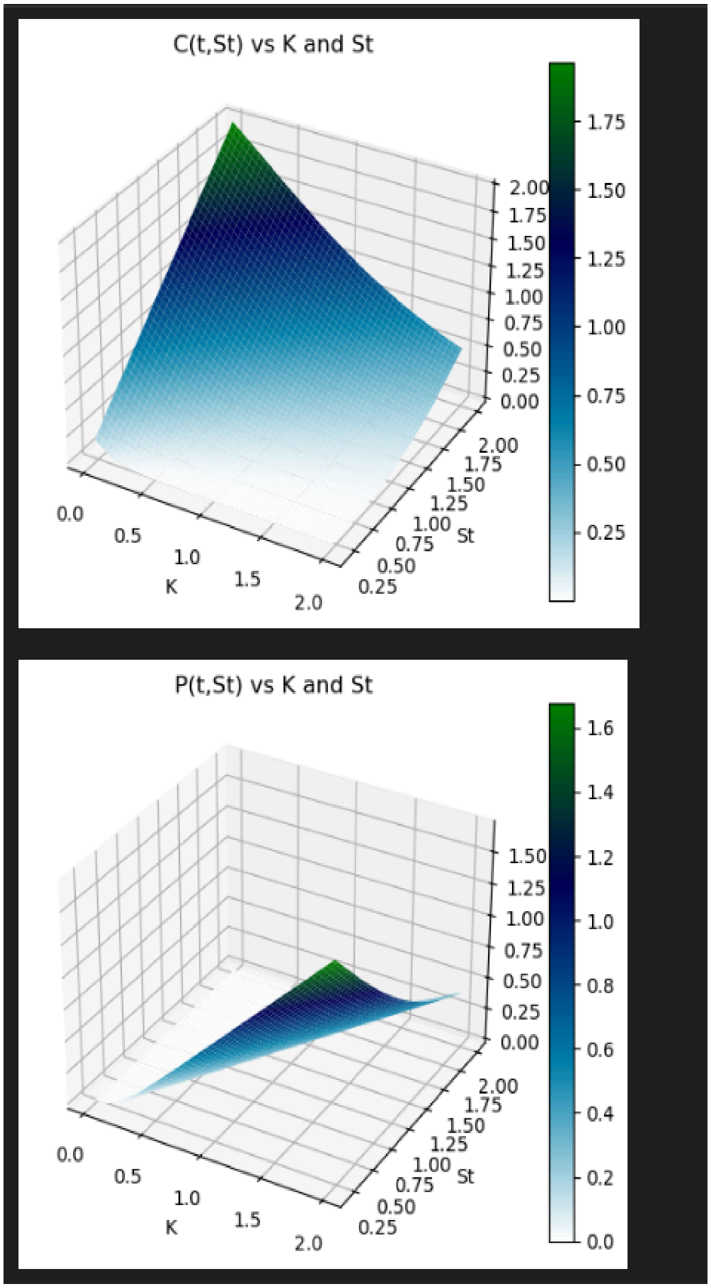


Sensitivity T vs sigma



The 3D plots show the variation of call and put option prices with respect to time to maturity T and volatility σ\sigma. Both option prices increase as sigma rises due to higher uncertainty. Additionally, as T increases, the call option price generally increases, while the put option price shows a more complex behavior.

K vs St



The 3D plots illustrate how call and put option prices vary with respect to strike price KKand current stock price St. The call option price decreases as K increases, while the put option price increases. Both prices are higher for larger St, with the call option being more sensitive to changes in St.

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sensitivity_x_and_r(0, 1, 1, 0.6)
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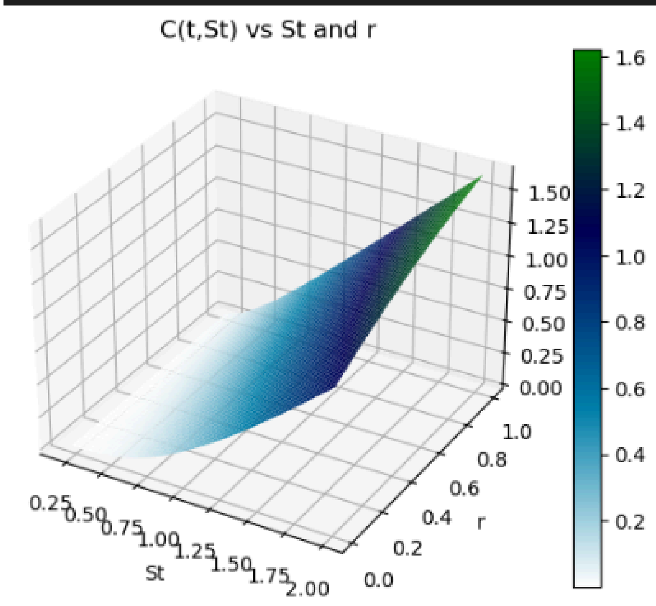
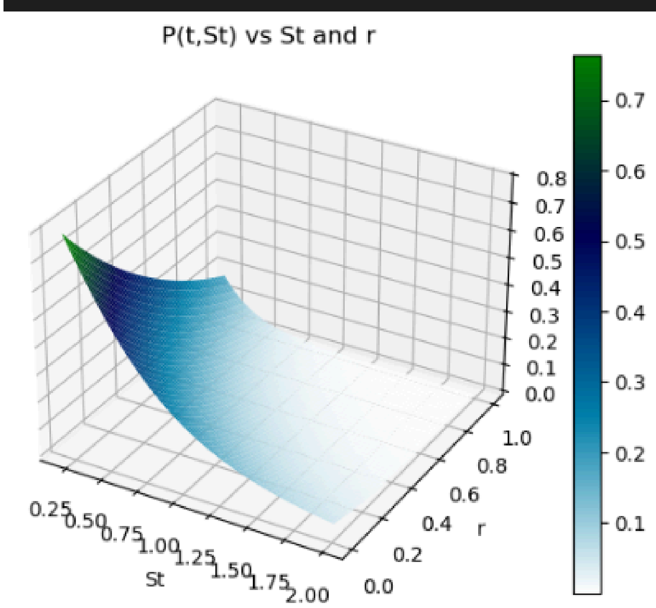
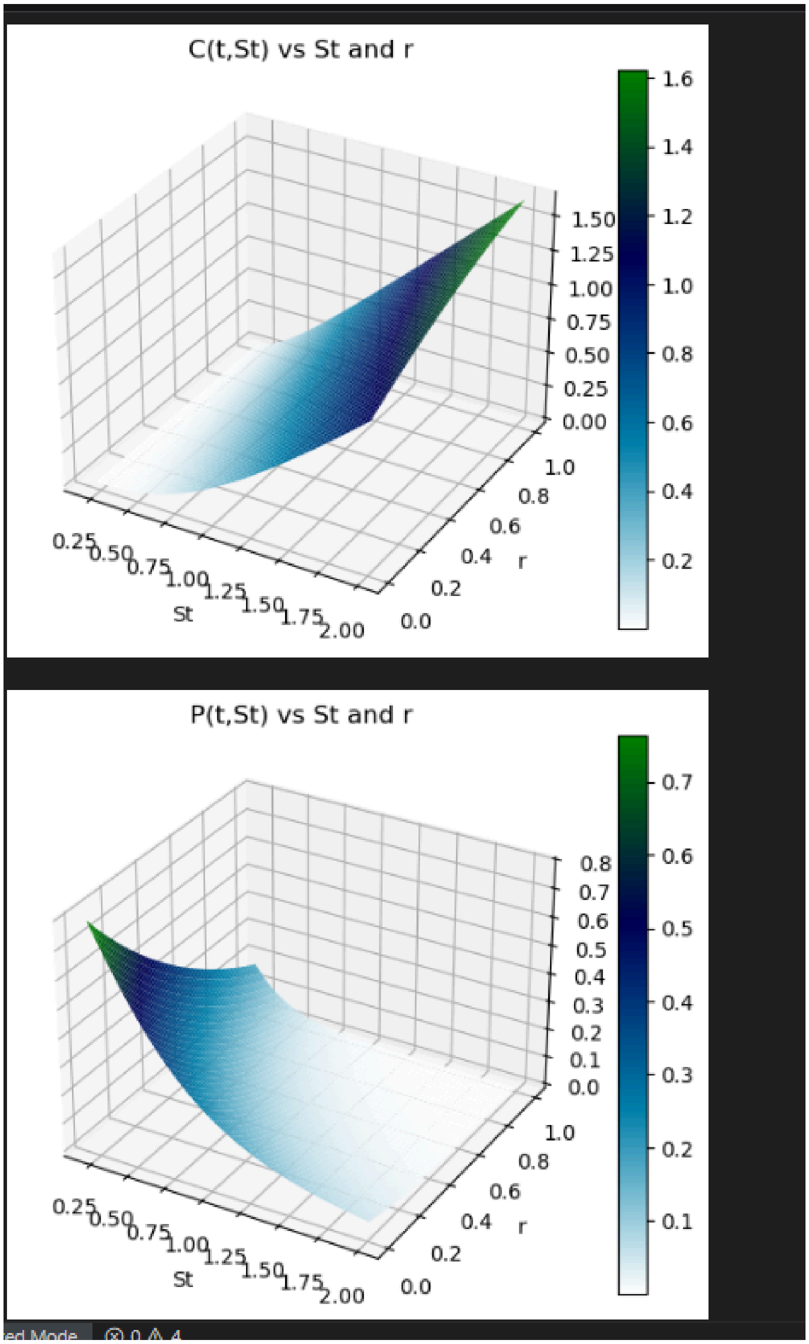
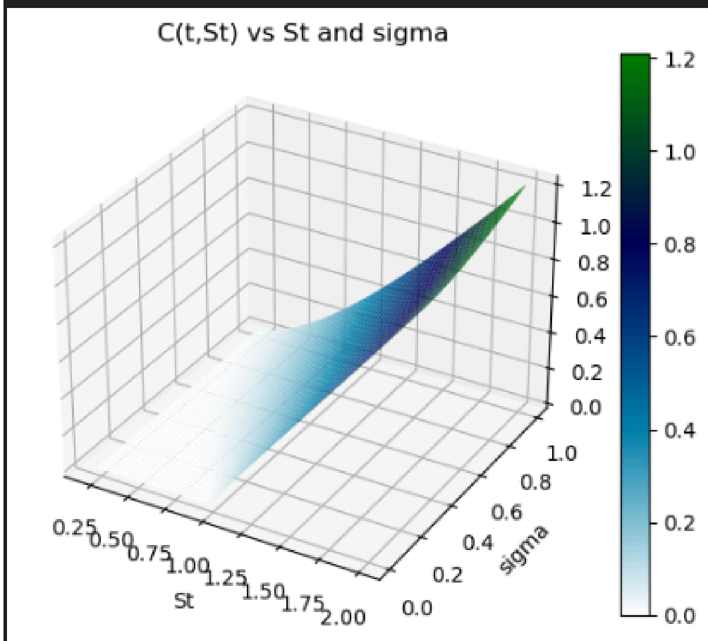
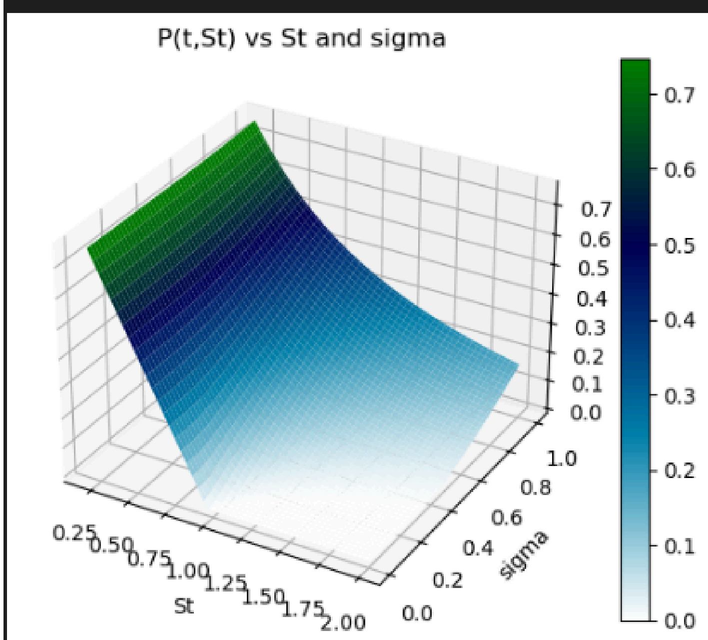




St vs r


C(t,St) vs St and r
St
r
0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00
0.0 0.2 0.4 0.6 0.8 1.0
0.00 0.25 0.50 0.75 1.00 1.25 1.50
0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6
P(t,St) vs St and r
St
r
0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00
0.0 0.2 0.4 0.6 0.8 1.0
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
0.1 0.2 0.3 0.4 0.5 0.6 0.7


C(t,St) vs St and sigma
St
sigma
0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00
0.0 0.2 0.4 0.6 0.8 1.0
0.0 0.2 0.4 0.6 0.8 1.0 1.2


P(t,St) vs St and sigma
St
sigma
0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00
0.0 0.2 0.4 0.6 0.8 1.0
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7