# MA374 Financial II Lab 12

Name Rigzen Chosdon
Roll Number : 220120353

Question1: Vasicek model

**Bond Price Formula:**

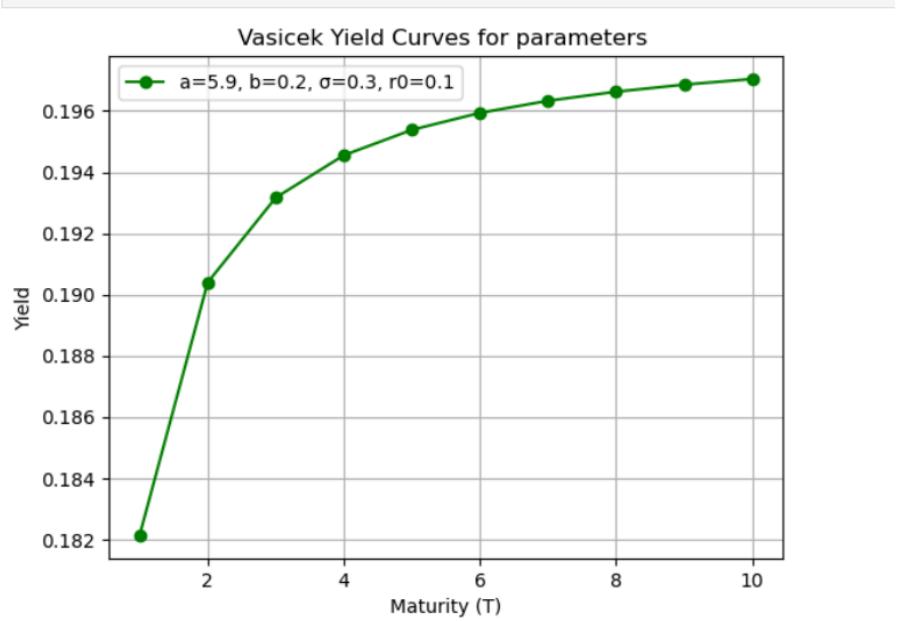
$$P(t,T) = A(T-t) \cdot e^{-B(T-t)\cdot r_t}$$

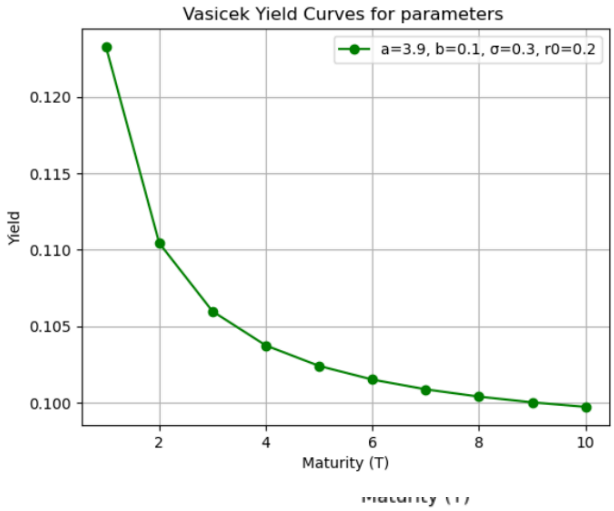
Where:

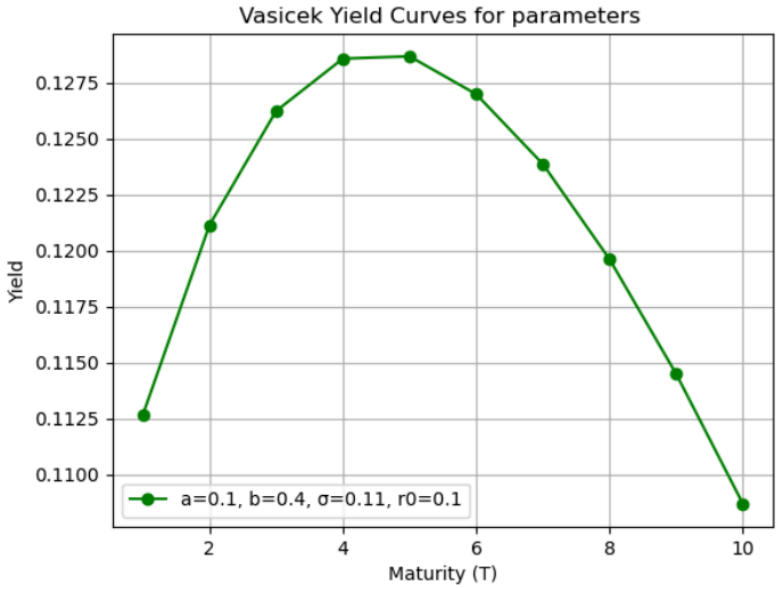
- $B(\tau) = \frac{1-e^{-a\tau}}{a}$
- $A(\tau) = \exp\left[\left(b - \frac{\sigma^2}{2a^2}\right)(B(\tau)-\tau) - \frac{\sigma^2}{4a}B(\tau)^2\right]$
- $\tau = T - t$, current time $t = 0$

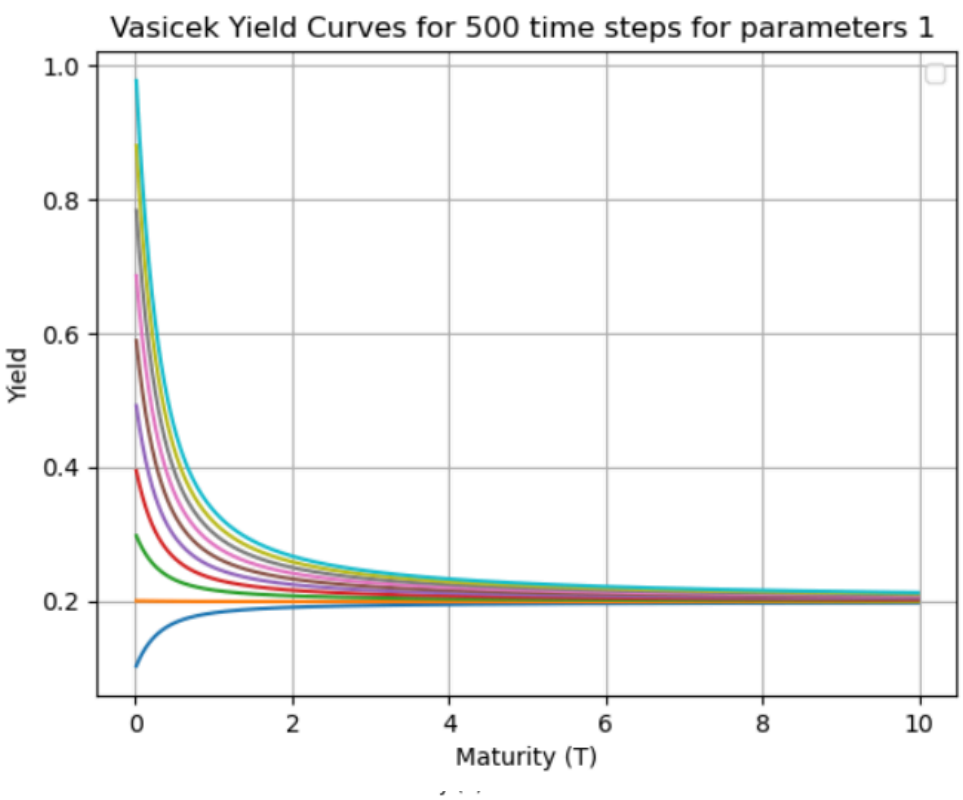
**Yield Formula:**

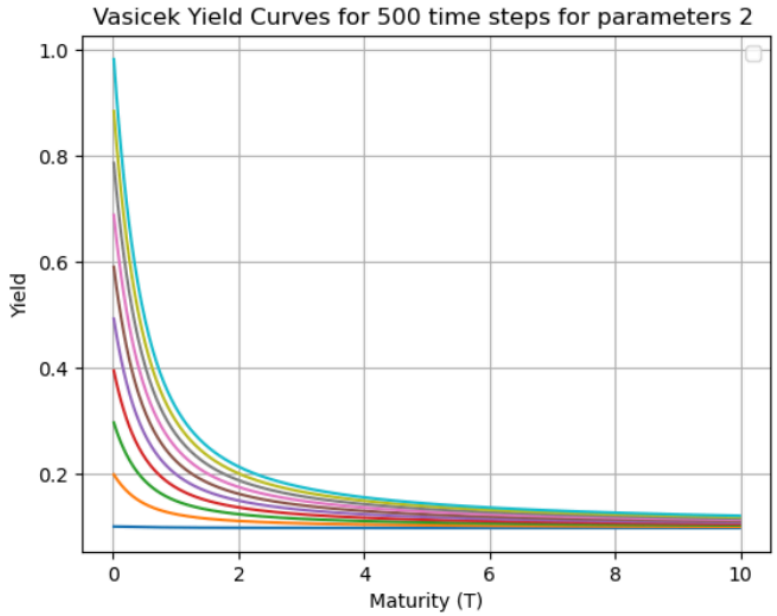
$$y(0,T) = -\frac{1}{T}\ln P(0,T)$$

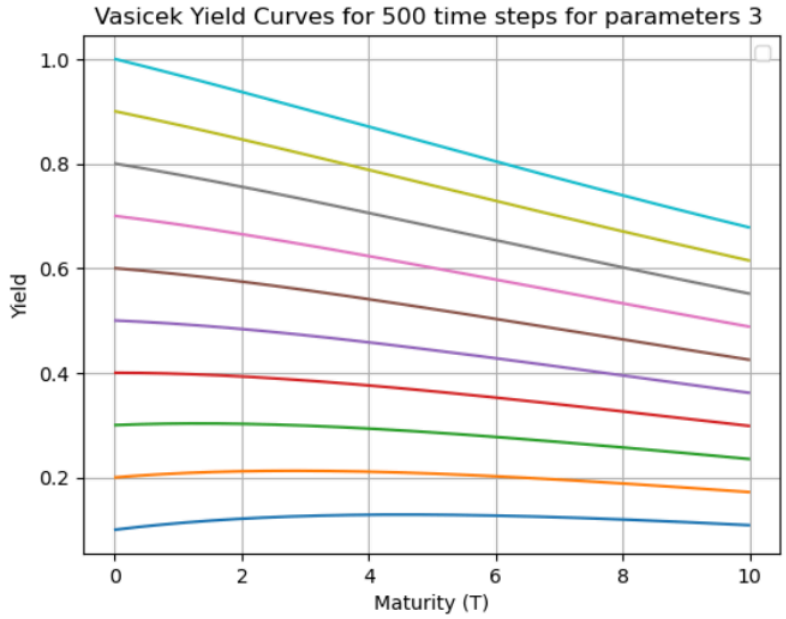



Vasicek Yield Curves for parameters
a=3.9, b=0.1, σ=0.3, r0=0.2
Yield
Maturity (T)
0.120
0.115
0.110
0.105
0.100
2
4
6
8
10


Vasicek Yield Curves for parameters
a=0.1, b=0.4, σ=0.11, r0=0.1
Yield
Maturity (T)
0.1275
0.1250
0.1225
0.1200
0.1175
0.1150
0.1125
0.1100
2
4
6
8
10


Vasicek Yield Curves for 500 time steps for parameters 1
Yield
Maturity (T)
1.0
0.8
0.6
0.4
0.2
0
2
4
6
8
10


Vasicek Yield Curves for 500 time steps for parameters 2
Yield
Maturity (T)
1.0
0.8
0.6
0.4
0.2
0
2
4
6
8
10




OBSERVATIONS

1. The yield of the bond price converges to a particular value as the maturity period is increased to
sufficiently high value, irrespective of the value of r(0) taken.
2. The term structure of parameters set for 10 time units show strikingly different behaviour. For the first parameter set, the yield increases and then converges. For the second one, the yield decreases and then converges, while for the last one, the yield curve has a “hump” in it.
3. The phenomenon of mean reversion is observed since high interest rate has negative trend while the low interest rate has positive trend to the reversion level. This is due to the fact that the Vasicek Model incorporates mean reversion.

Question 2 : CIR Model

### Zero-Coupon Bond Price in CIR:

Let $P(t,T)$ be the price at time $t$ of a zero-coupon bond maturing at time $T$. In the CIR model, the bond price has a closed-form solution:

$$P(t,T) = A(t,T) \cdot e^{-B(t,T)\cdot r(t)}$$

Where:

- $B(t,T) = \frac{2(e^{\gamma(T-t)}-1)}{(a+\gamma)(e^{\gamma(T-t)}-1)+2\gamma}$
- $A(t,T) = \left[\frac{2\gamma e^{(a+\gamma)(T-t)/2}}{(a+\gamma)(e^{\gamma(T-t)}-1)+2\gamma}\right]^{2ab/\sigma^2}$
- $\gamma = \sqrt{a^2+2\sigma^2}$

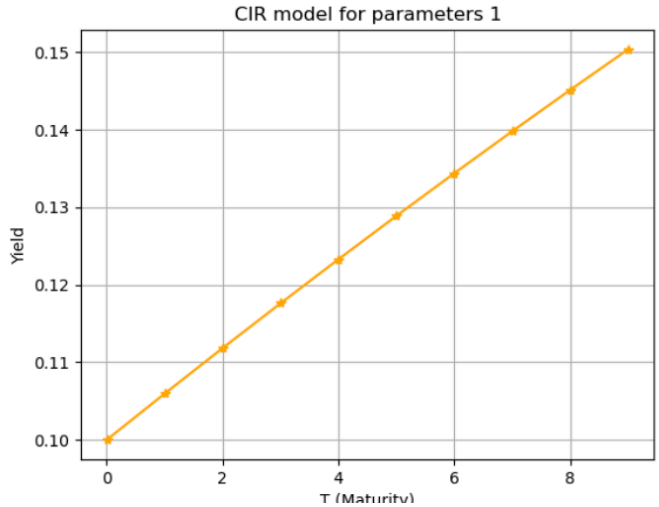
:

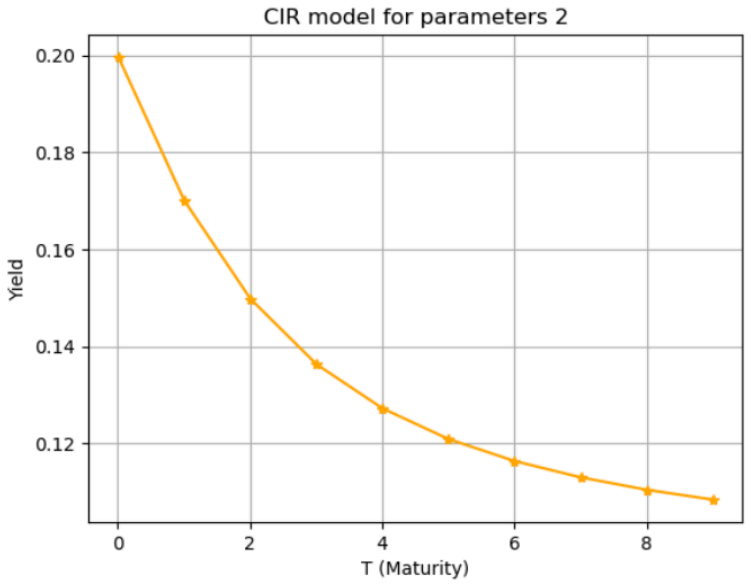
## Yield Curve (Term Structure):

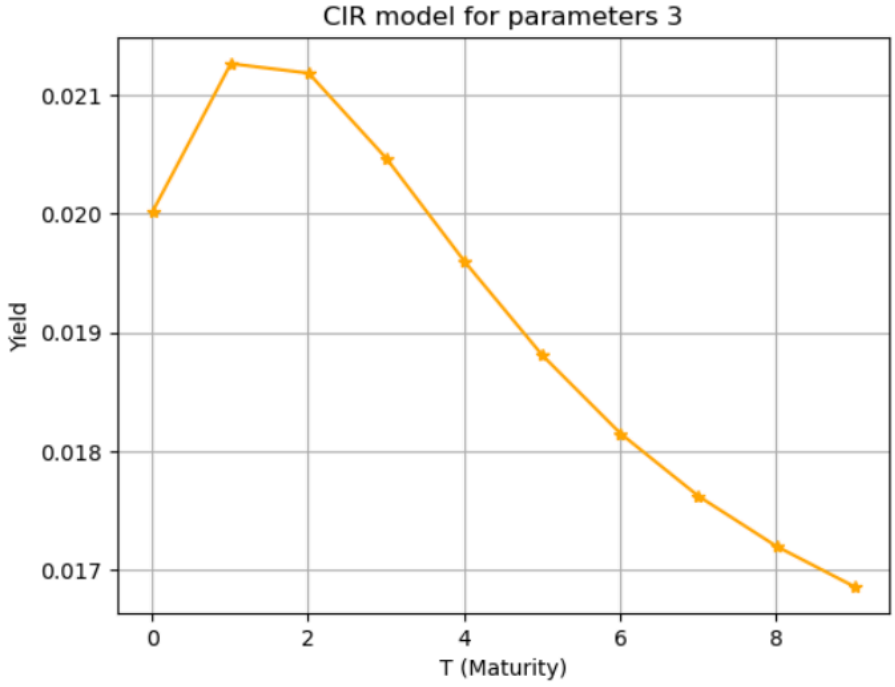
The **yield** for maturity $T$ is:

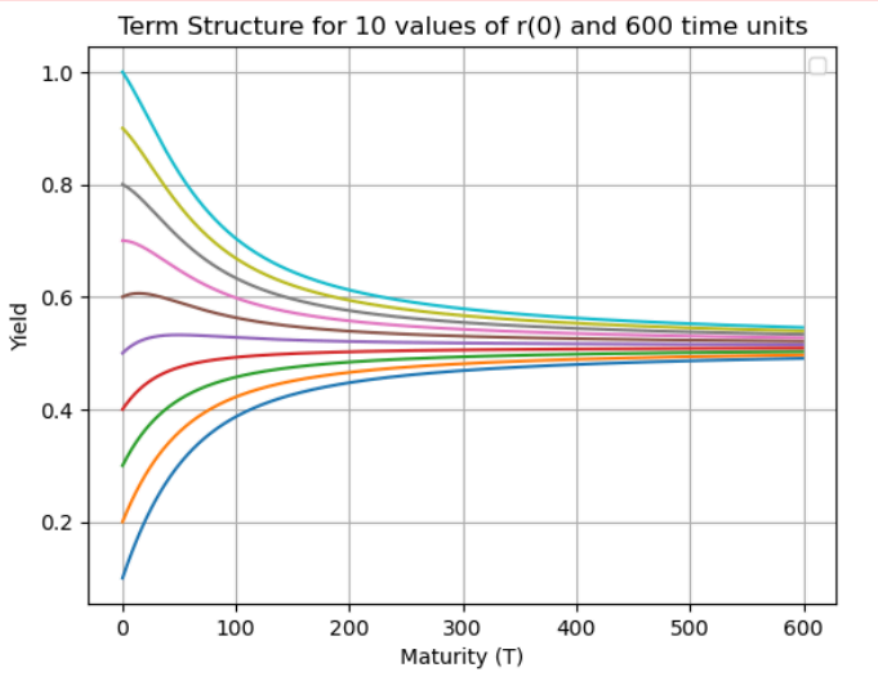
$$y(t,T) = -\frac{1}{T-t}\ln P(t,T)$$

This function $y(t,T)$ gives the **term structure** under CIR.




CIR model for parameters 2
0.20
0.18
0.16
0.14
0.12
Yield
0
2
4
6
8
T (Maturity)


CIR model for parameters 3
0.021
0.020
0.019
0.018
0.017
Yield
0
2
4
6
8
T (Maturity)




Observation:

1. The yield of the bond price converges to a particular value as the maturity period is increased to
sufficiently high value, irrespective of the value of r(0) taken.
2. The term structure of parameters set for 10 time units show strikingly different behaviour. For
the first parameter set, the yield increases and then converges. For the second one, the yield
decreases and then converges, while for the last one, the yield curve has a “hump” in it.
3. The phenomenon of mean reversion from the plots is observed. This is due to the fact that the
model assumes mean reversion towards a long-term normal interest rate level.