점화식

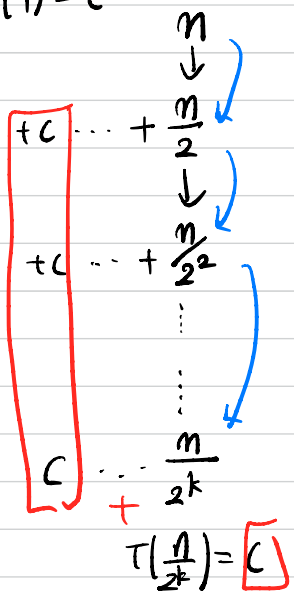
1. 이진 탐색 점화식

$T(n) = T(\frac{n}{2}) + c$

$T(1) = c$

가정 $n = 2^k$

$\Rightarrow k = \log_2 n$



k번 재귀함

$\Rightarrow$ c도 k개 생성

$= kc$개 생성

$T(n) = kc$

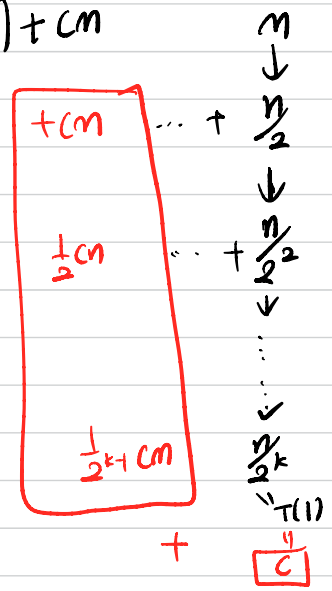
$= O(k)$

$= O(\log_2 n)$

2. Quick select best case

$T(n) = T(\frac{n}{2}) + cn$

$T(\frac{n}{2}) = T(\frac{n}{4}) + \frac{1}{2}cn$



$cn(1 + \frac{1}{2} + \cdots \frac{1}{2^{k-1}}) + c$

≤ 2

$= 2cn + c$

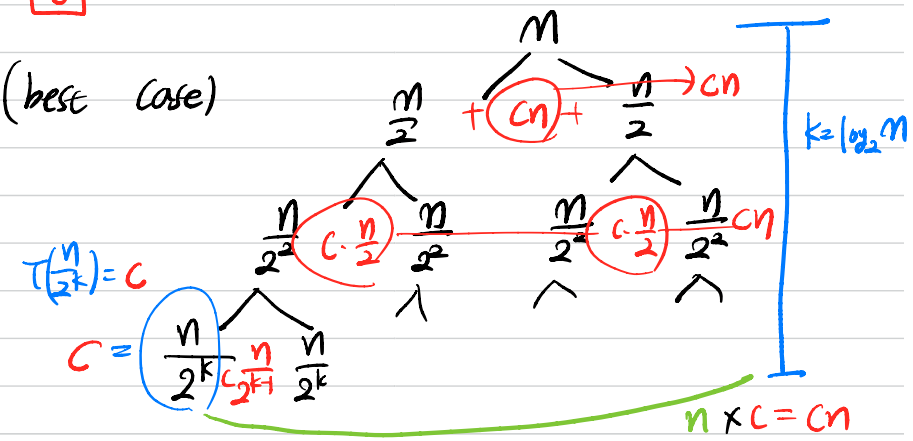
$= O(n)$

3. Quick Sort, merge sort (best case)

$T(n) = 2 \cdot T(\frac{n}{2}) + cn$

$T(n) = cn \times \log_2 n$

$= O(n \log n)$



Karatsuba 알고리즘

$T(n) = 3 \cdot T(\frac{n}{2}) + cn$

$T(1) =$

$$cn\left(1 + \frac{3}{2} + \frac{3^2}{2^2} + \cdots + \left(\frac{3}{2}\right)^{k-1}\right) \quad k = \log_2 n$$

$$+ c \cdot 3^k$$

$$= cn\left(\frac{\left(\frac{3}{2}\right)^k - 1}{\frac{3}{2} - 1}\right) + c \cdot 3^k$$

$$= O(3^k) = O(3^{\log_2 n}) = O(n^{\log_2 3}) = O(n^{1.5})$$