# EN811100 LINEAR CIRCUIT ANALYSIS

#### Chapter 10 Sinusoidal Steady-State Analysis Mar 10, 2563

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Fundamentals of Electric Circuits, 5<sup>th</sup> Edition, The McGraw-Hill Companies 2013
J. A. Svoboda – R. C. Dorf
Introduction to Electric Circuits, 9<sup>th</sup> edition, John Wiley & Sons, Inc. 2014

### Sinusoidal Steady-State Analysis- Chapter 10

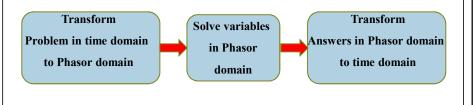
- 10.1 Basic Approach
- 10.2 Nodal Analysis
- 10.3 Mesh Analysis
- 10.4 Superposition Theorem
- 10.5 Source Transformation
- 10.6 Thevenin and Norton Equivalent Circuits

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## 10.1 Basic Approach

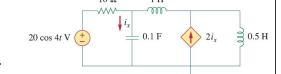
### Steps to Analyze AC Circuits:

- 1. <u>Transform</u> the circuit to the <u>phasor or frequency domain</u>.
- 2. <u>Solve</u> the problem using circuit techniques (nodal analysis, mesh analysis, superposition, etc.).
- 3. <u>Transform</u> the resulting phasor to the time domain.



10.2 Nodal Analysis

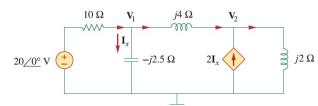
Example 10.1 Find  $i_x$  in the circuit of Fig. 10.1 using nodal analysis.



#### วิธีทำ 1. แปลงเป็น Phasor

$$\begin{array}{rcl}
20 \cos 4t & \Rightarrow & 20/0^{\circ}, & \omega = 4 \text{ rad/s} \\
1 \text{ H} & \Rightarrow & j\omega L = j4 \\
0.5 \text{ H} & \Rightarrow & j\omega L = j2
\end{array}$$

$$0.1 \text{ F} & \Rightarrow & \frac{1}{j\omega C} = -j2.5$$



Applying KCL at node 1,

$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} \implies (1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20$$

At node 2,

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$
 โดย  $\mathbf{I}_x = \frac{\mathbf{V}_1}{-j2.5}$ 

จะได้ 
$$\frac{2\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2} \implies 11\mathbf{V}_1 + 15\mathbf{V}_2 = 0$$

$$(1+j1.5)\mathbf{V}_1+j2.5\mathbf{V}_2=20$$
 วัดรูป Matrix 
$$\begin{bmatrix}1+j1.5 & j2.5\\11 & 15\end{bmatrix}\begin{bmatrix}\mathbf{V}_1\\\mathbf{V}_2\end{bmatrix}=\begin{bmatrix}20\\0\end{bmatrix}$$

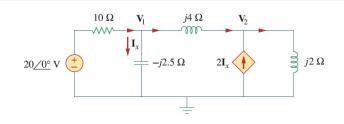
$$\Delta_{1} = \begin{vmatrix} 20 & j2.5 \end{vmatrix} = 300$$
  $\Rightarrow$   $V_{2} = \frac{\Delta_{1}}{2} = \frac{300}{2} = 18.97/18.439$ 

$$\Delta = \begin{vmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = 15 - j5$$

$$\Delta_{1} = \begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix} = 300 \implies \mathbf{V}_{1} = \frac{\Delta_{1}}{\Delta} = \frac{300}{15 - j5} = 18.97 / 18.43^{\circ} \, \mathbf{V}$$

$$\Delta_{2} = \begin{vmatrix} 1 + j1.5 & 20 \\ 11 & 0 \end{vmatrix} = -220$$

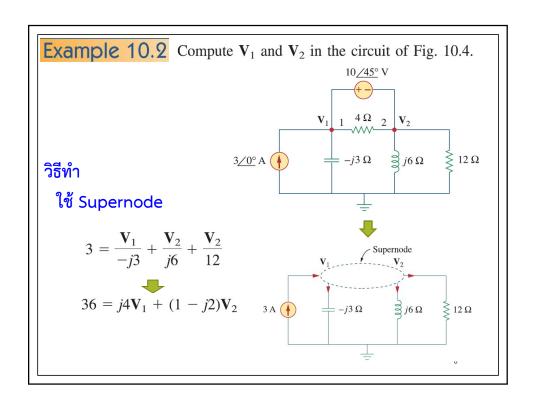
$$\mathbf{V}_{2} = \frac{\Delta_{2}}{\Delta} = \frac{-220}{15 - j5} = 13.91 / 198.3^{\circ} \, \mathbf{V}_{6}$$

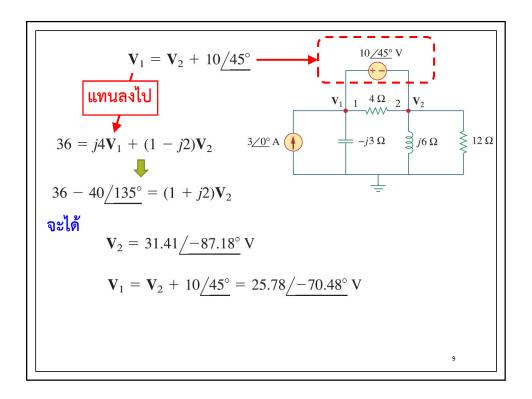


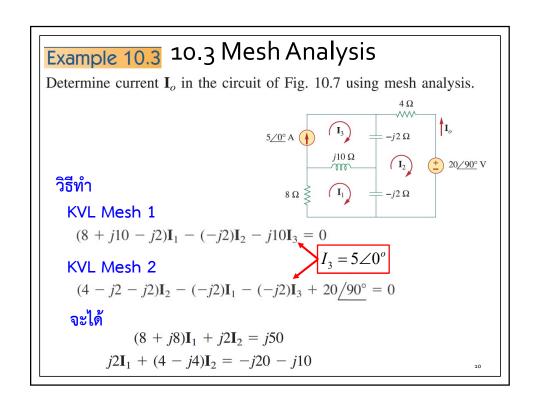
$$\mathbf{I}_x = \frac{\mathbf{V}_1}{-j2.5} = \frac{18.97/18.43^\circ}{2.5/-90^\circ} = 7.59/108.4^\circ \,\mathrm{A}$$

Transforming this to the time domain,

$$i_x = 7.59\cos(4t + 108.4^\circ) \text{ A}$$







$$(8+j8)\mathbf{I}_1+j2\mathbf{I}_2=j50$$

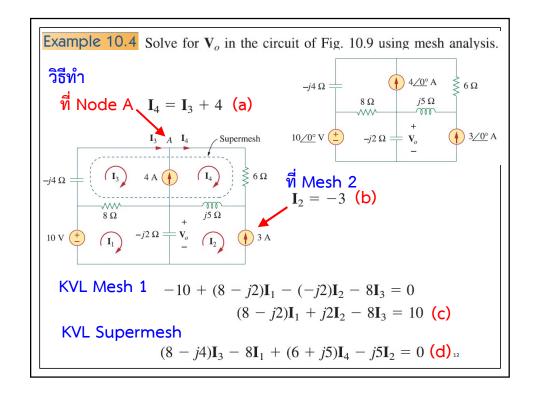
$$j2\mathbf{I}_1+(4-j4)\mathbf{I}_2=-j20-j10$$
จัดรูป Matrix
$$\begin{bmatrix} 8+j8 & j2 \\ j2 & 4-j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8+j8 & j2 \\ j2 & 4-j4 \end{vmatrix} = 32(1+j)(1-j)+4=68$$

$$\Delta_2 = \begin{vmatrix} 8+j8 & j50 \\ j2 & -j30 \end{vmatrix} = 340-j240=416.17/-35.22^\circ$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{416.17/-35.22^\circ}{68} = 6.12/-35.22^\circ \mathbf{A}$$

$$\mathbf{I}_o = -\mathbf{I}_2 = 6.12/144.78^\circ \mathbf{A}$$



$$\begin{bmatrix} 8 - j2 & -8 \\ -8 & 14 + j \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 10 + j6 \\ -24 - j35 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8 - j2 & -8 \\ -8 & 14 + j \end{vmatrix} = 112 + j8 - j28 + 2 - 64 = 50 - j20$$

$$\Delta_1 = \begin{vmatrix} 10 + j6 & -8 \\ -24 - j35 & 14 + j \end{vmatrix} = 140 + j10 + j84 - 6 - 192 - j280$$

$$= -58 - j186$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{-58 - j186}{50 - j20} = 3.618 / 274.5^{\circ} \text{ A}$$

$$\mathbf{V}_o = -j2(\mathbf{I}_1 - \mathbf{I}_2) = -j2(3.618 / 274.5^{\circ} + 3)$$

$$= -7.2134 - j6.568 = 9.756 / 222.32^{\circ} \text{ V}$$

### 

>> B = [10 -3 0 4]';

>> I = inv(A)\*B

```
MATLAB Code
  \Rightarrow A = [(8-j*2) j*2 -8 0;
                1
                      0
          0
                               0;
          -8
                -j*5 (8-j*4) (6+j*5);
              0
                       -1
                              1];
  >> B = [10 -3 0 4]';
  >> I = inv(A)*B
   0.2828 - 3.6069i
    -3.0000
    -1.8690 - 4.4276i
    2.1310 - 4.4276i
  >> Vo = -2*j*(I(1) - I(2))
    -7.2138 - 6.5655i
```

### 10.4 Superposition Theorem

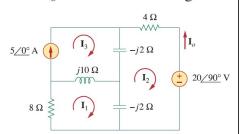
When a circuit has sources operating at different frequencies,

- The <u>separate</u> phasor circuit for each frequency must be solved independently, and
- The total response is the <u>sum of time-domain</u> responses of all the individual phasor circuits.

#### Example 10.5

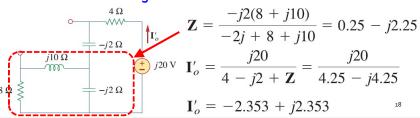
Use the superposition theorem to find  $I_o$  in the circuit in Fig. 10.7.

วิธีทำ วงจรนี้มี Source 2 ตัวคือ



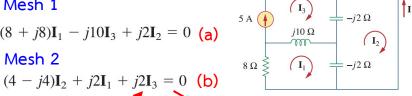
#### 1. ให้ Voltage source ON

และ Current source OFF (Open circuit) และหากระแส  $\mathbf{I}_0'$ อันเนื่องมาจาก Voltage source ทำงาน



2. ให้ Voltage source OFF (Short circuit) และ Current source ON และหากระแส  $\mathbf{I}_0''$  อันเนื่องมาจาก Current source ทำงาน KVL Mesh 1

 $(8+j8)\mathbf{I}_1 - j10\mathbf{I}_3 + j2\mathbf{I}_2 = 0$  (a) **KVL Mesh 2** 



Current Mesh 3 =  $I_3 = 5$   $(4 - j4)I_2 + j2I_1 + j10 = 0$ 

$$(8+j8)\mathbf{I}_1 - j10\mathbf{I}_3 + j2\mathbf{I}_2 = 0$$
 (a)

$$(8 + j8)[(2 + j2)\mathbf{I}_2 - 5] - j50 + j2\mathbf{I}_2 = 0$$

จะได้  $\mathbf{I}_2 = \frac{90 - j40}{34} = 2.647 - j1.176$ 

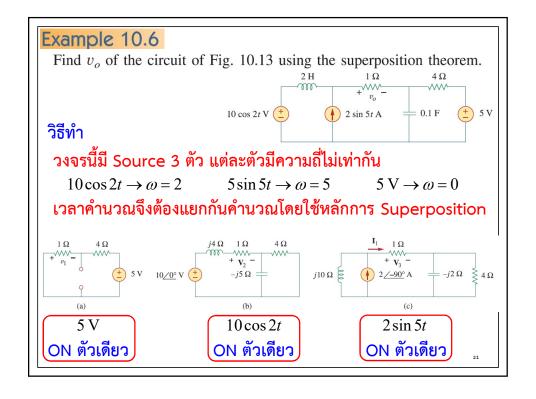
จะได้  $\mathbf{I}_{o}'' = -\mathbf{I}_{2} = -2.647 + j1.176$ 

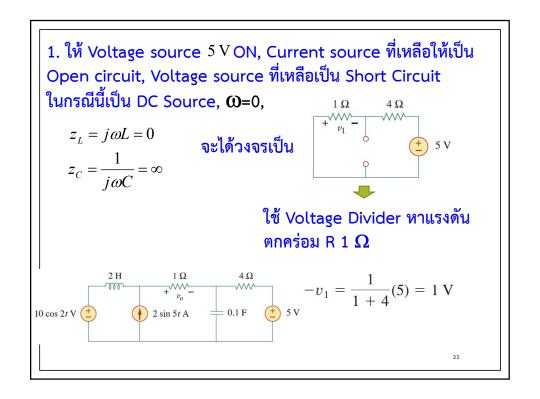
กระแส  $\mathbf{I}_0$  โดยรวมคือกระแส  $\mathbf{I}_0'$  อันเนื่องมาจาก Voltage source บวกกับกระแส  $\mathbf{I}_0''$  อันเนื่องมาจาก Current source

จะได้

$$\mathbf{I}_o = \mathbf{I}'_o + \mathbf{I}''_o = -5 + j3.529 = 6.12/144.78^{\circ} \,\mathrm{A}$$

คำตอบนี้ตรงกับคำตอบในตัวอย่างที่ 10.3





2. ให้ Voltage source 
$$10\cos(2t)$$
 ON, Current source ที่เหลือ ให้เป็น Open circuit, Voltage source ที่เหลือเป็น Short Circuit ในกรณีนี้  $\omega$ =2,

$$z_{L} = j\omega L = j2 \times 2 = j4\Omega$$

$$z_{C} = \frac{1}{j\omega C} = \frac{1}{j2 \times 0.1} = -j5\Omega$$

$$10/0^{\circ} \text{ V} \stackrel{+}{\longrightarrow} \frac{-j5\Omega}{}$$

$$\mathbf{Z} = -j5 \parallel 4 = \frac{-j5 \times 4}{4 - j5} = 2.439 - j1.951$$

ใช้ Voltage Divider หาแรงดันตกคร่อม R 1  $\Omega$ 

$$\mathbf{V}_2 = \frac{1}{1 + j4 + \mathbf{Z}} (10 / 0^{\circ}) = \frac{10}{3.439 + j2.049} = 2.498 / -30.79^{\circ}$$

$$v_2 = 2.498\cos(2t - 30.79^\circ)$$

3. ให้ Current source  $2\sin(5t)$  ON, Current source ที่เหลือ ให้เป็น Open circuit, Voltage source ที่เหลือเป็น Short Circuit Eutivisian  $\omega = 5$ ,  $z_L = j\omega L = j5 \times 2$   $+ v_3 - j10 \Omega$   $= j10 \Omega$   $= j10 \Omega$   $= j2 \Omega$   $= j2 \Omega$ ในกรณีนี้  $\omega$ =5,

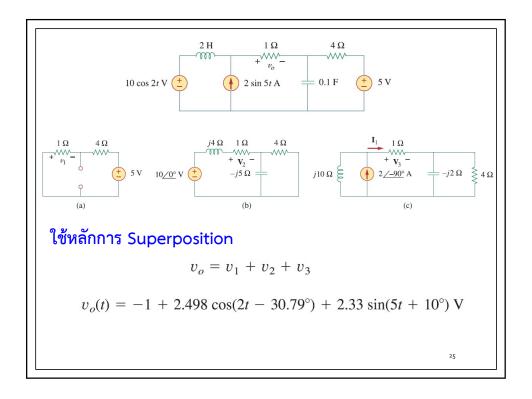
$$\mathbf{Z}_{1} = -j2\Omega$$

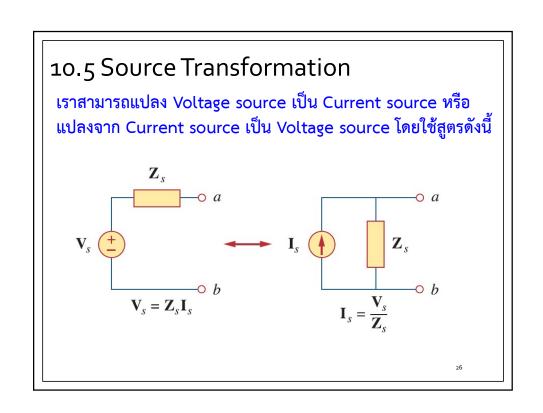
$$\mathbf{Z}_{1} = -j2 \parallel 4 = \frac{-j2 \times 4}{4 - j2} = 0.8 - j1.6 \Omega$$

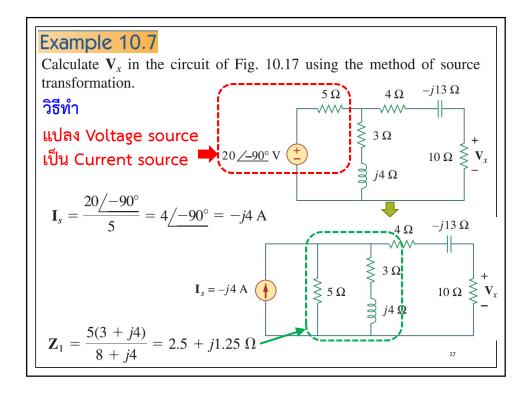
ใช้ Current Divider  $\mathbf{I}_1 = \frac{j10}{j10 + 1 + \mathbf{Z}_1} (2 / -90^\circ) \, \mathbf{A}$ 

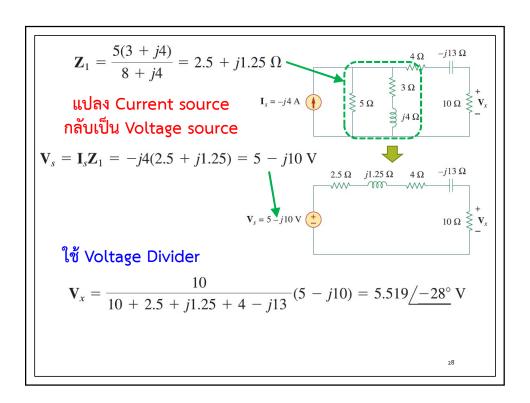
$$\mathbf{V}_3 = \mathbf{I}_1 \times 1 = \frac{j10}{1.8 + j8.4} (-j2) = 2.328 / -80^{\circ} \,\mathrm{V}$$

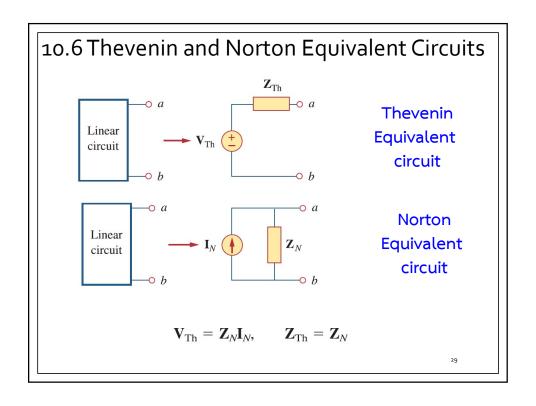
$$v_3 = 2.33 \cos(5t - 80^\circ) = 2.33 \sin(5t + 10^\circ) \text{ V}$$

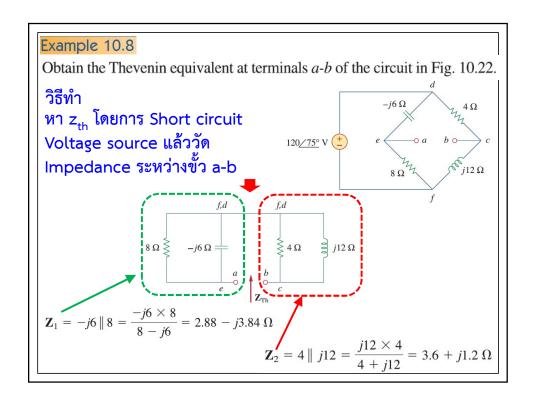


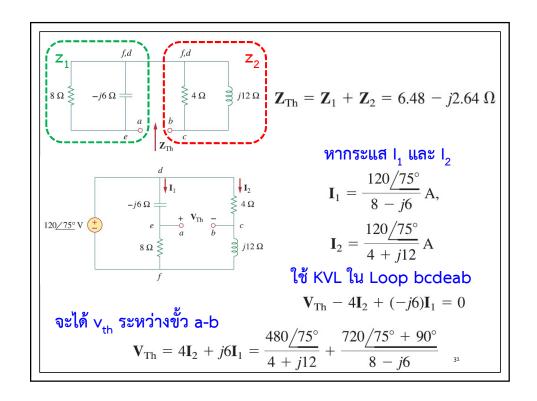


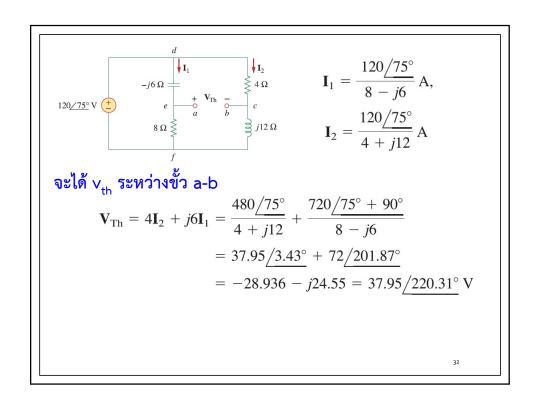






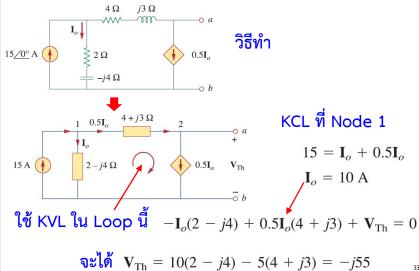




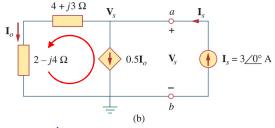




Find the Thevenin equivalent of the circuit in Fig. 10.25 as seen from terminals a-b.



หาค่า  $z_{th}$  โดยเอา Independent source ออกไป แล้วนำ Current Source  $\mathbf{I}=3\angle0^o$  A มาต่อที่ขั้ว a-b แล้ววัดแรงดันที่ขั้ว a-b

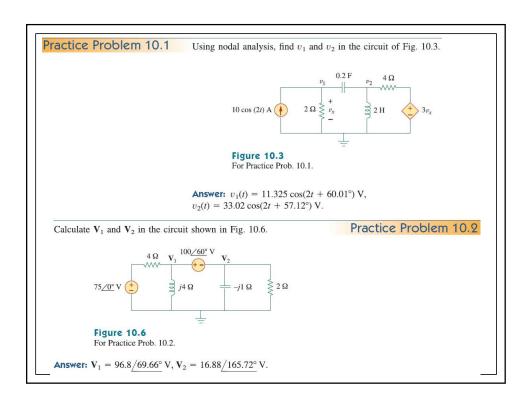


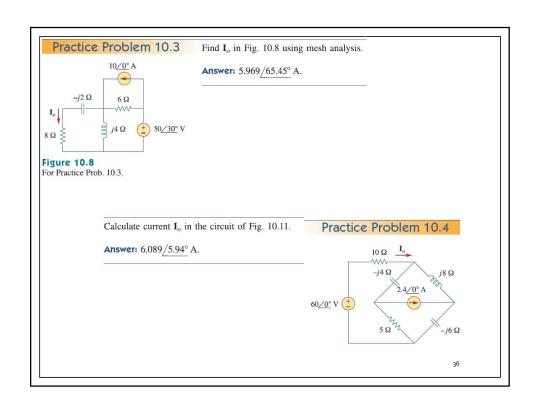
KCL  $\vec{N}$  Node a  $3 = \mathbf{I}_o + 0.5\mathbf{I}_o$   $\Rightarrow$   $\mathbf{I}_o = 2 \text{ A}$ 

KCL 
$$\vec{N}$$
 Loop 1  $V_s = I_o(4 + j3 + 2 - j4) = 2(6 - j)$ 

$$Z_{Th} = \frac{V_s}{I_s} = \frac{2(6 - j)}{3} = 4 - j0.6667 \Omega$$

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Practice Problem 10.5

Find current 
$$I_o$$
 in the circuit of Fig. 10.8 using the superposition theorem.

Answer:  $5.97/65.45^\circ$  A.

Practice Problem 10.6

Calculate  $v_o$  in the circuit of Fig. 10.15 using the superposition theorem.

8  $\Omega$ 

\*\*Properties 10.15\*\*

Figure 10.15\*\*

For Practice Problem 10.6.

Answer:  $11.577 \sin(5t - 81.12^\circ) + 3.154 \cos(10t - 86.24^\circ)$  V.

