EN811100 LINEAR CIRCUIT ANALYSIS

Chapter 6

Capacitors and Inductors

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C. K. Alexander – M. N. O. Sadiku
Fundamentals of Electric Circuits, 5th Edition, The McGraw-Hill Companies 2013
J. A. Svoboda – R. C. Dorf
Introduction to Electric Circuits, 9th edition, John Wiley & Sons, Inc. 2014

Chapter 6 Capacitors and Inductors

6.1 Capacitors

6.2 Series and Parallel Capacitors

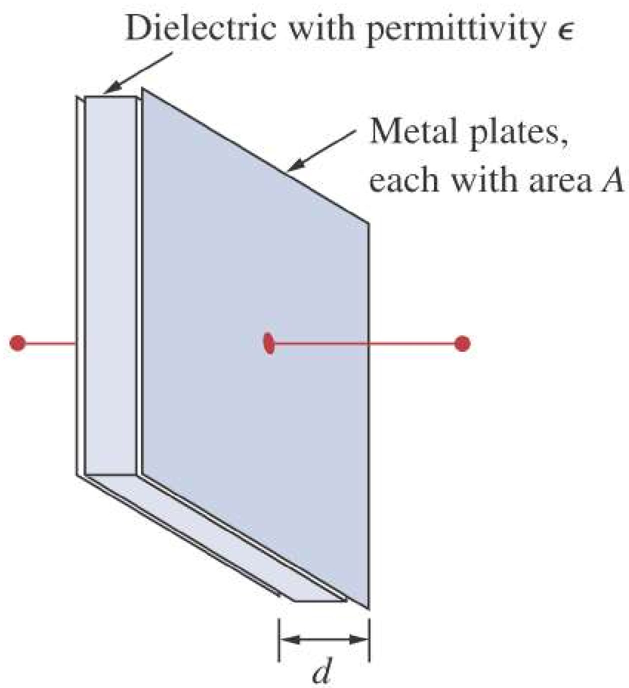
6.3 Inductors

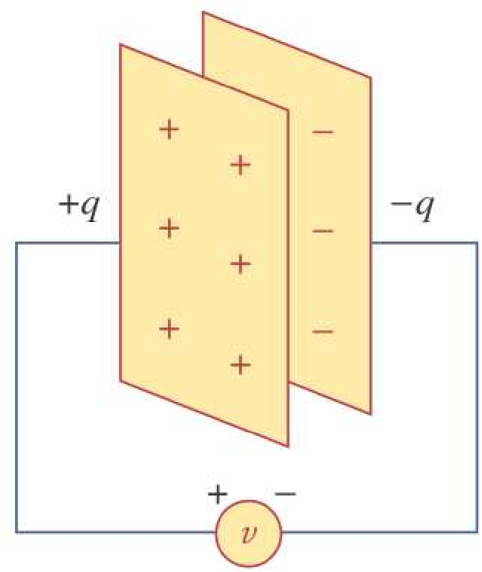
6.4 Series and Parallel Inductors

6.5 Initial Conditions of Switched Circuits

6.1 Capacitors

- A capacitor is a passive element designed to **store energy** in its **electric field.**

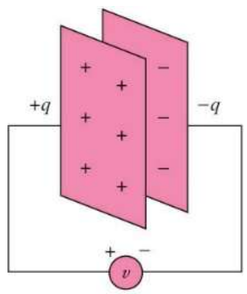




- A **capacitor** consists of two conducting plates separated by an insulator (or dielectric).

6.1 Capacitors

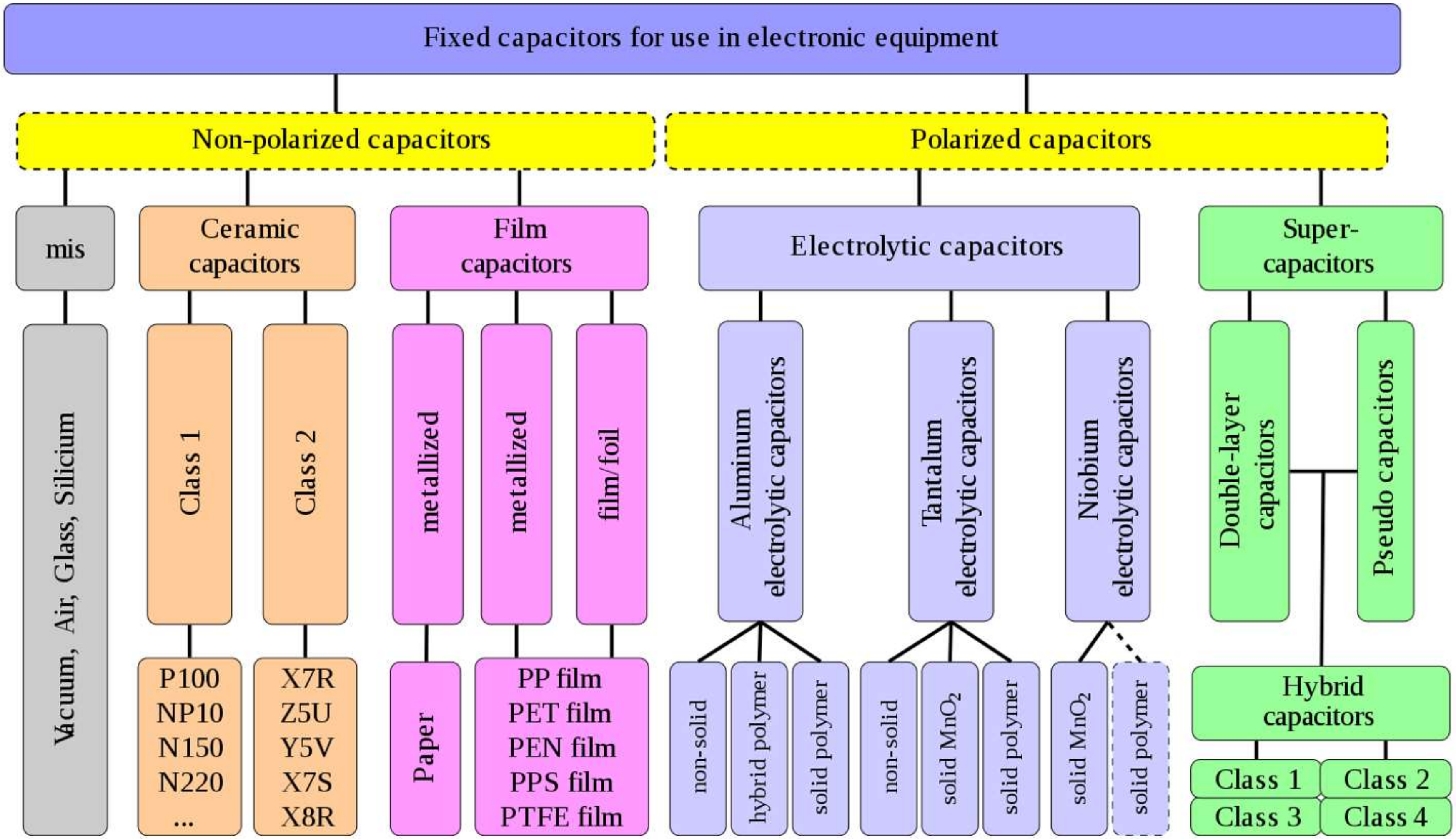
- **Capacitance** *C* is the ratio of the charge *q* on one plate of a capacitor to the voltage difference *v* between the two plates, measured in farads (F).



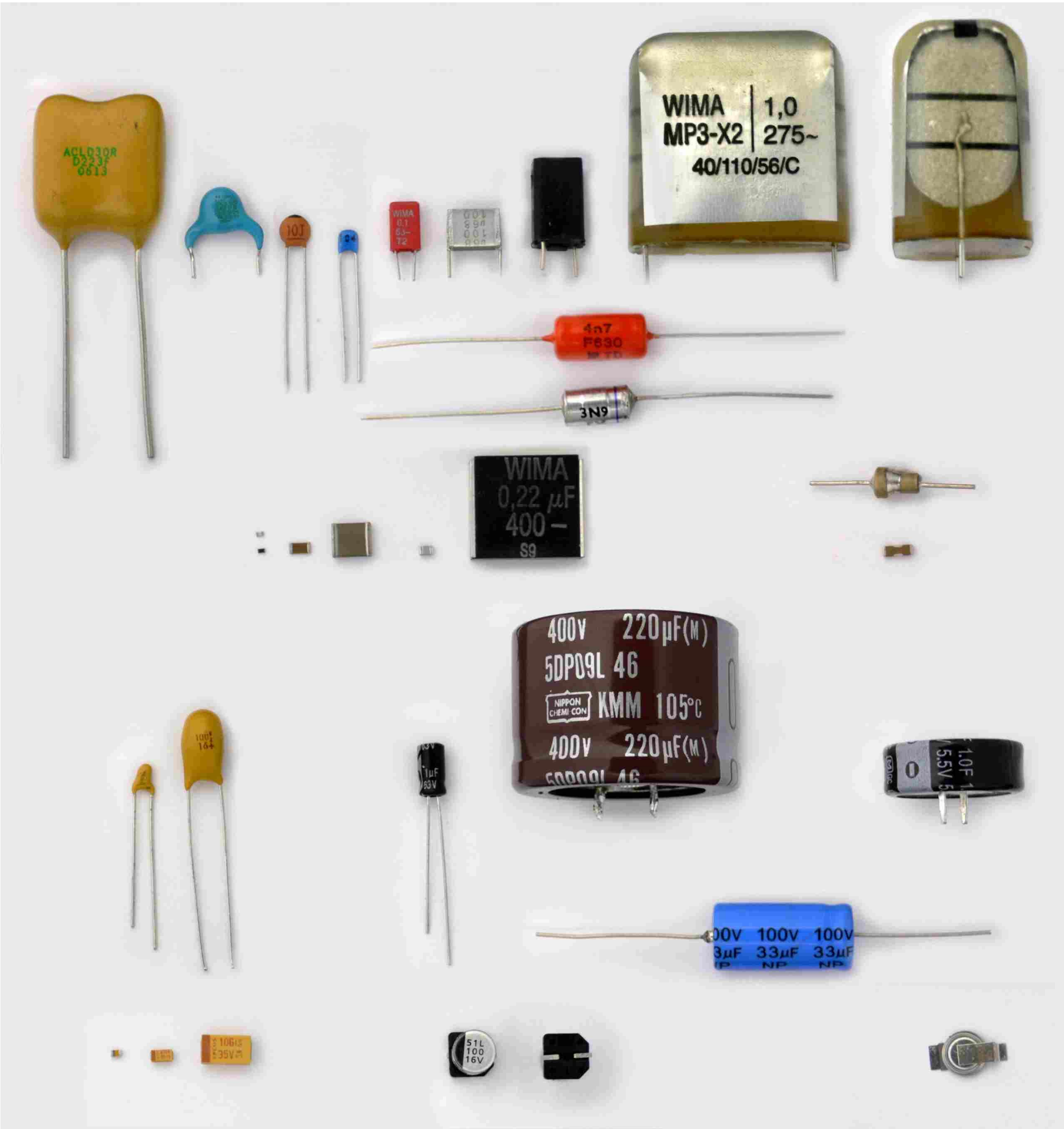
$$q = C\,v \quad \text{and} \quad C = \frac{\varepsilon\, A}{d}$$

- Where ε is the permittivity of the dielectric material between the plates, A is the surface area of each plate, d is the distance between the plates.
- Unit: F, pF (10^{-12}), nF (10^{-9}), and μF (10^{-6})

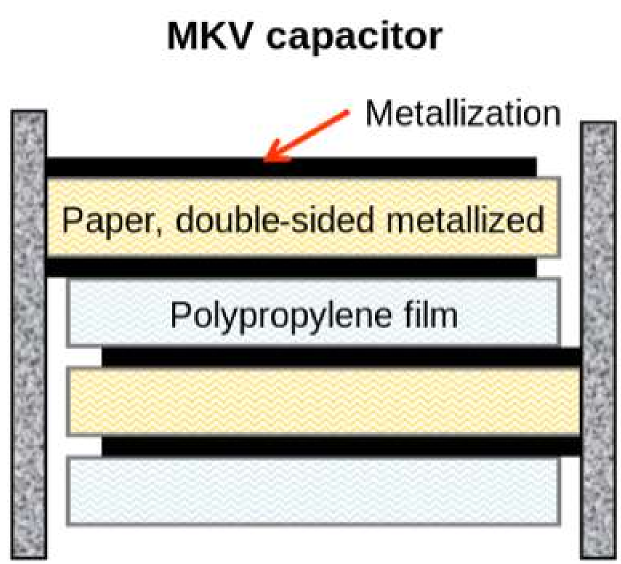
ชนิดของ Capacitor

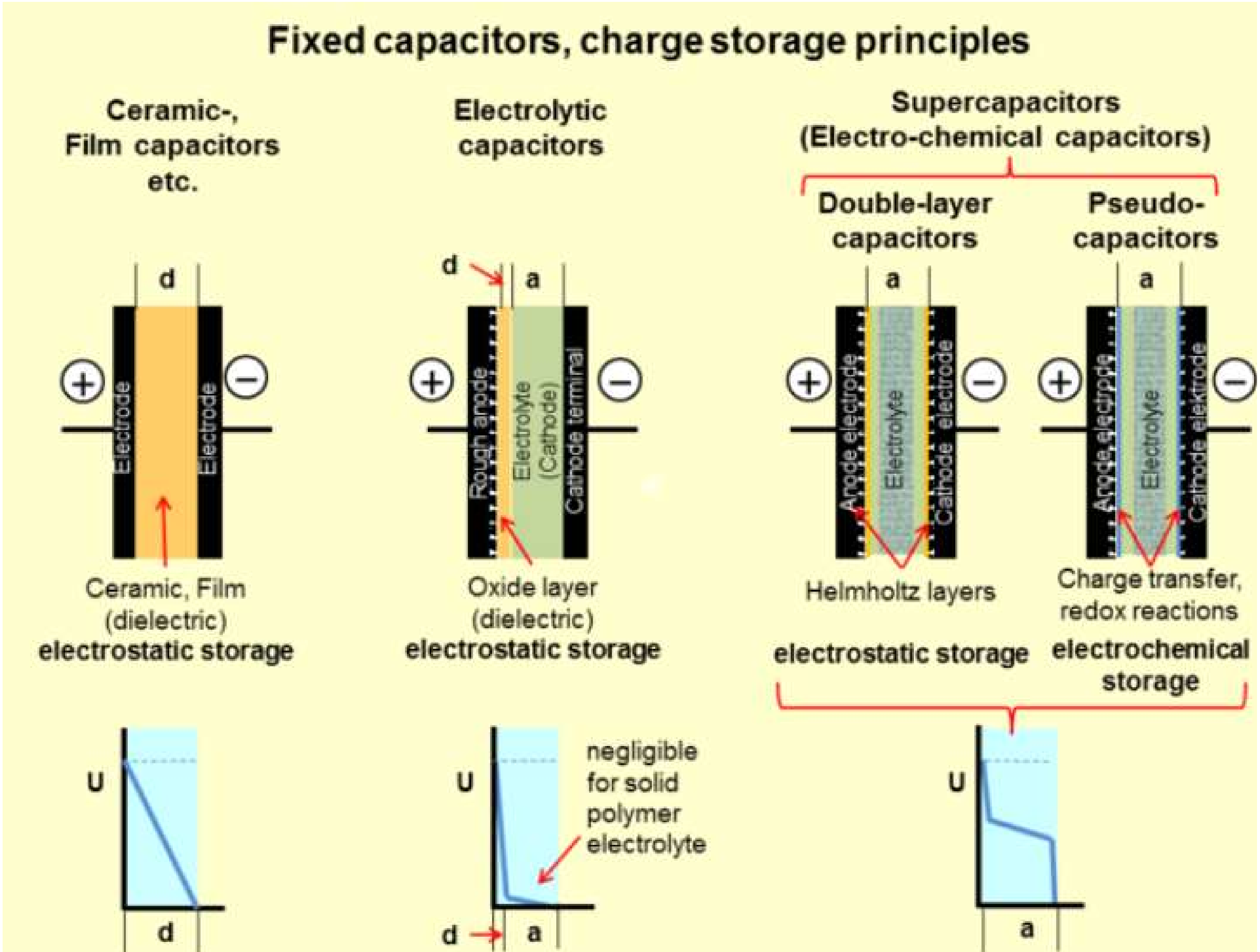


ลักษณะของ Capacitor ในวงจรอิเล็กทรอนิกส์



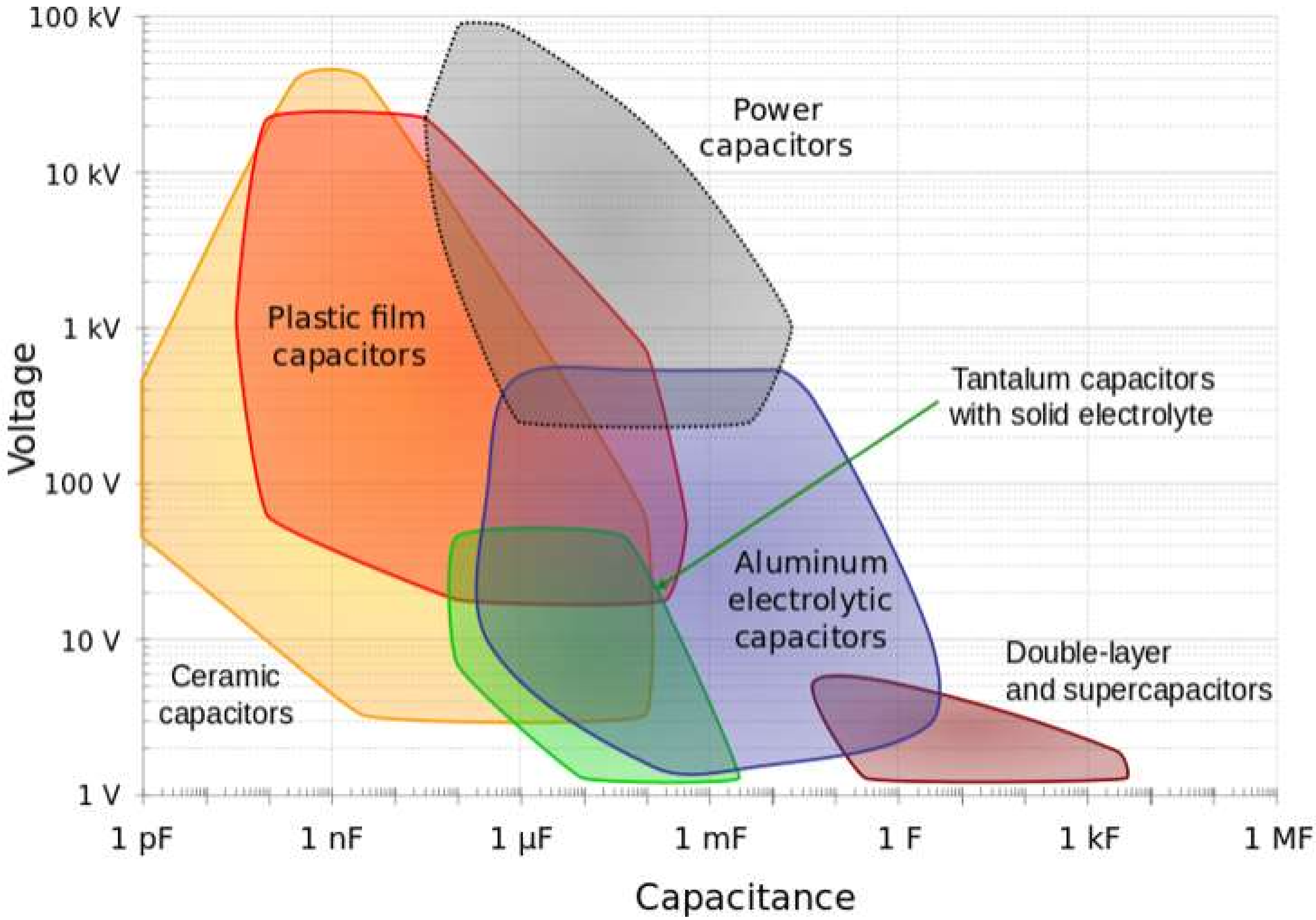
Power capacitor
ในระบบไฟฟ้ากำลัง



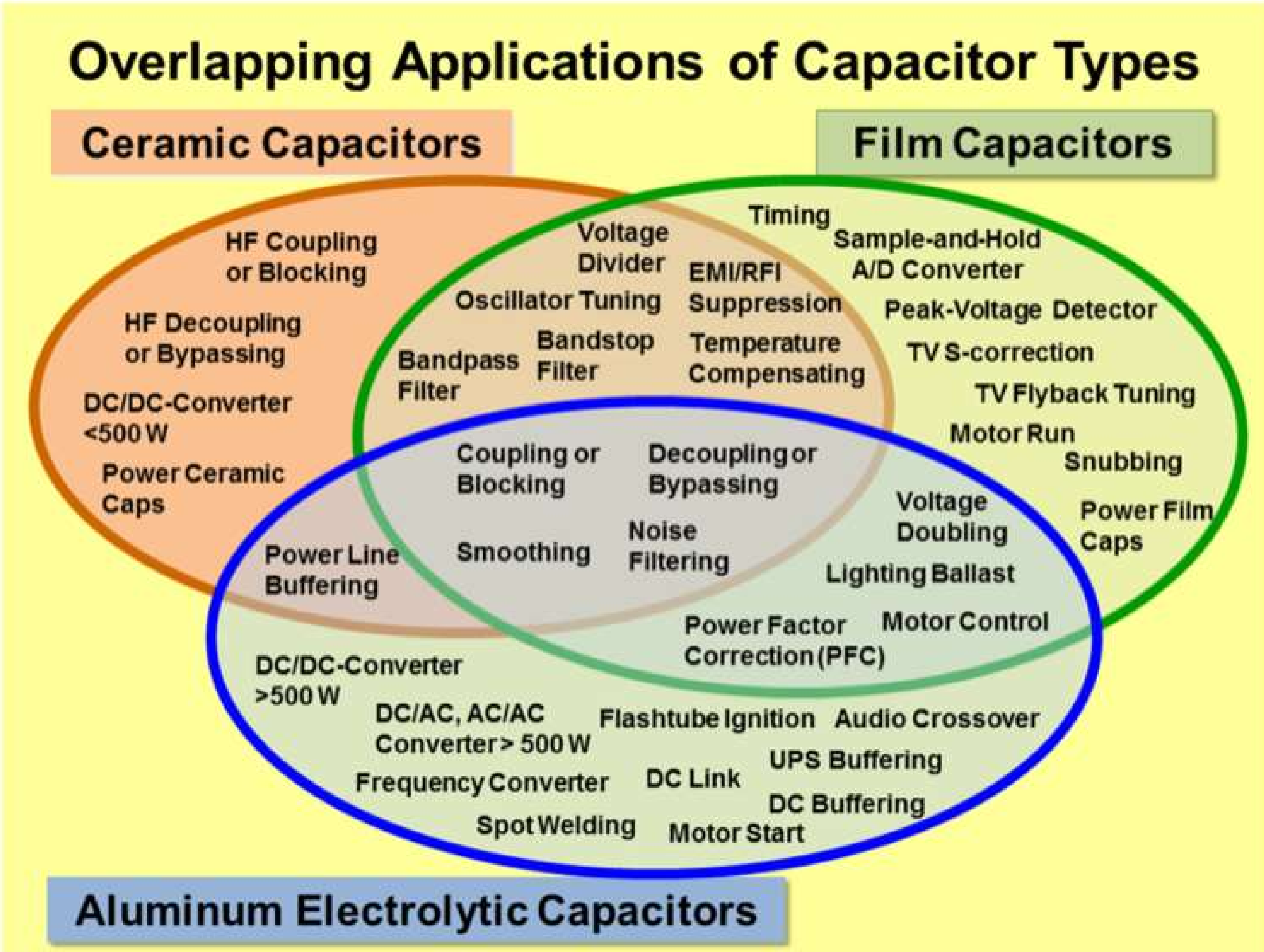


Fixed capacitors, charge storage principles
Ceramic-, Film capacitors etc.
Electrolytic capacitors
Supercapacitors (Electro-chemical capacitors)
Double-layer capacitors
Pseudo-capacitors
d
a
Electrode
Electrode
Rough anode
Electrolyte (Cathode)
Cathode terminal
Anode electrode
Electrolyte
Cathode electrode
Cathode elektrode
Ceramic, Film (dielectric)
electrostatic storage
Oxide layer (dielectric)
electrostatic storage
Helmholtz layers
electrostatic storage
Charge transfer, redox reactions
electrochemical storage
U
negligible for solid polymer electrolyte

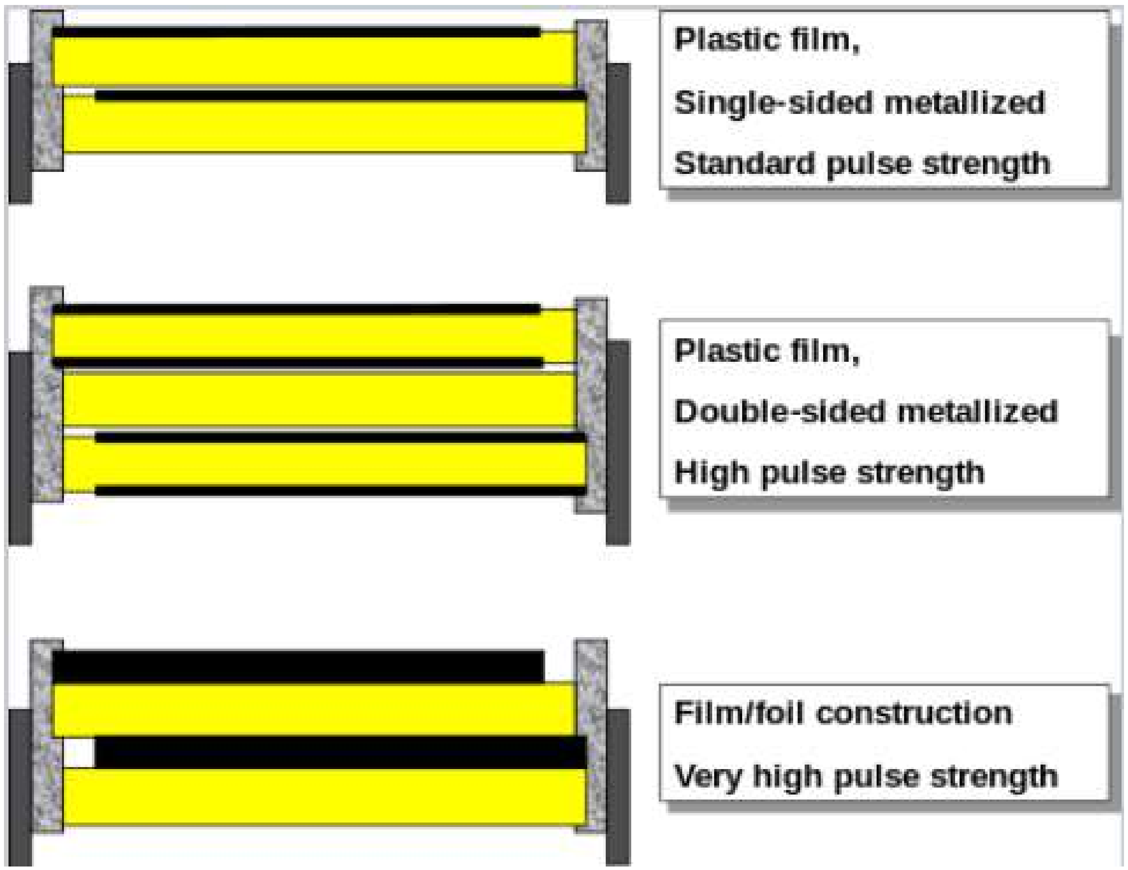
Capacitor แต่ละชนิดในช่วงแรงดันและความจุต่างๆ

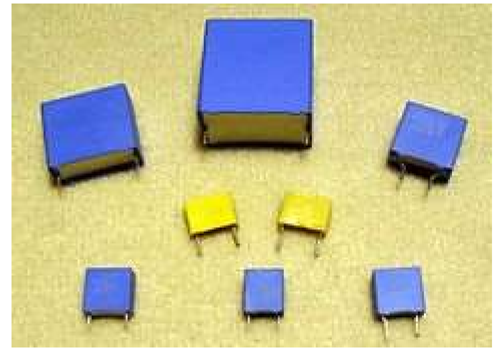


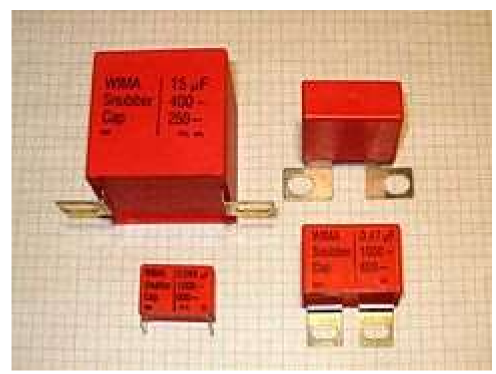
ลักษณะการใช้งาน Capacitor ชนิดต่างๆ



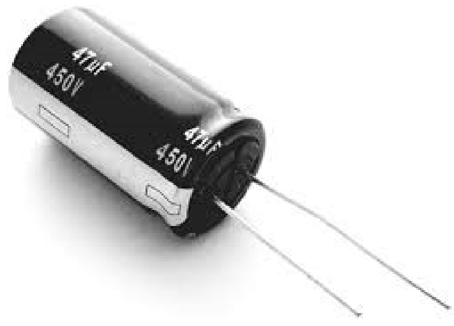
โครงสร้างภายในของ Film capacitor

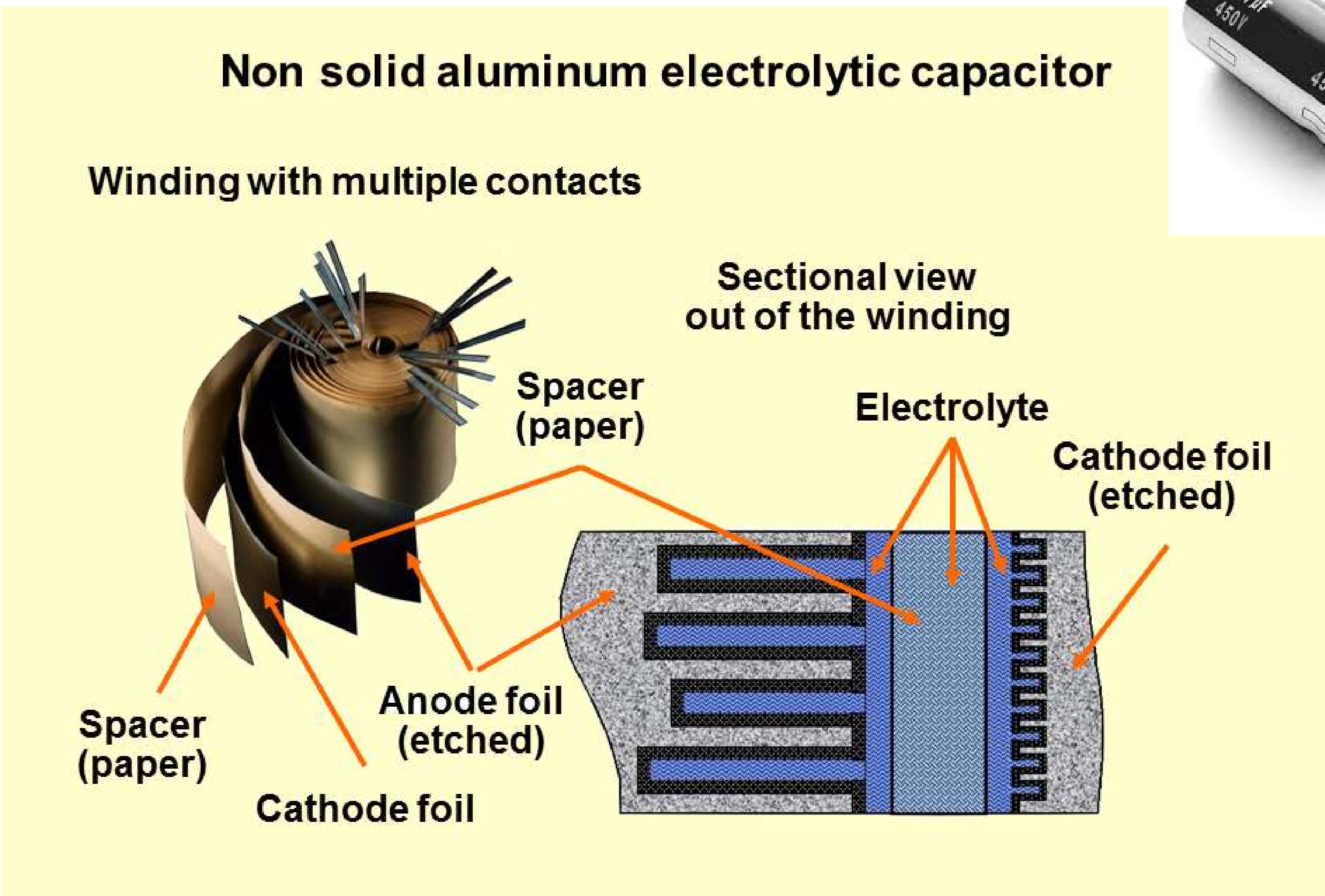






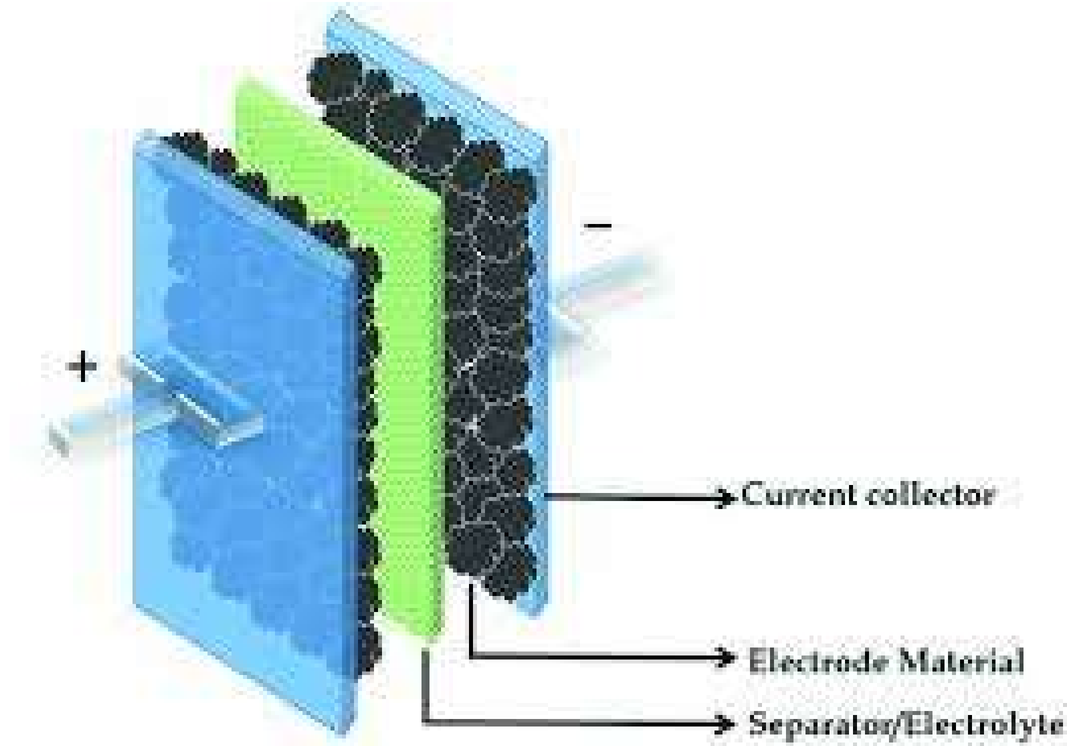
โครงสร้างภายในของ Electrolytic capacitor



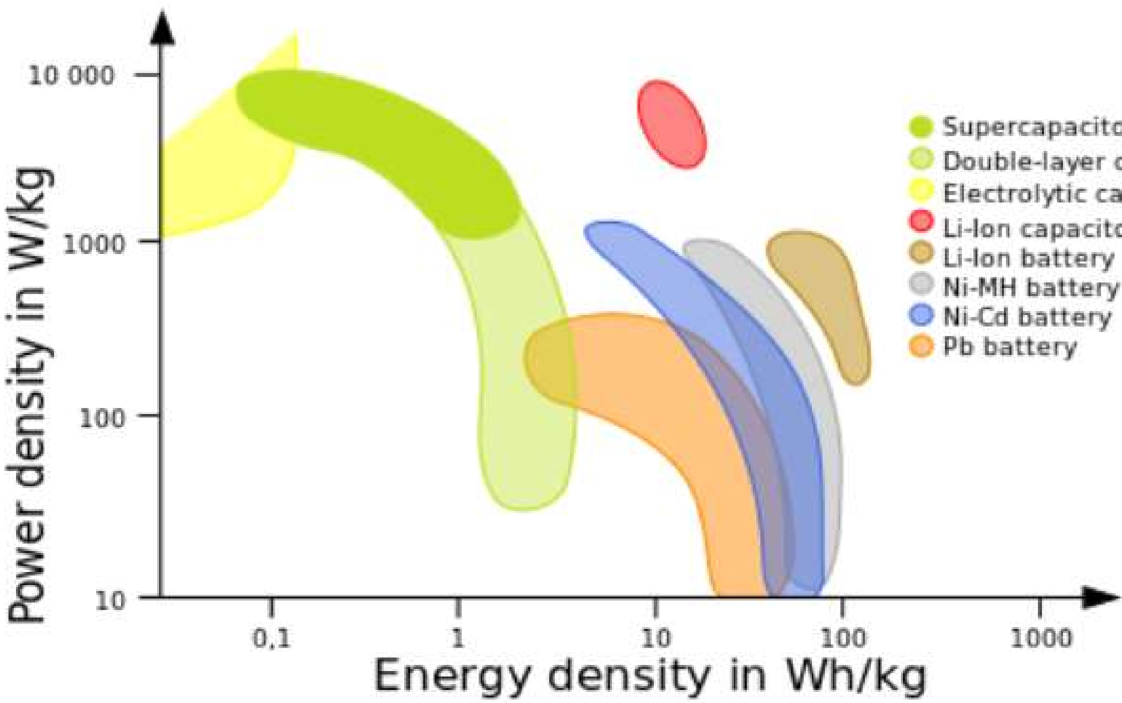


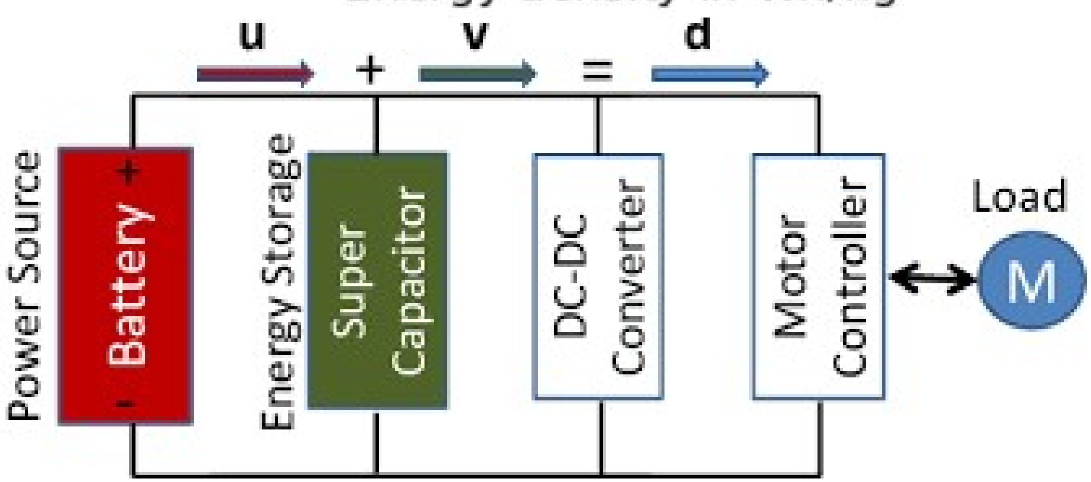
Super capacitor

Graphene supercapacitor
2.7V 100,000 F

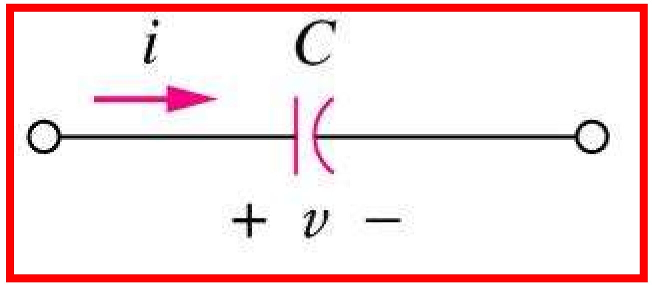


การใช้งาน Supercapacitor ร่วมกับ Battery





6.1 Capacitors



- If *i* is flowing into the +ve terminal of C
  - Charging => *i* is +ve
  - Discharging => *i* is –ve
- The current-voltage relationship of capacitor according to above convention is

$$\boxed{i = C\frac{dv}{dt}} \quad \text{and} \quad \boxed{v = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)}$$

- A capacitor is
  - an **open circuit** to dc (dv/dt = 0).
  - its voltage **cannot change abruptly**.

Capacitor จะต่อต้านการเปลี่ยนแปลงแรงดันไฟฟ้า (ในทางไฟฟ้ากำลัง เราจึงนิยมใช้ Capacitor ในการรักษาระดับแรงดันไฟฟ้า)

เปรียบเทียบ Capacitor ได้กับ Pressure tank ในระบบส่งน้ำประปา

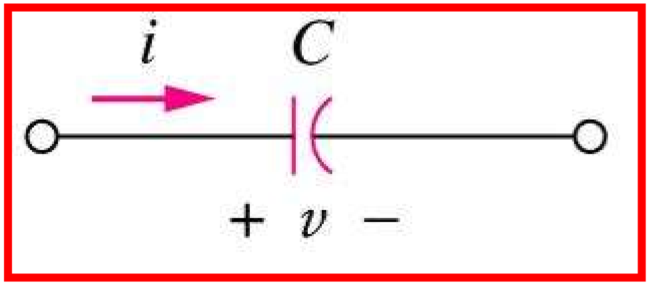
Pressure tank

Capacitor ใช้รักษาระดับแรงดันไฟฟ้าในสายส่ง (ไม่ให้แรงดันไฟฟ้าตก)

Pressure tank ใช้รักษาระดับแรงดันน้ำ ในท่อประปา

6.1 Capacitors

- The energy, ***w***, stored in the capacitor is



$$w=\int_{-\infty}^{t} p(\tau)\,d\tau = C\int_{-\infty}^{t} v\frac{dv}{d\tau}d\tau = C\int_{v(-\infty)}^{v(t)} v\,dv = \frac{1}{2}Cv^2\Big|_{v(-\infty)}^{v(t)}$$

$$p = vi = Cv\frac{dv}{dt}$$

$$v(-\infty)=0$$

$$\boxed{w=\frac{1}{2}C\,v^2}$$

พลังงานที่เก็บสะสมใน Capacitor

$$w = \frac{1}{2}Cv^2 \quad \boxed{\text{อีกสูตร}} \quad w = \frac{q^2}{2C}$$

$$q = Cv$$

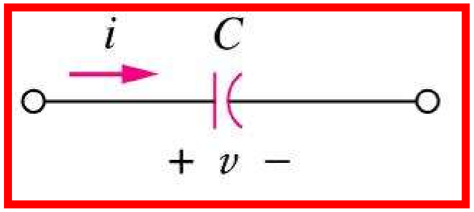
$$v = \frac{q}{C}$$

Supercapacitor

16V 500F

$$w = \frac{1}{2}500 \times 16^2 = 64{,}000\,J$$

6.1 Capacitors



Example 1

The current through a 2-μF capacitor is

$$i(t) = 6e^{-3000t}\ \text{mA}$$

Determine the voltage across this capacitor. Assume the initial capacitor voltage is zero.

$$\boxed{v = \frac{1}{C}\int_{t_0}^{t} i\, d\, t + v(t_0)}$$

$$v = \frac{1}{C}\int_0^t i\, dt + v(0) \text{ and } v(0) = 0,$$

$$v = \frac{1}{2 \times 10^{-6}}\int_0^t 6e^{-3000t}\, dt \cdot 10^{-3}$$

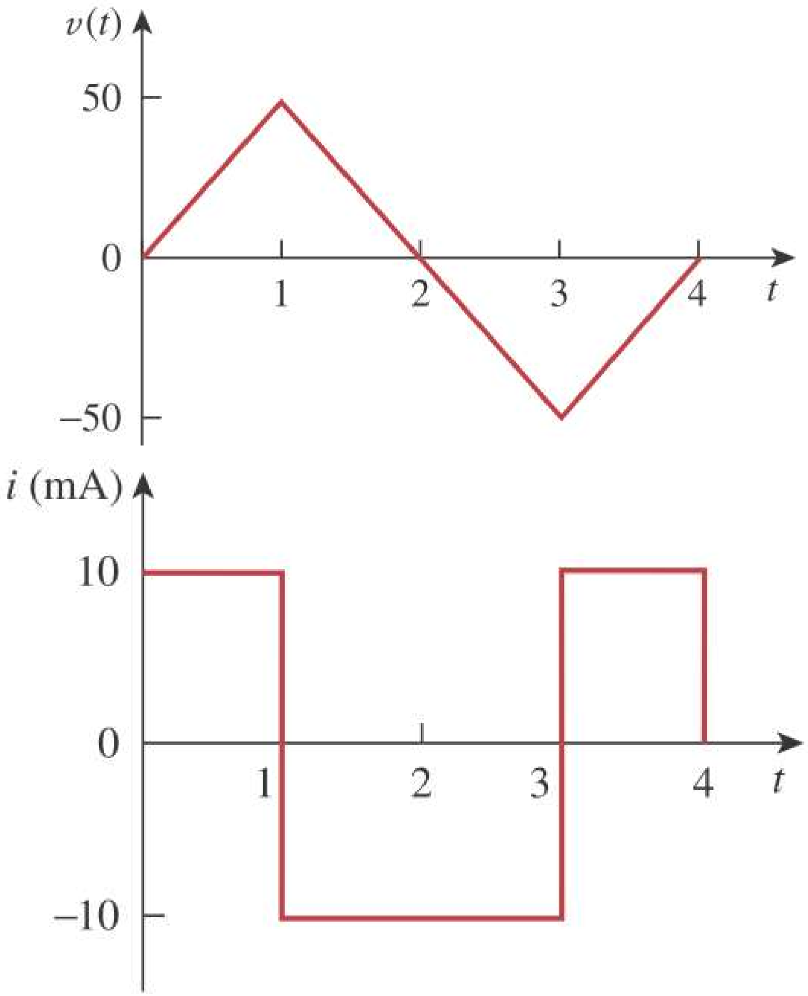
$$= \frac{3 \times 10^3}{-3000} e^{-3000t}\Big|_0^t = (1 - e^{-3000t})\ \text{V}$$

6.1 Capacitors

$$\boxed{i = C\frac{dv}{dt}}$$

Example 2

Determine the current through a 200-μF capacitor whose voltage is shown below.



$$v(t) = \begin{cases} 50t \text{ V} & 0 < t < 1 \\ 100 - 50t \text{ V} & 1 < t < 3 \\ -200 + 50t \text{ V} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

dv/dt

C

$$i(t) = 200 \times 10^{-6} \times \begin{cases} 50 & 0 < t < 1 \\ -50 & 1 < t < 3 \\ 50 & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} 10 \text{ mA} & 0 < t < 1 \\ -10 \text{ mA} & 1 < t < 3 \\ 10 \text{ mA} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

Example 6.2

$$i = C\frac{dv}{dt}$$

The voltage across a 5-μF capacitor is

$$v(t) = 10 \cos 6000t \text{ V}$$

Calculate the current through it.

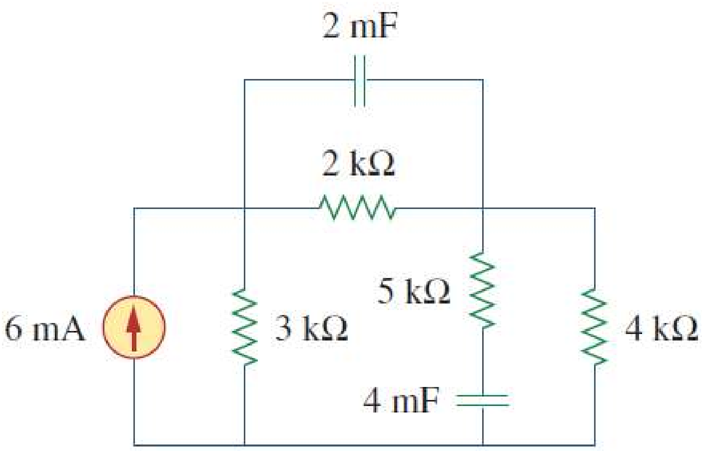
Solution:

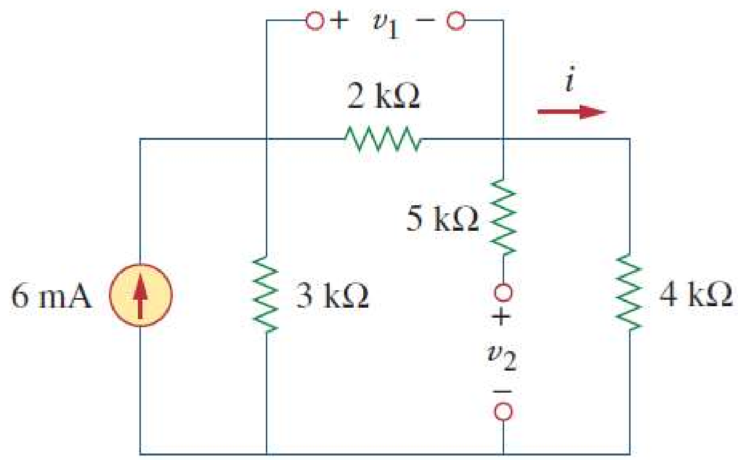
By definition, the current is

$$i(t) = C\frac{dv}{dt} = 5 \times 10^{-6} \frac{d}{dt}(10 \cos 6000t)$$
$$= -5 \times 10^{-6} \times 6000 \times 10 \sin 6000t = -0.3 \sin 6000t \text{ A}$$

Example 6.5

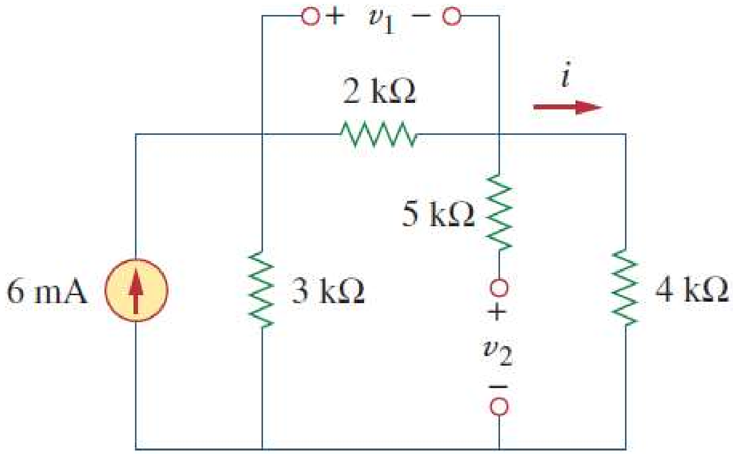
Obtain the energy stored in each capacitor in Fig. 6.12(a) under dc conditions.





DC Condition ของ Capacitor = อัดประจุจนเต็ม

จนแรงดันไฟฟ้าตกคร่อม C คงที่



Current divider

$$i = \frac{3}{3+2+4}(6\text{ mA}) = 2\text{ mA}$$

the voltages v_1 and v_2 across the capacitors are

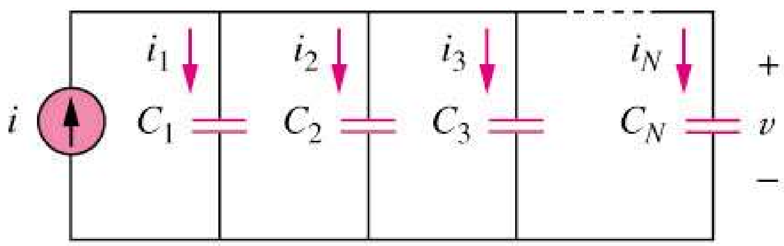
$$v_1 = 2000i = 4\text{ V} \qquad v_2 = 4000i = 8\text{ V}$$

$$w_1 = \frac{1}{2}C_1v_1^2 = \frac{1}{2}(2\times 10^{-3})(4)^2 = 16\text{ mJ}$$

$$w_2 = \frac{1}{2}C_2v_2^2 = \frac{1}{2}(4\times 10^{-3})(8)^2 = 128\text{ mJ}$$

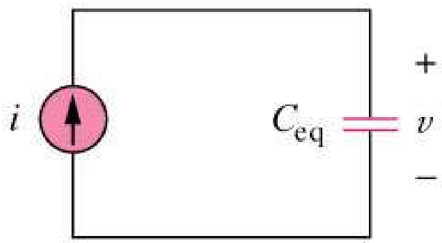
6.2 Series and Parallel Capacitors

- The equivalent capacitance of *N* **parallel-connected** capacitors is the sum of the individual capacitances.

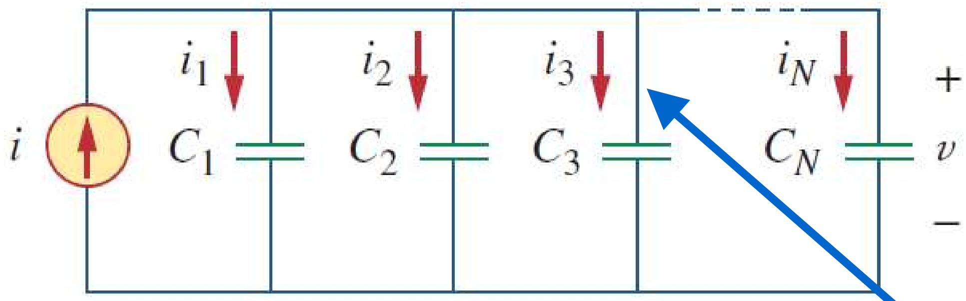


(a)

$$C_{eq} = C_1 + C_2 + ... + C_N$$



(b)



พิสูจน์

$$i_k = C_k \frac{dv}{dt}$$

$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

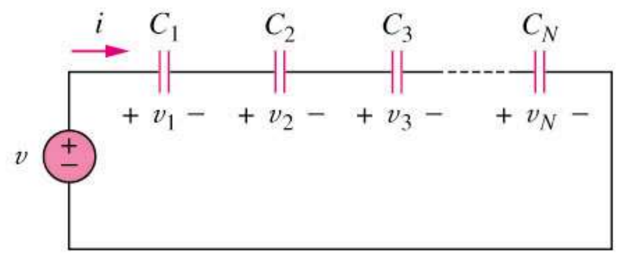
$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + C_3\frac{dv}{dt} + \cdots + C_N\frac{dv}{dt}$$

$$= \left(\sum_{k=1}^{N} C_k\right)\frac{dv}{dt} = C_{eq}\frac{dv}{dt}$$

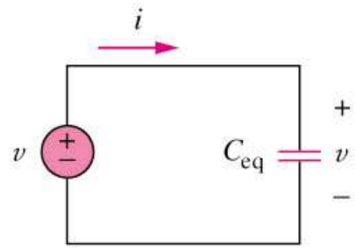
$$\boxed{C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N}$$

6.2 Series and Parallel Capacitors

- The equivalent capacitance of *N* **series-connected** capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.



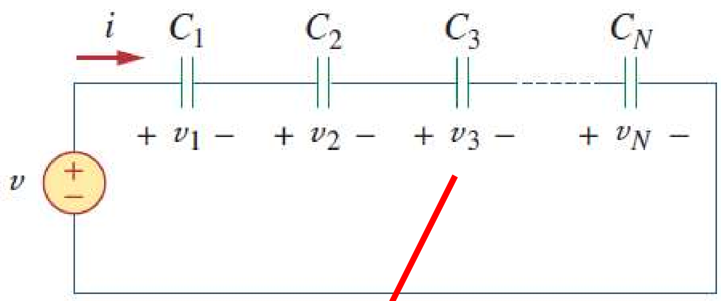
(a)



(b)

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_N}$$

พิสูจน์



$$\boxed{v_k = \frac{1}{C_k}\int_{t_0}^{t} i(\tau)\,d\tau + v_k(t_0).}$$

$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

$$\begin{aligned}
v &= \frac{1}{C_1}\int_{t_0}^{t} i(\tau)\,d\tau + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i(\tau)\,d\tau + v_2(t_0) \\
&\quad + \cdots + \frac{1}{C_N}\int_{t_0}^{t} i(\tau)\,d\tau + v_N(t_0) \\
&= \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}\right)\int_{t_0}^{t} i(\tau)\,d\tau + v_1(t_0) + v_2(t_0) \\
&\quad + \cdots + v_N(t_0)
\end{aligned}$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}\right)\int_{t_0}^{t} i(\tau)\,d\tau + v_1(t_0) + v_2(t_0) + \cdots + v_N(t_0)$$

$$= \frac{1}{C_{eq}}\int_{t_0}^{t} i(\tau)\,d\tau + v(t_0)$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}}$$

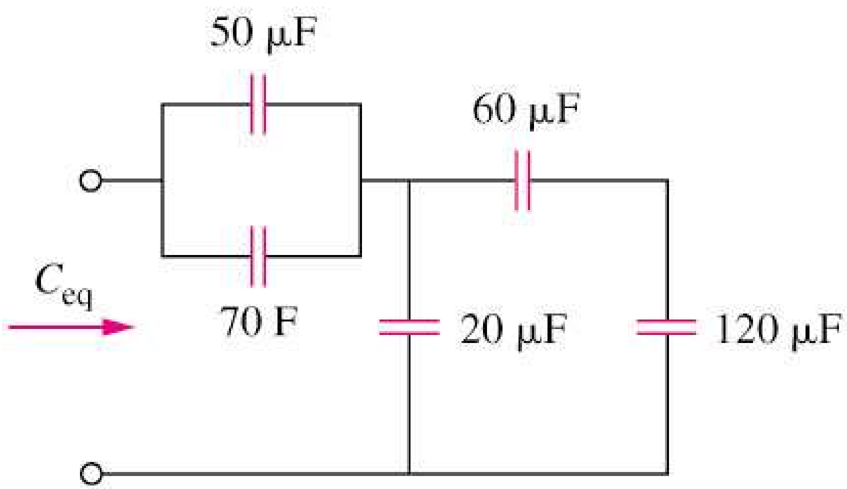
กรณี C_1 และ C_2 ต่ออนุกรมกัน

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

6.2 Series and Parallel Capacitors

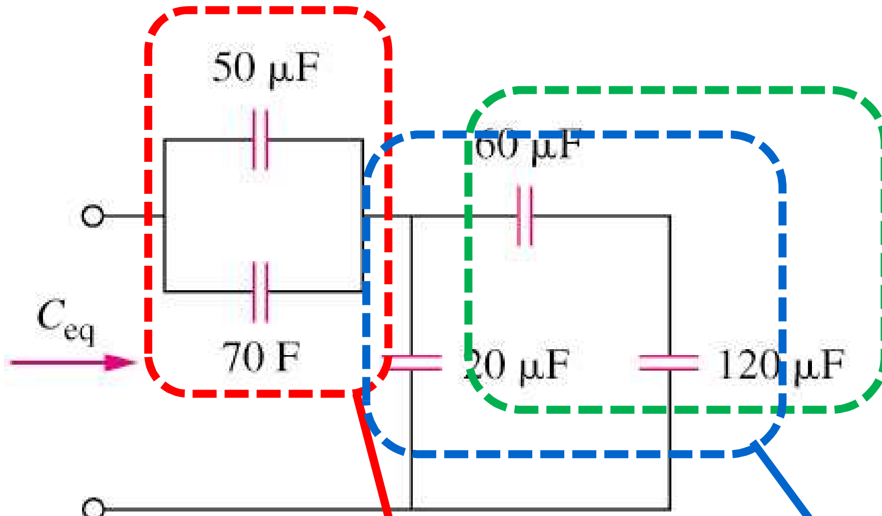
Example 3

Find the equivalent capacitance seen at the terminals of the circuit in the circuit shown below:



Answer:

C_{eq} = 40μF



$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

(2) $$\frac{60 \times 120}{60 + 120} = 40\ \mu F$$

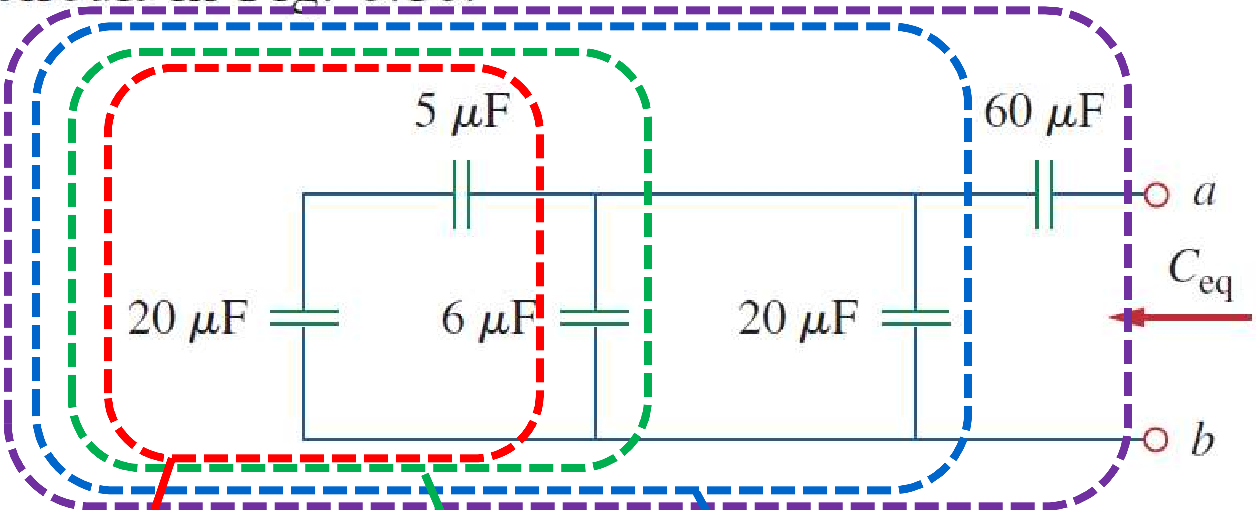
(3) $20 + 40 = 60\ \mu F$

(1) $50 + 70 = 120\ \mu F$

(4) $$\frac{120 \times 60}{120 + 60} = 40\ \mu F$$

Example 6.6

Find the equivalent capacitance seen between terminals a and b of the circuit in Fig. 6.16.



(1)

$$\frac{20\times 5}{20+5}=4\ \mu F$$

(2)

$$4+6=10\ \mu F$$

(3)

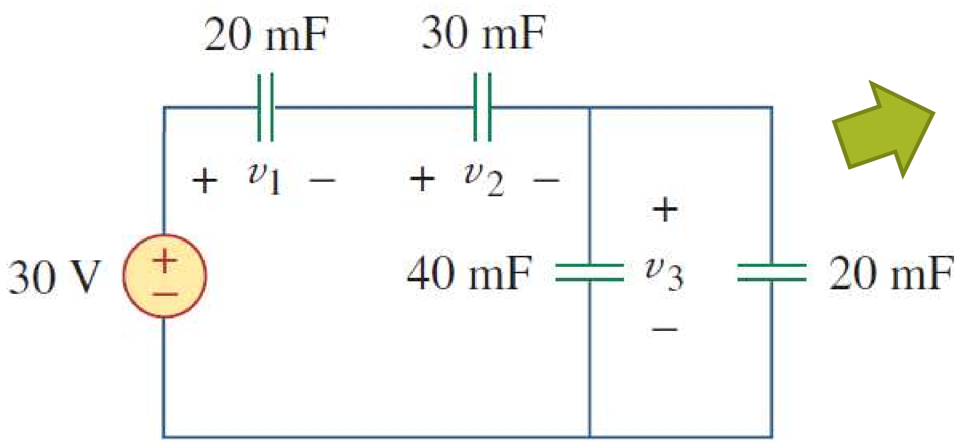
$$10+20=30\ \mu F$$

(4)

$$\frac{30\times 60}{30+60}=20\ \mu F$$

Example 6.7

For the circuit in Fig. 6.18, find the voltage across each capacitor.



$$C_{eq} = \frac{1}{\frac{1}{60} + \frac{1}{30} + \frac{1}{20}}\ \text{mF} = 10\ \text{mF}$$

$$q = C_{eq}v = 10 \times 10^{-3} \times 30 = 0.3\ \text{C}$$

กระแสที่ไหลผ่าน C_1 และ C_2 เท่ากัน ประจุของ C_1 และ C_2 จึงเท่ากัน

$$v_1 = \frac{q}{C_1} = \frac{0.3}{20 \times 10^{-3}} = 15\ \text{V} \qquad v_2 = \frac{q}{C_2} = \frac{0.3}{30 \times 10^{-3}} = 10\ \text{V}$$

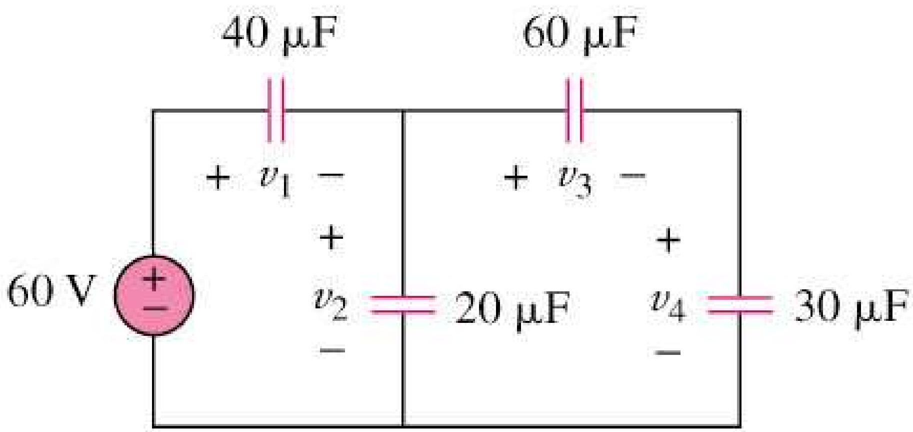
KVL

$$v_3 = 30 - v_1 - v_2 = 5\ \text{V}$$

6.2 Series and Parallel Capacitors

Example 4

Find the voltage across each of the capacitors in the circuit shown below:



Answer:

v_1 = 30V

v_2 = 30V

v_3 = 10V

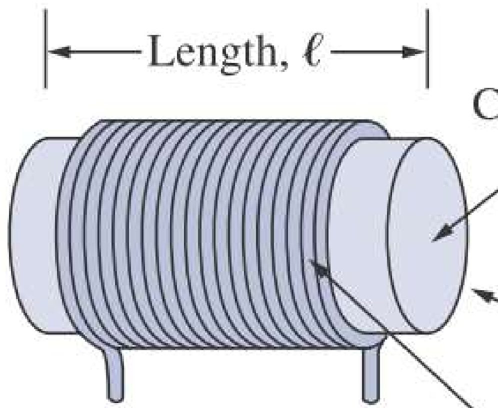
v_4 = 20V

ไปทำเอง

6.3 Inductors

มอเตอร์ไฟฟ้ามีส่วนประกอบเป็น
Inductor และ Resistor

- An inductor is a passive element designed to store energy in its magnetic field.



- An inductor consists of a coil of conducting wire.

Inductor หรือ ตัวเหนี่ยวนำ ประกอบด้วยขดลวดตัวนำ เมื่อจ่ายกระแสไฟฟ้าให้ Inductor จะเกิดสนามแม่เหล็กขึ้น เป็นการเก็บพลังงานในรูปสนามแม่เหล็ก




Inductor ในวงจรอิเล็กทรอนิกส์


Inductor ในสถานีไฟฟ้าแรงสูง (เรียกว่า Reactor)

6.3 Inductors

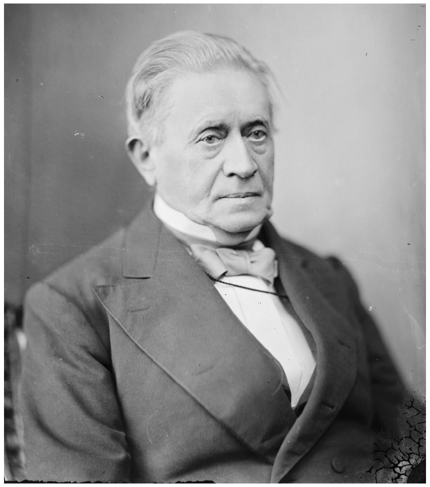
- Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).

$$v = L\frac{di}{dt} \quad \text{and} \quad L = \frac{N^2 \mu A}{l}$$

- The unit of inductors is Henry (H), mH (10^{-3}) and μH (10^{-6}).

- An inductor acts like a short circuit to dc (di/dt = 0) and its current cannot change abruptly.

Joseph Henry (1797 – 1878)



an American scientist who served as the first Secretary of the Smithsonian Institution. Henry discovered the electromagnetic phenomenon of self-inductance. He also discovered mutual inductance before, independently of Michael Faraday, though Faraday was the first to make the discovery and publish his results.

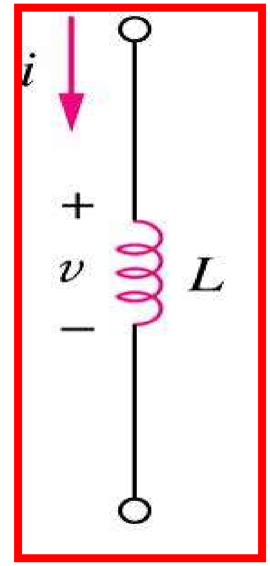
$$L = \frac{N^2 \mu A}{l}$$

N = จำนวนรอบของขดลวด, A = พื้นที่หน้าตัดของแกนขดลวด

L = ความยาวของขดลวด, μ = ค่า Permeability ของแกนขดลวด

6.3 Inductors

- The current-voltage relationship of an inductor:



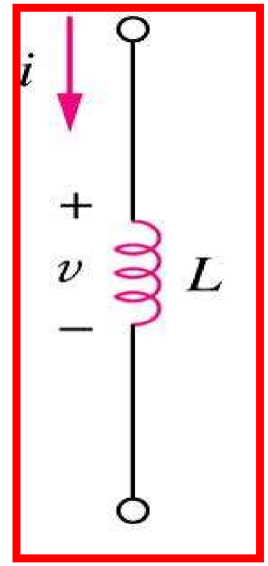
$$i = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)$$

$$v = L\frac{di}{dt}$$

Inductor จะต่อต้านการเปลี่ยนแปลงกระแสไฟฟ้า (ไม่เหมือนกับ Capacitor ที่ต่อต้านการเปลี่ยนแปลงแรงดันไฟฟ้า)

6.3 Inductors

- The energy stored by an inductor:



$$w = \int_{-\infty}^{t} p(\tau)\, d\tau = L \int_{-\infty}^{t} \frac{di}{d\tau}\, i d\tau$$

$$p = vi = \left(L\frac{di}{dt}\right)i$$

$$= L \int_{-\infty}^{t} i\, di = \frac{1}{2}Li^2(t) - \frac{1}{2}Li^2(-\infty)$$

$$w = \frac{1}{2} L\, i^2$$

$$i(-\infty) = 0$$

กระแสที่เวลา infinity มีค่าเป็น 0

ความคล้ายคลึงกันในทางฟิสิกส์ เกี่ยวกับการเก็บกักพลังงาน

พลังงานจลน์
ในวัตถุที่เคลื่อนที่

$$W = \frac{1}{2} m V^2$$

พลังงานที่สะสมใน
Capacitor
ในรูปสนามไฟฟ้า

$$W = \frac{1}{2} C v^2$$

พลังงานที่สะสมใน
Inductor
ในรูปสนามแม่เหล็ก

$$W = \frac{1}{2} L i^2$$

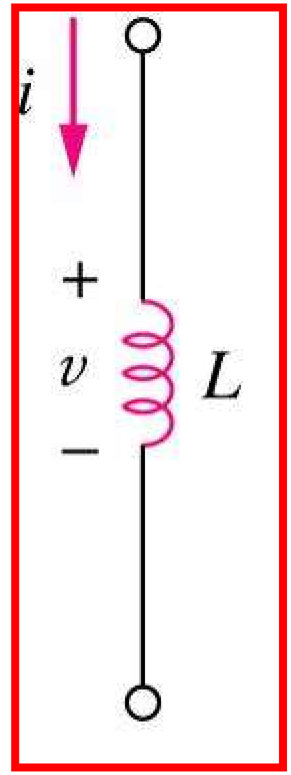
Example 5

The terminal voltage of a 2-H inductor is

v = 10(1-t) V

Find the current flowing through it at t = 4 s and the energy stored in it within 0 < t < 4 s.

Assume i(0) = 2 A.



$$i(4) = \frac{1}{L}\int_0^4 v(\tau)d\tau + i(0) = \frac{1}{2}\int_0^4 10(1-\tau)d\tau + 2$$

$$= 5(\tau - \frac{\tau^2}{2})\Big|_0^4 + 2 = 5(4 - \frac{4^2}{2}) + 2 = -18A$$

Answer:

i(4s) = -18V

w(4s) = 320 J

$$W = \frac{1}{2}Li(4)^2 - \frac{1}{2}Li(0)^2 = \frac{1}{2}\times 2\times 18^2 - \frac{1}{2}\times 2\times 2^2 = 320\,J$$

Example 6.8

The current through a 0.1-H inductor is $i(t) = 10te^{-5t}$ A. Find the voltage across the inductor and the energy stored in it.

Solution:

Since $v = L\,di/dt$ and $L = 0.1$ H,

$$v = 0.1\frac{d}{dt}(10te^{-5t}) = e^{-5t} + t(-5)e^{-5t} = e^{-5t}(1 - 5t) \text{ V}$$

The energy stored is

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.1)100t^2e^{-10t} = 5t^2e^{-10t} \text{ J}$$

Example 6.9

Find the current through a 5-H inductor if the voltage across it is

$$v(t) = \begin{cases} 30t^2, & t > 0 \\ 0, & t < 0 \end{cases}$$

Also, find the energy stored at $t = 5$ s. Assume $i(v) > 0$.

วิธีทำ Since $i = \dfrac{1}{L}\displaystyle\int_{t_0}^{t} v(t)\,dt + i(t_0)$ and $L = 5$ H,

$$i = \frac{1}{5}\int_0^t 30t^2\,dt + 0 = 6 \times \frac{t^3}{3} = 2t^3 \text{ A}$$

คำนวณพลังงาน

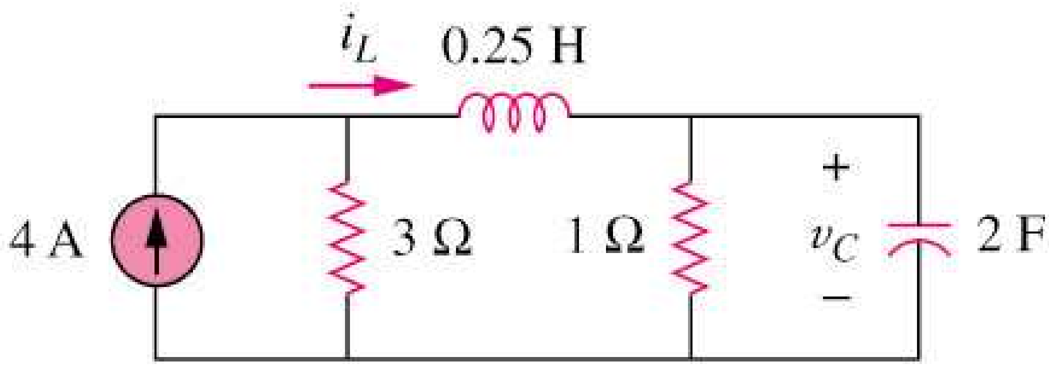
$$w\big|_0^5 = \frac{1}{2}Li^2(5) - \frac{1}{2}Li(0) = \frac{1}{2}(5)(2 \times 5^3)^2 - 0 = 156.25 \text{ kJ}$$

หรือ

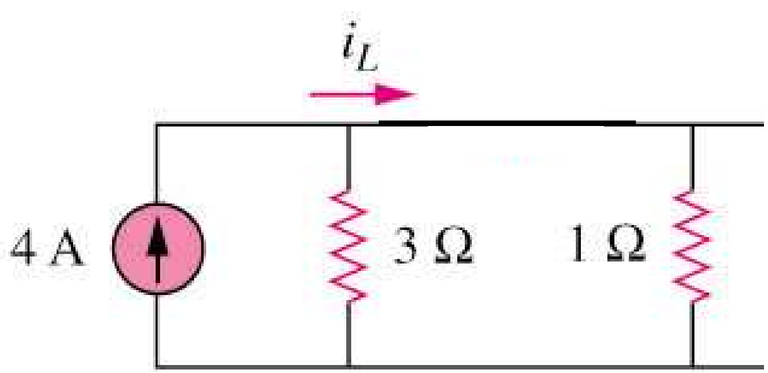
$$w = \int p\,dt = \int_0^5 60t^5\,dt = 60\frac{t^6}{6}\bigg|_0^5 = 156.25 \text{ kJ}$$

Example 6

Determine v_c, i_L, and the energy stored in the capacitor and inductor in the circuit of circuit shown below under dc conditions.



Under DC condition เราแทน C ด้วย Open circuit และแทน L ด้วย Short circuit



Answer:

$i_L = 3A$

$v_C = 3V$

$w_L = 1.125J$

$w_C = 9J$

หมายเหตุ: ที่ว่า Under DC condition (หมายถึงแรงดันไฟฟ้าและกระแสไฟฟ้ามีค่าคงที่)

เราแทน C ด้วย Open circuit เพราะว่าเมื่อแรงดันไฟฟ้าคงที่จะได้

$$i = C\frac{dv}{dt} = 0$$

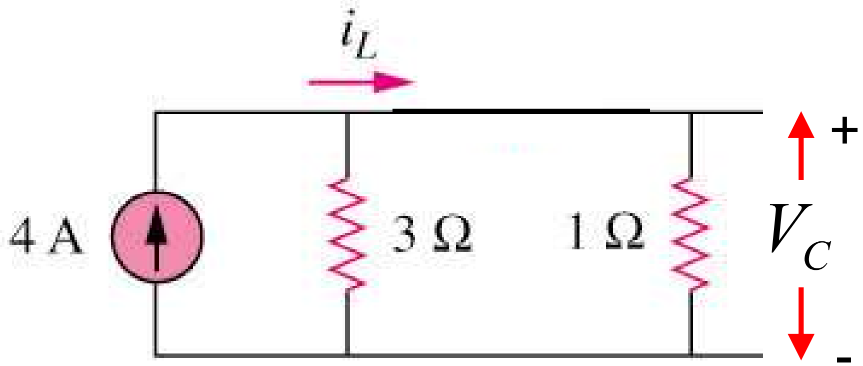
➡ กระแสเป็น 0 ก็คือไม่มีกระแสไหล คือเป็น Open circuit นั่นเอง

และที่เราแทน L ด้วย Short circuit เพราะว่า เมื่อกระแสคงที่จะได้

$$v = L\frac{di}{dt} = 0$$

➡ แรงดันเป็น 0 ก็คือการ short circuit นั่นเอง

หา i_L ใช้ Current divider



$$i_L = \frac{3}{3+1} \times 4 = 3 \text{ A}$$

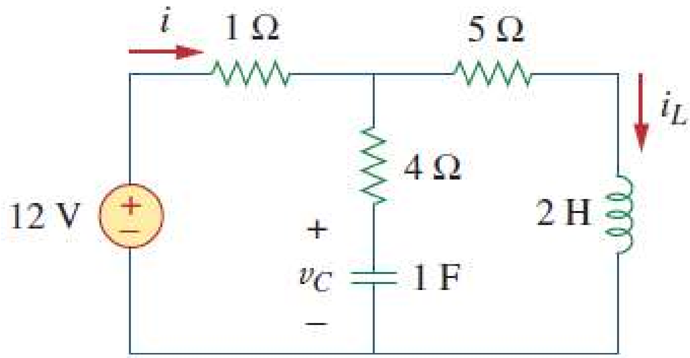
$$V_C = i_L \times R = 3 \times 1 = 3 \text{ V}$$

$$W_L = \frac{1}{2} L i^2 = \frac{1}{2} \times 0.25 \times 3^2 = 1.125 \text{ J}$$

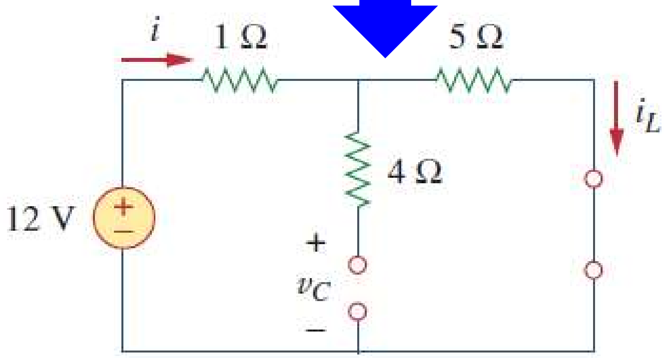
$$W_C = \frac{1}{2} C v^2 = \frac{1}{2} \times 2 \times 3^2 = 9 \text{ J}$$

Example 6.10

Consider the circuit in Fig. 6.27(a). Under dc conditions, find: (a) i, v_C, and i_L, (b) the energy stored in the capacitor and inductor.



Under DC condition
เราแทน C ด้วย Open circuit
และแทน L ด้วย Short circuit



$$i = i_L = \frac{12}{1+5} = 2 \text{ A}$$

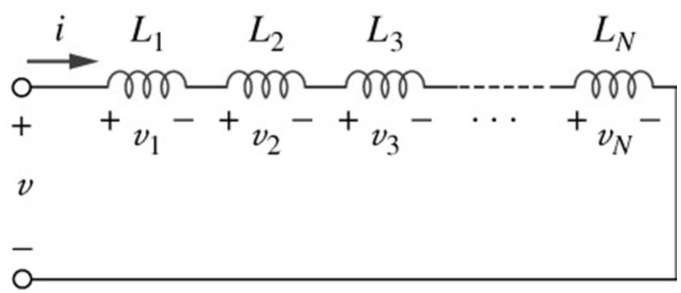
$$v_C = 5i = 10 \text{ V}$$

$$w_C = \frac{1}{2}Cv_C^2 = \frac{1}{2}(1)(10^2) = 50 \text{ J}$$

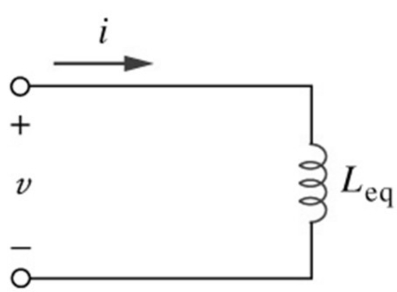
$$w_L = \frac{1}{2}Li_L^2 = \frac{1}{2}(2)(2^2) = 4 \text{ J}$$

6.4 Series and Parallel Inductors

- The equivalent inductance of **series-connected** inductors is the sum of the individual inductances.



(a)



(b)

$$L_{eq} = L_1 + L_2 + \ldots + L_N$$

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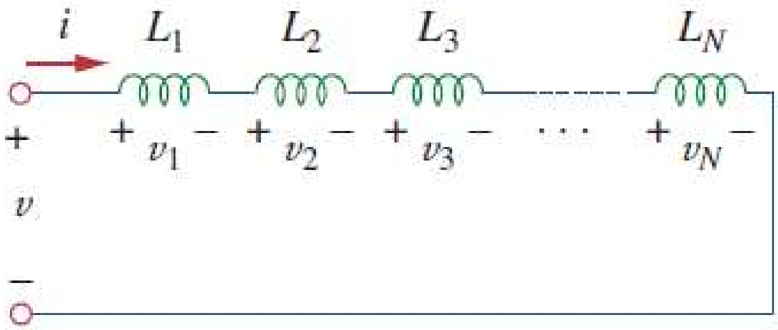


$$v_k = L_k \frac{di}{dt}$$

จาก KVL

$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

$$v = L_1\frac{di}{dt} + L_2\frac{di}{dt} + L_3\frac{di}{dt} + \cdots + L_N\frac{di}{dt}$$

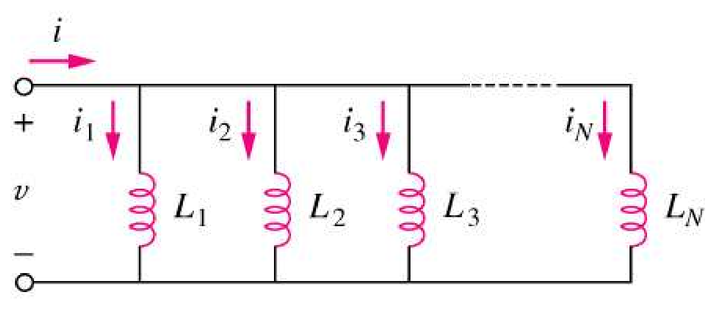
$$= (L_1 + L_2 + L_3 + \cdots + L_N)\frac{di}{dt}$$

$$= \left(\sum_{k=1}^{N} L_k\right)\frac{di}{dt} = L_{eq}\frac{di}{dt}$$

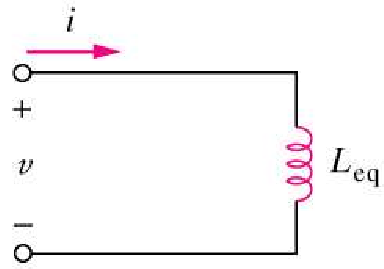
$$\boxed{L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N}$$

6.4 Series and Parallel Inductors

- The equivalent capacitance of **parallel** inductors is the reciprocal of the sum of the reciprocals of the individual inductances.



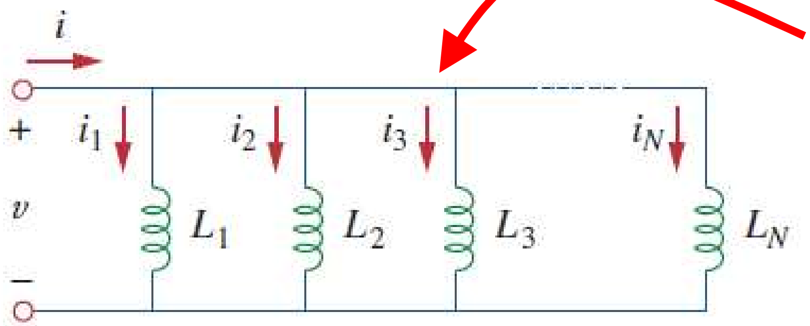
(a)



(b)

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_N}$$

พิสูจน์



$$i_k = \frac{1}{L_k}\int_{t_0}^{t} v(\tau)d\tau + i_k(t_0)$$

$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

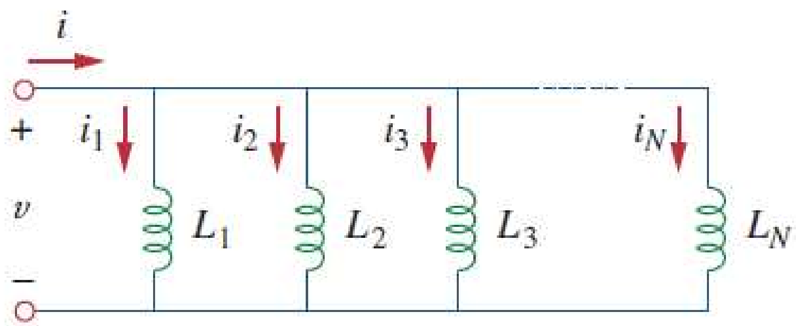
$$i = \frac{1}{L_1}\int_{t_0}^{t} v\,dt + i_1(t_0) + \frac{1}{L_2}\int_{t_0}^{t} v\,dt + i_2(t_0)$$

$$+ \cdots + \frac{1}{L_N}\int_{t_0}^{t} v\,dt + i_N(t_0)$$

$$= \left(\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}\right)\int_{t_0}^{t} v\,dt + i_1(t_0) + i_2(t_0)$$

$$+ \cdots + i_N(t_0)$$

$$= \left(\sum_{k=1}^{N}\frac{1}{L_k}\right)\int_{t_0}^{t} v\,dt + \sum_{k=1}^{N} i_k(t_0) = \frac{1}{L_{\text{eq}}}\int_{t_0}^{t} v\,dt + i(t_0)$$

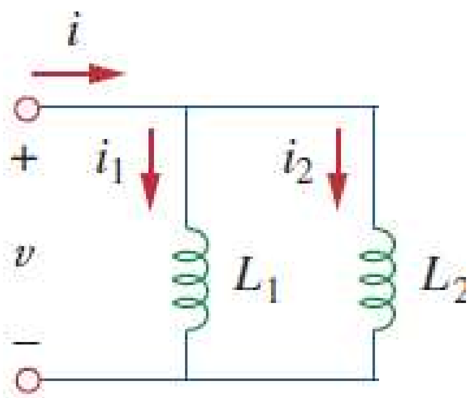


$$i = \left(\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}\right) \int_{t_0}^{t} v\,dt + i_1(t_0) + i_2(t_0) + \cdots + i_N(t_0)$$

$$= \left(\sum_{k=1}^{N} \frac{1}{L_k}\right) \int_{t_0}^{t} v\,dt + \sum_{k=1}^{N} i_k(t_0) = \frac{1}{L_{eq}} \int_{t_0}^{t} v\,dt + i(t_0)$$

$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}}$$

กรณี L สองตัวต่อขนานกัน



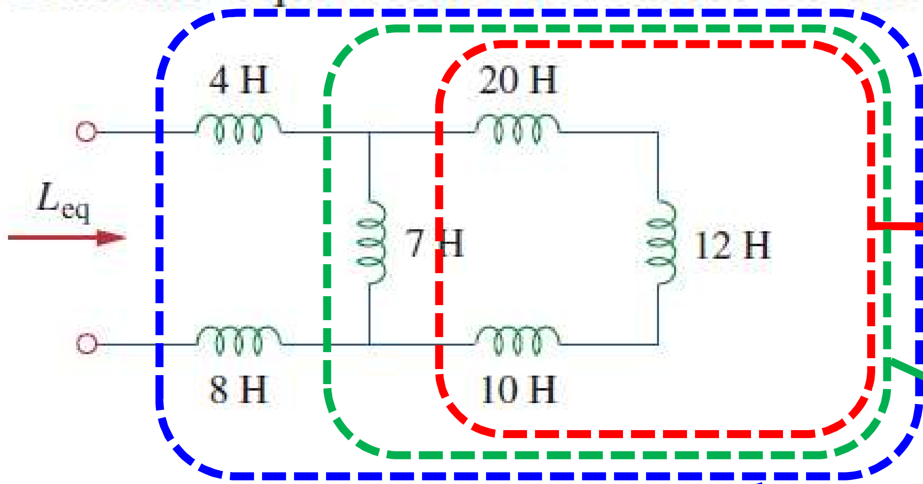
$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_1}$$

$$\Downarrow$$

$$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2}} = \frac{L_1 L_2}{L_1 + L_2}$$

Example 6.11

Find the equivalent inductance of the circuit shown in Fig. 6.31.



(1)

$$\boxed{20+12+10=42\ \text{H}}$$

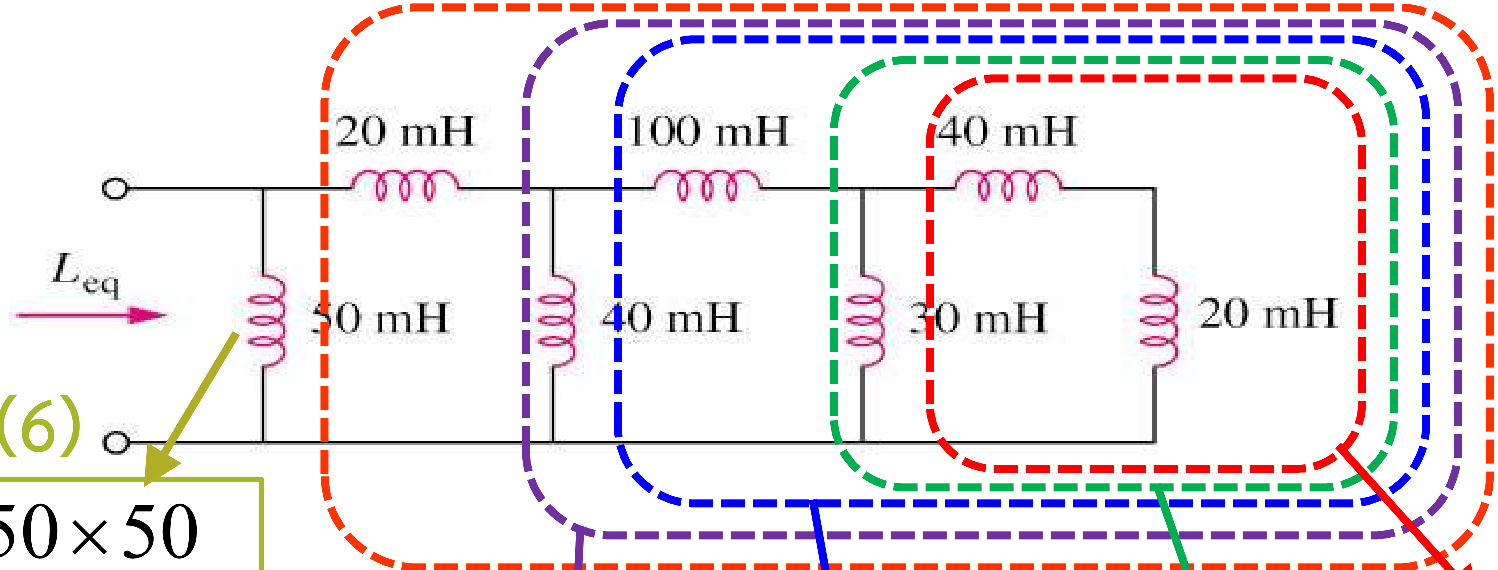
(2)

$$\boxed{\frac{7\times 42}{7+42}=6\ \text{H}}$$

(3)

$$\boxed{4+6+8=18\ \text{H}}$$

Example 7 Calculate the equivalent inductance for the inductive ladder network in the circuit shown below:



Answer:

L_{eq} = <u>25mH</u>

(1)

$$40 + 20 = 60 \text{ mH}$$

(2)

$$\frac{30 \times 60}{30 + 60} = 20 \text{ mH}$$

(3)

$$100 + 20 = 120 \text{ mH}$$

(4)

$$\frac{40 \times 120}{40 + 120} = 30 \text{ mH}$$

(5)

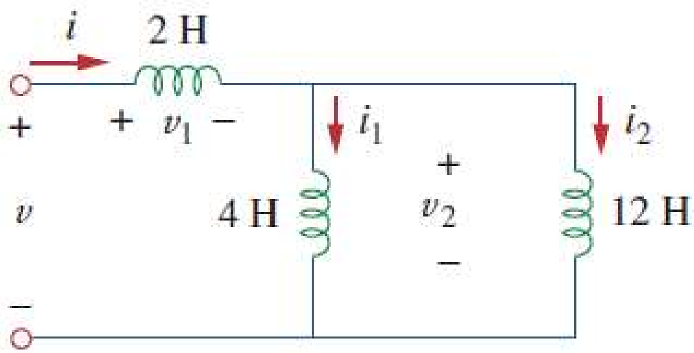
$$30 + 20 = 50 \text{ mH}$$

(6)

$$\frac{50 \times 50}{50 + 50} = 25 \text{ mH}$$

Example 6.12

For the circuit in Fig. 6.33, $i(t) = 4(2 - e^{-10t})$ mA. If $i_2(0) = -1$ mA, find: (a) $i_1(0)$; (b) $v(t)$, $v_1(t)$, and $v_2(t)$; (c) $i_1(t)$ and $i_2(t)$.



$$L_{eq} = 2 + 4 \| 12 = 2 + 3 = 5 \text{ H}$$

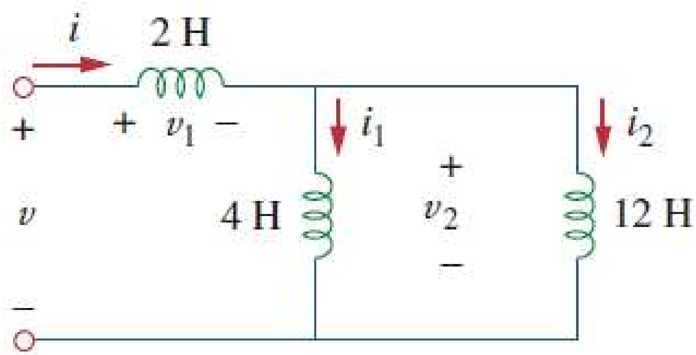
(a) หา $i_1(0)$

จาก $i(t) = 4(2 - e^{-10t})$ mA. ➡ $i(0) = 4(2 - 1) = 4$ mA

เนื่องจาก $i = i_1 + i_2$ ➡ $i_1 = i - i_2 = 4 - (-1) = 5$ mA

(b) หา $v(t)$

$$v = L_{eq}\frac{di}{dt} = 5\frac{d}{dt}\left(4(2 - e^{-10t})\right) = 5 \times (40e^{-10t}) = 200e^{-10t}$$



หา v_1(t) $$v_1(t) = 2\frac{di}{dt} = 2(-4)(-10)e^{-10t}\ \text{mV} = 80e^{-10t}\ \text{mV}$$

หา v_2(t) Since $v = v_1 + v_2$,

$$v_2(t) = v(t) - v_1(t) = 120e^{-10t}\ \text{mV}$$

(c) หา i_1(t) $$i_1(t) = \frac{1}{4}\int_0^t v_2\, dt + i_1(0) = \frac{120}{4}\int_0^t e^{-10t}\, dt + 5\ \text{mA}$$

$$= -3e^{-10t}\Big|_0^t + 5\ \text{mA} = -3e^{-10t} + 3 + 5 = 8 - 3e^{-10t}\ \text{mA}$$

หา i_2(t) $$i_2(t) = \frac{1}{12}\int_0^t v_2\, dt + i_2(0) = \frac{120}{12}\int_0^t e^{-10t}\, dt - 1\ \text{mA}$$

$$= -e^{-10t}\Big|_0^t - 1\ \text{mA} = -e^{-10t} + 1 - 1 = -e^{-10t}\ \text{mA}$$

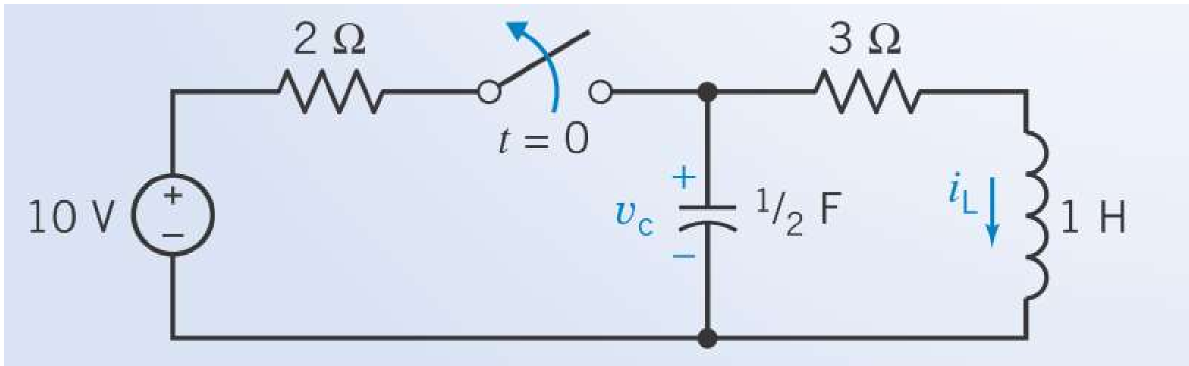
Important characteristics of the basic elements

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i

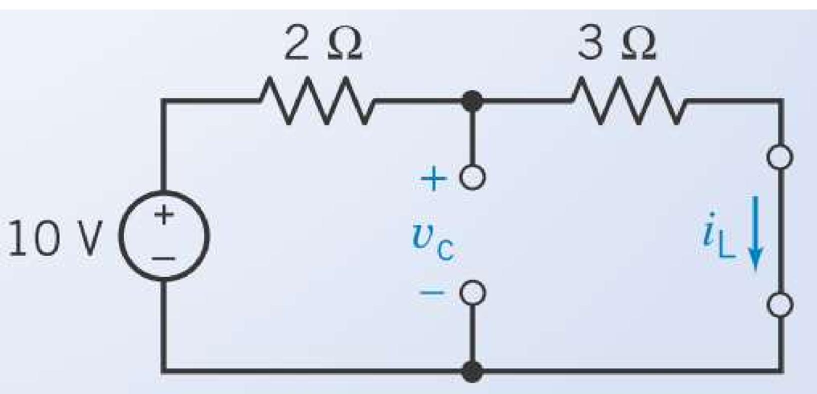
6.5 Initial Conditions of Switched Circuits

Example 8

Prior to t = 0, the switch has been closed for a long time. Determine the values of the capacitor voltage and inductor current immediately after the switch opens at time t = 0.



เมื่อ Switch เป็น Close เป็นเวลานาน C จะถูก Charge จนเต็มแรงดันของ C จะคงที่ ขณะที่กระแส L ก็จะคงที่ เราแทน C ด้วย Open circuit และแทน L ด้วย Short circuit



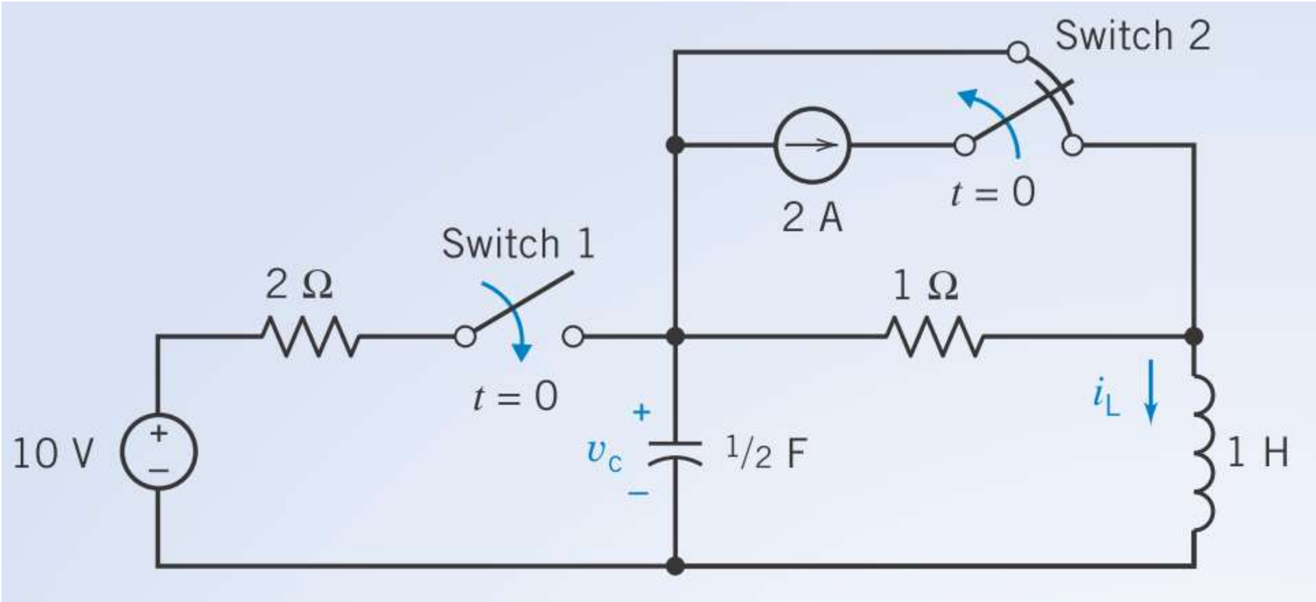
at $t = 0^-$ หมายถึงขณะก่อนสวิทต์เปลี่ยนสถานะ

$$i_L(0^-) = \frac{10}{5} = 2\ \text{A}$$

$$v_c(0^-) = \left(\frac{3}{5}\right)10 = 6\ \text{V}$$

6.5 Initial Conditions of Switched Circuits

Example 9



Find $i_L(0^+)$, $v_c(0^+)$, $dv_c(0^+)/dt$, and $di_L(0^+)/dt$

โดย

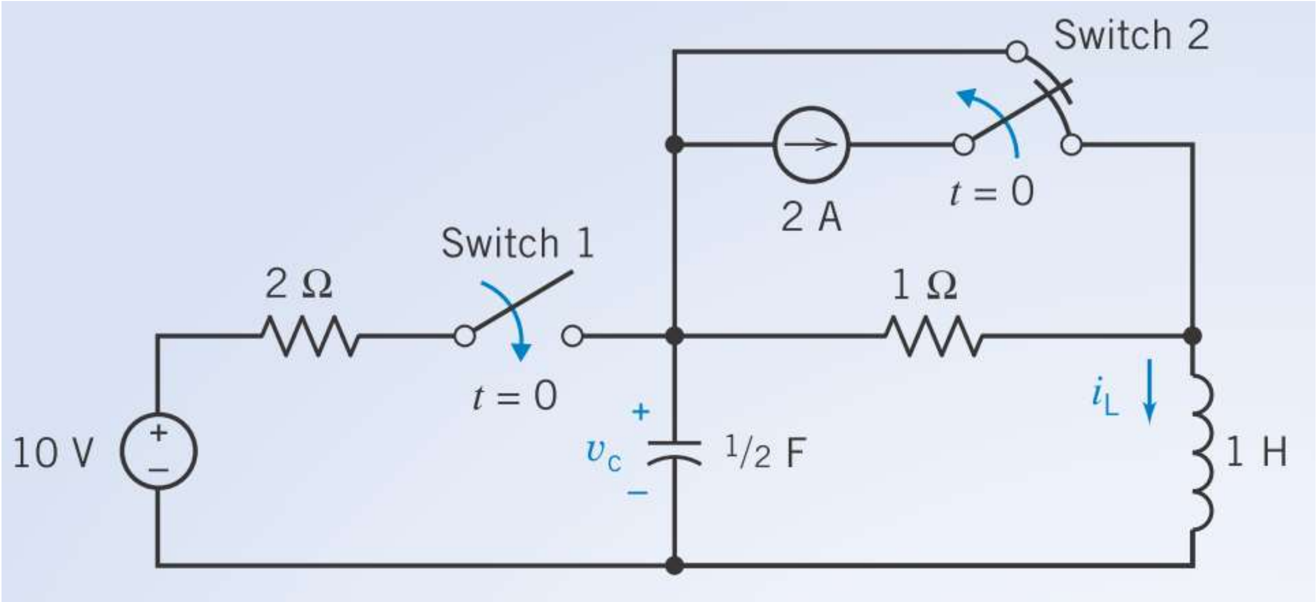
$$v_c(0^+) = v_c(0^-) = 6 \text{ V}$$

$$i_L(0^+) = i_L(0^-) = 2 \text{ A}$$

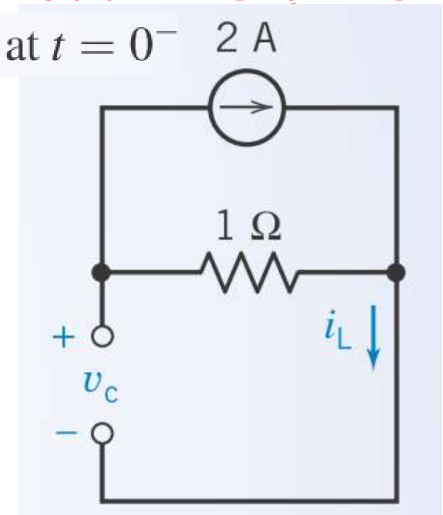
หมายเหตุ

t=0^+ หมายถึงขณะหลังสวิทต์เปลี่ยนสถานะ

The capacitor voltage and inductor current cannot change instantaneously

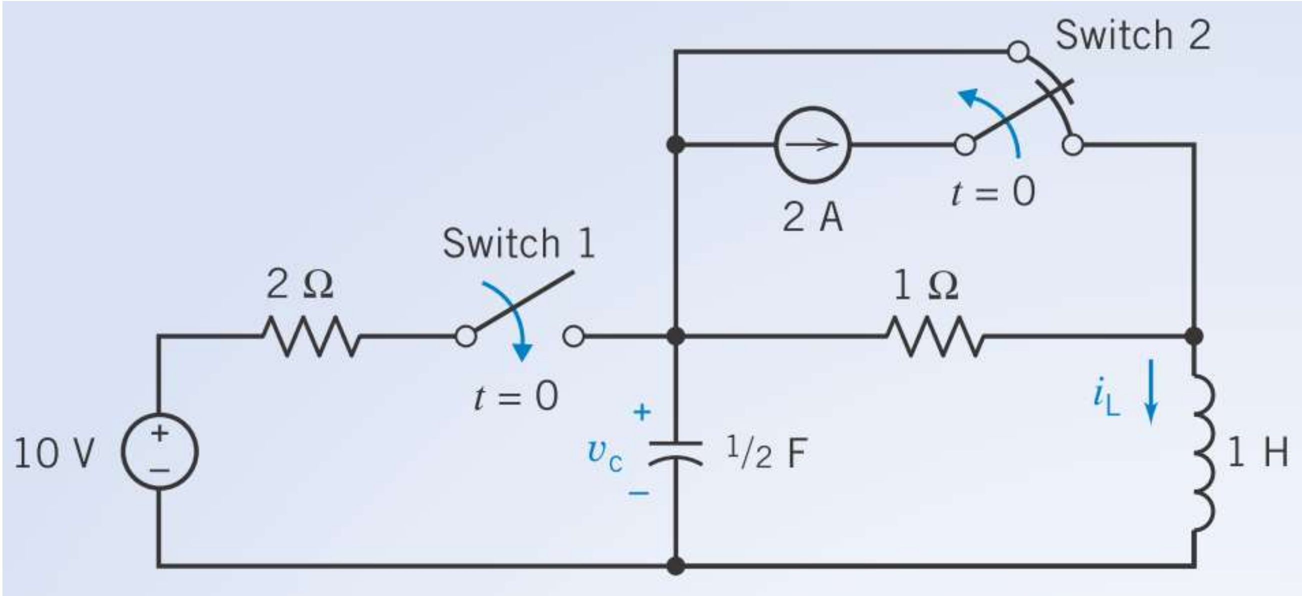


ก่อนสวิทต์เปลี่ยนสถานะ → Switch1 Open, Swith2 Close
ขณะที่ C ถูก Charge จนเต็ม จะได้วงจรเป็น

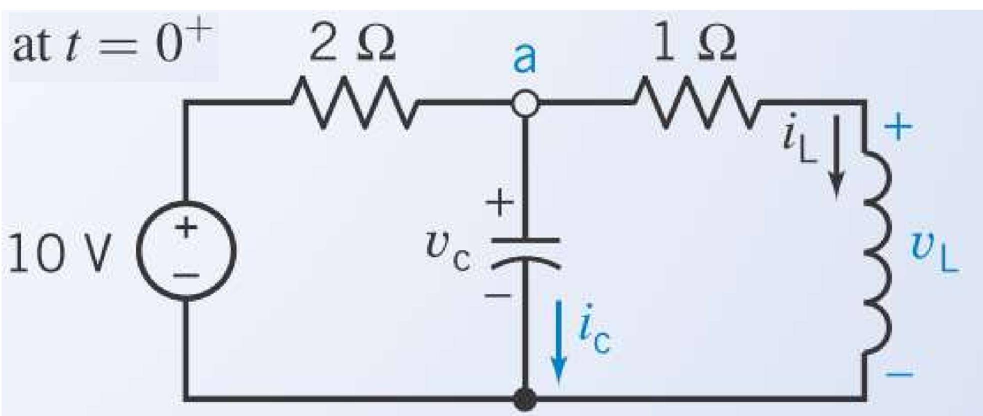


L เป็น Short circuit
C เป็น Open circuit

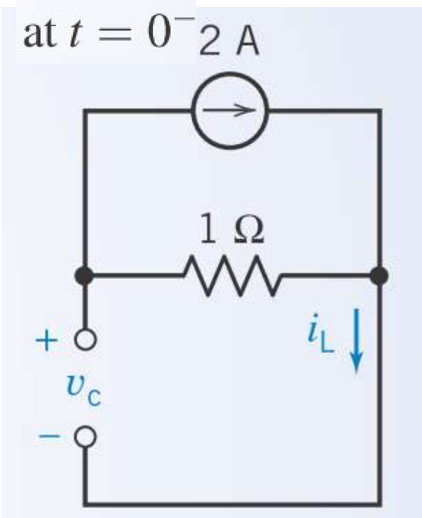
$$i_L(0^-) = 0$$
$$v_c(0^-) = -2\text{ V}$$



หลังสวิทต์เปลี่ยนสถานะ → Switch1 Close, Swith2 Open จะได้วงจรเป็น



ก่อนสวิทต์เปลี่ยนสถานะ



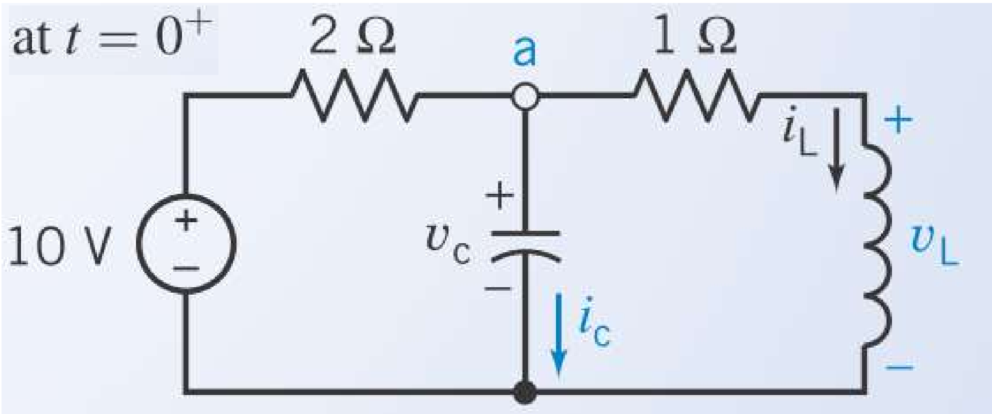
L เป็น Short circuit

C เป็น Open circuit

$$i_L(0^-) = 0$$
$$v_c(0^-) = -2\text{ V}$$

แรงดันของ C และกระแสของ L ไม่สามารถเปลี่ยนแปลงได้อย่างฉับพลัน ดังนั้น ช่วงรอยต่อระหว่างสวิทต์เปลี่ยนสถานะจะได้

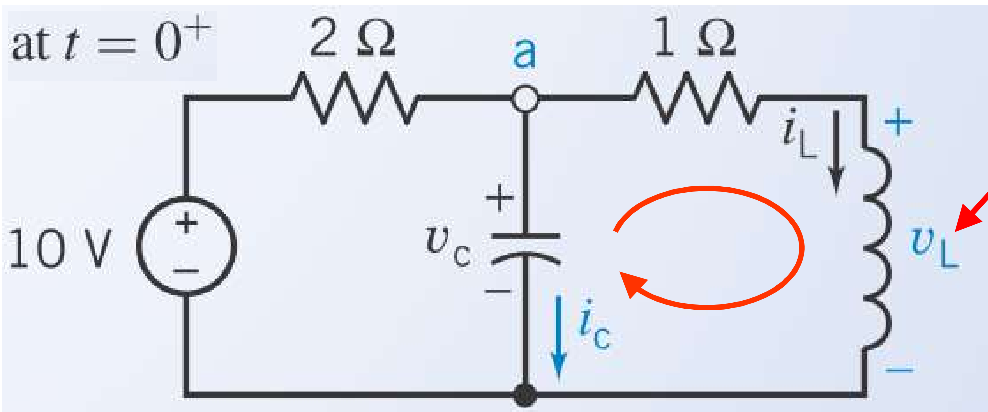
หลังสวิทต์เปลี่ยนสถานะ



$$i_L(0^+) = i_L(0^-) = 0$$
$$v_c(0^+) = v_c(0^-) = -2\text{ V}$$

หลังสวิทต์เปลี่ยนสถานะ

$$v_L = L\frac{di_L}{dt}$$



$$\frac{di_L(0^+)}{dt} = \frac{v_L(0^+)}{L}$$

KVL right hand mesh

$$v_c(0^+) = v_c(0^-) = -2\text{ V}$$

$$v_L - v_c + 1i_L = 0$$

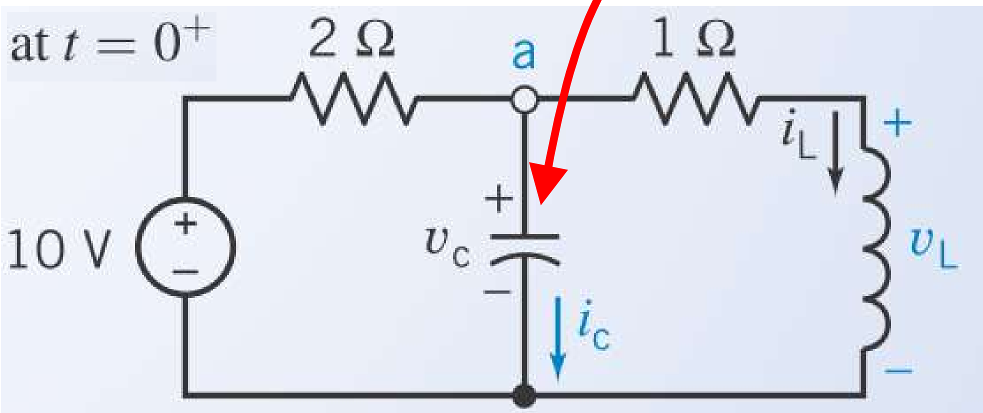
$$i_L(0^+) = i_L(0^-) = 0$$

at $t = 0^+$

$$v_L(0^+) = v_c(0^+) - i_L(0^+) = -2 - 0 = -2\text{ V}$$

$$\frac{di_L(0^+)}{dt} = \frac{v_L(0^+)}{L} = -2\text{ A/s}$$

หลังสวิทต์เปลี่ยนสถานะ



$$i_c = C\frac{dv_c}{dt}$$

$$\frac{dv_c(0^+)}{dt} = \frac{i_c(0^+)}{C}$$

$$\boxed{v_C(0^+) = -2\text{ V}}$$

$$\boxed{i_L(0^+) = 0\text{ A}}$$

KCL node a

$$i_c + i_L + \frac{v_c - 10}{2} = 0$$

at $t = 0^+$

$$i_c(0^+) = \frac{10 - v_c(0^+)}{2} - i_L(0^+) = 6 - 0 = 6\text{ A}$$

$$\frac{dv_c(0^+)}{dt} = \frac{i_c(0^+)}{C} = \frac{6}{1/2} = 12\text{ V/s}$$

Practice Problem 6.2

If a 10-μF capacitor is connected to a voltage source with

$$v(t) = 75 \sin 2000t \text{ V}$$

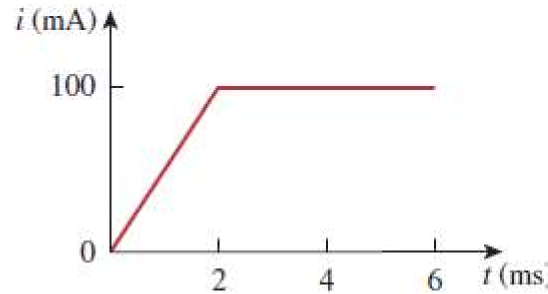
determine the current through the capacitor.

Answer: $1.5 \cos 2000t$ A.

Practice Problem 6.4

An initially uncharged 1-mF capacitor has the current shown in Fig. 6.11 across it. Calculate the voltage across it at $t = 2$ ms and $t = 5$ ms.

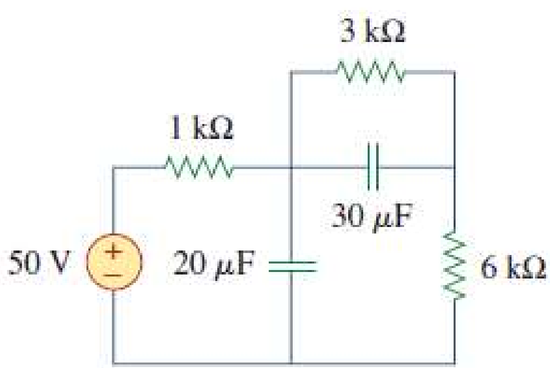
Answer: 100 mV, 400 mV.



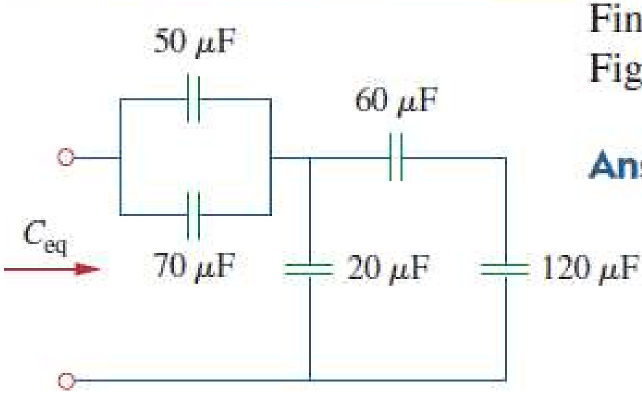
Practice Problem 6.5

Under dc conditions, find the energy stored in the capacitors in Fig. 6.13.

Answer: 20.25 mJ, 3.375 mJ.



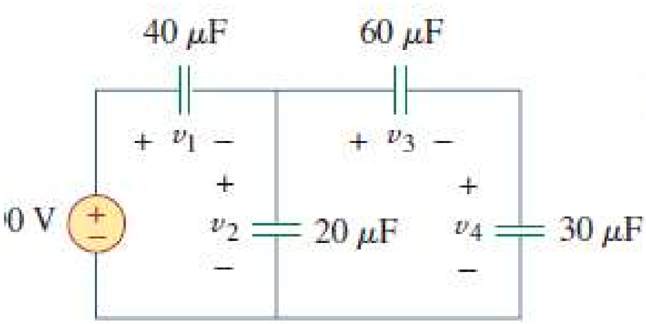
Practice Problem 6.6



Find the equivalent capacitance seen at the terminals of the circuit in Fig. 6.17.

Answer: 40 μF.

Practice Problem 6.7

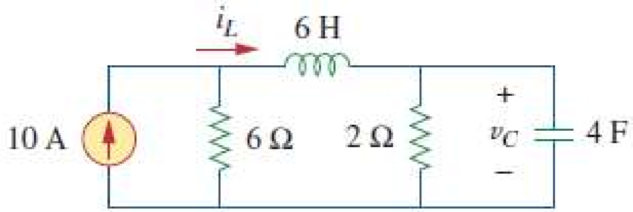


Find the voltage across each of the capacitors in Fig. 6.20.

Answer: $v_1 = 45$ V, $v_2 = 45$ V, $v_3 = 15$ V, $v_4 = 30$ V.

Practice Problem 6.10

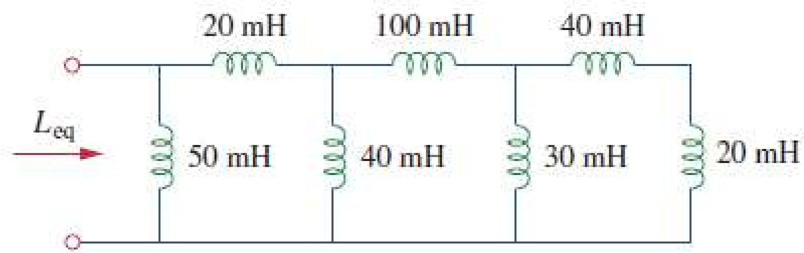
Determine v_C, i_L, and the energy stored in the capacitor and inductor in the circuit of Fig. 6.28 under dc conditions.



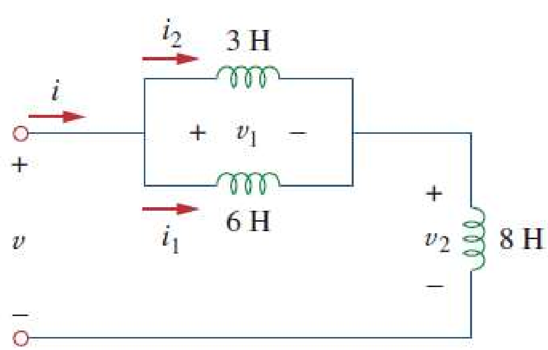
Answer: 15 V, 7.5 A, 450 J, 168.75 J.

Practice Problem 6.11

Calculate the equivalent inductance for the inductive ladder network in Fig. 6.32.



Practice Problem 6.12



In the circuit of Fig. 6.34, $i_1(t) = 0.6e^{-2t}$ A. If $i(0) = 1.4$ A, find: (a) $i_2(0)$; (b) $i_2(t)$ and $i(t)$; (c) $v_1(t)$, $v_2(t)$, and $v(t)$.

Answer: (a) 0.8 A, (b) $(-0.4 + 1.2e^{-2t})$ A, $(-0.4 + 1.8e^{-2t})$ A, (c) $-36e^{-2t}$ V, $-7.2e^{-2t}$ V, $-28.8e^{-2t}$ V.