EN811100
LINEAR CIRCUIT ANALYSIS

Chapter 3

Methods of Analysis

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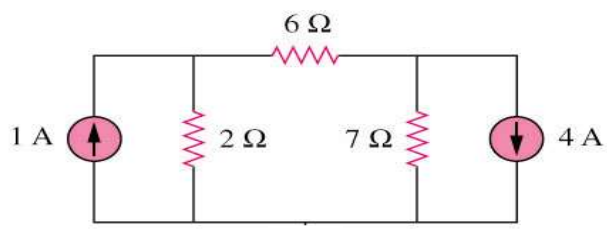
C. K. Alexander – M. N. O. Sadiku
Fundamentals of Electric Circuits, 5th Edition, The McGraw-Hill Companies 2013
J. A. Svoboda – R. C. Dorf
Introduction to Electric Circuits, 9th edition, John Wiley & Sons, Inc. 2014

Methods of Analysis - Chapter 3

3.1 Motivation

If you are given the following circuit, how can we determine (1) the voltage across each resistor, (2) current through each resistor. (3) power generated by each current source, etc.



What are the things which we need to know in order to determine the answers?

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3.1 Motivation

Things we need to know in solving any resistive circuit with current and voltage sources only:

- Kirchhoff's Current Laws (KCL)
- Kirchhoff's Voltage Laws (KVL)
- Ohm's Law

How should we apply these laws to determine the answers?

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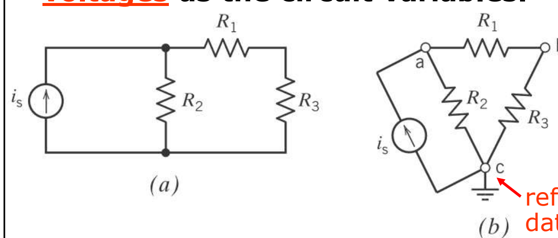
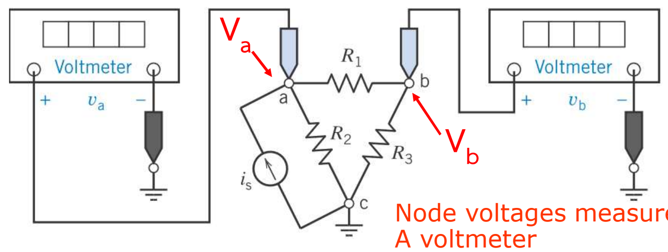
3.2 Nodal Analysis ➡ เป็นวิธีการที่ใช้หา Node Voltage

A general procedure for analyzing circuits using <u>node voltages</u> as the circuit variables.



Node Voltage คือแรงดันที่แต่ละ Node วัดเทียบกับ Reference node (จุด Ground หรือจุด 0V)

เรานิยมตั้งชื่อแรงดันตามชื่อ Node



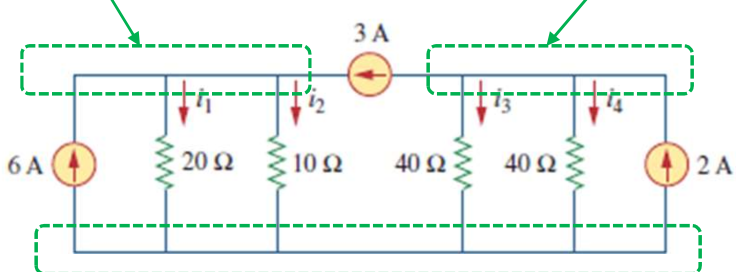
Node voltages measurement with
A voltmeter

Node พิจารณาอย่างไร?

Node คือจุดที่มีปลายของ Branch (หรือ Element) มาต่อกัน ถ้ามีอุปกรณ์หลายตัวมาต่อที่สายตัวนำเส้นเดียวกันโดยไม่มีอะไรมาคั่น ก็ถือว่าตัวนำเส้นนั้นเป็น Node เดียว

ถือว่า Node 1 Node

ถือว่า Node 1 Node

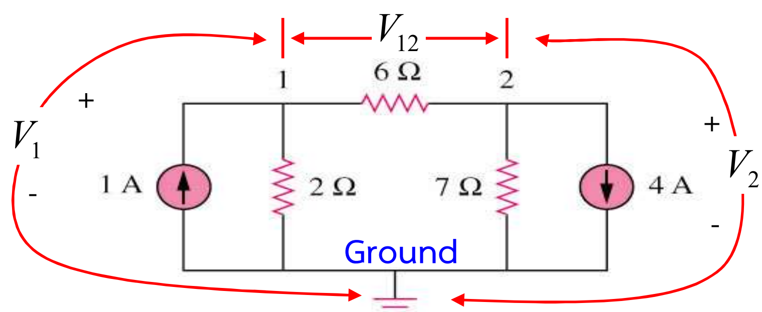


ถือว่า Node 1 Node

ธรรมเนียมการตั้งชื่อค่าแรงดันไฟฟ้า

$V_{12} = V_1 - V_2$ หมายถึงความต่างศักย์ระหว่างจุด 1 กับจุด 2

โดยแรงดัน V_1 หมายถึงความต่างศักย์ระหว่าง Node 1 กับจุดกราวน์ (Ground) หรือ Reference Node ซึ่งเรากำหนดให้จุดกราวน์มีแรงดันไฟฟ้าเป็น 0 Volt



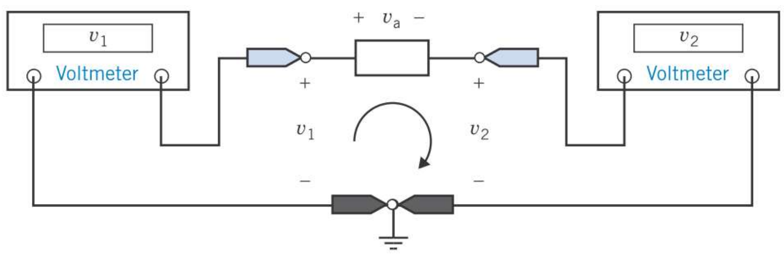
3.2 Nodal Analysis

Steps to determine the node voltages:

1. Select a node as the reference node.
2. Assign voltages $v_1, v_2, \ldots, v_{n-1}$ to the remaining n-1 nodes. The voltages are referenced with respect to the reference node.
3. Apply KCL to each of the n-1 non-reference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
4. Solve the resulting simultaneous equations to obtain the unknown node voltages.

3.2 Nodal Analysis

Consider the problem of expressing element currents as functions of the node voltages.



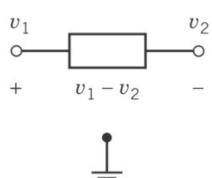
Node voltages v_1 and v_2 and element voltage v_a of a circuit element.

From KVL $$v_a = v_1 - v_2$$

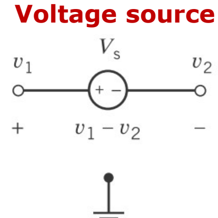
This equation expresses the element voltage v a as a function of the node voltages v_1 and v_2.

3.2 Nodal Analysis

Generic circuit element



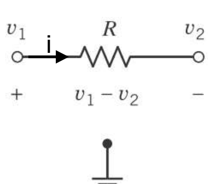
Voltage source



For a voltage source

$$V_S = v_1 - v_2$$

Resistor



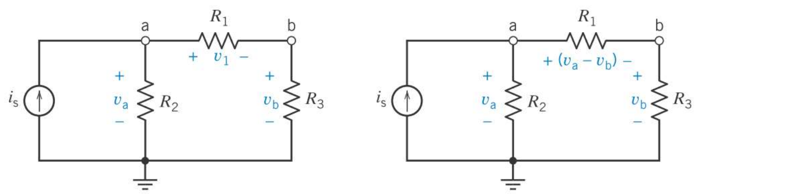
Ohm's law

$$i = \frac{v_1 - v_2}{R}$$

ในสูตรนี้ เราเอา v_1 เป็นตัวตั้ง และ v_2 เป็นตัวลบ ให้ตีความว่ากระแสไหลจาก Node 1 ไป Node 2

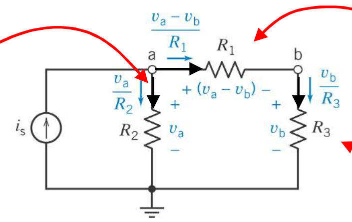
3.2 Nodal Analysis

Example 1 – circuit with independent current source only



(a) (b)

$$i_{R2} = \frac{v_a - 0}{R_2}$$



(c)

$$i_{R1} = \frac{v_a - v_b}{R_1}$$

$$i_{R3} = \frac{v_b - 0}{R_3}$$

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3.2 Nodal Analysis

KCL at node a gives

กระแสไหลเข้า $$i_s = \frac{v_a}{R_2} + \frac{v_a - v_b}{R_1}$$ กระแสไหลออก

KCL equation at node b

กระแสไหลเข้า $$\frac{v_a - v_b}{R_1} = \frac{v_b}{R_3}$$ กระแสไหลออก

If $R_1 = 1\ \Omega$, $R_2 = R_3 = 0.5\ \Omega$, and $i_s = 4$ A

จะได้

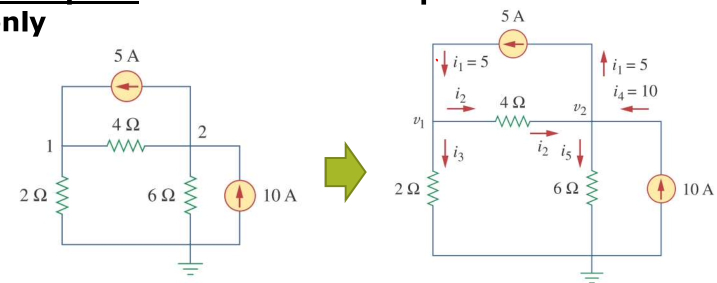
$$\left.\begin{aligned} 4 &= \frac{v_a - v_b}{1} + \frac{v_a}{0.5} \\ \frac{v_a - v_b}{1} &= \frac{v_b}{0.5} \end{aligned}\right\} \quad \begin{bmatrix} 3 & -1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$v_a = \frac{3}{2}\ \text{V}$$

$$v_b = \frac{1}{2}\ \text{V}$$

3.2 Nodal Analysis

<u>Example 2</u> – circuit with independent current source only



(a) (b)

At node 1,

$$i_1 = i_2 + i_3 \quad \Rightarrow \quad 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2}$$

Multiply by 4 $\quad 20 = v_1 - v_2 + 2v_1$

$$3v_1 - v_2 = 20$$

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3.2 Nodal Analysis

At node 2,

$$i_2 + i_4 = i_1 + i_5 \quad \Rightarrow \quad \frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6}$$

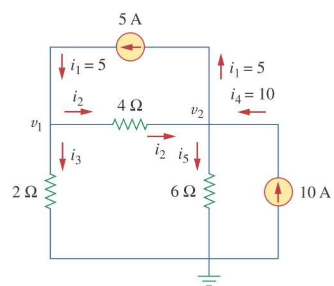
Multiply by 12

$$3v_1 - 3v_2 + 120 = 60 + 2v_2$$

$$-3v_1 + 5v_2 = 60$$

In matrix form

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$$



(b)

The determinant is $\quad \Delta = \begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix} = 15 - 3 = 12$

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3.2 Nodal Analysis

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$$

Using Cramer's rule

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 20 & -1 \\ 60 & 5 \end{vmatrix}}{\Delta} = \frac{100 + 60}{12} = 13.333\text{ V}$$

$$v_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 3 & 20 \\ -3 & 60 \end{vmatrix}}{\Delta} = \frac{180 + 60}{12} = 20\text{ V}$$

$$i_1 = 5\text{ A}, \quad i_2 = \frac{v_1 - v_2}{4} = -1.6668\text{ A}, \quad i_3 = \frac{v_1}{2} = 6.666\text{ A}$$

$$i_4 = 10\text{ A}, \quad i_5 = \frac{v_2}{6} = 3.333\text{ A}$$

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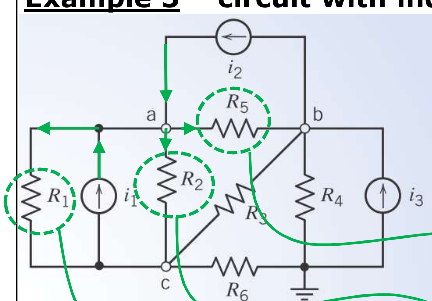
3.2 Nodal Analysis

Example 3 – circuit with independent current source only



$$i_1 = 1\text{ A}, i_2 = 2\text{ A}, i_3 = 3\text{ A}$$

$$R_1 = 5\,\Omega, R_2 = 2\,\Omega, R_3 = 10\,\Omega,$$

$$R_4 = 4\,\Omega, R_5 = 5\,\Omega, R_6 = 2\,\Omega,$$

กำหนดให้กระแส

ไหลเข้าเป็นบวก

ไหลออกเป็นลบ

KCL node a

$$-\left(\frac{v_a - v_c}{R_1}\right) + i_1 - \left(\frac{v_a - v_c}{R_2}\right) + i_2 - \left(\frac{v_a - v_b}{R_5}\right) = 0$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5}\right)v_a - \left(\frac{1}{R_5}\right)v_b - \left(\frac{1}{R_1} + \frac{1}{R_2}\right)v_c = i_1 + i_2$$

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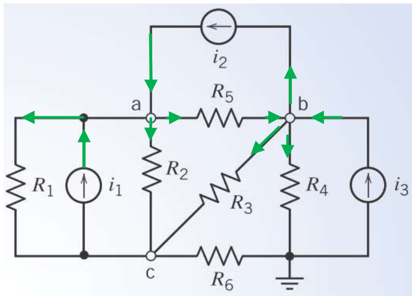


หมายเหตุ: ในการสร้างสมการ KCL ในตัวอย่างข้อนี้ เราต้องกำหนดทิศทางการไหลของกระแสในแต่ละ Branch ให้ชัดเจน และสอดคล้องกันทุก Node

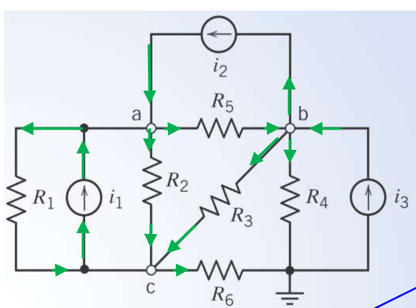
ไหลเข้าเป็นบวก

ไหลออกเป็นลบ

KCL node b

$$-i_2 + \left(\frac{v_a - v_b}{R_5}\right) - \left(\frac{v_b - v_c}{R_3}\right) - \left(\frac{v_b}{R_4}\right) + i_3 = 0$$

$$-\left(\frac{1}{R_5}\right)v_a + \left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right)v_b - \left(\frac{1}{R_3}\right)v_c = i_3 - i_2$$



ไหลเข้าเป็นบวก

ไหลออกเป็นลบ

KCL node c

$$-i_1 + \left(\frac{v_a - v_c}{R_1}\right) + \left(\frac{v_a - v_c}{R_2}\right) + \left(\frac{v_b - v_c}{R_3}\right) - \left(\frac{v_c - 0}{R_6}\right) = 0$$

$$-\left(\frac{1}{R_1}+\frac{1}{R_2}\right)v_a - \left(\frac{1}{R_3}\right)v_b + \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_6}\right)v_c = -i_1$$

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สรุปสมการทั้ง 3 Node

$$\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_5}\right)v_a-\left(\frac{1}{R_5}\right)v_b-\left(\frac{1}{R_1}+\frac{1}{R_2}\right)v_c=i_1+i_2$$

$$-\left(\frac{1}{R_5}\right)v_a+\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right)v_b-\left(\frac{1}{R_3}\right)v_c=i_3-i_2$$

$$-\left(\frac{1}{R_1}+\frac{1}{R_2}\right)v_a-\left(\frac{1}{R_3}\right)v_b+\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_6}\right)v_c=-i_1$$

จะได้

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_5} & -\frac{1}{R_5} & -\frac{1}{R_1}-\frac{1}{R_2} \\ -\frac{1}{R_5} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} & -\frac{1}{R_3} \\ -\frac{1}{R_1}-\frac{1}{R_2} & -\frac{1}{R_3} & \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_6} \end{bmatrix}\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}=\begin{bmatrix} i_1+i_2 \\ i_3-i_2 \\ -i_1 \end{bmatrix}$$

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Cramer's rule : ใช้แก้สมการเชิงเส้น

$$\begin{aligned} a_1x+b_1y+c_1z &= d_1 \\ a_2x+b_2y+c_2z &= d_2 \\ a_3x+b_3y+c_3z &= d_3 \end{aligned} \Rightarrow \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$x=\frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}},\quad y=\frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}},\text{ and } z=\frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$$

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Matrix determinant formula

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \left|\begin{array}{ccc|cc} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{array}\right.$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = aei + bfg + cdh - gec - hfa - idb$$

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แทนค่า $i_1 = 1\,\text{A}, i_2 = 2\,\text{A}, i_3 = 3\,\text{A}$ $\quad R_1 = 5\,\Omega, R_2 = 2\,\Omega, R_3 = 10\,\Omega, R_4 = 4\,\Omega, R_5 = 5\,\Omega, R_6 = 2\,\Omega,$

จะได้

$$\left(\frac{1}{5}+\frac{1}{2}+\frac{1}{5}\right)v_a - \left(\frac{1}{5}\right)v_b - \left(\frac{1}{5}+\frac{1}{2}\right)v_c = 1+2$$

$$-\left(\frac{1}{5}\right)v_a + \left(\frac{1}{10}+\frac{1}{5}+\frac{1}{4}\right)v_b - \left(\frac{1}{10}\right)v_c = -2+3$$

$$-\left(\frac{1}{5}+\frac{1}{2}\right)v_a - \left(\frac{1}{10}\right)v_b + \left(\frac{1}{5}+\frac{1}{2}+\frac{1}{10}+\frac{1}{2}\right)v_c = -1$$

$$0.9v_a - 0.2v_b - 0.7v_c = 3$$
$$-0.2v_a + 0.55v_b - 0.1v_c = 1$$
$$-0.7v_a - 0.1v_b + 1.3v_c = -1$$

In matrix form

$$[A][v] = [i_S] \Rightarrow [v] = [A]^{-1}[i_S] \Rightarrow v = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} 7.1579 \\ 5.0526 \\ 3.4737 \end{bmatrix}$$

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3.2 Nodal Analysis — General Node Equations

$$[G][v]=[i_S] \Rightarrow$$

[G] is a conductance matrix
[v] is the unknown node voltages
[i_s] is the known current sources matrix

ในกรณีที่ไม่มี Dependent source และ Voltage source, Conductance matrix จะเป็นเมทริกซ์สมมาตร (เทียบกับแนวทะแยง)

Current source matrix ประกอบด้วยกระแสที่ไหลเข้าแต่ละ Node (เข้า +, ออก -)

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_5} & -\frac{1}{R_5} & -\frac{1}{R_1}-\frac{1}{R_2} \\ -\frac{1}{R_5} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} & -\frac{1}{R_3} \\ -\frac{1}{R_1}-\frac{1}{R_2} & -\frac{1}{R_3} & \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_6} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} i_1+i_2 \\ i_3-i_2 \\ -i_1 \end{bmatrix}$$

Conductance ที่ต่อกับ Node a

-Conductance ระหว่าง a กับ b

-Conductance ระหว่าง a กับ c

$$\begin{bmatrix} \boxed{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_5}} & \boxed{-\frac{1}{R_5}} & \boxed{-\frac{1}{R_1}-\frac{1}{R_2}} \\ \boxed{-\frac{1}{R_5}} & \boxed{\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}} & \boxed{-\frac{1}{R_3}} \\ \boxed{-\frac{1}{R_1}-\frac{1}{R_2}} & \boxed{-\frac{1}{R_3}} & \boxed{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_6}} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} i_1+i_2 \\ i_3-i_2 \\ -i_1 \end{bmatrix}$$

-Conductance ระหว่าง c กับ a

-Conductance ระหว่าง c กับ b

กระแสเข้าแต่ละ Node

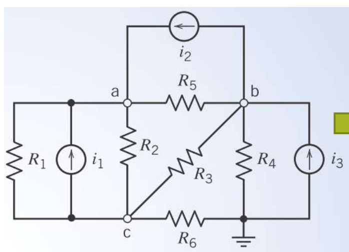
Conductance ที่ต่อกับ Node b

Conductance ที่ต่อกับ Node c

สรุป Conductance matrix
(ในกรณีที่ไม่มี dependent source, voltage source)

1. Element (i,i) คือ Conductance รวมที่ต่อกับ Node i

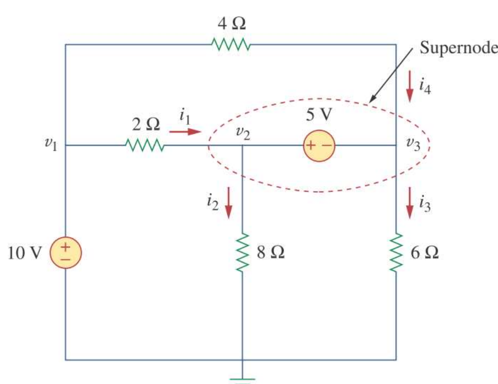
2. Element (i,j) โดย i≠j คือ –Conductance ที่ต่อระหว่าง Node i กับ Node j.



$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_5} & -\frac{1}{R_5} & -\frac{1}{R_1}-\frac{1}{R_2} \\ -\frac{1}{R_5} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} & -\frac{1}{R_3} \\ -\frac{1}{R_1}-\frac{1}{R_2} & -\frac{1}{R_3} & \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_6} \end{bmatrix}$$

3.3 Nodal Analysis with Voltage Source

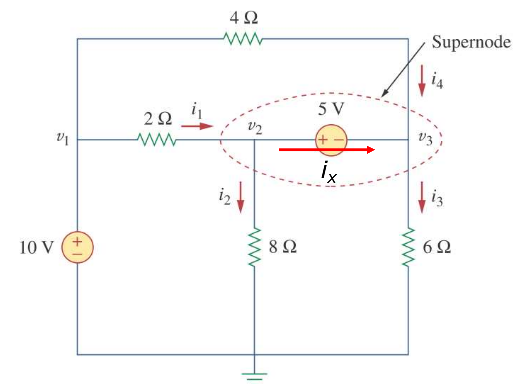
Circuit with independent voltage sources



หลักการ: เราจะยุบ Branch ที่มี Voltage source ทำให้เสมือนว่า Node 2 Node ที่อยู่ปลาย 2 ข้างของ Branch นั้นกลายเป็น Node เดียว เรียกว่า Supernode

A supernode is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.

3.3 Nodal Analysis with Voltage Source



KCL node 2 $\quad i_1 = i_2 + i_x$

KCL node 3 $\quad i_x + i_4 = i_3$

$\therefore i_x = i_3 - i_4$

KCL supernode

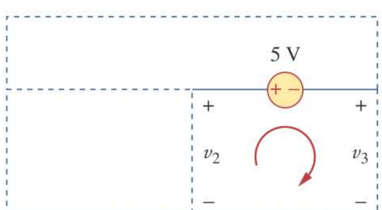
$\Rightarrow \quad i_1 + i_4 = i_2 + i_3$

3.3 Nodal Analysis with Voltage Source

KCL at supernode $\quad i_1 + i_4 = i_2 + i_3$

or
$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2 - 0}{8} + \frac{v_3 - 0}{6}$$

$v_1 = 10$ V จะได้ $\quad \frac{5}{8}v_2 + \frac{5}{12}v_3 = 7.5 \Rightarrow 3v_2 + 2v_3 = 36 \quad$ **(1)**



KVL constraint

$$-v_2 + 5 + v_3 = 0$$

$$v_2 - v_3 = 5 \qquad \textbf{(2)}$$

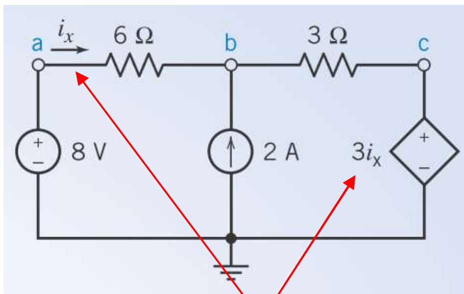
$$\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 36 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}^{-1}\begin{bmatrix} 36 \\ 5 \end{bmatrix} = \begin{bmatrix} 9.2 \\ 4.2 \end{bmatrix}$$

3.3 Nodal Analysis with Voltage Source

Circuit with dependent sources

When a circuit contains a dependent source the controlling current or voltage of that dependent source must be expressed as a function of the node voltages.

Example 4 – circuit with dependent source



i_x = Controlling current

$$i_x = \frac{v_a - v_b}{6}$$

$$v_a = 8\text{ V}$$

$$i_x = \frac{8 - v_b}{6}$$

$$v_c = 3i_x = 3\left(\frac{8 - v_b}{6}\right)$$

$$= 4 - \frac{v_b}{2}$$

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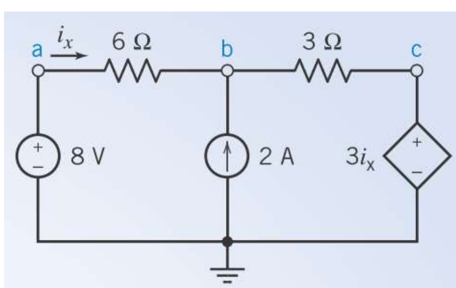
3.3 Nodal Analysis with Voltage Source

KCL at node b



$$\frac{8 - v_b}{6} + 2 = \frac{v_b - v_c}{3}$$

แทนค่า $v_c = 4 - \dfrac{v_b}{2}$

จะได้

$$\frac{8 - v_b}{6} + 2 = \frac{v_b - \left(4 - \frac{v_b}{2}\right)}{3}$$

$$4 - \frac{v_b}{2} + 6 = \frac{3}{2}v_b - 4$$

$$2v_b = 14$$

$$v_b = 7\text{ V}, \quad v_c = 0.5\text{ V}$$

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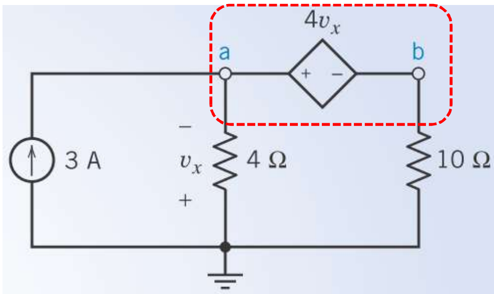
3.3 Nodal Analysis with Voltage Source

Circuit with dependent sources

When a circuit contains a dependent source the controlling current or voltage of that dependent source must be expressed as a function of the node voltages.

Example 5 – circuit with dependent source



$$v_x = -v_a$$

$$v_a - v_b = 4\,v_x = 4(-v_a)$$

$$= -4\,v_a$$

จะได้

$$v_b = 5\,v_a$$

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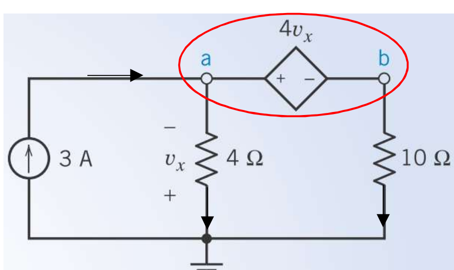
3.3 Nodal Analysis with Voltage Source



KCL supernode

$$3 = \frac{v_a}{4} + \frac{v_b}{10}$$

แทนค่า $v_b = 5\,v_a$

$$3 = \frac{v_a}{4} + \frac{5v_a}{10} = \frac{3}{4}v_a$$

$$v_a = 4\text{ V}$$

$$v_b = 5\,v_a = 20\text{ V}$$

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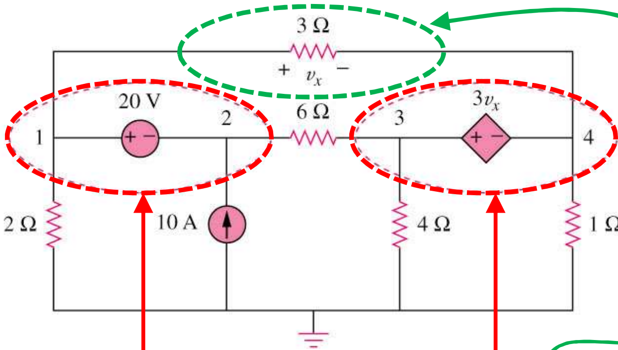
3.3 Nodal Analysis with Voltage Source

<u>Example 6</u> – circuit with two voltage sources

4 Unknowns: v_1, v_2, v_3, v_4



$$v_1 - v_4 = v_x$$

(1) $\boxed{v_1 - v_2 = 20}$

$$v_3 - v_4 = 3v_x = 3(v_1 - v_4)$$

(2) $\boxed{3v_1 - v_3 - 2v_4 = 0}$

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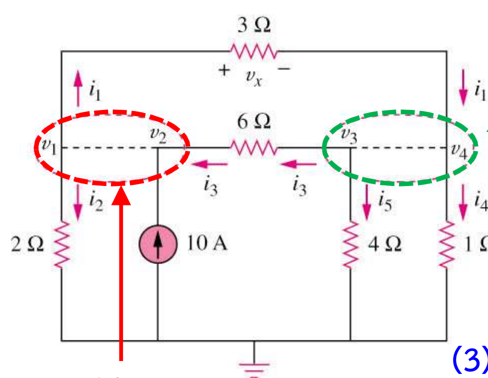


$$i_1 = i_3 + i_4 + i_5$$

$$\frac{v_1 - v_4}{3} = \frac{v_3 - v_2}{6} + \frac{v_4}{1} + \frac{v_3}{4}$$

$$4v_1 - 4v_4 = 5v_3 - 2v_2 + 12v_4$$

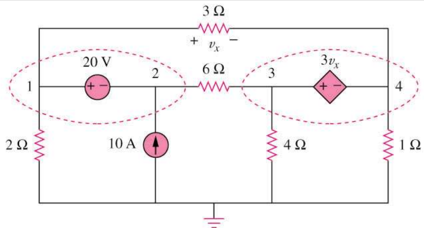
(3) $\boxed{4v_1 + 2v_2 - 5v_3 - 16v_4 = 0}$

$$i_3 + 10 = i_1 + i_2$$

$$\frac{v_3 - v_2}{6} + 10 = \frac{v_1 - v_4}{3} + \frac{v_1}{2}$$

$$v_3 - v_2 + 60 = 5v_1 - 2v_4$$

$\boxed{5v_1 + v_2 - v_3 - 2v_4 = 60}$ (4)



$$v_1 - v_2 = 20$$

$$3v_1 - v_3 - 2v_4 = 0$$

$$4v_1 + 2v_2 - 5v_3 - 16v_4 = 0$$

$$5v_1 + v_2 - v_3 - 2v_4 = 60$$

Unknowns มี 4 ตัวคำนวณยาก ให้กำจัดบางตัวออกไป จะคำนวณง่ายขึ้น ในที่นี้ให้กำจัด v_2 ออกไปโดยแทน $v_2 = v_1 - 20$

$$4v_1 + 2(v_1 - 20) - 5v_3 - 16v_4 = 0 \Rightarrow 6v_1 - 5v_3 - 16v_4 = 40$$

$$5v_1 + (v_1 - 20) - v_3 - 2v_4 = 60 \Rightarrow 6v_1 - v_3 - 2v_4 = 80$$

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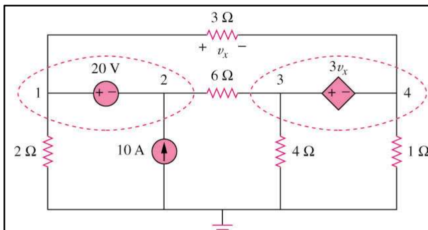

$$3v_1 - v_3 - 2v_4 = 0$$

$$6v_1 - 5v_3 - 16v_4 = 40$$

$$6v_1 - v_3 - 2v_4 = 80$$

$$\begin{bmatrix} 3 & -1 & -2 \\ 6 & -5 & -16 \\ 6 & -1 & -2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 40 \\ 80 \end{bmatrix} \Rightarrow \begin{vmatrix} 3 & -1 & -2 \\ 6 & -5 & -16 \\ 6 & -1 & -2 \end{vmatrix} = -18$$

$$\begin{vmatrix} 0 & -1 & -2 \\ 40 & -5 & -16 \\ 80 & -1 & -2 \end{vmatrix} = -480 \qquad \begin{vmatrix} 3 & 0 & -2 \\ 6 & 40 & -16 \\ 6 & 80 & -2 \end{vmatrix} = -3120 \qquad \begin{vmatrix} 3 & -1 & 0 \\ 6 & -5 & 40 \\ 6 & -1 & 80 \end{vmatrix} = 840$$

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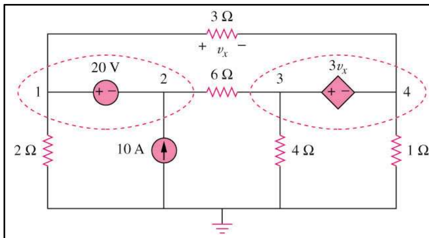

$$\Delta = \begin{vmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \end{vmatrix} = -18,$$

$$\Delta_3 = \begin{vmatrix} 3 & 0 & -2 \\ 6 & 80 & -2 \\ 6 & 40 & -16 \end{vmatrix} = -3120,$$

$$\Delta_1 = \begin{vmatrix} 0 & -1 & -2 \\ 80 & -1 & -2 \\ 40 & -5 & -16 \end{vmatrix} = -480,$$

$$\Delta_4 = \begin{vmatrix} 3 & -1 & 0 \\ 6 & -1 & 80 \\ 6 & -5 & 40 \end{vmatrix} = 840$$

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{-480}{-18} = 26.67\text{ V}, \qquad v_3 = \frac{\Delta_3}{\Delta} = \frac{-3120}{-18} = 173.33\text{ V},$$

$$v_4 = \frac{\Delta_4}{\Delta} = \frac{840}{-18} = -46.67\text{ V} \qquad v_2 = v_1 - 20 = 6.667\text{ V}.$$

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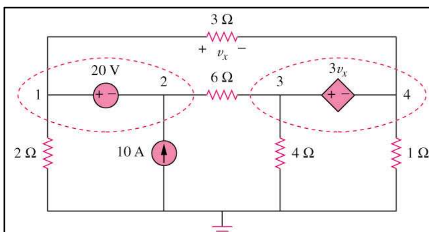

$$v_1 - v_2 = 20$$
$$3v_1 - v_3 - 2v_4 = 0$$
$$4v_1 + 2v_2 - 5v_3 - 16v_4 = 0$$
$$5v_1 + v_2 - v_3 - 2v_4 = 60$$

ถ้าใช้ Computer ก็สามารถแก้สมการ 4 ตัวแปรได้เลย

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 3 & 0 & -1 & -2 \\ 4 & 2 & -5 & -16 \\ 5 & 1 & -1 & -2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \\ 0 \\ 60 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 26.67 \\ 6.67 \\ 173.33 \\ -46.67 \end{bmatrix}$$

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3.4 Mesh Analysis ใช้คำนวณกระแสที่ไหลผ่าน Mesh

1. Mesh analysis provides another general procedure for analyzing circuits using mesh currents as the circuit variables.
2. Nodal analysis applies KCL to find unknown voltages in a given circuit, while mesh analysis applies KVL to find unknown currents.
3. A mesh is a loop which does not contain any other loops within it.
4. A loop is a closed path with no node passed more than once.

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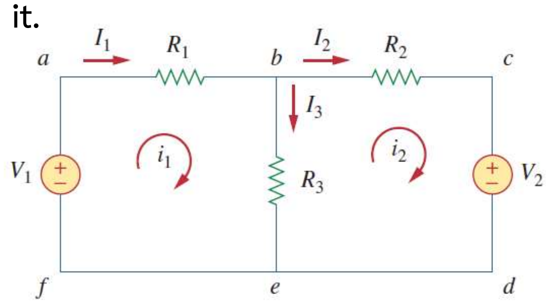
A mesh is a loop which does not contain any other loops within it.



abefa, bcdeb เป็น Mesh

abcdef ไม่ใช่ Mesh เพราะมี Loop abefa อยู่ภายใน

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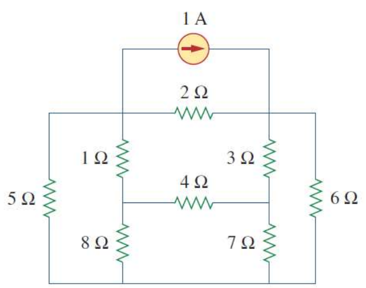
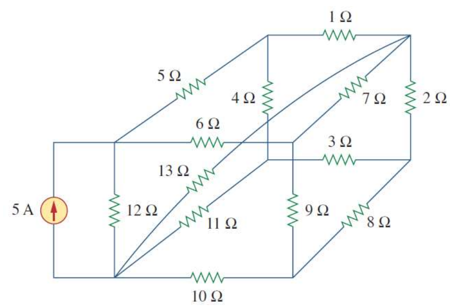
3.4 Mesh Analysis

-Mesh analysis is only applicable to a circuit that is planar.

-A planar circuit is one that can be drawn in a plane with no branches crossing one another.



Planar Circuit



Nonplanar Circuit

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3.4 Mesh Analysis

Steps to determine the mesh currents:

1. Assign mesh currents i_1, i_2, …, in to the n meshes.
   เรานิยมกำหนดให้ Mesh current ไหลในทิศตามเข็มนาฬิกา
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

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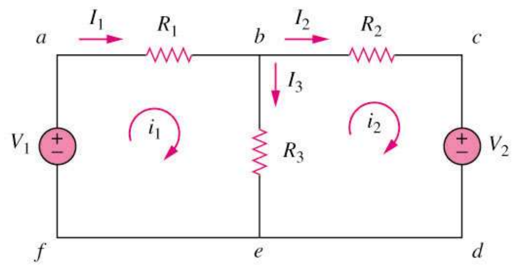
3.4 Mesh Analysis

<u>Example 7</u> – circuit with independent voltage sources



<u>Note</u>:

i_1 and i_2 are mesh current (imaginative, not measurable directly)

I_1, I_2 and I_3 are branch current (real, measurable directly)

$I_1 = i_1$; $I_2 = i_2$; $I_3 = i_1 - i_2$

เราใช้ Mesh analysis หา Mesh current ก่อน แล้วจึงหา Branch current ภายหลัง (จำนวน Unknown ที่เป็น Mesh current จะน้อยกว่า Unknown ที่เป็น Branch current)

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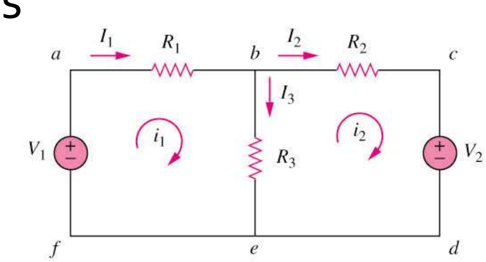
3.4 Mesh Analysis



Mesh 1

$$-V_1 + R_1 i_1 + R_3(i_1 - i_2) = 0$$

$$(R_1 + R_3)i_1 - R_3 i_2 = V_1$$

Mesh 2

$$R_2 i_2 + V_2 + R_3(i_2 - i_1) = 0$$

$$-R_3 i_1 + (R_2 + R_3)i_2 = -V_2$$

In matrix form

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$

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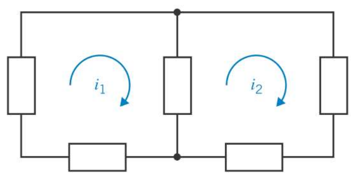
3.4 Mesh Analysis

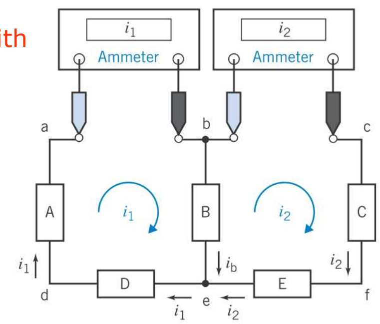
General mesh equations

$$[R][i]=[v_S]$$

[R] is a resistance matrix
[i] is the unknown mesh currents
[v_s] is the known voltage sources matrix

Mesh currents measurement with an ammeter





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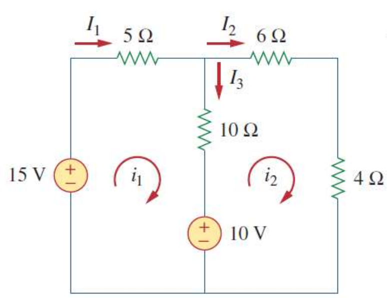
Example 3.5



Mesh 1:

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

$$3i_1 - 2i_2 = 1 \quad \text{(A)}$$

Mesh 2:

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

$$i_1 = 2i_2 - 1 \quad \text{(B)}$$

$$\begin{bmatrix} 3 & -2 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}=\begin{bmatrix} 1 \\ -1 \end{bmatrix} \Rightarrow \left.\begin{matrix} i_2 = 1A \\ i_1 = 1A \end{matrix}\right\} \quad \begin{matrix} I_1 = i_1 = 1A \\ I_2 = i_2 = 1A \\ I_3 = i_1 - i_2 = 0 \end{matrix}$$

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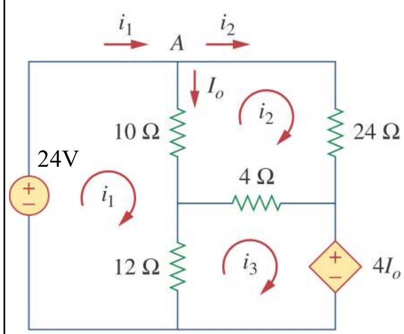
3.4 Mesh Analysis

Example 8 – circuit with dependent voltage source



Mesh 1

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$

$$11i_1 - 5i_2 - 6i_3 = 12 \quad \textbf{(1)}$$

Mesh 2

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

$$-5i_1 + 19i_2 - 2i_3 = 0 \quad \textbf{(2)}$$

Mesh 3 $4I_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$

$$I_o = i_1 - i_2$$

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

$$-i_1 - i_2 + 2i_3 = 0 \quad \textbf{(3)}$$

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3.4 Mesh Analysis

Mesh 1 $11i_1 - 5i_2 - 6i_3 = 12$ **(1)**

Mesh 2 $-5i_1 + 19i_2 - 2i_3 = 0$ **(2)**

Mesh 3 $-i_1 - i_2 + 2i_3 = 0$ **(3)**

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

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3.4 Mesh Analysis

In matrix form

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

Obtain the determinant

$$\Delta = \begin{array}{c|ccc|c} & 11 & -5 & -6 & \\ & -5 & 19 & -2 & \\ & -1 & -1 & 2 & \\ - & 11 & -5 & -6 & + \\ - & -5 & 19 & -2 & + \\ - & & & & + \end{array}$$

$$= 418 - 30 - 10 - 114 - 22 - 50 = 192$$

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3.4 Mesh Analysis

$$\Delta_1 = \begin{array}{c|ccc|c} & 12 & -5 & -6 & \\ & 0 & 19 & -2 & \\ & 0 & -1 & 2 & \\ - & 12 & -5 & -6 & + \\ - & 0 & 19 & -2 & + \\ - & & & & + \end{array} = 456 - 24 = 432$$

$$\Delta_2 = \begin{array}{c|ccc|c} & 11 & 12 & -6 & \\ & -5 & 0 & -2 & \\ & -1 & 0 & 2 & \\ - & 11 & 12 & -6 & + \\ - & -5 & 0 & -2 & + \\ - & & & & + \end{array} = 24 + 120 = 144$$

$$\Delta_3 = \begin{array}{c|ccc|c} & 11 & -5 & 12 & \\ & -5 & 19 & 0 & \\ & -1 & -1 & 0 & \\ - & 11 & -5 & 12 & + \\ - & -5 & 19 & 0 & + \\ - & & & & + \end{array} = 60 + 228 = 288$$

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3.4 Mesh Analysis

Calculate the mesh currents using Cramer's rule

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{432}{192} = 2.25 \text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = \frac{144}{192} = 0.75 \text{ A},$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{288}{192} = 1.5 \text{ A}$$

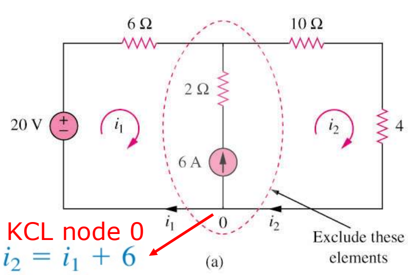
$$I_o = i_1 - i_2 = 1.5 \text{ A}.$$

3.5 Mesh Analysis with Current Source

Circuit with current source

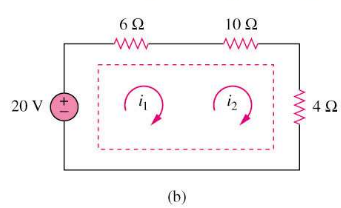
KVL supermesh

$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$



KCL node 0

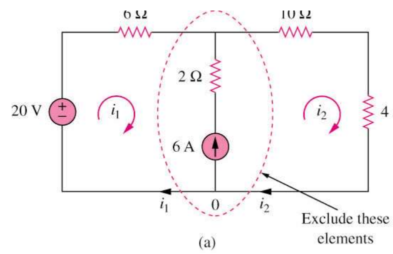
$i_2 = i_1 + 6$



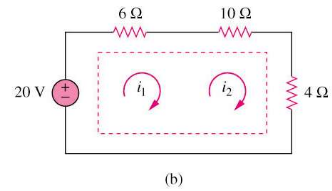
$i_1 = -3.2$ A, $\quad i_2 = 2.8$ A

A **super-mesh** results when two meshes have a (dependent or independent) current source in common as shown in (a). We create a super-mesh by excluding the current source and any elements connected in series with it as shown in (b).

หลักการ: เราจะตัด Branch ที่มี Current source ออกไป ทำให้ Mesh 2 Mesh รวมเป็น Mesh เดียว เรียกว่า Super Mesh



KVL of Super Mesh



$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

$$6i_1 + 14i_2 = 20$$

KCL at Node 0

$$i_2 = i_1 + 6$$

$$\begin{bmatrix} 6 & 14 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 20 \\ -6 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 6 & 14 \\ 1 & -1 \end{bmatrix}^{-1}\begin{bmatrix} 20 \\ -6 \end{bmatrix} = \begin{bmatrix} -3.2 \\ 2.8 \end{bmatrix}$$

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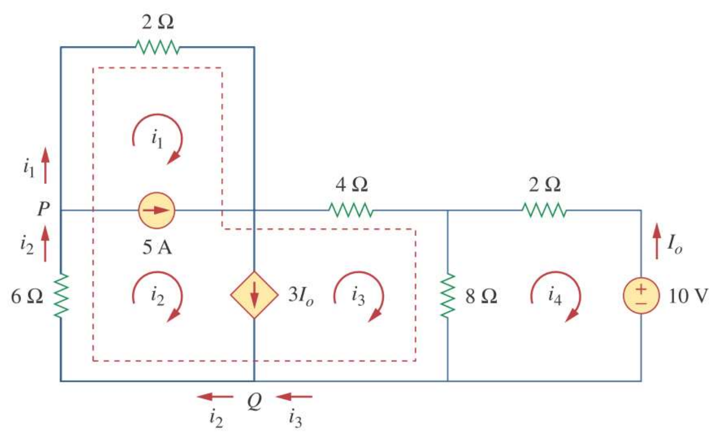
3.5 Mesh Analysis with Current Source

<u>Example 9</u> – circuit with current sources



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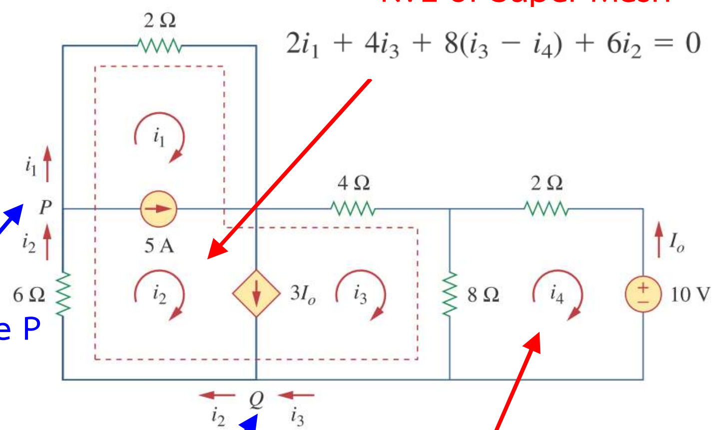
KVL of Super Mesh

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$



KCL of Node P

$$i_2 = i_1 + 5$$

KVL of Mesh 4

$$2i_4 + 8(i_4 - i_3) + 10 = 0$$

KCL of Node Q

$$i_2 = i_3 + 3I_o$$

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KVL supermesh $2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$

$$i_1 + 3i_2 + 6i_3 - 4i_4 = 0 \quad (1)$$

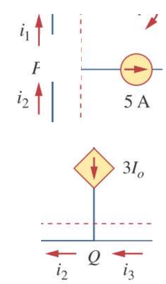
KCL node P $i_2 = i_1 + 5$ (2)

KCL node Q $i_2 = i_3 + 3I_o \leftarrow I_o = -i_4$

$$i_2 = i_3 - 3i_4 \quad (3)$$

KVL mesh 4 $2i_4 + 8(i_4 - i_3) + 10 = 0$

$$5i_4 - 4i_3 = -5 \quad (4)$$



$$\begin{bmatrix} 1 & 3 & 6 & -4 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & -4 & 5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 0 \\ -5 \end{bmatrix} \Rightarrow$$

$i_1 = -7.5$ A, $i_2 = -2.5$ A,
$i_3 = 3.93$ A, $i_4 = 2.143$ A

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MATLAB, SciLAB code

```
a = [ 1   3   6 -4;
     -1   1   0   0;
      0   1  -1   3;
      0   0  -4   5]
b = [ 0
      5
      0
     -5]

  I = inv(a)*b
```

$\Rightarrow$ $i_1 = -7.5$ A, $i_2 = -2.5$ A, $i_3 = 3.93$ A, $i_4 = 2.143$ A

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3.6 Nodal and Mesh Analysis with Inspection

General node equations

$$\mathbf{Gv} = \mathbf{i}$$

$$\begin{bmatrix} G_{11} & G_{12} & \dots & G_{1N} \\ G_{21} & G_{22} & \dots & G_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ G_{N1} & G_{N2} & \dots & G_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix}$$

G_{kk} = Sum of the conductances connected to node k

$G_{kj} = G_{jk}$ = Negative of the sum of the conductances directly connecting nodes k and j, $k \neq j$

v_k = Unknown voltage at node k

i_k = Sum of all independent current sources directly connected to node k, with currents entering the node treated as positive

** ใช้สำหรับวงจรที่มีแต่ Independent Current Source เท่านั้น **

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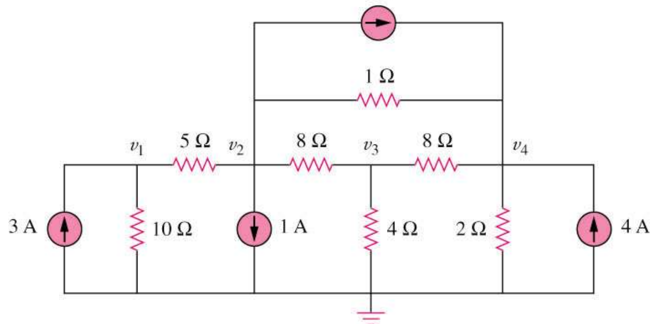
<u>Example 10</u> – By inspection, write the nodal voltage equations for the circuit



$$G_{11} = \frac{1}{5} + \frac{1}{10} = 0.3, \qquad G_{22} = \frac{1}{5} + \frac{1}{8} + \frac{1}{1} = 1.325$$

$$G_{33} = \frac{1}{8} + \frac{1}{8} + \frac{1}{4} = 0.5, \qquad G_{44} = \frac{1}{8} + \frac{1}{2} + \frac{1}{1} = 1.625$$

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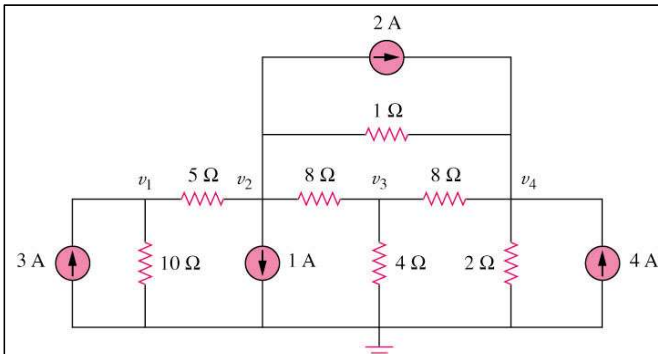

$$G_{12} = -\frac{1}{5} = -0.2,$$

$$G_{21} = -0.2,$$

$$G_{13} = G_{14} = 0$$

$$G_{31} = 0, \quad G_{41} = 0,$$

$$G_{23} = -\frac{1}{8} = -0.125,$$

$$G_{32} = -0.125,$$

$$G_{24} = -\frac{1}{1} = -1 \qquad G_{42} = -1, \qquad G_{34} = -\frac{1}{8} = -0.125 \qquad G_{43} = -0.125$$

$$i_1 = 3, \qquad i_2 = -1 - 2 = -3, \qquad i_3 = 0, \qquad i_4 = 2 + 4 = 6$$

$$\begin{bmatrix} 0.3 & -0.2 & 0 & 0 \\ -0.2 & 1.325 & -0.125 & -1 \\ 0 & -0.125 & 0.5 & -0.125 \\ 0 & -1 & -0.125 & 1.625 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 0 \\ 6 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 13.89 \\ 5.84 \\ 3.35 \\ 7.55 \end{bmatrix}$$

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MATLAB, SciLAB code

```
a = [ 0.3  -0.2    0.0     0.0;
     -0.2   1.325 -0.125  -1.0;
      0.0  -0.125  0.5    -0.125;
      0.0  -1.0   -0.125   1.625;]
b = [3
    -3
    0
    6]
v = inv(a)*b
```

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 13.89 \\ 5.84 \\ 3.35 \\ 7.55 \end{bmatrix}$$

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3.6 Nodal and Mesh Analysis with Inspection

General mesh equations

$$\mathbf{Ri} = \mathbf{v}$$

$$\begin{bmatrix} R_{11} & R_{12} & \dots & R_{1N} \\ R_{21} & R_{22} & \dots & R_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ R_{N1} & R_{N2} & \dots & R_{NN} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}$$

R_{kk} = Sum of the resistances in mesh k

$R_{kj} = R_{jk}$ = Negative of the sum of the resistances in common with meshes k and j, $k \neq j$

i_k = Unknown mesh current for mesh k in the clockwise direction

v_k = Sum taken clockwise of all independent voltage sources in mesh k, with voltage rise treated as positive

** ใช้สำหรับวงจรที่มีแต่ Independent Voltage Source เท่านั้น **

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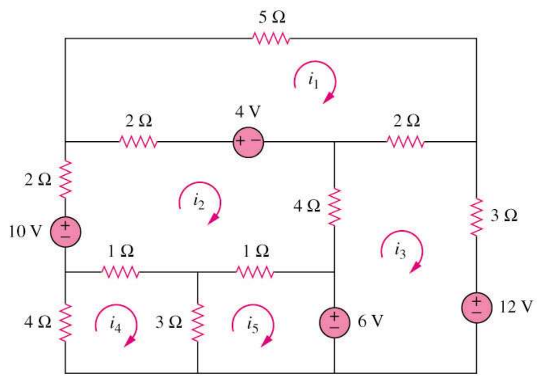
3.6 Nodal and Mesh Analysis with Inspection

<u>Example 11</u> – By inspection, write the mesh-current equations for the circuit



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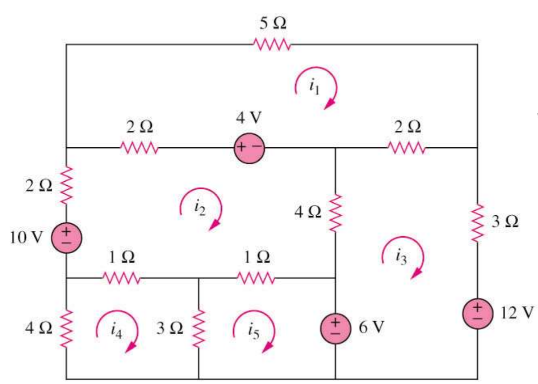

$$R_{11} = 5 + 2 + 2 = 9,$$

$$R_{22} = 2 + 4 + 1 + 1 + 2 = 10,$$

$$R_{33} = 2 + 3 + 4 = 9,$$

$$R_{44} = 1 + 3 + 4 = 8,$$

$$R_{55} = 1 + 3 = 4$$

$$R_{12} = -2, \quad R_{13} = -2, \quad R_{14} = 0 = R_{15},$$

$$R_{21} = -2, \quad R_{23} = -4, \quad R_{24} = -1, \quad R_{25} = -1,$$

$$R_{31} = -2, \quad R_{32} = -4, \quad R_{34} = 0 = R_{35},$$

$$R_{41} = 0, \quad R_{42} = -1, \quad R_{43} = 0, \quad R_{45} = -3,$$

$$R_{51} = 0, \quad R_{52} = -1, \quad R_{53} = 0, \quad R_{54} = -3$$

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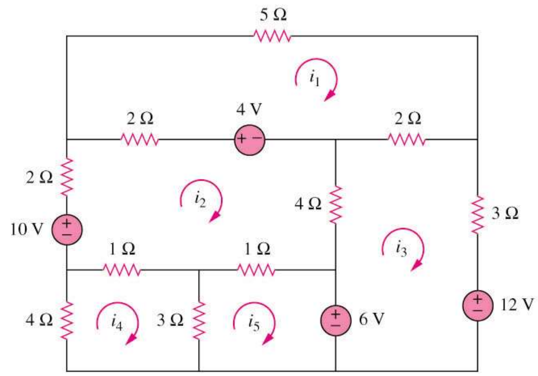

$v_1 = 4,$

$v_2 = 10 - 4 = 6,$

$v_3 = -12 + 6 = -6,$

$v_4 = 0, \quad v_5 = -6$

$$\begin{bmatrix} 9 & -2 & -2 & 0 & 0 \\ -2 & 10 & -4 & -1 & -1 \\ -2 & -4 & 9 & 0 & 0 \\ 0 & -1 & 0 & 8 & -3 \\ 0 & -1 & 0 & -3 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ -6 \\ 0 \\ -6 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 0.383 \\ 0.211 \\ -0.488 \\ -0.718 \\ -1.986 \end{bmatrix}$$

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MATLAB, SciLAB code

```
R=[ 9 -2  -2  0  0;
   -2  10 -4 -1 -1;
   -2 -4   9  0  0;
    0 -1   0  8 -3;
    0 -1   0 -3  4;]
V = [ 4
      6
     -6
      0
     -6]
  I = inv(R)*V
```

$$\Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 0.383 \\ 0.211 \\ -0.488 \\ -0.718 \\ -1.986 \end{bmatrix}$$

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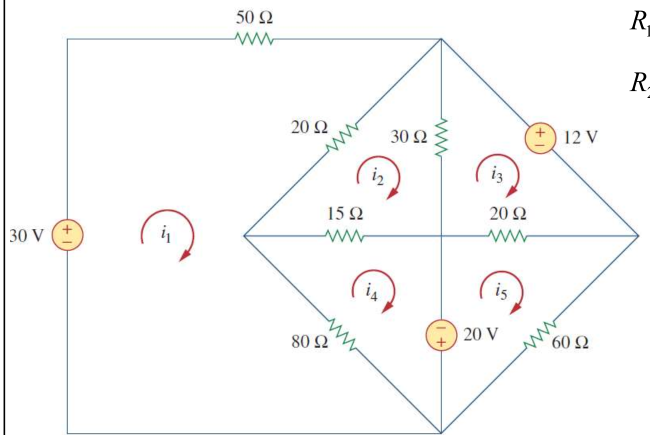
Practice Problem 3.9

By inspection, obtain the mesh-current equations for the circuit in Fig. 3.30.



$R_{11} = 50 + 20 + 80 = 150$

$R_{22} = 20 + 30 + 15 = 65$

$R_{33} = 20 + 30 = 50$

$R_{44} = 15 + 80 = 95$

$R_{55} = 20 + 60 = 80$

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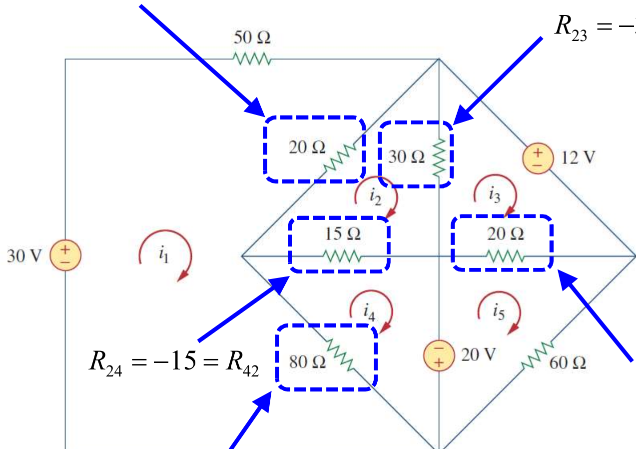
$R_{12} = -20 = R_{21}$

$R_{23} = -30 = R_{32}$



$R_{24} = -15 = R_{42}$

$R_{35} = -20 = R_{53}$

$R_{14} = -80 = R_{41}$

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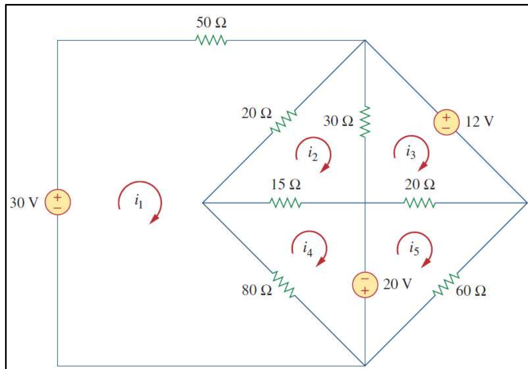

$V_1 = 30$

$V_2 = 0$

$V_3 = -12$

$V_4 = 20$

$V_5 = -20$

$$\begin{bmatrix} 150 & -20 & 0 & -80 & 0 \\ -20 & 65 & -30 & -15 & 0 \\ 0 & -30 & 50 & 0 & -20 \\ -80 & -15 & 0 & 95 & 0 \\ 0 & 0 & -20 & 0 & 80 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 30 \\ 0 \\ -12 \\ 20 \\ -20 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 0.704 \\ 0.347 \\ -0.146 \\ 0.858 \\ -0.287 \end{bmatrix}$$

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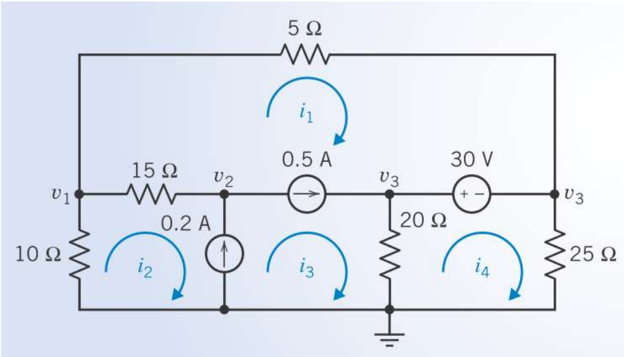
3.7 Nodal versus Mesh Analysis

To select the method that results in the smaller number of equations. For example:

1. Choose nodal analysis for circuit with fewer nodes than meshes.
   *Choose mesh analysis for circuit with fewer meshes than nodes.
   *Networks that contain many series connected elements, voltage sources, or supermeshes are more suitable for mesh analysis.
   *Networks with parallel-connected elements, current sources, or supernodes are more suitable for nodal analysis.
2. If node voltages are required, it may be expedient to apply nodal analysis. If branch or mesh currents are required, it may be better to use mesh analysis.

3.8 Circuit analysis with circuit simulator programs

1. Use schematic editor to draw your circuit.



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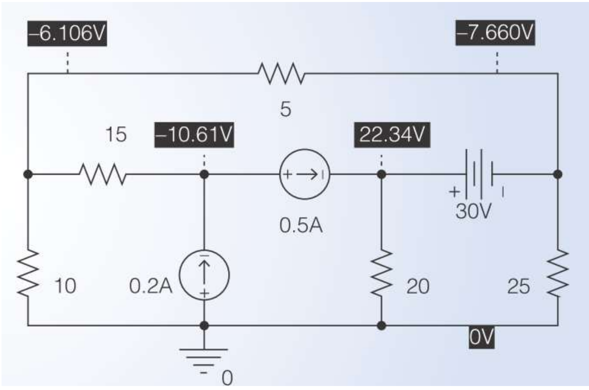
3.8 Circuit analysis with circuit simulator programs

2. Run simulation to obtain results.



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