Heat Diffusion Programming Task

Lavina Chotrani
Department of Mathematics
BITS-Pilani
Pilani, India
f2016194@pilani.bits-pilani.ac.in

***Abstract*—Solve the unsteady heat diffusion equation on a square domain [0,1] x [0,1] with an initial temperature distribution $T(t = 0, x, y) = 40\,C, if\ (x-0.5)^2 + (y-0.5)^2 < 0.2;\ 20\,C\ otherwise.$ The boundaries are maintained at 20 C all the time. First choose a discretization scheme (FD or FV or FE) and write the discrete heat equation. In case of FD, identify the order of the scheme, FV identify the control volume and for if using FE, write the weak form. Thereafter code the above in C++, compile and plot the transient response at time t=5s, 20s, 50s and 7200s.**

I. Introduction

The unsteady heat diffusion equation on a square domain [0,1]x[0,1] is given by

$$\frac{1}{\alpha} * \frac{\partial T}{\partial \tau} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}$$
$$\text{x} \in [0,1], y \in [0,1]$$

The Initial temperature distribution is defined by

$$T(t=0,x,y) = 40\,C, if\ (x-0.5)^2 + (y-0.5)^2 < 0.2;$$
$$20\ \text{C otherwise.}$$

II. Finite Difference Discretization

Let Δx be the finite difference between two points' x co-ordinates, Δy be the finite difference between two points' y co-ordinates and Δt be the time difference between two points. So,

$$x_1 = 0,\ x_{n+1} = 1,\ n \times \Delta x = 1$$
$$\text{y}_1 = 0,\ y_{m+1} = 1,\ m \times \Delta y = 1$$

Now, applying Finite Difference Discretization we get

$$\frac{T^{k+1}_{i,j} - T^k_{i,j}}{\Delta t} = \alpha \times [\frac{T^k_{i-1,j} - 2\times T^k_{i,j} + T^k_{i+1,j}}{\Delta x^2} + \frac{T^k_{i,j-1} - 2\times T^k_{i,j} + T^k_{i,j+1}}{\Delta y^2}]$$

For this problem, I have assumed $\Delta x = \Delta y = h$, so the equation becomes

$$\frac{T^{k+1}_{i,j} - T^k_{i,j}}{\Delta t} = \alpha \times [\frac{T^k_{i,j-1} + T^k_{i-1,j} - 4\times T^k_{i,j} + T^k_{i+1,j} + T^k_{i,j+1}}{h^2}]$$

From the above equation we get the discretized equation as

$$T^k_{i,j} = T^k_{i,j} \times (1 - \frac{4\alpha\Delta t}{h^2}) + \frac{\alpha\Delta t}{h^2}(T^k_{i,j-1} + T^k_{i,j+1} + T^k_{i-1,j} + T^k_{i+1,j})$$

From this, we get the condition for Δt as

$$1 - \frac{4\alpha\Delta t}{h^2} \geq 0$$
$$\Delta t \leq \frac{h^2}{4\alpha}$$

III. Results and Graphs

For calculative purposes, I have assumed the thermal diffusivity constant value as that of air. So till now, the assumptions taken are:

$$\Delta x = \Delta y = h$$
$$\alpha = 1.9 \times 10^{-5} m^2/sec$$

After putting everything to code, we get the transient analysis for the unsteady heat diffusion equation and the following are the graphs at few such instances.

A. Time: 2 secs

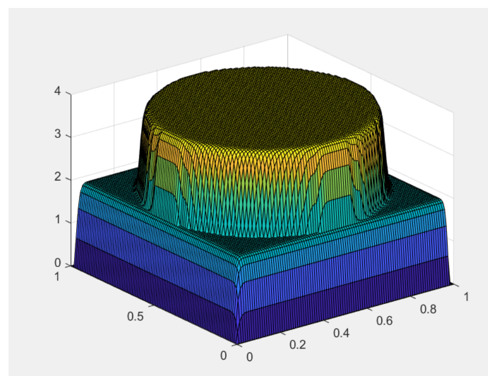

Fig. 1. Transient Analysis at T=2 secs

B. Time: 20 secs

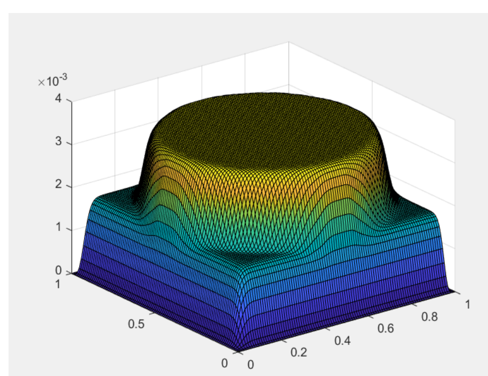

Fig. 2. Transient Analysis at T=20 secs

C. Time: 50 secs

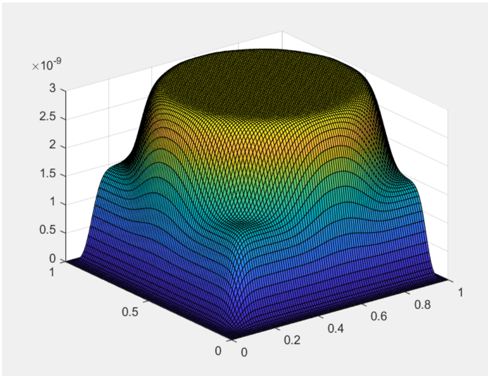

Fig. 3. Transient Analysis at T=50 secs

D. Time: 7200 secs

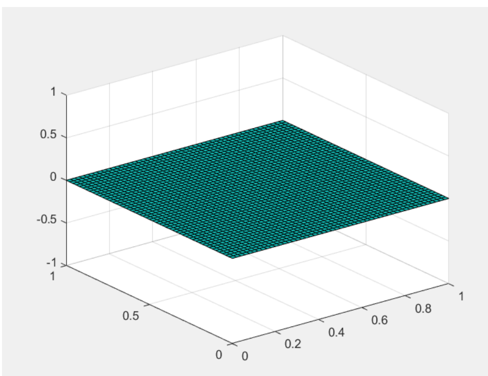

Fig. 4. Transient Analysis at T=7200 secs

Appendix A

Full Code available at: GitHub Profile