

1. Please write a program based on Berlekamp–Massey algorithm to find the shortest linear feedback shift register (LFSR) for the given sequence down below.

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1 def Berlekamp_Massey_algorithm(sequence):
2     N = len(sequence)
3     s = sequence[:]
4     for k in range(N):
5         if s[k] == 1:
6             break
7     f = set([k + 1, 0])
8     l = k + 1
9     g = set([0])
10    a = k
11    b = 0
12    for n in range(k + 1, N):
13        d = 0
14        for ele in f:
15            d ^= s[ele + n - 1]
16        if d == 0:
17            b += 1
18        else:
19            if 2 * l > n:
20                f ^= set([a - b + ele for ele in g])
21                b += 1
22            else:
23                temp = f.copy()
24                f = set([b - a + ele for ele in f]) ^ g
25                l = n + 1 - l
26                g = temp
27                a = b
28                b = n - l + 1
29
30    def print_poly(polynomial):
31        result = ''
32        lis = sorted(polynomial, reverse=True)
33        for i in lis:
34            if i == 0:
35                result += '1'
36            else:
37                result += 'x^%s' % str(i)
38            if i != lis[-1]:
39                result += ' + '
40        return result
41    return (print_poly(f), l)
42
43 if __name__ == '__main__':
44     seq = (0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 0, 1, 0,
45           0, 0, 1, 1, 1, 1, 0, 0, 1, 0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 1,
46           0, 1, 0, 0, 1, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 1, 0,
47           0, 1, 0, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 1, 1, 1, 0, 1, 1, 0,
48           0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 1)

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45 (poly, span) = Berlekamp_Massey_algorithm(seq)
46
47 print('The input sequence is %s.' % str(seq))
48 print('Its characteristic polynomial is (%s),' % poly,)
49 print('and linear span is %d.' % span)

```

Output:

```

1 The input sequence is (0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0,
  1, 0, 1, 0, 0, 0, 1, 1, 1, 1, 0, 0, 1, 0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 1,
  1, 0, 1, 0, 1, 0, 0, 1, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0,
  1, 0, 0, 1, 0, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 1, 1, 1, 0, 1, 1,
  0, 0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 1).
2 Its characteristic polynomial is (x^7 + x^1 + 1),
3 and linear span is 7.

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2. Find the sequence generation rule of 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610...

Sequence generation rule:

$$\begin{cases} a_{n+2} = a_{n+1} + a_n \\ a_1 = 1 \\ a_0 = 0 \end{cases}$$

3. Use Berlekamp–Massey algorithm to find out the sequence rule of 0, 1, 1, 2, 3, 5, 8, 13, 21, 34...

Sequence rule:  $x^2 + x$