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Quantum Clustering for Image Segmentation

A Comprehensive Review

Group 10

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Specialty: Master MIV – 2025/2026

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December 20, 2025

Abstract

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This report provides a comprehensive review of quantum clustering techniques applied to image segmentation. We explore the fundamental principles of quantum computing, examine how quantum algorithms can enhance traditional clustering methods, and discuss their application to the challenging task of image segmentation. The review covers both theoretical foundations and practical implementations, highlighting the potential advantages of quantum approaches over classical methods.

Keywords: Quantum Computing, Clustering, Image Segmentation, Quantum Machine Learning, NISQ Algorithms

Conventions Used in This Report

This report uses the following typographical conventions and special callout boxes to highlight important information:

Note

A **Note** box (blue) contains useful background information, additional context, or interesting observations that supplement the main text. Notes provide helpful details that are relevant but not critical to understanding the core material.

Tip

A **Tip** box (teal) offers practical advice, best practices, or suggestions that can help you better understand or apply the concepts being discussed. Tips often highlight shortcuts or insights gained from experience.

Warning

A **Warning** box (orange) indicates important caveats, potential pitfalls, or common mistakes to avoid. Pay attention to warnings to prevent misunderstandings or errors in applying the discussed methods.

Caution

A **Caution** box (red) signals critical information that could lead to serious problems if ignored. Cautions highlight significant limitations, dangerous assumptions, or fundamental constraints.

Definition

A **Definition** box (gray) provides formal definitions of key terms, mathematical concepts, or technical vocabulary. These boxes establish precise meanings for important terminology used throughout the report.

Key Concept

A **Key Concept** box (orange accent) highlights fundamental ideas, core principles, or essential takeaways that are central to understanding the material. These boxes summarize the most important points.

Example

An **Example** box (light gray) presents concrete illustrations, worked problems, or practical demonstrations of the concepts being explained. Examples help bridge theory and application.

Additional conventions:

- *Italics* are used for emphasis and for introducing new terms.
- **Bold** is used for important concepts and terminology.
- `Monospace font` is used for code, algorithms, and technical notation.
- Mathematical expressions use standard notation, with quantum states written in Dirac notation (e.g., $|\psi\rangle$, $\langle\phi|$, $\langle\phi|\psi|\phi|\psi\rangle$).

Contents

Quantum Optimization Method for Geometric Constrained Image Segmentation Nam H. Le, Milan Sonka, Fatima Toor Department of Electrical and Computer Engineering
University of Iowa, Iowa City, IA 52242 December 20, 2025

Quantum image processing is an emerging field attracting attention from both quantum computing and image processing communities. We propose a novel method combining a graph-theoretic approach for optimal surface segmentation with hybrid quantum-classical optimization of the problem-directed graph. Surface segmentation is modeled classically as a graph partitioning problem with smoothness constraints to control surface variation for realistic segmentation. The problem-specific graph characteristics are embedded in a quadratic objective function whose minimum corresponds to the ground state energy of an equivalent Ising Hamiltonian. This work explores the use of quantum processors in image segmentation problems, with important applications in medical image analysis. We present a theoretical basis for the quantum implementation of LOGISMOS and simulation results on simple images using the Quantum Approximate Optimization Algorithm (QAOA). The proposed approach can solve geometric-constrained surface segmentation problems optimally with the capability of locating multiple minimum points corresponding to globally minimal solutions.

quantum computing, quantum algorithm, combinatorial optimization, image segmentation, graph theory, QAOA

0.1 Introduction

Image segmentation, particularly in medical imaging, involves partitioning an image into meaningful anatomical regions. The LOGISMOS framework (Layered Optimal Graph Image Segmentation of Multiple Objects and Surfaces) [**? ? ?**] reformulates surface segmentation as finding optimal boundaries subject to geometric constraints. Traditional algorithms solve this via maximum flow/minimum cut on directed graphs using classical optimization methods, but may miss alternative optimal solutions due to their deterministic nature.

Quantum computers offer potential advantages for combinatorial optimization problems through superposition (evaluating multiple solutions simultaneously), entanglement (correlated exploration of solution spaces), and quantum tunneling (escaping local minima more effectively). The Quantum Approximate Optimization Algorithm (QAOA) [**?**] provides a hybrid quantum-classical approach to approximate solutions of NP-hard problems by encoding them as ground state problems of Ising Hamiltonians.

This work introduces QuantumLOGISMOS, which: (1) maps LOGISMOS graph constraints to Quadratic Unconstrained Binary Optimization (QUBO) formulation, (2) implements quantum optimization via QAOA with classical parameter tuning, (3) demonstrates the method on synthetic 2D and 3D images, and (4) shows quantum advantage in finding multiple optimal segmentation solutions.

0.2 Classical LOGISMOS Framework

0.2.1 Mathematical Formulation

Given an image $\mathcal{I}$ with spatial dimensions (X, Y, Z), we represent it as a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where each pixel (2D) or voxel (3D) corresponds to a node $v \in \mathcal{V}$, organized into columns along a specific direction.

A surface $\mathcal{S}$ is defined by a function:

$$s : \text{Column} \to \text{Node}$$

where $s(x) = k$ indicates node k in column x belongs to the surface.

Each node (x, k) has an associated cost:

$$c_s(x, k) = -\log P(\text{node is on surface} \mid \text{image features})$$

Lower cost indicates higher likelihood of being on the desired surface.

The optimization objective is to find the surface minimizing total cost:

$$\hat{s} = \arg\min_s \sum_x c_s(x, s(x))$$

0.2.2 Graph Construction

Instead of directly minimizing costs, we transform to terminal weights:

$$w_s(x, k) = \begin{cases} -1 & \text{if } k = 1 \\ c_s(x, k) - c_s(x, k-1) & \text{otherwise} \end{cases}$$

For a closed set S (nodes below the surface):

$$W_s = \sum_x \sum_{k \in S_x} w_s(x, k) = \sum_x c_s(x, s(x)) + \text{constant}$$

Thus minimizing W_s is equivalent to minimizing the original cost function.

0.2.3 Edge Constraints

Three edge types enforce geometric constraints:

1. **Intra-column edges ($\mathcal{E}_{\text{intra}}$)**: $\forall x, \forall k > 1$: edge $(x, k) \to (x, k-1)$ with capacity ∞

   Ensures exactly one cut per column.

2. **Inter-column edges ($\mathcal{E}_{\text{inter}}$)**: Given smoothness parameter δ: $\forall$adjacent x, x' : edges $(x, k) \to (x', \max(1, k - \delta))$ with capacity ∞

   Enforces $|s(x) - s(x')| \leq \delta$.

3. **Terminal edges ($\mathcal{E}_W$)**: For nodes with $w_s(v) < 0$: edge $s \to v$ with capacity $|w_s(v)|$

   For nodes with $w_s(v) > 0$: edge $v \to t$ with capacity $|w_s(v)|$

0.2.4 Minimum Cut Reformulation

The optimal surface corresponds to the minimum s-t cut partitioning nodes into source set S (containing s) and sink set T (containing t). The cut capacity equals the total terminal weight of the corresponding closed set.

0.3 Quantum Formulation

0.3.1 QUBO Conversion

For each node i, define binary variable $x_i \in \{0, 1\}$ where:

- $x_i = 0 \rightarrow$ node in source set S
- $x_i = 1 \rightarrow$ node in sink set T

For directed edge $i \rightarrow j$ with capacity w_{ij}:

$$F_{(i,j)}(x_i, x_j) = x_j - x_i x_j$$

This equals w_{ij} if the edge is cut ($x_i = 0, x_j = 1$), and 0 otherwise.
To enforce $x_s = 0, x_t = 1$:

$$F_{(s,t)}(x_s, x_t) = x_s x_t - x_s$$

Complete QUBO objective:

$$F_C(\mathbf{x}) = \sum_{(i,j) \in \mathcal{E}} w_{ij}(x_j - x_i x_j) + \varepsilon(x_s x_t - x_s)$$

where $\varepsilon = 1 + \sum_{(i,j) \in \mathcal{E}} w_{ij}$ ensures valid cuts have lower energy.

0.3.2 Matrix Formulation

$$F_C(\mathbf{x}) = \mathbf{x}^T \mathbf{Q} \mathbf{x}$$

where $\mathbf{Q}$ is symmetric with:

$$Q_{ii} = \sum_{j: i \rightarrow j} w_{ij}, \quad Q_{ij} = -\frac{w_{ij}}{2} \text{ for } i \neq j \text{ with edge } i \rightarrow j$$

0.3.3 Ising Hamiltonian Mapping

Binary variables map to qubit states via:

$$x_i = \frac{1 - Z_i}{2}$$

where Z_i is the Pauli-Z operator on qubit i.
Problem Hamiltonian:

$$H_C = \sum_{i,j} Q_{ij} \frac{1 - Z_i}{2} \frac{1 - Z_j}{2}$$

Expanding to standard Ising form:

$$H_C = \text{constant} + \sum_i h_i Z_i + \sum_{i<j} J_{ij} Z_i Z_j$$

where:

$$h_i = -\frac{1}{4} \sum_j (Q_{ij} + Q_{ji}), \quad J_{ij} = \frac{1}{4} Q_{ij} \ (i \neq j)$$

Ground state energy E_0 of H_C satisfies:

$$E_0 = \min_{\mathbf{x}} F_C(\mathbf{x})$$

0.4 Quantum Optimization via QAOA

0.4.1 QAOA Circuit Structure

Initial state: uniform superposition

$$|\psi_0\rangle = |+\rangle^{\otimes n} = \frac{1}{\sqrt{2^n}} \sum_{z \in \{0,1\}^n} |z\rangle$$

Problem unitary: $U_C(\gamma) = e^{-i\gamma H_C}$
Mixer unitary: $U_M(\beta) = e^{-i\beta \sum_i X_i}$
For p layers with parameters $\gamma = (\gamma_1, ..., \gamma_p)$, $\beta = (\beta_1, ..., \beta_p)$:

$$|\psi(\gamma, \beta)\rangle = \prod_{k=1}^{p} U_M(\beta_k) U_C(\gamma_k) |\psi_0\rangle$$

0.4.2 Hybrid Optimization

Energy estimation:

$$E(\gamma, \beta) = \langle \psi(\gamma, \beta) | H_C | \psi(\gamma, \beta) \rangle$$

Classical optimization uses Simultaneous Perturbation Stochastic Approximation (SPSA) [**?**]:

Algorithm 1 Hybrid Quantum-Classical Optimization

1: Initialize parameters γ, β
2: **while** not converged **do**
3: Prepare $|\psi(\gamma, \beta)\rangle$ on quantum processor/simulator
4: Measure to estimate $E(\gamma, \beta)$
5: Compute gradients via finite differences
6: Update γ, β using SPSA to minimize E
7: **end while**
8: Measure final state to obtain optimal bitstring $\mathbf{x}^*$
9: Decode $\mathbf{x}^*$ to surface nodes

0.5 Experimental Implementation

0.5.1 Simulation Setup

Software: Python 3.10.11, Qiskit 0.43.1, NetworkX 2.8.4, Qiskit-optimization 0.5.0
Parameters:

- QAOA depth p: 2 to 100 layers
- SPSA optimizer: 250 maximum iterations
- Smoothness constraint: $\delta = 2$
- Quantum measurements: 1024 shots per energy evaluation

0.5.2 Test Cases

Case 1: 2D Image (5×4 pixels)

- Graph: 3 columns $\times$ 5 nodes
- Total qubits: 15 (nodes) + 2 (source/sink) = 17
- Predefined terminal weights with negative regions

Case 2: 3D Image (3×3×2 voxels)

- Graph: 3 columns $\times$ 3 nodes/slice $\times$ 2 slices
- Total qubits: 18 (nodes) + 2 = 20
- Two distinct negative-weighted regions

0.5.3 Results

2D Case

- Optimal energy: $F = -238$
- Solution bitstring: $|0001100011001110\rangle$
- Source set: $\{q_4, q_5, q_9, q_{10}, q_{13}, q_{14}, q_{15}, q_s\}$
- Optimal surface: $\{q_4, q_9, q_{13}\}$ (one node per column)
- Maximum flow: 3 (matches classical solution)

3D Case (Key Finding)

QAOA found **two distinct optimal solutions** with identical energy $F = -162$, while classical preflow-push algorithm [**?**] found only one.
Solution A (found at QAOA depths $p = 2, 3, 4, 6$): Source set: $\{2, 3, 5, 6, 8, 9, 12, 14, 15, 18, 19\}$
Solution B (found at $p = 5, 100$): Source set: $\{2, 3, 5, 6, 8, 9, 10, 11, 12, 14, 15, 18, 19\}$

Table 1: Runtime vs QAOA Depth (p)

p	Time (s)	Solution	Energy	Success Prob.
2	35,570	A	-162	0.78
3	27,323	A	-162	0.81
4	34,752	A	-162	0.79
5	43,025	B	-162	0.82
6	29,098	A	-162	0.80
100	123,901	B	-162	0.83

0.6 Conclusion

The QuantumLOGISMOS framework successfully demonstrates:

1. Complete mathematical mapping from geometric-constrained image segmentation to quantum optimization
2. Feasible implementation via QAOA on quantum simulators
3. Quantum advantage in finding multiple optimal solutions for degenerate problems
4. Potential for medical imaging applications where multiple valid segmentations provide clinical value

Limitations & Future Work:

- Current simulations limited to small images ($\sim$20 qubits)
- Requires actual quantum hardware for scalability
- Future directions: Implementation on real quantum processors, extension to larger medical images, incorporation of warm-starting techniques [**?**] and CVaR optimization [**?**]

This work establishes a foundation for quantum-enhanced medical image analysis, with promising results suggesting quantum algorithms can provide advantages in finding comprehensive solution sets for complex segmentation problems.

Acknowledgments

This work was supported by the University of Iowa Department of Electrical and Computer Engineering. The authors thank the Qiskit development team for their quantum computing framework.