ADVANCE STATISTICS

PROJECT REPORT

By

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Problem 1

A physiotherapist with a male football team is interested in studying the relationship between foot injuries and the positions at which the players play from the data collected

	Striker	Forward	Attacking Midfielder	Winger	**Total**
Players Injured	45	56	24	20	**145**
Players Not Injured	32	38	11	9	**90**
Total	**77**	**94**	35	**29**	**235**

Solution:

1.1 What is the probability that a randomly chosen player would suffer an injury?

The number of players injured is 145, and the total number of players is 235.

Hence the probability that a randomly chosen player is injured is;

P (Injured Player) = 145/235 = 0.617

1.2 What is the probability that a player is a forward or a winger?

Total number of players that are forwards or wingers is (94 + 29) = 123

Total number of players is 235.

Hence, the probability that a randomly chosen player is a forward or a winger is;

P (Forward OR Winger) = (94+29)/235 = 0.5234

1.3 What is the probability that a randomly chosen player plays in a striker position and has a foot injury?

The number of players who have a foot injury and who play in striker position is 45.

The total number of players is 235.

This is a **joint probability** problem,

Hence the probability that a player is a striker and has foot injury is:

P (Striker AND Injured) = 45/235 = 0.1915

1.4 What is the probability that a randomly chosen injured player is a striker?

This is a **conditional probability problem**, here not all players and considered.

Given that the chosen player is injured we need to find the probability that he is a striker.

Total No of Injured player is 145. The conditional probability is;

P (Striker | Injured) = 45/145 = 0.3103

1.5 What is the probability that a randomly chosen injured player is either a forward or an attacking midfielder?

Total No of injured player who is either a forward or an attacking midfielder is 56 + 24 = 80

This is again a **Conditional Probability** Problem

P (Forward OR Attacking Midfielder | Injured) = (56+24)/145 = 0.5517

Problem 2

An independent research organization is trying to estimate the probability that an accident at a nuclear power plant will result in radiation leakage. The types of accidents possible at the plant are, fire hazards, mechanical failure, or human error. The research organization also knows that two or more types of accidents cannot occur simultaneously.

According to the studies carried out by the organization, the probability of a radiation leak in case of a fire is 20%, the probability of a radiation leak in case of a mechanical 50%, and the probability of a radiation leak in case of a human error is 10%. The studies also showed the following;

The probability of a radiation leak occurring simultaneously with a fire is 0.1%.

The probability of a radiation leak occurring simultaneously with a mechanical failure is 0.15%.

The probability of a radiation leak occurring simultaneously with a human error is 0.12%.

On the basis of the information available, answer the questions below:

2.1 What are the probabilities of a fire, a mechanical failure, and a human error respectively?

2.2 What is the probability of a radiation leak?

2.3 Suppose there has been a radiation leak in the reactor for which the definite cause is not known. What is the probability that it has been caused by

A Fire.
A Mechanical Failure.

A Human Error.

Solutions: Let's Define the events as:

F stands for Fire Hazards

M stands for Mechanical Failure

H Stands for Human Error

R stands for Radiation Leakage

The Information provided as per question are following:

- Probability of Radiation leak in case of Fire, $P(R|F) = 0.2$
- Probability of Radiation leak in case of Mechanical Hazards, $P(R|M) = 0.5$
- Probability of Radiation leak in case of Human Error, $P(R|H) = 0.1$
- probability of a radiation leak occurring simultaneously with a fire, P (R and F) = 0.001
- probability of a radiation leak occurring simultaneously with a Mechanical failure,

  P (R and M) = 0.0015
- probability of a radiation leak occurring simultaneously with a Human Error,

  P (R and H) = 0.0012

2.1 What are the probabilities of a fire, a mechanical failure, and a human error respectively?
Solution:

1. According to Conditional Probability, P(R|F) = P (R and F) / P(F)

Hence, P(F) = P (R and F) / P(R|F) = 0.001/0.2 = 0.005

Probability of fire: 0.005

2. According to Conditional Probability, P(R|M) = P (R and M)/P(M)

Hence, P(M) = P (R and M)/P(R|M) = 0.0015/0.5 = 0.003

Probability of Mechanical Failure: 0.003

3. According to Conditional Probability, P(R|H) = P (R and H)/P(H)

Hence, P(H) = P (R and H)/P(R|H) = 0.0012/0.1 = 0.012

Probability of Human Error: 0.012

2.2 What is the probability of a radiation leak?

Solution:

P(R) = P (R and F) + P (R and M) + P (R and H) = 0.001 + 0.0015 + 0.0012 = 0.0037

Probability of Radiation Leakage: 0.0037

2.3 Suppose there has been a radiation leak in the reactor for which the definite cause is not known. What is the probability that it has been caused by:

1. **A Fire**
2. **A Mechanical Failure**
3. **A Human Error**

Solution:

Here we have to find P(F|R), P(M|R) and P(H|R)

According to ***Baye's Theorem,***

P (R and F) = P (F and R), P (R and M) =P (M and R) and P (R and H) =P (H and R)

Hence,

P(F|R) = P (F and R) / P(R)

P(M|R) = P (M and R) / P(R)

P(H|R) = P (H and R) / P(R)

1. probability of a radiation leak in the reactor due to a Fire:

   P(F|R) = P (F and R) / P(R) = 0.001/0.0037 = 0.2703

   Probability of a radiation leak in the reactor due to a Fire: 0.2703

2. probability of a radiation leak in the reactor due to a Mechanical Failure:

   P(M|R) = P (M and R)/P(R) = 0.0015/0.0037 = 0.4054

   Probability of a radiation leak in the reactor due to a Mechanical Failure: 0.4054

3. probability of a radiation leak in the reactor due to a Human Error:

   P(H|R) = P (H and R)/P(R) = 0.0012/0.0037 = 0.3243

   probability of a radiation leak in the reactor due to a Human Error: 0.3243

Problem 3:

The breaking strength of gunny bags used for packaging cement is normally distributed with a mean of 5 kg per sq. centimetre and a standard deviation of 1.5 kg per sq. centimetre. The quality team of the cement company wants to know the following about the packaging material to better understand wastage or pilferage within the supply chain; Answer the questions below based on the given information; **(Provide an appropriate visual representation of your answers, without which marks will be deducted)**

Solution:

3.1 What proportion of the gunny bags have a breaking strength less than 3.17 kg per sq. cm?

Using **Stats.norm.cdf** we can find the required probability

P (Breaking strength < 3.17) = 0.1112

proportion of the gunny bags have a breaking strength less than 3.17 kg per sq. cm is: 0.1112

Visual Representation:

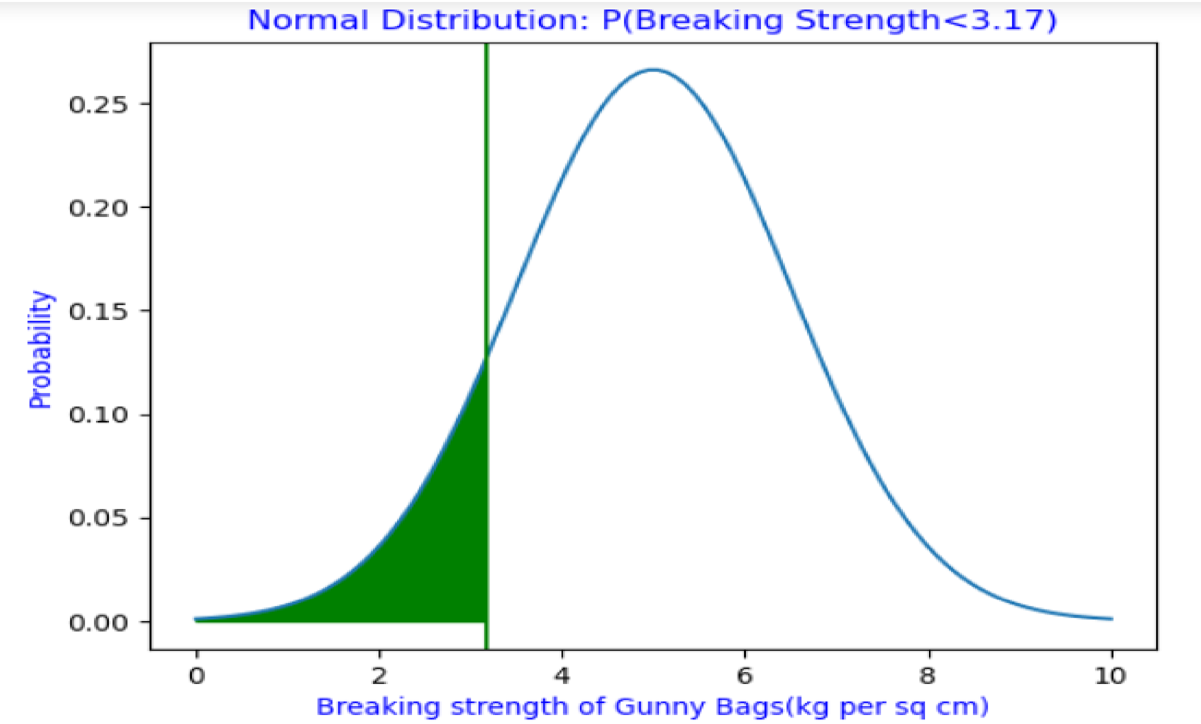

Fig 1. Shaded Area gives Required Probability

3.2 What proportion of the gunny bags have a breaking strength at least 3.6 kg per sq. cm.?

Using **Stats.norm.cdf** we can find the required probability

P (Breaking strength > 3.6) = 1 - P (Breaking strength <3.6)

= 1 - 0.1753 = 0.8247

proportion of the gunny bags have a breaking strength at least 3.6 kg per sq. cm is: 0.8247

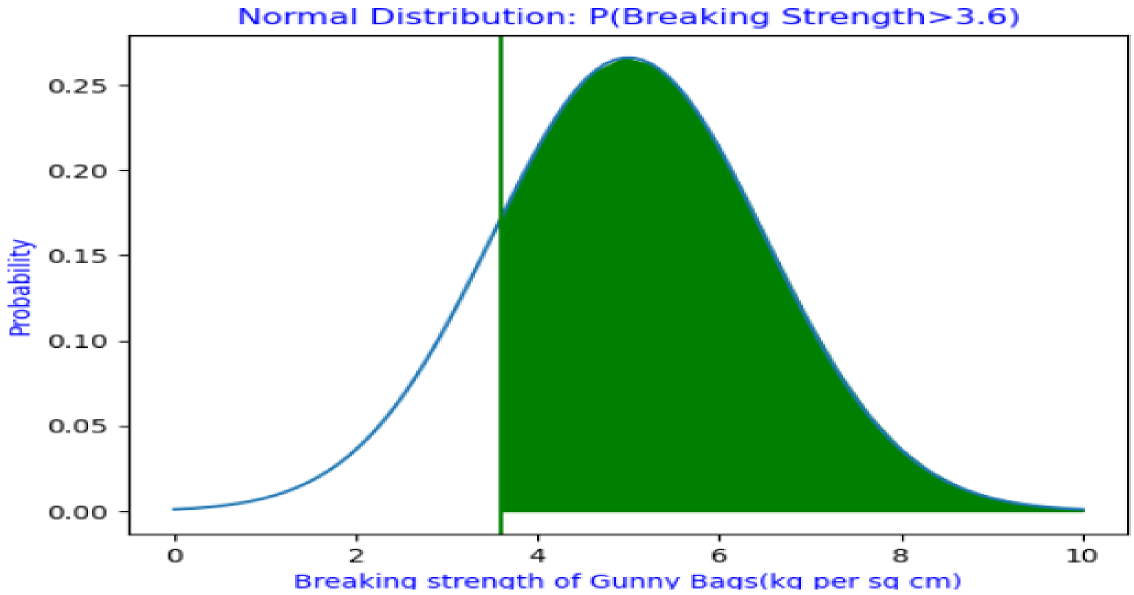

Fig 2. Shaded Area gives Required Probability

3.3 What proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq. cm.?

Using **Stats.norm.cdf** we can find the required probability

P (5 < Breaking strength < 5.5) = 0.6306 - 0.5 = 0.1306

proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq. cm is: 0.1306

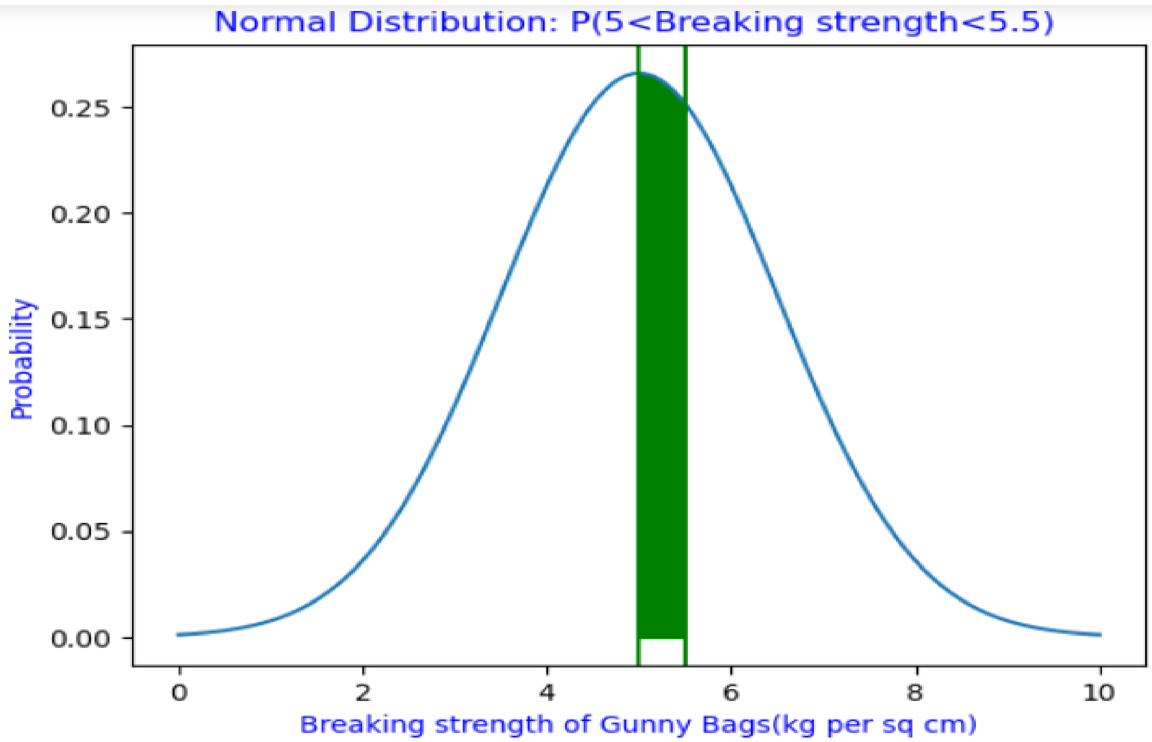

Fig 3. Shaded Area gives Required Probability

3.4 What proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq. cm.?

Using **Stats.norm.cdf** we can find the required probability

P (Breaking strength Not between 3 and 7.5) = 1 - P (3 < Breaking strength < 7.5)

= 1 – 0.861 = 0.139

proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq. cm. is: 0.139

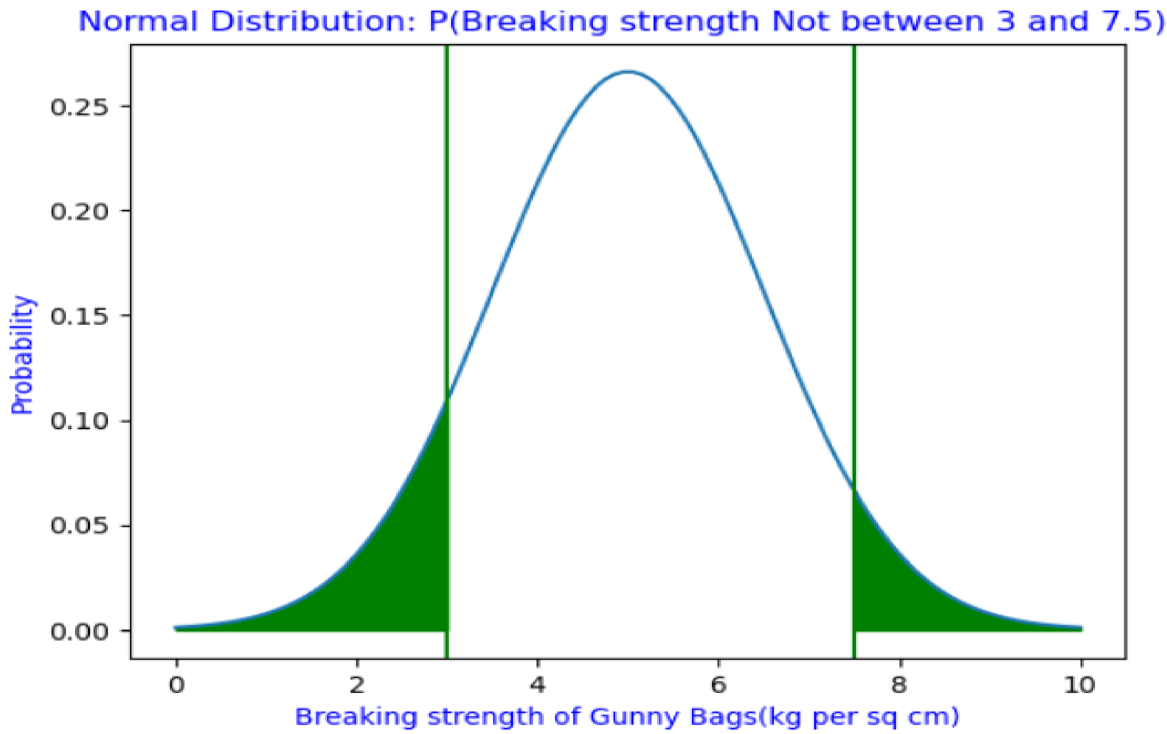

Fig 4. Shaded Area gives Required Probability

Problem 4:

Grades of the final examination in a training course are found to be normally distributed, with a mean of 77 and a standard deviation of 8.5. Based on the given information answer the questions below.

4.1 What is the probability that a randomly chosen student gets a grade below 85 on this exam?

Solution: Using **Stats.norm.cdf** we can find the required probability

P (Grade < 85) = 0.8267

The probability that a randomly chosen student gets a grade below 85 on this exam is: 0.8267

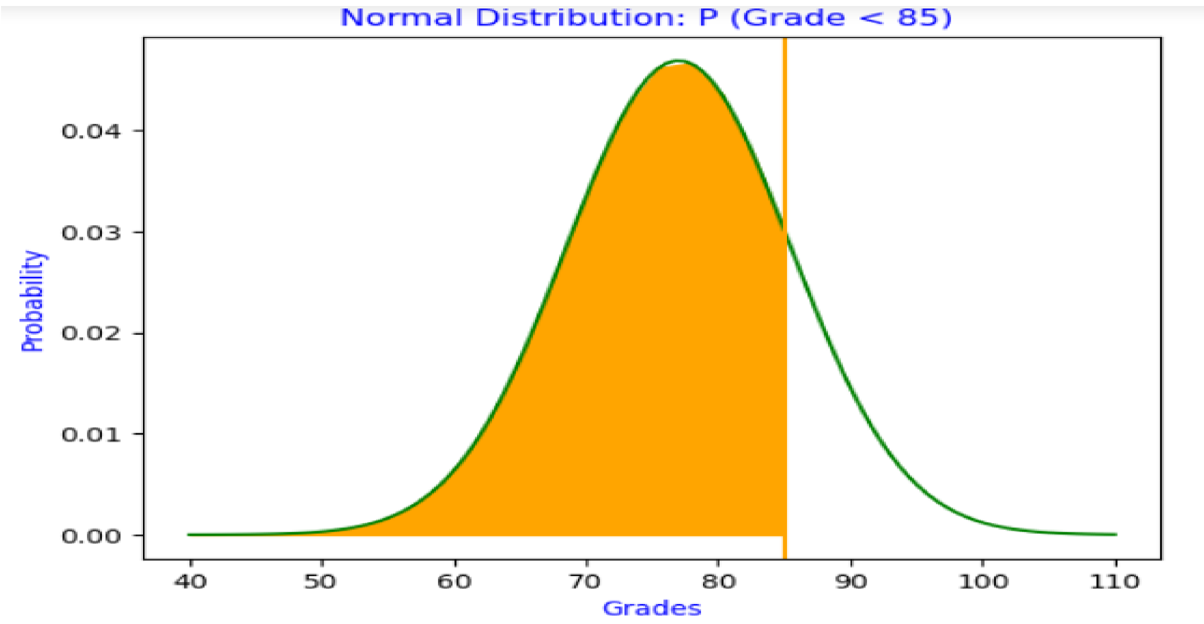

Fig 5. Shaded Area gives Required Probability

4.2 What is the probability that a randomly selected student scores between 65 and 87?

Solution: Using **Stats.norm.cdf** we can find the required probability

$$P\ (65 < \text{Grade} < 87) = P\ (\text{Grade} < 87) - P\ (\text{Grade} < 65)$$

$$= 0.8803 - 0.0790 = 0.8013$$

The probability that a randomly selected student scores between 65 and 87 is: 0.8013

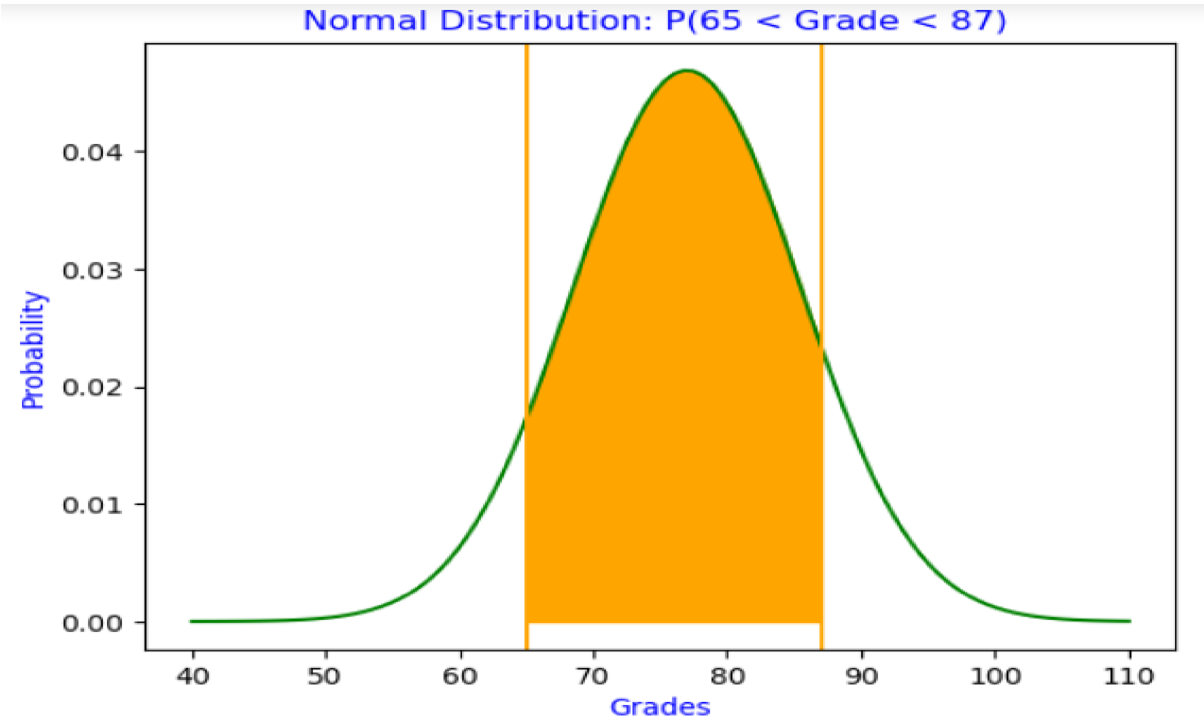

Fig 6. Shaded Area gives Required Probability

4.3 What should be the passing cut-off so that 75% of the students clear the exam?

Solution: To find the Passing Cut-off Mark we will use **stats.norm.ppf** function

The passing cut-off so that 75% of the students clear the exam is: 71.2668

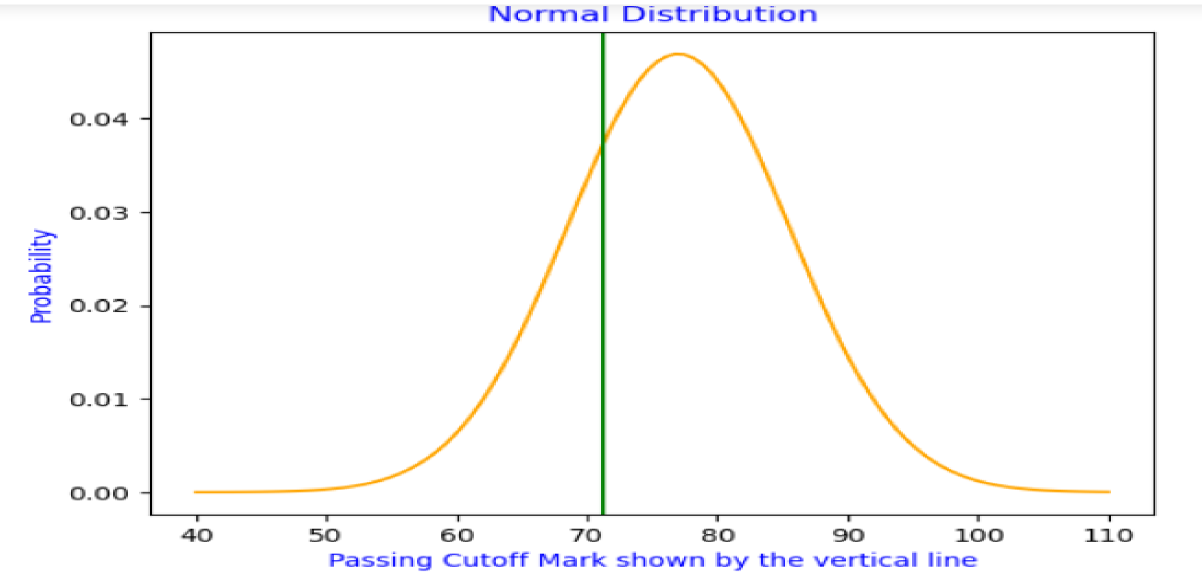

Fig 7. Shaded Area gives Required Probability

Problem 5:

Zingaro stone printing is a company that specializes in printing images or patterns on polished or unpolished stones. However, for the optimum level of printing of the image the stone surface has to have a Brinell's hardness index of at least 150. Recently, Zingaro has received a batch of polished and unpolished stones from its clients. Use the data provided to answer the following (assuming a 5% significance level);

5.1 Earlier experience of Zingaro with this particular client is favourable as the stone surface was found to be of adequate hardness. However, Zingaro has reason to believe now that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so?

Sample of Dataset:

	Unpolished	Treated and Polished
0	164.481713	133.209393
1	154.307045	138.482771
2	129.861048	159.665201
3	159.096184	145.663528
4	135.256748	136.789227

Solution:

Step 1: Define null and alternative hypotheses:

Here we need to test the hardness of unpolished stone. As per Zingano's believe unpolished stones may not be suitable for printing,

we can formulate the hypothesis as:

- Null Hypothesis: The mean hardness of the 'Unpolished stone' is greater than 150
- Alternative Hypothesis: The mean hardness of the 'Unpolished stone' is less than 150

H0: mu >= 150

Ha: mu < 150

Step 2: Decide the significance level:

Here we select α= 0.05 and the population standard deviation is not known.

Step 3: Identify the test statistic:

- We have two samples and we do not know the population standard deviation.
- Sample sizes for both samples are same.
- The sample is large sample, n > 30. So we use the t distribution and the $tSTAT$ test statistic.

Step 4: Calculate the p - value and test statistic:

- We will use **scipy. stats. ttest_1samp** t_test to calculate the mean of one sample given the sample observations and the expected value in the null hypothesis.
- This function returns t _statistic and the two-tailed p value.

Output: t-Statistic: -4.164629601426757

P-Value: 4.171286997419652e-05

Step 5: Decide to reject or accept null hypothesis:
Since P_value < alpha (0.05)

- We have enough Evidence to Reject the Null hypothesis against alternative hypothesis.
- The mean hardness of the 'Unpolished stone' is less than 150
- Hence the unpolished stones may not be suitable for printing.

With 95% confidence, we can say that we have enough evidence to say that the hardness for unpolished stones is less than 150. Thus Zingaro is right in its thinking that unpolished stones are not fit for printing.

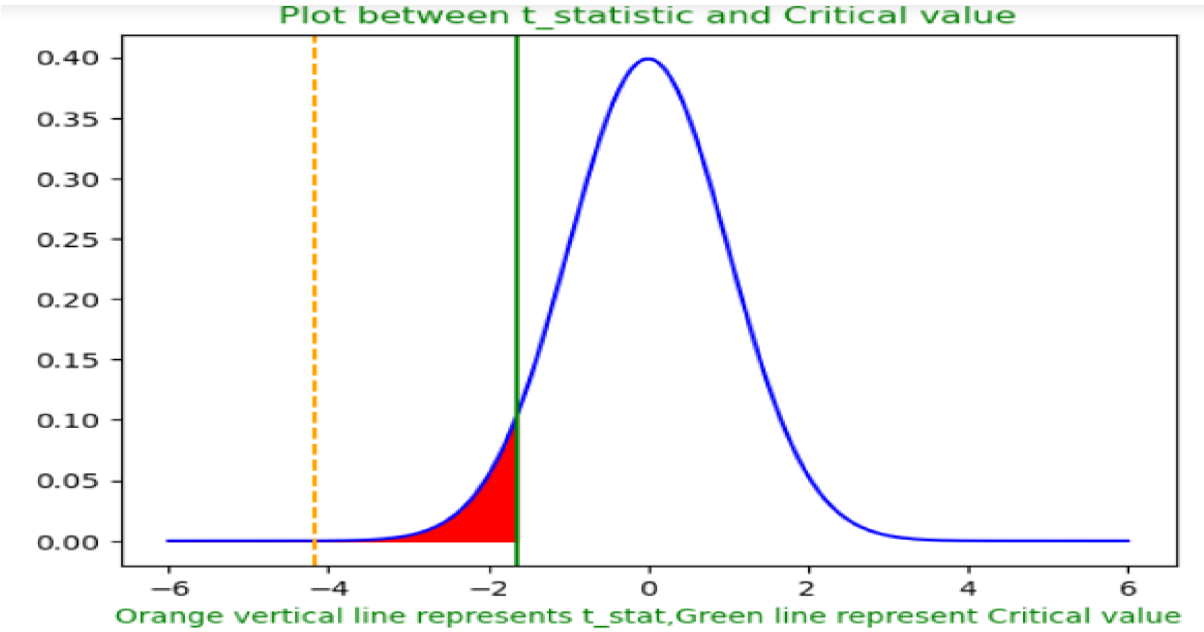

Fig 8. Shaded Area gives Required Probability

❖ We Reject the Null Hypothesis since test_statistic value lies in the Rejection Region.

5.2 Is the mean hardness of the polished and unpolished stones the same?

Solution:

Step 1: Define null and alternative hypotheses:

- H0: The mean hardness of the polished and unpolished stones is same
- Ha: The mean hardness of the polished and unpolished stones is not same.

Step 2: Decide the significance level:

Here we select α= 0.05 and the population standard deviation is not known.

Step 3: Identify the test statistic:

❖ We have two samples and we do not know the population standard deviation.

❖ Sample sizes for both samples are same.

❖ The sample is large sample, n > 30. So we use the t distribution and the $tSTAT$ test statistic.

❖ This is a two tailed test.

Step 4: Calculate the p - value and test statistic:

- We will use the **scipy. stats. ttest_ind** to calculate the t-test for the means of Two Independent samples.
- This function returns t statistic and two-tailed p value.

Output:

t_stat: -3.2422320501414053

P Value: 0.0014655150194628353

Step 5: Decide to reject or accept null hypothesis:

Since P_value < alpha (0.05)

- We can say that The mean hardness of the polished and unpolished stones is not same.

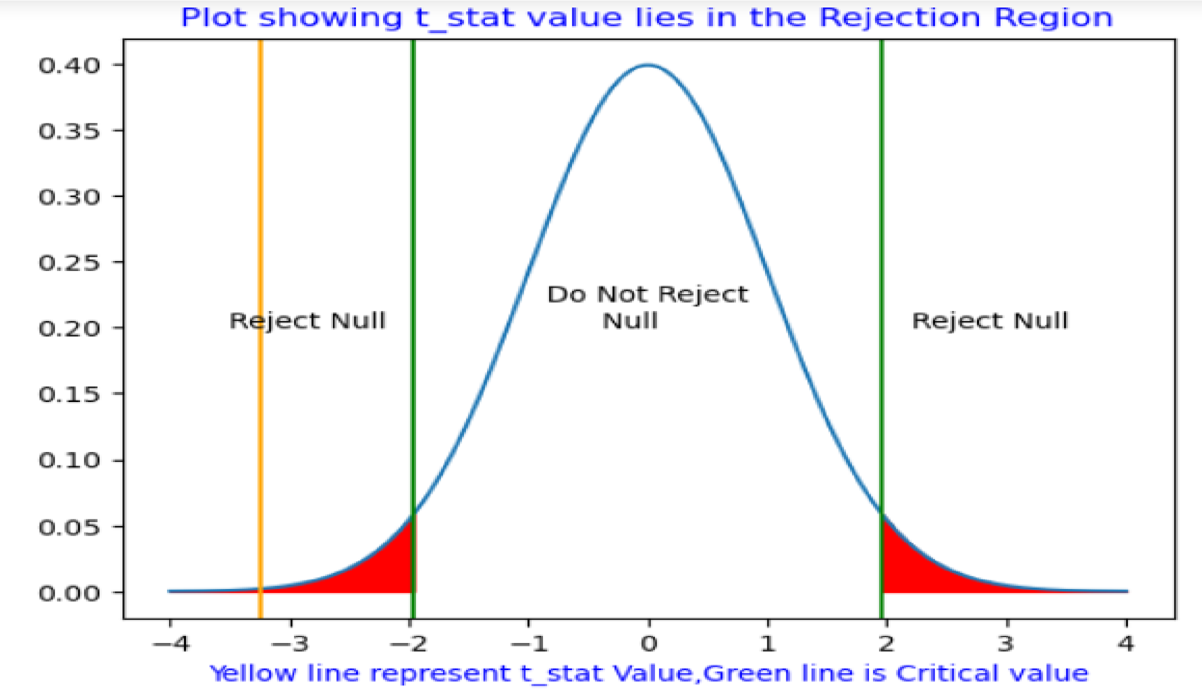

Fig 9. Shaded Area gives Required Probability

- As our test statistic value -3.24 lies in the rejection region, we can reject the null hypothesis and Accept the Alternate hypothesis that The mean hardness of the polished and unpolished stones is not same.

	count	mean	std	min	25%	50%	75%	max
Unpolished	75.0	134.110527	33.041804	48.406838	115.329753	135.597121	158.215098	200.161313
Treated and Polished	75.0	147.788117	15.587355	107.524167	138.268300	145.721322	157.373318	192.272856

- ❖ From the summary above we can also see that the mean hardness of Polished stones is 134.11 whereas mean hardness of Treated and Polished stone is 147.78 which is not same.

Problem 6:

Aquarius health club, one of the largest and most popular cross-fit gyms in the country has been advertising a rigorous program for body conditioning. The program is considered successful if the candidate is able to do more than 5 push-ups, as compared to when he/she enrolled in the program. Using the sample data provided can you conclude whether the program is successful? (Consider the level of Significance as 5%)

Note that this is a problem of the paired-t-test. Since the claim is that the training will make a difference of more than 5, the null and alternative hypotheses must be formed accordingly.

Sample of the Dataset:

	Sr no.	Before	After
0	1	39	44
1	2	25	25
2	3	39	39
3	4	6	13
4	5	40	44

Solution:

Step 1: Define null and alternative hypotheses

Let µ1 be the Mean push ups done by candidate before enrolment and µ2 be the Mean push ups done by candidate after enrolment.

- Null Hypothesis: Difference of Before and After push-ups less than or equal to 5.
- Alternate Hypothesis: Difference of Before and After push-ups more than 5.
  - H0: µ2 - µ1 <= 5 # The program is Not Successful
  - Ha: µ2 - µ1 > 5 # The Program is Successful

Step 2: Decide the significance level

Here we select α = 0.05 as given in the question.

Step 3: Identify the test statistic:

- Sample sizes for both samples are same.
- We have two paired samples and we do not know the population standard deviation.
- The sample is large sample, n > 30. So we use the t distribution and the $tSTAT$ test statistic for two sample paired test.
- Degree of Freedom: Since the sample is the same for both Sampling tests, we have N-1 degrees of freedom: 100-1=99
- Since the sole purpose of the test is to check whether The program is successful or not, we would prefer a One-sided T-test.

Step 4: Calculate the p - value and test statistic:

- We use the scipy. stats. ttest_rel to calculate the T-test on Two paired samples.
- Here we give the two sample observations as input. This function returns t statistic and two-tailed p value.

Output: t_stat -19.323

p-value for one-tail: 1.1460209626255983e-35

Step 5: Decide to reject or accept null hypothesis:

Since P_value < alpha (0.05)

- ❖ We have enough Evidence to Reject the Null hypothesis against alternative hypothesis.
- ❖ The program is considered successful since the candidate is able to do more than 5 push-ups.

With 95% confidence, we can say that we have enough evidence to say that The program is considered successful.

Problem 7:

Dental implant data: The hardness of metal implant in dental cavities depends on multiple factors, such as the method of implant, the temperature at which the metal is treated, the alloy used as well as on the dentists who may favour one method above another and may work better in his/her favourite method. The response is the variable of interest.

7.1 Test whether there is any difference among the dentists on the implant hardness. State the null and alternative hypotheses. Note that both types of alloys cannot be considered together. You must state the null and alternative hypotheses separately for the two types of alloys.?

Solution:

Since we have to consider both the type of alloys separately we will first subset the dataset based on Alloy 1 and Alloy 2.

After sub setting the dataset, we get two separate dataset based on Alloy 1 and Alloy 2 and now we can formulate null and alternative hypothesis separately on each dataset.

Hypothesis Formulation: For Alloy 1 Null and Alternate Hypothesis:

- ❖ H0: The Mean Implant hardness is same across all Dentist for alloy 1.
- ❖ Ha: The Mean Implant hardness is not same for at least one pair of Dentist

  For alloy1.

For Alloy 2 Null and Alternate Hypothesis:

- ❖ H0: The Mean Implant hardness is same across all Dentist for alloy 2.
- ❖ Ha: The Mean Implant hardness is not same for at least one pair of Dentist for

  alloy

ANOVA output Summary for Alloy 1

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	106683.688889	26670.922222	1.977112	0.116567
Residual	40.0	539593.555556	13489.838889	NaN	NaN

Conclusion:

- As P_value > 0.05(alpha), we fail to Reject the Null Hypothesis and Hence we can say that the mean implant hardness is same across all the Dentist for Alloy 1

ANOVA output Summary for Alloy 2

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	5.679791e+04	14199.477778	0.524835	0.718031
Residual	40.0	1.082205e+06	27055.122222	NaN	NaN

Conclusion:

- As P_value > 0.05(alpha), we fail to Reject the Null Hypothesis and Hence we can say that the mean implant hardness is same across all the Dentist for Alloy 2.

7.3 Irrespective of your conclusion in 2, we will continue with the testing procedure. What do you conclude regarding whether implant hardness depends on dentists? Clearly state your conclusion. If the null hypothesis is rejected, is it possible to identify which pairs of dentists differ?

Solution:

Hypothesis Formulation:

For Alloy 1 Null and Alternate Hypothesis:

- H0: The Mean Implant hardness is same across all Dentist for Alloy 1.
- Ha: The Mean Implant hardness is not same for at least one pair of Dentist for Alloy 1.

For Alloy 2 Null and Alternate Hypothesis:

- H0: The Mean Implant hardness is same across all Dentist for Alloy 2.
- Ha: The Mean Implant hardness is not same for at least one pair of Dentist for Alloy 2

ANOVA Output Summary for Alloy 1

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	106683.688889	26670.922222	1.977112	0.116567
Residual	40.0	539593.555556	13489.838889	NaN	NaN

- As P_value > 0.05(alpha) we fail to Reject the Null Hypothesis and Hence we can say that The mean implant hardness is same across all the Dentist for Alloy 1
- Since the null hypothesis is not Rejected it is not possible to identify which pair of dentists differ. The mean hardness is same across all the dentist.

ANOVA Output Summary for Alloy 2

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	5.679791e+04	14199.477778	0.524835	0.718031
Residual	40.0	1.082205e+06	27055.122222	NaN	NaN

- As P_value > 0.05(alpha) we fail to Reject the Null Hypothesis and Hence we can say that The mean implant hardness is same across all the Dentist for Alloy 2
- Since the null hypothesis is not Rejected it is not possible to identify which pair of dentists differ. The mean hardness is same across all the dentist.

7.4 Now test whether there is any difference among the methods on the hardness of dental implant, separately for the two types of alloys. What are your conclusions? If the null hypothesis is rejected, is it possible to identify which pairs of methods differ?
Solution:

Hypothesis Formulation:

For Alloy 1 Null and Alternate Hypothesis:

- H0: The Mean Implant hardness is same across all the Methods for Alloy 1.
- Ha: The Mean Implant hardness is not same for at least one pair of Method for

  Alloy 1.

For Alloy 2 Null and Alternate Hypothesis:

- H0: The Mean Implant hardness is same across all the Methods for Alloy 2.
- Ha: The Mean Implant hardness is not same for at least one pair of Method for
  - Alloy 2.

ANOVA Output Summary for Alloy 1

	df	sum_sq	mean_sq	F	PR(>F)
C(Method)	2.0	148472.177778	74236.088889	6.263327	0.004163
Residual	42.0	497805.066667	11852.501587	NaN	NaN

Conclusions:

- As P_value < 0.05(alpha), We Reject the Null Hypothesis and hence we can say that
- The Mean Implant hardness is not same for at least one pair of Method for alloy 1.

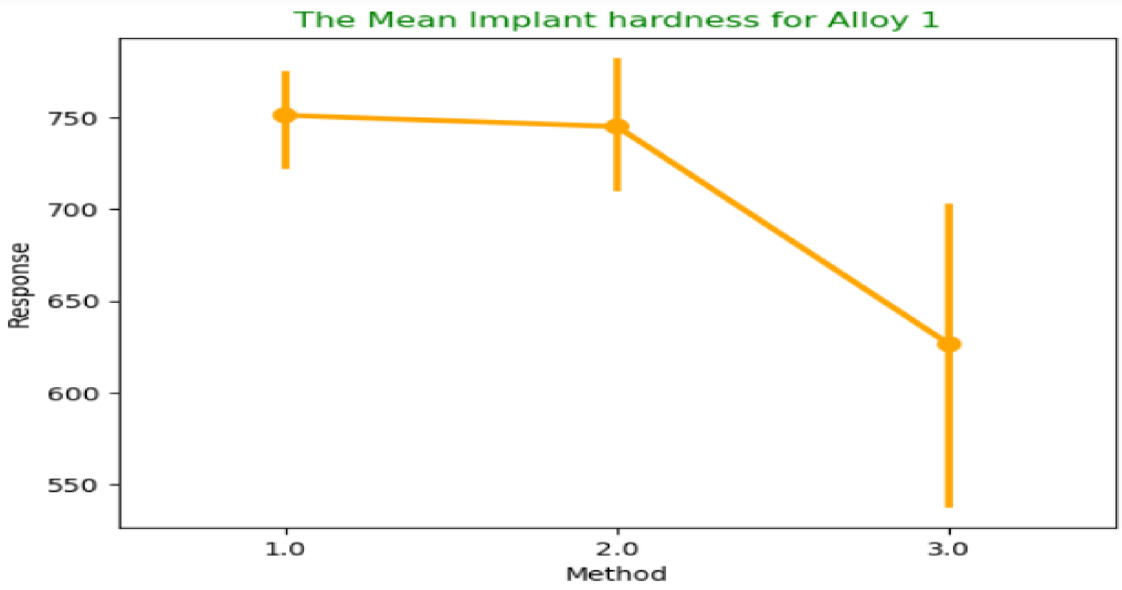

Fig 10: The mean implant hardness for Method on Alloy 1

Insights: It is now clear that for Alloy 1 mean Implant Hardness for Method 1.0 and 2.0 is similar but Implant hardness for Method 3.0 is significantly different from these two.

ANOVA Output Summary for Alloy 2

	df	sum_sq	mean_sq	F	PR(>F)
C(Method)	2.0	499640.4	249820.200000	16.4108	0.000005
Residual	42.0	639362.4	15222.914286	NaN	NaN

- As P_value < 0.05(alpha), We Reject the Null Hypothesis and hence we can say that The Mean Implant hardness is not same for at least one pair of Method for alloy 2.

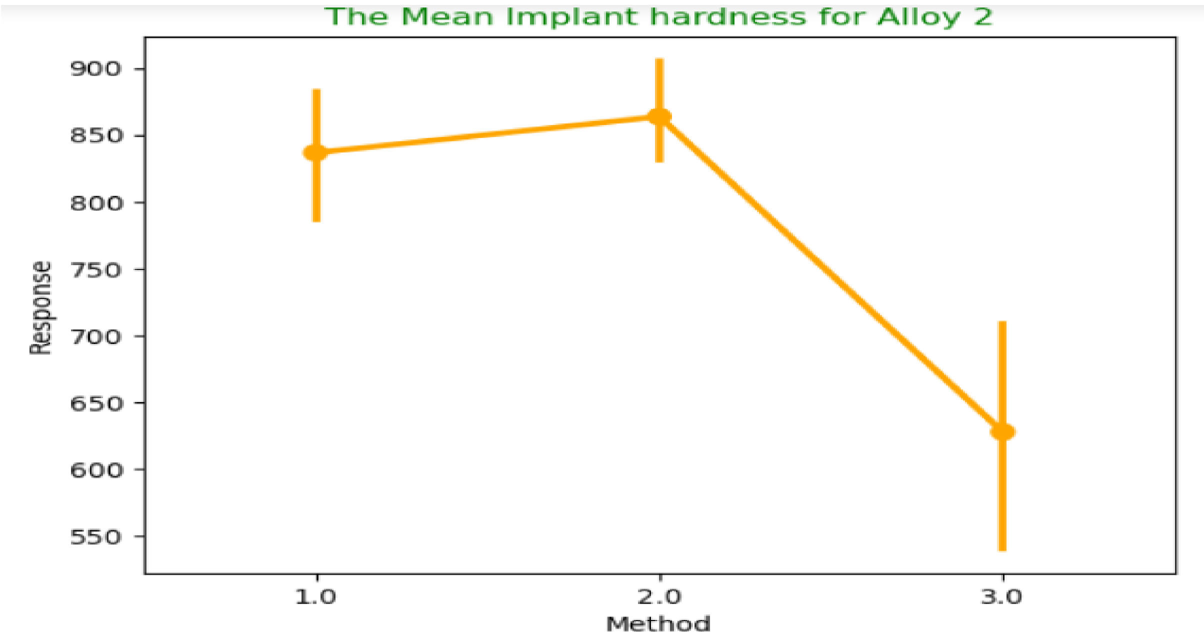

Fig 11: The mean implant hardness for Method on Alloy 2

Insights:

- It is now clear that for Alloy 2 mean Implant Hardness for Method 1.0 and 2.0 is similar but Implant hardness for Method 3.0 is significantly different from these two.
- As the null hypothesis is rejected, now it is possible to identify which pairs of methods differ.

To identify which pair of Methods Differ we will use Tukey HSD Test:

For Alloy 1:

Multiple Comparison of Means - Tukey HSD, FWER=0.05

group1	group2	meandiff	p-adj	lower	upper	reject
1.0	2.0	-6.1333	0.987	-102.714	90.4473	False
1.0	3.0	-124.8	0.0085	-221.3807	-28.2193	True
2.0	3.0	-118.6667	0.0128	-215.2473	-22.086	True

Conclusions:

- P-value is significant for comparing Implant Hardness mean levels for the pair 1.0 - 1.3 and 2.0 - 3.0, but not for 1.0 - 2.0
- The null hypothesis of equality of all population means is rejected.
- It is now clear that mean Implant Hardness for Method 1.0 and 2.0 is similar but Implant hardness for Method 3.0 is significantly different from these two.

For Alloy 2:

```
  Multiple Comparison of Means - Tukey HSD, FWER=0.05
=======================================================
group1 group2 meandiff p-adj   lower     upper   reject
-------------------------------------------------------
   1.0    2.0     27.0 0.8212  -82.4546 136.4546  False
   1.0    3.0   -208.8 0.0001 -318.2546  -99.3454   True
   2.0    3.0   -235.8    0.0 -345.2546 -126.3454   True
-------------------------------------------------------
```

Conclusions:

- P-value is significant for comparing Implant Hardness mean levels for the pair 1.0 - 1.3 and 2.0 - 3.0, but not for 1.0 - 2.0
- The null hypothesis of equality of all population means is rejected.
- It is now clear that mean Implant Hardness for Method 1.0 and 2.0 is similar but Implant hardness for Method 3.0 is significantly different from these two.

7.5 Now test whether there is any difference among the temperature levels on the hardness of dental implant, separately for the two types of alloys. What are your conclusions? If the null hypothesis is rejected, is it possible to identify which levels of temperatures differ?

Solution:

Hypothesis Formulation:

For Alloy 1 Null and Alternate Hypothesis:

- H0: The Mean Implant hardness is same across all the Temperature for alloy 1.
- Ha: The Mean Implant hardness is not same for at least one pair of Temperature for alloy 1.

For Alloy 2 Null and Alternate Hypothesis:

- H0: The Mean Implant hardness is same across all the Temperature for alloy 2.
- Ha: The Mean Implant hardness is not same for at least One pair of Temperature for alloy 2.

ANOVA Output Summary for Alloy 1

	df	sum_sq	mean_sq	F	PR(>F)
Temp	2.0	10154.444444	5077.222222	0.335224	0.717074
Residual	42.0	636122.800000	15145.780952	NaN	NaN

Conclusions:

- As P_value > 0.05(alpha) we fail to Reject the Null Hypothesis and Hence we can say that The mean implant hardness is same across all the Temperature level for Alloy 1
- Since the null hypothesis is not Rejected it is not possible to identify which pair of Temperature differs. The mean hardness is same across all the Temperature levels.

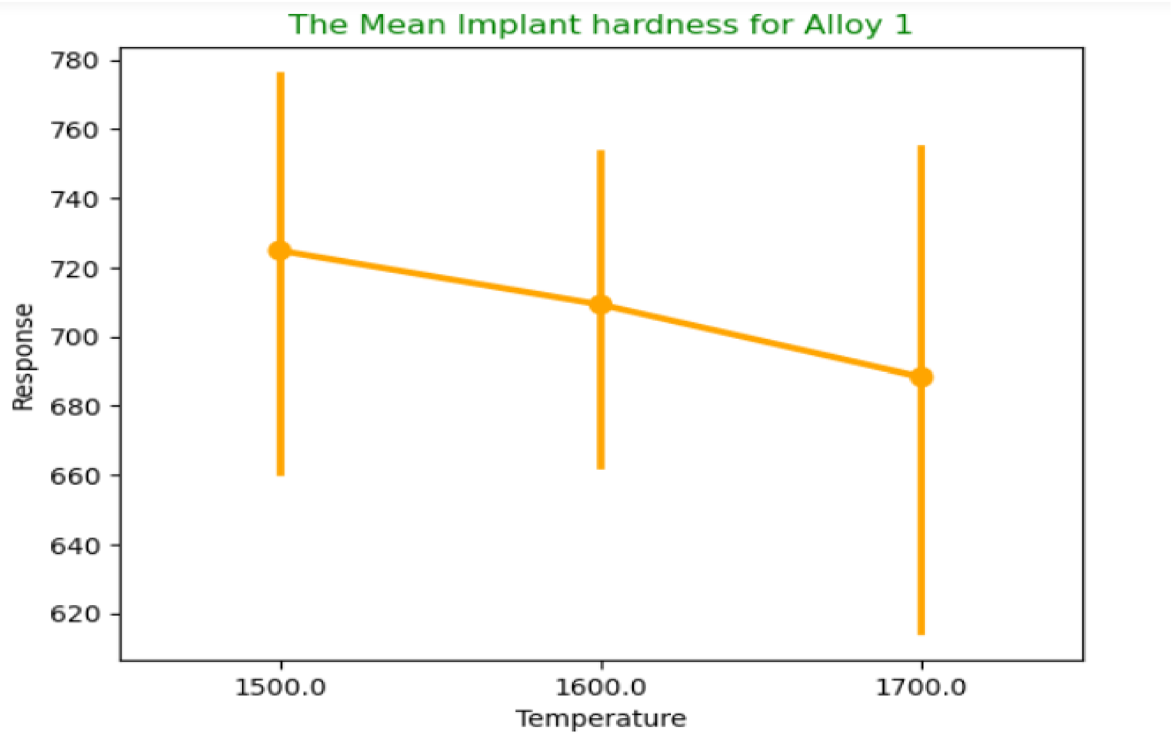

Fig 12: The mean implant hardness for Temperature on Alloy 1

ANOVA Summary for Alloy 2

	df	sum_sq	mean_sq	F	PR(>F)
Temp	2.0	9.374893e+04	46874.466667	1.883492	0.164678
Residual	42.0	1.045254e+06	24886.996825	NaN	NaN

Conclusions:

- As P_value > 0.05(alpha) we fail to Reject the Null Hypothesis and Hence we can say that The mean implant hardness is same across all the Temperature level for Alloy 2
- Since the null hypothesis is not Rejected it is not possible to identify which pair of Temperature differs. The mean hardness is same across all the Temperature levels.

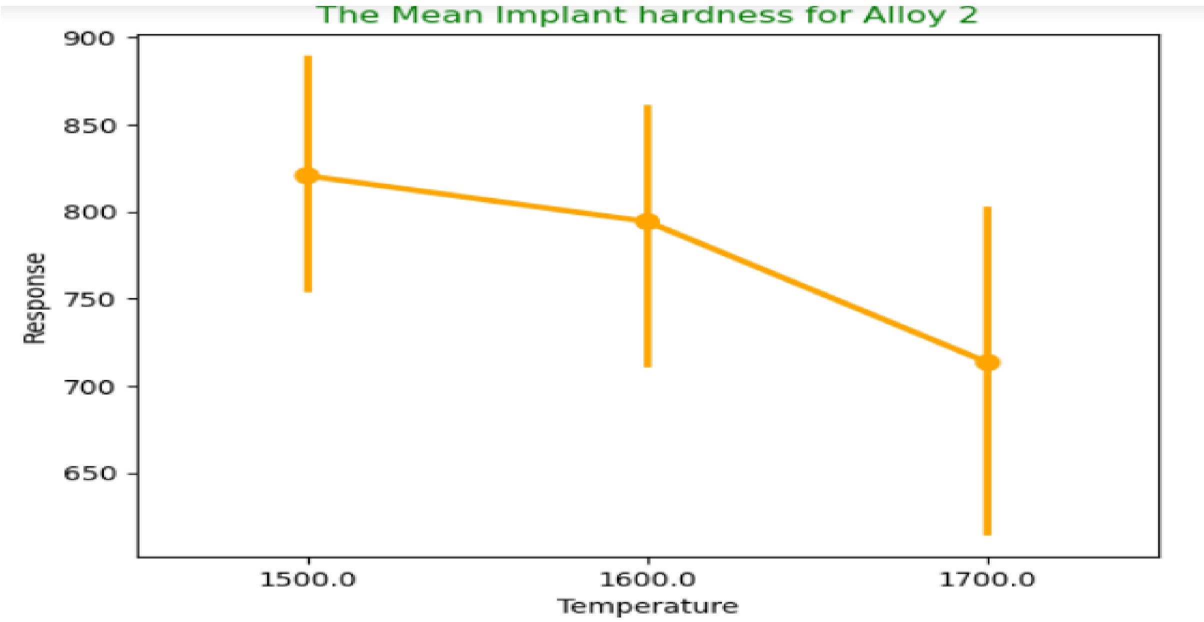

Fig 13: The mean implant hardness of Temperature on Alloy 2

7.6 Consider the interaction effect of dentist and method and comment on the interaction plot, separately for the two types of alloys?

Solution:

Interaction effect of Dentist and Method on Alloy 1

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	106683.688889	26670.922222	3.899638	0.011484
C(Method)	2.0	148472.177778	74236.088889	10.854287	0.000284
C(Dentist):C(Method)	8.0	185941.377778	23242.672222	3.398383	0.006793
Residual	30.0	205180.000000	6839.333333	NaN	NaN

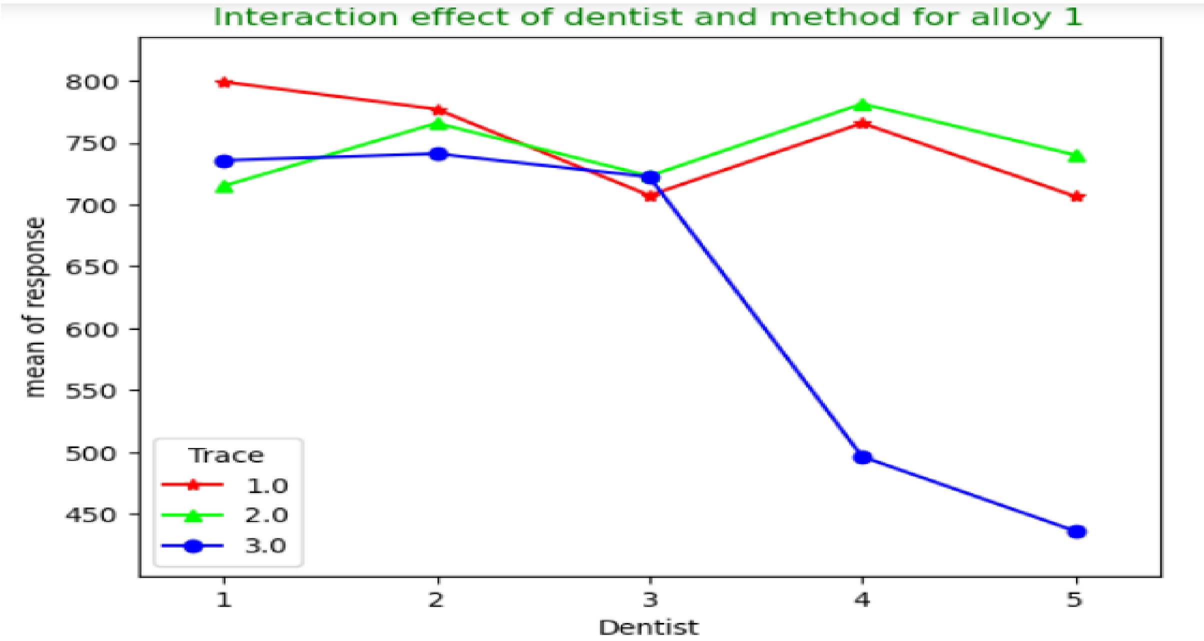

Fig 14: Interaction effect of Dentist and Method on Alloy 1

Conclusions:

❖ We can conclude that the mean hardness of dental implant is not same when different dentists are using different methods for alloy 1. This interaction is very significant here.

Interaction effect of Dentist and Method on Alloy 2

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	56797.911111	14199.477778	1.106152	0.371833
C(Method)	2.0	499640.400000	249820.200000	19.461218	0.000004
C(Dentist):C(Method)	8.0	197459.822222	24682.477778	1.922787	0.093234
Residual	30.0	385104.666667	12836.822222	NaN	NaN

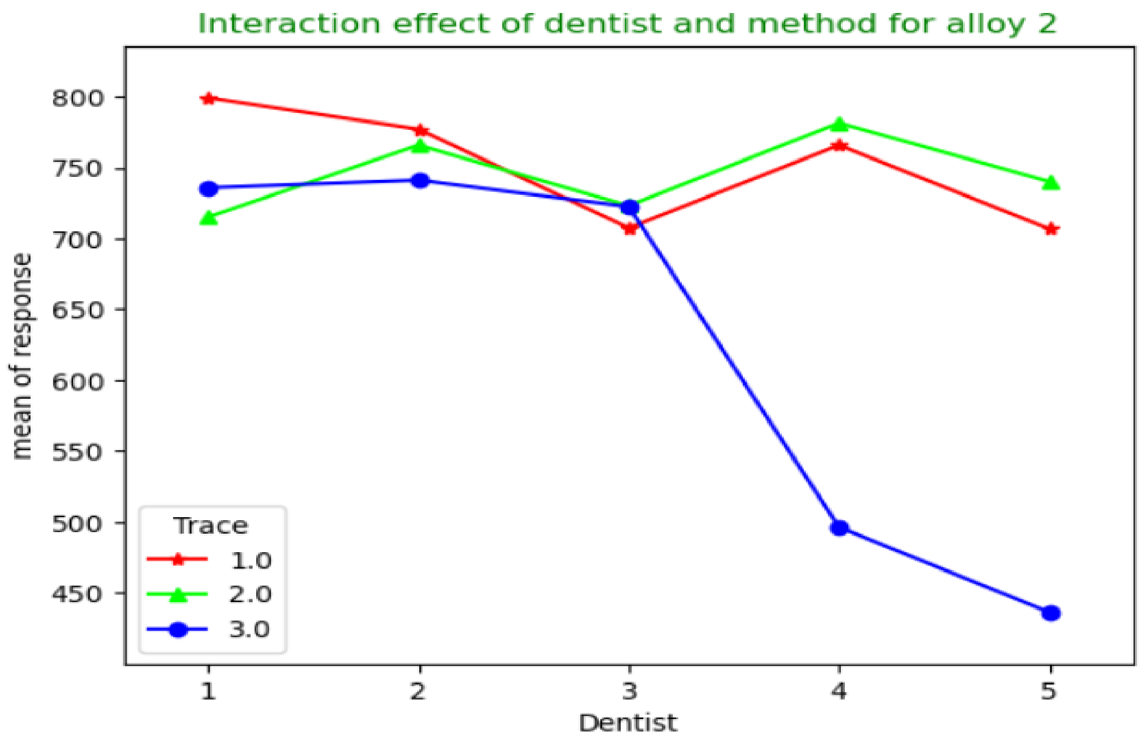

Fig 15: Interaction effect of Dentist and Method on Alloy 2

Conclusions:

❖ We can conclude that the mean hardness of dental implant is not same when different dentists are using different methods for alloy 2. This interaction is very significant here.

7.7 Now consider the effect of both factors, dentist, and method, separately on each alloy. What do you conclude? Is it possible to identify which dentists are different, which methods are different, and which interaction levels are different?

Solution:

Hypothesis test formulation for Alloy 1:

❖ H0: The mean implant hardness is same across different Dentists for Alloy 1.

❖ HA: Mean implant hardness is not same for at least one pair of Dentist for Alloy 1.

❖ H0: The mean implant hardness is same across different Methods for Alloy 1.

- HA: Mean implant hardness is not same for at least one pair of Methods for Alloy 1.
- H0: There is no interaction between Dentist and Method types for Alloy 1.
- HA: There is interaction between Dentist and Method types for Alloy 1.

ANOVA Output Summary for Alloy 1

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	106683.688889	26670.922222	2.591255	0.051875
C(Method)	2.0	148472.177778	74236.088889	7.212522	0.002211
Residual	38.0	391121.377778	10292.667836	NaN	NaN

Conclusions:

- As P> 0.05 for Dentist, there is no evidence that there is any difference in Dental implant hardness due to Dentists.
- we fail to Reject the Null Hypothesis and Hence we can say that the mean implant hardness is same across all the Dentist for Alloy 1
- As P<0.05 for Methods, We Reject the Null Hypothesis and hence we can say that The Mean Implant hardness is not same for at least one pair of Method for alloy 1.
- Methods is a significant cause for Dental hardness, Dentists is not a significant cause for Dental Hardness.

Let's Find which method is different using Tukey HDS test.

Tukey HSD Output Summary for Alloy 1

Multiple Comparison of Means - Tukey HSD, FWER=0.05

group1	group2	meandiff	p-adj	lower	upper	reject
1.0	2.0	-6.1333	0.987	-102.714	90.4473	False
1.0	3.0	-124.8	0.0085	-221.3807	-28.2193	True
2.0	3.0	-118.6667	0.0128	-215.2473	-22.086	True

Conclusion:

- P-value is significant for comparing Implant Hardness mean levels for the pair 1.0 - 1.3 and 2.0 - 3.0, but not for 1.0 - 2.0
- The null hypothesis of equality of all population means is rejected.
- It is now clear that mean Implant Hardness for Method 1.0 and 2.0 is similar but Implant hardness for Method 3.0 is significantly different from these two.

With Interaction:

ANOVA Output Summary with Interaction Effect for Alloy1

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	106683.688889	26670.922222	3.899638	0.011484
C(Method)	2.0	148472.177778	74236.088889	10.854287	0.000284
C(Dentist):C(Method)	8.0	185941.377778	23242.672222	3.398383	0.006793
Residual	30.0	205180.000000	6839.333333	NaN	NaN

Conclusions:

- As P_value < 0.05, We reject the null hypothesis and accept the alternate hypothesis that There is interaction between Dentist and Method types with type 1 Alloy.
- Dentist, Method and their interaction all are significant.

Hypothesis test formulation for Alloy 2

- H0: The mean implant hardness is same across different Dentists for Alloy 2.
- HA: Mean implant hardness is not same for at least one pair of Dentist for Alloy 2.
- H0: The mean implant hardness is same across different Methods for Alloy 2.
- HA: Mean implant hardness is not same for at least one pair of Methods for Alloy 2.
- H0: There is no interaction between Dentist and Method for Alloy 2.
- HA: There is interaction between Dentist and Method for Alloy 2.

ANOVA Output Summary for Alloy 2

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	56797.911111	14199.477778	0.926215	0.458933
C(Method)	2.0	499640.400000	249820.200000	16.295479	0.000008
Residual	38.0	582564.488889	15330.644444	NaN	NaN

Conclusions:

- As P> 0.05 for Dentist, there is no evidence that there is any difference in Dental implant hardness due to Dentists.
- we fail to Reject the Null Hypothesis and Hence we can say that the mean implant hardness is same across all the Dentist for Alloy 2
- As P<0.05 for Methods, We Reject the Null Hypothesis and hence we can say that The Mean Implant hardness is not same for at least one pair of Method for alloy 2.
- Methods is a significant cause for Dental hardness, Dentists is not a significant cause for Dental Hardness.

Let's Find which method is different using Tukey HDS test.

Tukey HSD Output Summary for Alloy 2

```
 Multiple Comparison of Means - Tukey HSD, FWER=0.05
=========================================================
group1 group2 meandiff p-adj    lower     upper   reject
---------------------------------------------------------
   1.0    2.0     27.0 0.8212  -82.4546  136.4546  False
   1.0    3.0   -208.8 0.0001 -318.2546  -99.3454   True
   2.0    3.0   -235.8    0.0 -345.2546 -126.3454   True
---------------------------------------------------------
```

Conclusion:

- P-value is significant for comparing Implant Hardness mean levels for the pair 1.0 - 1.3 and 2.0 - 3.0, but not for 1.0 - 2.0
- The null hypothesis of equality of all population means is rejected.
- It is now clear that mean Implant Hardness for Method 1.0 and 2.0 is similar but Implant hardness for Method 3.0 is significantly different from these two.

With interaction:

ANOVA Output Summary with Interaction Effect for Alloy 2

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	56797.911111	14199.477778	1.106152	0.371833
C(Method)	2.0	499640.400000	249820.200000	19.461218	0.000004
C(Dentist):C(Method)	8.0	197459.822222	24682.477778	1.922787	0.093234
Residual	30.0	385104.666667	12836.822222	NaN	NaN

- Here P_value > 0.05, For interaction effect we fail to reject the null hypothesis and hence conclude that There is no interaction between Dentist and Method types with type 2 Alloy.

Thank you