DATA 609 HW Week 5 & 6

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Week 5

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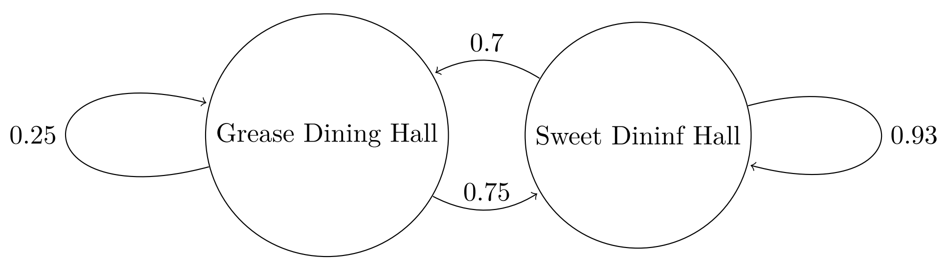
Consider a model for the long-term dining behavior of the students at College USA. It is found that 25% of the students who eat at the college's Grease Dining Hall return to eat there again, whereas those who eat at Sweet Dining Hall have a 93% return rate. These are the only two dining halls available on campus, and assume that all students eat at one of these halls. Formulate a model to solve for the long-term percentage of students eating at each hall.

Solution



Let,

G_n = the percentage of students who eat at Grease Dining Hall in period n

S_n = the percentage of students who eat at Sweet Dining Hall in period n

Probabilistic model:

$G_{n+1} = 0.25G_n + 0.07S_n$

$S_{n+1} = 0.75G_n + 0.93S_n$

```
# Probabilistic model
g_n <- function(g, s) {
  .25*g + .07*s
}

s_n <- function(g, s) {
  .75*g + .93*s
```

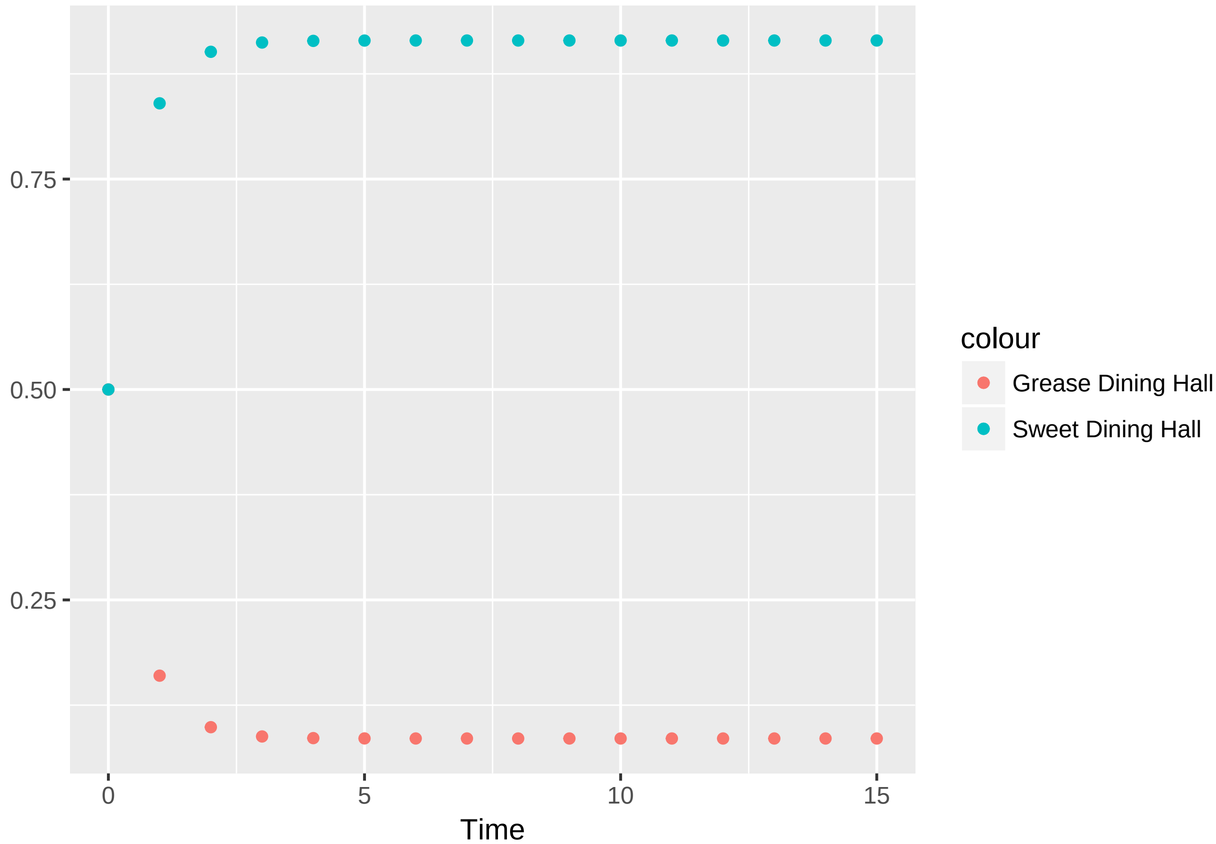
```
}


# Start with same probability and run 15 times
dine <- data.frame(n = 0:15, g = .5, s = .5)
for (i in 1:15){
  dine$g[i + 1] <- g_n(dine$g[i], dine$s[i])
  dine$s[i + 1] <- s_n(dine$g[i], dine$s[i])
}

knitr::kable(dine)
```

n	g	s
0	0.5000000	0.5000000
1	0.1600000	0.8400000
2	0.0988000	0.9012000
3	0.0877840	0.9122160
4	0.0858011	0.9141989
5	0.0854442	0.9145558
6	0.0853800	0.9146200
7	0.0853684	0.9146316
8	0.0853663	0.9146337
9	0.0853659	0.9146341
10	0.0853659	0.9146341
11	0.0853659	0.9146341
12	0.0853659	0.9146341
13	0.0853659	0.9146341
14	0.0853659	0.9146341
15	0.0853659	0.9146341

```
library(ggplot2)
```

```
## Warning: package 'ggplot2' was built under R version 3.3.3
```

```
ggplot(dine, aes(n)) +
  geom_point(aes(y = g, colour = "Grease Dining Hall")) + geom_point(aes(y = s, color = "Sweet Dining Ha
  theme(axis.title.y=element_blank())  +
  ylab("Percent of Student") + xlab ("Time")
```



After 9 iteration we have reached a steady state, 8.54 percent of student eat at the Grease Dinning Hall and 91.46 percent of student will eat at the Sweet Dinnig hall.

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Consider a stereo with CD player, FM-AM radio tuner, speakers (dual), and power amplifier (PA) components, as displayed with the reliabilities shown below. Determine the system's reliability. What assumptions are required in your model

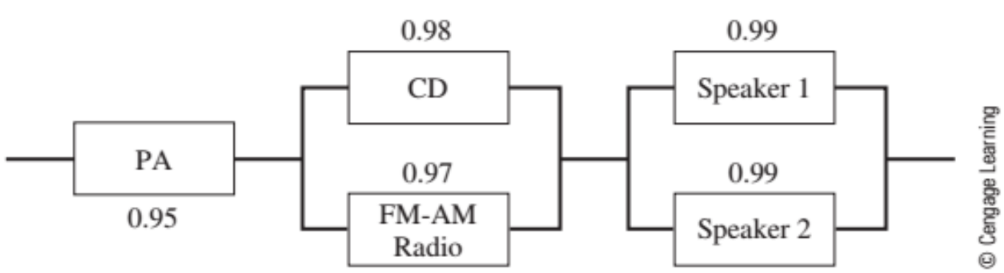

Figure 1:

Solution

Parllel system 1 reliability

$R_{s1}(t) = 0.95$

Parllel system 2 reliability

R_{s2}(t) $= R_{cd} + R_{fm-am} - R_{cd}.R_{fm-am} = 0.9994$

Parllel system 3 reliability

R_{s3}(t) $= R_{sp1} + R_{sp2} - R_{sp1}.R_{sp2} = 0.9999$

Total System reliability

$R_s(t) = R_{s1}(t).R_{s2}(t).R_{s3}(t) = 0.9493351$

Assumptions: We assumed there are three parallel systems, 1) the PA system, 2) CD and AM-FM Radio system and, 3) Speaker1 and Speaker2 system. We also assumed in the 2nd and 3rd parallel systems, which have two components each, if one component fails the system will still work.

Week 6

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***Nutritional Requirements* – A rancher has determined that the minimum weekly nutritional requirements for an average-sized horse include 40 lb of protein, 20 lb of carbohydrates, and 45 lb of roughage. These are obtained from the following sources in varying amounts at the prices indicated:**

source	protein	carbohydrates	roughage	cost
hay(per bale)	0.5	2.0	5.0	1.8
oats(per sack)	1.0	4.0	2.0	3.5
feeding blocks(per block)	2.0	0.5	1.0	0.4
high-protein cincentrate(per sack)	6.0	1.0	2.5	1.0
requirement per horse(per week)	40.0	20.0	45.0	0.0

Formulate a mathematical model to determine how to meet the minimum nutritional requirements at minimum cost.

Solution

Objective:

$Minimize\, cost : (1.8 \times Hay) + (3.5 \times Oats) + (0.4 \times FB) + (1.0 \times HPC)$

Subject to following constraints

$Protein : (0.5 \times Hay) + (1 \times Oats) + (2 \times FB) + (6 \times HPC) \geq 40$

$Carbohydrates : (92 \times Hay) + (4 \times Oats) + (0.5 \times FB) + (1 \times HPC) \geq 20$

$Roughage : (5 \times Hay) + (2 \times Oats) + (1 \times FB) + (2.5 \times HPC) \geq 45$

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Solve problem using graphical analysis:

Maximize $10x + 35y$

subject to

$$8x + 6y \leq 48 \; (board - feet \, of \, lumber)$$
$$4x + y \leq 20 \; (hours \, of \, capacity)$$
$$y \leq 5 \; (demand)$$
$$x, y \leq 0 \; (nonnegativity)$$

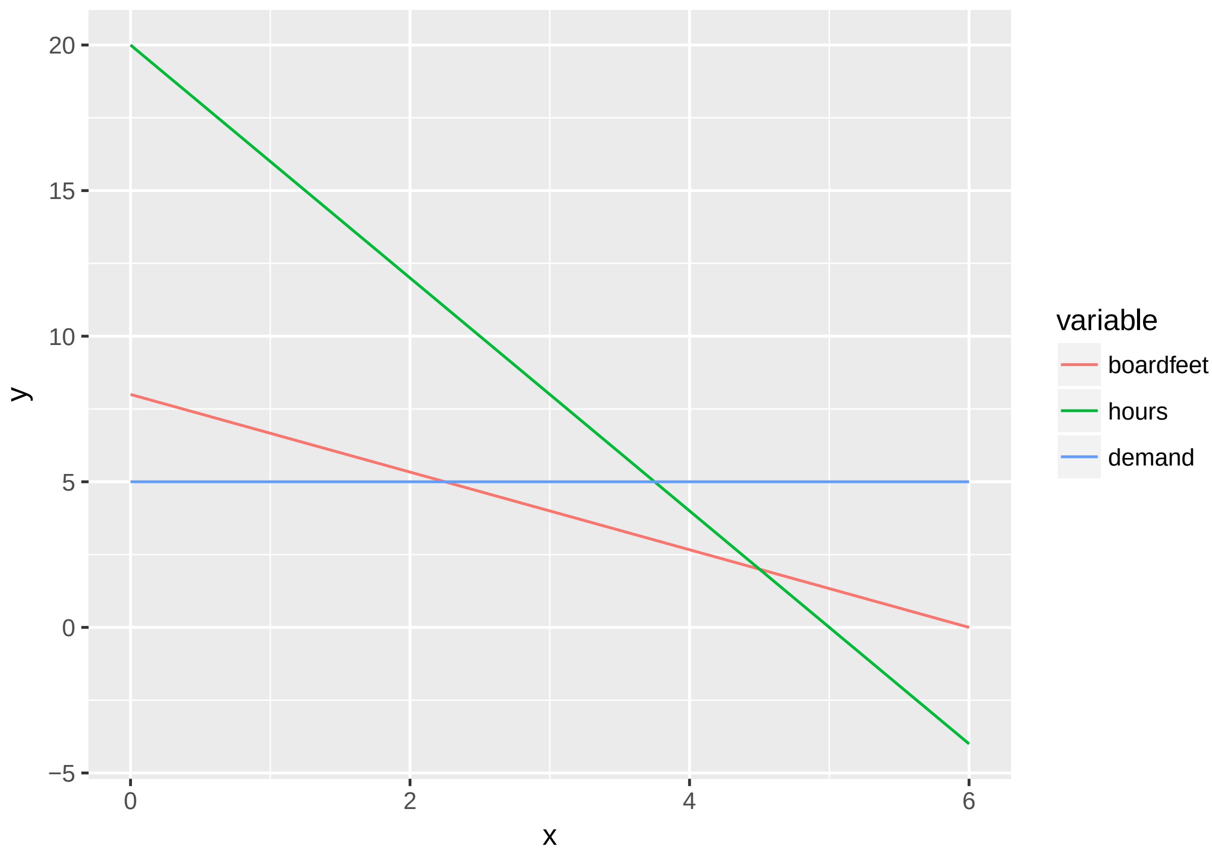
```
x <- seq(0, 6, by=0.1)
df1 <- data.frame(x = x, boardfeet = (48-8*x)/6, hours = 20-4*x, demand = 5)

library(reshape2)
```

```
## Warning: package 'reshape2' was built under R version 3.3.3
```

```
df1 <- melt(df1, id.vars="x", value.name="y")

ggplot(df1, aes(x=x, y=y, color=variable)) + geom_line()
```



Based on the feasible are in out graph we narrow our solution to possible three intersection points (0,5), (0,8), (2.2,5)

```
obj_f <- function(x,y){
  10*x + 35*y
}

(is1 <- obj_f(0,5))
```

```
## [1] 175
```

```r
(is2 <- obj_f(0,8))
```

```
## [1] 280
```

```r
(is3 <- obj_f(2.2,5))
```

```
## [1] 197
```

Objective function is maximized at intersection point (0,8) with a value of 280.