DATA 609 HW Week 9 & 11

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Contents

Week 9

Page 385: #1

Using the definition provided for the movement diagram, determine whether the following zero-sum games have a pure strategy Nash equilibrium. If the game does have a pure strategy Nash equilibrium, state the Nash equilibrium. Assume the row player is maximizing his payoffs which are shown in the matrices below.

a.

a.

		Colin	
		C1	C2
Rose	R1	10	10
	R2	5	0

Figure 1: 1.a

Solution

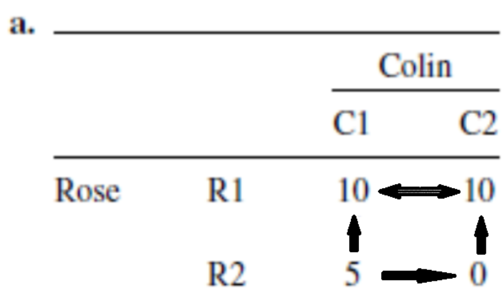


Figure 2: 1.a Solution

The game has a pure strategy Nash equilibrium, no matter what Colin does (C1 or C2) the payoff for Rose in !0 as long as she choses R1.

c.

c.

		Pitcher	
		Fastball	Knuckleball
Batter	Guesses fastball	.400	.100
	Guesses knuckleball	.300	.250

Figure 3: 1.c

Solution

c.

		Pitcher	
		Fastball	Knuckleball
Batter	Guesses fastball	.400	.100
	Guesses knuckleball	.300	.250

Figure 4: 1.c Solution

The game has a pure strategy Nash equilibrium, all the arrows points to .250 and no arrow exits that outcome. The pitcher has a dominant strategy of throwing a knuceball.

Page 404: #2.a

Build a linear programming model for each player's decisions and solve it both geometrically and algebraically. Assume the row player is maximizing his payoffs shown in the matrices below.

a.

		Colin	
		C1	C2
Rose	R1	10	10
	R2	5	0

Figure 5: 2.a

Solution

For Rose, we define the following variables:

$$
\begin{aligned}
P &= \text{Payoff} \\
x &= \text{Portion of time playing R1} \\
(1-x) &= \text{Portion of time playing R2}
\end{aligned}
$$

If Colin plays a pure $C1$ strategy, the expected value is,

$$EV(c1) = 10x + 5(1-x) = 5x + 5$$

If Colin plays a pure $C2$ strategy, the expected value is,

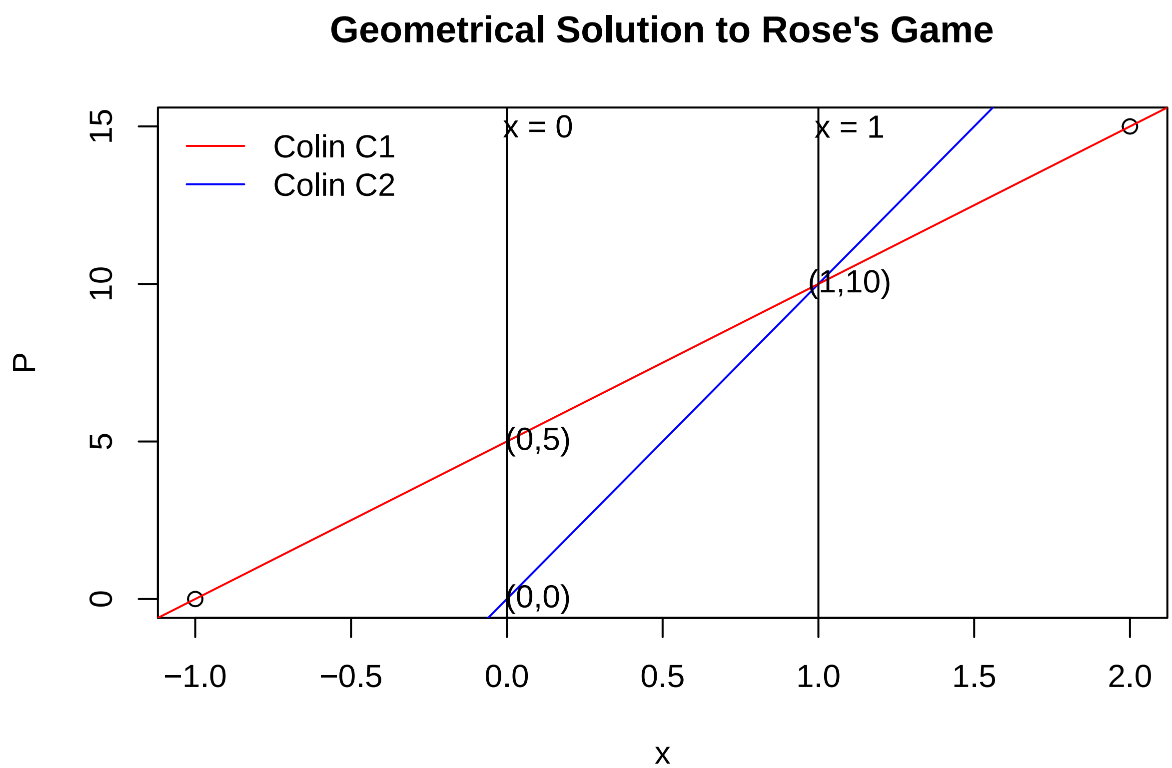
$$EV(c2) = 10x + 0(1-x) = 10x$$

We have the following linear program for Rose:

$$
\begin{aligned}
&\text{Maximize P} \\
&P \leq 5x + 5 \\
&P \leq 10x \\
&x \geq 0 \\
&x \leq 1
\end{aligned}
$$

```
plot(c(-1,2), c(0,15),xlab="x", ylab="P", main = "Geometrical Solution to Rose's Game")
abline(0, 10, col = "blue")
abline(5, 5, col = "red")
abline(v=0)
abline(v=1)
text(0.1, 15, label="x = 0")
text(1.1,15, label="x = 1")
```

```r
text(1.1,10, label="(1,10)")
text(0.1,5, label="(0,5)")
text(0.1,0, label="(0,0)")
legend("topleft", legend = c("Colin C1", "Colin C2"), col=c("red", "blue"), lty=1:1, bty = "n")
```



$$x = 0: \quad P = 10 \times 0 + 5 \times (1 - 0) = 5 \tag{1}$$
$$x = 1: \quad P = 10 \times 1 + 5 \times (1 - 1) = 10 \tag{2}$$
$$x = 0: \quad P = 10 \times 0 = 0 \tag{3}$$
$$x = 1: \quad P = 10 \times = 10 \tag{4}$$

For Colin, we define the following variables:

$$y = \text{Portion of time playing C1}$$
$$(1 - Y) = \text{Portion of time playing C2}$$

We can see from both geometrically and algebraically P takes maximum value of 10 when x =1, in other words, Rose plays R1.

We have the following linear program for Colin:

Minimize P

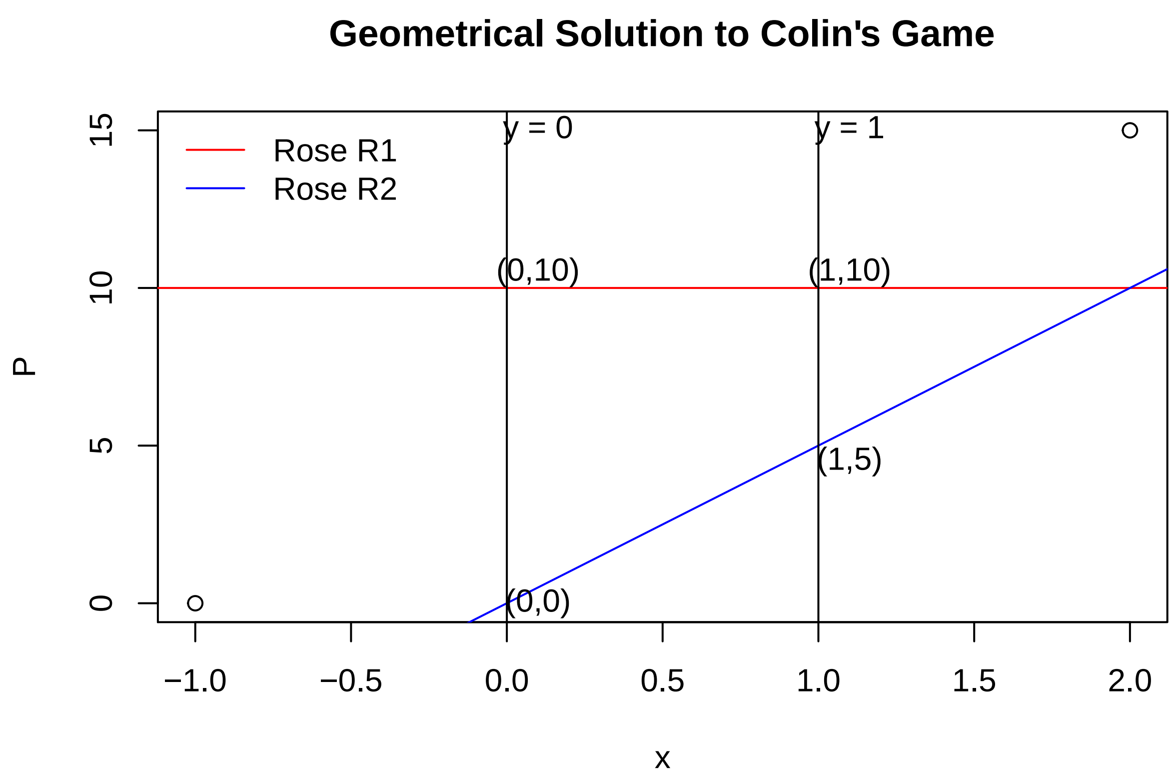
$P \leq 10$

$P \leq 5y$

$y \geq 0$

$y \leq 1$

```
plot(c(-1,2), c(0,15),xlab="x", ylab="P", main = "Geometrical Solution to Colin's Game")
abline(10, 0, col = "red")
abline(0, 5, col = "blue")
abline(v=0)
abline(v=1)
text(0.1, 15, label="y = 0")
text(1.1,15, label="y = 1")
text(1.1,10.5, label="(1,10)")
text(0.1,10.5, label="(0,10)")
text(1.1,4.5, label="(1,5)")
text(0.1,0, label="(0,0)")
legend("topleft", legend = c("Rose R1", "Rose R2"), col=c("red", "blue"), lty=1:1, bty = "n")
```



$$
\begin{align}
y = 0: &\qquad P = 10 \tag{5}\\
y = 1: &\qquad P = 10 \tag{6}\\
y = 0: &\qquad P = 5 \times 0 = 0 \tag{7}\\
y = 1: &\qquad P = 5 \times 1 = 5 \tag{8}
\end{align}
$$

Given our constraints we can see from both geometrically and algebraically it does not matter if Colin play pure C1 or pure C2 if Rose plays R1, P remains the same at 10.

Week 8

Page 469: #3

The following data were obtained for the growth of a sheep population introduced into a new environment on the island of Tasmania (adapted from J. Davidson, "On the Growth of the Sheep Population in Tasmania," *Trans. R. Soc. S. Australia* 62(1938): 342-346).

t (year)	1814	1824	1834	1844	1854	1864
P(t)	125	275	830	1200	1750	1650

a. Make an estimate of M by graphing $P(t)$.

b. Plot $ln[(P/(M-P)]$ against t. If a logistic curve seems reasonable, estimate rM and $t^{\star}$.

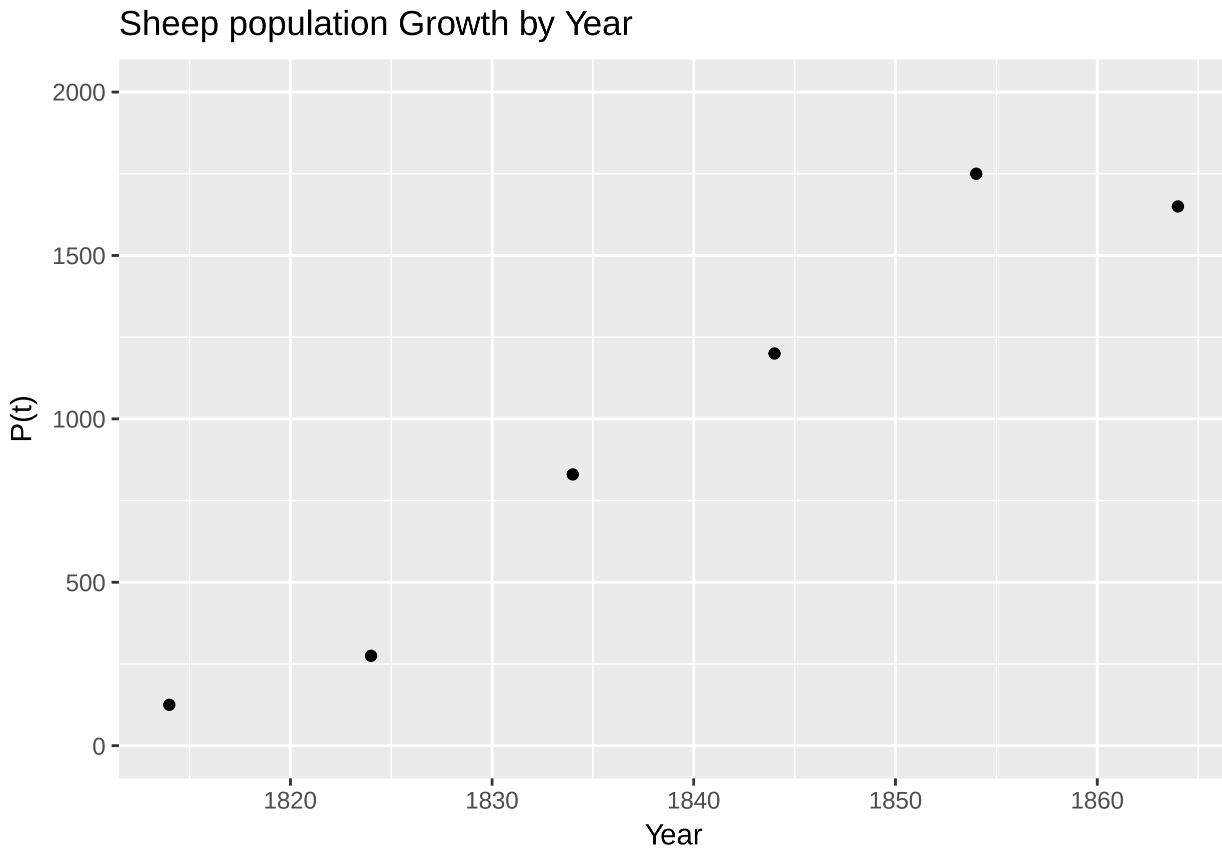
Solution

```
if (!require('ggplot2')) (install.packages('ggplot2'))
```

```
## Loading required package: ggplot2
```

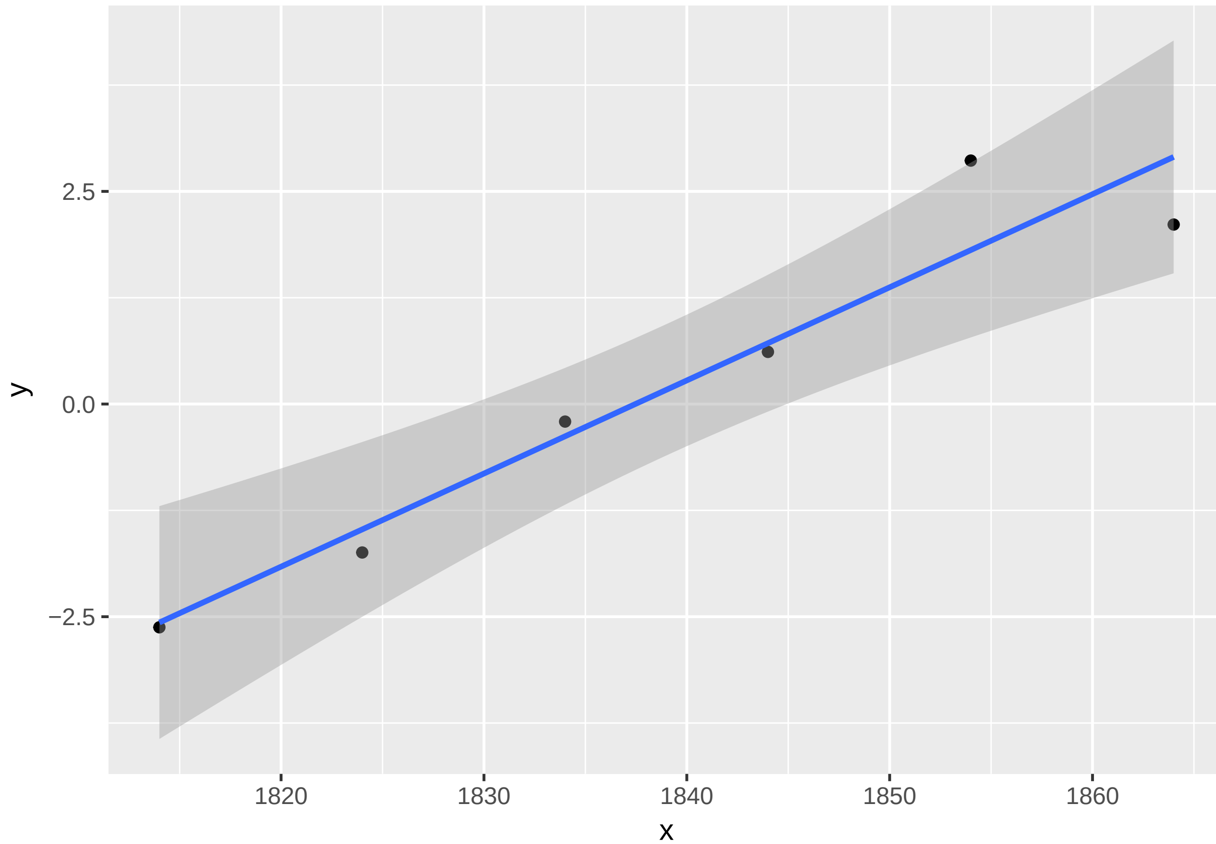
```
## Warning: package 'ggplot2' was built under R version 3.3.3
```

```
t <- c(1814, 1824, 1834, 1844, 1854, 1864)
p <- c(125, 275, 830, 1200, 1750, 1650)
mydf <- data.frame(t,p)

ggplot(mydf, aes(x=t, y=p)) + geom_point() +
  ggtitle('Sheep population Growth by Year') +
  labs(x = "Year", y = "P(t)") + ylim(0, 2000)
```



The population never exceeds 1750, after reaching 1750 it drops to 1650. There are only six data points and we wold assume 1750 is not the peak and we would estimate the maximum population $M \approx 1850$.

```
m <- 1850
lnp_mp <- log(p/(m-p))

ggplot(data.frame(x=t,y=lnp_mp),aes(x=x,y=y)) + geom_point() +
  stat_smooth(method="lm")
```



The linear model fits the data well, so we accept the logistic growth model is a reasonable approximation.

Now lets estimate the slope rM and $t^{\star}$

```
rm <- lm(lnp_mp~t)$coefficients["t"]
rm
```

```
##         t
## 0.1094742
```

```
tstar <- -(1/rm)*log(p[1]/(m-p[1]))
tstar + t[1]
```

```
##        t
## 1837.975
```

Half of our estimate is 925, which is our population estimate for 1838. The population of 1834 is 830 and 1844 is 1200, therefore the logistical model checks out.

Page 481 #1

a. Using the estimate that $d_b = 0.054v^2$, where 0.054 has dimension $ft.hr^2/mi^2$, show that the constant k in equation (11.29) has the value $19.9ft/sec^2$.

Equation (11.9):

$$d_b = \frac{-v_0^2}{2k} + \frac{v_0^2}{k} = \frac{v_0^2}{2k}$$

b. Using the data in Table 4.4 plot d_b in ft versus $v^2/2\ in\ ft^2/sec^2$ to estimate $1/k$ directly.

Solution

a.

From the equation we get:

$k = \frac{1}{2d_b}$

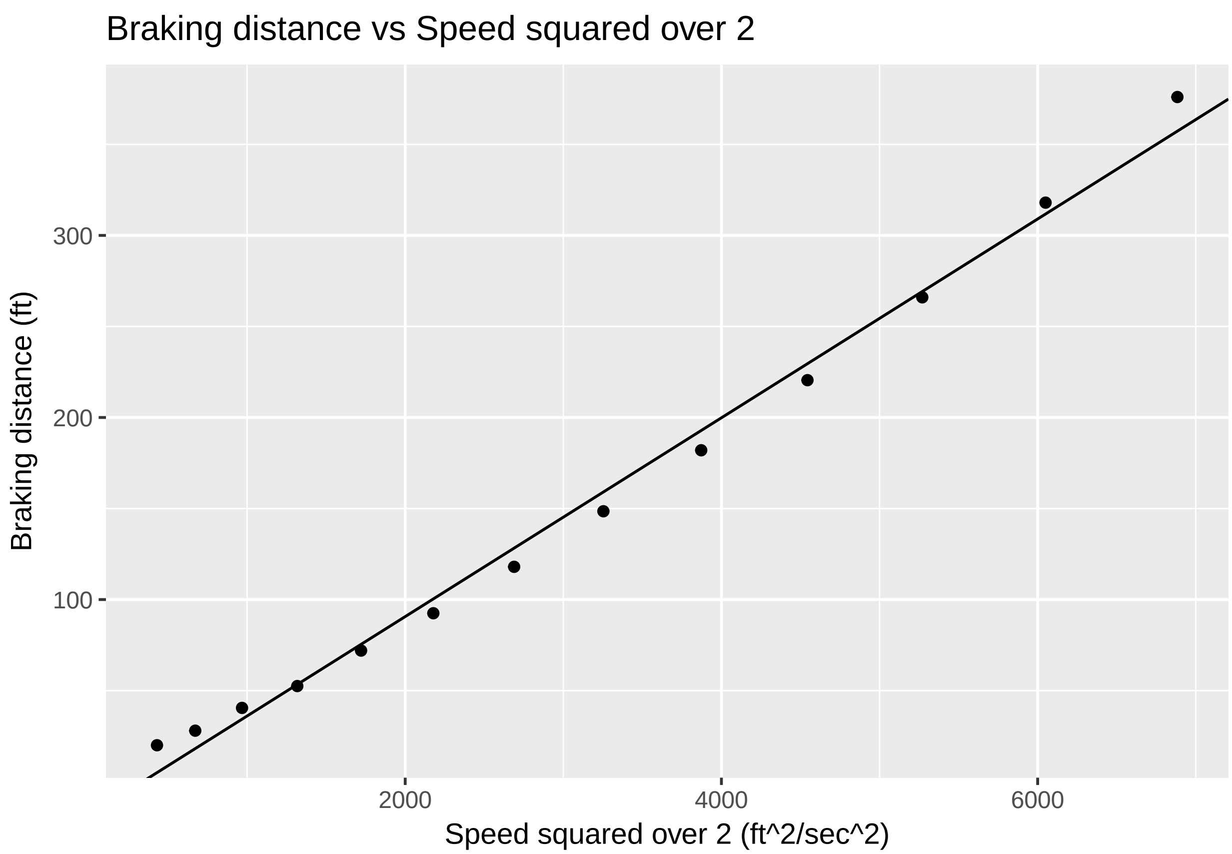
```
# convert units
sec_per_hr <- 60^2
ft_per_mile <- 5280

# convert 0.054
my_db <- 0.054 * sec_per_hr^2 / ft_per_mile^2

# equation
k = round(1/(2*my_db), 1)
k
```

```
## [1] 19.9
```

b.

The table 4.4 contains data on harvesting blue crabs, so we would assume there is a typo in the question and the correct table will be table 2.4 which contains data on braking distance.

```
speed <- c(20,25,30,35,40,45,50,55,60,65,70,75,80)
db <- c(20,28,40.5,52.5,72,92.5,118,148.5,182,220.5,266,318,376)

# convert unit
ft_per_sec <- speed * (5280/60^2)

# speed squared over 2
v2_2 <- ft_per_sec^2/2

my_df <- data.frame(ft_per_sec, db)

# calculate slope intercept for line of best fit
my_model <- lm(db~v2_2, data = my_df)

# plot
ggplot(my_df, aes(v2_2, db)) + geom_point() +
  ggtitle("Braking distance vs Speed squared over 2") +
  xlab("Speed squared over 2 (ft^2/sec^2)") +
  ylab("Braking distance (ft)") +
  geom_abline(intercept = coef(my_model)[1] , slope = coef(my_model)[2] )
```




```
k <- 1/coef(my_model)[2]
k
```

```
##     v2_2
## 18.31807
```