

# 重積分的變數變換

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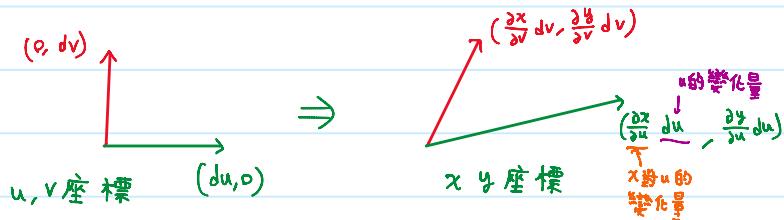
單變數：若  $x = g(u)$

$$\int_{x=g(a)}^{x=g(b)} f(x) dx = \int_{u=a}^{u=b} f(g(u)) g'(u) du$$

多變數：若  $x = g(u, v)$ ,  $y = h(u, v)$

$$\iint_R f(x, y) dx dy = \iint_G f(g(u, v), h(u, v)) \frac{J(u, v)}{\text{Jacobian}} du dv$$

Jacobian



$$\begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix} \begin{bmatrix} du & 0 \\ 0 & dv \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial u} du & \frac{\partial x}{\partial v} dv \\ \frac{\partial y}{\partial u} du & \frac{\partial y}{\partial v} dv \end{bmatrix}$$

← 座標轉換矩陣

$$\text{Jacobian} = J(u, v) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

行列式值，代表面積的改變量