積分技巧-三角代換

2021年5月3日 星期一　　下午5:25

① $\int \sqrt{a^2-x^2}\,dx$ ⇒ 用 $x=a\sin\theta$，因為 $1-\sin^2\theta=\cos^2\theta$

② $\int \sqrt{a^2+x^2}\,dx$ ⇒ 用 $x=a\tan\theta$，因為 $1+\tan^2\theta=\sec^2\theta$

③ $\int \sqrt{x^2-a^2}\,dx$ ⇒ 用 $x=a\sec\theta$，因為 $\sec^2x-1=\tan^2x$

有用的三角函數規則

(1) $\sin^2x+\cos^2x=1$

(2) $\sec^2x-\tan^2x=1$

(3) $\csc^2x-\cot^2x=1$

(4) $\cos 2x=\cos^2x-\sin^2x=1-2\sin^2x=2\cos^2x-1$

$$\int \frac{\sqrt{4-x^2}}{x^2}\,dx$$

$$\Rightarrow \int \frac{\sqrt{2^2-x^2}}{x^2}\,dx$$

let $x=2\sin\theta$

$dx=2\cos\theta\,d\theta$

$$\int \frac{\sqrt{2^2-(2\sin\theta)^2}}{(2\sin\theta)^2}\,2\cos\theta\,d\theta$$

$$=\int \frac{\sqrt{4-4\sin^2\theta}}{4\sin^2\theta}\,2\cos\theta\,d\theta$$

$$=\int \frac{2\sqrt{1-\sin^2\theta}}{2\sin^2\theta}\cos\theta\,d\theta$$

$$=\int \frac{\cos\theta}{\sin^2\theta}\cos\theta\,d\theta$$

$$=\int \frac{\cos^2\theta}{\sin^2\theta}\,d\theta$$

$$=\int \frac{1-\sin^2\theta}{\sin^2\theta}\,d\theta$$

$$=\int \left[\frac{1}{\sin^2\theta}-1\right]d\theta$$

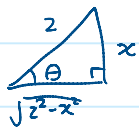
$$=\int (\csc^2\theta-1)\,d\theta$$

$$=-\cot\theta-\theta+C$$

$$=-\frac{\sqrt{2^2-x^2}}{x}-\arcsin\left(\frac{x}{2}\right)+C$$

$x=2\sin\theta$

$\sin\theta=\frac{x}{2}$



$\Rightarrow \cot\theta=\frac{\sqrt{2^2-x^2}}{x}$