多變數函式

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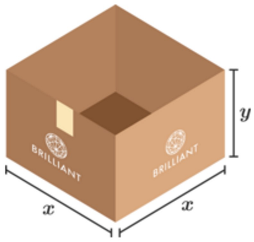
Imagine a box with base length x and height y. The box doesn't have a lid. The cost of manufacturing is directly proportional to the amount of material (or surface area) of the box.



What is the surface area of the outside of the box as a function of base width x and height y?

$$A(x,y) = x^2 + 4xy$$

The surface area of the box A depends on the base width x and the height y.

We express this relationship explicitly as

$$A(x, y) = x^2 + 4xy.$$

Writing the surface area as $A(x, y)$ tells us that it is a function of both base width and height.

A company wants to produce such a box with base width at least 4 units and a height of at least 1 unit. If box material is 4 dollars per unit area, what's the cost of the cheapest box that can be produced?

$$C(x,y) = 4 \times A(x,y)$$

$$= 4(x^2+4xy) = 4x^2+16xy$$

$$\Rightarrow C(4,1) = 4(4)^2+16(4)(1) = 128$$

Now let's put our calculus to good use.

Suppose the company wants to produce the cheapest possible box with a fixed volume of $x^2y = 4$ cubic units.

Besides being positive, there's no restriction on the base width x or the height y this time.

If the cost is $C(x, y) = 4x^2 + 16xy$, what is the minimum output in this case?

$x^2 y = 4 \Rightarrow y = \frac{4}{x^2}$

$C(x,y) = 4x^2 + 16xy$

$\Rightarrow c(x) = 4x^2 + 16x\left(\frac{4}{x^2}\right)$

$= 4x^2 + \frac{64}{x} = 4x^2 + 64x^{-1}$

微分 $\Rightarrow c'(x) = 8x - 64x^{-2}$

找極值 $\rightarrow 8x - 64x^{-2} = 0 \quad 8x^3 = 64 \quad x^3 = 8$

$\Rightarrow x = 2$

$c''(x) = 8 + 128x^{-3} \Rightarrow c''(2) > 0$ → 確定是 minimum

代回原式 → $C(2) = 4(2^2) + 16(2)\left(\frac{4}{4}\right) = 16 + 32 = 48$