



Tutorial: Third Normal Form

1. Consider the schema $R = \{A, B, C, D\}$ with the set of functional dependencies $\Sigma = \{\{A\} \rightarrow \{B, C, D\}, \{C\} \rightarrow \{D\}\}$.
 - (a) Is the decomposition into $\sigma = \{R_1(A, B, D), R_2(A, C)\}$ a dependency preserving decomposition?

Solution: NO.

The projection of Σ on R_1 (we can denote this as $\Sigma|_{R_1}$) is $\{\{A\} \rightarrow \{B, D\}\}$. The projection $\Sigma|_{R_2}$ is $\{\{A\} \rightarrow \{C\}\}$.

The union of the projection is $\Sigma_{\cup} = \Sigma|_{R_1} \cup \Sigma|_{R_2} = \{\{A\} \rightarrow \{B, D\}, \{A\} \rightarrow \{C\}\}$. We need to check if it is equivalent to Σ .

Note that by construction Σ_{\cup} is logically entailed by Σ . So we only need to show that Σ is logically entailed by Σ_{\cup} .

- Consider $\{\{A\} \rightarrow \{B, C, D\}\}$.
 - $\{A\}^+$ with respect to Σ_{\cup} is $\{A, B, C, D\}$. $\{A\} \rightarrow \{B, C, D\}$ is logically entailed by Σ_{\cup} .
- Consider $\{\{C\} \rightarrow \{D\}\}$.
 - $\{C\}^+$ with respect to Σ_{\cup} is $\{C\}$. $\{C\} \rightarrow \{D\}$ is not logically entailed by Σ_{\cup} .

Therefore, $\Sigma \not\equiv \Sigma_{\cup}$. The decomposition is not dependency preserving.

- (b) Is the decomposition into $\sigma = \{R_1(A, B, C), R_2(C, D)\}$ a dependency preserving decomposition?

Solution: YES.

The projection of Σ on R_1 (we can denote this as $\Sigma|_{R_1}$) is $\{\{A\} \rightarrow \{B, C\}\}$. The projection $\Sigma|_{R_2}$ is $\{\{C\} \rightarrow \{D\}\}$.

The union of the projection is $\Sigma_{\cup} = \Sigma|_{R_1} \cup \Sigma|_{R_2} = \{\{A\} \rightarrow \{B, C\}, \{C\} \rightarrow \{D\}\}$. We need to check if it is equivalent to Σ .

- Consider $\{\{A\} \rightarrow \{B, C, D\}\}$.
 - $\{A\}^+$ with respect to Σ_{\cup} is $\{A, B, C, D\}$. $\{A\} \rightarrow \{B, C, D\}$ is logically entailed by Σ_{\cup} .
- Consider $\{\{C\} \rightarrow \{D\}\}$.
 - $\{C\}^+$ with respect to Σ_{\cup} is $\{C, D\}$. $\{C\} \rightarrow \{D\}$ is logically entailed by Σ_{\cup} .

Therefore, $\Sigma \equiv \Sigma_{\cup}$. The decomposition is dependency preserving.

2. Consider the schema $R = \{A, B, C, D, E\}$ with a set of functional dependencies $\Sigma = \{\{A\} \rightarrow \{B, D, E\}, \{C, D\} \rightarrow \{A\}, \{E\} \rightarrow \{B, D\}\}$.

(a) Is R in third normal form with respect to Σ ?

Solution: R is not in third normal form with respect to Σ . Consider the following functional dependency $\{E\} \rightarrow \{B\}$. This violates the third normal form property of R with respect to Σ because

- $\{B\} \not\subseteq \{E\}$ so $\{E\} \rightarrow \{B\}$ is non-trivial.
- $\{E\}^+ = \{B, D, E\}$ so $\{E\}$ is not the superkey of R .
- The keys of R are $\{A, C\}$, $\{C, D\}$, and $\{C, E\}$. So B is not a prime attribute of R with respect to Σ .

Therefore, R is not in third normal form with respect to Σ .

- (b) If R is not in third normal form with respect to Σ , find a third normal form decomposition of R with respect to Σ that are both dependency preserving and lossless join.

Solution: To guarantee both properties, we need to use the 3NF synthesis algorithm. The first step is to find the minimal cover.

- $\Sigma_1 = \{\{A\} \rightarrow \{B\}, \{A\} \rightarrow \{D\}, \{A\} \rightarrow \{E\}, \{C, D\} \rightarrow \{A\}, \{E\} \rightarrow \{B\}, \{E\} \rightarrow \{D\}\}$
- $\Sigma_2 = \{\{A\} \rightarrow \{B\}, \{A\} \rightarrow \{D\}, \{A\} \rightarrow \{E\}, \{C, D\} \rightarrow \{A\}, \{E\} \rightarrow \{B\}, \{E\} \rightarrow \{D\}\}$
- $\Sigma_3 = \{\{A\} \rightarrow \{E\}, \{C, D\} \rightarrow \{A\}, \{E\} \rightarrow \{B\}, \{E\} \rightarrow \{D\}\}$

Then we find the canonical cover.

- $\Sigma_4 = \{\{A\} \rightarrow \{E\}, \{C, D\} \rightarrow \{A\}, \{E\} \rightarrow \{B, D\}\}$

Using canonical cover, we construct the table from each functional dependency.

- $\delta = \{R_1(A, E), R_2(A, C, D), R_3(B, D, E)\}$

Lastly, to ensure lossless join decomposition, we need to check if at least one of the key is already in one of the fragments. Here, either $\{A, C\}$ or $\{C, D\}$ is in R_2 , so we do not need to construct a table for the key.

- (c) Is your decomposition in part (b) above in Boyce-Codd normal form with respect to Σ ?

Solution: R_2 is not in Boyce-Codd normal form because $\{A\} \rightarrow \{D\}$.

- $\{D\} \not\subseteq \{A\}$ so $\{A\} \rightarrow \{D\}$ is non-trivial.
- $\{A\}^+ = \{A, B, D, E\}$ so $\{A\}$ is not the superkey of R_2 .

However, do note that there is a possibility that by using 3NF synthesis algorithm, the decomposed schema is in Boyce-Codd normal form. In which case, we are lucky because this is dependency preserving Boyce-Codd normal form decomposition. Unfortunately, it is not always the case.