Assignments II

Exercise 1 *Write a* MATLAB*-function which determines the zero of a function using the Secant method. This is an iterative method in which the* next *point is determined by drawing a straight line through the* previous two *points on the graph and taking the zero of that line.*

The input consists of a function of a real variable, and two starting points. The output is, first, the calculated zero and, second (i.e. optional), a vector of approximate values obtained in the iterations.

1. It is probably a smart idea to have a good look at the function `newton.m`.
2. Make the function so that the maximum number of iteration steps is an optional input.
3. Make the function so that a user can give the desired precision as optional input parameter. What do you choose as a measure for this precision?
4. Can you compare the order of convergence to that of Newton's method?
5. What are the advantages and drawbacks of the Secant method compared to Newton's method?

Exercise 2 *Write a* MATLAB*-function which determines the zero of a function using the modified Newton method.*

The inputs are a function, its derivative, and a starting point. The outputs are the calculated zero, an estimate of the order of the zero, and a vector of all the iterations. Optional inputs are a measure for the precision and the maximum number of iterations.

1. You can start out from the function `newton.m`.
2. You should estimate the order of the zero from the sequence of values in the iterations.
3. Can you, based on an example, say something about the order of convergence?
4. Compare with the classical Newton method.

Exercise 3 *Write a* MATLAB*-function which applies Newton's method to the complex function* $f(z) = z^3 - 1$ *with initial data in a given rectangle in the complex plane.*

The input consists of a vector of two complex numbers which form the corners of a rectangle in the complex plane. Furthermore, there is an input vector indicating the number of discretization steps in both directions in the complex plane. The number of steps in the Newton iteration is an optional input. There is no output, but the function produces a picture with colored points in a rectangle, where the color of the point z_0 indicates towards which of the three zeros of $z^3 - 1$ the Newton iterations converge with starting value z_0.

1. Determine for each starting point to which zero the Newton iterations converge.
2. It may be useful to have the Newton iterations run on a matrix of starting values all at once.
3. For each of the three limiting zeros, give the corresponding starting points a color.
4. Keep everything well-organized and efficient.