Assignments I

Exercise 1 *Write a* MATLAB*-function which calculates the sine of a number using the Taylor series around* 0.

The input is a number x, the output is an approximation of $\sin(x)$.

1. You are *not* meant to use the built-in function `sin`, nor `pi`.
2. How many terms of the series do you need, i.e., how accurate is the answer?
3. Which term in the series is largest?
4. What may go wrong?
5. For which values of x does the function produce reliable results?
6. Finally, adapt the function so that it also works for vectors (as input and output).

Exercise 2 *Write a* MATLAB*-function which calculates the solution of the differential equation*

$$m\frac{d^2x}{dt^2}(t) + \beta\frac{dx}{dt}(t) + \alpha x(t) = 0$$

with initial conditions $x(0) = x_0$ *and* $\frac{dx}{dt}(0) = v_0$.

The equation describes a damped harmonic oscillator. The idea is to use the analytic solution (*not* the built-in ODE solvers). The inputs are the mass $m > 0$, the friction coefficient $\beta \geq 0$, the spring constant $\alpha \geq 0$, the initial position $x_0 \in \mathbb{R}$ and the initial velocity $v_0 \in \mathbb{R}$. The output is of the form `[t,x]` with t and x column vectors.

1. Which different cases for the parameter values do you distinguish?
2. What is a good choice for the time interval $[0, T]$ on which you want to depict the solution?
3. Finally, adapt the function so that it also works for (column or/and row) vectors x_0 and v_0 (for fixed parameter values). You still want to be able to plot the output with `plot(t,x)`.

Exercise 3 *The logistic map is given by*

$$x_{n+1} = \lambda x_n(1 - x_n)$$

with $x_0 \in [0, 1]$ *and* $\lambda \in [0, 4]$. *Write a* MATLAB*-function which produces a picture of the "attractor" for a sequence of values* $\lambda \in [\lambda_{\min}, \lambda_{\max}]$.

The attractor is (for fixed λ) the collection of limiting values of the sequence x_n for large n. The inputs are $\lambda_{\min}$, $\lambda_{\max}$ and N, the number of values of λ between $\lambda_{\min}$ and $\lambda_{\max}$ for which the attractor is calculated. There is no output, but the function produces a picture with $\lambda \in [\lambda_{\min}, \lambda_{\max}]$ along the horizontal axis and values of x_n for "large" n along the vertical axis.

1. Which value of x_0 do you choose?
2. Which values of n do you use in the picture?
3. Check that the input satisfies $0 \leq \lambda_{\min} \leq \lambda_{\max} \leq 4$.
4. Make sure the figure looks nice.
5. Make the input variable N optional: the function should also work if only $\lambda_{\min}$ and $\lambda_{\max}$ are given as input variables.