bacs_hw12

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110034002 walked me through the parameters of the principal(), and i realizes that h2 stands for "commonality", u2 "uniqueness", and com "item complexity"

109048231 notified me that in Question 3e, the factor loadings themselves reflected the correlation-like relationship. And clearly, we got different meanings from the component (from 3 to 2 in Question 3 case) since the greater-than-0.7 loadings shifted significantly.

Question 1

Earlier, we examined a dataset from a security survey sent to customers of e-commerce websites. However, we only used the "eigenvalue > 1" criteria and the "elbow rule" on the screeplot to find a suitable number of components. Let's perform a parallel analysis as well this week

```
sq <- read.csv('security_questions.csv')
pca_sq <- prcomp(sq, scale. = TRUE)
summary(pca_sq)
```

```
## Importance of components:
##                           PC1     PC2     PC3     PC4     PC5     PC
6     PC7
## Standard deviation     3.0514 1.26346 1.07217 0.87291 0.82167 0.7820
9 0.70921
## Proportion of Variance 0.5173 0.08869 0.06386 0.04233 0.03751 0.0339
8 0.02794
## Cumulative Proportion  0.5173 0.60596 0.66982 0.71216 0.74966 0.7836
5 0.81159
##                            PC8     PC9    PC10    PC11    PC12    PC1
3    PC14
## Standard deviation     0.68431 0.67229 0.6206 0.59572 0.54891 0.5406
3 0.51200
## Proportion of Variance 0.02602 0.02511 0.0214 0.01972 0.01674 0.0162
4 0.01456
## Cumulative Proportion  0.83760 0.86271 0.8841 0.90383 0.92057 0.9368
1 0.95137
##                           PC15   PC16   PC17   PC18
## Standard deviation     0.48433 0.4801 0.4569 0.4489
## Proportion of Variance 0.01303 0.0128 0.0116 0.0112
## Cumulative Proportion  0.96440 0.9772 0.9888 1.0000
```

1a

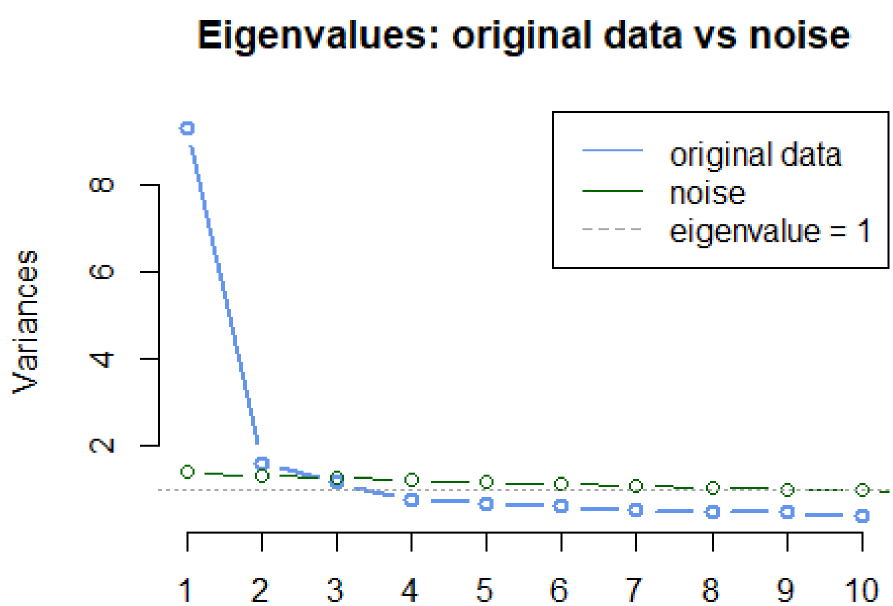
Show a single visualization with scree plot of data, scree plot of simulated noise (use average eigenvalues of ≥ 100 noise samples), and a horizontal line showing the eigenvalue = 1 cutoff.

```
set.seed(1111313131)

sim_noise_ev <- function(n,p){
  noise <- data.frame(replicate(p, rnorm(n)))
  eigen(cor(noise))$values
}
evalues_noise <- replicate(100 ,sim_noise_ev(406,18))
evalues_mean <- apply(evalues_noise,1,mean)


# Plotting
screeplot(pca_sq, col= "cornflowerblue", type = "lines", lwd = 2, main
= "Eigenvalues: original data vs noise")
lines(evalues_mean, type = "b",col = "darkgreen")
abline(h = 1,col = "darkgray", lty = "dotted")

legend("topright", legend=c("original data", "noise", "eigenvalue = 1"),
       col=c("cornflowerblue", "darkgreen", "darkgray"), lty=c(1, 1, 2))
```



1b

Question

How many dimensions would you retain if we used Parallel Analysis?

Answer

As observed in the screeplot above, we should only **take PC1 and PC2**. Starting from PC3, the variances of the garbage data (noise) start surpassing those of the original data.

Question 2

Earlier, we treated the underlying dimensions of the security dataset as composites and examined their eigenvectors (weights). Now, let's treat them as factors and examine factor loadings (use the principal() method from the psych package)

2a

Question

Looking at the loadings of the first 3 principal components, to which components does each item seem to best belong?

```
sq <- read.csv('security_questions.csv')
library(psych)
sq_principal <- principal(sq, nfactor = 3, rotate= "none", scores = TRU
E)
sq_principal$loadings[,1:3]
```

```
##            PC1         PC2          PC3
## Q1  0.8169846 -0.13941235 -0.002115927
## Q2  0.6726084 -0.01375526  0.089174403
## Q3  0.7655215 -0.03269651  0.089686106
## Q4  0.6233733  0.64307826  0.108031860
## Q5  0.6900841 -0.03126466 -0.542354570
## Q6  0.6828029 -0.10462094  0.207232000
## Q7  0.6566249 -0.31763196  0.324176779
## Q8  0.7861054  0.04235983 -0.343212951
## Q9  0.7230295 -0.23164618  0.203556038
## Q10 0.6861529 -0.09868038 -0.532678749
## Q11 0.7529735 -0.26100673  0.172516196
## Q12 0.6303505  0.63753124  0.121522834
## Q13 0.7119085 -0.06463837  0.084335919
## Q14 0.8114677 -0.09970016  0.156787046
## Q15 0.7040428  0.01057936 -0.332546876
## Q16 0.7575616 -0.20281591  0.183170175
## Q17 0.6175336  0.66426051  0.110061160
## Q18 0.8067284 -0.11360432 -0.065189145
```

Answer

The first principal component

2b

Question

How much of the total variance of the security dataset do the first 3 PCs capture?

```
sq_principal$loadings

## 
## Loadings:
##     PC1    PC2    PC3   
## Q1   0.817 -0.139       
## Q2   0.673              
## Q3   0.766              
## Q4   0.623  0.643  0.108
## Q5   0.690        -0.542
## Q6   0.683 -0.105  0.207
## Q7   0.657 -0.318  0.324
## Q8   0.786        -0.343
## Q9   0.723 -0.232  0.204
## Q10  0.686        -0.533
## Q11  0.753 -0.261  0.173
## Q12  0.630  0.638  0.122
## Q13  0.712              
## Q14  0.811         0.157
## Q15  0.704        -0.333
## Q16  0.758 -0.203  0.183
## Q17  0.618  0.664  0.110
## Q18  0.807 -0.114       
## 
##                  PC1   PC2   PC3
## SS loadings    9.311 1.596 1.150
## Proportion Var 0.517 0.089 0.064
## Cumulative Var 0.517 0.606 0.670
```

Answer

PC1: 0.517 PC2: 0.089 PC3: 0.064 (individual)

0.670 (cumulative)

2c

Question

Looking at commonality and uniqueness, which items are less than adequately explained by the first 3 principal components?

```
sq_principal$communality

##        Q1        Q2        Q3        Q4        Q5        Q6        Q
7        Q8
## 0.6869041 0.4605433 0.5951359 0.8138147 0.7713420 0.5201104 0.637136
9 0.7375512
##        Q9       Q10       Q11       Q12       Q13       Q14       Q1
5       Q16
## 0.6178667 0.7642903 0.6648554 0.8185557 0.5181043 0.6930021 0.606375
6 0.6485852
##       Q17       Q18
## 0.8347032 0.6679663
```

Answer

Commonality (h2) + uniqueness(u2) =1 , and high uniqueness(u2) means data being less explained. It is known that the first three PCs capture 0.67 of total variance, and items whose commonality is less than 0.67 is exactly what we are looking for: Q2, 3, 6, 7, 9, 11, 13, 15, 16, 18.

2d

Question

How many measurement items share similar loadings between 2 or more components?

```
sq_principal$complexity

##       Q1       Q2       Q3       Q4       Q5       Q6       Q7
 Q8
## 1.058202 1.035995 1.031144 2.055762 1.899001 1.233397 1.959957 1.374
540
##       Q9      Q10      Q11      Q12      Q13      Q14      Q15
Q16
## 1.373796 1.932541 1.351862 2.072501 1.044775 1.105810 1.425577 1.266
376
##      Q17      Q18
## 2.047594 1.052956
```

Answer

It can be seen that Q4, 12, 17 have their item complexity value greater than 2.

2e

Question

Can you interpret a ‘meaning’ behind the first principal component from the items that load best upon it? (see the wording of the questions of those items)

Answer

The patterns looks no clear for interpretation, but my guess is that the first principal component vaguely captures “confidentiality”

Question 3

To improve interpretability of loadings, let’s rotate our principal component axes using the varimax technique to get rotated components (extract and rotate only three principal components)

3a

Question

Individually, does each rotated component (RC) explain the same, or different, amount of variance than the corresponding principal components (PCs)?

Answer

Different.

PC1 = 0.517 , RC1 = 0.312

PC2 = 0.089 , RC2 = 0.164

PC3 = 0.064 , RC3 = 0.194

```
sq_pca_rot <- principal(sq, nfactor = 3, rotate = "varimax", scores = T
RUE)
sq_pca_rot$loadings

##
## Loadings:
##     RC1   RC3   RC2
## Q1  0.660 0.450 0.221
## Q2  0.544 0.286 0.288
## Q3  0.621 0.337 0.311
## Q4  0.218 0.193 0.854
## Q5  0.244 0.828 0.162
## Q6  0.652 0.199 0.234
## Q7  0.790 0.103
## Q8  0.382 0.706 0.305
## Q9  0.738 0.234 0.138
## Q10 0.277 0.823 0.102
## Q11 0.757 0.278 0.118
## Q12 0.233 0.186 0.854
## Q13 0.593 0.315 0.259
## Q14 0.719 0.310 0.283
## Q15 0.342 0.656 0.244
## Q16 0.740 0.267 0.174
## Q17 0.205 0.187 0.870
```

```
## Q18 0.609 0.495 0.227
##
##                   RC1   RC3   RC2
## SS loadings    5.613 3.490 2.954
## Proportion Var 0.312 0.194 0.164
## Cumulative Var 0.312 0.506 0.670
```

3b

Question

Together, do the three rotated components explain the same, more, or less cumulative variance as the three principal components combined?

Answer

The same. Proven by the fact that cumulative variance = 0.67.

3c

Question

Looking back at the items that shared similar loadings with multiple principal components (#2d), do those items have more clearly differentiated loadings among rotated components?

Answer

Yes.

```
sq_pca_rot$loadings

##
## Loadings:
##     RC1   RC3   RC2
## Q1  0.660 0.450 0.221
## Q2  0.544 0.286 0.288
## Q3  0.621 0.337 0.311
## Q4  0.218 0.193 0.854
## Q5  0.244 0.828 0.162
## Q6  0.652 0.199 0.234
## Q7  0.790 0.103
## Q8  0.382 0.706 0.305
## Q9  0.738 0.234 0.138
## Q10 0.277 0.823 0.102
## Q11 0.757 0.278 0.118
## Q12 0.233 0.186 0.854
## Q13 0.593 0.315 0.259
## Q14 0.719 0.310 0.283
## Q15 0.342 0.656 0.244
## Q16 0.740 0.267 0.174
## Q17 0.205 0.187 0.870
```

```
## Q18 0.609 0.495 0.227
##
##                    RC1   RC3   RC2
## SS loadings      5.613 3.490 2.954
## Proportion Var 0.312 0.194 0.164
## Cumulative Var 0.312 0.506 0.670
```

3d

Question

Can you now more easily interpret the "meaning" of the 3 rotated components from the items that load best upon each of them? (see the wording of the questions of those items)

Answer

RC1: Q7, Q9, Q11, Q14, Q16 –> "personal information"

RC2: Q4,Q12,Q17 –> "evidence showing the transaction is not denied"

RC3: Q5,Q8,Q10 –> "authenticity & user-to-website security"

3e

Question

If we reduced the number of extracted and rotated components to 2, does the meaning of our rotated components change?

Answer

Yes. It can be observed that both RC1 and RC2 (column) have different questions (row) yield correlations greater than 0.7, so the underlying meanings might have changed.

```r
sq_pca_rot <- principal(sq, nfactor = 2, rotate = "varimax", scores = T
RUE)
sq_pca_rot$loadings
```

```
##
## Loadings:
##     RC1   RC2
## Q1  0.783 0.271
## Q2  0.596 0.312
## Q3  0.687 0.340
## Q4  0.236 0.864
## Q5  0.620 0.305
## Q6  0.649 0.237
## Q7  0.728
## Q8  0.668 0.416
## Q9  0.745 0.145
```

```
## Q10 0.649 0.244
## Q11 0.786 0.134
## Q12 0.245 0.862
## Q13 0.655 0.286
## Q14 0.759 0.304
## Q15 0.612 0.348
## Q16 0.762 0.187
## Q17 0.221 0.880
## Q18 0.762 0.289
##
##                  RC1   RC2
## SS loadings    7.521 3.387
## Proportion Var 0.418 0.188
## Cumulative Var 0.418 0.606
```