bacs_hw8

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Question 1

We will use the interactive_regression() function from CompStatsLib again.

Question

Comparing scenarios 1 and 2, which do we expect to have a stronger R^2 ?

Answer

Scenario 1

1b

Question

Comparing scenarios 3 and 4, which do we expect to have a stronger R^2 ?

Answer

Scenario 3

1c

Question

Comparing scenarios 1 and 2, which do we expect has bigger/smaller SSE, SSR, and SST? (intuitively)

Answer

SSE : Scenario 2 has larger SSE than Scenario 1 does

SSR : Scenario 1 has larger SSE than Scenario 2 does

SST : likely to be slightly higher in Scenario 2 due to greater overall variance

1d

Question

Comparing scenarios 3 and 4, which do we expect has bigger/smaller SSE, SSR, and SST? (intuitively)

Answer

SSE : Scenario 4 has larger SSE than Scenario 3 does

SSR : Scenario 3 has larger SSE than Scenario 4 does

SST : likely to be slightly higher in Scenario 4 due to greater variance

Question 2

2a

Use the lm() function to estimate the regression model Salary ~ Experience + Score + Degree. Show the beta coefficients, R^2, and the first 5 values of y ($fitted.values) and ($residuals)

```
salary_data <- read.table("programmer_salaries.txt", header=TRUE, sep="
\t")
regr <- lm(Salary ~ Experience + Score + Degree, data=salary_data)
summary(regr)

##
## Call:
## lm(formula = Salary ~ Experience + Score + Degree, data = salary_dat
a)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.8963 -1.7290 -0.3375  1.9699  5.0480
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   7.9448     7.3808   1.076   0.2977
## Experience    1.1476     0.2976   3.856   0.0014 **
## Score         0.1969     0.0899   2.191   0.0436 *
## Degree        2.2804     1.9866   1.148   0.2679
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.396 on 16 degrees of freedom
## Multiple R-squared:  0.8468, Adjusted R-squared:  0.8181
## F-statistic: 29.48 on 3 and 16 DF,  p-value: 9.417e-07

# Extract the first 5 fitted values from the model
fitted_values <- head(fitted(regr), 5)
print("First 5 Fitted Values:")
print(fitted_values)

# Extract the first 5 residuals from the model
residuals <- head(residuals(regr), 5)
```

```
print("First 5 Residuals:")
print(residuals)

## [1] "First 5 Fitted Values:"
##        1        2        3        4        5
## 27.89626 37.95204 26.02901 32.11201 36.34251
## [1] "First 5 Residuals:"
##          1          2          3          4          5
## -3.8962605  5.0479568 -2.3290112  2.1879860 -0.5425072
```

2b

Use only linear algebra and the geometric view of regression to estimate the regression yourself

(i)(ii)

```
# Create an X matrix that has a first column of 1s followed by columns
of the independent variables
X <- cbind(1, salary_data$Experience, salary_data$Score, salary_data$De
gree)

# Create a y vector with the Salary values
y <- salary_data$Salary
```

(iii)

```
# Compute the beta_hat vector of estimated regression coefficients
beta_hat <- solve(t(X) %*% X) %*% t(X) %*% y
beta_hat

##          [,1]
## [1,] 7.944849
## [2,] 1.147582
## [3,] 0.196937
## [4,] 2.280424
```

(iv)

```
# Using the above, compute a y_hat vector of estimated y values, and a
res vector of residuals
y_hat <- X %*% beta_hat
res <- y - y_hat

# Show the code and the first 5 values of y_hat and res
print(head(y_hat, 5))
print(head(res, 5))

##          [,1]
## [1,] 27.89626
## [2,] 37.95204
## [3,] 26.02901
## [4,] 32.11201
## [5,] 36.34251
```

```
##              [,1]
## [1,] -3.8962605
## [2,]  5.0479568
## [3,] -2.3290112
## [4,]  2.1879860
## [5,] -0.5425072
```

(v)

```
# Using only the results from (i) - (iv), compute SSR, SSE and SST
SSR <- sum((y_hat - mean(y))^2)
SSE <- sum((y - y_hat)^2)
SST <- sum((y - mean(y))^2)

cat("SSR:", SSR, "\n")
cat("SSE:", SSE, "\n")
cat("SST:", SST, "\n")

## SSR: 507.896
## SSE: 91.88949
## SST: 599.7855
```

2c

Compute R^2 for in two ways, and confirm you get the same results

(i)

Use any combination of SSR, SSE, and SST

```
SSR / SST

## [1] 0.8467961
```

(ii)

Use the squared correlation of vectors y and y_hat

```
cor(y, y_hat)^2

##             [,1]
## [1,] 0.8467961
```

Observation

Two methods yield the same result indeed.

Question 3

We're going to look back at the early, heady days of global car manufacturing, when American, Japanese, and European cars competed to rule the world. Take a look at the data set in file auto-data.txt. We are interested in explaining what kind of cars have higher fuel efficiency (mpg).

mpg/ miles-per-gallon (dependent variable)
cylinders/ cylinders in engine
displacement/ size of engine
horsepower/ power of engine
weight/ weight of car
acceleration/ acceleration ability of car
model_year/ year model was released
origin/ place car was designed (1: USA, 2: Europe, 3: Japan)
car_name/ make and model names

```
auto <- read.table("auto-data.txt", header=FALSE, na.strings = "?")
names(auto) <- c("mpg", "cylinders", "displacement", "horsepower", "wei
ght",
                 "acceleration", "model_year", "origin", "car_name")
```

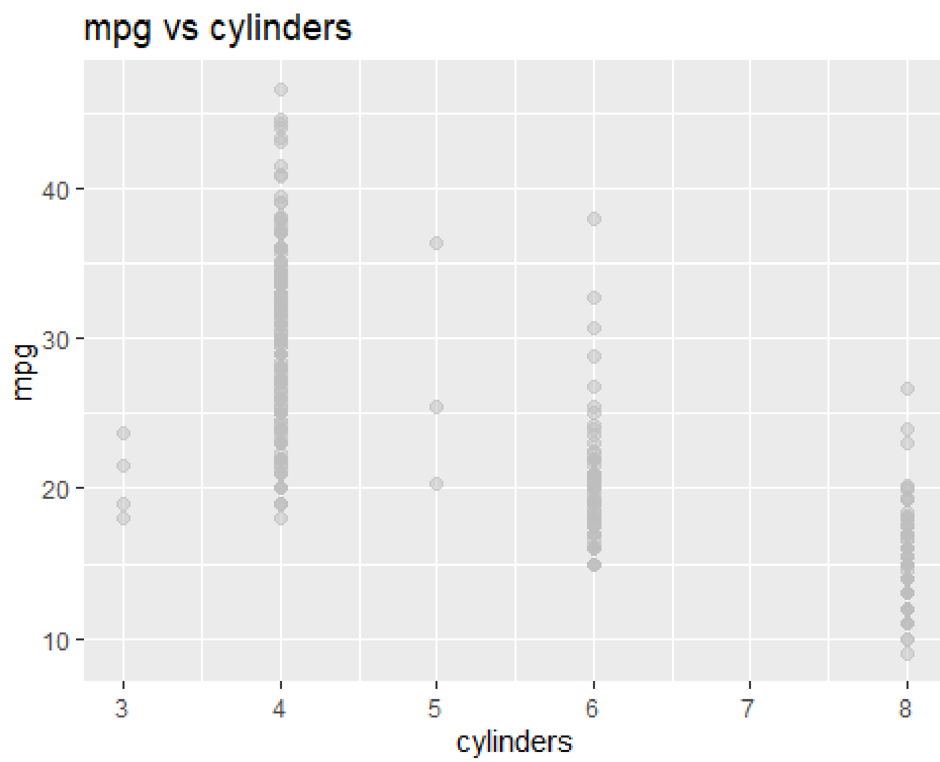
3a

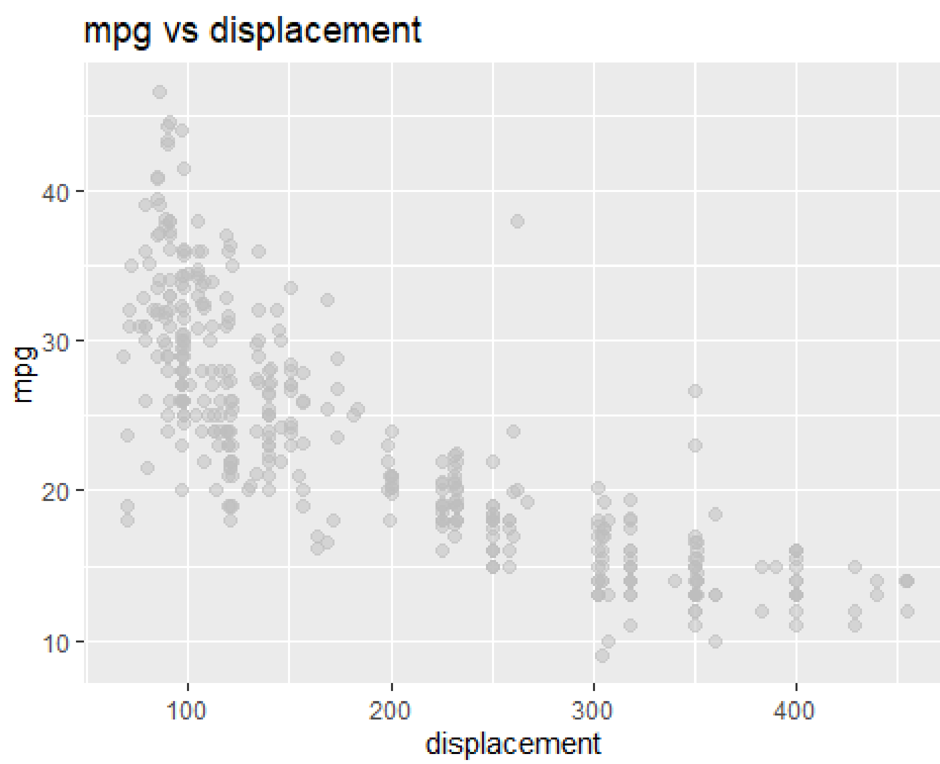
Let's first try exploring this data and problem.

(i)

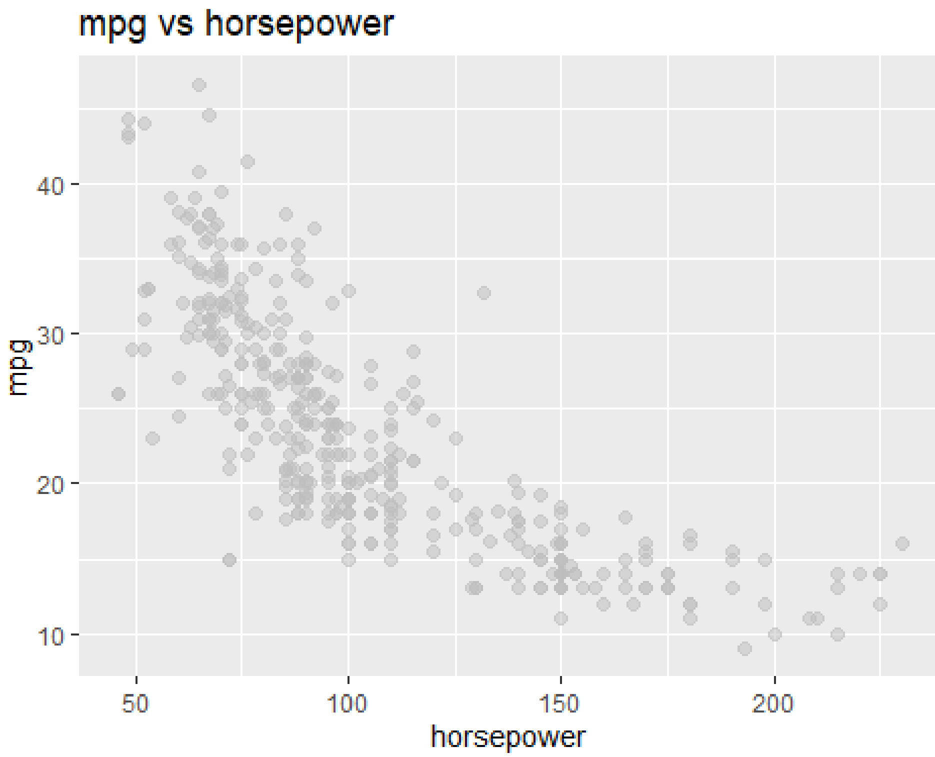
Visualize the data as you wish (report only relevant/interesting plots)

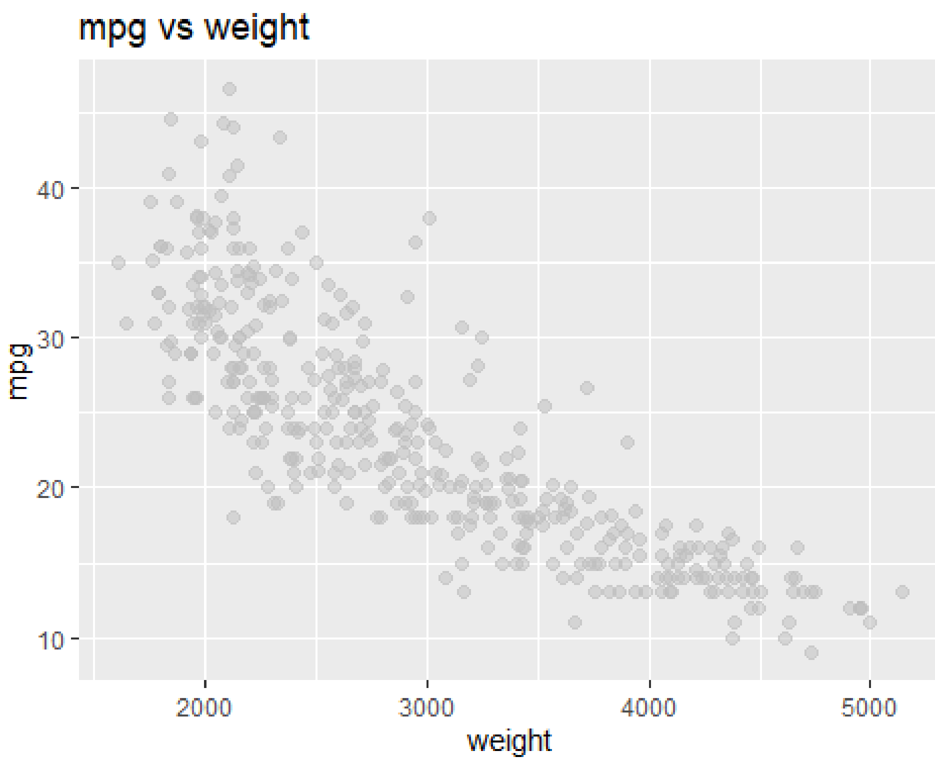
```
library(ggplot2)

vars <- c("cylinders","displacement", "horsepower", "weight", "accelera
tion","model_year","origin")
for (var in vars) {
  p <- ggplot(auto, aes_string(x = var, y = "mpg")) +
    geom_point(alpha = 0.5, color = "grey",cex = 2) +
    labs(title = paste("mpg vs", var), x = var, y = "mpg")
  print(p)
}

## Warning: `aes_string()` was deprecated in ggplot2 3.0.0.
## ℹ Please use tidy evaluation idioms with `aes()`.
## ℹ See also `vignette("ggplot2-in-packages")` for more information.
## This warning is displayed once every 8 hours.
## Call `lifecycle::last_lifecycle_warnings()` to see where this warnin
g was
## generated.
```



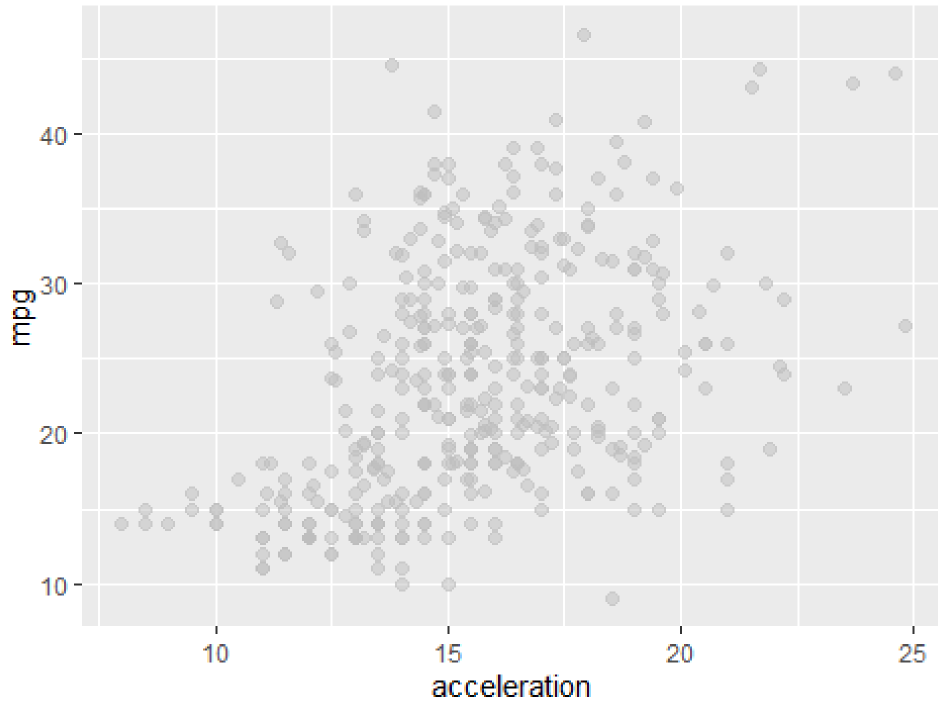



```
## Warning: Removed 6 rows containing missing values or values outside
the scale range
## (`geom_point()`).
```

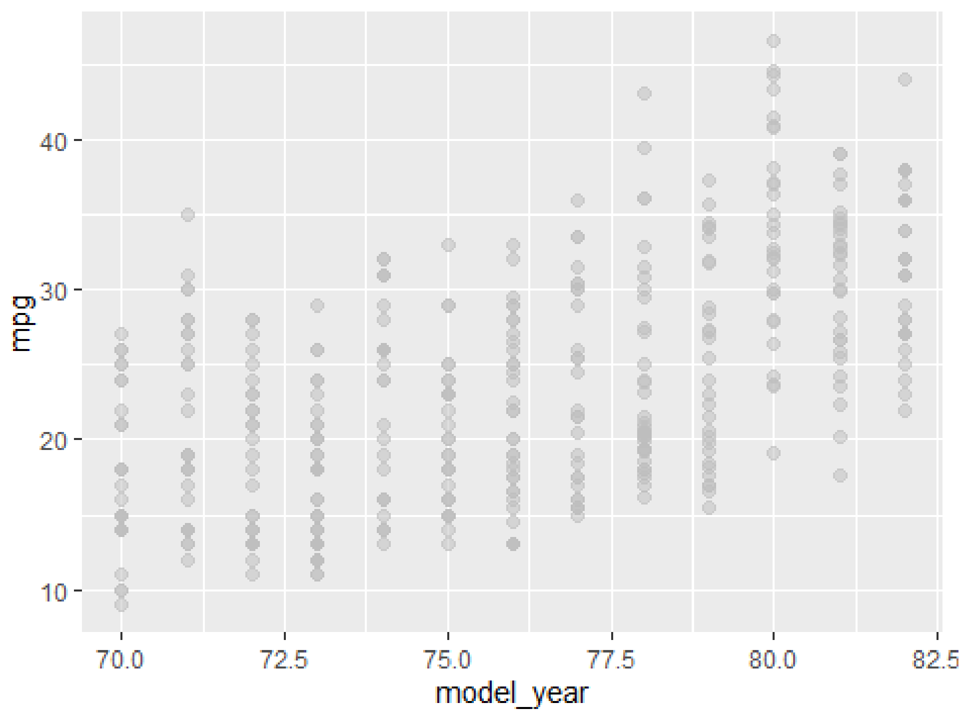

mpg vs horsepower
mpg
40
30
20
10
50
100
150
200
horsepower

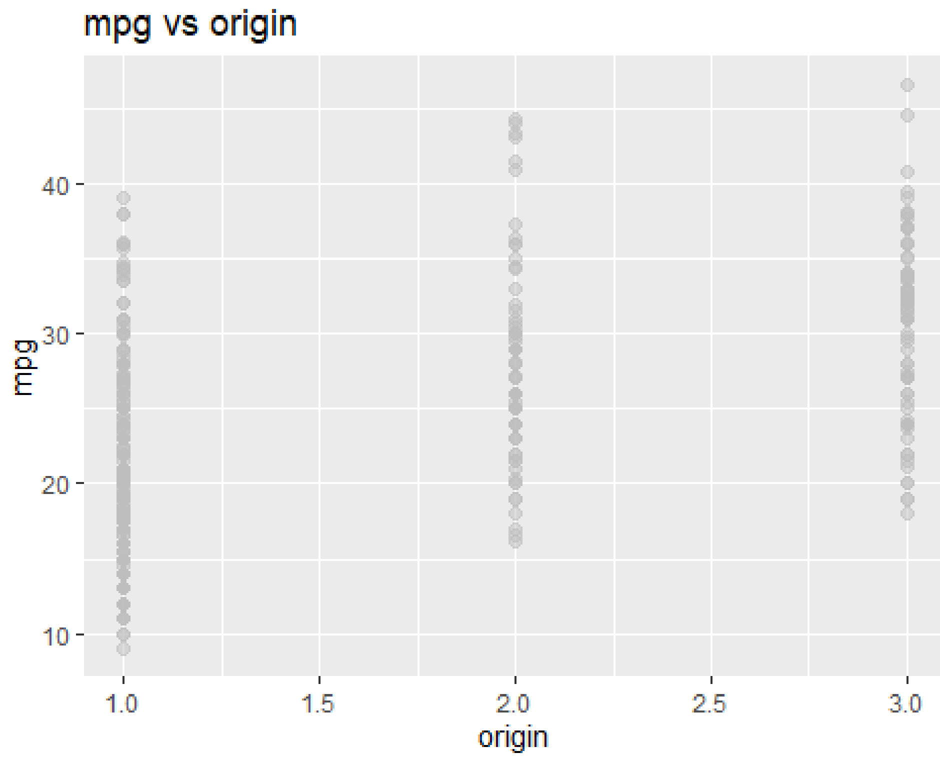

mpg vs weight
mpg
40
30
20
10
2000
3000
4000
5000
weight

mpg vs acceleration



mpg vs model_year





(ii)

Report a correlation table of all variables, rounding to two decimal places

```
# Compute the correlation matrix
cor_matrix <- cor(auto[, sapply(auto, is.numeric)], use = "pairwise.com
plete.obs")
round(cor_matrix, 2)

##                mpg cylinders displacement horsepower weight accelera
tion
## mpg           1.00     -0.78        -0.80      -0.78  -0.83
0.42
## cylinders    -0.78      1.00         0.95       0.84   0.90         -
0.51
## displacement -0.80      0.95         1.00       0.90   0.93         -
0.54
## horsepower   -0.78      0.84         0.90       1.00   0.86         -
0.69
## weight       -0.83      0.90         0.93       0.86   1.00         -
0.42
## acceleration  0.42     -0.51        -0.54      -0.69  -0.42
1.00
## model_year    0.58     -0.35        -0.37      -0.42  -0.31
0.29
## origin        0.56     -0.56        -0.61      -0.46  -0.58
0.21
```

```
##              model_year origin
## mpg                0.58   0.56
## cylinders         -0.35  -0.56
## displacement      -0.37  -0.61
## horsepower        -0.42  -0.46
## weight            -0.31  -0.58
## acceleration       0.29   0.21
## model_year         1.00   0.18
## origin             0.18   1.00
```

(iii)

Question

From the visualizations and correlations, which variables appear to relate to mpg?

Answer

displacement, horsepower and weight

(iv)

Question

Which relationships might not be linear?

Answer

displacement, horsepower, and weight

(v)

Question

Are there any pairs of independent variables that are highly correlated (r > 0.7)?

Answer

cylinders, displacement, horsepower, and weight

3b

Let's create a linear regression model where mpg is dependent upon all other suitable variables

(i)

Question

Which independent variables have a 'significant' relationship with mpg at 1% significance?

```r
# Fit the linear regression model
regr <- lm(mpg ~ cylinders + displacement + horsepower + weight + accel
eration + model_year + factor(origin), data = auto)
summary(regr)
```

```
##
## Call:
## lm(formula = mpg ~ cylinders + displacement + horsepower + weight +
##     acceleration + model_year + factor(origin), data = auto)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.0095 -2.0785 -0.0982  1.9856 13.3608
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)     -1.795e+01  4.677e+00  -3.839 0.000145 ***
## cylinders       -4.897e-01  3.212e-01  -1.524 0.128215
## displacement     2.398e-02  7.653e-03   3.133 0.001863 **
## horsepower      -1.818e-02  1.371e-02  -1.326 0.185488
## weight          -6.710e-03  6.551e-04 -10.243  < 2e-16 ***
## acceleration     7.910e-02  9.822e-02   0.805 0.421101
## model_year       7.770e-01  5.178e-02  15.005  < 2e-16 ***
## factor(origin)2  2.630e+00  5.664e-01   4.643 4.72e-06 ***
## factor(origin)3  2.853e+00  5.527e-01   5.162 3.93e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.307 on 383 degrees of freedom
## Multiple R-squared:  0.8242, Adjusted R-squared:  0.8205
## F-statistic: 224.5 on 8 and 383 DF,  p-value: < 2.2e-16
```

Answer

weight, model_year

(ii)

Question

Looking at the coefficients, is it possible to determine which independent variables are the most effective at increasing mpg? If so, which ones, and if not, why not? (hint: units!)

Answer

No. The data are comparable only when they are standardized, or we are simply juggling with different units.

3c

Let's try to resolve some of the issues with our regression model above.

(i)

Question

Create fully standardized regression results: are these slopes easier to compare?

```
auto_std <- data.frame(cbind(scale(auto[,1:7]), origin = auto[,8]))
regr_std <- lm(mpg ~ cylinders + displacement + horsepower + weight + a
cceleration + model_year + factor(origin), data = auto_std)
summary(regr_std)

##
## Call:
## lm(formula = mpg ~ cylinders + displacement + horsepower + weight +
##     acceleration + model_year + factor(origin), data = auto_std)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.15270 -0.26593 -0.01257  0.25404  1.70942
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)       -0.13323    0.03174  -4.198 3.35e-05 ***
## cylinders         -0.10658    0.06991  -1.524  0.12821
## displacement       0.31989    0.10210   3.133  0.00186 **
## horsepower        -0.08955    0.06751  -1.326  0.18549
## weight            -0.72705    0.07098 -10.243  < 2e-16 ***
## acceleration       0.02791    0.03465   0.805  0.42110
## model_year         0.36760    0.02450  15.005  < 2e-16 ***
## factor(origin)2    0.33649    0.07247   4.643 4.72e-06 ***
## factor(origin)3    0.36505    0.07072   5.162 3.93e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.423 on 383 degrees of freedom
## Multiple R-squared:  0.8242, Adjusted R-squared:  0.8205
## F-statistic: 224.5 on 8 and 383 DF,  p-value: < 2.2e-16
```

Answer

Yes. All variables get uniform units, so they become comparable.

(ii)

Question

Regress mpg over each nonsignificant independent variable, individually. Which ones become significant when we regress mpg over them individually?

Observation

According to the summary table above, three variables (cylinders, horsepower, and acceleration) are not significant as their p-value are greater than 0.01.

```
regr_cylinders <- lm(mpg ~ cylinders, data = auto_std)
summary(regr_cylinders)

##
## Call:
## lm(formula = mpg ~ cylinders, data = auto_std)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.82455 -0.43297 -0.08288  0.32674  2.29046
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.834e-15  3.169e-02    0.00        1
## cylinders   -7.754e-01  3.173e-02  -24.43   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6323 on 396 degrees of freedom
## Multiple R-squared:  0.6012, Adjusted R-squared:  0.6002
## F-statistic: 597.1 on 1 and 396 DF,  p-value: < 2.2e-16

regr_horsepower <- lm(mpg ~ horsepower, data = auto_std)
summary(regr_horsepower)

##
## Call:
## lm(formula = mpg ~ horsepower, data = auto_std)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.73632 -0.41699 -0.04395  0.35351  2.16531
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.008784   0.031701  -0.277    0.782
## horsepower  -0.777334   0.031742 -24.489   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6277 on 390 degrees of freedom
##   (因為不存在，6 個觀察量被刪除了)
```

```
## Multiple R-squared:  0.6059, Adjusted R-squared:  0.6049
## F-statistic: 599.7 on 1 and 390 DF,  p-value: < 2.2e-16
```

```r
regr_acceleration <- lm(mpg ~ acceleration, data = auto_std)
summary(regr_acceleration)
```

```
##
## Call:
## lm(formula = mpg ~ acceleration, data = auto_std)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.3039 -0.7210 -0.1589  0.6087  2.9672
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.004e-16  4.554e-02   0.000        1
## acceleration 4.203e-01  4.560e-02   9.217   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9085 on 396 degrees of freedom
## Multiple R-squared:  0.1766, Adjusted R-squared:  0.1746
## F-statistic: 84.96 on 1 and 396 DF,  p-value: < 2.2e-16
```

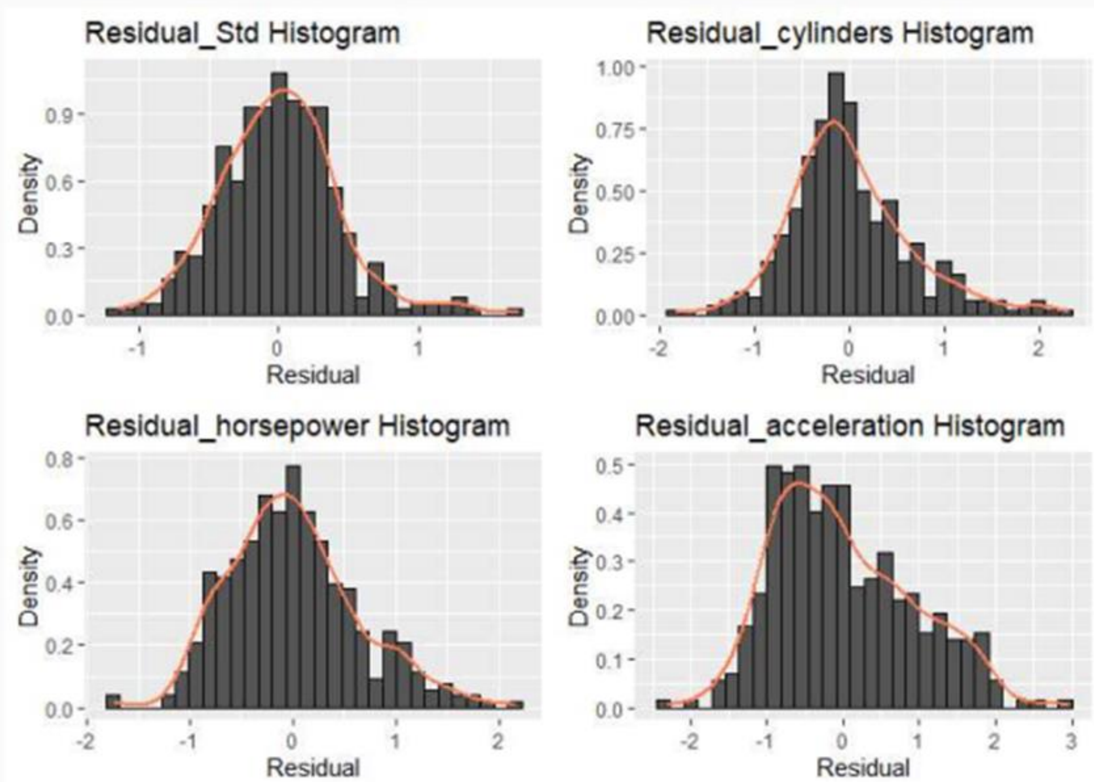
(iii)

Question

Plot the distribution of the residuals: are they normally distributed and centered around zero?
(get the residuals of a fitted linear model, e.g. regr <- lm(…), using regr$residuals

```r
residual1 <- data.frame(residual = regr_std$residuals)
residual2 <- data.frame(residual = regr_cylinders$residuals)
residual3 <- data.frame(residual = regr_horsepower$residuals)
residual4 <- data.frame(residual = regr_acceleration$residuals)

plot1 <- ggplot(x = residual1)+
  geom_histogram(aes(y = after_stat(density)), color = "grey" , bins =
40)+
  labs(title = "STD Residuals Histogram", x = "residuals", y = "frequen
cy")+
  geom_density(aes(x = residual1), color = "darkgrey", lwd = 1)
plot2 <- ggplot(x = residual2)+
  geom_histogram(aes(y = after_stat(density)), color = "grey" , bins =
40)+
  labs(title = "CYLINDERS Residuals Histogram", x = "residuals", y = "f
requency")+
  geom_density(aes(x = residual2), color = "darkgrey", lwd = 1)
plot3 <- ggplot(x = residual3)+
```

```
  geom_histogram(aes(y = after_stat(density)), color = "grey" , bins =
40)+
  labs(title = "HORSEPOWER Residuals Histogram", x = "residuals", y = "
frequency")+
  geom_density(aes(x = residual3), color = "darkgrey", lwd = 1)
plot4 <- ggplot(x = residual4)+
  geom_histogram(aes(y = after_stat(density)), color = "grey" , bins =
40)+
  labs(title = "ACCELERATION Residuals Histogram", x = "residuals", y =
 "frequency")+
  geom_density(aes(x = residual4), color = "darkgrey", lwd = 1)
```



Answer

According to the graphs presented above, the residuals are clearly normally distributed and centered around zero as well.