bacs_hw9

2024-04-21

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110034002 has done research and discussed with me how the Stepwise VIF Selection can be stimulated as some part of topic was left for the next lecture.

Question 1

Let's deal with **non-linearity** first. Create a new dataset that log-transforms several variables from our original dataset (called cars in this case)

```
cars <- read.table("auto-data.txt", header = FALSE, na.strings = "?")
names(cars) <- c("mpg", "cylinders", "displacement", "horsepower", "wei
ght",
                 "acceleration", "model_year", "origin", "car_name")

cars_log <- with(cars, data.frame(log(mpg), log(cylinders), log(displac
ement),
                                  log(horsepower), log(weight), log(acc
eleration),
                                  model_year, origin))
```

1a

Run a new regression on the cars_log dataset, with mpg.log. dependent on all other variables

i

Question

Which log-transformed factors have a significant effect on log.mpg. at 10% significance?

Answer

Acceleration, weight, model year, factor(origin), and horsepower

```
log_regr <- summary(lm(formula = log.mpg. ~ log.cylinders. + log.displa
cement. +
    log.horsepower. + log.weight. + log.acceleration. + model_year +
    factor(origin), data = cars_log, na.action = na.exclude))
log_regr
```

```
##
## Call:
## lm(formula = log.mpg. ~ log.cylinders. + log.displacement. +
##     log.horsepower. + log.weight. + log.acceleration. + model_year +

##     factor(origin), data = cars_log, na.action = na.exclude)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.39727 -0.06880  0.00450  0.06356  0.38542
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)        7.301938   0.361777  20.184  < 2e-16 ***
## log.cylinders.    -0.081915   0.061116  -1.340  0.18094
## log.displacement.  0.020387   0.058369   0.349  0.72707
## log.horsepower.   -0.284751   0.057945  -4.914 1.32e-06 ***
## log.weight.       -0.592955   0.085165  -6.962 1.46e-11 ***
## log.acceleration. -0.169673   0.059649  -2.845  0.00469 **
## model_year         0.030239   0.001771  17.078  < 2e-16 ***
## factor(origin)2    0.050717   0.020920   2.424  0.01580 *
## factor(origin)3    0.047215   0.020622   2.290  0.02259 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.113 on 383 degrees of freedom
## Multiple R-squared:  0.8919, Adjusted R-squared:  0.8897
## F-statistic:   395 on 8 and 383 DF,  p-value: < 2.2e-16
```

ii

Question

Do some new factors now have effects on mpg, and why might this be?

Answer

Acceleration and horsepower are new factors. This may arise because the original data could be skewed, and now log-transformed to display the otherwise hidden patterns.

iii

Question

Which factors still have insignificant or opposite (from correlation) effects on mpg? Why might this be?

Answer

Cylinders and displacement. This may arise due to the data’s multicollinearity.

1b

i

Create a regression (call it regr_wt) of mpg over weight <u>from the original cars dataset</u>

```
regr_wt <- summary(lm(mpg ~ weight, data=cars))
regr_wt

##
## Call:
## lm(formula = mpg ~ weight, data = cars)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -12.012  -2.801  -0.351   2.114  16.480
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept) 46.3173644  0.7952452   58.24   <2e-16 ***
## weight      -0.0076766  0.0002575  -29.81   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.345 on 396 degrees of freedom
## Multiple R-squared:  0.6918, Adjusted R-squared:  0.691
## F-statistic: 888.9 on 1 and 396 DF,  p-value: < 2.2e-16
```

ii

Create a regression (call it regr_wt_log) of log.mpg. on log.weight. from cars_log

```
regr_wt_log <- summary(lm(log.mpg. ~ log.weight., data=cars_log, na.act
ion=na.exclude))
regr_wt_log

##
## Call:
## lm(formula = log.mpg. ~ log.weight., data = cars_log, na.action = na.
exclude)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.52408 -0.10441 -0.00805  0.10165  0.59384
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  11.5219     0.2349   49.06   <2e-16 ***
## log.weight.  -1.0583     0.0295  -35.87   <2e-16 ***
## ---
```

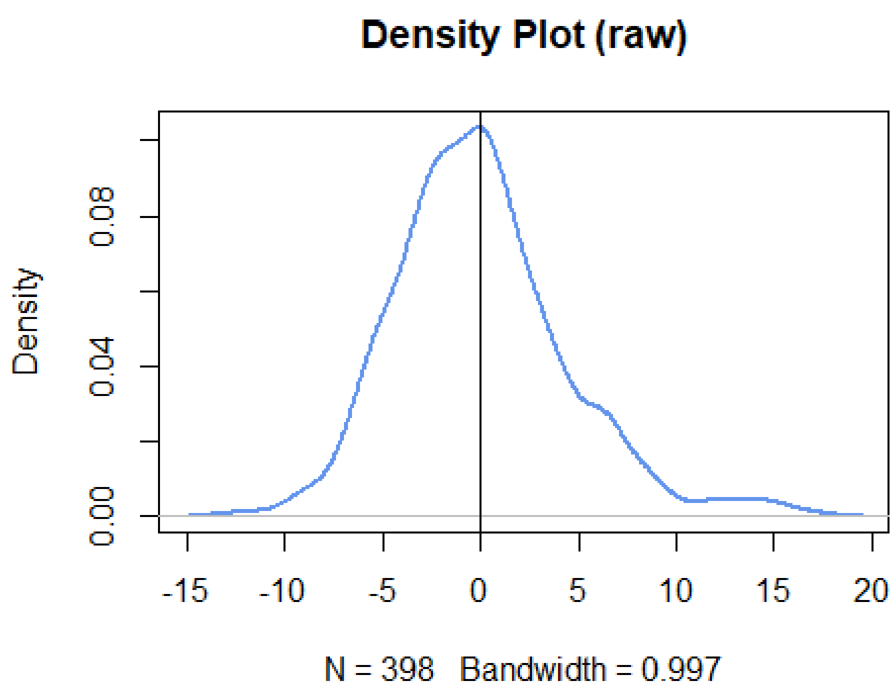
```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.165 on 396 degrees of freedom
## Multiple R-squared:  0.7647, Adjusted R-squared:  0.7641
## F-statistic:  1287 on 1 and 396 DF,  p-value: < 2.2e-16
```

iii

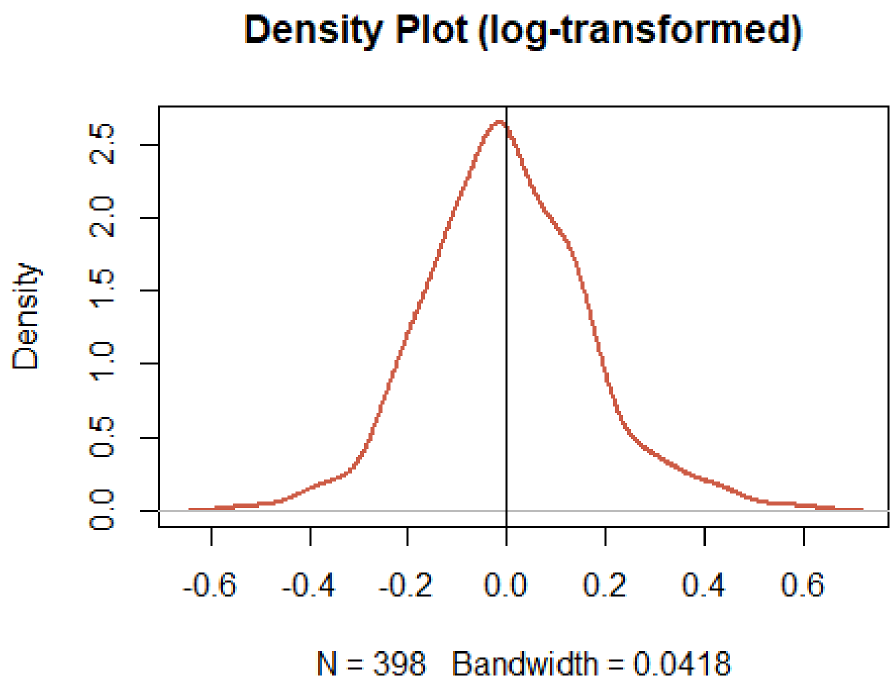
Visualize the residuals of both regression models (raw and log-transformed)

(1) density plot of residuals

```
plot(density(regr_wt$residuals), lwd = 2, col = "cornflowerblue", main
= "Density Plot (raw)")
abline(v= mean(regr_wt$residuals))
```

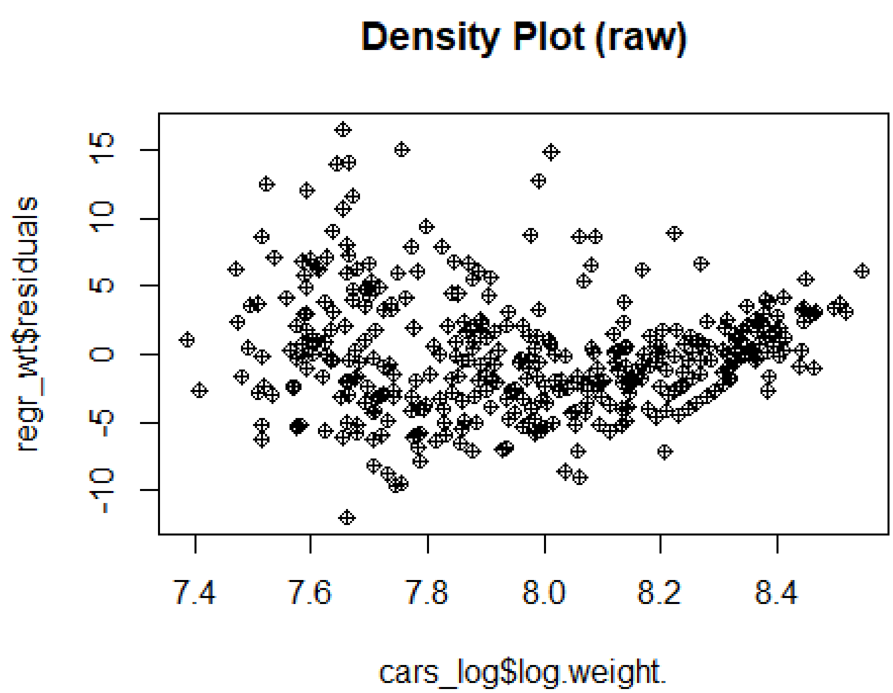



```
plot(density(regr_wt_log$residuals), lwd = 2, col = "coral3", main = "D
ensity Plot (log-transformed)")
abline(v= mean(regr_wt_log$residuals))
```

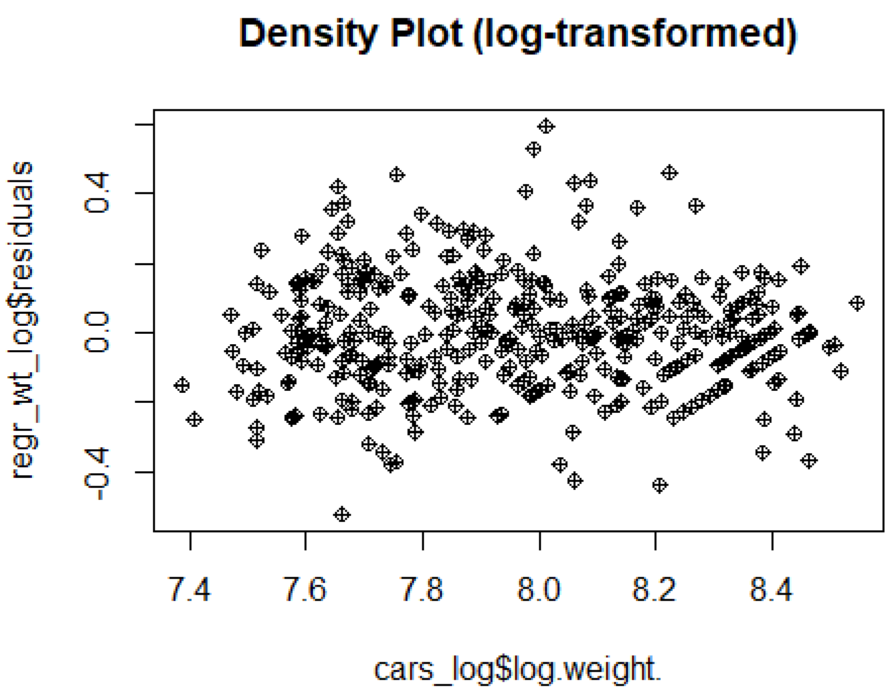


(2) scatterplot of log.weight. vs. residuals

```
plot(cars_log$log.weight., regr_wt$residuals, pch=10,main = "Density Pl
ot (raw)")
```



```r
plot(cars_log$log.weight., regr_wt_log$residuals, pch=10, main = "Densi
ty Plot (log-transformed)")
```



iv

Question

Which regression produces better distributed residuals for the assumptions of regression?

Answer

The log-transformed regression.

v

Question

How would you interpret the slope of log.weight. vs log.mpg. in simple words?

Answer

Based on the summary tables in (i) and (ii), it is clear that one percent change in log.weight. leads to -1.0583 percent change in log.mpg.

vi

Question

From its standard error, what is the 95% confidence interval of the slope of log.weight. vs log.mpg.?

Answer

```
regr_wt_log

##
## Call:
## lm(formula = log.mpg. ~ log.weight., data = cars_log, na.action = na.
exclude)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.52408 -0.10441 -0.00805  0.10165  0.59384
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  11.5219     0.2349   49.06   <2e-16 ***
## log.weight.  -1.0583     0.0295  -35.87   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.165 on 396 degrees of freedom
## Multiple R-squared:  0.7647, Adjusted R-squared:  0.7641
## F-statistic:  1287 on 1 and 396 DF,  p-value: < 2.2e-16

slope_estimate <- regr_wt_log$coefficients["log.weight.", "Estimate"]
slope_se <- regr_wt_log$coefficients["log.weight.", "Std. Error"]

CI_lower <- slope_estimate - 1.96 * slope_se
CI_upper <- slope_estimate + 1.96 * slope_se

cat("CI_upperbound:", CI_upper, "\n")
cat("CI_lowerbound:", CI_lower)

## CI_upperbound: -1.000448
## CI_lowerbound: -1.116088
```

Question 2

Let's tackle **multicollinearity** next. Consider the regression model:

```
regr_log <- lm(log.mpg. ~ log.cylinders. + log.displacement. + log.hors
epower. +
                          log.weight. + log.acceleration. + model_year
+
                          factor(origin), data=cars_log)
```

2a

Using regression and R2, compute the VIF of log.weight. using the approach shown in class

```
logweight_regr <- lm(log.weight.~log.cylinders.+log.displacement.+log.h
orsepower.+log.acceleration.+model_year, data=cars_log, na.action = na.
exclude)
r2_logweight_regr <- summary(logweight_regr)$r.squared
vif_logweight <- 1 / (1 - r2_logweight_regr)
vif_logweight

## [1] 16.07917
```

2b

Let's try a procedure called Stepwise VIF Selection to remove highly collinear predictors.

i

Use vif(regr_log) to compute VIF of the all the independent variables

```
#install.packages('car')
library('car')

## 載入需要的套件：carData

regr_log <- lm(log.weight. ~ log.cylinders.+log.displacement.+log.horse
power.+log.acceleration.+model_year,
                 data=cars_log, na.action=na.exclude)
vif(regr_log)

##    log.cylinders. log.displacement.   log.horsepower. log.accelerati
on.
##          9.748860         13.412802          7.013535          2.253
283
##        model_year
##          1.198164
```

ii

Eliminate from your model the single independent variable with the largest VIF score that is also greater than 5

```
# log.displacement scores the largest VIF
regr_log <- lm(log.weight. ~ log.cylinders.+log.horsepower.+log.acceler
ation.+model_year,
                 data=cars_log, na.action=na.exclude)
vif(regr_log)
```

```
##    log.cylinders.   log.horsepower. log.acceleration.         model_y
ear
##          3.326803          5.208472          2.167932          1.190
458
```

iii

Repeat steps (i) and (ii) until no more independent variables have VIF scores above 5

```
# Keep on to eliminate log.horsepower, whose VIF is at once the largest
 and greater than 5
regr_log <- lm(log.weight. ~ log.cylinders.+log.acceleration.+model_yea
r,
                 data=cars_log, na.action=na.exclude)
vif(regr_log)
```

```
##    log.cylinders. log.acceleration.        model_year
##          1.412557          1.382461          1.165129
```

iv

Report the final regression model and its summary statistics

```
summary(regr_log)
```

```
##
## Call:
## lm(formula = log.weight. ~ log.cylinders. + log.acceleration. +
##     model_year, data = cars_log, na.action = na.exclude)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.35101 -0.09406 -0.00256  0.09311  0.41564
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)       6.398532   0.198541  32.228   <2e-16 ***
## log.cylinders.    0.835451   0.026327  31.734   <2e-16 ***
## log.acceleration. 0.035708   0.043451   0.822    0.412
## model_year        0.001084   0.001950   0.556    0.579
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1331 on 394 degrees of freedom
## Multiple R-squared:  0.7769, Adjusted R-squared:  0.7752
## F-statistic: 457.3 on 3 and 394 DF,  p-value: < 2.2e-16
```

2c

Question

Using stepwise VIF selection, have we lost any variables that were previously significant? If so, how much did we hurt our explanation by dropping those variables?

Answer

We eliminated displacement and horsepower, which used to seem significant. In dropping these variables, the explanation of the fit model can be hurt due to the R-squared change.

2d

From only the formula for VIF, try deducing/deriving the following:

i

Question

If an independent variable has no correlation with other independent variables, what would its VIF score be?

Answer

By the VIF formula 1 / (1 - R-squared), no correlation means R-squared to be 0, and VIF in turn becomes 1.

ii

Question

Given a regression with only two independent variables (X1 and X2), how correlated would X1 and X2 have to be, to get VIF scores of 5 or higher? To get VIF scores of 10 or higher?

Answer

Correlation would have to be above 0.894 to get VIF scores of 5 or higher.

Correlation would have to be above 0.948 to get VIF scores of 10 or higher.

Question 3

Might the relationship of weight on mpg be different for cars from different origins? Let's try visualizing this. First, plot all the weights, using different colors and symbols for the three origins

3a

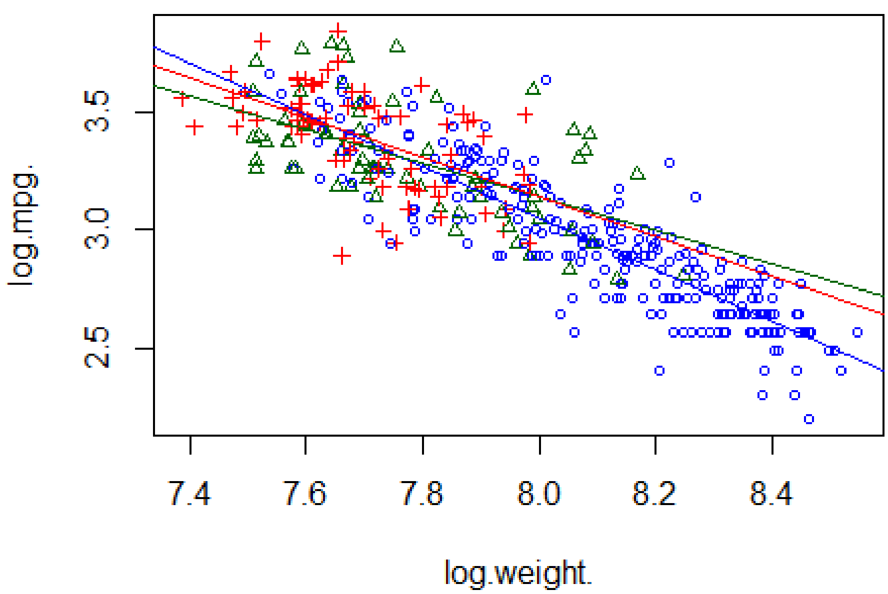
Let's add three separate regression lines on the scatterplot, one for each of the origins. Here's one for the US to get you started:

```r
origin_colors = c("blue", "darkgreen", "red")
with(cars_log, plot(log.weight., log.mpg., pch=origin, col=origin_color
s[origin], cex = 0.7))

cars_us <- subset(cars_log, origin == 1)
wt_regr_us <- lm(log.mpg. ~ log.weight., data=cars_us)
abline(wt_regr_us, col=origin_colors[1], lwd=1)

cars_jp <- subset(cars_log, origin == 2)
wt_regr_jp <- lm(log.mpg. ~ log.weight., data=cars_jp)
abline(wt_regr_jp, col=origin_colors[2], lwd=1)

cars_eu <- subset(cars_log, origin == 3)
wt_regr_eu <- lm(log.mpg. ~ log.weight., data=cars_eu)
abline(wt_regr_eu, col=origin_colors[3], lwd=1)
```



3b

Question

Do cars from different origins appear to have different weight vs. mpg relationships?

Answer

Yes, each of their data points seems clustered.