4 Simulation

4.3 Results for the simulation study

After repeating sampling with and without replacement, sampling distribution of each candidate estimate can be constructed and their Biase, Variance and MSE can also be obtained. Figure 4.3.1, Figure 4.3.2, Table 4.3.1 and Table 4.3.2 shows that when sampling "with" replacement, MLE and Second Order Taylor's Approximation are always the best esitmate for θ_A and θ_P and when sample size becomes larger, their performance on Variance would also become better. As for the performance of Arithmetric Mean, like our expectation, severely overestimates the true mean no matter how sample size changes. Conversly, when it is sampling "without" replacement, the performance of MLE and Second Order Taylor's Approximation estimate are not worse only when the sample size compared to population size is small because when sample size is small, sampling without replacement is similar to sampling with replacement. For Arithmetic Mean, even though its expected value approximates to the true mean as sample size becomes larger, it is still a biased estimate as long as the sample size is not equal to the population size.

To sum up, in this simulation, when it is sampling "with" replacement, MLE and the 2nd Order Taylor's Approximation estimate perform best and Arithmetic Mean has the worst performance. Nevertheless, MLE and 2nd Order Taylor's Approximation estimate can only be obtained under strong assumptions on distributions of Area and Circularity. Hence, we also include Weighted Mean in our following distribution for its sufficient performance and its nonparametric assumption.

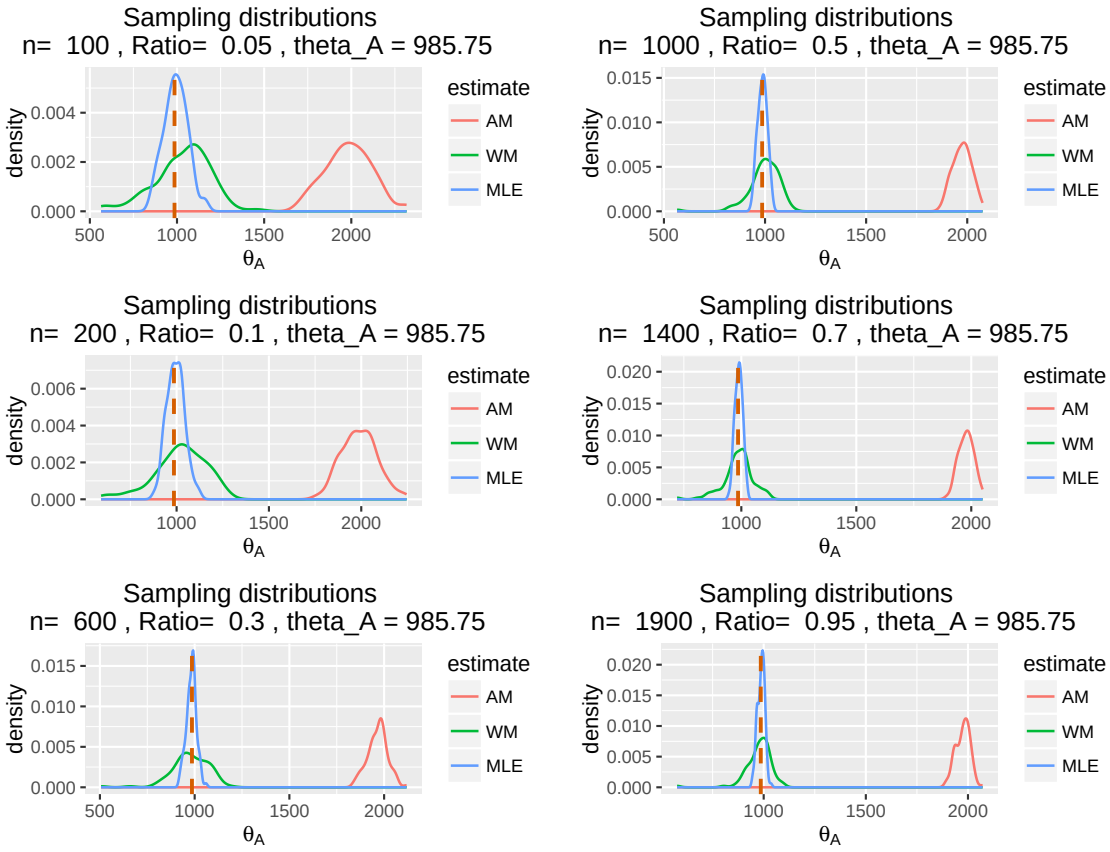

Figure 4.3.1: Sampling Distributions for Estimates of Mean Size (WR)

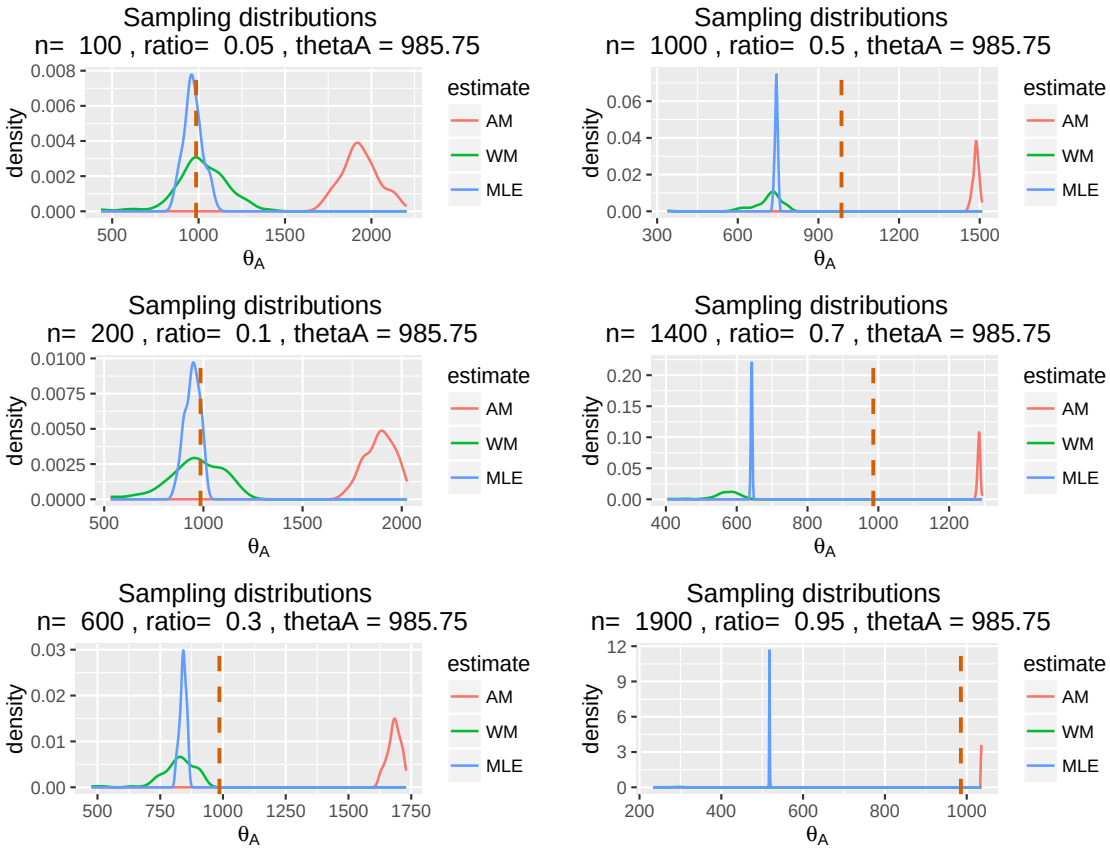

Figure 4.3.2: Sampling Distributions for Estimates of Mean Size (WOR)

Sampling with Replacement					Sampling without Replacement				
n	Estimate	Bias	Variance	MSE	n	Estimate	Bias	Variance	MSE
100	AM	1000	17929.4	1017830.2	100	AM	951.1	11438	916068.9
	WM	45.2	26254.8	28299.9		WM	33.7	22616.9	23753.2
	MLE	7.1	4482.4	4532.8		MLE	-17.3	2859.5	3159.3
200	AM	1003.9	9268.9	1017014.8	200	AM	903.1	5635.2	821268.8
	WM	35	19713.5	20939.4		WM	-25.5	18871.3	19523.7
	MLE	9.1	2317.2	2399.3		MLE	-41.3	1408.8	3115.5
600	AM	980.2	2764.6	963566.6	600	AM	696	686.8	485126.4
	WM	-8.7	10112.5	10187.5		WM	-163.5	5522.6	32246.4
	MLE	-2.8	691.2	698.8		MLE	-144.9	171.7	21157.8
1000	AM	988.4	2118.8	979145	1000	AM	500.3	119.8	250416.6
	WM	11.2	6076.7	6202.9		WM	-274.8	3718	79210.9
	MLE	1.3	529.7	531.5		MLE	-242.7	30	58945.8
1400	AM	991.7	1100.9	984636.9	1400	AM	298.4	12.4	89071.2
	WM	0.9	4239.2	4239.9		WM	-416.2	1691.9	174916.1
	MLE	3	275.2	284.2		MLE	-343.7	3.1	118105.7
1900	AM	989.2	1196.4	979657.2	1900	AM	49.9	0	2493.2
	WM	-8.9	4230.7	4310.8		WM	-697.6	332.4	486925.7
	MLE	1.7	299.1	302		MLE	-467.9	0	218938.4

Table 4.3.1: Performance Table for Area

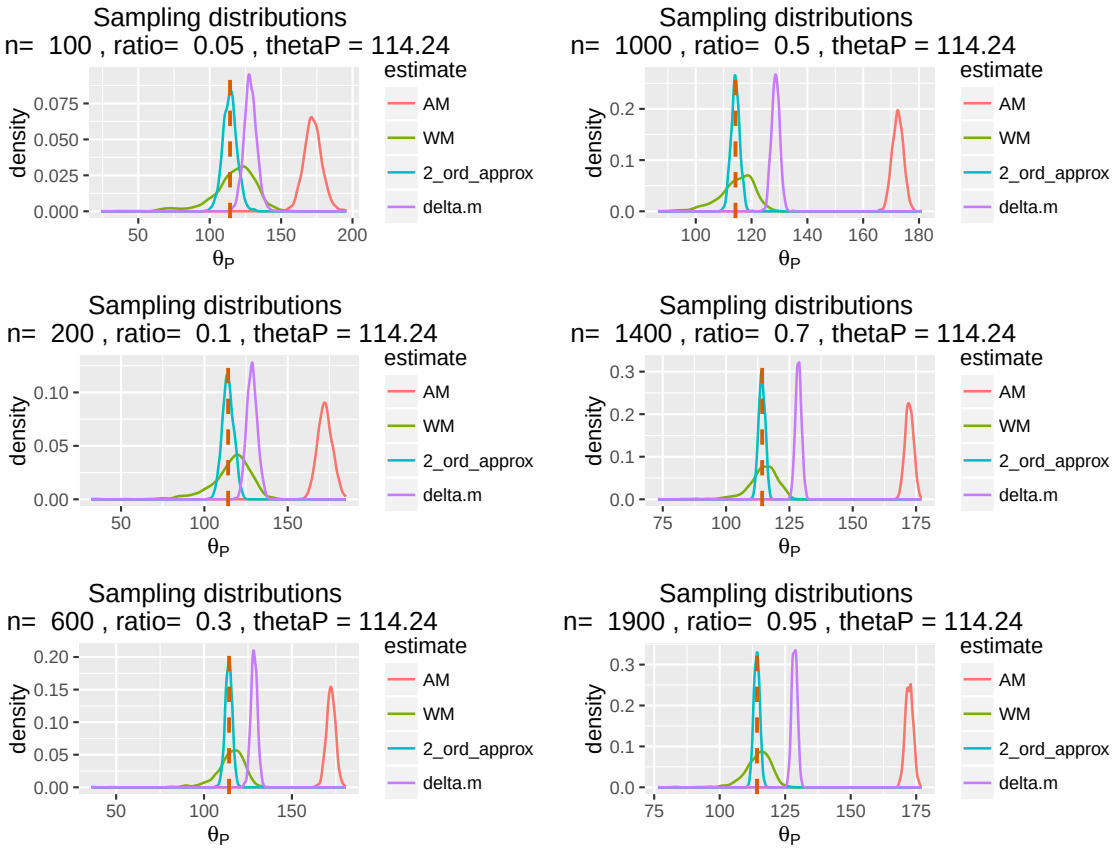

Figure 4.3.3: Sampling Distributions for Estimates of Mean Perimeter (WR)

```
## Error in array.mean.per[, 1, i] = apply(sample.mean.per[, , i], 2, mean) - :  number

of items to replace is not a multiple of replacement length
```

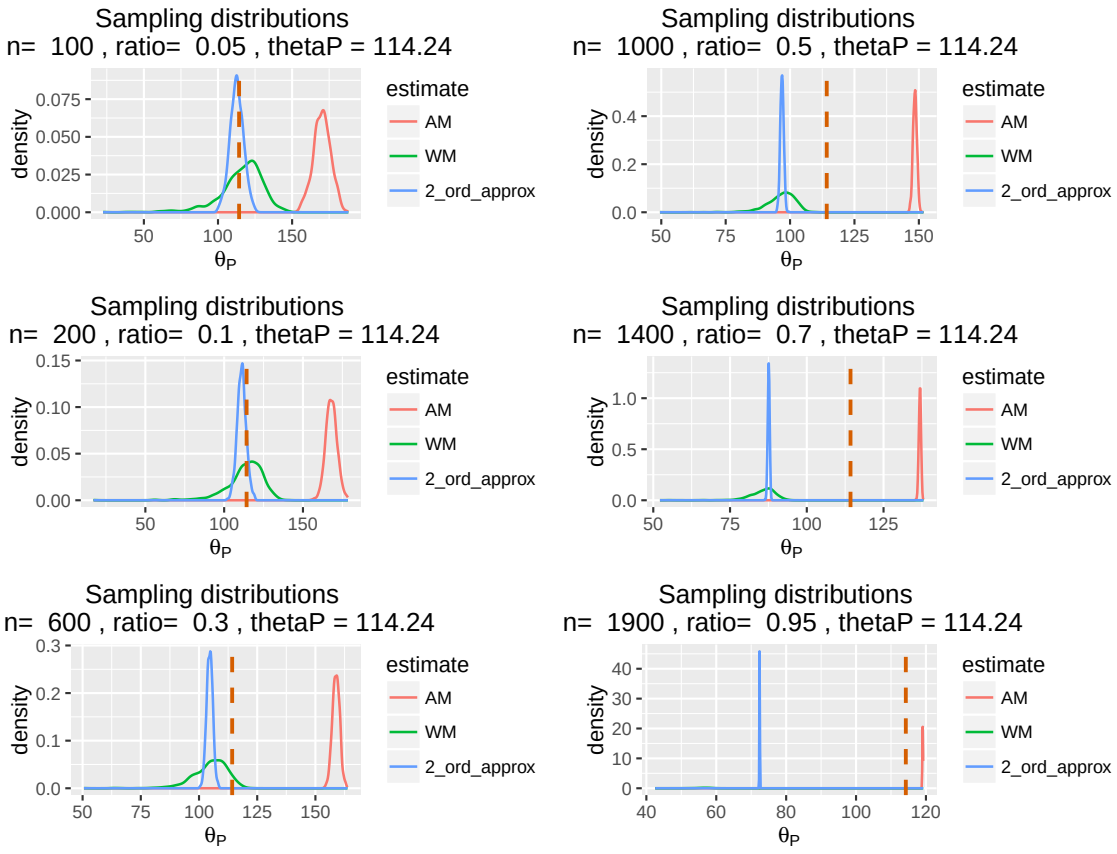

Figure 4.3.4: Sampling Distributions for Estimates of Mean Perimeter (WOR)

Sampling with Replacement					Sampling without Replacement				
n	Estimate	Bias	Variance	MSE	n	Estimate	Bias	Variance	MSE
100	AM				100	AM	56	33.7	3164.6
	WM					WM	1.6	217.9	220.6
	2nd Approx.					2nd Approx.	-1.6	20.9	23.5
200	AM				200	AM	53.8	12.8	2905.8
	WM					WM	-0.9	137.1	137.9
	2nd Approx.					2nd Approx.	-3.2	8.1	18.1
600	AM				600	AM	44.6	2.4	1992.2
	WM					WM	-9.4	56.6	144.1
	2nd Approx.					2nd Approx.	-9.7	1.7	95.1
1000	AM				1000	AM	34.2	0.6	1172.9
	WM					WM	-17.8	32.4	348.5
	2nd Approx.					2nd Approx.	-17.4	0.4	302.4
1400	AM				1400	AM	22.6	0.1	511
	WM					WM	-28.7	21.7	845.9
	2nd Approx.					2nd Approx.	-26.6	0.1	710.1
1900	AM				1900	AM	4.8	0	23.5
	WM					WM	-58.4	8.1	3420.3
	2nd Approx.					2nd Approx.	-41.8	0	1749.4

Table 4.3.2: Performance Table for Perimeter