# Parallelizing Irregular Gauss-Seidel Using Full Sparse Tiling


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FRACTAL

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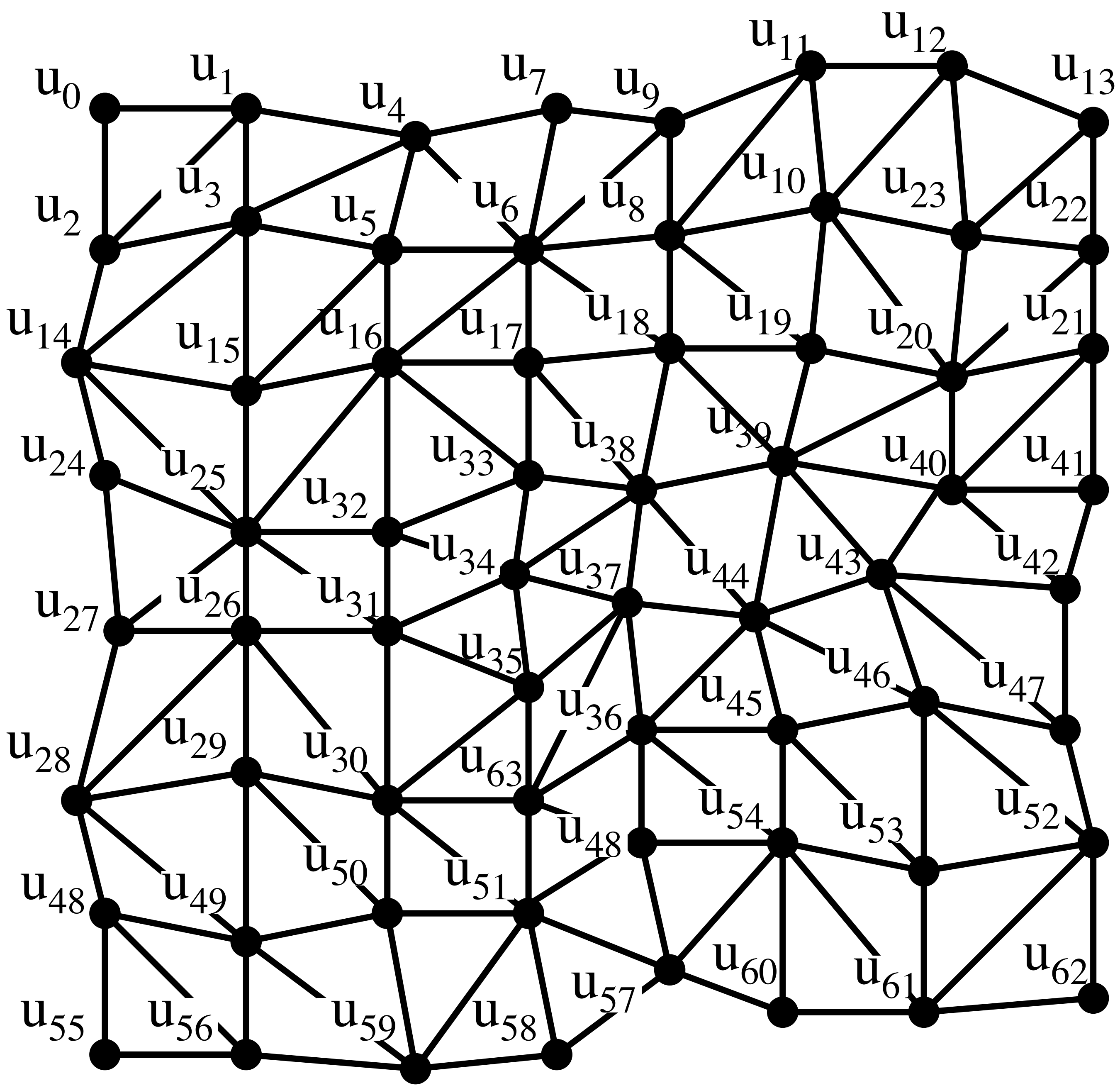
# Goal: Make computations over meshes fast



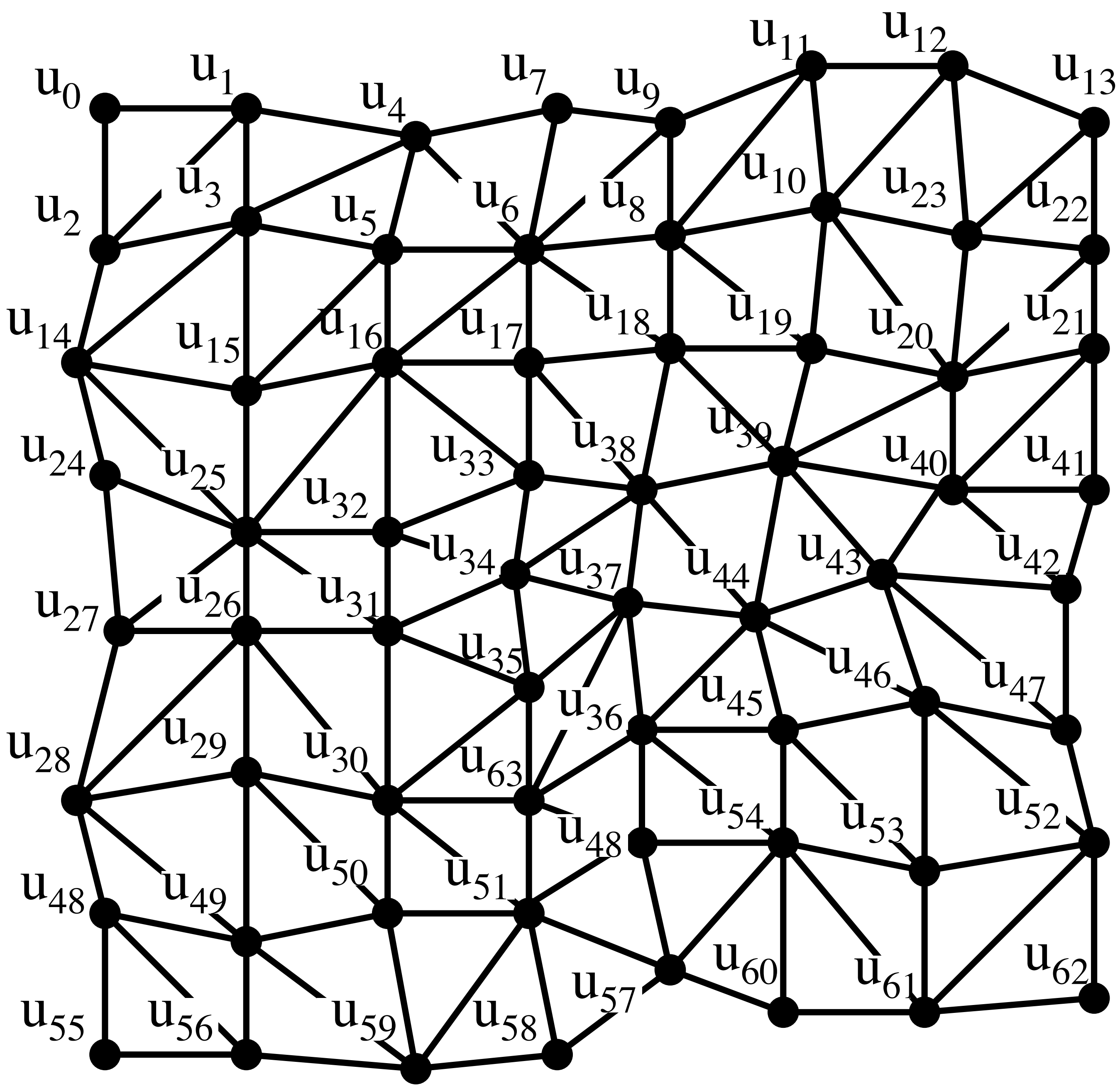

- Iterative smoothers such as Gauss-Seidel dominate the execution time of finite element applications
- Need to effectively utilize the memory hierarchy
- Performance should scale to multiple processors

# Full Sparse Tiling to the Rescue!

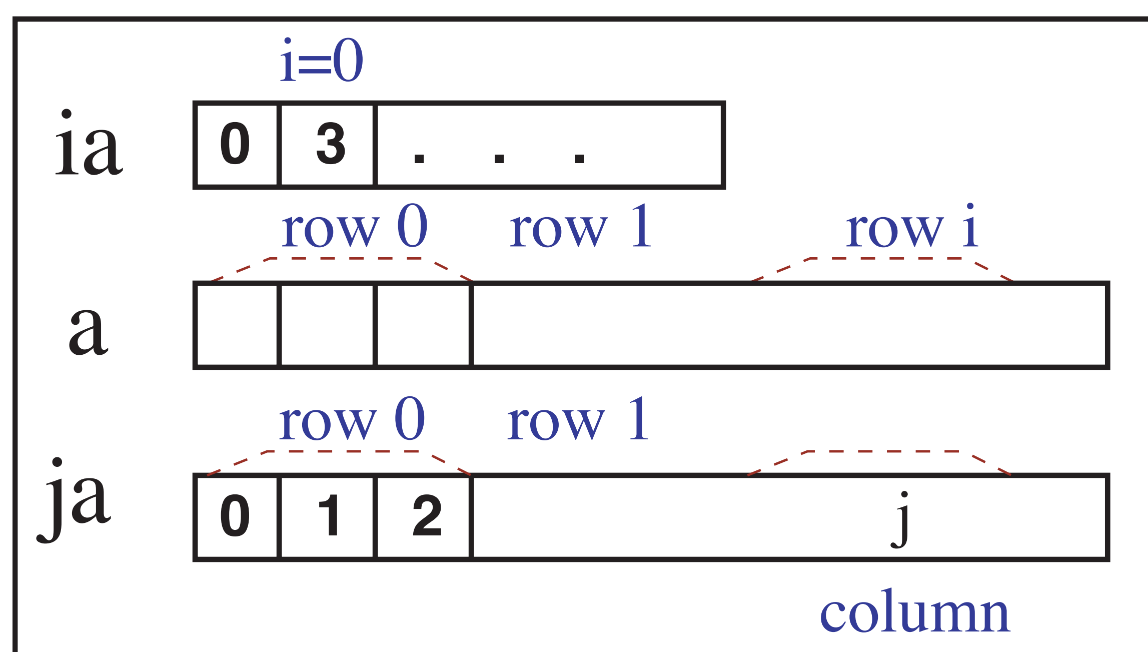
- **Problem**
  - Compile-time data reordering and computation scheduling to improve data locality and parallelism is not possible due to irregular memory references A[B[i]]
- **Solution**
  - Inspector/executor strategies perform data and computation reordering at runtime
  - Full sparse tiling is one such strategy that improves performance by exploiting parallelism, intra-iteration data reuse, and inter-iteration data reuse

# Gauss-Seidel Iteratively Solves Au = f

- u is a vector of unknowns
- A is a sparse matrix stored in the compressed sparse row format (CSR)

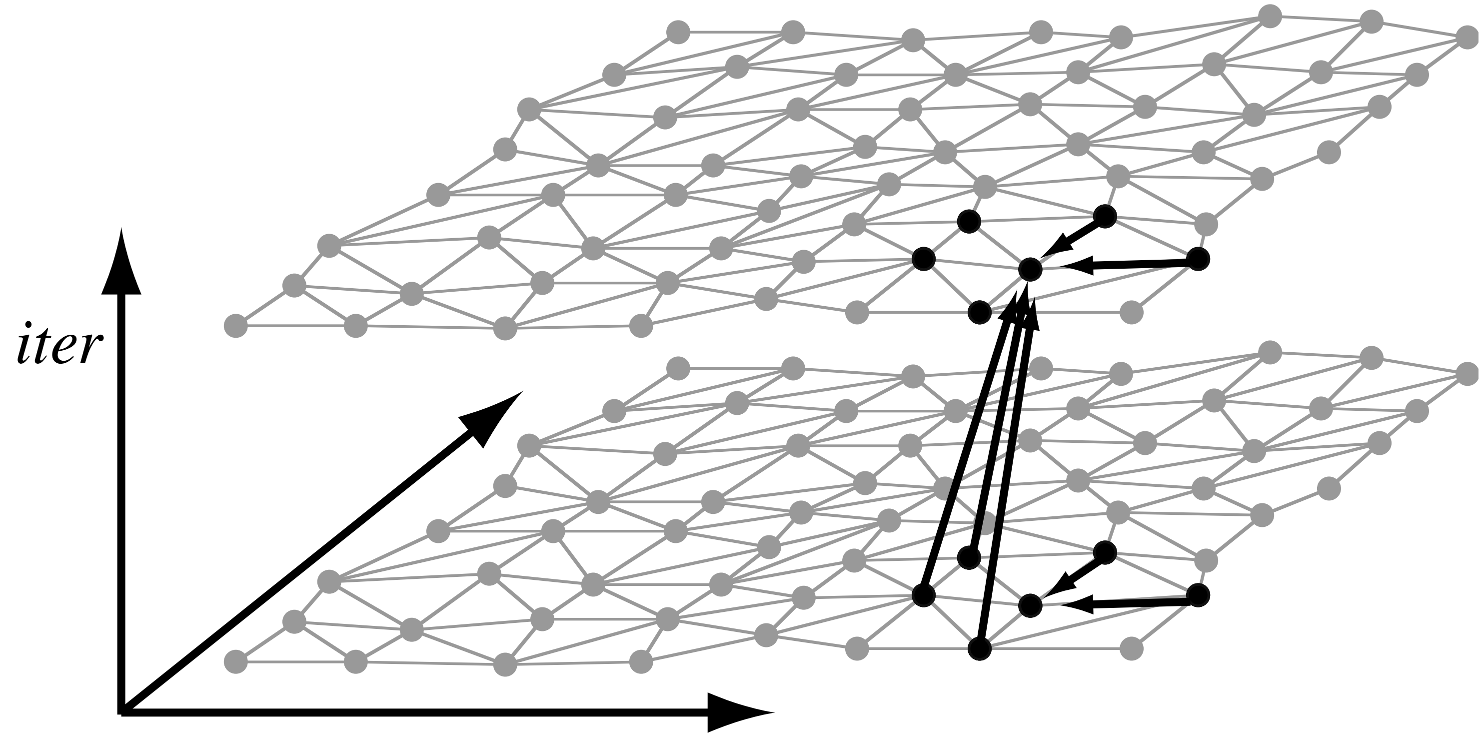



Matrix Graph

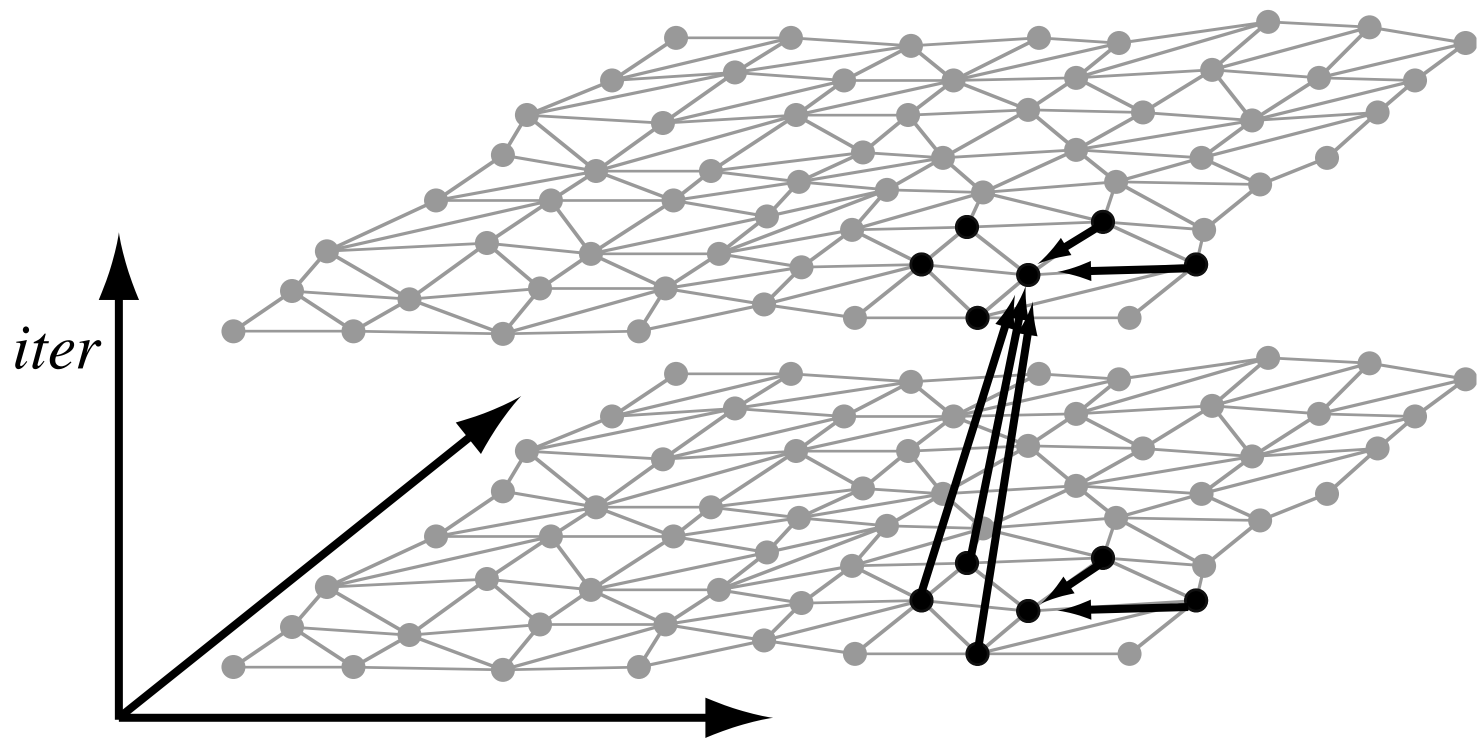


# Gauss-Seidel Iteration Space

```
do iter = 1, T
  do i = 1, R
    u(i) = f(i)
    do p = ia(i), ia(i+1) - 1
      j = ja(p)
      if (j != i)
        u(i) -= a(p) * u (ja(p))
      else
        diag = a(p)
      endif
    enddo
    u(i) = u(i) / diag
  enddo
enddo
```



# Performance Improvement Opportunities

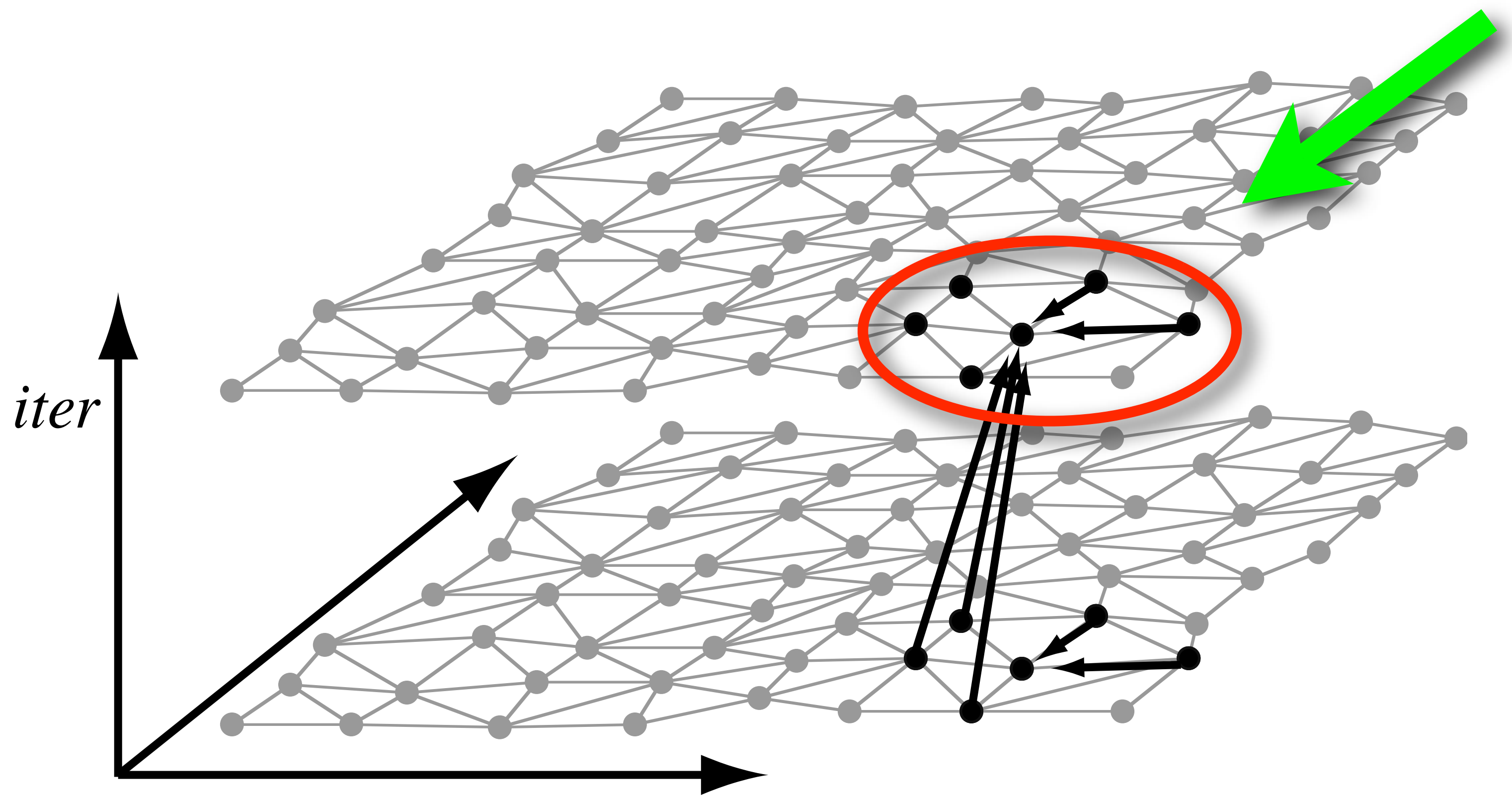



*Intra-iteration reuse*

*Inter-iteration reuse*

*Parallelism*

# Performance Improvement Opportunities

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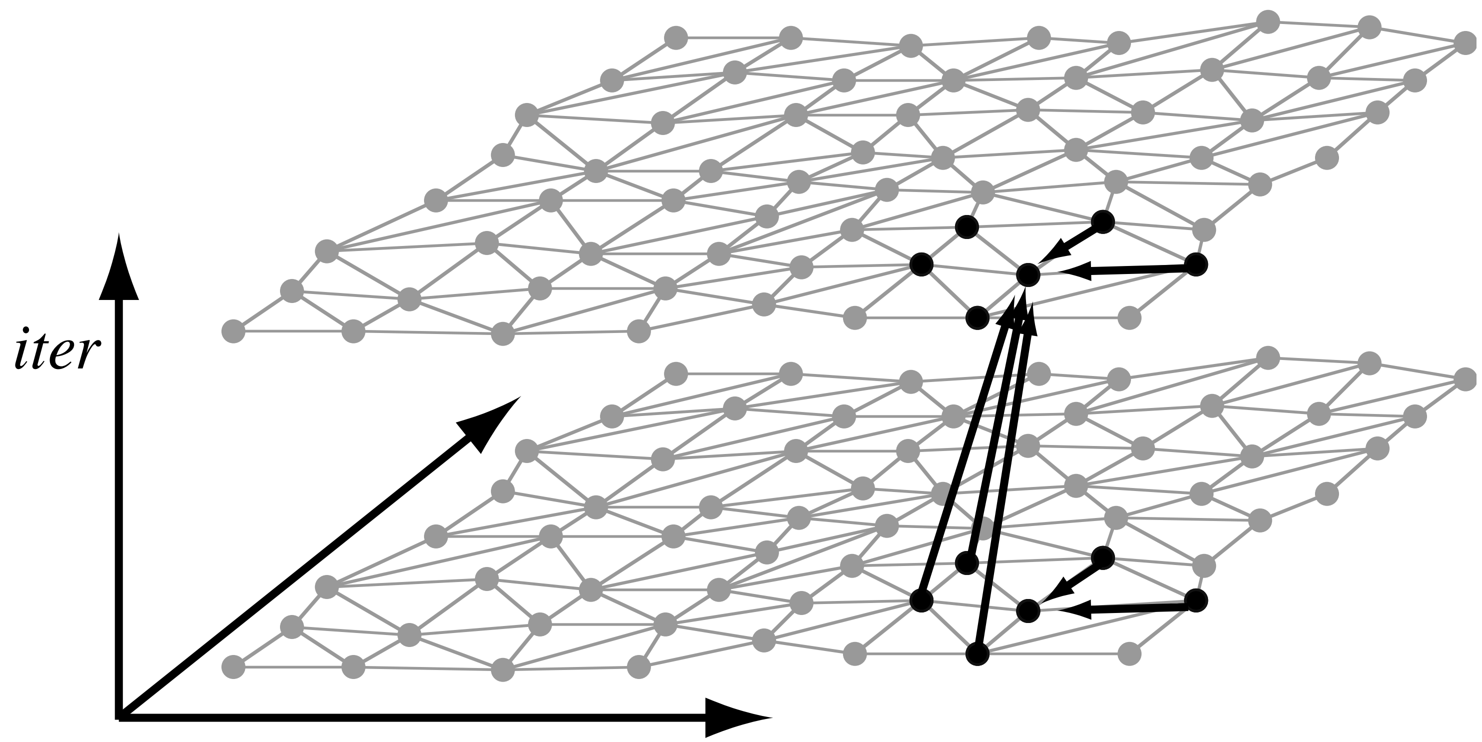


*Intra-iteration reuse*

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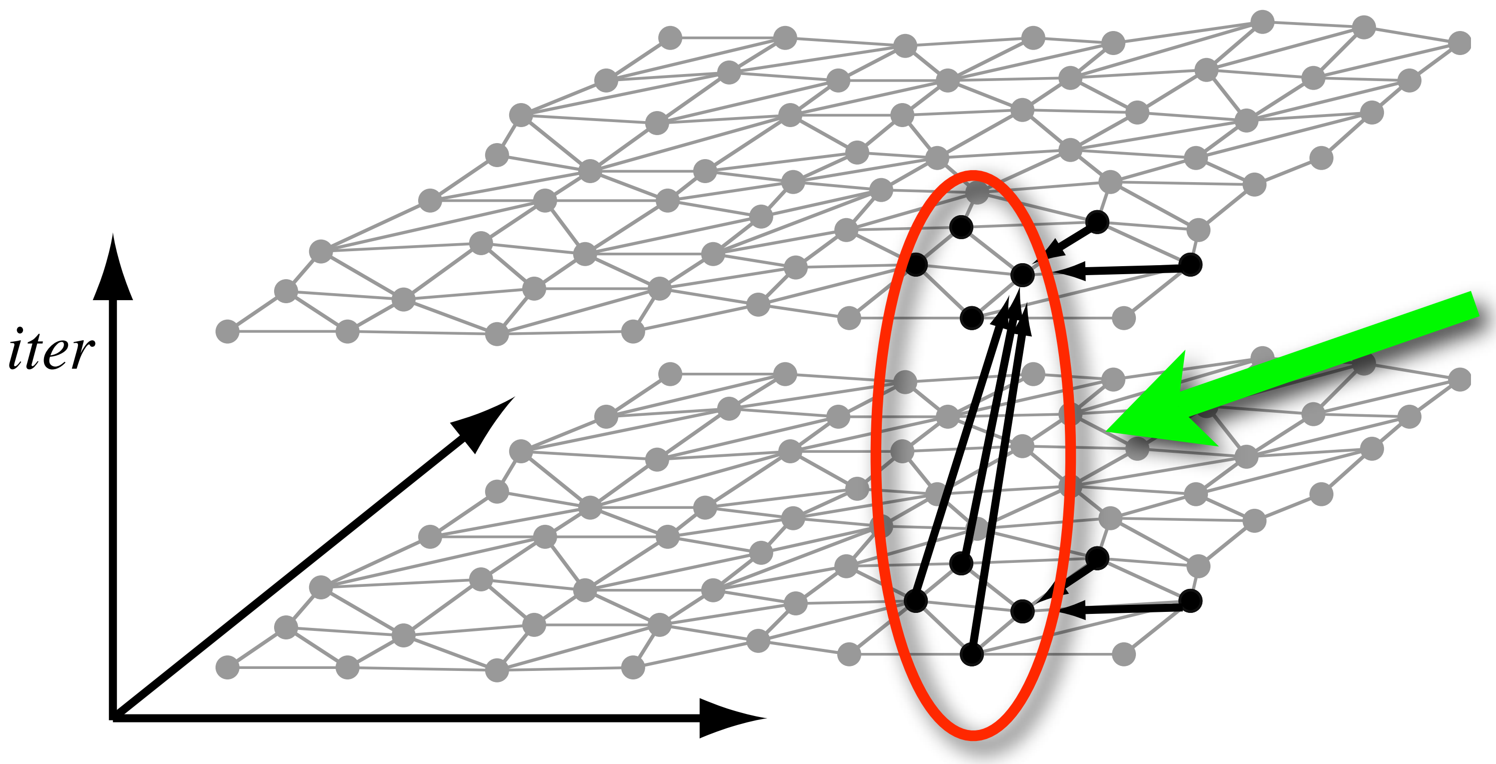


*Intra-iteration reuse*

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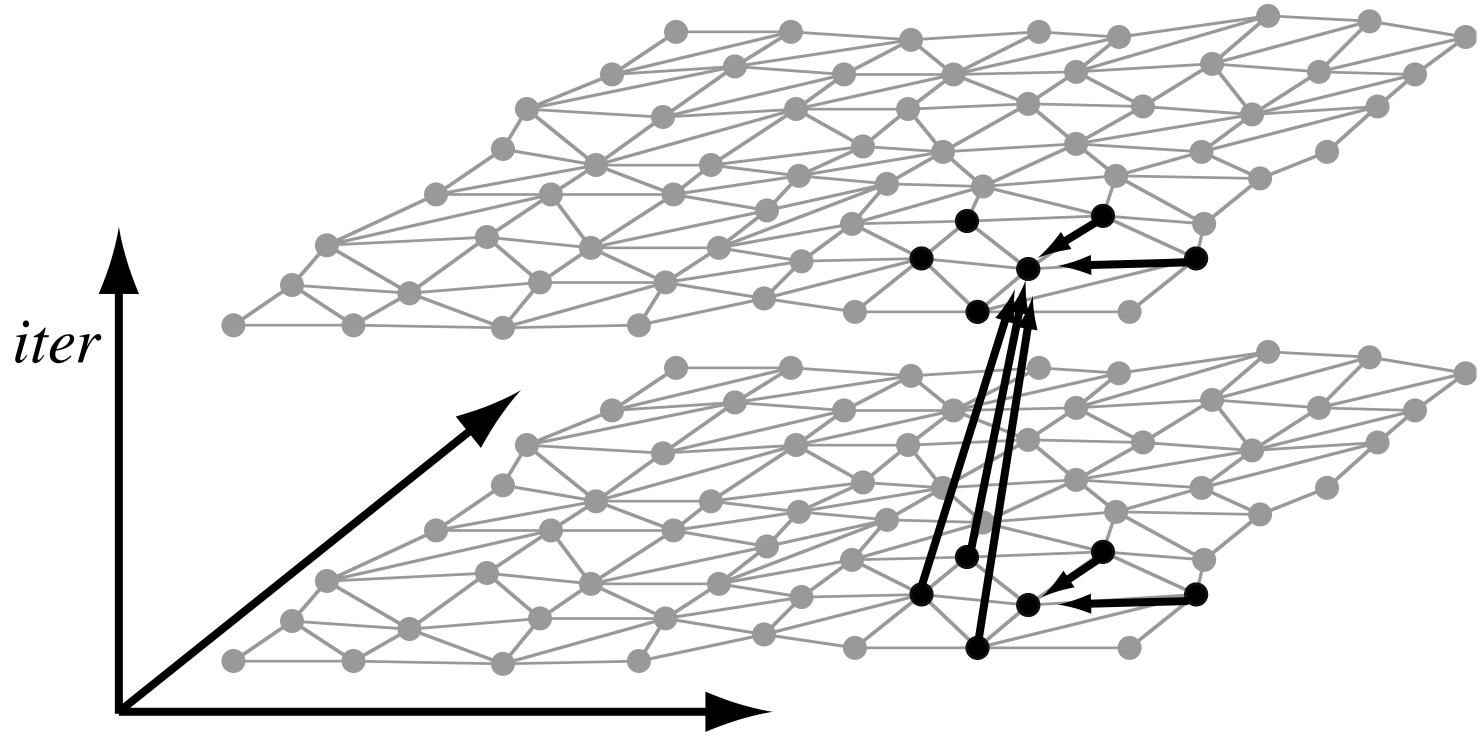
# Performance Improvement Opportunities

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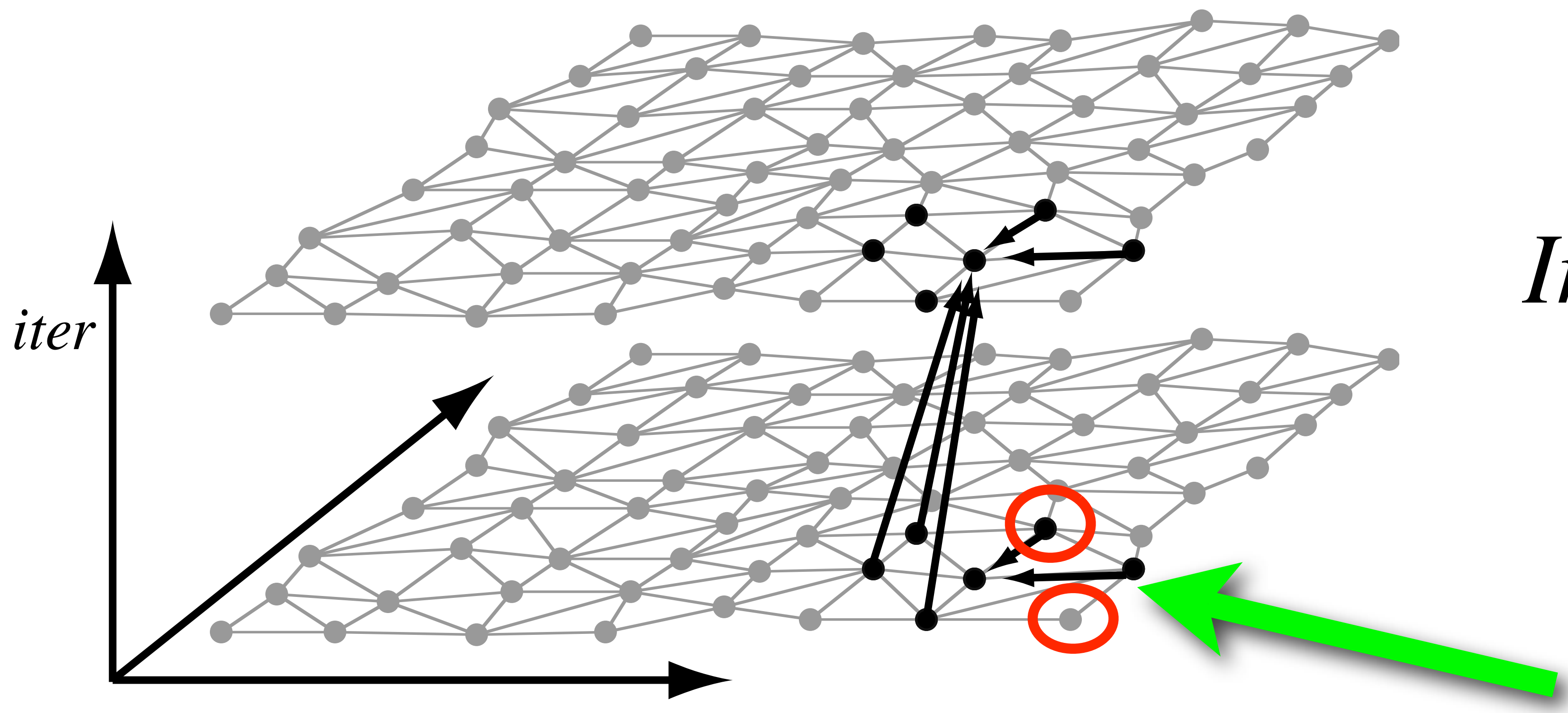
# Performance Improvement Opportunities




*Intra-iteration reuse*

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*Parallelism*

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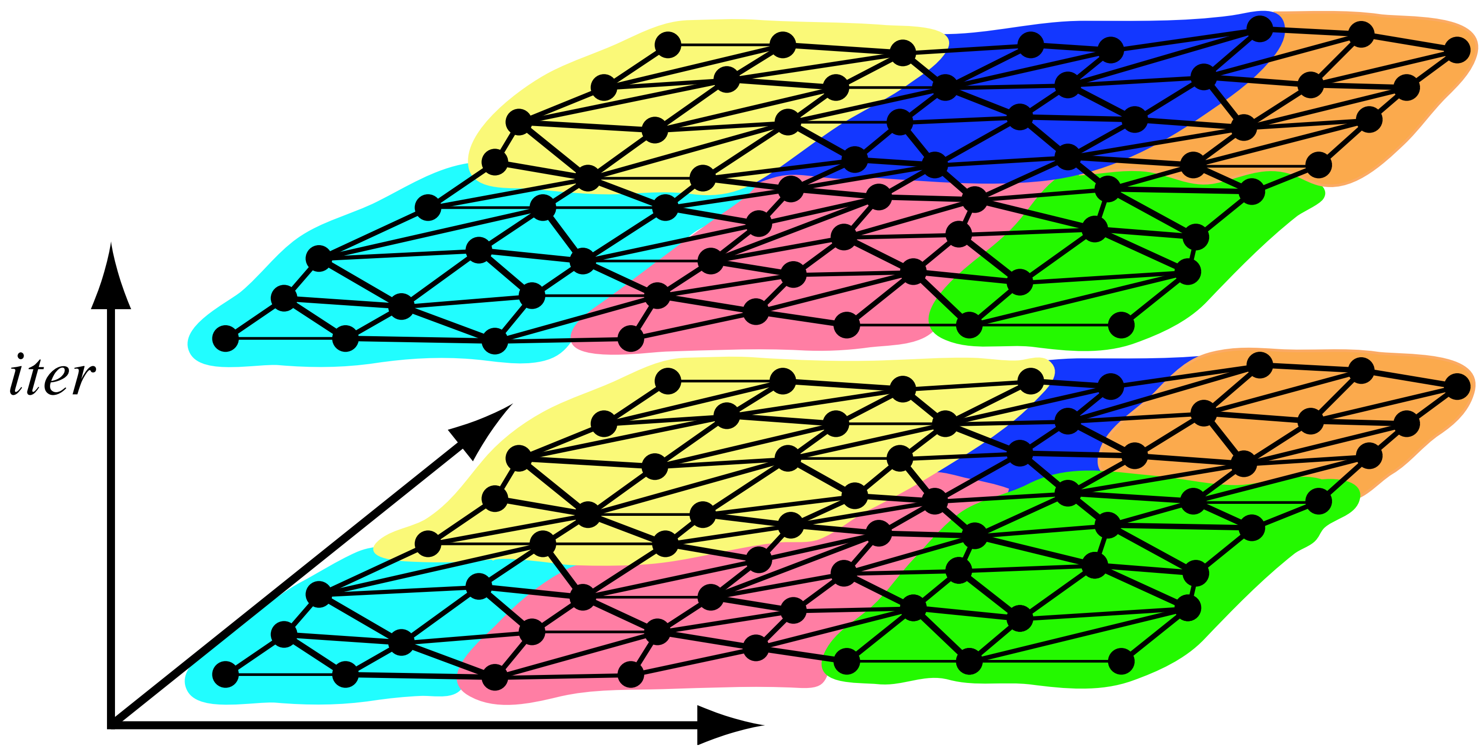
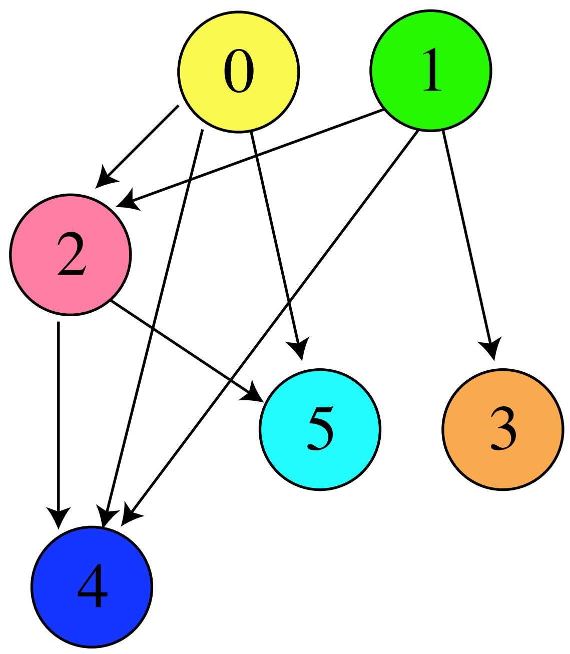
# Punchline: Full Sparse Tiling




- Breaks up computation into pieces
- Each piece has intra and inter-iteration data locality
- Some pieces can be executed in parallel

# Talk Outline

- Overview
- Parallelizing Jacobi with the owner computes method xor full sparse tiling
- Increasing the parallelism in full sparse tile schedules
- Extra work needed to handle Gauss-Seidel
- Experimental Results

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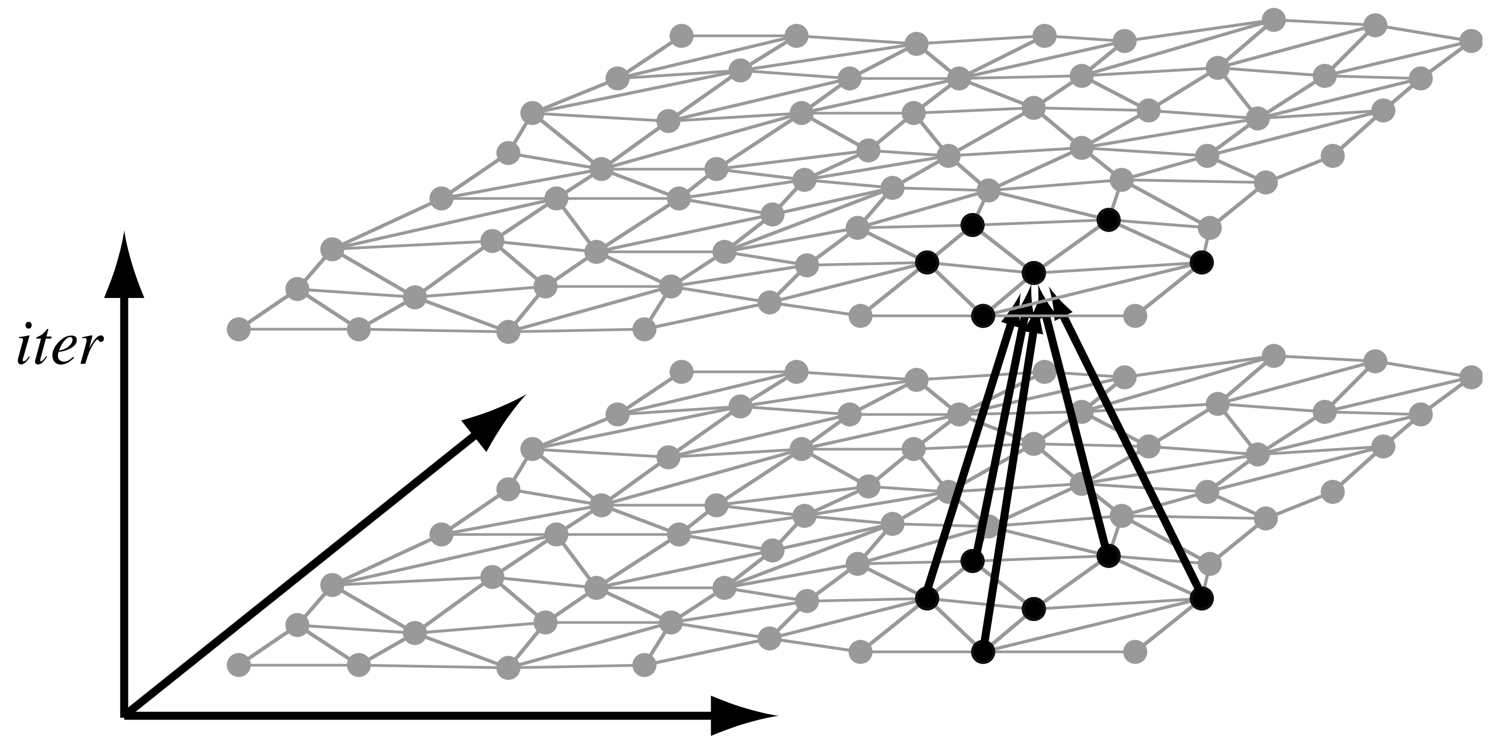
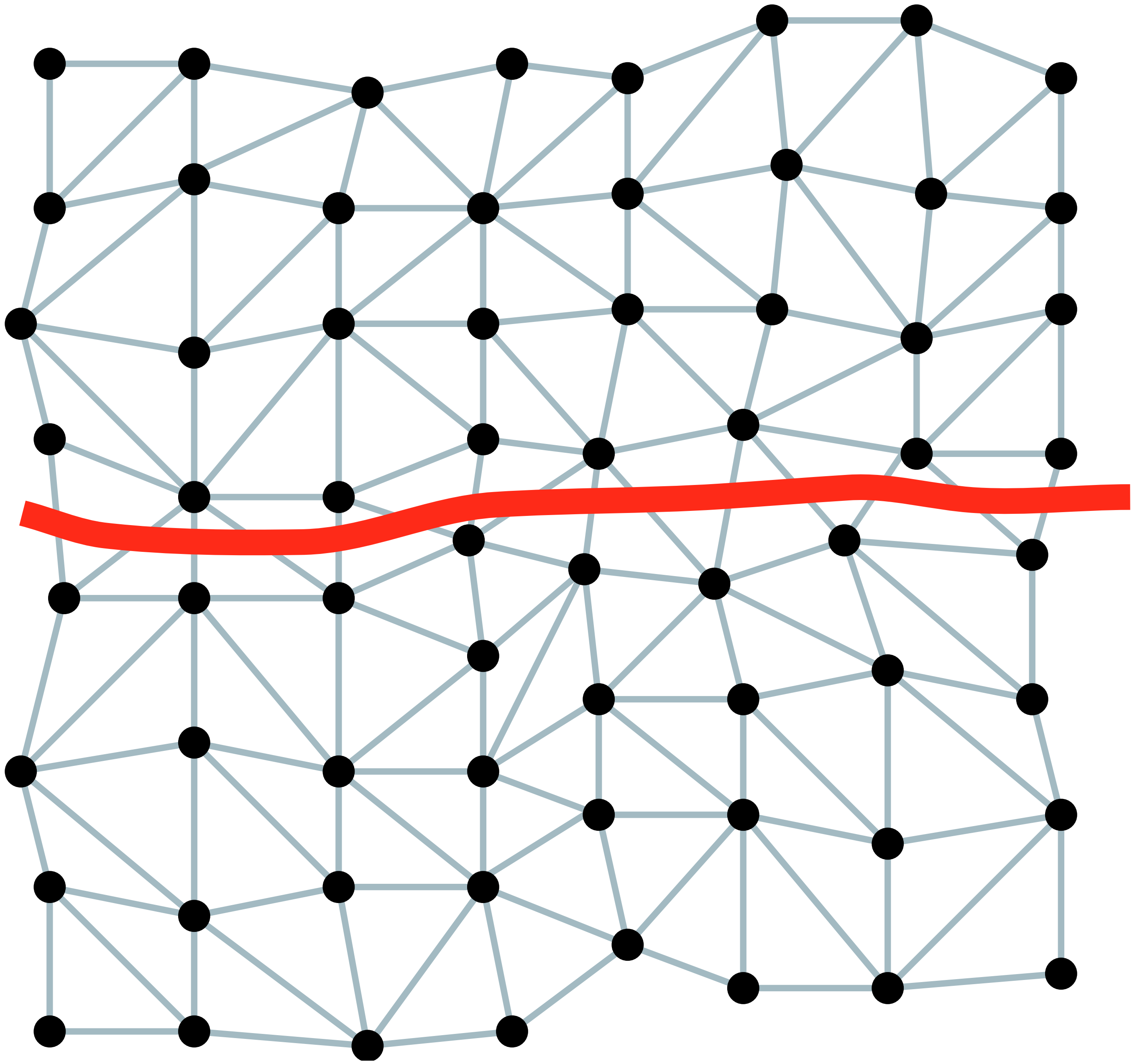
# Jacobi: similar yet simpler

```
do iter = 1, T
   do i = 1, R
      u(i) = f(i)
      do p = ia(i), ia(i+ 1) - 1
         j = ja(p)
         if (j != i) then
            u(i) -= a(p)*tmp (j)
         else
            diag = a(p)
         endif
      enddo
      u(i) = u(i)/diag
   enddo
   do i = 1, R
      tmp (i) = u(i)
   enddo
enddo
```




- data dependences not known until runtime
- no intra-iteration dependences
- i loop is a reduction, therefore parallelizable

# Parallelize with Owner Computes Method



At each convergence iteration ...

- each partition receives data from previous convergence iteration
- each partition executes in parallel
- each partition sends data

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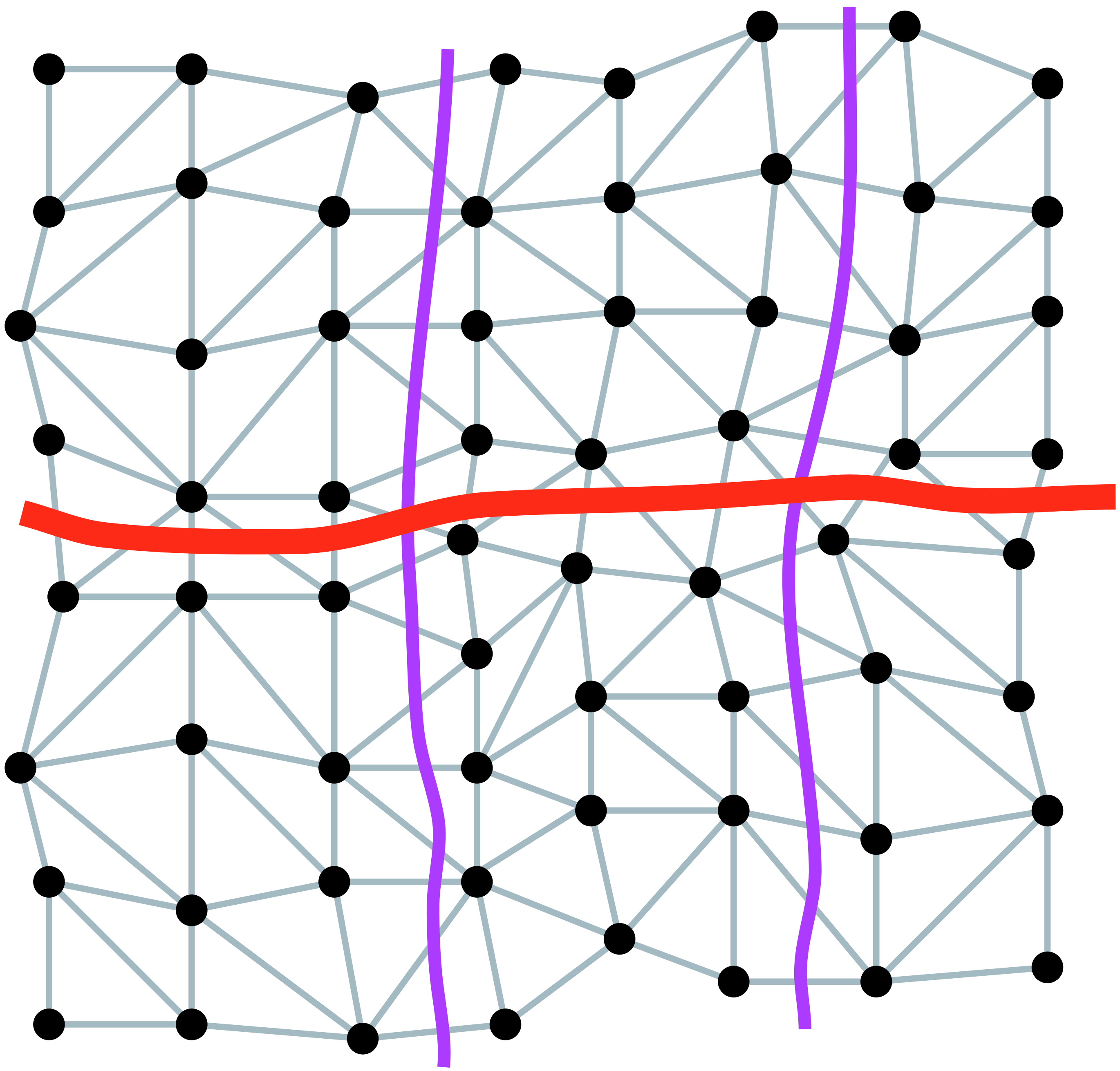
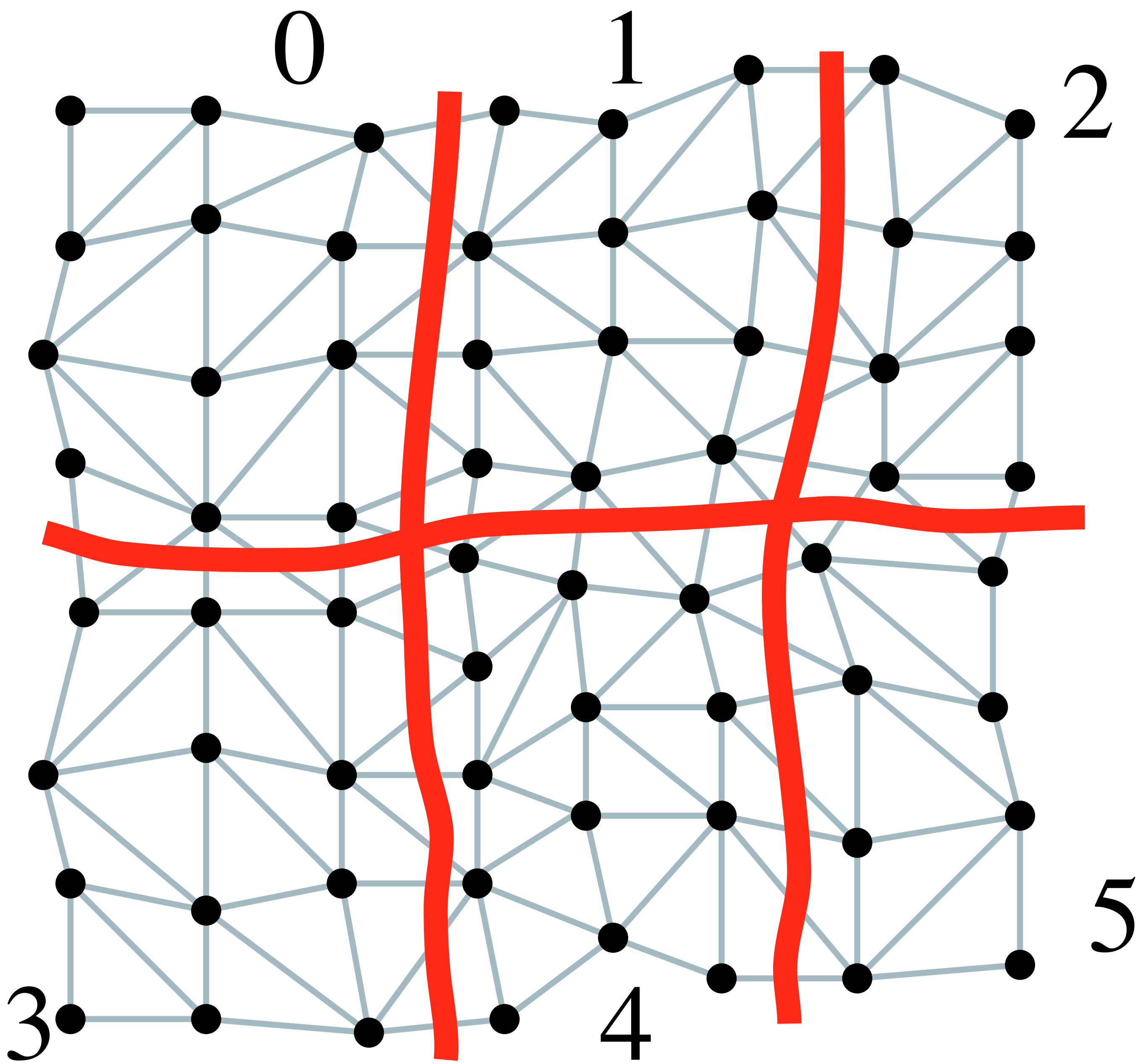
# Locality in Owner Computes Method



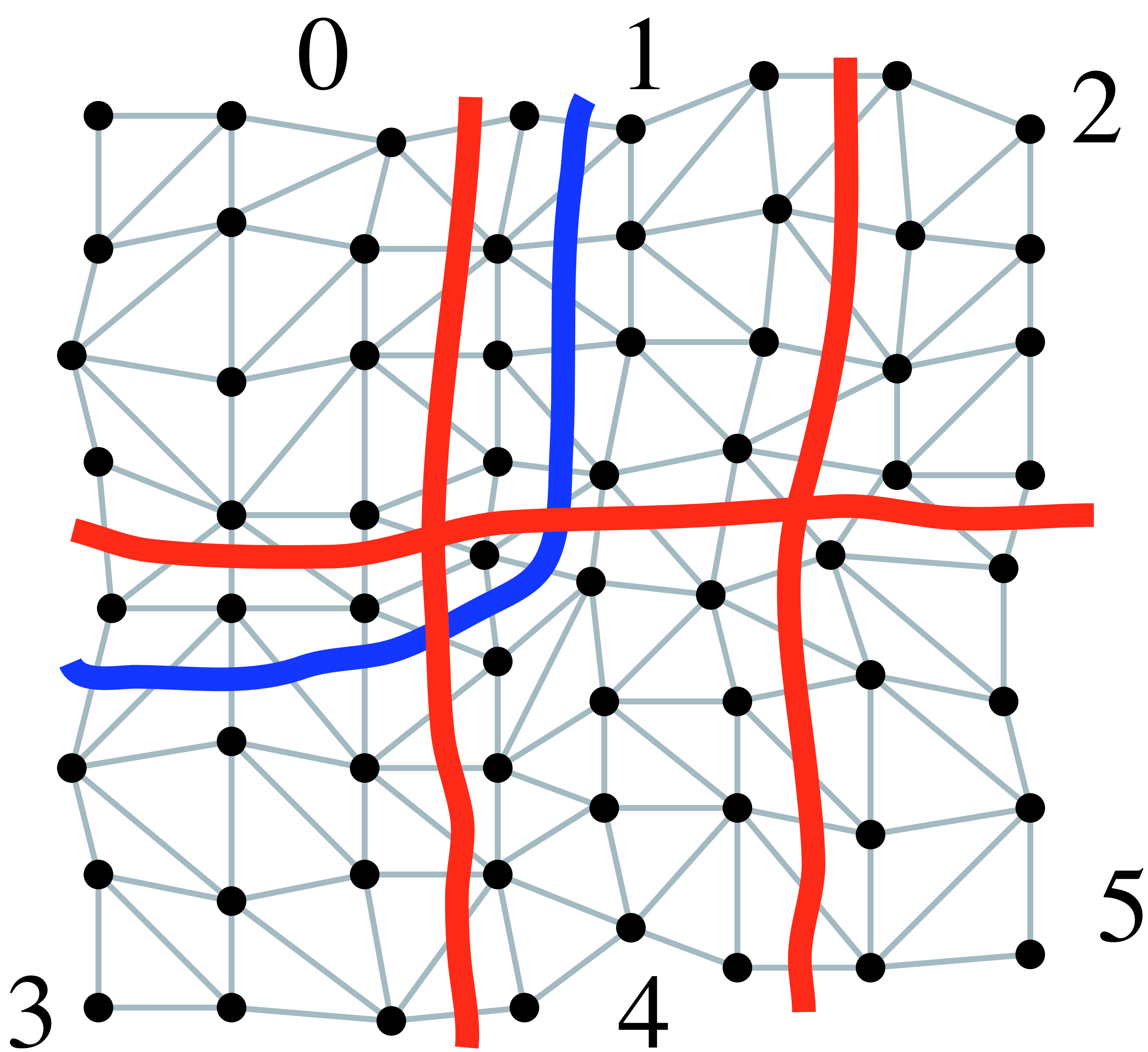
- Intra-iteration locality
  - schedule by sub-part
  - reorder data for consecutive order in sub-parts
- NO Inter-iteration locality, main partitions don't fit in cache

# Parallelize with Full Sparse Tiling



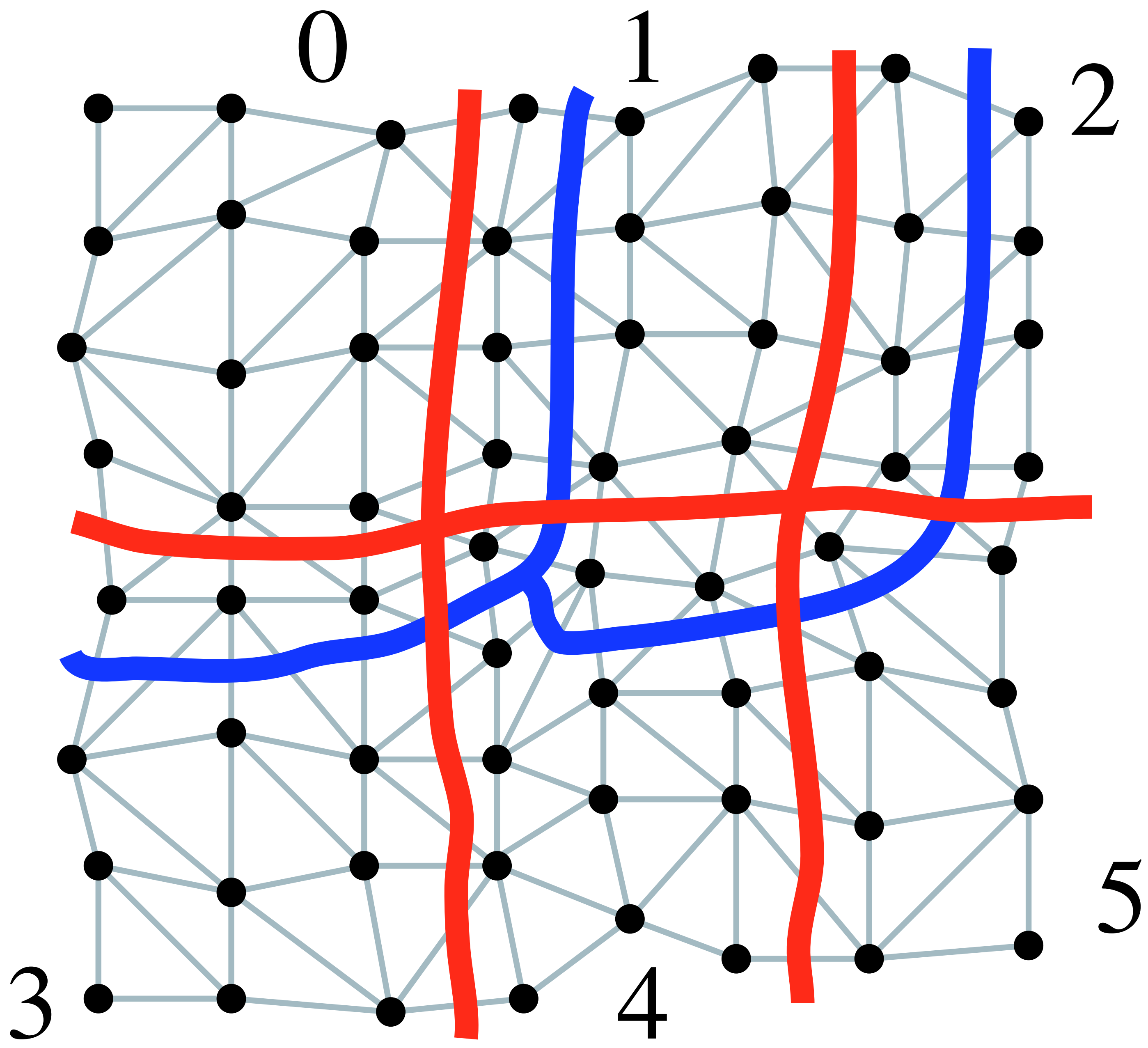

- Create seed partitioning at *iter* = 2

# Parallelize with Full Sparse Tiling



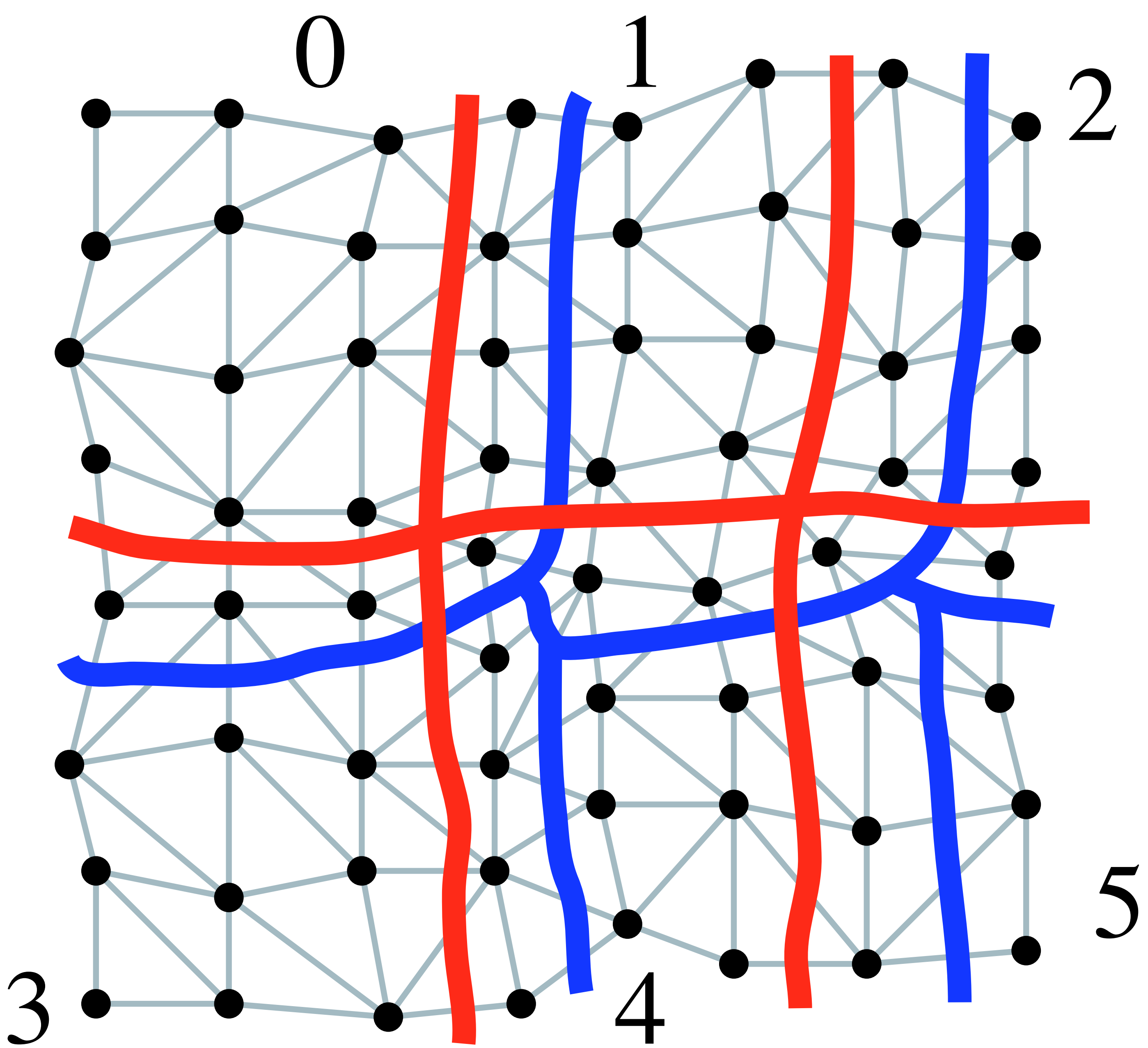

- Create seed partitioning at *iter* = 2
- Grow tiles to *iter* = 1 based on ordering of partitions

# Parallelize with Full Sparse Tiling



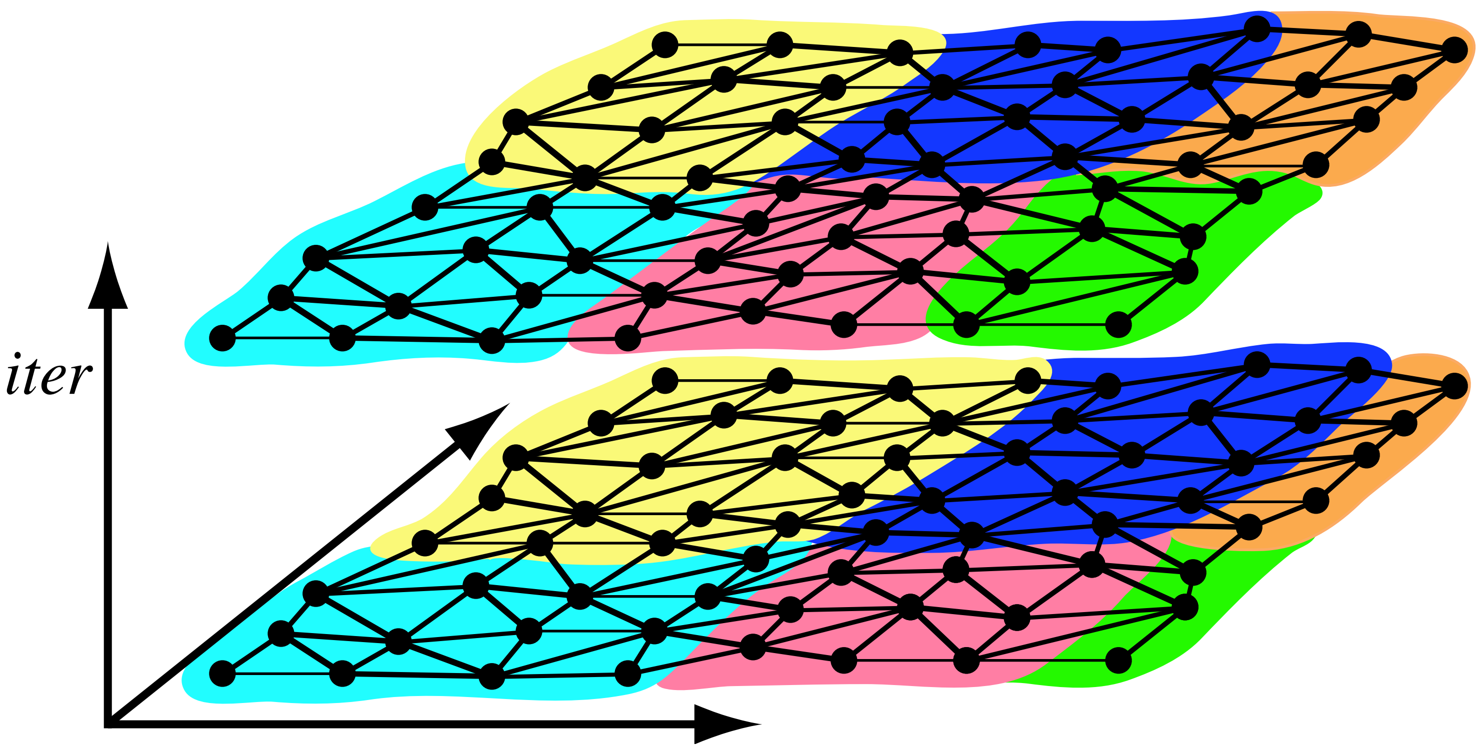

- Create seed partitioning at *iter* = 2
- Grow tiles to *iter* = 1 based on ordering of partitions

# Parallelize with Full Sparse Tiling




- Create seed partitioning at *iter* = 2
- Grow tiles to *iter* = 1 based on ordering of partitions

# Locality in Full Sparse Tiled Jacobi




- **Intra-iteration** locality achieved by consecutively ordering matrix graph nodes within a seed partition
- **Inter-iteration** locality achieved with tile-by-tile execution

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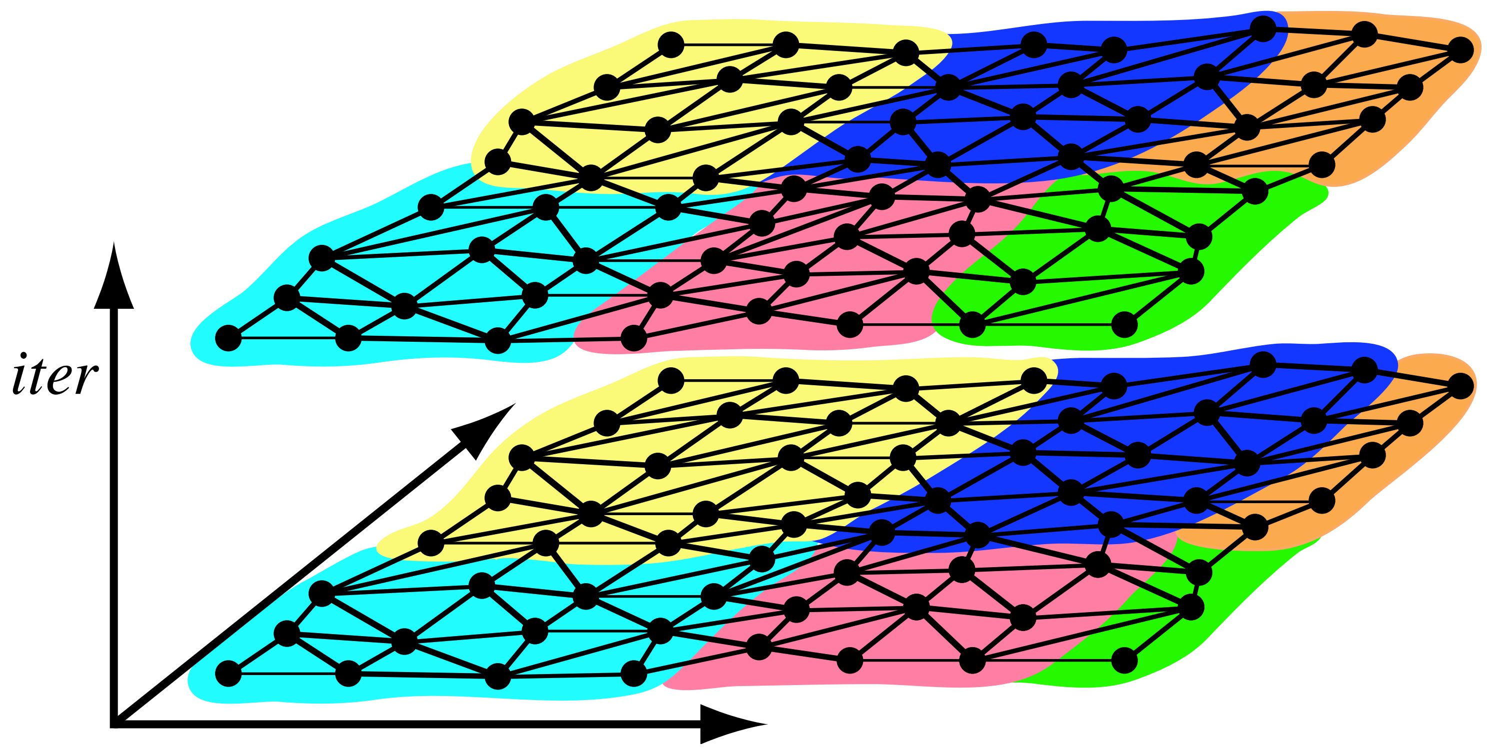
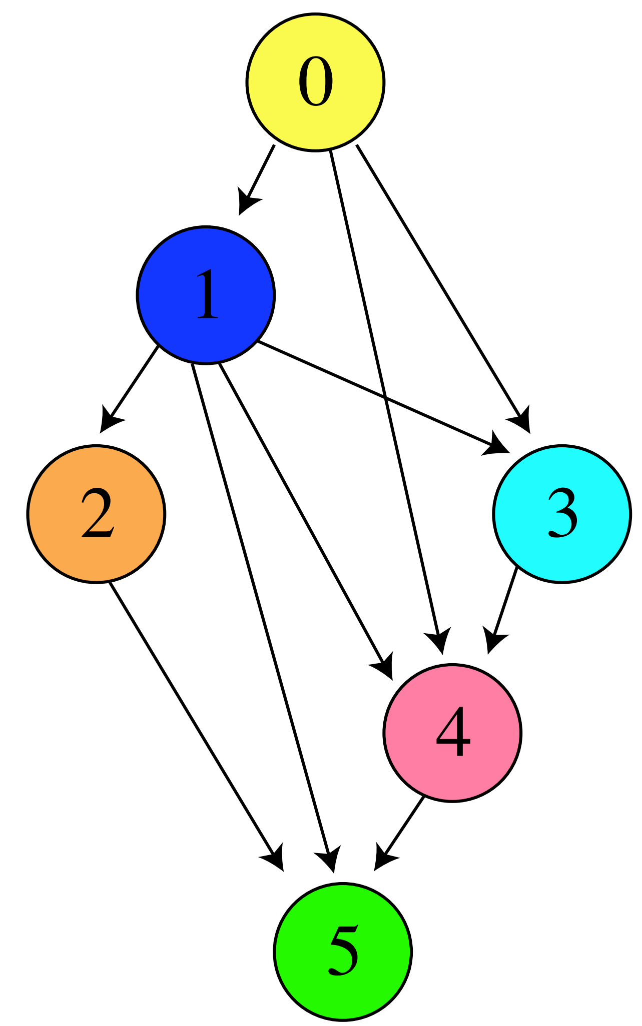
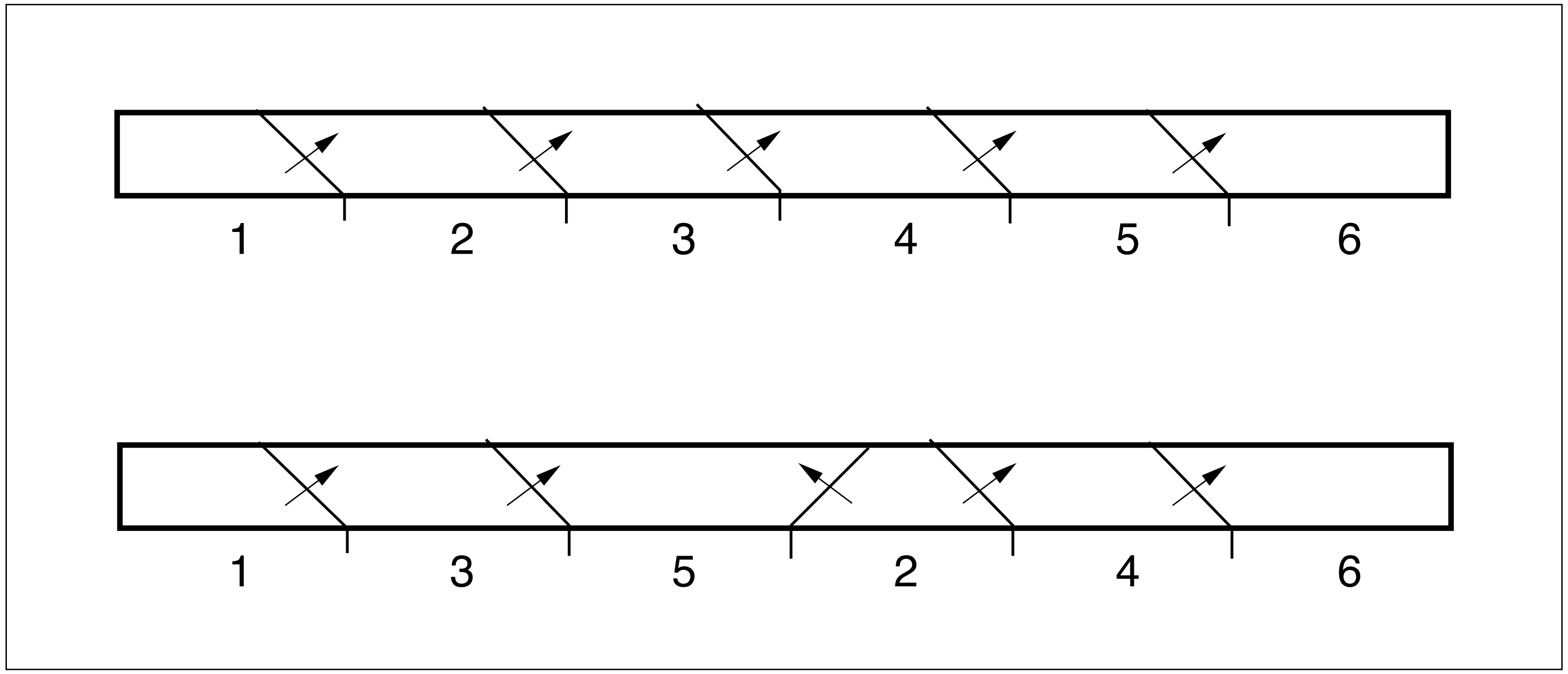
# Parallelism in Full Sparse Tiled Jacobi



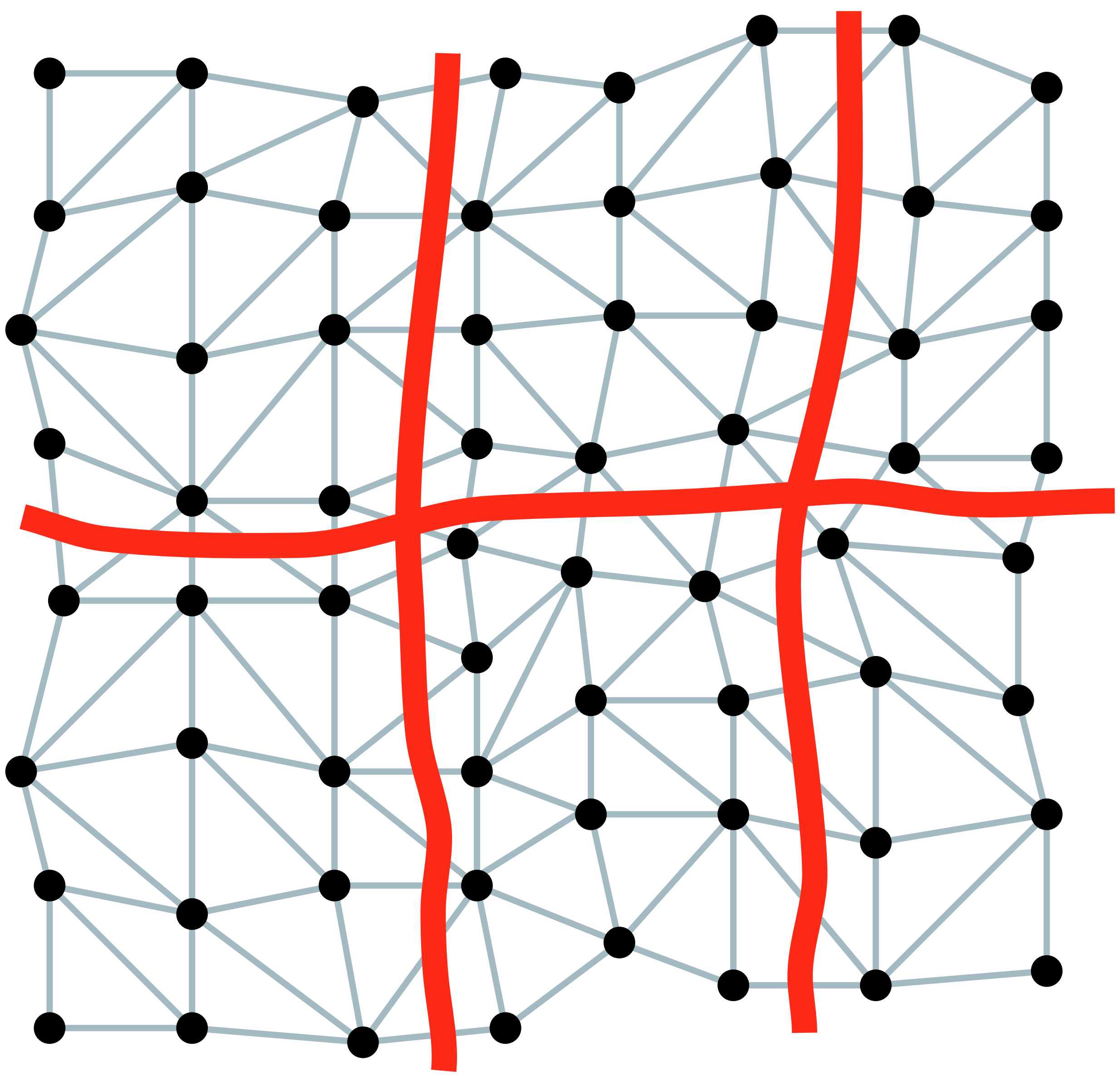

Average parallelism = (# tiles) / (# tiles in critical path)
= 6 / 5 = 1.2

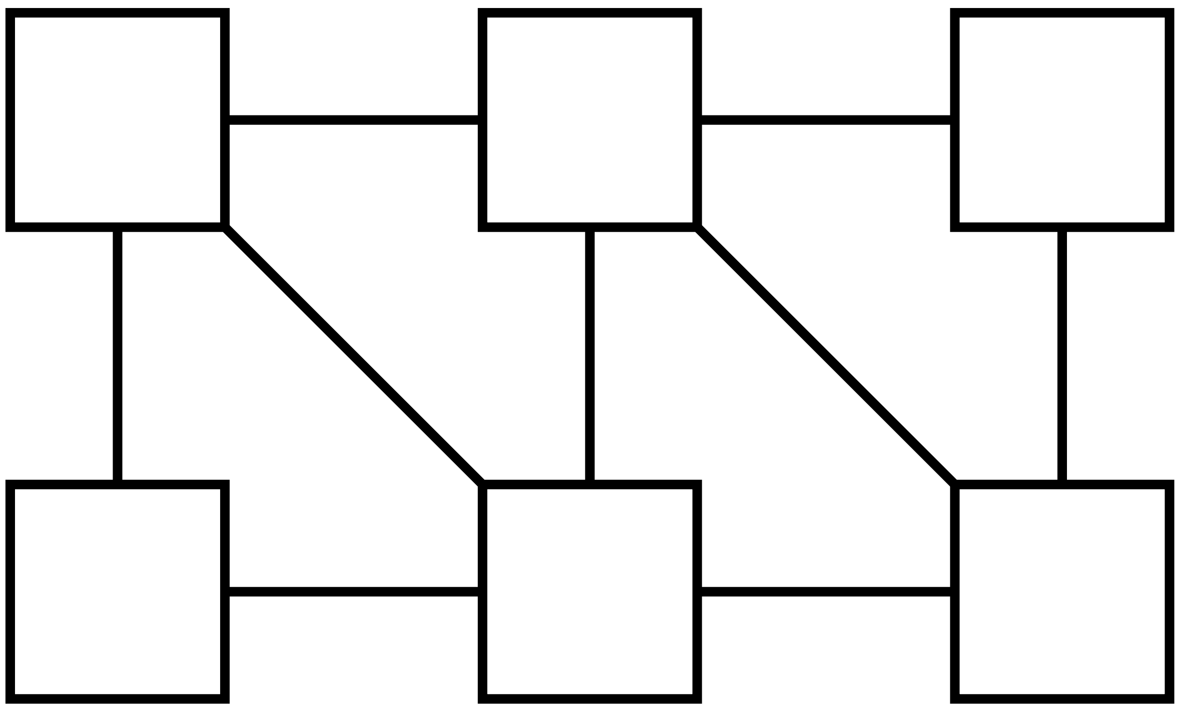
# How can we increase parallelism between tiles?




- Order that tile growth is performed matters
- Best is to first grow tiles whose seed partitions are not adjacent

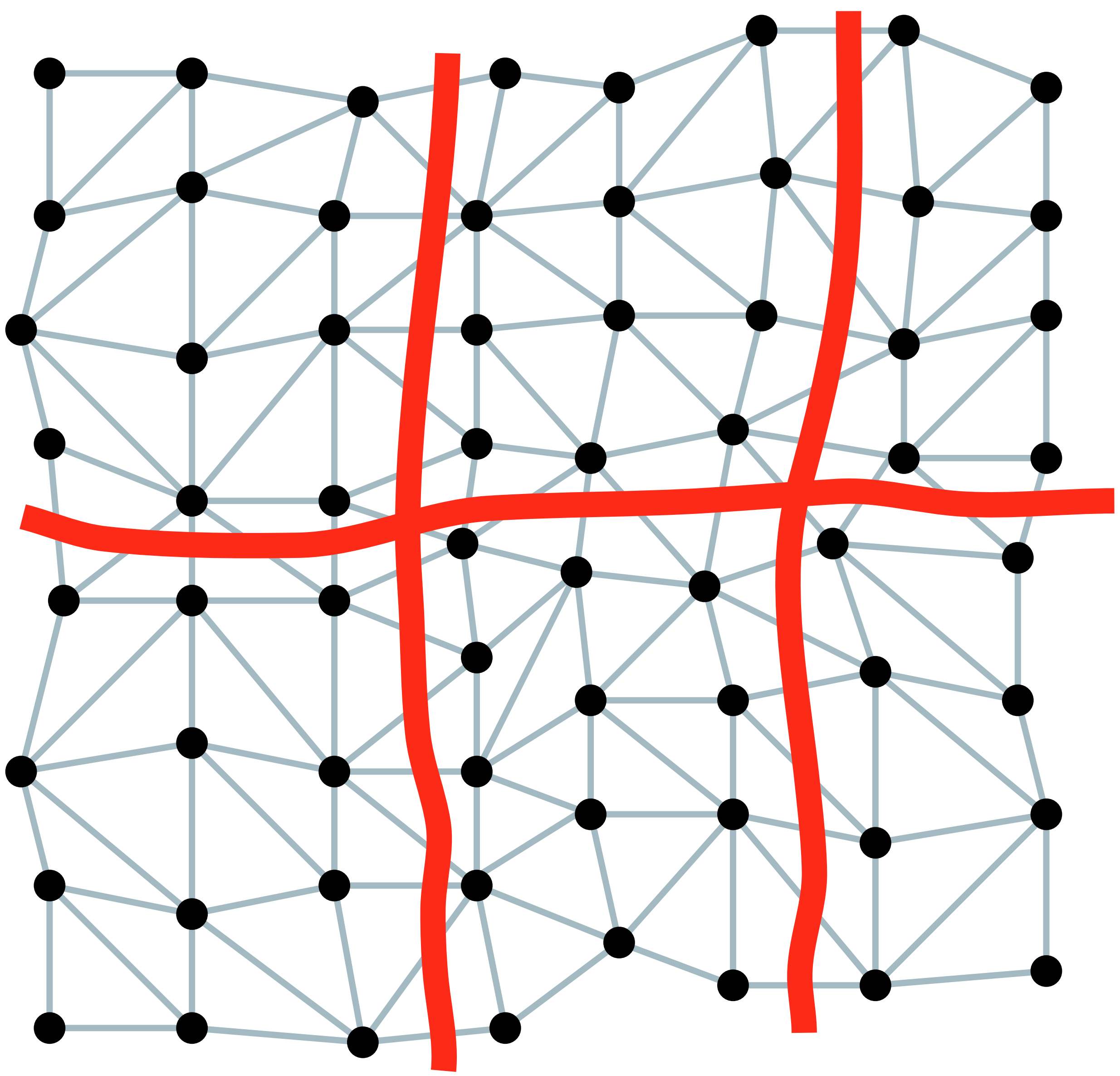
# Improving Average Parallelism Using Coloring

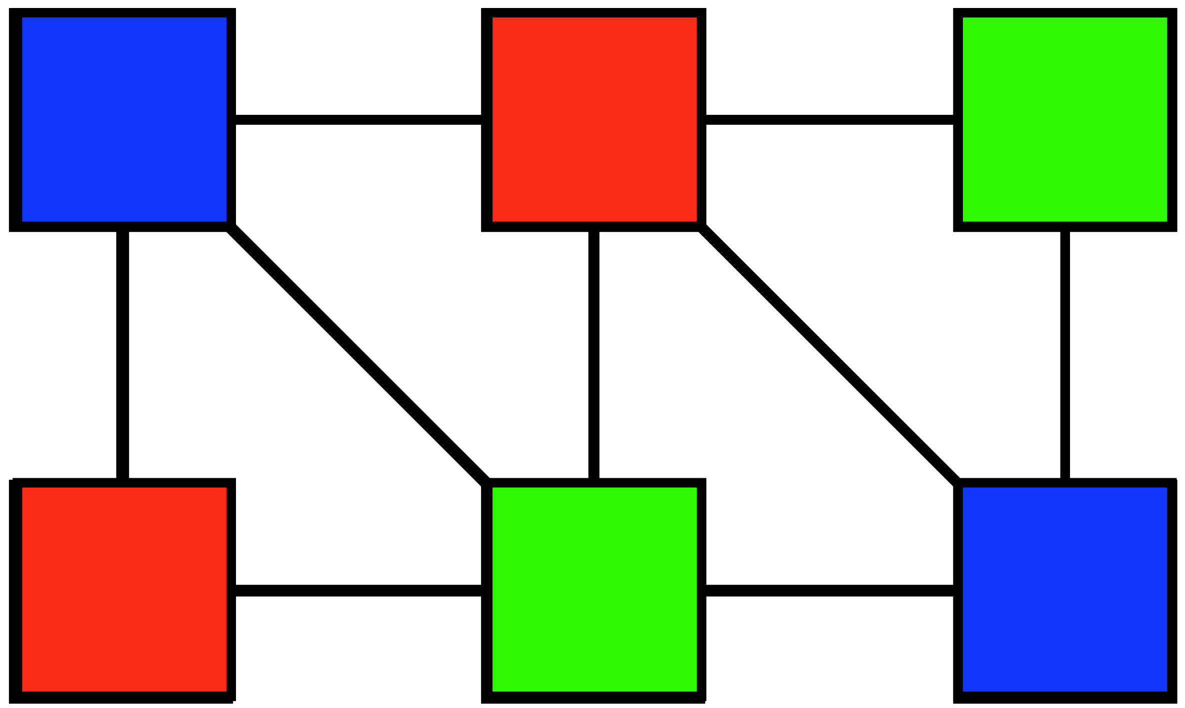




- Create a partition graph

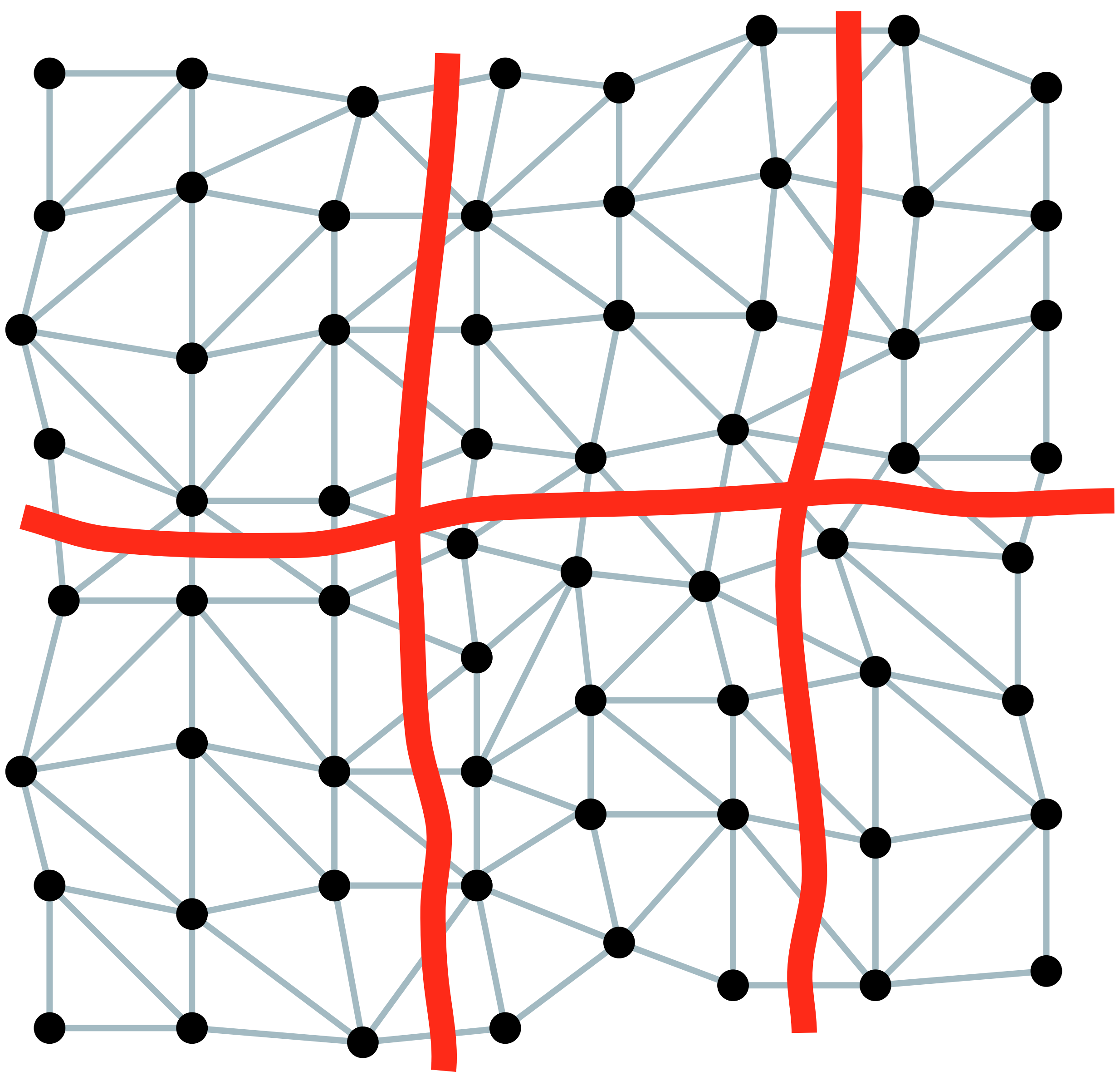
# Improving Average Parallelism Using Coloring

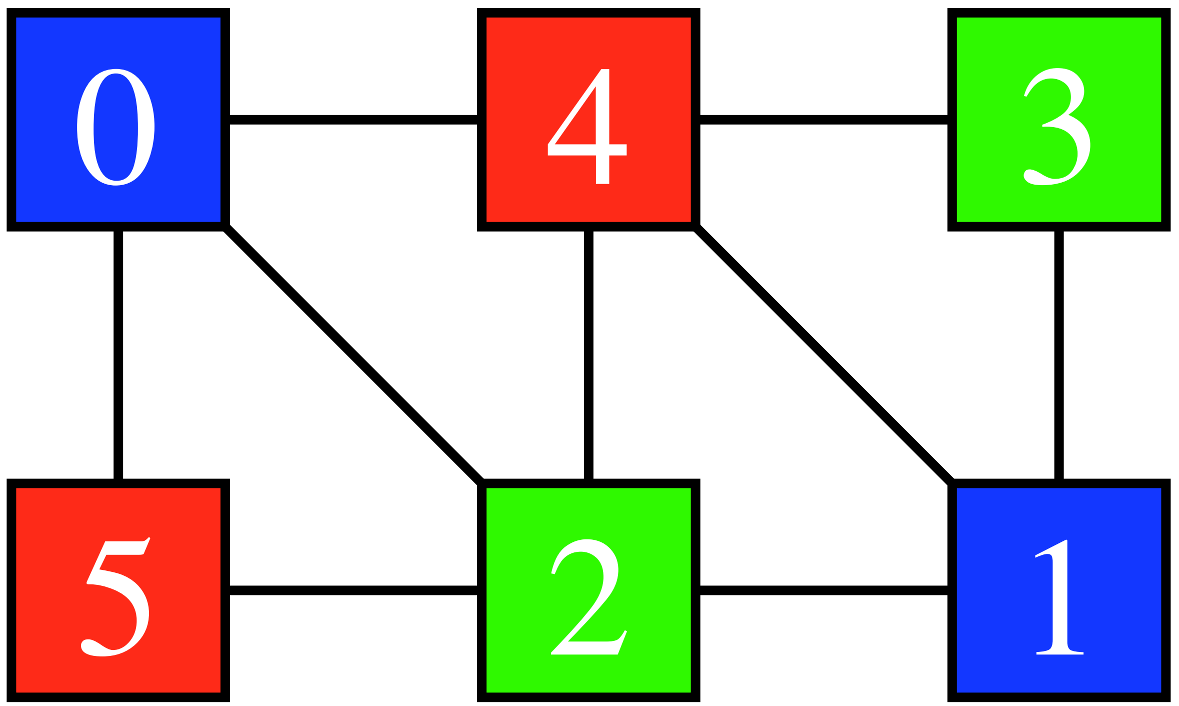




- Create a partition graph
- Color the partition graph

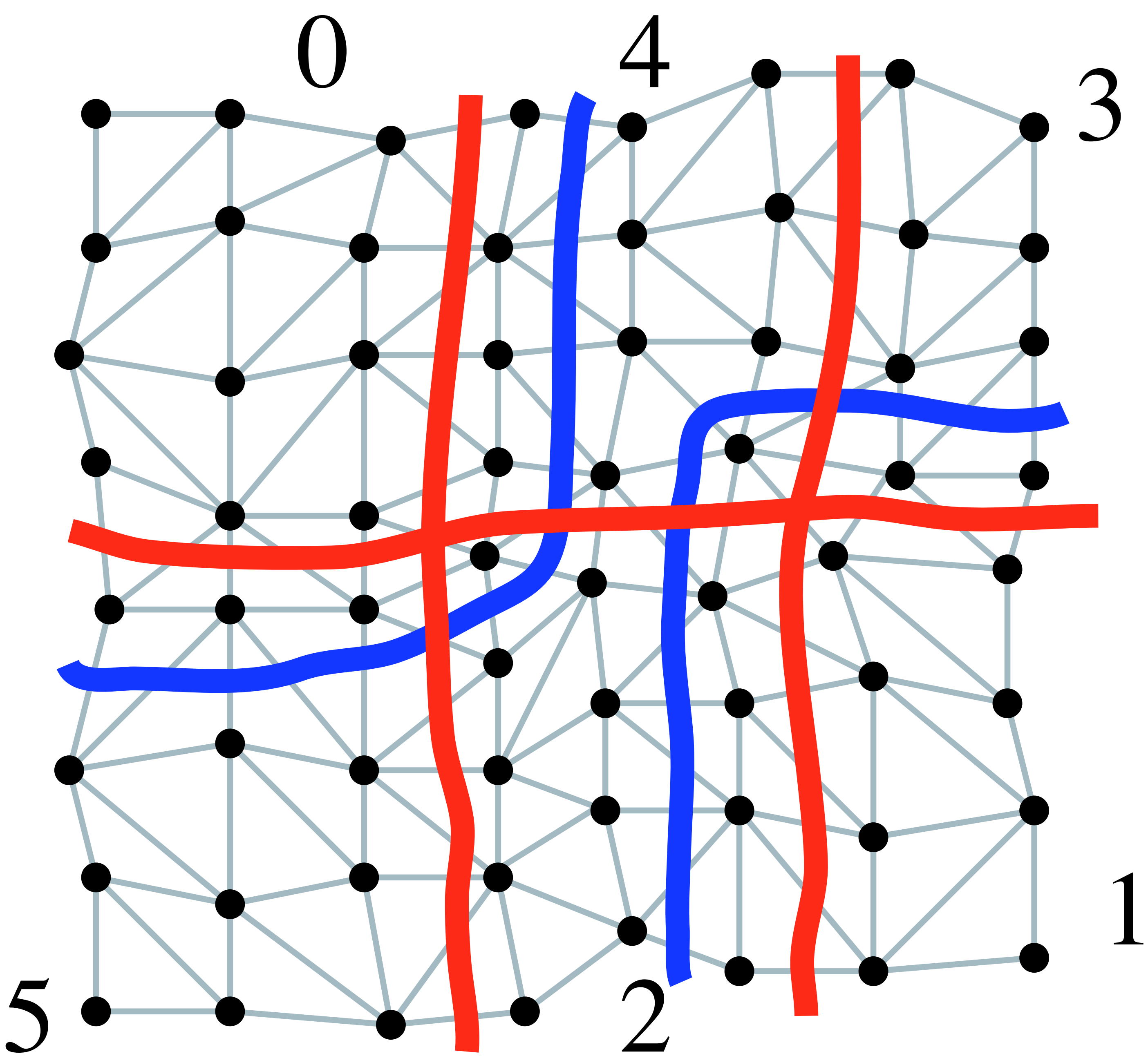
# Improving Average Parallelism Using Coloring






- Create a partition graph
- Color the partition graph
- Renumber partitions consecutively by color

# Grow Using New Partition Order




- Renumber the seed partition cells based on coloring
- Grow tiles using new ordering
- Notice that tiles 0 and 1 may be executed in parallel

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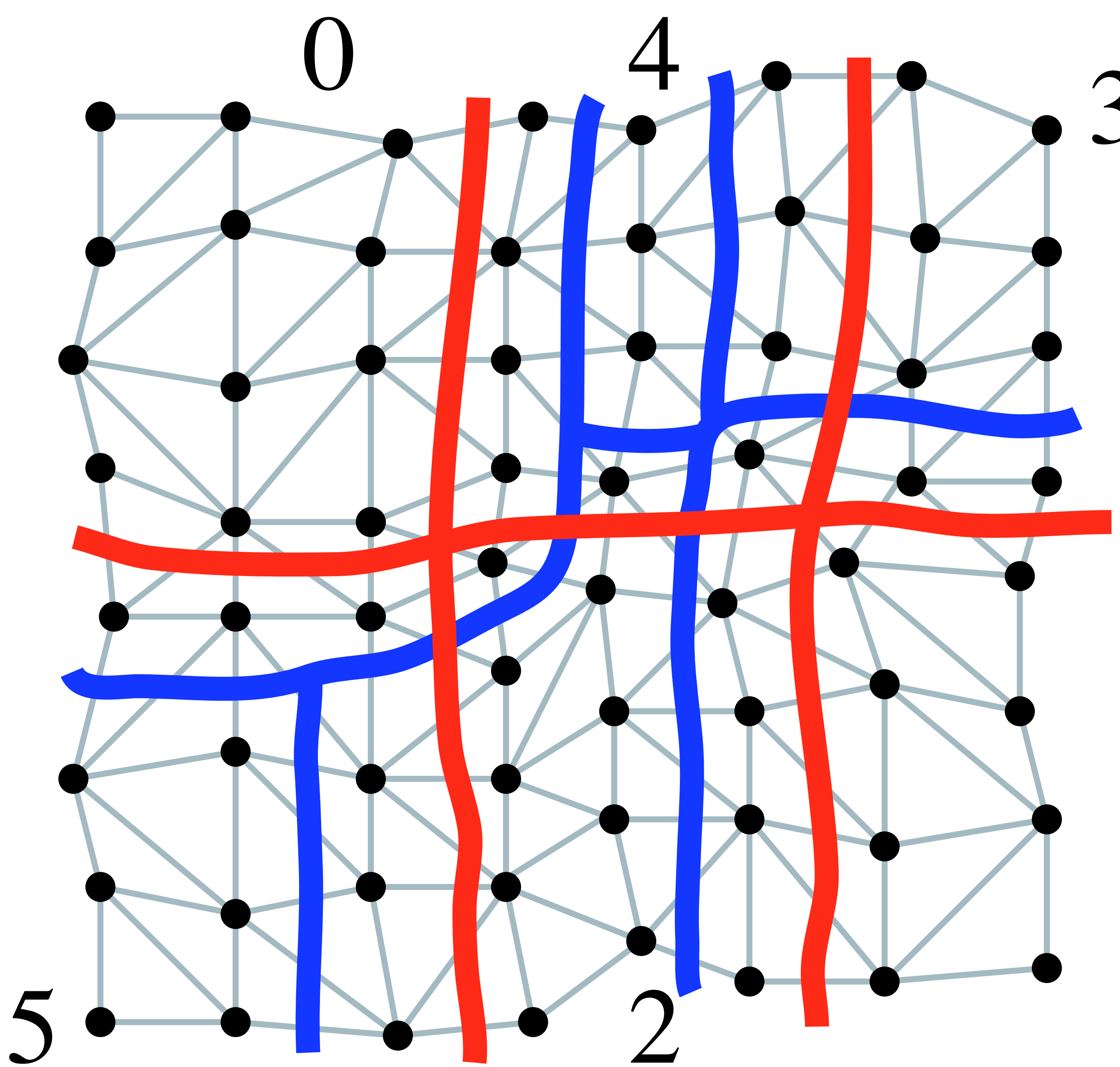
# Re-grow Using New Partition Order




- Renumber the seed partition cells based on coloring
- Grow tiles using new ordering
- Notice that tiles 0 and 1 may be executed in parallel
- Tiles 4 and 5 may also be executed in parallel

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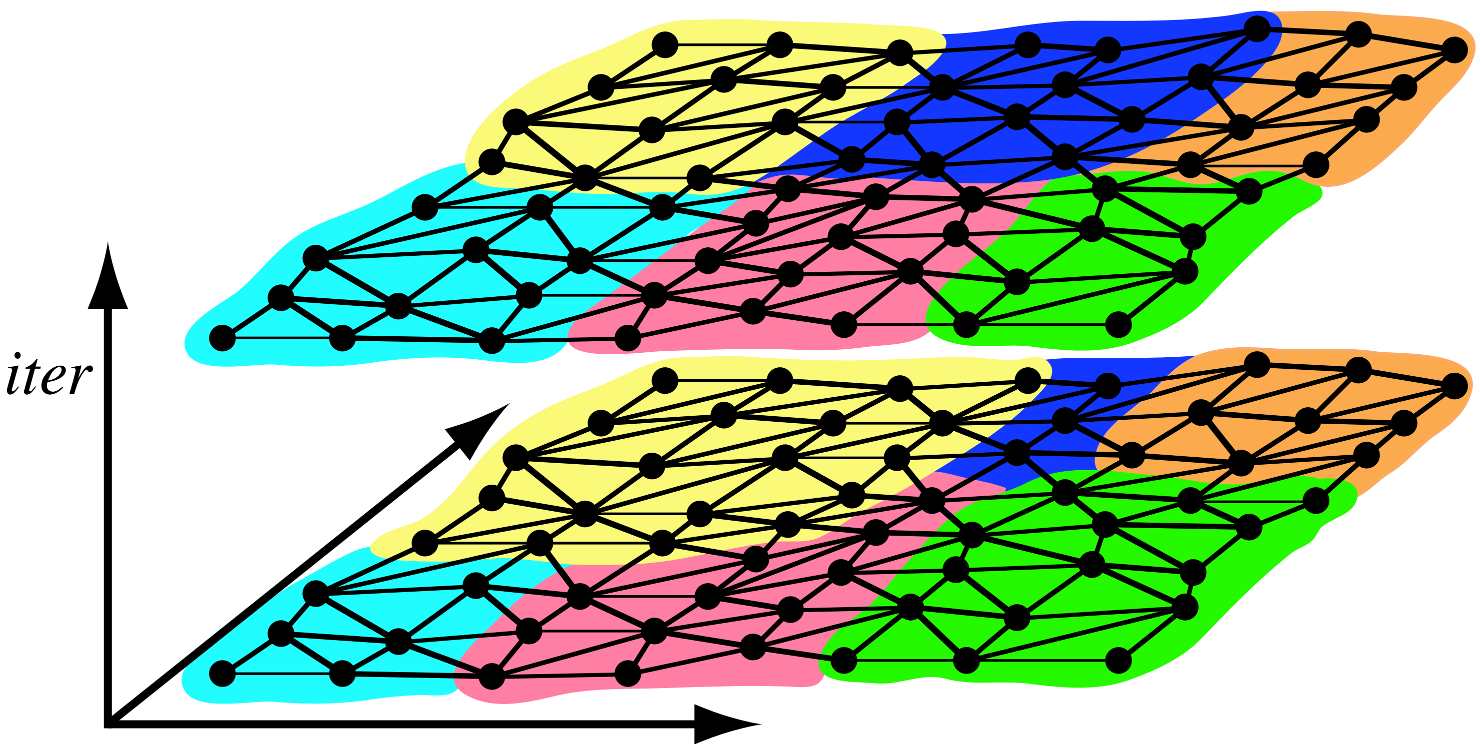
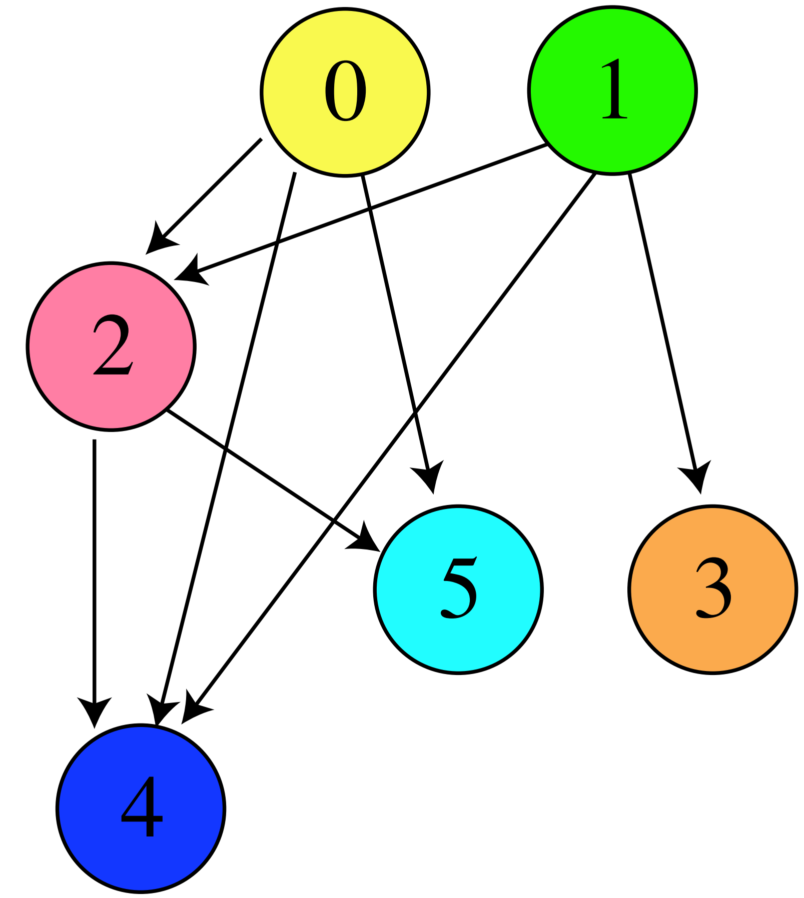
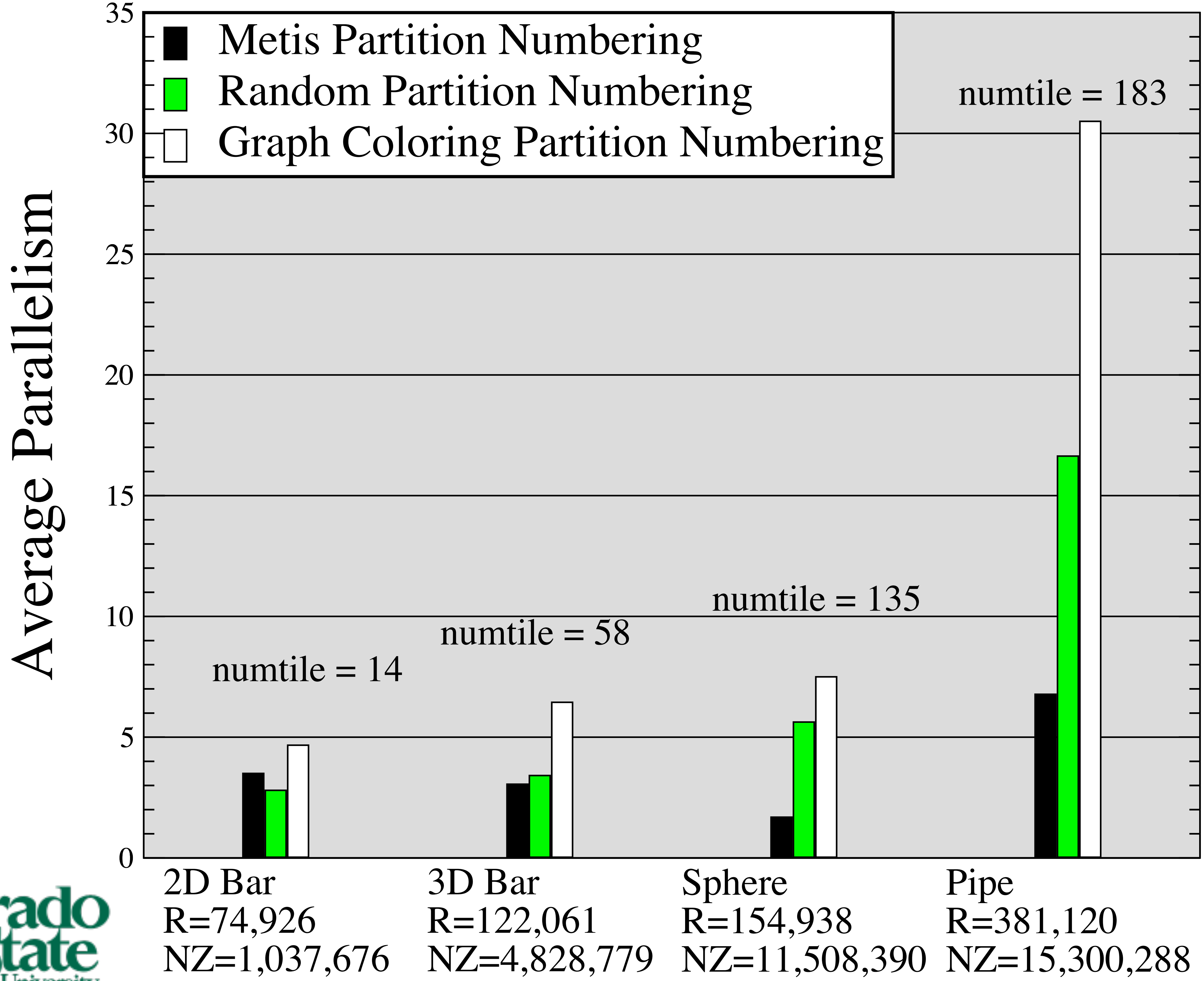
# Average Parallelism is Improved




Average parallelism = (# tiles) / (# tiles in critical path)
= 6 / 3 = 2

# Improvement with Real Matrix Graphs

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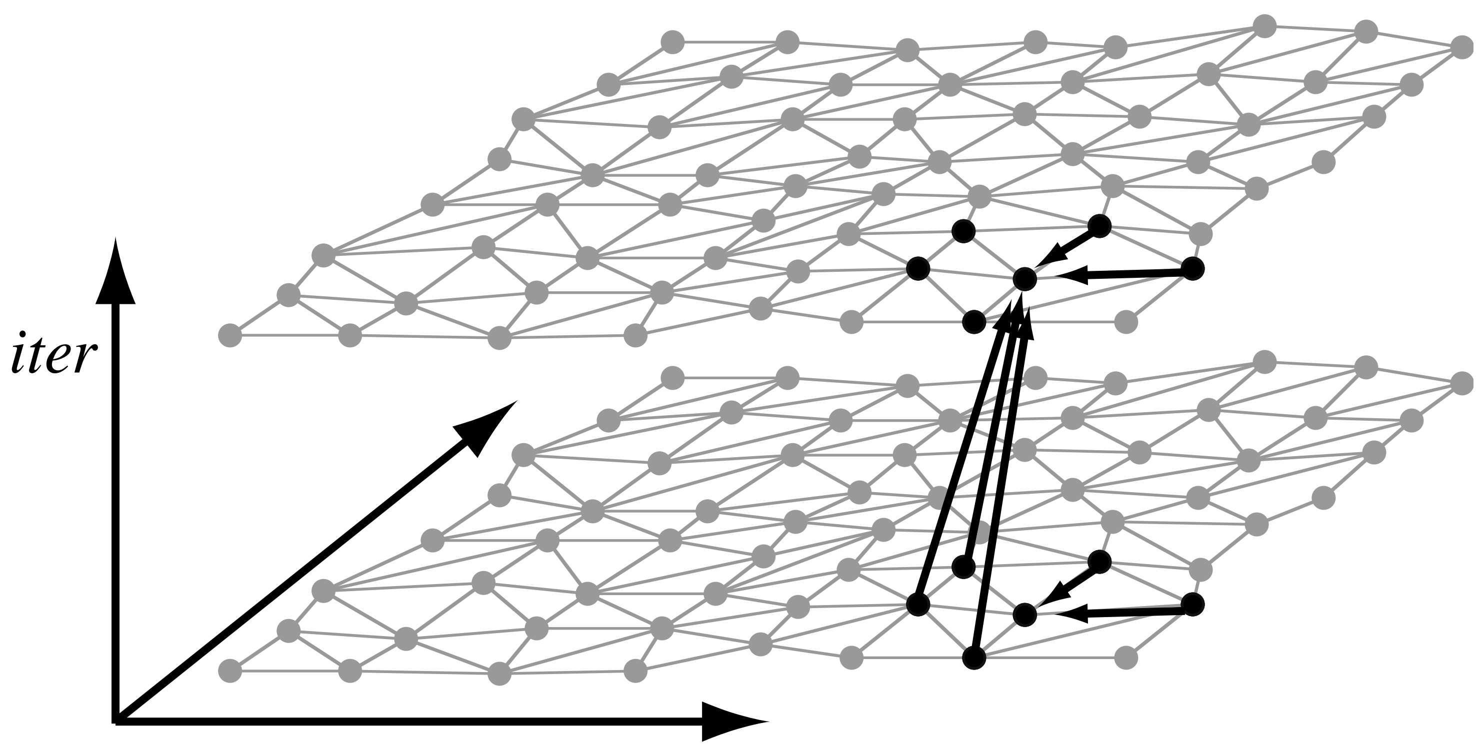
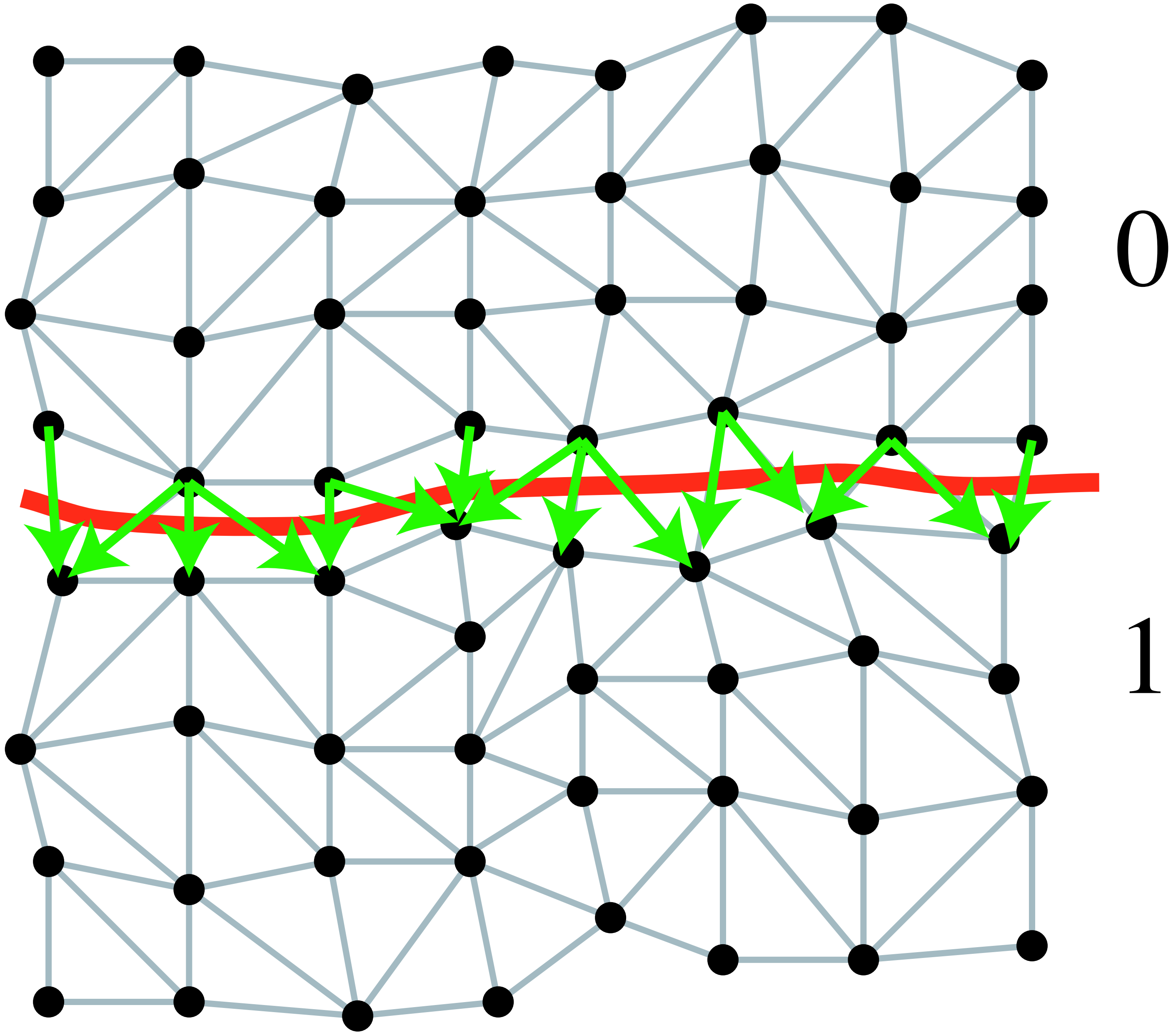
# Gauss-Seidel



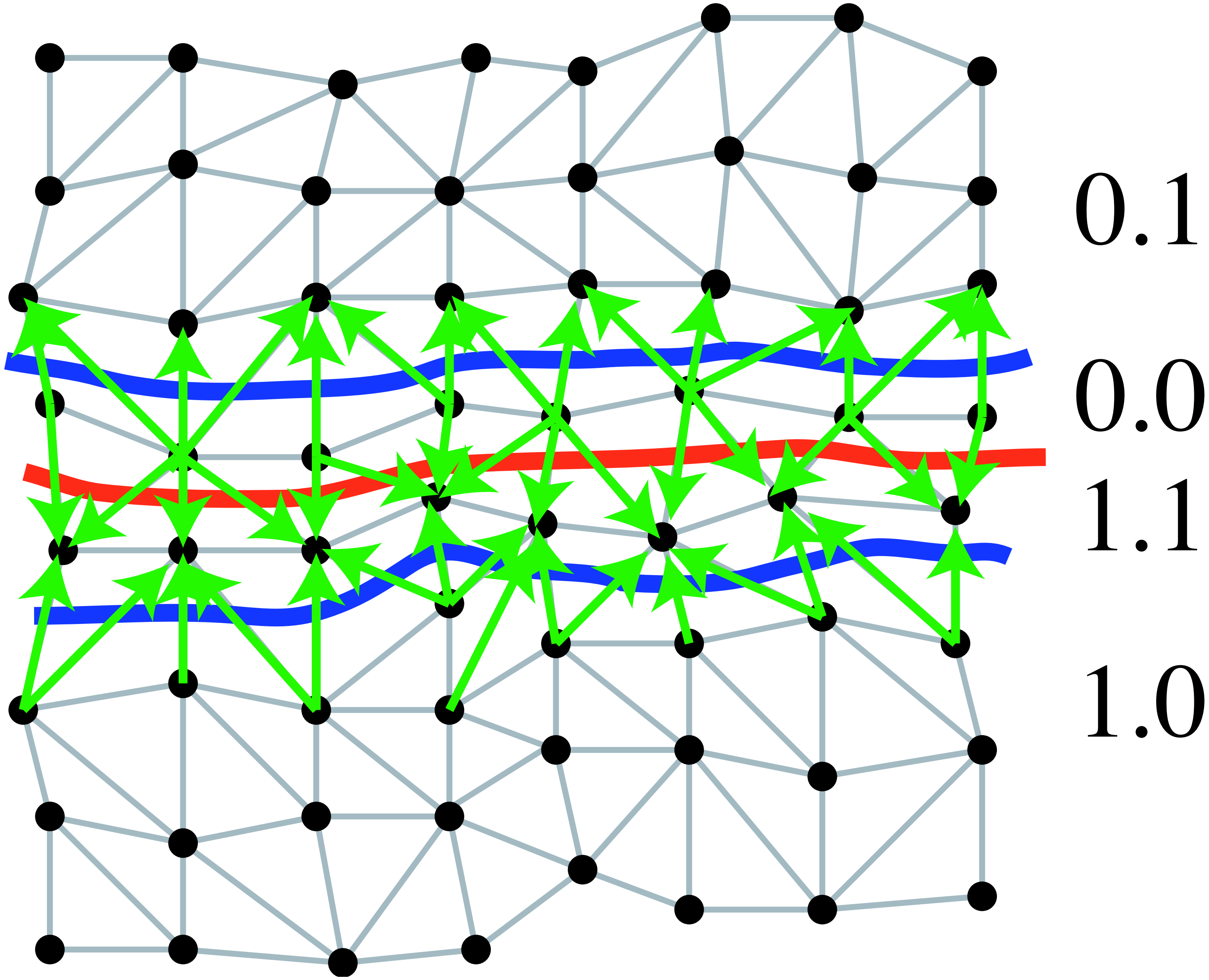

- Loop carried dependences within convergence iteration as well as between them
- Dependences depend on the ordering of the nodes
- Nodes can be reordered apriori

# Owner Computes Method Nodal Gauss-Seidel [Adams 2001]




- Renumbers nodes (rows/columns in sparse matrix)
- Make data dependences between main partitions consistent

# Owner Computes Method
# Nodal Gauss-Seidel [Adams 2001]




- Renumbers nodes (rows/columns in sparse matrix)
- Make data dependences between main partitions consistent
- Subpartition for better parallelism

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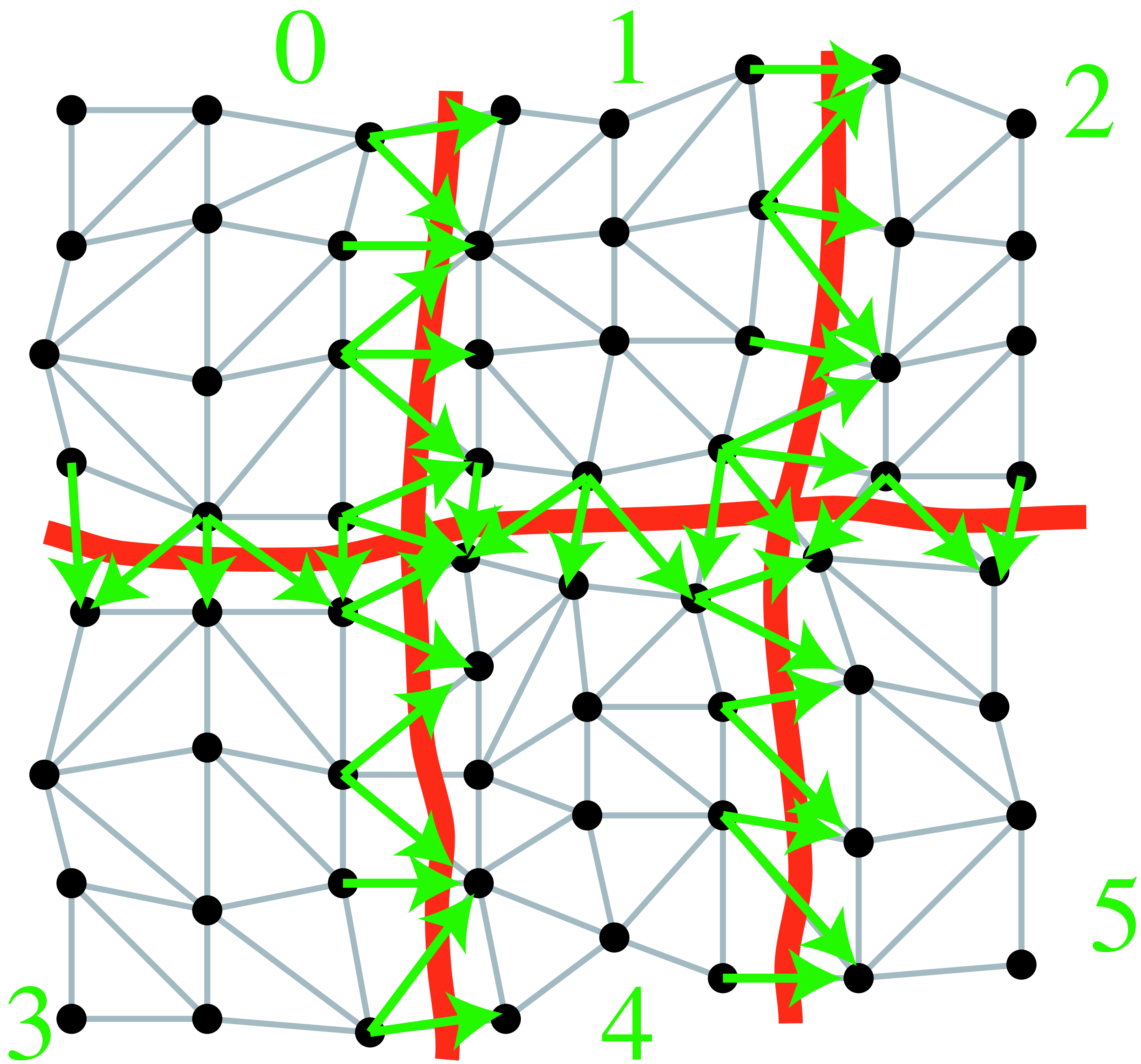
# Full Sparse Tiled Gauss-Seidel




- Tile growth creates and maintains a partial ordering between nodes in matrix graph
- Reorder nodes so that partial ordering is maintained

# Experimental Methodology Part 1

- **Baseline is Gauss-Seidel implemented CSR and uses provided ordering**
- **Owner computes implementation**
  - Partition matrix graph into equal-sized cells for each processor
  - Sub-partition and reorder on each processor for intra-iteration locality
  - Violate intra-iteration dependences for an idealistic parallel efficiency
- **Full sparse tiling implementation**

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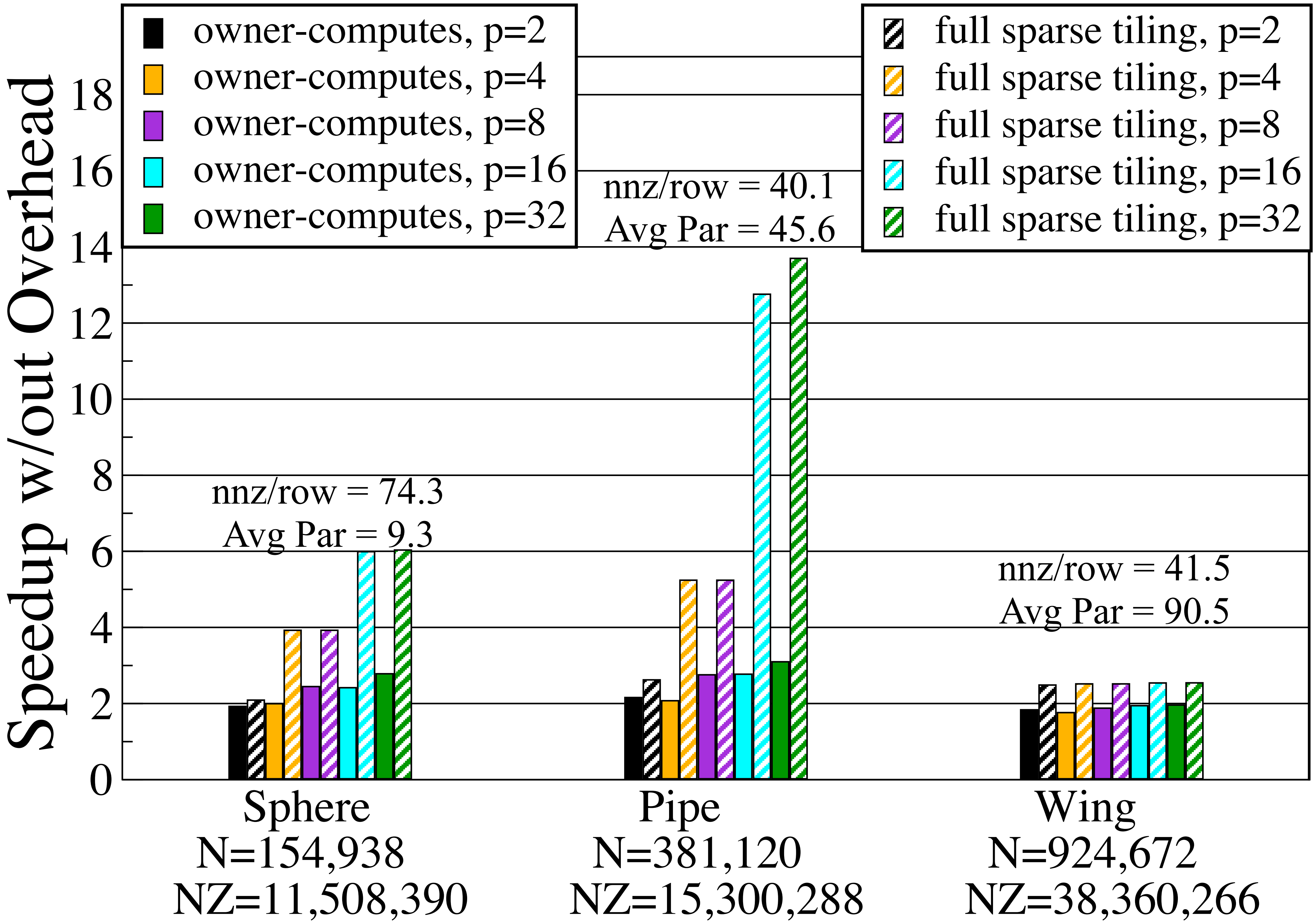
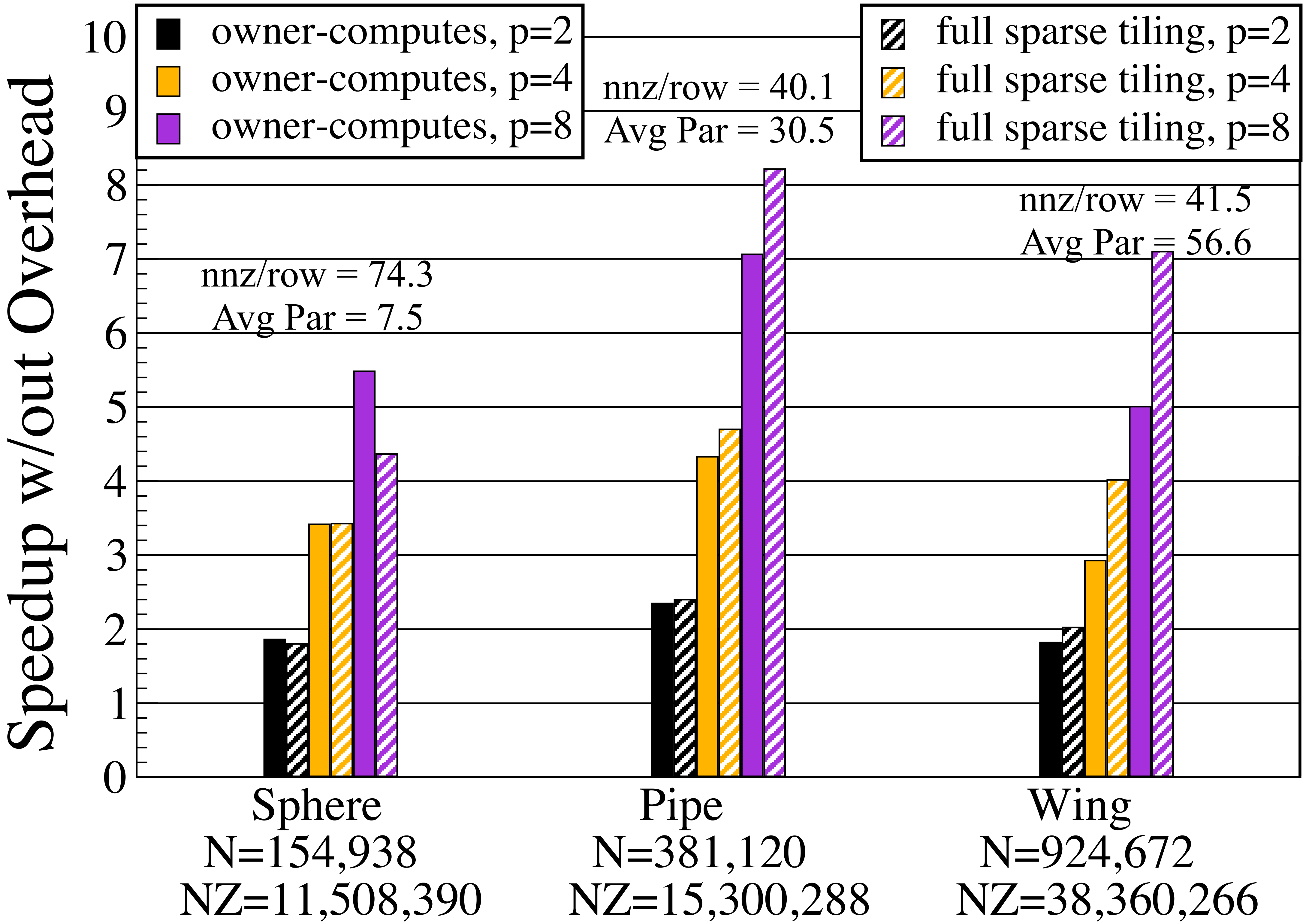
# Experimental Results

SUN HPC10000, 36 UltraSPARC II, 400MHz, 4MB L2



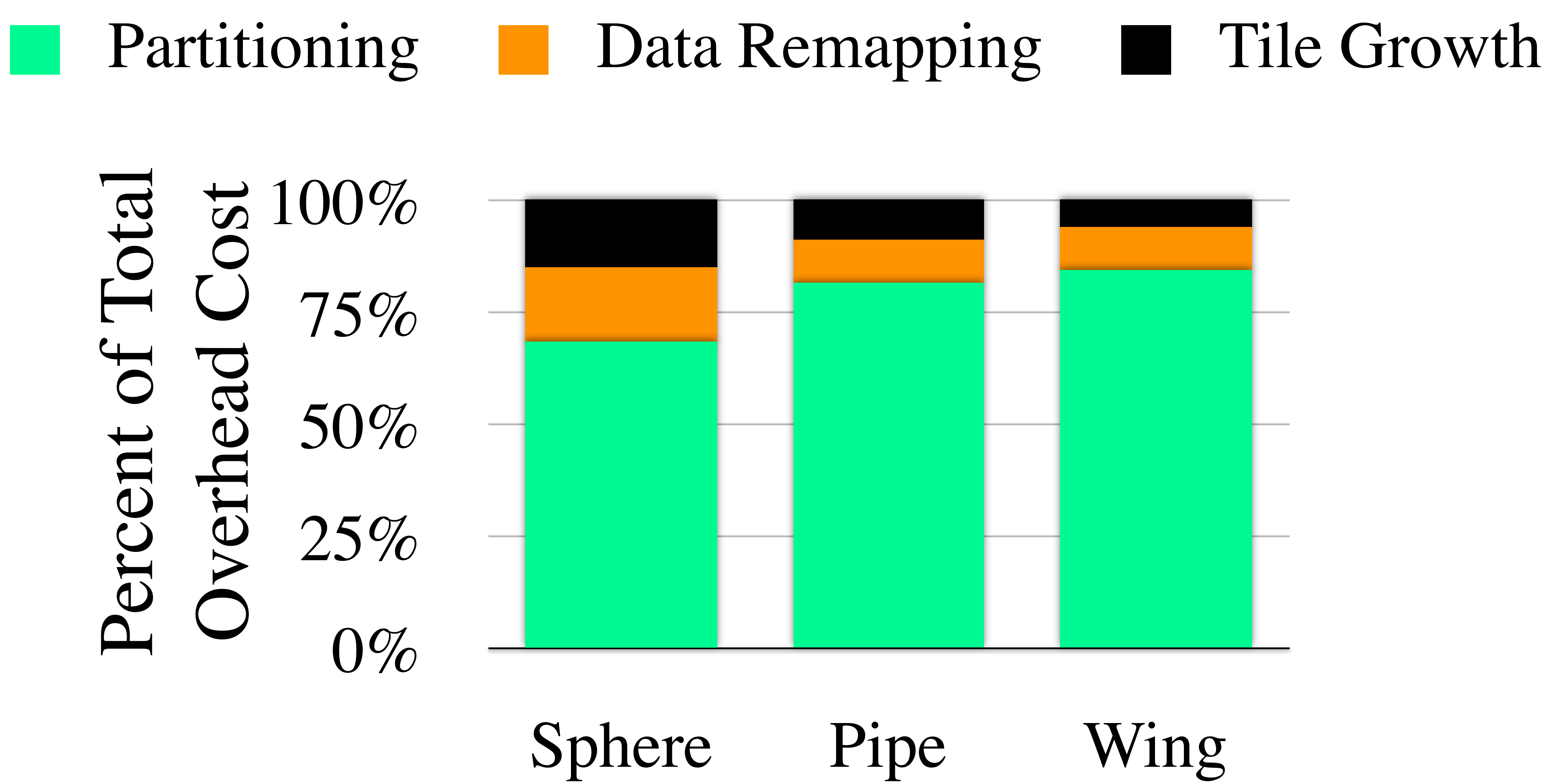
# Experimental Results

## IBM Power 3, 375MHz, 8MB L2 cache



# Overhead of Full Sparse Tiling on IBM Power 3

- Break even at 114 thru 400 convergence iterations
- Partitions created with Metis package [Karypis98]



# Experimental Methodology Part 2

- **Baseline is one processor version of full sparse tiled Gauss-Seidel**
- **Owner computes implementation**
  - one convergence iteration is partitioned and a task graph is constructed between partitions, T = 1
- **Full sparse tiling implementation**
  - one convergence iteration is partitioned and a task graph is constructed between partitions, T = 2

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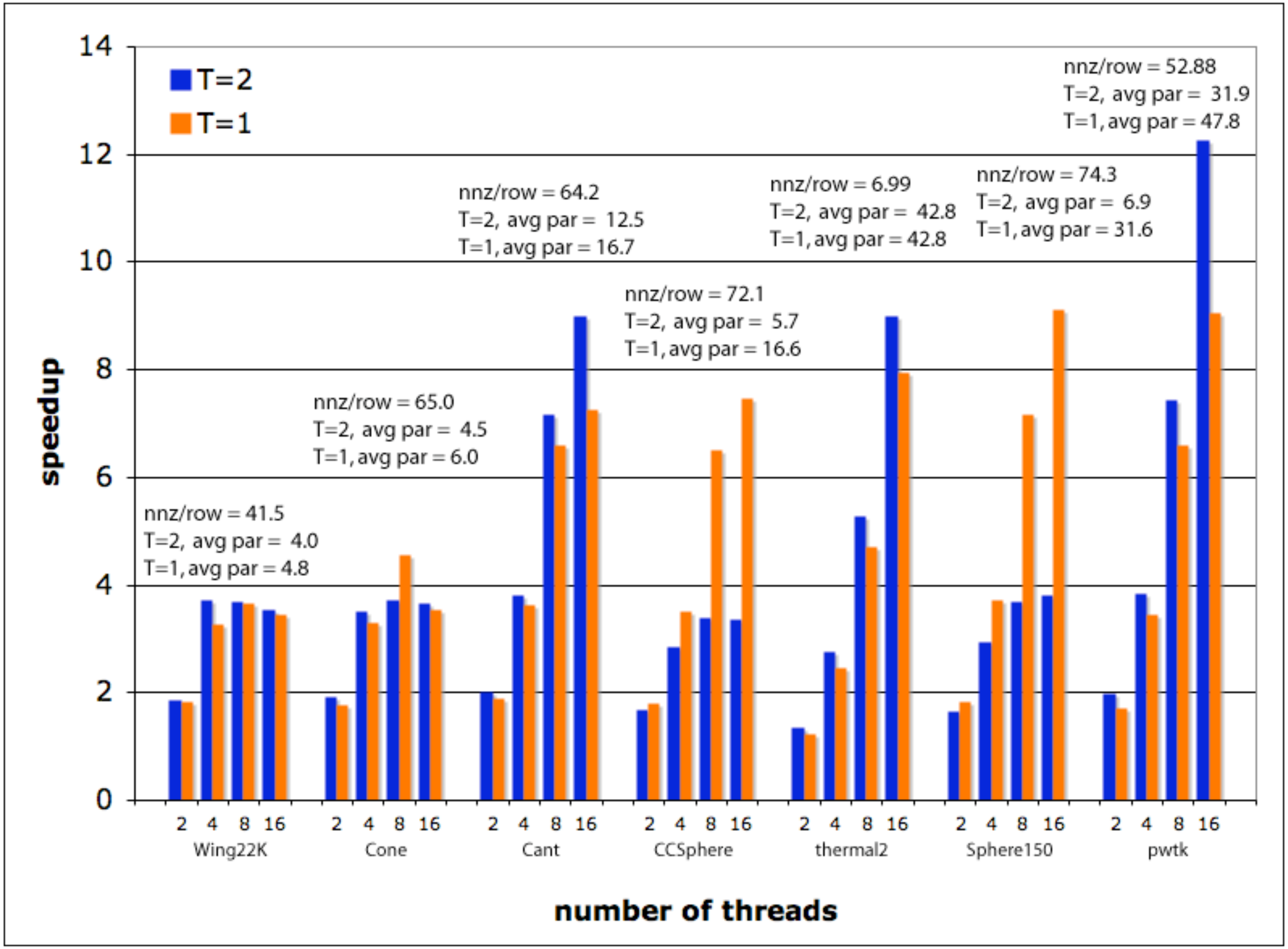
# Experimental Results for Blue Ice

IBM Power 5+, 1.9GHz, 1.9MB L2 cache, 36MB L3 cache



# Parameters Identified for Possible Auto-tuning

- Given mesh characteristics such as the average non-zeros per row, size, etc., model and tune for the following:
  - seed partition size
  - number of iterations to tile across
  - static or dynamic tile scheduling for parallelism
  - parameters for either scheduling strategy
- Would overlapping tiles remove the parallelism versus data locality tradeoff?

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# Conclusions

- Full sparse tiling exploits parallelism, intra-iteration data reuse, and inter-iteration data reuse
- Coloring the partition graph to number seed partitions improves the average parallelism in the tile dependence graph
- On shared memory processors, full sparse tiling can outperform owner-computes methods when enough parallelism is present
- More study needed of temporal data locality versus parallelism tradeoff

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