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Data: 24/03/2021

$a = 5 \qquad b = 5$

1)

$T(5,0,1) = (5,-5)$

$T(0,-1,-5) = (-5,-5)$

$T(5,0,0) = (5,-10)$

a. $(x,y,z) = a(5,0,1) + b(0,-1,-5) + c(5,0,0)$

$$\begin{cases} 5a + 5c = x \\ -b = y \\ a - 5b = z \end{cases}$$

$b = -y$

$a - 5(-y) = z$

$a + 5y = z$

$a = z - 5y$

$5(z - 5y) + 5c = x$

$5z - 2Sy + 5c = x$

$5c = x - 5z + 25y$

$c = \frac{x}{5} - z + 5y$

$(x,y,z) = (z-5y)(5,0,1) + (-y)(0,-1,-5) + \left(\frac{x}{5} - z + 5y\right)(5,0,0)$

$T(x,y,z) = T\left((z-5y)(5,0,1) + (-y)(0,-1,-5) + \left(\frac{x}{5} - z + 5y\right)(5,0,0)\right)$

$T(x,y,z) = (z-5y)T(5,0,1) + (-y)T(0,-1,-5) + \left(\frac{x}{5} - z + 5y\right)$

$T(5,0,0)$

$T(x,y,z) = (z-5y)(5,-5) + (-y)(-5,-5) +$

$(\frac{x}{5} - z + 5y)(5,-10)$

$(5z - 25y, -5z + 25y) + (5y, 5y) + (x - 5z + 25y,$
$-2x + 10z - 50y)$

$T(x,y,z) = (x + 5y, -2x - 20y + 5z)$

b.

Considere: $A \in \mathbb{R}^3$, $B \in \mathbb{R}^3$ e $\alpha \in \mathbb{R}$

$A = (a,b,c)$ $\quad A + B = (a+x, b+y, c+z)$

$B = (x,y,z)$

$T(A) = (a + 5b, -2a - 20b + 5c)$

$T(B) = (x + 5y, -2x - 20y + 5z)$

$T(A) + T(B) = (a + x + 5(b+y), -2(a+x) - 20(b+y) + 5(c+z))$

$T(A+B) = (a + x + 5(b+y), -2(a+x) - 20(b+y) + 5(c+z))$

$\alpha A = (\alpha a, \alpha b, \alpha c)$

$T(\alpha A) = (\alpha a + 5(\alpha b), -2(\alpha a) - 20(\alpha b) + 5(\alpha c))$

$= (\alpha a + 5\alpha b, -2\alpha a - 20\alpha b + 5\alpha c)$

$\alpha T(A) = \alpha(a + 5b, -2a - 20b + 5c)$

$= (\alpha a + 5\alpha b, -2\alpha a - 20\alpha b + 5\alpha c)$

Como $T(A) + T(B) = T(A+B)$ e $\alpha T(A) = T(\alpha A)$ podemos afirmar que T é uma transformação linear

c. $(0,0) = (x+5y, -2x-20y+5z)$

$$x + 5y = 0$$
$$-2x - 20y + 5z = 0$$

$x = -5y$

$-2(-5y) - 20y + 5z = 0$

$10y - 20y + 5z = 0$

$5z = 10y$

$z = 2y$

$N(T) = \{(x, y, z \in \mathbb{R}^3 ; x = -5y \text{ e } z = 2y\} = \{(-5y, y, 2y); y \in \mathbb{R}\}$

$u = (x, y, z) = (-5y, y, 2y) = y(-5, 1, 2)$, logo

$\text{Base}(N(T)) = \beta = \{(-5, 1, 2)\}$ e

$\text{Dim}(N(T)) = 1$

d. $\alpha = \{(5,1), (1,-5)$ e $\beta = \left\{ \begin{bmatrix} 5 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 5 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 5 & 0 \end{bmatrix}, \begin{bmatrix} 5 & 10 \\ 10 & -15 \end{bmatrix} \right\}$

$S(5,1) = \begin{bmatrix} 4 & -1 \\ -3 & 5 \end{bmatrix}$

$$\begin{bmatrix} 4 & -1 \\ -3 & 5 \end{bmatrix} = a\begin{bmatrix} 5 & -1 \\ 0 & 0 \end{bmatrix} + b\begin{bmatrix} 0 & -1 \\ 5 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 0 \\ 5 & 0 \end{bmatrix} + d\begin{bmatrix} 5 & 10 \\ 10 & -15 \end{bmatrix}$$

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$4 = 5a + 5d \Rightarrow 4 = 5a - \frac{5}{3}$

$-1 = -a - b + 10d \quad \frac{17}{3} = 5a$

$-3 = 5b + 5c + 10d \quad a = \frac{17}{15}$

$5 = -15d$

$d = -\frac{1}{3}$

$-1 = -\frac{17}{15} - b - \frac{10^{\cdot 5}}{3^{\cdot 5}} \Rightarrow -1 = -\frac{17}{15} - \frac{50}{15} - b$

$-1 + b = -\frac{67}{15} \Rightarrow b = -\frac{67}{15} + \frac{15}{15} \Rightarrow b = -\frac{52}{15}$

$-3 = 5.\left(-\frac{52}{15}\right) + 5c + 10\left(-\frac{1}{3}\right)$

$-3 = -\frac{260}{15} + 5c - \frac{10^{\cdot 5}}{3 \cdot 5} \Rightarrow -3 = -\frac{260}{15} + 5c - \frac{50}{15}$

$-3 = -\frac{310}{15} + 5c$

$-\frac{45}{15} + \frac{310}{15} = 5c$

$\frac{265}{15} \cdot \frac{1}{5} = c \Rightarrow c = \frac{53}{15}$

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$$S(1,-5) = \begin{bmatrix} 6 & 5 \\ -11 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 5 \\ -11 & 1 \end{bmatrix} = a\begin{bmatrix} 5 & -1 \\ 0 & 0 \end{bmatrix} + b\begin{bmatrix} 0 & -1 \\ 5 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 0 \\ 5 & 0 \end{bmatrix} + d\begin{bmatrix} 5 & 10 \\ 10 & -15 \end{bmatrix}$$

$6 = 5a + 5d \Rightarrow 6 = -\frac{1}{3} + 5a \Rightarrow \frac{19}{3} = 5a \Rightarrow a = \frac{19}{15}$

$5 = -a - b + 10d$

$-11 = 5b + 5c + 10d$ $\quad 5 = -\frac{19}{15} - b - \frac{10}{15}$

$1 = -15d$ $\quad 5 + \frac{29}{15} = -b \Rightarrow -\frac{104}{15} = b$

$d = -\frac{1}{15}$

$-11 = -\frac{104}{3} + 5c - \frac{2}{3}$

$-11 = -\frac{106}{3} + 5c$

$-\frac{33}{3} + \frac{106}{3} = 5c$

$\frac{73}{3} = 5c \Rightarrow c = \frac{73}{15}$

Com isso concluímos que a matriz $[S]^{\alpha}_{\beta}$ é:

$$\begin{bmatrix} 17/15 & 19/15 \\ -52/15 & -104/15 \\ 53/15 & 73/15 \\ -1/3 & -1/15 \end{bmatrix}$$

e. $S \circ T \quad S(T(a,b,c)) =$

$S(a+5b, -2a-20b+5c) =$

$$\begin{bmatrix} a+5b-(-2a-20b+5c) & -(-2a-20b+5c) \\ 2(-2a-20b+5c)-(a+5b) & a+5b \end{bmatrix} =$$

$$\begin{bmatrix} a+5b+2a+20b-5c & +2a+20b-5c \\ -4a-40b+10c-a-5b & a+5b \end{bmatrix} =$$

$$\begin{bmatrix} 3a+25b-5c & 2a+20b-5c \\ -5a-45b+10c & a+5b \end{bmatrix}$$

$x = 3a+25b-5c$

$y = 2a+20b-5c$

$z = -5a-45b+10c$

$w = a+5b$

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$x+y+z=0$

$w+y=x$

$Im(S \circ T) = \left\{ \begin{bmatrix} x & y \\ z & w \end{bmatrix} ; x+y+z=0 \text{ e } w+y=x \right\}$

$z = -x-y$

$z = -(w+y)-y$

$z = -w-2y$

Base:

$$\begin{bmatrix} w+y & y \\ -w-2y & w \end{bmatrix} = w\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} + y\begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & -2 & 0 \\ 1 & 0 & 0 \end{array}\right] \begin{array}{l} L_3 \leftarrow L_3 + L_1 \\ L_4 \leftarrow L_4 - L_1 \end{array} = \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & 0 \end{array}\right] \begin{array}{l} L_3 \leftarrow L_3 + L_2 \\ L_4 \leftarrow L_4 - L_2 \end{array}$$

$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

Como temos um sistema L.I, chegamos que:

$Base(Im(S \circ T)) = \left\{ \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix} \right\}$

$Dim(Im(S \circ T)) = 2$

2. a)

$T(v) = 2p + v$

$p = \left(\frac{u \cdot v}{u \cdot u}\right) \cdot u$

$u = (3,1)$

$T(v) = 2\left(\frac{(3,1) \cdot v}{10}\right) \cdot (3,1) - v$

Pontos:

A(-7,5), B(0,-2), C(-6,6), D(14,6), E(4,2), F(8,-2), G(8,0), H(15,5), I(4,4) e J(0,0)

$T(A) = 2\left(\frac{-21+5}{10}\right) \cdot (3,1) - (-7,5) = -3{,}2 \cdot (3,1) - (-7,5)$

$= (-2{,}6, 8{.}2)$

$T(B) = 2\left(\frac{(3,1) \cdot (0,-2)}{10}\right) \cdot (3,1) - (0,-2) = (-1{.}2, 1{.}6)$

$T(C) = 2\left(\frac{(3,1) \cdot (-6,6)}{10}\right) \cdot (3,1) - (-6,6) = (-1{.}2, -8{.}4)$

$T(D) = 2\left(\frac{(3,1) \cdot (14,6)}{10}\right) \cdot (3,1) - (14,6) = (14{.}8, 3{.}6)$

$T(E) = 2\left(\frac{(3,1) \cdot (4,2)}{10}\right) \cdot (3,1) - (4,2) = (4{.}4, 0{.}8)$

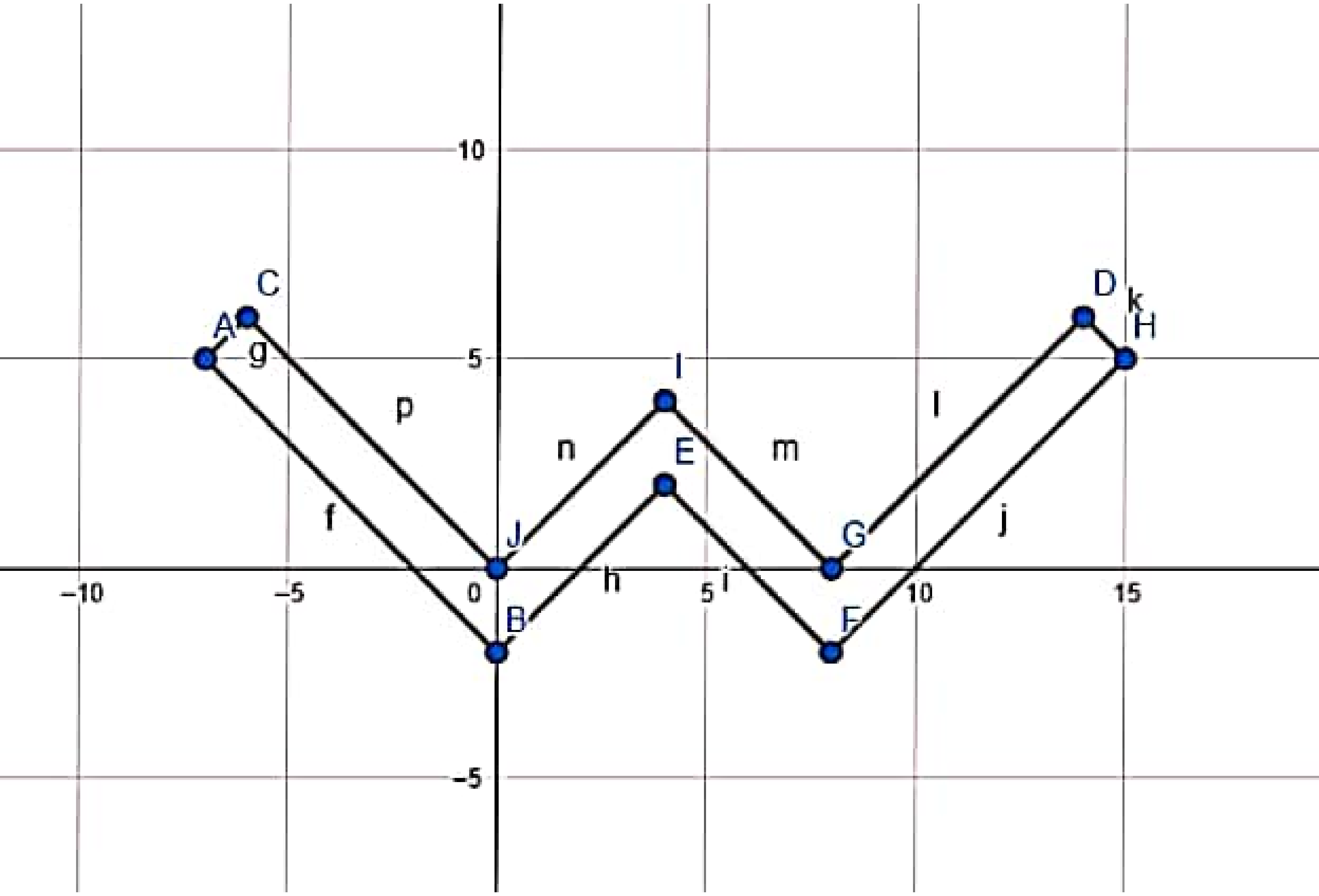
$T(F) = 2\left(\frac{(3,1) \cdot (8,-2)}{10}\right) \cdot (3,1) - (8,-2) = (5{.}2, 6{.}4)$

$$T(G) = 2\left(\frac{(3,1) \cdot (8,0)}{10}\right) \cdot (3,1) - (8,0) = (6.4, 4.8)$$

$$T(H) = 2\left(\frac{(3,1) \cdot (15,5)}{10}\right) \cdot (3,1) - (15,5) = (15,5)$$

$$T(I) = 2\left(\frac{(3,1) \cdot (4,4)}{10}\right) \cdot (3,1) - (4,4) = (5.6, -0.8)$$

$$T(J) = 2\left(\frac{(3,1) \cdot (0,0)}{10}\right) \cdot (3,1) - (0,0) = (0,0)$$



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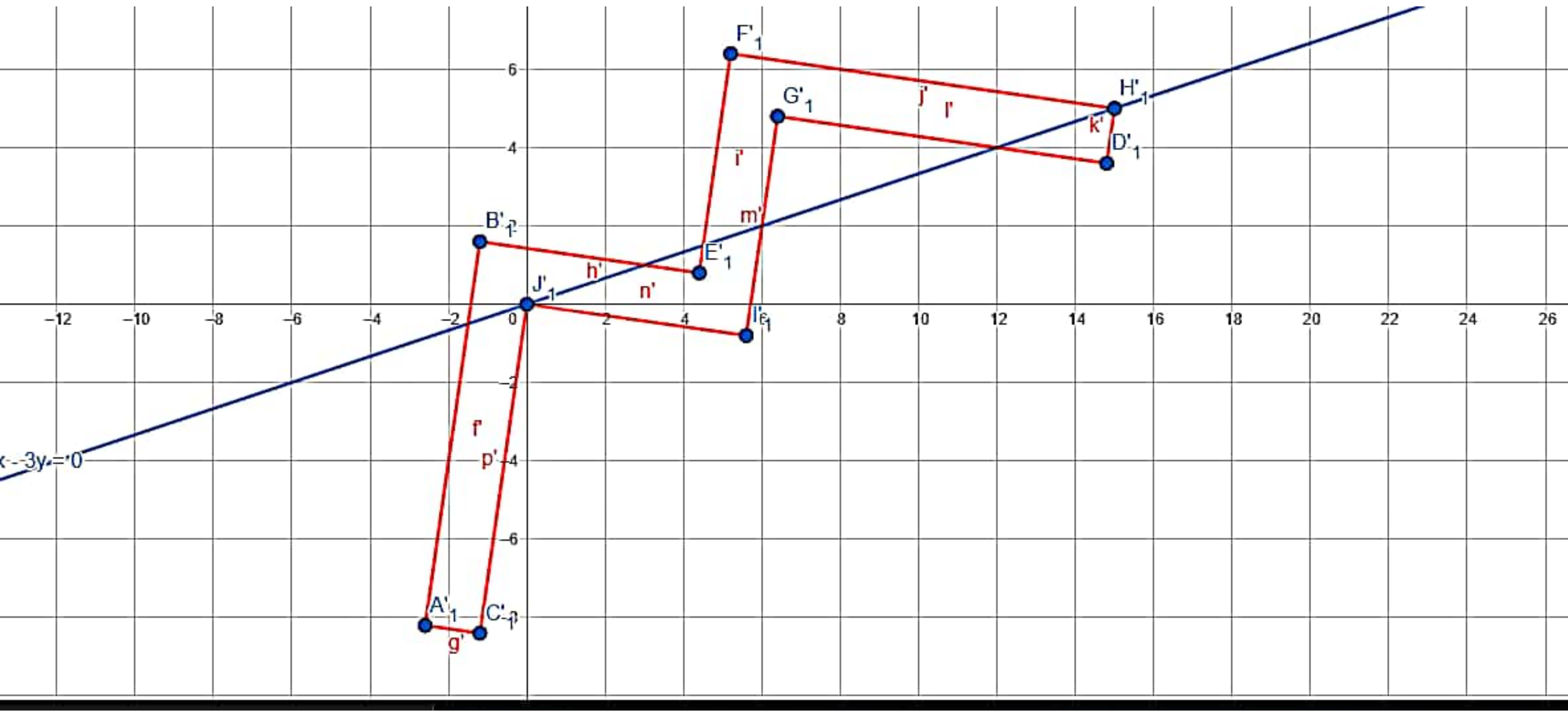

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x - 3y = 0

2.b)

$$\boxed{T(x,y) = (x + k \cdot y, y)}$$

Pontos para formar a letra W:

$k = -2$

A(-7,5), B(0,-2), C(-6,6), D(14,6), E(4,2), F(8,-2), G(8,0), H(15,5), I(4,4) e J(0,0)

$T(-7,5) = (-7+(-2.5), 5) = (-7-10, 5) = (-17,5)$

$T(0,-2) = (0+(-2 \cdot -2), -2) = (4,-2)$

$T(-6,6) = (-6+(-2.6), 6) = (-6-12, 6) = (-18,6)$

$T(14,6) = (14+(-2.6), 6) = (14-12, 6) = (2,6)$

$T(4,2) = (4+(-2.2), 2) = (0,2)$

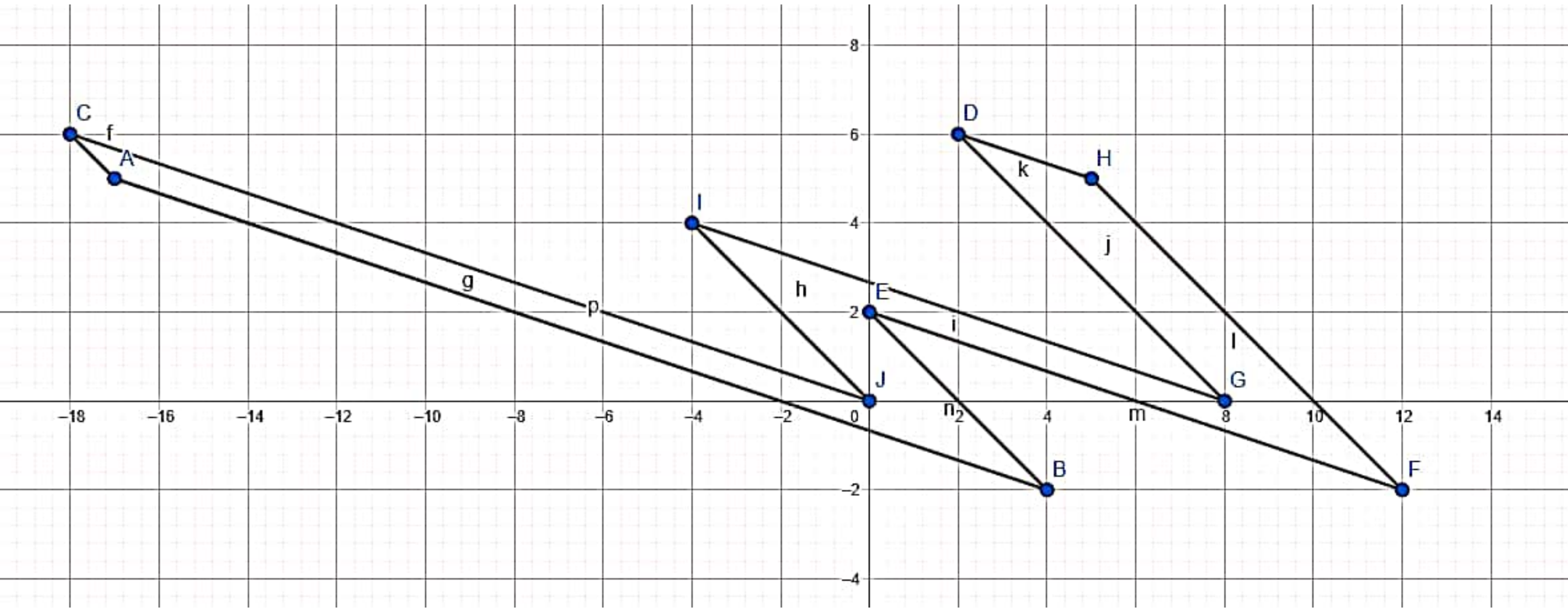
$T(8,-2) = (8+(-2 \cdot -2), -2) = (12,-2)$

$T(8,0) = (8+(-2.0), 0) = (8,0)$

$T(15,5) = (15+(-2 \cdot 5), 5) = (5,5)$

$T(4,4) = (4+(-2.4), 4) = (-4, 4)$

$T(0,0) = (0+(-2 \cdot 0), 0) = (0,0)$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2.c) $\theta = 60°$

$T(x,y) = \left(x \cdot \frac{1}{2} - y \cdot \frac{\sqrt{3}}{2}, x\frac{\sqrt{3}}{2} + y \cdot \frac{1}{2}\right)$

$T(-7,5) = \left(\frac{-7}{2} - \frac{5\sqrt{3}}{2}, \frac{-7\sqrt{3}}{2} + \frac{5}{2}\right)$

$T(0,-2) = (\sqrt{3}, -1)$

$T(-6,6) = (-3 - 3\sqrt{3}, -3\sqrt{3} + 3)$

$T(14,6) = (7 - 3\sqrt{3}, 7\sqrt{3} + 3)$

$T(4,2) = (2 - \sqrt{3}, 2\sqrt{3} + 1)$

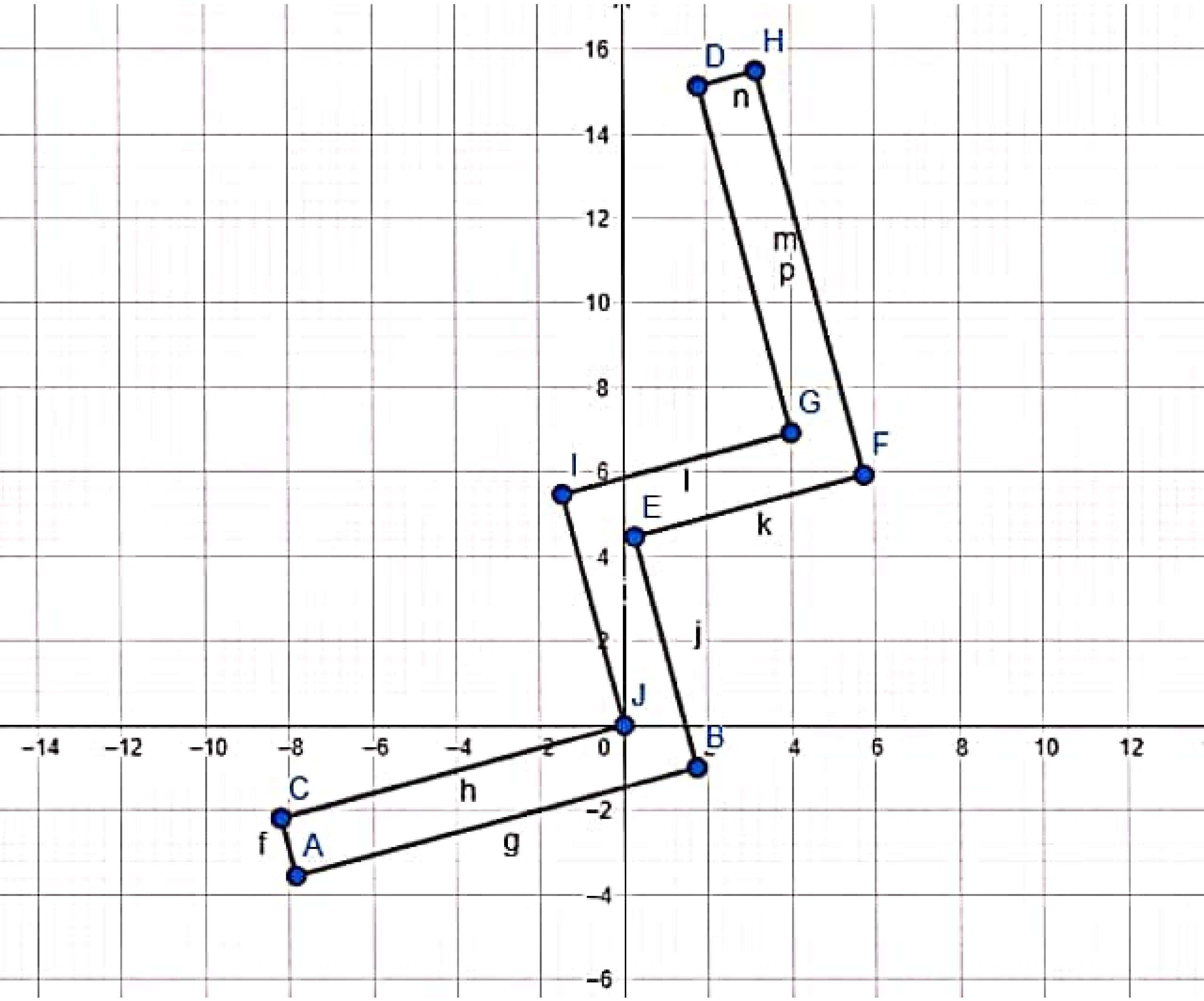
$T(8,-2) = (4 + \sqrt{3}, 4\sqrt{3} - 1)$

$T(8,0) = (4, 4\sqrt{3})$

$T(15,5) = \left(\frac{15}{2} - \frac{5\sqrt{3}}{2}, \frac{15\sqrt{3} + 5}{2}\right)$

$T(4,4) = (2 - 2\sqrt{3}, 2\sqrt{3} + 2)$

$T(0,0) = (0,0)$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