D S T Q Q S S **Harry Potter** ___/___/___

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Lista 5

$\vec{u} = (3, -1, -2)$
$\vec{v} = (2, 4, -1)$
$\vec{w} = (-1, 0, 1)$

a) $|\vec{v} \times \vec{v}|$

↳ $\vec{v} \times \vec{v}$ pela "propriedade" resulta em $\vec{0}$

$|\vec{0}| = 0$

b) $(2\vec{v}) \times (3\vec{v})$

↳ pela "propriedade", como são números proporcionais o resultado é um vetor $\vec{0}$

c) $(\vec{u} - \vec{v}) \times \vec{w}$

$(1, -5, -1) \times (-1, 0, 1)$

$$\begin{matrix} 1 & -5 & -1 & 1 & -5 \\ -1 & 0 & 1 & -1 & 0 \end{matrix}$$

$(-5, 0, +5)$

d) $(\vec{u} \times \vec{v}) \cdot \vec{v}$

$$\begin{pmatrix} 3 & -1 & -2 & 3 & -1 \\ 2 & 4 & -1 & 2 & 4 \end{pmatrix} \cdot \vec{v}$$

$(9, -1, 14) \cdot (2, 4, -1)$

$18 - 4 - 14$

0

e) $(\vec{u} \times \vec{v}) \cdot \vec{w}$

$(9, -1, 14) \cdot (-1, 0, 1)$

$-9 + 14$

0

2) $\vec{u} = (-3, 2, 0)$
$\vec{v} = (0, -1, -2)$

$\vec{u} + 2\vec{v} \quad \times \quad \vec{v} - \vec{u}$

$(-3, 2, 0) + (0, -2, -4) \times (0, 1, -2) - (-3, 2, 0)$

$(-3, 0, -4) \quad \times \quad (3, -3, -2)$

$$\begin{matrix} -3 & 0 & -4 & -3 & 0 \\ 3 & -3 & -2 & 3 & -3 \end{matrix}$$

$\boxed{(-12, -18, 9)}$

3) $\vec{u} = (3, 2, 2)$
$\vec{v} = (0, 1, 1)$

$\left(\dfrac{\vec{u} \times \vec{v}}{|\vec{u} \times \vec{v}|} \right) \cdot 2$ para ter módulo 2

$\vec{u} \times \vec{v}$

$$\begin{matrix} 3 & 2 & 2 & 3 & 2 \\ 0 & 1 & 1 & 0 & 1 \end{matrix}$$

$(0, -3, 3)$

$\dfrac{(0, -3, 3)}{\sqrt{0^2 + (-3)^2 + (3)^2}} \Rightarrow \dfrac{(0, -3, 3)}{\sqrt{18}} \Rightarrow \dfrac{(0, -3, 3)}{3\sqrt{2}}$

$\left(\dfrac{0}{3\sqrt{2}}, \dfrac{-\cancel{3}}{\cancel{3} \cdot \sqrt{2}}, \dfrac{\cancel{3}}{\cancel{3}\sqrt{2}} \right) \Rightarrow \left(0, \dfrac{-1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}} \right) \rightsquigarrow \dfrac{-1}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$\left(0, \dfrac{\sqrt{2}}{\cancel{2}}, \dfrac{\sqrt{2}}{\cancel{2}} \right) \cdot \cancel{2} \rightarrow \boxed{(0, \sqrt{2}, \sqrt{2})}$

4.a) $|\vec{AB} \times \vec{AD}| = |\vec{AB}|.|\vec{AD}|. \text{sen}\,\theta \rightsquigarrow \theta = 60°$

$2 . \cancel{2} . \frac{\sqrt{3}}{\cancel{2}}$

$\boxed{2\sqrt{3}}$

4.b) $|\vec{BA} \times \vec{BC}| = |\vec{BA}|.|\vec{BC}|. \text{sen}\,\theta \rightsquigarrow \theta = 120°$

$\cancel{2} . 2 . \frac{\sqrt{3}}{\cancel{2}}$

$\boxed{2\sqrt{3}}$

4.c) $|\vec{AB} \times \vec{DC}| = |\vec{AB}|.|\vec{DC}|. \text{sen}\,\theta \rightsquigarrow$ como $\vec{AB} = \vec{DC}$

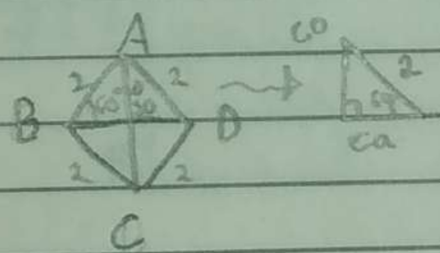
$2 . 2 . 0$ $\qquad \theta = 0°$

$\boxed{0}$

4.d) $|\vec{AB} \times \vec{CD}| = |\vec{AB}|.|\vec{CD}|. \text{sen}\,\theta \rightsquigarrow \theta = 180°$

$2 . 2 . 0$

$\boxed{0}$



$co = \text{sen}\,60 . 2 \rightarrow \sqrt{3}$

$ca = \cos 60 . 2 \rightarrow 1$

$|\vec{AC}|$ e $|\vec{CA}| = 2 . co = 2\sqrt{3}$

$|\vec{BD}|$ e $|\vec{DB}| = 2 . ca = 2$

4.e) $|\vec{BD} \times \vec{AC}|$

$|\vec{BD}| . |\vec{AC}| . \text{sen}\,\theta \rightarrow \theta = 90°$

$2 . 2\sqrt{3} . 1$

$\boxed{4\sqrt{3}}$

4.f) $|\vec{BD} \times \vec{CD}|$

$|\vec{BD}|.|\vec{CD}|. \text{sen}\,\theta \rightarrow \theta = 60°$

$2 . \cancel{2} . \frac{\sqrt{3}}{\cancel{2}}$

$\boxed{2\sqrt{3}}$

5) $|\vec{v}| = 1$

$|\vec{u}| = 13$

$|\vec{u} \times \vec{v}| = 12$

$|\vec{u} \times \vec{v}| = |\vec{u}| . |\vec{v}| . \operatorname{sen} \alpha$

$12 = 13 . 1 . \operatorname{sen} \alpha$

$\operatorname{sen} \alpha = \frac{12}{13}$

$\vec{u} . \vec{v} = \operatorname{sena}\left(\frac{12}{13}\right) . |\vec{u}| . |\vec{v}|$

$\vec{u} . \vec{v} = 5$

6) $A = (4, 1, 2)$

$B = (5, 0, 1)$

$C = (-1, 2, -2)$

$D = (-2, 3, -1)$

$\overrightarrow{AB} \,//\, \overrightarrow{CD}$

$|\overrightarrow{AB} \times \overrightarrow{AC}| = \text{Area}$

$\begin{vmatrix} 1 & -1 & -1 & 1 & -1 \\ -5 & 1 & -4 & -5 & 1 \end{vmatrix} = \text{Area}$

$|4-(-1), 5-(-4), 1+(-5)| = \text{Area}$

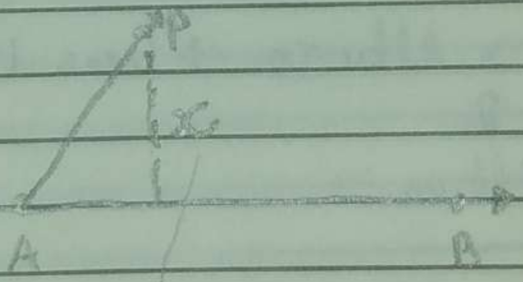
$|(5, 9, 4)| = \text{Area}$

$\text{Area} = \sqrt{122}$

7) $P(4,3,3)$

$A(1,2,-1)$

$B(3,1,1)$



$|\overrightarrow{AB} \times \overrightarrow{AP}|$

$\begin{vmatrix} 2 & -1 & 2 & 2 & -1 \\ 3 & 1 & 4 & 3 & 1 \end{vmatrix}$

$(-6, -2, 5)$

$\overrightarrow{AP} = (3, 1, 4)$

$\overrightarrow{AB} = (2, -1, 2)$

$x = \frac{|\overrightarrow{AB} \times \overrightarrow{AP}|}{|\overrightarrow{AB}|} \rightarrow \frac{\sqrt{65}}{\sqrt{9}}$

$|\overrightarrow{AP}| = \sqrt{(2)^2 + (-1)^2 + (2)^2} = \sqrt{9}$

$|\overrightarrow{AB} \times \overrightarrow{AP}| = \sqrt{(-6)^2 + (-2)^2 + (5)^2} = \sqrt{65}$

8) $A(-4,1,1)$
$B(1,0,1)$
$C(0,-1,+3)$

$|\vec{AB} \times \vec{AC}| = Area$
$|(5,-1,0) \times (4,-2,2)|$
$\begin{vmatrix} 5 & -1 & 0 & 5 & -1 \\ 4 & -2 & 2 & 4 & -2 \end{vmatrix}$
$|-2, 10, -6|$
$\sqrt{(-2)^2 + (10)^2 + (-6)^2}$
$\sqrt{4 + 100 + 36}$
$\sqrt{140}$ = Area do Paralelograma
$\frac{\sqrt{140}}{2}$ -> Area do Triangulo
$\frac{\sqrt{2} \cdot \sqrt{2} \cdot \sqrt{35}}{2} = \frac{2\sqrt{35}}{2} = \boxed{\sqrt{35}}$ = Area do Triangulo

$Area = \frac{base \times Altura}{2}$ -> base $|\vec{BC}| = \sqrt{6}$
$\frac{Area \cdot 2}{base} = Altura$

$\frac{2 \cdot \sqrt{35}}{\sqrt{6}}$ = Altura -> $\boxed{Altura = \frac{2 \cdot \sqrt{35}}{\sqrt{6}}}$

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9) $A = (2, 0, 0)$
$B = (0, 2, 0)$
$C = (0, 0, z)$

$|\vec{AB} \times \vec{AC}| = 6.2$

$\begin{vmatrix} -2 & 2 & 0 & -2 & 2 \\ -2 & 0 & z & -2 & 0 \end{vmatrix} = 6.2$

$|(2z, -2z, -4)| = 6.2$

$\sqrt{(2z)^2 + (-2z)^2 + (-4)^2} = 6.2$

$\sqrt{8z^2 + 16}^{\,2} = (6.2)^2$

6 = Área do triângulo
6.2 = Área do Módulo

$8z^2 = 144 - 16$

$z = \sqrt{\frac{128}{8}}$

$z = \sqrt{16}$

$\boxed{z = \pm 4}$