Panel Method Program Report

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Abstract

For this project, a function was created that could take the geometry of an airfoil and solve for a series of important values. By use of the Panel Method, the specific values that are solved for via this function include the coefficient of lift for varying values for the angle of attack, as well as an approximation for α_{ZL}. To add, this function was constructed such that the airfoil design would be plotted, and for each angle of attack, the coefficient of pressure on the top and bottom surfaces are plotted verses the x position along the length of the airfoil's chord.

Formulation

For the important formulations for the project, a few various conceptual mathematical models were utilized. These important formulas are given below:

$$1)\ Velocity\ given\ by\ mass\ source = \frac{\Lambda\,\hat{r}}{2\pi|r|}\ given\ \Lambda\ is\ the\ strength\ of\ the\ source$$

$$2)\ Velocity\ induced\ by\ vortex\ source = \frac{\Gamma\,\hat{t}}{2\pi|r|}\ given\ \Gamma\ is\ the\ strength\ of\ the\ vortex$$

$$3)\ Net\ velocity\ at\ a\ control\ point = \sum u_s + \sum u_v + U_\infty \hat{\alpha}$$

$$where\ u_s\ is\ the\ velocity\ from\ sources\ and\ u_v\ is\ the\ velocity\ from\ the\ vortices$$

$$4)\ One\ of\ the\ most\ important\ equations\ used\ was\ this:\ \vec{u} \bullet \hat{n} = 0$$

$$where\ \vec{u}\ is\ the\ net\ velocity\ vector\ and\ \hat{n}\ is\ the\ normal\ on\ the\ surface\ at\ that\ point$$

Equation 1 is based on the mathematical model for a source which is a mathematical object used to spew mass from a center radially outward, housing an infinite velocity strength at the center but gradually decreasing in strength as the radial distance out increases, expressed by the fact that as that occurs the magnitude increases which then decreases the velocity given by the equation. Equation 2 is similar to equation to in the sense that as you move radially outward, the velocity strength induced by the vortex decreases but now the direction of this velocity is given by the tangent vector found at any control point, that tangent vector being perpendicular to the radial r vector. Next is equation 3 and as it would be logical enough, the net velocity at any point is the sum of all velocities acting on that point, thus in this scenario the summation of all the velocities caused by sources, vortices and the free stream velocity come to find this value. Lastly, the simple but important statement from equation 4 essentially states no mass, thus velocity of fluid, can go through the surface of the airfoil or in other words it can't have a component of velocity parallel to the normal vector on the surfaces at any point. This physical concept is the basis for solving for the magnitudes of the sources and vortices. This concept is implemented by taking equations 1 through 4 and applying it at each control point along the surface of the airfoil, shown below:

$$5)\ \ \left(\sum \frac{\Lambda_i\, \hat{r}_{ki}}{2\pi|r_{ki}|} + \sum \frac{\Gamma_j\, \hat{t}_{kj}}{2\pi|r_{ki}|} + \ U_\infty \hat{\alpha}\right)\ \bullet \hat{n}_i = 0$$

With these sets tools, it is possible to use control points on the airfoils geometry to help build up linear equations and solve for all the Λ and Γ values. Using these values, you can plug these values into equation 3 and find the net velocity at each control point. After finding the net velocities, you can then use Bernoulli's Equation to come to an equation for finding the coefficient of pressure at each control point, that equation being:

$$6)\ \ C_P = 1 - \left(\frac{|\vec{u}_{net}|}{|U_\infty|}\right)^2$$

Now the coefficient of lift is found using the appropriate equation:

$$7)\ \ \ C_L = \int_{LE}^{TE} C_{P,bottom}(x) - \ C_{P,top}(x)\, dx$$

Given that, I found it was easiest to use the Least Squares method for function fitting and find polynomial fits for the coefficient of pressure distributions on the top and bottom and then use the Simpson's Method for Integration. Below are the basic ideas of each mathematical method:

$$8)\ \ Least\ Squares\ Method\!:\ A^T A\, \vec{\beta} = A^T \check{y}$$

$$such\ that\ A = \begin{bmatrix} 1 & x_1^{1/2} & x_1 & x_1^{3/2} & x_1^2 & \cdots & x_1^{m+1/2} & x_1^m \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_n^{1/2} & x_n & x_n^{3/2} & x_n^2 & \cdots & x_n^{m+1/2} & x_n^m \end{bmatrix}, \qquad \check{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

$$\text{which then finds } \vec{\beta} = \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_m \end{bmatrix},$$

$$\text{which is the constants in front of each polynomial to fit to the given } x \text{ and } y \text{ values}$$

9) *Simpson's Method of Integration Formula*:

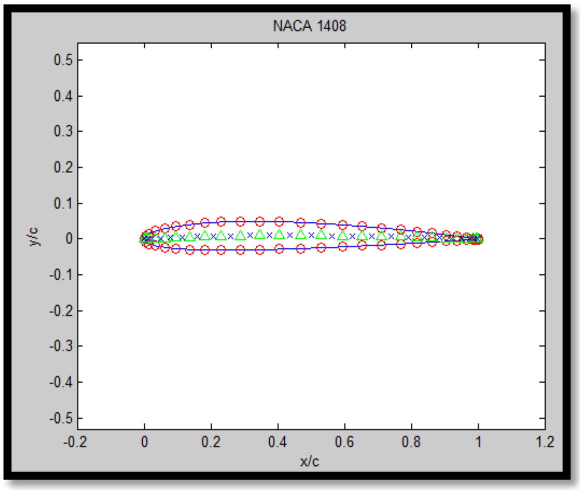
$$\int_{x_o}^{x_{2n}} f(x)dx \approx \frac{1}{3}\frac{x_{2n} - x_o}{n} \left[f_0 + 4\left(f_1 + f_3 + \ldots + f_{2n-1}\right) + 2(f_2 + f_4 + \ldots + f_{2n-2}) + f_{2n}\right]$$

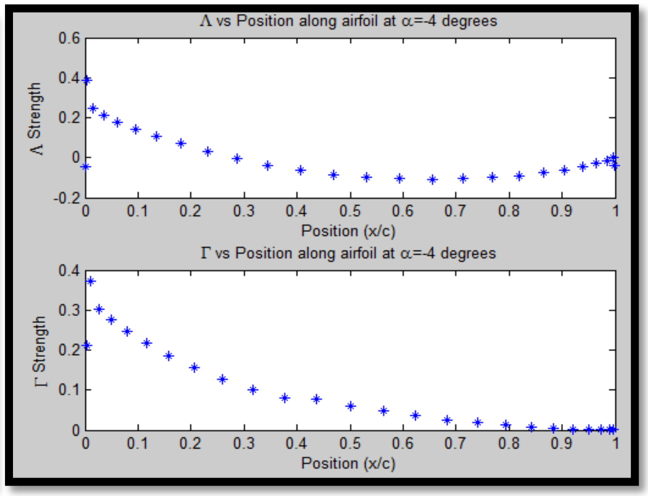
$$\text{where } n \text{ is the number of spaces between } x_o \text{ and } x_{2n}$$

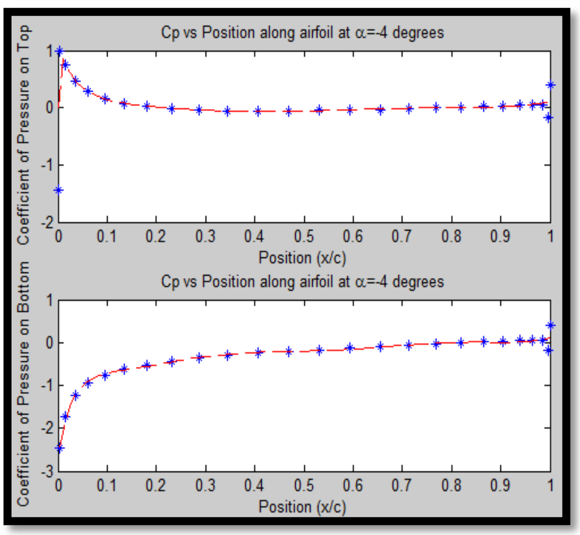
Now, after all is done with, the coefficient of lift is found. To find the α_{ZL}, I made MATLAB look at the output coefficients for lift and their angle of attack counterparts and fit a linear curve to the data. However, to account for the moments where the data begins to lose its linearity, I made the program not look at the outlier data points and keep deleting the outliers until the R^2 value is approximately 1, showing a basically perfect linear fit. After this, simple algebra was used to find α_{ZL}, essentially taking the fitted line and finding at what α value it crosses the C_L=0 axis. This now concludes the formulations used for this project.

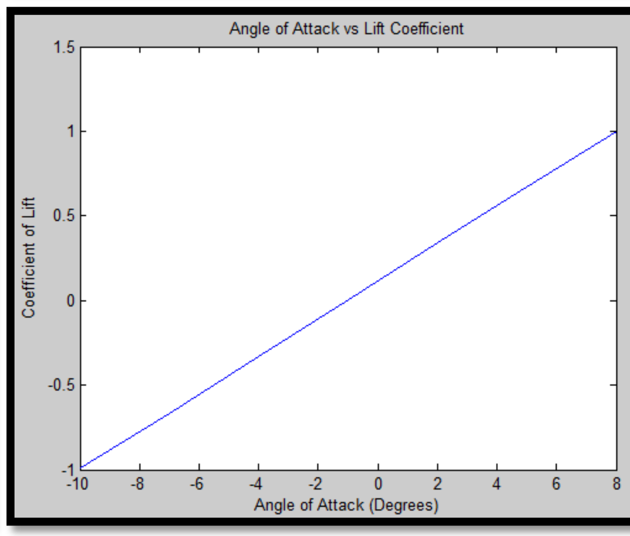
Results & Discussion

Example Output Plots



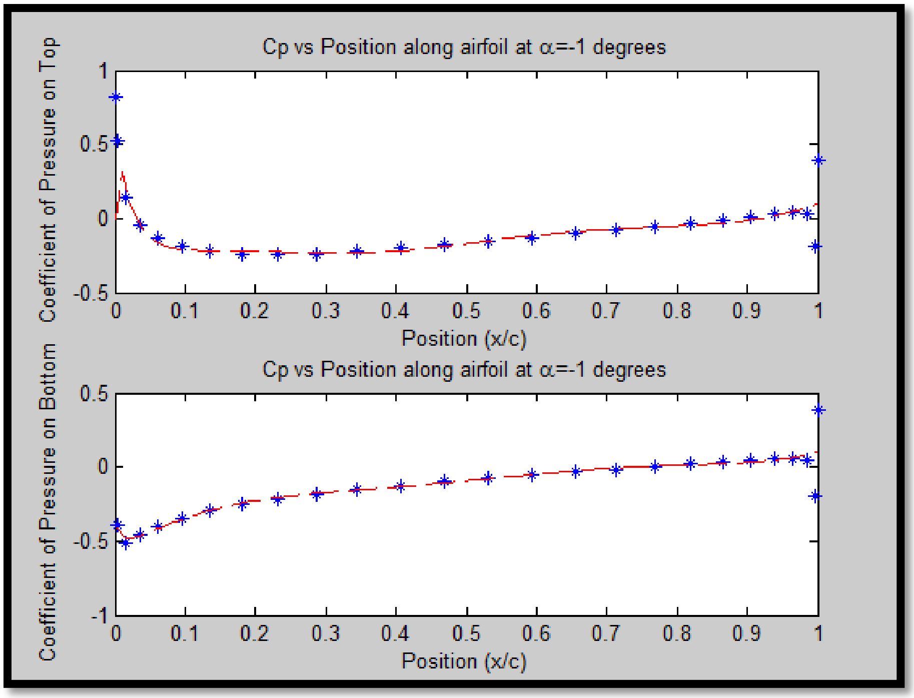






According to the data that was found via the function that was built, I found various interesting conclusions. First of all, I found that the coefficient of pressure, for negative angles of attack, along the bottom surface generally had a negative value along the airfoil, while the coefficient of pressure on the top of the wing had a generally positive value and then vice versa for the positive angle of attack case. This makes sense based on equation 6 from earlier because at negative angles of attack, the velocity on the bottom of the airfoil will be greater than the free stream, which then causes a lower pressure and thus a smaller coefficient of pressure; also at positive angles of attack, the velocity on the top of the airfoil will be greater than the free stream which causes the same result as I just mentioned. I also found that the strength of the vortices along the x position of the wing had a distribution that was higher towards the leading edge when an angle of attack was less than or equal to a 0 degree angle of attack and what looks like almost exponentially decreases as you approach the trailing edge. However, I also found that as the angle of attack increased, the vortex strength distribution swapped so the trailing edge had higher magnitudes than that of the leading edge. When it comes to the strength of the sources, I found their distribution generally housed values that were larger at the leading edge than the trailing edge but also, the trailing edge values were larger than the values toward the center of the airfoil's chord. In relation to the coefficient of lift verse angle of attack plots, I found the idea of flow separation becomes evident in lowering the coefficient of lift because as the angle of attack hits a particularly high angle of attack value, the linear line begins to curve and then cause a decrease in the coefficient of lift as the angle of attack values keep increasing. I see this relationship making sense because the area around the leading edge and the trailing edge will have the greater pressures on the wing, so the source needs to output a greater flow at these points to have the desired shape.

Now when it comes to some basic details about the NACA 1408 airfoil, I found a series of traits. First of all, the wing is thick due to the fact it had even a function for its thickness. Also, I found it was positively cambered, thus implying it is a non-symmetric airfoil. As I partially explained before, the curve I obtained for the C_L verse α plot sported a linear curve, at least until the angle of attack was too large magnitude-wise, which fits with what is found with thin airfoil theory and its equation of $C\ell = 2\pi(\alpha - \alpha_{ZL})$. The α_{ZL} that my program found was averaged at about -1^o. This value makes sense to me since the coefficient of pressure distributions of the top and bottom of the airfoil at $\alpha = -1^o$, shown below, look as though if you took the lower distribution and subtract it from the top one, the value will be about 0, which would make $C_L \approx 0$.



Now lastly, the resulting linear line found for lower magnitude angles of attack doesn't work for higher values of angles of attack because, as shown in the plot when you have an array containing larger valued angles of attack, the curve for the C_L verse α plot begins to curve and halt the increase in the magnitude of C_L, which is obviously occurring due to the sin(α) proportionality relationship with C_L we first looked at.

Conclusion

In conclusion, I found this project was almost like that of a fresh breath of air, allowing there to be some sort of use and application for the material that has been taught in a way that could be quite beneficial in the long run. I feel that through this project I have come to learn the concepts of the panel method greater than before and the analytical methods that going along with it. I have come to see, first hand, that the theory we have been taught is actually applicable and realistic to real world airfoils, such as the NACA 1408 airfoil. Lastly, thanks to this project I have been able to find a good, fun project to help practice MATLAB and to thus benefit me in the long run in schooling and quite possibly in the job field as well.