QUESTION:- 1

The random variable x and y have the following joint probability density

$$f_{xy}(x,y) = \begin{cases} e^{-x-y} & 0 \le x < \infty,\ 0 < y < \infty \\ 0 & \text{elsewhere} \end{cases}$$

what is $P(X<Y) = ?$

Solution :-

Given

$f(x,y) = e^{-x-y}$ $\quad 0 < x < \infty,\ 0 < y < \infty$

we have to find

$P[X < Y] = ?$

we have range of 'x' is $[0,\infty]$

range of 'y' is $[0,\infty]$

To find probability of $X<Y$; then the range becomes

$$0 \le X \le Y \le \infty$$

therefore

range of y becomes $\quad X < Y < \infty$

range of x becomes $\quad 0 < X < \infty$

$$P[X<Y] = \int_0^\infty \int_x^\infty e^{-(x+y)}\, dx\, dy$$

$$= \int_0^\infty \int_x^\infty e^{-x} \cdot e^{-y} \cdot dx\, dy$$

$$= \int_0^\infty e^{-x} \left[\int_x^\infty e^{-y}\, dy \right] dx$$

$$= \int_0^\infty e^{-x} \cdot \left[\frac{e^{-y}}{-1} \right]_x^\infty \cdot dx$$

$$= \int_0^\infty e^{-x} \left[-e^{-\infty} + e^{-x} \right] dx.$$

Here, $e^{-\infty} = 0$

$$\therefore \int_0^\infty e^{-x} \cdot e^{-x} \cdot dx$$

$$= \int_0^\infty e^{-2x}\, dx$$

$$= \frac{1}{-2} \left[e^{-\infty} - e^{-0} \right]$$

$$= -\frac{1}{2} [0 - 1]$$

$$= \frac{1}{2}$$

$$\therefore P[X<Y] = \frac{1}{2}$$