COMPSCI/SFWRENG 2FA3 Final Exam

DAY CLASS	**Dr. W. M. Farmer**
VERSION:	**A**
DURATION OF EXAMINATION:	**1.25 hours**
MCMASTER UNIVERSITY FINAL EXAMINATION	**April 22, 2020**

Academic integrity statement

By submitting this work, I certify that the work represents solely my own independent efforts. I confirm that I am expected to exhibit honesty and use ethical behaviour in all aspects of the learning process. I confirm that it is my responsibility to understand what constitutes academic dishonesty under the Academic Integrity Policy available at

`https://secretariat.mcmaster.ca/app/uploads/Academic-Integrity-Policy-1-1.pdf`

General instructions

Make a copy of the file `Final_Exam_A.tex` attached to the Final Exam A folder and name it

`Final_Exam_A_`*`YourMacID`*`.tex`

where *`YourMacID`* is your MacID.

The exam consists of 20 short-answer questions. Answer each of the 20 questions in your `Final_Exam_A_`*`YourMacID`*`.tex`. Your answers should be one or at most two sentences long. Each question is worth 1 mark. An answer can receive 0, 0.5, or 1 mark.

After you have finished your `Final_Exam_A_`*`YourMacID`*`.tex` file, use `pdflatex` to produce a `Final_Exam_A_`*`YourMacID`*`.pdf` file. Submit both your `.tex` and `.pdf` files to the Final Exam A folder.

You have 75 minutes, starting at 12:30 PM on April 22, to complete the exam and submit your files. If your files are submitted to the Final Exam A folder after 1:45 PM, April 22, 2020, a penalty will be imposed (see below).

Special instructions

1. The use of notes, textbooks, and electronic devices is permitted.
2. **You are *not* allowed to communicate with *any* person about the exam questions.** This means, in particular, you are not allowed to obtain help from any person or provide help to other students. Violations of this requirement will be processed as a breach of the McMaster University Academic Integrity Policy.

3. Read each question carefully.
4. Try to allocate your time sensibly and divide it appropriately between the questions.
5. Only the first two sentences of an answer will be read; any additional sentences will be ignored.

Penalties

1. **If either your .tex file or .pdf file is missing or unusable, you will receive a 20% penalty (i.e., 6 points will be subtracted from your final exam mark).**
2. **If your files are submitted late, you will receive a 2% penalty times the number of minutes the files are late (i.e., 6 points will be subtracted from your final exam mark if your files are 10 minutes late).**

Question 1 [1 mark] Q1

What is the advantage of a traditional proof over a formal proof?

Answer:

The MAIN advantage of traditional proofs over formal proofs is Communication. In addition, it is good for organization, discovery, and beauty.

Question 2 [1 mark] Q2

What is the advantage of a formal proof over a traditional proof?

Answer:

The MAIN advantage of a formal proof over a traditional proof is certification because there is a very high assurance that the theorem is correct. Formal proofs are also good for organization and discovery.

Question 3 [1 mark] Q4

How does one usually prove an existential statement $\exists\, x \in S\ .\ A$?

Answer:

For an existential statement, you need to come up with an example for when 'x' holds in the property 'A'.

Question 4 [1 mark] Q9

What does it mean that an inductive set S has "no confusion"?

Answer:

If S has "no confusion", then each constructor you make with its elements are unique. In other words, there are no duplicates constructors (or constructors that are the same).

Question 5 [1 mark] Q11

The mathematical structure $(\mathbb{N}, <)$, where $<$ is the usual order on $\mathbb{N}$, is a well-order. What is the ordinal induction principle for $(\mathbb{N}, <)$?

Answer:

$\forall x \in \mathbb{N}.((\forall y \in \mathbb{N}.y < x \to P(y)) \to P(x)) \to \forall x \in \mathbb{N}.P(x)$

Question 6 [1 mark]

Give an example of a well-order $(U, <)$ such that U is infinite and the members of U are not natural numbers.

Answer:

The Integer (set of numbers) are a good example of this.

Question 7 [1 mark]

Let $\Sigma = (\mathcal{B}, \mathcal{C}, \mathcal{F}, \mathcal{P}, \tau)$ be a signature of MSFOL where $\mathcal{B} = \{\alpha, \beta\}$, $\mathcal{C} = \{a, b\}$, $\mathcal{F} = \{f, g\}$, $\mathcal{P} = \{p, q\}$, and τ is defined by the following table:

s	$\tau(s)$
a	α
b	β
f	$(\alpha \to \beta)$
g	$(\beta \to \alpha)$
p	$(\alpha \to \mathbb{B})$
q	$((\alpha \times \beta) \to \mathbb{B})$

Give an example of a **closed Σ-formula that contains the universal quantifier but no boolean operators**. Assume $\mathcal{V} = \{x, y, z, x', y', z', x'', y'', z'', \ldots\}$.

Answer:

???

Question 8 [1 mark]

Let $\Sigma = (\mathcal{B}, \mathcal{C}, \mathcal{F}, \mathcal{P}, \tau)$ be a signature of MSFOL; $\mathcal{M} = (\mathcal{D}, I)$ be a Σ-structure where $\mathcal{D} = \{D_\alpha \mid \alpha \in \mathcal{B}\}$; and ϕ be a variable assignment into $\mathcal{M}$. What can you say about the value of $V_\phi^{\mathcal{M}}(t)$ **if t is a Σ-term of type** $\alpha \in \mathcal{B}$?

Answer:

The value is satisfiable

Question 9 [1 mark]

What kind of automaton does the following transition table define? (You don't need to determine the language that the automaton accepts.)

	Q \ Σ	0	1
start →	p	$\{q, s\}$	$\{q\}$
final →	q	$\{r\}$	$\{q, r\}$
	r	$\{s\}$	$\{p\}$
final →	s	$\{\,\}$	$\{p\}$

Answer:

This is an NFA (but not with epsilon transitions. not.)

Question 10 [1 mark]

What is Thompson's construction used for?

Answer:

Thompson's constructor is used to convert a regular expression into an NFA, with or without epsilon transitions.

Question 11 [1 mark]

Who first showed that there are undecidable decision problems?

Answer:

Alonzo Church first showed this.

Question 12 [1 mark]

Who invented regular expressions?

Answer:

Stephen Kleene invented regular expressions.

Question 13 [1 mark]

What is an example of a language that is context-free but not regular?

Answer:

An example of context-free grammar is $L = \{a^n b^n | n > 0\}$

Question 14 [1 mark]

What is an example of a language that is linear but not regular?

Answer:

A language that is linear but not regular is: $L = \{a^n b^n c^n | n > 0\}$

Question 15 [1 mark]

What normal form is a context-free grammar in if all its productions have the form $A \to aB_1 \cdots B_k$ where $k \geq 0$.

Answer:

It is in greibach normal form

Question 16 [1 mark]

Let M be a nondeterministic push-down automaton (NPDA) $(Q, \Sigma, \Gamma, \delta, s, \bot, F)$. A configuration of M is a member of $Q \times \Sigma^* \times \Gamma^*$. What do the three components of a configuration designate?

Answer:

These three components make up the transition table, which shows you how the automaton outputs based on each input. For example: If you give it a value of '0', then the automaton will output 'x' or something else.

Question 17 [1 mark]

Which kind of languages do nondeterministic push-down automata (NPDAs) accept?

Answer:

They accept context-free grammars

Question 18 [1 mark]

What is the tape used for in a Turing machine?

Answer:

The tape in a Turing machine is basically memory and stores the program. You read from it and you can write to it, in either direction, and it is semi-infinite. Once you reach the END of the tape, the program is over, and the TM halts.

Question 19 [1 mark]

What kind of language is A if the decision problem "$x \in A$" is decidable?

Answer:

The language is recursive because there is some total Turing Machine that accepts the language, and thus the TM will always halt since it is total.

Question 20 [1 mark]

What kind of language is A if the decision problem "$x \in A$" is undecidable?

Answer:

The language CAN be recursively enumerable because the program is undecidable, and the TM may not halt.

THE END.