MATHEMATICS 1LS3 TEST 1

Day Class
Duration of Test: 60 minutes
McMaster University

E. Clements

24 January 2013

40

FIRST NAME (please print) : Sol^{N}S

FAMILY NAME (please print) : _____

Student No.: _____

THIS TEST HAS 6 PAGES AND 8 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE.

Total number of points is 40. Marks are indicated next to the problem number in square brackets. Any Casio fx991 (or lower, non-graphing) calculator is allowed.

USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL, YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

You need to show work to receive full credit, except for Multiple Choice.

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1. State whether each statement is **true or false** and then **explain** your reasoning.

(a) [2] The graph of every function passes both the vertical line test and the horizontal line test. True or false? Explain.

FALSE!
The graphs of all functions pass the vertical line test but only one-to-one functions pass the horizontal line test as well.

(b) [2] The half-life of caffeine is approximately 5 hours. Suppose that at 8 a.m. you drink a tall coffee from Second Cup (approximately 240 mg of caffeine). At 10 p.m., you will have less than 10 mg left in your body. True or false? Explain.

time	amount of caffeine
8 am	240 mg
1 pm	120 mg
6 pm	60 mg
11 pm	30 mg

FALSE!
At 10 pm, you will have 30mg – 60mg left in your body.

2. **Multiple Choice.** Clearly **circle** the one correct answer.

(a) [3] Which of the following statements is/are correct?

✗ (I) $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$ for $x, y \geq 0$

✓ (II) $\dfrac{x^2}{1+x^2} = 1 - \dfrac{1}{1+x^2}$ $= \dfrac{1+x^2}{1+x^2} - \dfrac{1}{1+x^2} = \dfrac{x^2}{1+x^2}$

✓ (III) $|2x-1| = 1-2x$ when $x \leq \frac{1}{2}$

$= -(2x-1)$ when $2x-1 \leq 0 \Rightarrow x \leq \frac{1}{2}$

(A) none (B) I only (C) II only (D) III only

(E) I and II (F) I and III (G) II and III (H) all three

(b) [3] If the graph of the function $f(x)$ was stretched vertically by 4, reflected in the y-axis, and translated to the right 3 units, then the equation of the new graph is given by

(A) $4f(3-x)$ (B) $4f(x-3)$ (C) $-f(4(x+3))$ (D) $f(3-4x)$

(c) [3] For the linear relationship $y = 2x + 5$, which of the following statements is/are true?

✓ (I) $\triangle y \propto \triangle x$

✗ (II) if x doubles, so does y

✓ (III) $y = 2x + 5$ is a one-to-one function

(A) none (B) I only (C) II only (D) III only

(E) I and II (F) I and III (G) II and III (H) all three

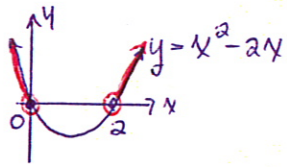
3. (a) [2] Find the domain of $f(x) = \dfrac{1}{\sqrt{x^2 - 2x}}$.

$x^2 - 2x > 0$

$x(x-2) > 0$

$x < 0$ or $x > 2$

$x \in (-\infty, 0) \cup (2, \infty)$



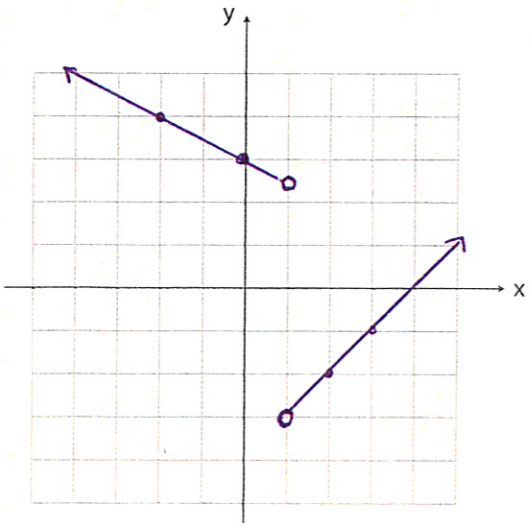
(b) [2] Solve $\ln(2x + 3) - 1 = 0$.

$\ln(2x+3) = 1$

$2x + 3 = e^1$

$x = \dfrac{e-3}{2}$

4. (a) [2] Sketch the graph of $f(x) = \begin{cases} -\frac{1}{2}x + 3 & \text{if } x < 1 \\ x - 4 & \text{if } x > 1 \end{cases}$.



(b) [2] State the domain and range of f.

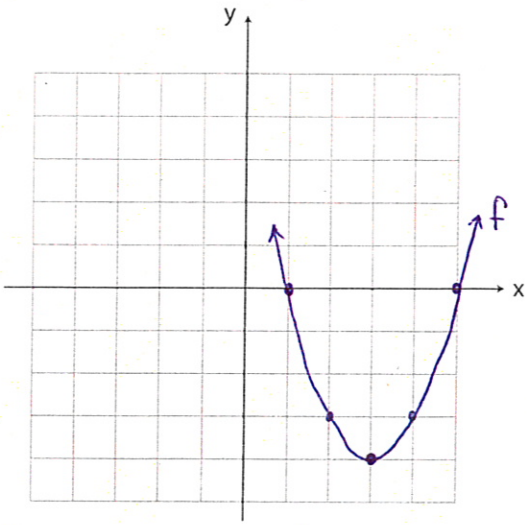
$x \neq 1$

$y > -3$

5. Consider the function $f(x) = x^2 - 6x + 5$.

$(-\frac{6}{2})^2 = (-3)^2 = 9$

(a) [3] Complete the square to find the vertex. Sketch the graph of f, clearly labelling at least three points on the graph.



$f(x) = x^2 - 6x + 9 - 9 + 5$

$= (x-3)^2 - 4$

$V(3,-4)$

(b) [2] Solve $x^2 - 6x + 5 = 0$. Explain how this shows that the function f is not one-to-one.

$(x-5)(x-1) = 0 \Rightarrow x = 1$ or $x = 5$.

When $y = 0$, $x = 1$ OR $x = 5$

This violates the def^N of a 1-1 f^N which states that for every y-value in the range, there is a UNIQUE x-value in the domain such that $y = f(x)$.

6. [3] The function $f(x) = 2(x-3)^3 + 1$ is one-to-one. Find f^{-1}.

$y = 2(x-3)^3 + 1$

$\frac{y-1}{2} = (x-3)^3$

$\sqrt[3]{\frac{y-1}{2}} = x - 3$

$x = 3 + \sqrt[3]{\frac{y-1}{2}}$ $\therefore f^{-1}(x) = 3 + \sqrt[3]{\frac{x-1}{2}}$.

7. Suppose that your average stress level during the day, rated on a scale of 1 (minimum) to 10 (maximum), is inversely proportional to the square root of the amount of sleep obtained the night before.

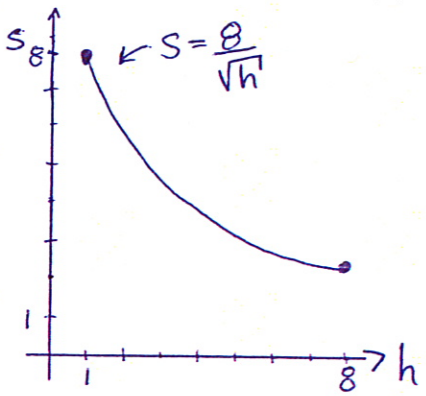
(a) [2] Express stress level S as a function of amount of sleep h, where h is measured in hours, for $1 \leq h \leq 8$. Suppose that your stress level is about 8 when you obtained only 1 hour of sleep the night before. Solve for the proportionality constant.

$$S \propto \frac{1}{\sqrt{h}} \Rightarrow S(h) = \frac{a}{\sqrt{h}}$$

$$S(1) = 8 \Rightarrow 8 = \frac{a}{\sqrt{1}} \Rightarrow a = 8$$

$$\therefore S(h) = \frac{8}{\sqrt{h}}$$

(b) [2] Sketch the relationship in part (a).



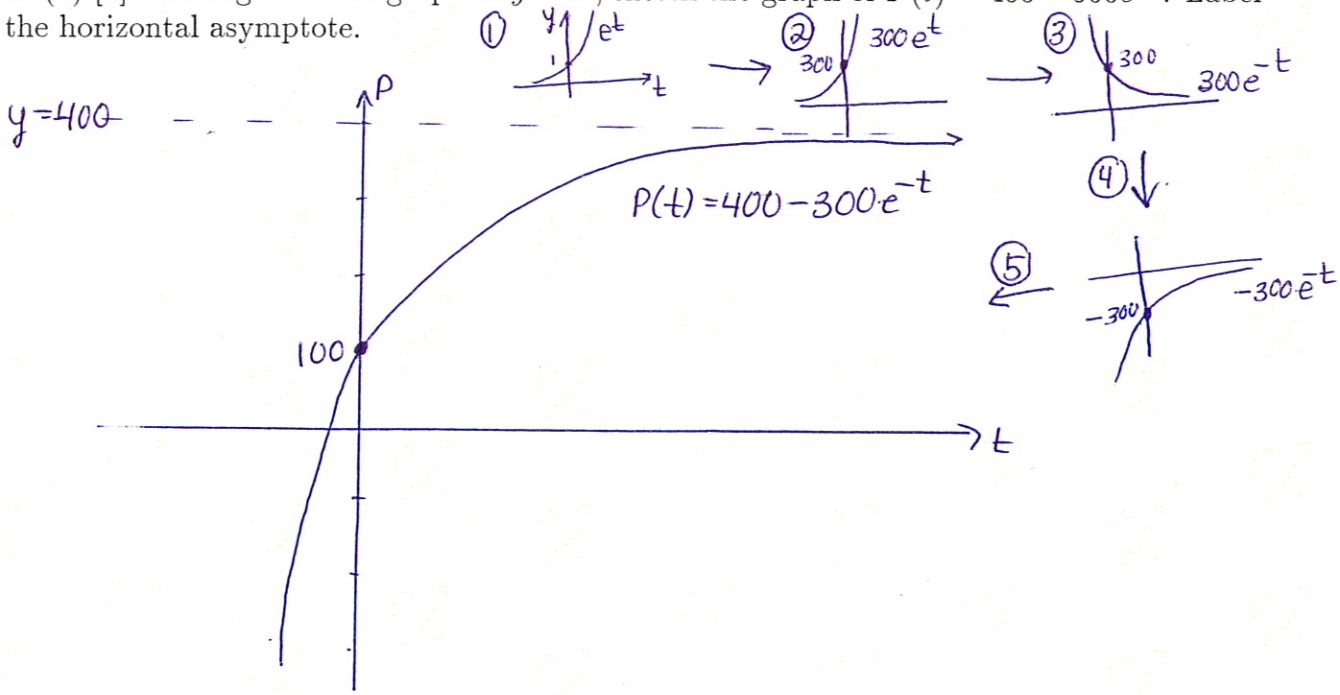
(c) [2] If you start sleeping 20% less each night, by what percentage will your daily stress levels increase?

$$h_{new} = 0.8 h_{old}$$

$$S_{new} = \frac{8}{\sqrt{h_{new}}} = \frac{8}{\sqrt{0.8 h_{old}}} = \frac{1}{\sqrt{0.8}} \cdot \underbrace{\frac{8}{\sqrt{h_{old}}}}_{S_{old}} \approx 1.12 S_{old}$$

$\therefore$ Your daily stress levels would increase by about 12%.

8. (a) [3] Starting with the graph of $y = e^t$, sketch the graph of $P(t) = 400 - 300e^{-t}$. Label the horizontal asymptote.



(b) [2] Assume that $P(t) = 400 - 300e^{-t}$ represents the population size of an island at time t, where $t \geq 0$. When will the population reach 250?

$P(t) = 250 \Rightarrow \quad 250 = 400 - 300e^{-t}$

$-150 = -300e^{-t}$

$0.5 = e^{-t}$

$\ln 0.5 = -t$

$t = -\ln\frac{1}{2} \left(= -[\underbrace{\ln 1}_{=1} - \ln 2] = \ln 2\right.$

$t \approx 0.693$ time units

THE END