Math 1A03/1ZA3 Test #1 (Version 1)
October 15th, 2018

Name:____
(Last Name) (First Name)

Student Number:______________________

This test consists of 20 multiple choice questions worth 1 mark each (no part marks), and 1 question worth 1 mark (no part marks) on proper computer card filling. Questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Only the McMaster standard calculator **Casio fx-991 MS or MS Plus** is allowed.

1. If $\csc\theta = -\frac{4}{3}$ and $\frac{3\pi}{2} < \theta < 2\pi$. Find $\cos(\theta)$.
(a) $\frac{3}{5}$ **(b)** $-\frac{5}{3}$ **(c)** $\frac{\sqrt{7}}{4}$ **(d)** $\frac{5}{3}$ **(e)** $-\frac{\sqrt{7}}{4}$

2. Which of the following is equal to $\cot^2\theta + \sec^2\theta$?
(a) $\frac{1}{\tan^2\theta+\cos^2\theta}$ **(b)** $\tan^2\theta + \csc^2\theta$ **(c)** $\tan^2\theta + \cos^2\theta$ **(d)** $\tan^2\theta + \sin^2\theta$
(e) $\csc^2\theta + \cos^2\theta$

3. Let

$$f(x) = \frac{1}{1+e^{-2x}}.$$

Find $f^{-1}(x)$.

(a) $-\frac{1}{2}\ln\left(\frac{1}{x}-1\right)$ **(b)** $\frac{1}{1+\ln(2x)}$ **(c)** $\frac{1}{1-2\ln x}$ **(d)** $\frac{1}{2}\ln\left(\frac{1}{x}+1\right)$ **(e)** $\ln\left(-\frac{1}{2x}+1\right)$

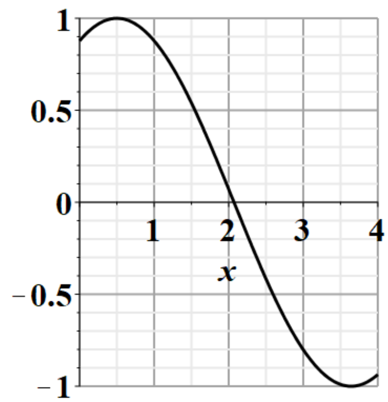
4. Consider the following equation: $x^2 - 12 = \ln x$. The Intermediate Value Theorem guarantees that the equation has a root on which of the following intervals?
(a) $(5,6)$ **(b)** $(4,5)$ **(c)** $(1,2)$ **(d)** $(3,4)$ **(e)** $(2,3)$

5. Find the slope of the tangent line to the curve $\sin(x+y) = 2x - 2y$ at the point (π,π).

(a) 2 **(b)** $\frac{1}{2}$ **(c)** $\frac{1}{3}$ **(d)** 1 **(e)** -1

6. Let $f(x)$ be the function whose graph is shown to the right. If Newton's method were used to find a root of f with initial approximation $x_1 = 1$, estimate the value of the second approximation x_2.

(a) 3 **(b)** $\frac{3}{2}$ **(c)** 2 **(d)** $\frac{3}{4}$ **(e)** 0



7. The following limit represents the derivative of some function f at some number a.

$$\lim_{h \to 0} \frac{e^{-2+h} - e^{-2}}{h}$$

Find f and a.

(a) $f(x) = e^x$, $a = -2$.
(b) $f(x) = e^{-x}$, $a = 2$
(c) $f(x) = e^{-(x+2)}$, $a = 0$
(d) $f(x) = e^{-(x+4)}$, $a = -2$
(e) all of the above

8. Solve the following equation for x: $\ln(1 + \ln x) = 2$.
(a) $e^2 - e^e$ **(b)** $e^{(e^2-1)}$ **(c)** $e(e-1)$ **(d)** $e^{e^2} - e$ **(e)** $e^{(e^2-e)}$

9. For what value of c is the line $y = 2x + 3$ tangent to the parabola $y = cx^2$?
(a) $\frac{1}{2}$ **(b)** $-\frac{1}{2}$ **(c)** $-\frac{1}{3}$ **(d)** 1 **(e)** -1

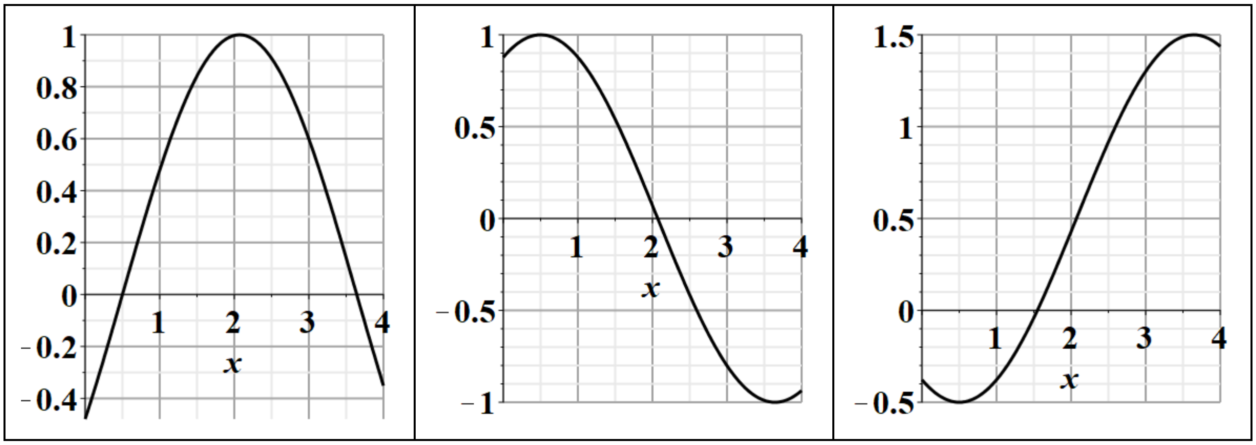
10. Find the value of the constant c so that the following function is continuous at $x = 9$.

$$f(x) = \begin{cases} \frac{\sqrt{x}-3}{x-9} & x \neq 9 \\ c & x = 9 \end{cases}$$

(a) $\frac{1}{12}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{1}{6}$ **(d)** $\frac{1}{3}$ **(e)** $\frac{1}{9}$

11. Suppose that f is one-to-one, $f(6) = 9$, and $f(9) = 4$. Find $f^{-1}(9)$.
(a) $\frac{1}{9}$ **(b)** $\frac{1}{6}$ **(c)** $\frac{1}{4}$ **(d)** 6 **(e)** 4

12. Consider the following graphs

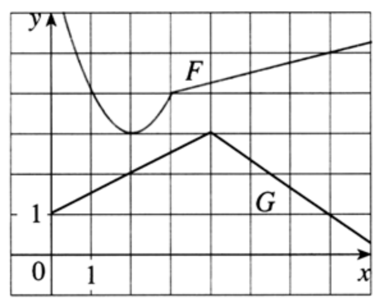


These are the graphs of a function $f(x)$ and its derivatives $f'(x)$ and $f''(x)$, in **random order**. Identify each graph, from left to right.

(a) $f'(x), f(x), f''(x)$ **(b)** $f'(x),\ f''(x),\ f(x)$ **(c)** $f''(x), f(x), f'(x)$
(d) $f''(x), f'(x), f(x)$ **(e)** $f(x), f''(x), f'(x)$

13. Let $Q(x) = F(x)/[G(x)]^2$, where F and G are the functions whose graphs are shown to the right. Find $Q'(2)$.

(a) $-\frac{5}{8}$ **(b)** $-\frac{3}{16}$ **(c)** $\frac{5}{8}$ **(d)** $-\frac{3}{8}$ **(e)** 0



14. Let $f(x) = 4\sin x\,\cos x$. Find the equation of the tangent to line to $f(x)$ when $x = \frac{\pi}{3}$.
(a) $y = -2x + (\frac{2\pi}{3} + \sqrt{3})$ **(b)** $y = 2x + (\frac{2\pi}{3} - \sqrt{3})$ **(c)** $y = 2x + (\frac{2\pi}{3} + \sqrt{3})$
(d) $y = 2x + (\frac{\pi}{3} + \sqrt{3})$ **(e)** $y = -2x + (\frac{\pi}{3} + \sqrt{3})$

15. Let $f(x) = cx + \ln(\sin x)$. For what value of c is $f'(\pi/4) = 6$?

(a) 7 **(b)** 6 **(c)** 4 **(d)** 5 **(e)** 3

16. Solve for x if $x > 0$ and $\sinh x = \frac{3}{4}$.

(a) $\ln 3$ **(b)** $2\ln 2$ **(c)** 4 **(d)** 2 **(e)** $\ln 2$

17. Find the critical numbers of the function

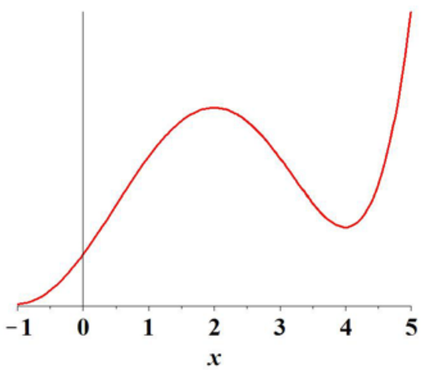
$$f(x) = \sqrt{x^3} - 2\sqrt{x}$$

(a) $2, 0$ **(b)** $3, 0$ **(c)** $\frac{1}{3}, \frac{1}{2}$ **(d)** $1, 0$ **(e)** $\frac{2}{3}, 0$

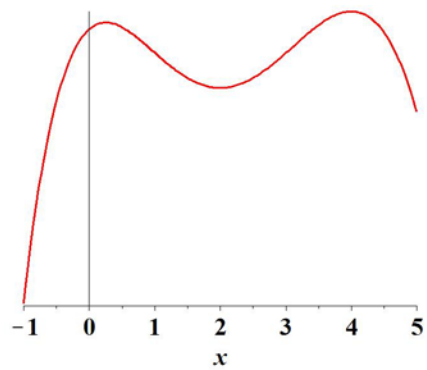
18. Which of the graphs below is an example of a function that satisfies the following conditions?

$$g'(0) = 1, g'(1) = 0, g'(2) = 1, g'(3) = 1, g'(4) = -1$$

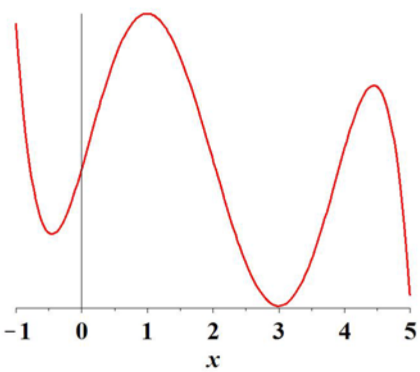
(a)



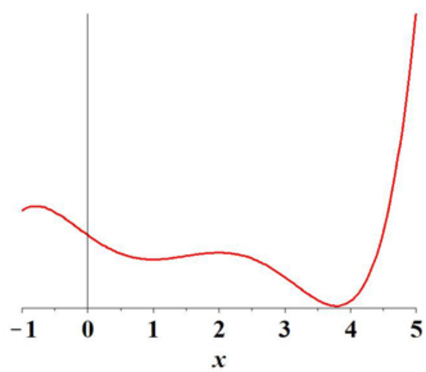
(b)



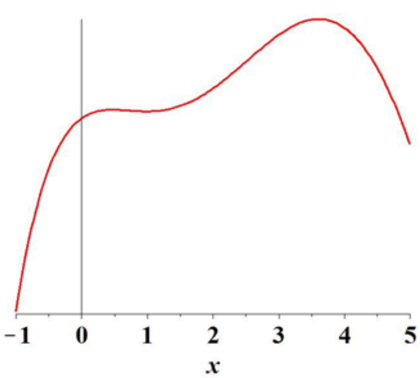
(c)



(d)



(e)



19. Consider the following function

$$f(x) = \begin{cases} x^2 - 2 & x < 2 \\ -x + 1 & x \geq 2 \end{cases}$$

Which of the following statements is/are true?

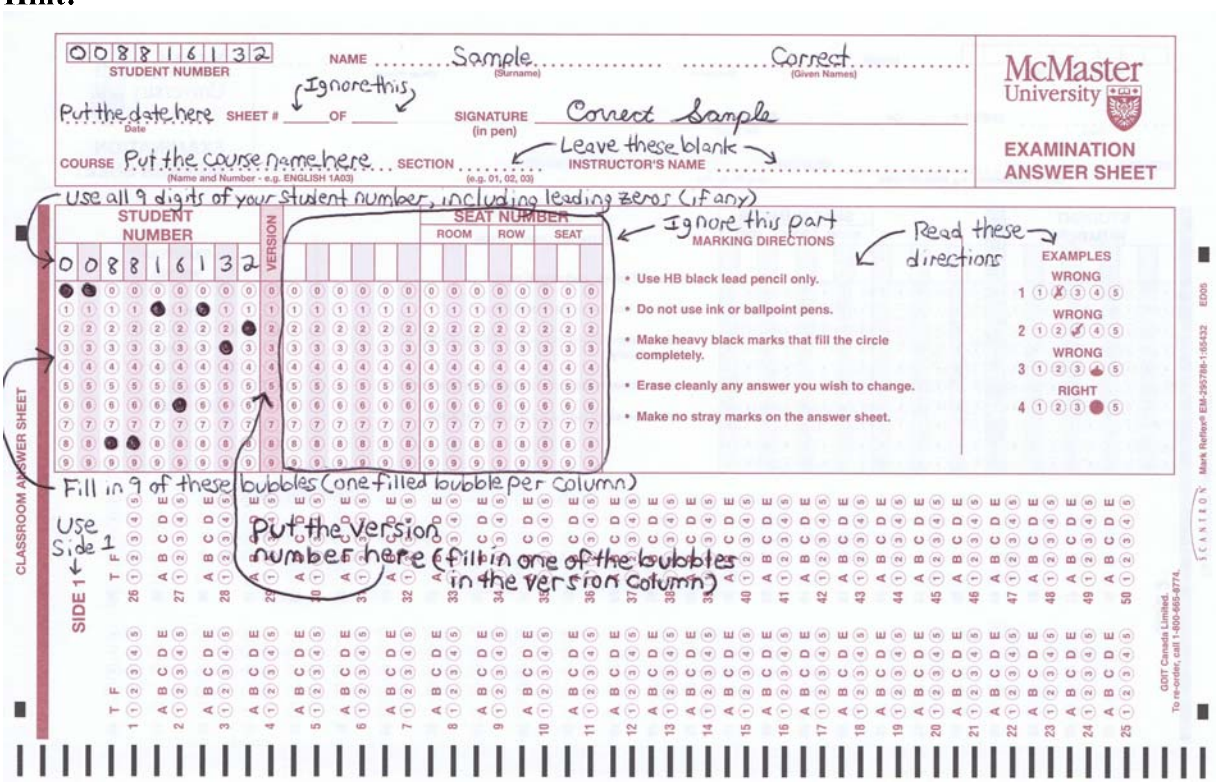
(i) f has a local minimum, but no absolute minimum
(ii) f has a local maximum, but no absolute maximum.
(iii) f is differentiable at $x = 2$.

(a) (ii) only **(b)** (i) only **(c)** (i) and (ii) only **(d)** (i) and (iii) only **(e)** (ii) and (iii) only

20. Suppose that $f(e^2) = 3$ and $f'(e^2) = 4$. Let $g(x) = f(e^{\sqrt{x}})$. Find $g'(4)$.

(a) $\frac{3}{4}e^2$ **(b)** e^2 **(c)** $2e^2$ **(d)** $4e^2$ **(e)** $3e^2$

21. Correctly fill out the bubbles corresponding to all 9 digits of your student number, as well as the version number of your test in the correct places on the computer card.
Note: You are writing **Version 1**.
Hint:



Answers (Version 1)**:**

1. c **2.** b **3.** a **4.** d **5.** c **6.** a **7.** a **8.** b **9.** c **10.** c
11. d **12.** b **13.** d **14.** a **15.** d **16.** e **17.** e **18.** e **19.** b **20.** b