MATHEMATICS 1LS3 TEST 2

Day Class F. Font, M. Lovrić, D. Lozinski
Duration of Examination: 60 minutes
McMaster University, 31 October 2016

First name (PLEASE PRINT): ______________________

Family name (PLEASE PRINT): ______________________

Student No.: ______________________

THIS TEST HAS 8 PAGES AND 6 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE.

Total number of points is 40. Marks are indicated next to the problem number. Any non-graphing calculator is allowed.

EXCEPT ON QUESTIONS 1 AND 2, YOU MUST SHOW CORRECT WORK TO EARN CREDIT.

USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

You must show work to receive full credit.

Problem	Points	Mark
1	4	
2	6	
3	7	
4	6	
5	6	
6	11	
TOTAL	40	

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1. **Multiple choice questions: circle ONE answer. No justification is needed.**

(a)[2] If $f(x) = \ln(ax)\ln(bx)$ then $f'(1)$ is equal to

(A) $\ln a \ln b$ (B) $\ln(a+b)$ (C) $\ln(ab)$ (D) $\dfrac{\ln(a+b)}{a+b}$

(E) $\dfrac{\ln(ab)}{a+b}$ (F) $\dfrac{\ln a \ln b}{a+b}$ (G) $\dfrac{1}{ab}$ (H) $\dfrac{1}{a}+\dfrac{1}{b}$

(b)[2] Which of the following functions has/have **no critical points**?

(I) $f(x) = 2.3x + 5$
(II) $f(x) = x^2 + 3$
(III) $f(x) = e^{0.04x}$

(A) none (B) I only (C) II only (D) III only

(E) I and II (F) I and III (G) II and III (H) all three

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2. **Identify each statement as true or false (circle your choice). You do not need to justify your answer.**

(a)[2] The function $y = -1$ is the linear approximation of $f(x) = \sec x$ at $x = \pi$.

TRUE FALSE

(b)[2] From $f''(x) = e^{-x-2}(3 - x)$ we conclude that the graph of $f(x)$ is concave down on the interval $(0, 3)$.

TRUE FALSE

(c)[2] The function $f(x)$ has a horizontal tangent at $x = 4$. Therefore, it must have a local maximum or a local minimum at $x = 4$.

TRUE FALSE

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Questions 3-6: You must show CORRECT work to receive full credit.

3. (a)[3] Using L'Hôpital's rule, calculate $\lim_{x\to 0^+} x^4 \ln x$.

(b)[4] Find $y'(x)$ if $x^3 \ln y = x - e^y + e$. Compute y' when $x = 0$ and $y = 1$.

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4. (a)[2] In the article *Migration behaviour of grizzly bears in Northern British Columbia: contribution to a modelling approach.* G. Brown et al, Bear Science 4 (June 2012), we find the formula

$$P(t) = \arctan(1.7t) + 4.7$$

where t represents time.

Next, we read "initially, $P(t) \approx 1.7t + 4.7$, which gives a linear relationship."Explain why this statement is correct. [Hint: Think in terms of the linear approximation at $t = 0$.]

(b)[4] A simple model of diffusion states that the concentration of a substance diffusing in air is given by

$$c(x) = e^{-x^2+0.2}$$

where x is the distance from the source. This formula is sometimes simplified using a quadratic approximation near $x = a = 0$. Find that approximation.

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5. The resistance R of the flow of blood through a blood vessel (assumed to have the shape of a cylindrical tube) is given by

$$R = \frac{K^{0.96} L(\gamma + 1)^2}{d^4}$$

where L is the length of the tube, d is its diameter and $\gamma \geq 0$ is the curvature. The positive constant K represents the viscosity of the blood (viscosity is a measure of the resistance of fluid to stress; water has low viscosity, honey has high viscosity).

(a)[3] Find the derivative of R with respect to K and interpret your answer, i.e., explain what your answer implies for the dependence of R on the viscosity of the blood.

(b)[3] Find the derivative of R with respect to d and interpret your answer, i.e., explain what your answer implies for the dependence of R on the diameter of a blood vessel.

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6. (a)[3] The function $f(x) = x^2 e^{4x}$ has two critical points. Find them.

(b)[2] Give a statement of the Extreme Value Theorem. Make sure to clearly identify assumptions and conclusions.

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(c)[3] Find the absolute maximum and the absolute minimum of the function $f(x) = x^2e^{4x}$ on the interval $[-1, 1]$. In each case, state what the value is, and where it occurs.

(d)[3] You have to find the absolute maximum and the absolute minimum of the function $f(x) = x^2e^{4x}$, this time on the interval $[1, 10]$. Without repeating the routine as in part (c), find the absolute maximum and the absolute minimum of $f(x)$, and explain why your answer makes sense.

THE END