ASSIGNMENT 21

1.(a) $\int_0^{\pi} (\sin x - 2\cos x\, dx) = (-\cos x + 2\sin x)\Big|_0^{\pi} = 1-(-1) = 2$

(b) $\int_1^9 x^{-1/2}dx = 2x^{1/2}\Big|_1^9 = 2\sqrt{9} - 2\sqrt{1} = 4$

(c) $\int_0^1 \frac{6}{1+x^2}dx = 6\arctan x\Big|_0^1 = 6\cdot\frac{\pi}{4} - 0 = \frac{3\pi}{2}$

(d) $\int_0^1 11e^x dx = 11e^x\Big|_0^1 = 11e - 11$

(e) $\int_{-2}^{-1} x^{-1}dx = \ln|x|\Big|_{-2}^{-1} = \ln 1 - \ln 2 = -\ln 2$

(f) $\int_1^2 \frac{x^2+1}{x}dx = \int_1^2 \left(x + \frac{1}{x}\right)dx = \left(\frac{x^2}{2} + \ln|x|\right)\Big|_1^2$

$= (2 + \ln 2) - \left(\frac{1}{2} + \ln 1\right) = \frac{3}{2} + \ln 2$

2.(a) $= \left\{ \begin{array}{l} u = x^2+x+2 \\ du = (2x+1)dx \end{array} \right\} = \int \frac{du}{u^2} = -\frac{1}{u} + C$

$= -\frac{1}{x^2+x+2} + C$

(b) $= \left\{ \begin{array}{l} u = 3-e^x \\ du = -e^x dx \end{array} \right\} = \int \frac{-du}{u} = -\ln|u| + C = -\ln|3-e^x| + C$

(c) $= \left\{ \begin{array}{l} u = 1+\sqrt{x} \\ du = \frac{1}{2\sqrt{x}}dx \to dx = 2\sqrt{x}du \end{array} \right\} = \int \frac{u^2}{\cancel{\sqrt{x}}} 2\cancel{\sqrt{x}}\, du$

$= 2\int u^2 du = \frac{2u^3}{3} + C = \frac{2}{3}(1+\sqrt{x})^3 + C$

(d) $= \left\{ \begin{array}{l} u = \ln x \\ du = \frac{1}{x}dx \end{array} \right\} = \int_0^{\ln 2} \frac{u^4}{1}du = \frac{u^5}{5}\Big|_0^{\ln 2} = \frac{(\ln 2)^5}{5}$

when $x=1 \to u = \ln 1 = 0$
when $x=2 \to u = \ln 2$

3.(a) $\int_0^1 \frac{1}{1+x^2}\,dx = \arctan x \Big|_0^1$

$= \arctan 1 - \arctan 0 = \frac{\pi}{4} - 0 = \frac{\pi}{4}$

(b) $\int_0^1 \frac{x}{1+x^2}\,dx = \left\{ \begin{array}{l} u = 1+x^2 \\ du = 2x\,dx \end{array} \right]$

$= \int_1^2 \frac{x}{u}\frac{du}{2x} = \frac{1}{2}\int_1^2 \frac{1}{u}\,du$

$= \frac{1}{2}\ln|u| \Big|_1^2 = \frac{1}{2}\ln 2 - \cancel{\frac{1}{2}\ln 1}^{\,0} = \frac{1}{2}\ln 2$

(c) $\int_0^1 \frac{x^2}{1+x^2}\,dx =$

$$\begin{array}{r} 1 \\ x^2+1 \,\overline{)\, x^2 } \\ x^2+1 \\ \hline -1 \end{array}$$

$= \int_0^1 \left(1 - \frac{1}{1+x^2}\right) dx$

$= (x - \arctan x)\Big|_0^1$

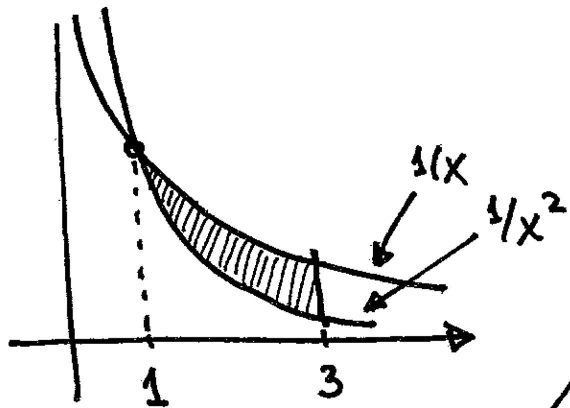
$= (1 - \arctan 1) - (0 - \underbrace{\arctan 0}_{0}) = 1 - \frac{\pi}{4}$

4. $$\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}} = \left\{ \begin{array}{l} u = \ln x \\ du = \frac{1}{x}dx \end{array} \right\}$$

$$= \int_{\ln e}^{\ln e^4} \frac{1}{\sqrt{u}}\,du = \int_1^4 u^{-1/2}\,du = 2\sqrt{u}\,\Big|_1^4$$

$$= 2\sqrt{4} - 2\sqrt{1} = 2$$

5.



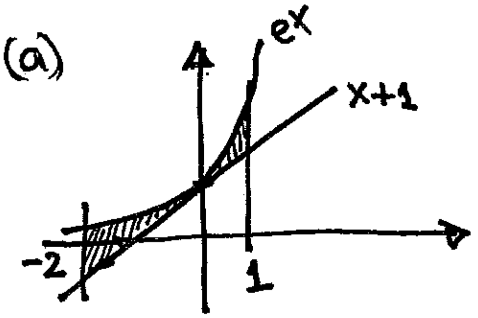
$$A = \int_1^3 \left(\frac{1}{x} - \frac{1}{x^2}\right)dx$$

$$\cancel{A = \left(\ln|x| - \frac{x^{-3}}{-3}\right)\Big|_1^3 = \ln|x| + \frac{1}{3x^3}}$$

$$A = \left(\ln|x| - \frac{x^{-1}}{-1}\right)\Big|_1^3 = \left(\ln|x| + \frac{1}{x}\right)\Big|_1^3$$

$$= \left(\ln 3 + \frac{1}{3}\right) - \left(\cancel{\ln 1}^{\,0} + 1\right) = \ln 3 - \frac{2}{3}$$

6.(a)



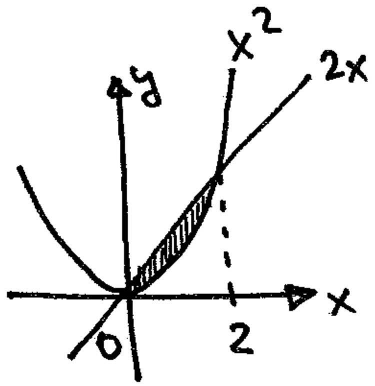
$$A = \int_{-2}^{1} \left(e^x - (x+1)\right)dx$$

(b) $$A = \left(e^x - \frac{x^2}{2} - x\right)\Big|_{-2}^{1}$$

$$= \left(e - \frac{1}{2} - 1\right) - \left(e^{-2} - \cancel{2} + \cancel{2}\right)$$

$$= e - e^{-2} - \frac{3}{2} \approx \underline{\underline{1.08}}$$

7.

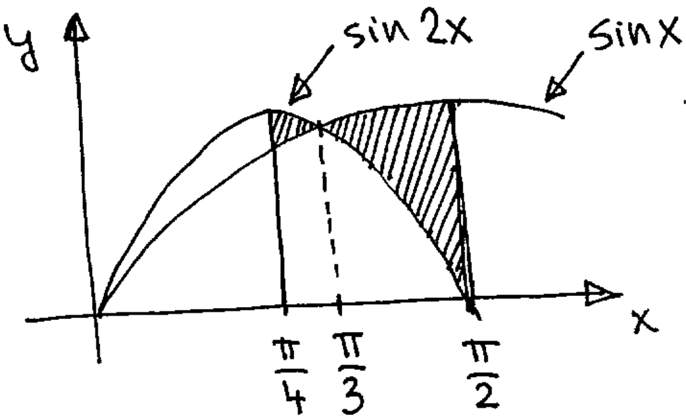


$x^2 = 2x \to x^2 - 2x = x(x-2) = 0$
$\to x = 0, 2$ are pts. of intersection

$$A = \int_0^2 (2x - x^2)\,dx$$

$$= \left(x^2 - \frac{x^3}{3}\right)\Big|_0^2 = 4 - \frac{8}{3} = \underline{\underline{\frac{4}{3}}}$$

8. (a)



$$\sin 2x = \sin x$$
$$2\sin x \cos x = \sin x$$
$$\rightarrow \sin x\,(2\cos x - 1) = 0$$

$$\sin x = 0 \rightarrow x = 0, \pi, 2\pi, \ldots$$
$$2\cos x - 1 = 0 \rightarrow \cos x = \frac{1}{2} \rightarrow x = \frac{\pi}{3}$$

(we need solutions between $\pi/4$ and $\pi/2$)

$$A = \int_{\pi/4}^{\pi/3} (\sin 2x - \sin x)\,dx + \int_{\pi/3}^{\pi/2} (\sin x - \sin 2x)\,dx$$

(b) $$= \left(-\frac{1}{2}\cos 2x + \cos x\right)\Big|_{\pi/4}^{\pi/3} + \left(-\cos x + \frac{1}{2}\cos 2x\right)\Big|_{\pi/3}^{\pi/2}$$

$\cos\frac{\pi}{3} = \frac{1}{2}$

$\cos\frac{2\pi}{3} = -\frac{1}{2}$

$$= \left(-\frac{1}{2}\cos\frac{2\pi}{3} + \cos\frac{\pi}{3}\right) - \left(-\frac{1}{2}\cos\frac{\pi}{2} + \cos\frac{\pi}{4}\right)$$
$$+ \left(-\cos\frac{\pi}{2} + \frac{1}{2}\cos\pi\right) - \left(-\cos\frac{\pi}{3} + \frac{1}{2}\cos\frac{2\pi}{3}\right)$$

$$= \left(-\frac{1}{2}\left(-\frac{1}{2}\right) + \frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)$$
$$+ \left(-\frac{1}{2}\right) - \left(-\frac{1}{2} + \frac{1}{2}\left(-\frac{1}{2}\right)\right)$$

$$= \frac{3}{4} - \frac{\sqrt{2}}{2} - \frac{1}{2} + \frac{3}{4} = 1 - \frac{\sqrt{2}}{2}$$

END