Math 1A03/1ZA3

Sample Test Questions for Test #1

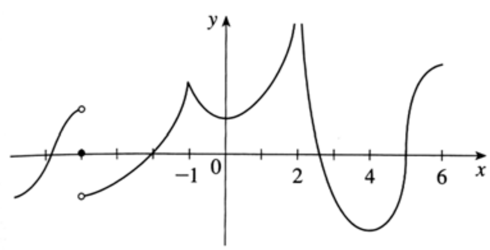
Name:____
(Last Name) (First Name)

Student Number:____________________ **Tutorial Number:**____________________

This test consists of 84 multiple choice questions worth 1 mark each (no part marks), and 1 question worth 1 mark (no part marks) on proper computer card filling. All questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Only the McMaster standard calculator Casio fx-991 is allowed.

1. Which of the following is equal to $\frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta}$?
(a) $\frac{1}{1-\tan\theta}$ **(b)** $\frac{1}{1+\sin\theta}$ **(c)** $\frac{1}{1+\tan\theta}$ **(d)** $2\csc^2\theta$ **(e)** $2\sec^2\theta$

2. For the function f, whose graph is given to the right evaluate $\lim\limits_{x\to -4} f(x)$
(a) 1 **(b)** -1 **(c)** 0 **(d)** ∞ **(e)** d.n.e



3. For the function f, whose graph is given to the right
(a) f is continuous from the left at $x = -4$
(b) f is continuous from the right at $x = -4$
(c) $f(-4)$ does not exist
(d) none of the above

4. If f and g are continuous functions with $g(4) = 3$ and $\lim\limits_{x\to 4}\left[\frac{[f(x)]^2}{f(x)-g(x)}\right] = 12$, find $f(4)$.
(a) 2 **(b)** 4 **(c)** 6 **(d)** 8 **(e)** 1

5. If $\csc\theta = 5,\ 0 < \theta < \frac{\pi}{2},$ find $\sin 2\theta$.
(a) $\frac{2}{25}\sqrt{24}$ **(b)** $\frac{2}{5}$ **(c)** $\frac{\sqrt{24}}{5}$ **(d)** $\sqrt{24}$ **(e)** $\frac{1}{\sqrt{24}}$

6. Find the exact value of $\tan^{-1}\sqrt{3}$
(a) $\pi/3$ **(b)** $\pi/6$ **(c)** $\pi/2$ **(d)** π **(e)** $\pi/4$

7. If $f(x) = x^2 + 2\ln\left(\frac{4}{x}\right) - e^{x-4}$, find $f^{-1}(15)$.
(a) 0 **(b)** 1 **(c)** 2 **(d)** 3 **(e)** 4

8. State the domain of $y = \ln\left(5 - \sqrt{1-x}\right)$.
(a) $\{x \mid -5 < x \le 1\}$ **(b)** $\{x \mid 1 \le x \le 5\}$ **(c)** $\{x \mid x \le 5\}$
(d) $\{x \mid -24 < x \le 1\}$ **(e)** $\{x \mid x \le 1\}$

9. Find $\sec\left(\frac{5\pi}{3}\right)$ (angle given in radians)
(a) $\frac{2}{\sqrt{3}}$ **(b)** -2 **(c)** 2 **(d)** $-\frac{2}{\sqrt{3}}$ **(e)** $-\sqrt{3}$

10. Simplify the expression $\cos(\sin^{-1}x)$.
(a) $\sqrt{1-x^2}$ **(b)** $\dfrac{\sqrt{1-x^2}}{x}$ **(c)** $\dfrac{x}{\sqrt{1-x^2}}$ **(d)** $\dfrac{1}{\sqrt{1-x^2}}$ **(e)** x

11. Evaluate $\lim\limits_{x\to 0} \frac{2-\sqrt{4-x^2}}{3x}$
(a) 0 **(b)** $\frac{1}{3}$ **(c)** $\frac{1}{6}$ **(d)** $\frac{1}{12}$ **(e)** d.n.e

12. Solve for x: $\ln(5^x-1)=3$
(a) $\frac{3}{\ln 5}$ **(b)** $(e^3+1)^{1/5}$ **(c)** e^{e^3+1} **(d)** $\frac{\ln(e^3+1)}{\ln 5}$ **(e)** $\frac{\ln(e^3+1)}{e^5}$

13. Find all values of θ in the interval $[0, 2\pi]$ that satisfy the equation $2\cos^2\theta = 1$
(a) $0, \frac{\pi}{2}, \pi$ **(b)** $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ **(c)** $\frac{\pi}{4}, \frac{3\pi}{4}$ **(d)** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}$ **(e)** $\frac{\pi}{3}, \frac{2\pi}{3}$

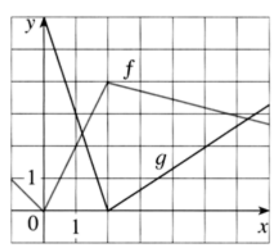
14. Find a formula for the inverse of the function $f(x)=e^{-(x+3)}$
(a) $(-\ln x)+3$ **(b)** $(\ln x)-3$ **(c)** $(\ln x)+3$ **(d)** e^{x+3} **(e)** $(-\ln x)-3$

15. The following limit represents the derivative of some function f at some number a. State such an f and a. $\lim\limits_{h\to 0}\frac{\ln(1+h)}{h}$
(a) $f(x)=\ln(1+x),\ a=0$ **(b)** $f(x)=\ln x,\ a=1$
(c) $f(x)=2+\ln x,\ a=1$ **(d)** $f(x)=1+\ln(1+x),\ a=0$
(e) all of the above

16. Suppose that $f(5)=-1,\ f'(5)=2,\ g(5)=5,$ and $g'(5)=-3.$ Find $[g/(f^2+g)]'(5)$
(a) $\frac{16}{36}$ **(b)** $\frac{15}{36}$ **(c)** $\frac{17}{6}$ **(d)** $\frac{17}{36}$ **(e)** $\frac{15}{6}$

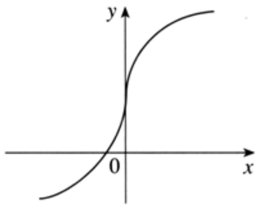
17. If $h(4)=-3$ and $h'(4)=7$, find $\frac{d}{dx}(x^2h(x))|_{x=4}$
(a) 88 **(b)** 53 **(c)** 66 **(d)** 77 **(e)** 82

18. If f and g are the functions whose graphs are shown, let $v(x)=g(f(x))$. Find $v'(1)$, if it exists.
(a) 6 **(b)** -6 **(c)** 2 **(d)** 0 **(e)** d.n.e.

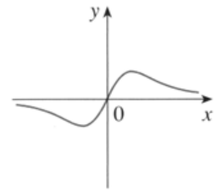


19. Find $\lim\limits_{x\to\infty}\tan^{-1}\left(\frac{4x^2+3x}{6x^3+x}\right)$
(a) 0 **(b)** $\frac{\pi}{2}$ **(c)** ∞ **(d)** $\tan^{-1}\left(\frac{2}{3}\right)$ **(e)** $-\frac{\pi}{2}$

20. Match the graph of the function below with the graph of its derivative in I-IV.



I



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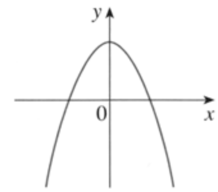
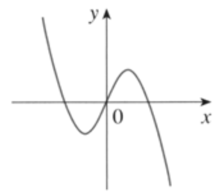
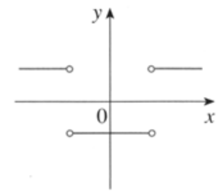


III



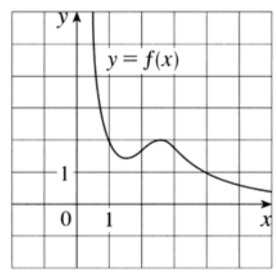
IV



(a) I **(b)** II **(c)** III **(d)** IV
(e) none of them

21. Find an equation of the tangent line to the curve $y = \sec x - 2\cos x$ at the point $(\pi/3, 1)$.
(a) $y = 2\sqrt{3}x + (1 - 2\sqrt{3}\pi)$ **(b)** $y = \frac{\sqrt{3}}{2}x + (1 - \sqrt{3}\pi)$
(c) $y = \frac{1}{2}x + (1 - \pi/2)$ **(d)** $y = 3\sqrt{3}x + (1 - \sqrt{3}\pi)$
(e) $y = \sqrt{3}x + (1 - 1/\sqrt{3}\pi)$

22. Use the given graph to the right to estimate the value of $f'(4)$
(a) 1 **(b)** -1 **(c)** -3 **(d)** 2 **(e)** $\frac{1}{4}$



23. For what values of r does the function $y = e^{rx}$ satisfy the equation $y'' + 6y' - 7y = 0$?
(a) $0, 6, 7$ **(b)** $0, 1, -7$ **(c)** $1, 7$ **(d)** $1, -7$ **(e)** $6, 7$

24. Find all points on the graph of the function $f(x) = 2\sin x + \sin^2 x$ at which the tangent line is horizontal.
(a) $(2n+1)\pi \pm \frac{1}{3}\pi$ **(b)** $\frac{(2n+1)\pi}{2}$ and $\frac{3\pi}{2} + 2n\pi$ **(c)** $\frac{3\pi}{2} + 2n\pi$
(d) $\frac{(2n+1)\pi}{2}$ and $\frac{4\pi}{3} + 2n\pi$ **(e)** $2n\pi \pm \frac{1}{3}\pi$
(n is any integer)

25. Find a *simplified* expression for the derivative of $y = \frac{\sqrt{x^3} - 2\sqrt{x}}{x}$ without using the quotient or product rule
(a) $\frac{3x-2}{2\sqrt{x}}$ **(b)** $\frac{3-x^{1/6}}{3\sqrt{x^3}}$ **(c)** $\frac{3x^{7/6}+8}{6x^{5/3}}$ **(d)** $-\frac{1}{2\sqrt{x}}$ **(e)** $\frac{(x+2)}{2\sqrt{x^3}}$

26. Find y' if $y = 2^{x\cos x}$
(a) $2^{x\cos x}\ln 2 \cdot (\cos x - x\sin x)$ **(b)** $x\cos x\, 2^{x\cos x - 1}\ln 2 \cdot (\cos x - x\sin x)$
(c) $2^{x\cos x}(\cos x - x\sin x)$ **(d)** $2^{x\cos x}(\cos x + x\sin x)$
(e) $x\cos x\, 2^{x\cos x - 1}(\cos x - x\sin x)$

27. Find $\lim_{x \to \pi/2^+} e^{\tan x}$

(a) 0 **(b)** 1 **(c)** ∞ **(d)** d.n.e. **(e)** $e^{\pi/2}$

28. Let $f(x) = 3x^2 - 5x$. Which of the following is equal to $f'(x)$?

(a) $\lim_{h \to 0} \frac{3(x+h)^2 - 5(x+h) - 3x^2 + 5x}{h}$ **(b)** $\lim_{h \to 0} \frac{3(x+h)^2 - 5x - h - 3x^2 + 5x}{h}$

(c) $\lim_{h \to 0} \frac{6(x+h) - 5 - 3x^2 + 5x}{h}$ **(d)** $\lim_{h \to x} \frac{3h^2 - 5h - 3x^2 + 5x}{h}$

(e) $\lim_{h \to 0} \frac{3(x+h)^2 - 5(x+h) - 3x^2 + 5x}{h - x}$

29. Evaluate $\lim_{h \to 0} \frac{\frac{x+h-1}{x+h-2} - \frac{x-1}{x-2}}{h}$

(a) $\frac{1}{x-2}$ **(b)** $-\frac{2}{(x-2)^2}$ **(c)** $-\frac{1}{(x-2)^2}$ **(d)** $\frac{1}{(x-2)^2}$ **(e)** $\frac{2}{(x-2)^2}$

30. Suppose f is differentiable on $\mathbb{R}$. Let $F(x) = f(\tan^{-1}[e^{x^2}])$. Find an expression for $F'(x)$

(a) $f'(\tan^{-1}[e^{x^2}] \cdot \frac{2xe^{x^2}}{1+e^{2x^2}})$ **(b)** $f'(\tan^{-1}[e^{x^2}]) \cdot 2xe^{x^2}$

(c) $f'(\tan^{-1}[e^{x^2}]) \cdot \frac{2xe^{x^2}}{1+e^{x^4}}$ **(d)** $f'(\tan^{-1}[e^{x^2}]) \cdot \frac{2xe^{x^2}}{1+e^{x^2}}$

(e) $f'(\tan^{-1}[e^{x^2}]) \cdot \frac{2xe^{x^2}}{1+e^{2x^2}}$

31. Find $\frac{dy}{dx}$ by implicit differentiation. $\sin^{-1}(\sqrt{xy}) = 1 + x^2y$

(a) $\frac{2xy + \frac{y}{\sqrt{1-xy}} \cdot \frac{1}{2}(xy)^{-1/2}}{\frac{x}{\sqrt{1-xy}} \cdot \frac{1}{2}(xy)^{-1/2} + x^2}$ **(b)** $\frac{2xy}{\frac{x}{\sqrt{1-xy}} \cdot \frac{1}{2}(xy)^{-1/2} - x^2}$ **(c)** $\frac{2xy + \frac{y}{1+xy} \cdot \frac{1}{2}(xy)^{-1/2}}{\frac{x}{1+xy} \cdot \frac{1}{2}(xy)^{-1/2} + x^2}$

(d) $\frac{2xy - \frac{y}{\sqrt{1-xy}} \cdot \frac{1}{2}(xy)^{-1/2}}{\frac{x}{\sqrt{1-xy}} \cdot \frac{1}{2}(xy)^{-1/2} - x^2}$ **(e)** $\frac{2xy - \frac{y}{1+xy} \cdot \frac{1}{2}(xy)^{-1/2}}{\frac{x}{1+xy} \cdot \frac{1}{2}(xy)^{-1/2} - x^2}$

32. Let $f(x) = \frac{6}{\pi}x + \sin(x - \frac{\pi}{2})$. Find $(f^{-1})'(3)$.

(a) $\frac{\pi}{6+\pi}$ **(b)** $\frac{6}{\pi} + 1$ **(c)** $\frac{\pi}{2}$ **(d)** $\frac{\pi}{6}$ **(e)** $\frac{6}{\pi}$

33. If $f(x) + x^2[f(x)]^3 = 10$ and $f(1) = 2$, find $f'(1)$.

(a) $-\frac{6}{13}$ **(b)** $\frac{5}{13}$ **(c)** $-\frac{14}{13}$ **(d)** $-\frac{15}{13}$ **(e)** $-\frac{16}{13}$

34. Find the numerical value of $\cosh(\ln 4)$

(a) $\frac{e^4 - e^{-4}}{2}$ **(b)** 4 **(c)** $\frac{15}{8}$ **(d)** $\frac{1}{4}\sinh(\ln 4)$ **(e)** $\frac{17}{8}$

35. Evaluate $\lim_{x \to \infty} \tanh x$

(a) 1 **(b)** 0 **(c)** ∞ **(d)** $-\infty$ **(e)** -1

36. Evaluate $\lim_{x \to -\infty} \tanh x$

(a) 1 **(b)** 0 **(c)** ∞ **(d)** $-\infty$ **(e)** -1

37. If $f(x) = \ln(\ln x)$, find $f'(e^2)$.

(a) $\frac{1}{2e^2}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{2}{e}$ **(d)** $\frac{1}{\ln 2}$ **(e)** $\frac{4}{e^2}$

38. Find y' if $y = (\cos x)^{\ln x}$

(a) $\ln x(\cos x)^{(\ln x)-1}(-\sin x)$ **(b)** $(\cos x)^{\ln x}\ln(\cos x) \cdot \frac{1}{x}$

(c) $(\cos x)^{\ln x}\left(\frac{1}{x}\ln(\cos x) - \tan x \ln x\right)$ **(d)** $(\cos x)^{\ln x}\left(\frac{1}{x}\cos x - \sin x \ln x\right)$

(e) $\left(\frac{1}{x}\ln(\cos x) - \tan x \ln x\right)^{-1}$

39. Which of the following is equal to $\frac{1+\tanh x}{1-\tanh x}$?

(a) $\frac{e^x-1}{e^x+1}$ **(b)** $\frac{1+e^x+e^{-x}}{1-e^x-e^{-x}}$ **(c)** $\frac{1+e^{2x}}{1-e^{2x}}$ **(d)** e^x **(e)** e^{2x}

40. Let $f(x) = 2 - x^2$. Starting with $x_1 = 2$, compute x_3 using Newton's method.

(a) $\frac{1}{2}$ **(b)** $\frac{47}{20}$ **(c)** $\frac{17}{12}$ **(d)** $\frac{3}{2}$ **(e)** $\frac{5}{2}$

41. Which of the following is equal to $\csc y - \sin y$?

(a) $\tan y \sin y$ **(b)** $\cot y \sin y$ **(c)** $\cot y \cos y$ **(d)** $\tan y \cos y$ **(e)** $\cos y \sin y$

42. A function is given by the table of values below. Find $f^{-1}(2)$.

x	1	2	3	4	5	6
$f(x)$	3.6	2.4	3.7	1.5	2.8	2.0

(a) 2.4 **(b)** 2 **(c)** 4 **(d)** 1.5 **(e)** 6

43. Consider the following function,

$$f(x) = \begin{cases} 5 - 3x & x \le 2 \\ 3 - (x-2)^2 & x > 2 \end{cases}$$

Which of the below statements is (are) true?

(i) f is right continuous at 2

(ii) f is one-to-one

(a) (i) only **(b)** (ii) only **(c)** (i) and (ii) **(d)** neither of them

44. Evaluate $\lim\limits_{x\to 16} \frac{4-\sqrt{x}}{x-16}$

(a) $-\frac{1}{8}$ **(b)** $\frac{1}{8}$ **(c)** $-\infty$ **(d)** ∞ **(e)** d.n.e.

45. Find $\cot\left(\frac{7\pi}{2}\right)$ (angle given in radians)

(a) π **(b)** $\pi/2$ **(c)** $\pi/3$ **(d)** 0 **(e)** $\pi/6$

46. Find the exact value of the expression $2\log_5 \frac{1}{5} + \ln e + 4\ln 2 - \ln 2 - \ln 8$.

(a) -1 **(b)** 1 **(c)** e **(d)** 2 **(e)** -2

47. Find all values of x in the interval $[0, 2\pi]$ that satisfy $\cos x + \sin 2x = 0$.

(a) $\frac{2\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **(b)** $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{2\pi}{3}, \frac{5\pi}{3}$ **(c)** $\frac{2\pi}{3}, \frac{5\pi}{3}$ **(d)** $\frac{7\pi}{6}, \frac{11\pi}{6}$ **(e)** $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

48. Find a formula for the inverse of $f(x) = 2^{\ln x}$

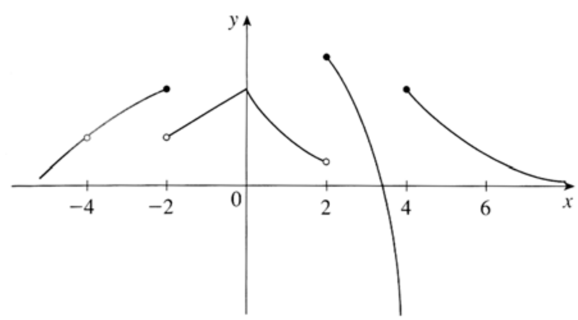
(a) $e^{\frac{\ln x}{\ln 2}}$ **(b)** $e^{\ln(x-2)}$ **(c)** $\frac{x}{2}$ **(d)** $2^{\ln\left(\frac{1}{x}\right)}$ **(e)** $\ln x^2$

49. Solve for x. $\ln x + \ln\left(\dfrac{1}{x} + 1\right) = 3$

(a) $1 - e^3$ **(b)** 1 **(c)** e^3 **(d)** $e^3 - 1$ **(e)** $\frac{e^3 - 1 \pm \sqrt{(1-e^3)^2 - 4}}{2}$

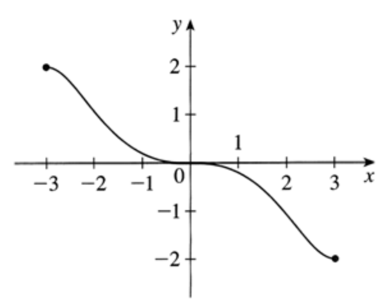
50. From the graph of f, given to the right,

(a) f is continuous from the left at $x = -2$
(b) f is continuous from the right at $x = -2$
(c) f is one-to-one
(d) $f(-2)$ does not exist
(e) none of the above



51. The graph of f is given to the right. Find the value of $f^{-1}(-2)$

(a) -1 **(b)** 2 **(c)** 3 **(d)** 1 **(e)** 0



52. Evaluate $\displaystyle\lim_{h \to 0} \frac{(2+h)^4 - 16}{h}$

(a) 16 **(b)** 32 **(c)** 4 **(d)** 64 **(e)** d.n.e.

53. Evaluate $\displaystyle\lim_{t \to 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right)$

(a) 0 **(b)** $\frac{1}{4}$ **(c)** $\frac{1}{2}$ **(d)** $-\frac{1}{2}$ **(e)** $-\frac{1}{4}$

54. The following limit represents the derivative of some function f at some number a. State such an f and a. $\lim_{t \to 1} \frac{\ln t + e^t - e}{t-1}$

(a) $f(t) = \ln t + e^t$, $a = 1$ **(b)** $f(t) = \ln(t+1) + e^t - e$, $a = 1$
(c) $f(t) = \ln(t-1) + e^t$, $a = 1$ **(d)** $f(t) = \ln(t+1) + e^{t+1}$, $a = 0$
(e) all of the above

55. A table of values for f, g, f', and g' is given.

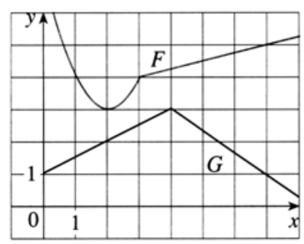
x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	7	8	5
2	8	1	6	9
3	2	2	12	4

If $G(x) = g(g(f(x)))$, find $G'(3)$.
(a) 380 **(b)** 760 **(c)** 540 **(d)** 440 **(e)** 575

56. If the tangent line to $y = f(x)$ at $(2, 0)$ passes through the point $(3, 4)$, find $f(2)$ and $f'(2)$.
(a) $2, -4$ **(b)** $2, 4$ **(c)** $0, -4$ **(d)** $0, 4$ **(e)** $4, -4$

57. Let $r(x) = f(g(h(x)))$, where $h(1) = 2$, $g(2) = 3$, $h'(1) = 4$, $g'(2) = 5$, and $f'(3) = 6$. Find $r'(1)$.
(a) 720 **(b)** 360 **(c)** 24 **(d)** 30 **(e)** 120

58. Let $Q(x) = F(x)/[G(x)]^2$, where F and G are the functions whose graphs are shown. Find $Q'(2)$, if it exists
(a) $-\frac{5}{8}$ **(b)** $-\frac{3}{16}$ **(c)** $-\frac{3}{8}$ **(d)** $\frac{5}{8}$ **(e)** 0



59. Find constants A and B so that the function $y = A\sin x + B\cos x$ satisfies the equation $y'' + 2y' - y = \cos x$
(a) $1, -1$ **(b)** $1, 1$ **(c)** $\frac{1}{4}, -\frac{1}{4}$ **(d)** $4, -4$ **(e)** $0, -1$

60. Suppose f and g are differentiable on $\mathbb{R}$. Let $F(x) = f(e^{g(x)})$. Find $F'(x)$.
(a) $f'(e^{g(x)})e^{g'(x)}g(x)$ **(b)** $f'(e^{g(x)})e^{g(x)}g'(x)$ **(c)** $f(e^{g(x)})e^{g(x)}g'(x)$
(d) $f(g(x))e^{g(x)}g'(x)$ **(e)** $f'(e^{g(x)})e^{g'(x)}$

61. Find $g'(x)$ if $g(x) = \sqrt{2}\cos(3x) + \sqrt{3\tan^3 x}$

(a) $-\sqrt{2}\sin(3x) + \frac{1}{2}(3\tan^3 x)^{-1/2} \cdot 9\tan^2 x$

(b) $-\sqrt{2}\sin(3x) \cdot 3 + \frac{1}{2}(3\tan^3 x)^{-1/2} \cdot 9\tan^2 x \cdot \sec x$

(c) $-\sqrt{2}\sin(3x) \cdot 3 + \frac{1}{2}(3\tan^3 x)^{-1/2} \cdot 9\tan^2 x \cdot 2\tan x \sec^2 x$

(d) $-\sqrt{2}\sin(3x) \cdot 3 + \frac{1}{2}(3\tan^3 x)^{-1/2} \cdot 9\tan^2 x \cdot \sec^2 x$

(e) $-\sqrt{2}\sin(3x) + \frac{1}{2}(3\tan^3 x)^{-1/2} \cdot 9\tan^2 x \cdot \sec^2 x$

62. Find a *simplified* expression for the derivative of $f(x) = x^2 e^{-1/x^2}$

(a) $\frac{4}{x^2}e^{-1/x^2}$ **(b)** $e^{-1/x^2}(2x - e^{-1})$ **(c)** $\frac{2}{x}e^{-1/x^2}(x^2+1)$

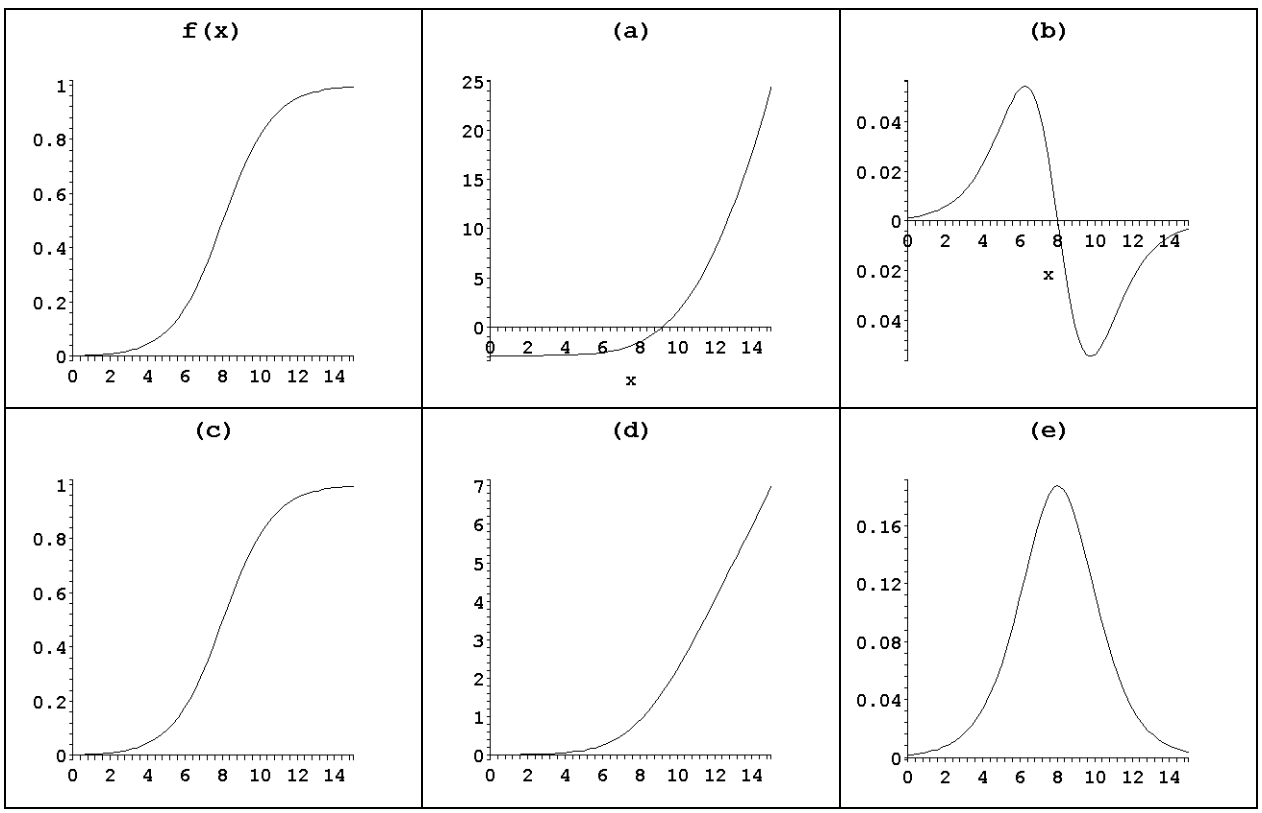
(d) $xe^{-1/x^2}(2+x)$ **(e)** $e^{-1/x^2}(2x-1)$

63. Find an equation of the tangent line to the curve $y = \cos(\cos x)$ at the point $(\pi/2, 1)$.

(a) $y = 1 - \pi$ **(b)** $y = 1$ **(c)** $y = x + (1 - \frac{\pi}{2})$ **(d)** $y = -x + (\frac{\pi}{2} + 1)$

(e) $y = x + 1$

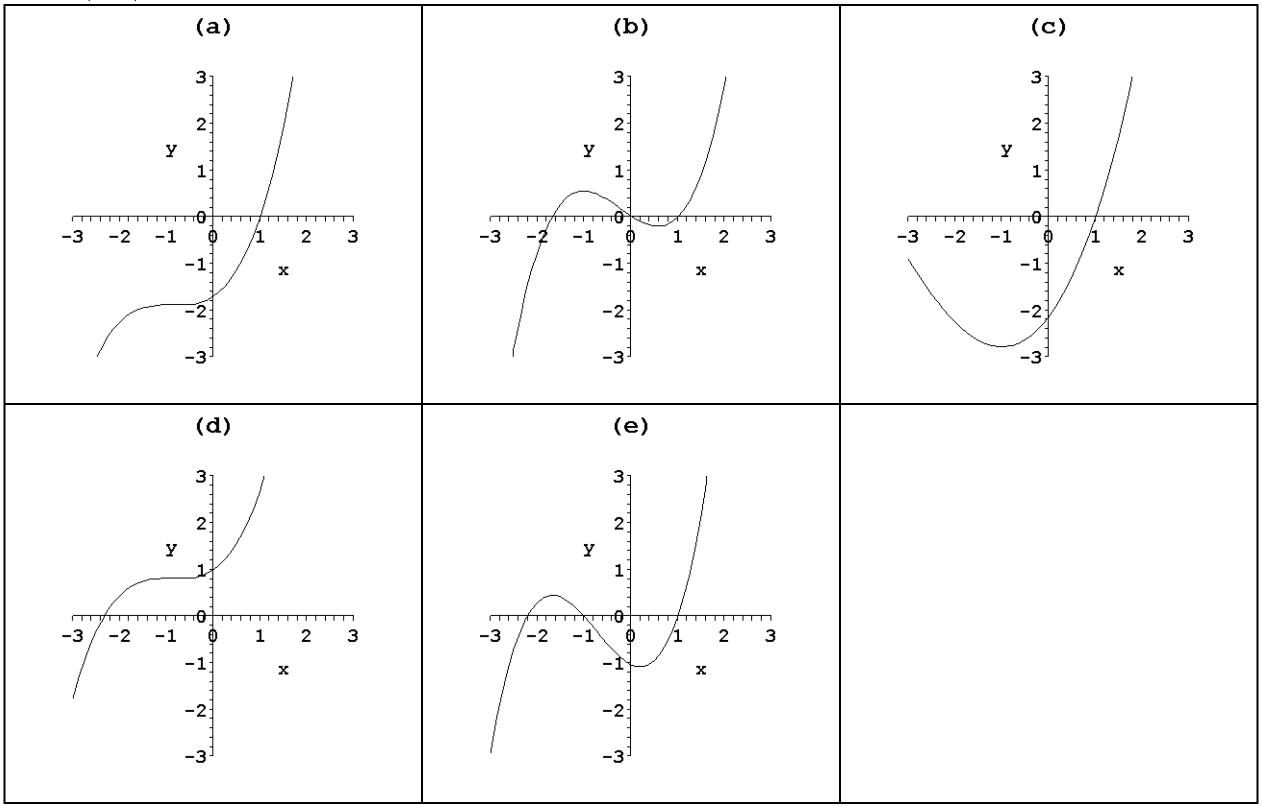
64. The graph of $f(x)$ is given. Sketch the graph of $f'(x)$.



65. Find the 999th derivative of $f(x) = \sin 2x$.

(a) $2^{1000}\cos 2x$ **(b)** $-2^{999}\sin 2x$ **(c)** $2^{999}\cos 2x$ **(d)** $-2^{999}\cos 2x$ **(e)** $-2^{1000}\cos 2x$

66. Sketch the graph of a function f for which $f(1) = 0,\ f'(1) = 3,\ f'(-1) = 0$, and $f'(-2) = 1$.



67. Let $f(x) = \frac{x-1}{x-2}$. Which of the following is equal to $f'(a)$?

(a) $\lim\limits_{h \to 0} \frac{\frac{x+h-1}{x+h-2} - \frac{a-1}{a-2}}{h}$ **(b)** $\lim\limits_{x \to a} \frac{\frac{x-1}{x-2} - \frac{a-1}{a-2}}{x-a}$ **(c)** $\lim\limits_{x \to a} \frac{\frac{x+h-1}{x+h-2} - \frac{a-1}{a-2}}{x-a}$

(d) $\lim\limits_{h \to 0} \frac{\frac{x+h-1}{x+h-2} - \frac{x-1}{x-2}}{x-a}$ **(e)** $\lim\limits_{x \to a} \frac{\frac{a+h-1}{a+h-2} - \frac{a-1}{a-2}}{x-a}$

68. Evaluate $\lim\limits_{t \to a} \frac{\frac{2t+1}{t+3} - \frac{2a+1}{a+3}}{t-a}$.

(a) $\frac{4}{a+3}$ **(b)** $\frac{4}{(a+3)^2}$ **(c)** $\frac{5}{a+3}$ **(d)** $\frac{5}{(a+3)^2}$ **(e)** $\frac{5}{(a+3)^3}$

69. Find a simplified expression for y' if $y = \dfrac{r}{\sqrt{r^2+1}}$

(a) $\frac{1}{(r^2+1)^{1/2}}$ **(b)** $\frac{1}{(r^2+1)^{3/2}}$ **(c)** $\frac{r}{(r^2+1)^{3/2}}$ **(d)** $\frac{1}{r^2+1}$ **(e)** $\frac{r}{r^2+1}$

70. If $f(\frac{1}{3}) = 2,\ f(4) = \frac{1}{3},\ f'(\frac{1}{3}) = 6,$ and $f'(4) = 8$, find $(f^{-1})'(\frac{1}{3})$.

(a) $\frac{1}{8}$ **(b)** $\frac{3}{8}$ **(c)** $\frac{1}{6}$ **(d)** $\frac{8}{3}$ **(e)** $\frac{1}{2}$

71. Find $f'(t)$ if $f(t) = \tan^{-1}\left(\frac{1}{\sqrt[4]{t^3}}\right)$

(a) $\frac{1}{1+t^{-3/4}}\left(-\frac{3}{4}t^{-7/4}\right)$ **(b)** $\frac{1}{1+t^{-3/2}}\left(-\frac{4}{3}t^{-7/3}\right)$ **(c)** $\frac{1}{1+t^{-3/4}}$

(d) $\frac{1}{1+t^{-3/2}}$ **(e)** $\frac{1}{1+t^{-3/2}}\left(-\frac{3}{4}t^{-7/4}\right)$

72. Find $\frac{dy}{dx}$ by implicit differentiation. $e^{x-y^2} = \frac{y}{1+e^{x^2}}$

(a) $\frac{e^{x-y^2} - \frac{2xye^{x^2}}{(1+e^{x^2})^2}}{\frac{1}{1+e^{x^2}} + 2ye^{x-y^2}}$ **(b)** $\frac{e^{x-y^2} + \frac{2xye^{x^2}}{(1+e^{x^2})^2}}{\frac{1}{1+e^{x^2}} - 2ye^{x-y^2}}$ **(c)** $\frac{e^{x-y^2}}{\frac{1}{1+e^{x^2}} + 2ye^{x-y^2}}$

(d) $\frac{e^{x-y^2} + \frac{2xye^{x^2}}{(1+e^{x^2})^2}}{\frac{1}{1+e^{x^2}} + 2ye^{x-y^2}}$ **(e)** $\frac{\frac{2xye^{x^2}}{(1+e^{x^2})^2}}{\frac{1}{1+e^{x^2}} + 2ye^{x-y^2}}$

73. Find the numerical value of $\sinh(\ln 5)$.

(a) $\frac{1}{5}\cosh(\ln 5)$ **(b)** $\frac{13}{5}$ **(c)** $\frac{12}{5}$ **(d)** $\frac{1}{5}$ **(e)** 5

74. Evaluate $\lim_{x\to\infty} \sinh x$

(a) 1 **(b)** 0 **(c)** ∞ **(d)** $-\infty$ **(e)** d.n.e

75. Let $f(x) = x^{x^2}$. Find the value of $f'(2)$.

(a) $2^3[\ln 2 + 1]$ **(b)** $2[\ln 2 + 1]$ **(c)** $\ln 2 + 1$ **(d)** $2^5[2\ln 2 + 1]$ **(e)** $2^4[2\ln 2 + 1]$

76. Differentiate $f(x) = \ln\left(\frac{1}{2-\sqrt{\ln x}}\right)$.

(a) $\frac{1}{2x\sqrt{\ln x}(2-\sqrt{\ln x})}$ **(b)** $2 - \sqrt{\ln x}$ **(c)** $\frac{2-\sqrt{\ln x}}{2x\sqrt{\ln x}}$ **(d)** $\frac{1}{2x\sqrt{\ln x}(2-\sqrt{\ln x})^3}$

(e) $2x\sqrt{\ln x}(2 - \sqrt{\ln x})$

77. Find the domain of $f(x) = \ln\left(\frac{1}{2-\sqrt{\ln x}}\right)$.

(a) $\{x \,|\, 0 < x < e^4\}$ **(b)** $\{x \,|\, x \geq 1\}$ **(c)** $\{x \,|\, x > 0\}$

(d) $\{x \,|\, 1 \leq x < e^{-4}\}$ **(e)** $\{x \,|\, 1 \leq x < e^4\}$

78. Find $f'(t)$ if $f(t) = e^{\sqrt{1+\ln(\sinh t)}}$

(a) $e^{\sqrt{1+\ln(\sinh t)}} \frac{1}{2}(1 + \ln(\sinh t))^{-1/2}\coth t$

(b) $e^{\sqrt{1+\ln(\sinh t)}} \frac{1}{2}(1 + \ln(\sinh t))^{-1/2} \frac{\ln(\cosh t)}{\sinh t}$

(c) $e^{\sqrt{1+\ln(\sinh t)}} \frac{1}{2}(1 + \ln(\sinh t))^{-1/2}\operatorname{csch} t$

(d) $e^{\frac{1}{2}(1 + \ln(\sinh t))^{-1/2}} \coth t$

(e) $e^{\frac{1}{2}(1 + \ln(\sinh t))^{-1/2}} \frac{\ln(\cosh t)}{\sinh t}$

79. Find an equation of the tangent line to $f(x) = \ln(\ln x)$ at the point $(e, 0)$.

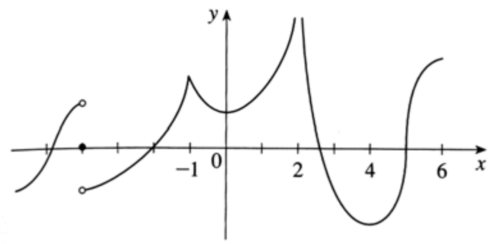
(a) $y = \frac{1}{e}x - 1$ **(b)** $y = x - e$ **(c)** $y = \frac{1}{e}x + e$ **(d)** $y = 0$ **(e)** $y = \frac{1}{e}x$

80. Which of the following is equal to $\cosh x + \sinh x$?

(a) $\frac{e^{2x} - e^{-2x}}{2}$ **(b)** 1 **(c)** e^x **(d)** e^{-x} **(e)** $\sqrt{2}$

81. Suppose the line $y = 8x - 1$ is tangent to the curve $y = f(x)$ when $x = 5$. If Newton's method is used to locate a root of the equation $f(x) = 0$ and the initial approximation is $x_1 = 5$, find the second approximation x_2.
(a) -1 **(b)** 8 **(c)** $\frac{1}{8}$ **(d)** 1 **(e)** 0

82. Consider the function $f(x)$ given in the graph below.



Which of the following statements is true for the function f on the interval $[-5, 0]$ (ignore the rest of the graph of f)?
(a) On the interval $[-5, 0]$, f has an absolute minimum and absolute maximum.
(b) On the interval $[-5, 0]$, f has no absolute minimum but has an absolute maximum.
(c) On the interval $[-5, 0]$, f has an absolute minimum but no absolute maximum.
(d) On the interval $[-5, 0]$, f has no absolute minimum and no absolute maximum.

83. Find all of the critical numbers of the function $f(x) = x^{3/5}(4 - x)$
(a) $0, 4$ **(b)** $0, 1$ **(c)** $0, \frac{3}{2}$ **(d)** 0 **(e)** $\frac{3}{2}$

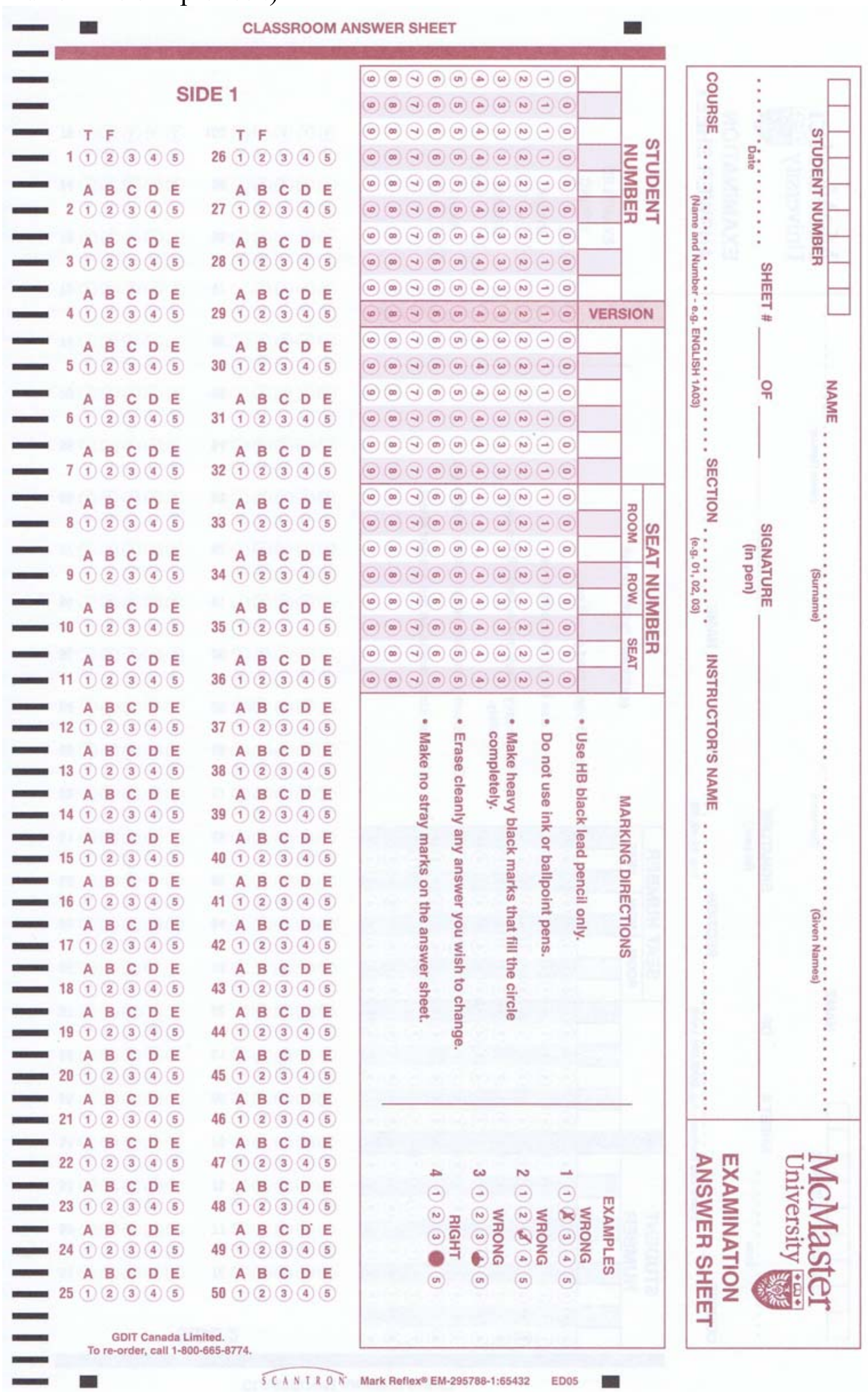
84. Find the absolute maximum and absolute minimum of

$$f(x) = x^{2/3}(5 + 2x)$$

on the interval $[-2, 1]$.

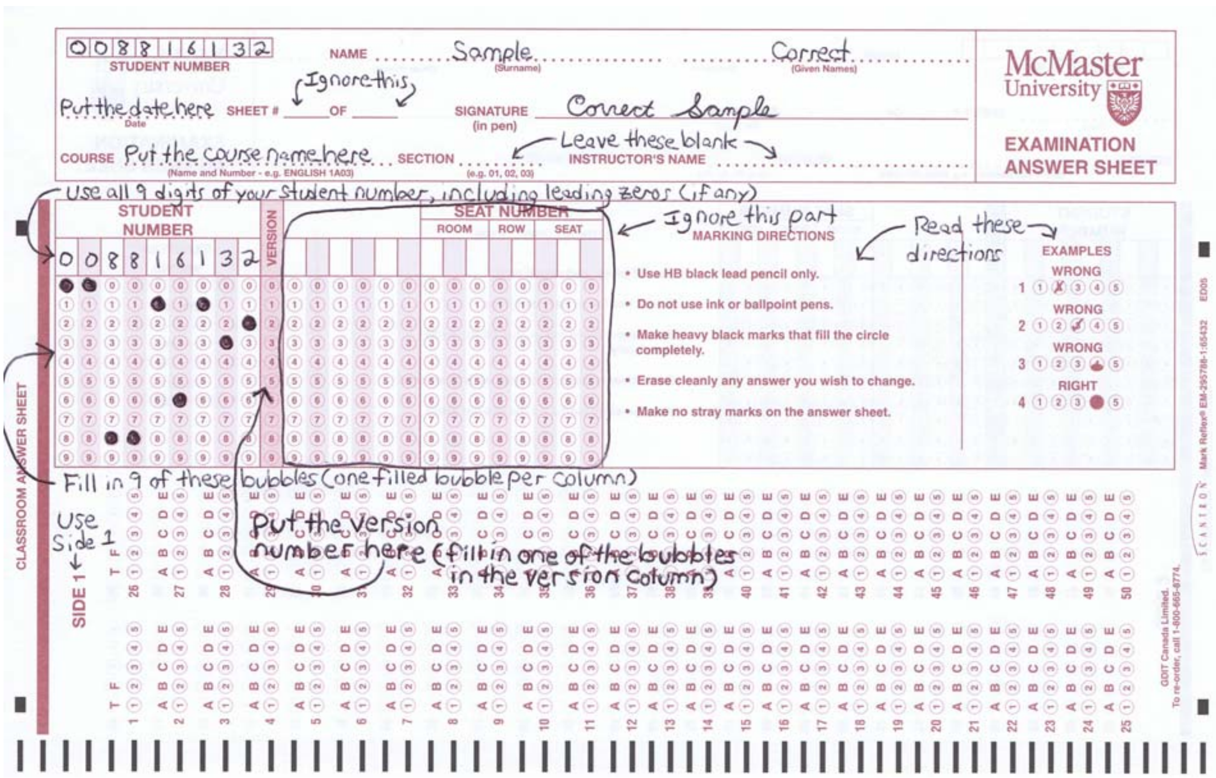
(a) $7, 0$ **(b)** $7, 4^{1/3}$ **(c)** $7, 3$ **(d)** $3, 0$ **(e)** $7, -3$

85. Correctly fill out the bubbles corresponding to all 9 digits of your student number, as well as the version number of your test in the correct places on the computer card. (Use the below computer card for this sample test.)



Answers

1. d **2.** e **3.** d **4.** c **5.** a **6.** a **7.** e **8.** d **9.** c **10.** a
11. a **12.** d **13.** b **14.** e **15.** e **16.** d **17.** a **18.** e **19.** a **20.** e
21. d **22.** b **23.** d **24.** b **25.** e **26.** a **27.** a **28.** a **29.** c **30.** e
31. d **32.** a **33.** e **34.** e **35.** a **36.** e **37.** a **38.** c **39.** e **40.** c
41. c **42.** e **43.** d **44.** a **45.** d **46.** a **47.** e **48.** a **49.** d **50.** a
51. c **52.** b **53.** d **54.** a **55.** c **56.** d **57.** e **58.** c **59.** c **60.** b
61. d **62.** c **63.** b **64.** e **65.** d **66.** a **67.** b **68.** d **69.** b **70.** a
71. e **72.** d **73.** c **74.** c **75.** d **76.** a **77.** e **78.** a **79.** a **80.** c
81. c **82.** b **83.** c s**84.** a
85.


0 0 8 8 1 6 1 3 2
STUDENT NUMBER
NAME ....Sample.............. ....Correct.............
(Surname) (Given Names)
McMaster University
EXAMINATION ANSWER SHEET
Ignore this
Put the date here SHEET # _____ OF _____
Date
SIGNATURE (in pen) Correct Sample
Leave these blank
COURSE Put the course name here... SECTION .... INSTRUCTOR'S NAME ....
(Name and Number - e.g. ENGLISH 1A03) (e.g. 01, 02, 03)
Use all 9 digits of your student number, including leading zeros (if any)
STUDENT NUMBER
VERSION
SEAT NUMBER
ROOM ROW SEAT
Ignore this part
MARKING DIRECTIONS
Read these directions
• Use HB black lead pencil only.
• Do not use ink or ballpoint pens.
• Make heavy black marks that fill the circle completely.
• Erase cleanly any answer you wish to change.
• Make no stray marks on the answer sheet.
EXAMPLES
WRONG
WRONG
WRONG
RIGHT
Fill in 9 of these bubbles (one filled bubble per column)
Put the version number here (fill in one of the bubbles in the version column)
Use Side 1
CLASSROOM ANSWER SHEET
SIDE 1

NOTE: On the sample tests, a version number is not given. On the actual tests, it will say "Version X" at the top, where X is the version number that you will have to fill in on the computer card. The sample above assumes that your student number is 008816132. On the actual test, you will have to fill in the bubbles corresponding to YOUR student number (not 008816132).