Math 1B03 Term 2/1ZC3

1st Sample Test #1

Name:____

(Last Name) (First Name)

Student Number:______________________ **Tutorial Number:**_____________________

This test consists of 20 multiple choice questions worth 1 mark each (no part marks), and 1 question worth 1 mark (no part marks) on proper computer card filling. All questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Calculators are NOT allowed.

1. Which of the following matrices are in reduced row echelon form?

$$\text{(i)} \begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{(ii)} \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 4 \end{bmatrix} \text{(iii)} \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 2 & 4 \end{bmatrix}$$

$$\text{(iv)} \begin{bmatrix} 1 & -7 & 5 & 5 \\ 0 & 1 & 3 & 2 \end{bmatrix}$$

(a) (i), (iii), and (iv) only
(b) (iii) only
(c) (i), (ii), and (iii) only
(d) (i) and (iii) only
(e) none of them

2. Let $A = \begin{bmatrix} 2 & 1 & -1 & 0 \\ 1 & 3 & 5 & 1 \\ 3 & -1 & -7 & 2 \end{bmatrix}$. Find the reduced row echelon form of A.

$$\textbf{(a)} \begin{bmatrix} 1 & 0 & -\frac{8}{5} & 0 \\ 0 & 1 & \frac{9}{5} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \textbf{(b)} \begin{bmatrix} 1 & 0 & \frac{8}{5} & 0 \\ 0 & 1 & \frac{11}{5} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \textbf{(c)} \begin{bmatrix} 1 & 0 & -\frac{8}{5} & 0 \\ 0 & 1 & \frac{11}{5} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\textbf{(d)} \begin{bmatrix} 1 & 0 & -\frac{8}{5} & 0 \\ 0 & 1 & -\frac{11}{5} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \textbf{(e)} \begin{bmatrix} 1 & 0 & -\frac{7}{5} & 0 \\ 0 & 1 & \frac{11}{5} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the given reduced row echelon form. Solve the system.

$$\begin{bmatrix} 1 & -6 & 0 & 0 & 3 & -2 \\ 0 & 0 & 1 & 0 & 4 & 7 \\ 0 & 0 & 0 & 1 & 5 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) $x_1 = -2s - 3t + 6u$
$x_2 = u$
$x_3 = 7s - 4t$
$x_4 = 8s - 5t$
$x_5 = t$

(b) $x_1 = -2 - 3s + 6t$
$x_2 = t$
$x_3 = 7 - 4t$
$x_4 = 8 - 5t$
$x_5 = s$

(c) $x_1 = 2 - 3t + 6s$
$x_2 = s$
$x_3 = -7 - 4t$
$x_4 = -8 - 5t$
$x_5 = t$

(d) $x_1 = -2 - 3t + 6$
$x_2 = 0$
$x_3 = 7 - 4t$
$x_4 = 8 - 5t$
$x_5 = t$

(e) $x_1 = -2 - 3t + 6s$
$x_2 = s$
$x_3 = 7 - 4t$
$x_4 = 8 - 5t$
$x_5 = t$

4. Solve the following system of equations

$$\begin{aligned} 2x_1 - x_2 + x_3 + x_4 - 2x_5 &= 1 \\ 3x_1 - 3x_2 + 2x_3 + 3x_5 &= 0 \\ 3x_2 - x_3 + 3x_4 - 12x_5 &= -1 \end{aligned}$$

(a) no solution

(b) $x_1 = 3 - 2s - 4t$
$x_2 = -1 + 2t$
$x_3 = -5 + 3s - t$
$x_4 = s$
$x_5 = t$

(c) $x_1 = 3 - 2s + 8t$
$x_2 = s$
$x_3 = -6 + 3s - 15t$
$x_4 = 1 + 2s - 12t$
$x_5 = t$

(d) $x_1 = 3 - 2s + 8t$
$x_2 = -1 - t$
$x_3 = -6 + 3s - 15t$
$x_4 = s$
$x_5 = t$

(e) $x_1 = -1 + s + 4t$
$x_2 = -1 - 3s + t$
$x_3 = s$
$x_4 = 2s + 6t$
$x_5 = t$

5. If ABC^T can be formed, A is 3×2, and C is 4×5, what size is B?
(a) 2×2 **(b)** 2×5 **(c)** 3×4 **(d)** 2×4 **(e)** 3×5

6. Find conditions on a and b such that the following system has exactly one solution

$$\begin{aligned} x + by &= -1 \\ 2ax + 2y &= 5 \end{aligned}$$

(a) $ab = 1$ and $a \neq -\frac{5}{2}$
(b) $ab \neq 1$
(c) $a = -\frac{5}{2},\ b = -\frac{2}{5}$
(d) $a = 3b,\ b \neq -\frac{2}{5}$
(e) $ab = 2,\ a \neq -\frac{5}{2}$

7. Consider the following system.

$$\begin{aligned} 2x - y + 2z &= 5 \\ x - y + 3z &= 1 \\ x + 2y + 4z &= 6 \end{aligned}$$

Given that the inverse of $\begin{bmatrix} 2 & -1 & 2 \\ 1 & -1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$ is equal to $\begin{bmatrix} \frac{10}{13} & -\frac{8}{13} & \frac{1}{13} \\ \frac{1}{13} & -\frac{6}{13} & \frac{4}{13} \\ -\frac{3}{13} & \frac{5}{13} & \frac{1}{13} \end{bmatrix}$, which of the following gives a solution to the above system?

(a) $\begin{bmatrix} \frac{10}{13} & \frac{1}{13} & -\frac{3}{13} \\ -\frac{8}{13} & -\frac{6}{13} & \frac{5}{13} \\ \frac{1}{13} & \frac{4}{13} & \frac{1}{13} \end{bmatrix} \begin{bmatrix} 5 \\ 1 \\ 6 \end{bmatrix}$ **(b)** $\begin{bmatrix} \frac{10}{13} & -\frac{8}{13} & \frac{1}{13} \\ \frac{1}{13} & -\frac{6}{13} & \frac{4}{13} \\ -\frac{3}{13} & \frac{5}{13} & \frac{1}{13} \end{bmatrix} \begin{bmatrix} 5 \\ 1 \\ 6 \end{bmatrix}$

(c) $\begin{bmatrix} \frac{10}{13} & \frac{1}{13} & -\frac{3}{13} \\ -\frac{8}{13} & -\frac{6}{13} & \frac{5}{13} \\ \frac{1}{13} & \frac{4}{13} & \frac{1}{13} \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ 1 \\ 6 \end{bmatrix}$ **(d)** $\begin{bmatrix} \frac{10}{13} & -\frac{8}{13} & \frac{1}{13} \\ \frac{1}{13} & -\frac{6}{13} & \frac{4}{13} \\ -\frac{3}{13} & \frac{5}{13} & \frac{1}{13} \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ 1 \\ 6 \end{bmatrix}$

(e) none of the above

8. Find the matrix A if

$$(A^T - 2I)^{-1} = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$$

(a) $A = \begin{bmatrix} 0 & -2 \\ -1 & 1 \end{bmatrix}$ **(b)** $A = \begin{bmatrix} \frac{5}{2} & -\frac{1}{2} \\ -\frac{1}{4} & \frac{11}{4} \end{bmatrix}$ **(c)** $A = \frac{1}{4}\begin{bmatrix} 1 & -2 \\ -1 & 5 \end{bmatrix}$

(d) $A = \begin{bmatrix} 1 & -2 \\ -1 & 5 \end{bmatrix}$ **(e)** $A = \begin{bmatrix} -\frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{4} & -\frac{5}{4} \end{bmatrix}$

9. Find an elementary matrix E such that $B = EA$.

$$A = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}, \; B = \begin{bmatrix} -1 & -2 \\ 2 & 4 \end{bmatrix}$$

(a) $\begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & -2 \\ 2 & -2 \end{bmatrix}$ **(c)** $\begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}$ **(d)** $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ **(e)** $\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

10. Which of the following matrices are *always* symmetric.
(i) $A + A^T$ (ii) AA^T (iii) kA for any scalar k (iv) $A - A^T$

(a) (i), (ii), and (iii) only
(b) (i), (ii), and (iv) only
(c) (ii) and (iv) only
(d) (i) and (ii) only
(e) (i), (ii), (iii), and (iv)

11. Given that $\det \begin{bmatrix} r & s & t \\ u & v & w \\ x & y & z \end{bmatrix} = 4$, compute $\det \begin{bmatrix} r & s & t \\ x - 8r & y - 8s & z - 8t \\ 8u & 8v & 8w \end{bmatrix}$.

(a) 32 **(b)** -32 **(c)** 256 **(d)** -256 **(e)** 0

12. If $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = -3$, calculate $\det \begin{bmatrix} 2 & -2 & 0 \\ c+1 & -1 & 2a \\ d-2 & 2 & 2b \end{bmatrix}$.

(a) 4 **(b)** -12 **(c)** 12 **(d)** -4 **(e)** -3

13. If A is 3×3 and $\det(2A^{-1}) = -3 = \det(A^3(B^{-1})^T)$, find $\det B$.
(a) $\frac{3^2}{8^3}$ **(b)** $\frac{8^3}{3^4}$ **(c)** $\frac{8^3}{3^2}$ **(d)** $\frac{2^3}{3^4}$ **(e)** $\frac{2^3}{3^2}$

14. Compute the determinant of the following matrix,

$$\begin{bmatrix} 3 & 1 & -5 & 2 \\ 1 & 3 & 0 & 1 \\ 1 & 0 & 5 & 2 \\ 1 & 1 & 2 & -1 \end{bmatrix}$$

(a) -31 **(b)** -132 **(c)** -131 **(d)** -130 **(e)** 0

15. Find the adjoint of the following matrix $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$.

(a) $\begin{bmatrix} 1 & -1 & -4 \\ 9 & 1 & 0 \\ 0 & 1 & 4 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & 1 & -4 \\ -9 & 1 & 0 \\ 0 & -1 & 4 \end{bmatrix}$ **(c)** $\begin{bmatrix} 1 & 1 & -2 \\ 3 & 1 & -6 \\ -3 & -1 & 4 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & -1 & -2 \\ -3 & 1 & 6 \\ -3 & 1 & 4 \end{bmatrix}$ **(e)** $\begin{bmatrix} 1 & 3 & 0 \\ -1 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$

16. Find the eigenvalues of $A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & -1 \\ 1 & 3 & -2 \end{bmatrix}$.

(a) $2, 1, -1$ **(b)** $1, -1$ **(c)** $2, 1$ **(d)** $2, -1$ **(e)** $2, 1, 0$

17. Suppose that λ_1 is an eigenvalue of A with eigenvector **x**, and λ_2 is an eigenvalue of B with the same eigenvector **x**. Consider the following statements.
(i) $\lambda_1 + \lambda_2$ is an eigenvalue of the matrix $(A + B)$
(ii) $\lambda_1 \lambda_2$ is an eigenvalue of the matrix BA
(iii) λ_1^3 is an eigenvalue of the matrix A^3

Which of the above statements are always true?
(a) (i), (ii), and (iii)
(b) (i) and (ii) only
(c) (i) and (iii) only
(d) (ii) only
(e) (i) only

18. Suppose that a matrix A (not given) has eigenvalues $\lambda = 1, -2, 3$ with eigenvectors $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, respectively. Find P and D so that $P^{-1}AP = D$.

(a) $P = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

(b) $P = \begin{bmatrix} 0 & 1 & -1 \\ 0 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

(c) $P = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(e) $P = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

19. Suppose $p(\lambda) = (\lambda - 1)^3$ for some diagonalizable 3×3 matrix A (not given). Calculate A^{25}.

(a) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ **(b)** $\begin{bmatrix} -25 & 0 & 0 \\ 0 & -25 & 0 \\ 0 & 0 & -25 \end{bmatrix}$ **(c)** $\begin{bmatrix} 25 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 25 \end{bmatrix}$

(d) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ **(e)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

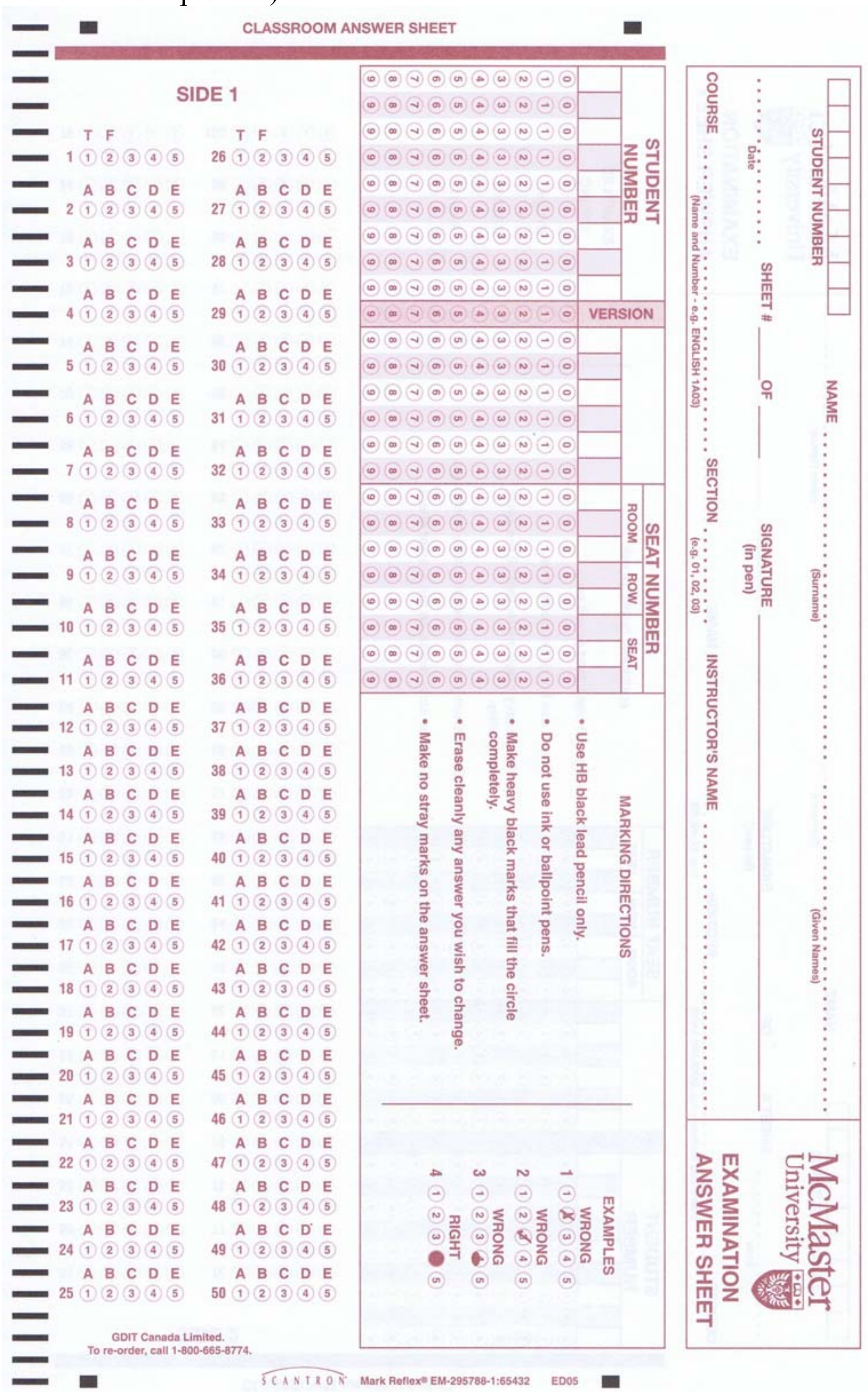
20. In Matlab what command could be used to create the row vector $(3, 5, 7, 9, 11, 13, 15, 17, 19)$?

(a) `>>[3 by 2 to 19]` **(b)** `>>3:2:19` **(c)** `>>[3 to 19 by 2]`

(d) `>>for (i = 3 to 19 by 2) x[i] = i end`

(e) `>>[3;5;7;9;11;13;15;17;19]`

21. Correctly fill out the bubbles corresponding to all 9 digits of your student number, as well as the version number of your test in the correct places on the computer card. (Use the below computer card for this sample test.)



Math 1B03 Term 2/1ZC3

2nd Sample Test #1

Name:____

(Last Name) (First Name)

Student Number:______________________ **Tutorial Number:**_____________________

This test consists of 20 multiple choice questions worth 1 mark each (no part marks), and 1 question worth 1 mark (no part marks) on proper computer card filling. All questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Calculators are NOT allowed.

1. Find matrices A, X and B that express the given system of linear equations as a single matrix equation $AX = B$.

$$\begin{aligned} 4x_1 - 3x_3 + x_4 &= 1 \\ 5x_1 + x_2 - 8x_4 &= 3 \\ 2x_1 - 5x_2 + 9x_3 - x_4 &= 0 \\ 3x_2 - x_3 + 7x_4 &= 2 \end{aligned}$$

(a) $A = \begin{bmatrix} 0 & 4 & -3 & 1 \\ 0 & 5 & 1 & -8 \\ 2 & -5 & 9 & -1 \\ 0 & 3 & -1 & 7 \end{bmatrix}$ $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 2 \end{bmatrix}$

(b) $A = \begin{bmatrix} 4 & 0 & -3 & 1 & 1 \\ 5 & 1 & 0 & -8 & 3 \\ 2 & -5 & 9 & -1 & 0 \\ 0 & 3 & -1 & 7 & 2 \end{bmatrix}$ $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

(c) $A = \begin{bmatrix} 4 & 0 & -3 & 1 & 1 \\ 5 & 1 & 0 & -8 & 3 \\ 2 & -5 & 9 & -1 & 0 \\ 0 & 3 & -1 & 7 & 2 \end{bmatrix}$ $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 2 \end{bmatrix}$

(d) $A = \begin{bmatrix} 4 & 0 & -3 & 1 \\ 5 & 1 & 0 & -8 \\ 2 & -5 & 9 & -1 \\ 0 & 3 & -1 & 7 \end{bmatrix}$ $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 2 \end{bmatrix}$

(e) $A = \begin{bmatrix} 0 & 4 & -3 & 1 & 1 \\ 0 & 5 & 1 & -8 & 3 \\ 2 & -5 & 9 & -1 & 0 \\ 0 & 3 & -1 & 7 & 2 \end{bmatrix}$ $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

2. Consider the matrices

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}, \quad E = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

Compute $C^T A^T + 2E^T$, if possible.

(a) $\begin{bmatrix} 15 & 7 & 10 \\ 10 & 0 & 9 \\ 14 & 10 & 13 \end{bmatrix}$ **(b)** $\begin{bmatrix} 15 & 3 & 12 \\ 14 & 0 & 7 \\ 12 & 12 & 13 \end{bmatrix}$ **(c)** undefined **(d)** $\begin{bmatrix} 15 & 14 & 12 \\ 3 & 0 & 12 \\ 12 & 7 & 13 \end{bmatrix}$
(e) $\begin{bmatrix} 15 & 10 & 14 \\ 7 & 0 & 10 \\ 10 & 9 & 13 \end{bmatrix}$

3. Solve the following system of equations

$$\begin{aligned} 2x_1 - x_2 + x_3 + x_4 - 2x_5 &= 1 \\ 3x_1 - 3x_2 + 2x_3 + 3x_5 &= 0 \\ 2x_1 + x_2 + x_3 + x_4 &= -1 \end{aligned}$$

(a) no solution **(b)** $\begin{aligned} x_1 &= 3 - 2s - 4t \\ x_2 &= -1 + 2t \\ x_3 &= -5 + 3s - t \\ x_4 &= s \\ x_5 &= t \end{aligned}$ **(c)** $\begin{aligned} x_1 &= 3 - 2s + 8t \\ x_2 &= s \\ x_3 &= -6 + 3s - 15t \\ x_4 &= 1 + 2s - 12t \\ x_5 &= t \end{aligned}$

(d) $\begin{aligned} x_1 &= 3 - 2s + 8t \\ x_2 &= -1 - t \\ x_3 &= -6 + 3s - 15t \\ x_4 &= s \\ x_5 &= t \end{aligned}$ **(e)** $\begin{aligned} x_1 &= -1 + s + 4t \\ x_2 &= -1 - 3s + t \\ x_3 &= s \\ x_4 &= 2s + 6t \\ x_5 &= t \end{aligned}$

4. Use determinants to find all of the possible real values of a which make the following matrix *not* invertible.

$$A = \begin{bmatrix} 1 & 1 & a \\ -a & 1 & -a \\ a & -1 & 2 \end{bmatrix}$$

(a) 2 and -1 **(b)** ± 1 **(c)** -1 **(d)** ± 2 **(e)** 0

5. Find conditions on a, b, and c such that the system has infinitely many solutions

$$\begin{aligned} -cx + 3y + 2z &= -8 \\ x + z &= 2 \\ 3x + 3y + az &= b \end{aligned}$$

(a) $a - c - 5 \neq 0$
(b) $a - c = 0$ and $b - 2c + 2 = 5$
(c) $a - c - 5 = 0$ and $b - 2c + 2 = 0$
(d) $a - c = 0$ and $b - 2c + 2 \neq 5$
(e) $a - c - 5 = 0$ and $b - 2c + 2 \neq 0$

6. Find the diagonal entries of the inverse of $\begin{bmatrix} 3 & 1 & 2 \\ 1 & -1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$.

(a) $\begin{bmatrix} \frac{2}{5} & * & * \\ * & -\frac{2}{5} & * \\ * & * & \frac{4}{25} \end{bmatrix}$ **(b)** $\begin{bmatrix} \frac{2}{5} & * & * \\ * & \frac{2}{5} & * \\ * & * & \frac{4}{25} \end{bmatrix}$ **(c)** $\begin{bmatrix} -\frac{2}{5} & * & * \\ * & -\frac{2}{5} & * \\ * & * & \frac{4}{25} \end{bmatrix}$
(b) $\begin{bmatrix} \frac{2}{5} & * & * \\ * & -\frac{2}{5} & * \\ * & * & -\frac{4}{25} \end{bmatrix}$ **(b)** $\begin{bmatrix} -\frac{2}{5} & * & * \\ * & \frac{2}{5} & * \\ * & * & -\frac{4}{25} \end{bmatrix}$

7. Consider the following matrix,

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$$

Note that A can be reduced to I using the following row operations:

(i) $\mathrm{r}_2 \text{ −> } \frac{1}{3}\mathrm{r}_2$
(ii) $\mathrm{r}_1 \text{ −> } \mathrm{r}_1 - 2\mathrm{r}_2$

Using the above two row operations in the above order, find elementary matrices E_1 and E_2 such that $A = E_1^{-1}E_2^{-1}$.

(a) $E_1 = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{bmatrix}, E_2 = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$ **(b)** $E_1 = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}, E_2 = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$
(c) $E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}, E_2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ **(d)** $E_1 = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}, E_2 = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$
(e) $E_1 = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{bmatrix}, E_2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

8. Suppose that A and B are symmetric matrices. Which of the following matrices are *always* symmetric?
(i) A^{-1} (ii) AB (iii) $AB - BA$

(a) (i) only **(b)** (i) and (ii) only **(c)** (i) and (iii) only **(d)** (ii) and (iii) only
(e) none of them

9. If $A^3 = 0$, which of the following is equal to $(I - A)^{-1}$?

(a) $I + A$ **(b)** $I + A + A^2$ **(c)** $I - A$ **(d)** $I - A - A^2$ **(e)** $I - A + A^2$

10. A matrix A is **skew-symmetric** if $A^T = -A$. Suppose that A and B are both skew-symmetric. Which of the following matrices are *always* skew-symmetric?
(i) $A + B$ (ii) AB (iii) kA

(a) (i) only
(b) (i) and (iii) only
(c) (iii) only
(d) (i), (ii), and (iii)
(e) (ii) only

11. Consider the following statements,
(i) $(A - B)^2 = (B - A)^2$ for all $n \times n$ matrices A and B.
(ii) $\det(A + B^T) = \det(A^T + B)$
(iii) If $AB = 0$ then $A = 0$ or $B = 0$.

Which of the above statements are always true?
(a) (i) only
(b) (i) and (ii) only
(c) (i) and (iii) only
(d) (ii) and (iii) only
(e) all of them

12. Let $A = \begin{bmatrix} 1 & 2 & 4 & -1 \\ 4 & 0 & -1 & 2 \\ 3 & 3 & 7 & 0 \\ 3 & 5 & 6 & -4 \end{bmatrix}$. Given that $\det A = -4,$ use the adjoint to find the entry in row 1 column 2 of A^{-1}.

(a) $\frac{9}{4}$ **(b)** $-\frac{9}{4}$ **(c)** 9 **(d)** $-\frac{65}{2}$ **(e)** -9

13. A square matrix P is called **idempotent** if $P^2 = P$. If P is idempotent, which of the following matrices are also idempotent?
(i) $I - P$ (ii) $I + P$ (iii) $I - 2P$

(a) (i) only
(b) (i) and (ii)
(c) (i) and (iii)
(d) (ii) only
(e) (i), (ii), and (iii)

14. If A is 3×3 and det $A = 2$, find det$(A^{-1} + 4\,\text{adj}\,A)$.

(a) 364 **(b)** $\frac{729}{2}$ **(c)** 365 **(d)** 729 **(e)** $\frac{365}{2}$

15. Let $A = \begin{bmatrix} a & b & c \\ p & q & r \\ u & v & w \end{bmatrix}$ and assume that det $A = 2$. Compute det$(2B^{-1})$ where
$B = \begin{bmatrix} 4u & 2a & -p \\ 4v & 2b & -q \\ 4w & 2c & -r \end{bmatrix}$.

(a) -1 **(b)** $-\frac{1}{2}$ **(c)** -16 **(d)** -2 **(e)** $-\frac{1}{4}$

16. Let A and B be $n \times n$ matrices. Consider the following statements.
(i) det$(AB) =$ det(BA)
(ii) det$(A + B) =$ det $A +$ det B
(iii) det$(-A) = -$det(A)

Which of the above statements are always true?
(a) (i) only
(b) (i) and (ii) only
(c) (i) and (iii) only
(d) (i), (ii), and (iii)
(e) (iii) only

17. Given that the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ has $\lambda = -1$ as one of its eigenvalues, find the corresponding eigenvector(s).

(a) $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ **(c)** $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ **(d)** $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$
(e) $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

18. Find a matrix P such that $P^{-1}AP$ is diagonal. $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 2 \\ 0 & -1 & 0 \end{bmatrix}$

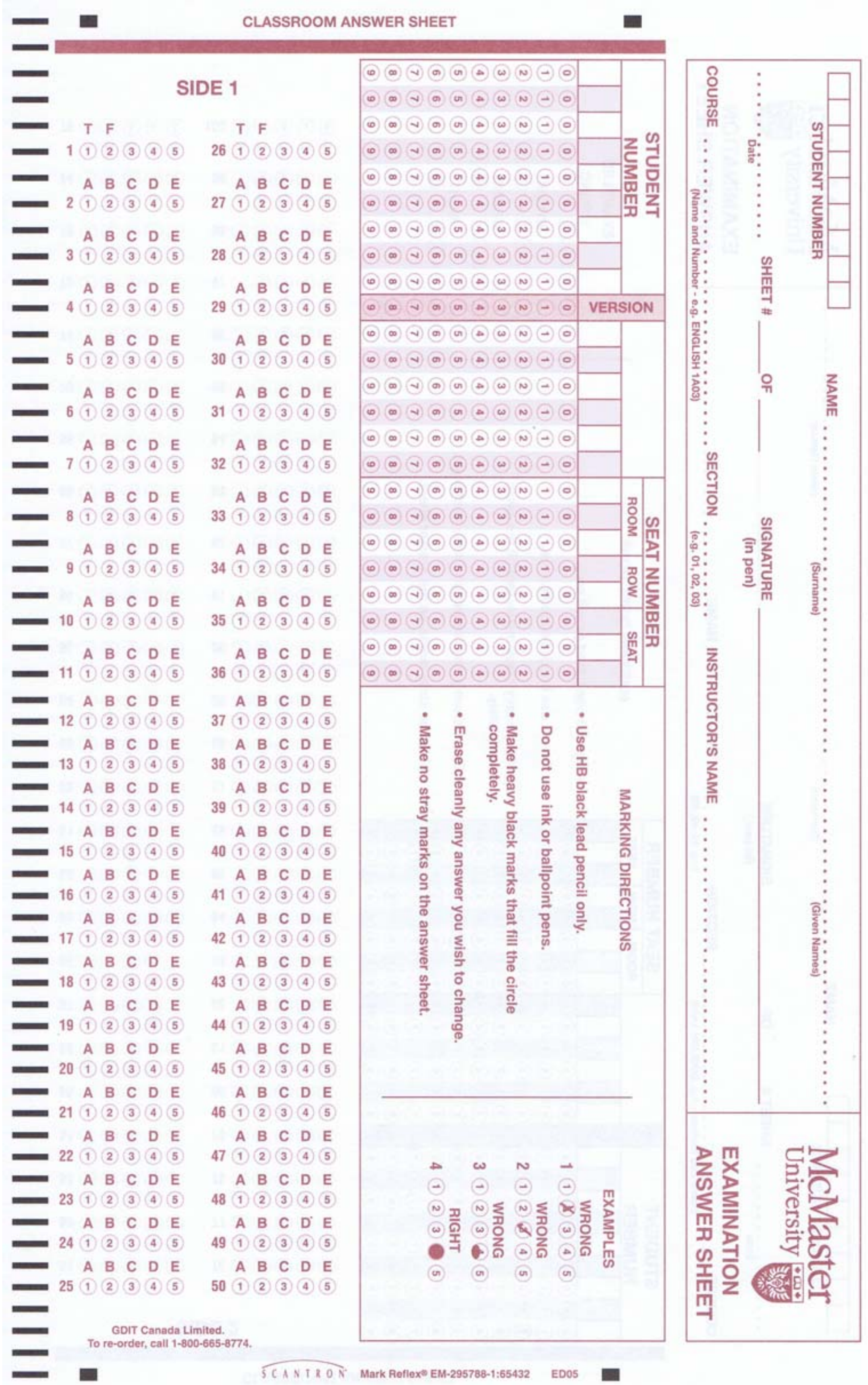
(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 1 & 1 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & 1 & 1 \end{bmatrix}$ **(c)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -2 \\ 0 & 1 & 1 \end{bmatrix}$
(d) $\begin{bmatrix} 1 & 0 & 0 \\ -1 & -1 & -2 \\ 0 & 1 & 1 \end{bmatrix}$ **(e)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -2 \\ 1 & 1 & 1 \end{bmatrix}$

19. Consider the following statements.
(i) If $P^{-1}AP$ is diagonal, and $P^{-1}BP$ is diagonal, then AB diagonalizable.
(ii) If A is diagonalizable then $\det(A) = \lambda_1\lambda_2\cdots\lambda_n$, where $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the (not necessarily distinct) eigenvalues of A.
(iii) If A is diagonalizable then A must be invertible.

Which of the above statements are always true?
(a) (ii) only
(b) (ii) and (iii) only
(c) all of them
(d) (i) and (iii) only
(e) (i) and (ii) only

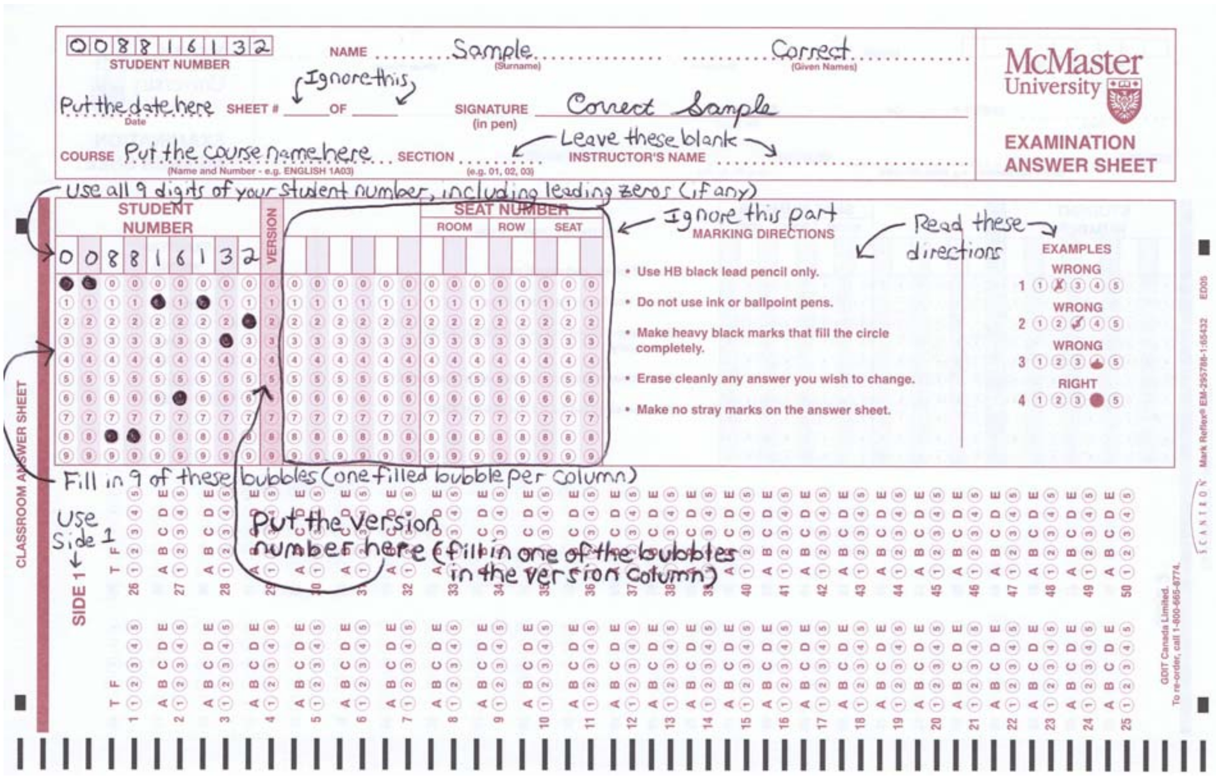
20. In Matlab, suppose that we have defined a vector **x**, and we want to square every component of the vector **x**. Which command could accomplish this?
(a) `>>x^2` **(b)** `>>square(x)` **(c)** `>>x[1]^2,x[2]^2,...,x[n]^2`
(d) `>>x.^2` **(d)** `>>for i = 1 to size(x) x[i] = x[i]^2 endfor`

21. Correctly fill out the bubbles corresponding to all 9 digits of your student number, as well as the version number of your test in the correct places on the computer card. (Use the below computer card for this sample test.)



Answers for 1st Sample Test #1

1. d **2.** c **3.** e **4.** a **5.** b **6.** b **7.** b **8.** b **9.** e **10.** d
11. b **12.** c **13.** b **14.** b **15.** d **16.** a **17.** a **18.** b **19.** e **20.** b
21.



NOTE: On the sample tests, a version number is not given. On the actual tests, it will say "Version X" at the top, where X is the version number that you will have to fill in on the computer card. The sample above assumes that your student number is 008816132. On the actual test, you will have to fill in the bubbles corresponding to YOUR student number (not 008816132).

Answers for 2nd Sample Test #1

1. d **2.** b **3.** d **4.** a **5.** c **6.** a **7.** a **8.** a **9.** b **10.** b
11. b **12.** a **13.** a **14.** b **15.** b **16.** a **17.** a **18.** c **19.** e **20.** d
21. see the answer to #21 on the first sample test above.