Assignment 8

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Assignment 8 consists of two problems. You must write your solutions to the problems using LaTeX.

Please submit Assignment 8 as two files, `Assignment_8_YourMacID.tex` and `Assignment_8_YourMacID.pdf`, to the Assignment 8 folder on Avenue under Assessments/Assignments. *YourMacID* must be your personal MacID (written without capitalization). The `Assignment_8_YourMacID.tex` file is a copy of the LaTeX source file for this assignment (`Assignment_8.tex` found on Avenue under Contents/Assignments) with your solution entered after each problem. The `Assignment_8_YourMacID.pdf` is the PDF output produced by executing

```
pdflatex Assignment_8_YourMacID
```

This assignment is due **Sunday, March 22, 2020 before midnight.** You are allow to submit the assignment multiple times, but only the last submission will be marked. **Late submissions and files that are not named exactly as specified above will not be accepted!** It is suggested that you submit your preliminary `Assignment_8_YourMacID.tex` and `Assignment_8_YourMacID.pdf` files well before the deadline so that your mark is not zero if, e.g., your computer fails at 11:50 PM on March 22.

Although you are allowed to receive help from the instructional staff and other students, your submission must be your own work. Copying will be treated as academic dishonesty! If any of the ideas used in your submission were obtained from other students or sources outside of the lectures and tutorials, you must acknowledge where or from whom these ideas were obtained.

Problems

1. **[10 points]** Let $G = (N, \Sigma, P, S)$ be the CFG where $N = \{S\}$, $\Sigma = \{a, b\}$, and P contains the following productions:

$$S \to aSb \mid \epsilon.$$

For $x \in \Sigma^*$, let $Q(x)$ be the property that $S \xrightarrow[G]{*} x$ iff $x = a^n b^n$ for some $n \geq 0$. Prove

$$\forall x \in \Sigma^* \, . \, Q(x)$$

by weak induction on the length of the derivation $S \xrightarrow[G]{*} x$ for the $(\Rightarrow)$ direction and by strong induction on the length of x for the $(\Leftarrow)$ direction.

Jatin Chowdhary — *chowdhaj* — **March 22nd**

Put your proof here.

First step is to prove the base case.

Base Case:

$S \xrightarrow[G]{1} x$. So x must be ϵ, which makes $x = a^0 b^0$ and the result is ϵ, because $a \, \& \, b$ are being repeated 0 times; not repeated at all. Thus, $Q'_{\Rightarrow}(1)$ holds.

Second step is to prove the weak induction step.

Weak Induction Step:

$S \xrightarrow[G]{n+1} x$ where $1 \leq n$. I will assume $Q'_{\Rightarrow}(n)$. Then $S \xrightarrow[G]{1} \alpha \xrightarrow[G]{n} x$ where $\alpha = aSb$. Now, $x = ayb$. Since this implies that $S \xrightarrow[G]{n} y$, the induction hypothesis implies that $y = a^{n-1} b^{n-1}$, which further implies that $x = a^n b^n$. In conclusion, the statement $Q'_{\Rightarrow}(n+1)$ holds.

$\therefore \forall n \in \mathbb{N}$, the property $Q'_{\Rightarrow}(n)$ holds.

The third step is to prove the strong induction case on the length of x

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2. [**10 points**] Let $\Sigma = (\mathcal{B}, \mathcal{C}, \mathcal{F}, \mathcal{P}, \tau)$ be a signature of MSFOL where:

$\mathcal{B} = \{\alpha, \beta\}$.

$\mathcal{C} = \{a, b\}$ with $\tau(a) = \alpha$ and $\tau(b) = \beta$.

$\mathcal{F} = \{f, g\}$ with $\tau(f) = \alpha \times \beta \rightarrow \alpha$ and $\tau(g) = \beta \rightarrow \beta$.

$\mathcal{P} = \{p, q\}$ with $\tau(p) = \alpha \rightarrow \mathbb{B}$ and $\tau(q) = \beta \times \beta \rightarrow \mathbb{B}$.

Write a context-free grammar in BNF form that generates the set of Σ-formulas. Assume $\mathcal{V} = \{u, v, w, x, y, z\}$.

Jatin Chowdhary — *chowdhaj* — March 22nd

Put your BNF here.

$\langle \Sigma - Formula \rangle ::= \langle Negation \rangle \mid \langle Implication \rangle \mid \langle Equality \rangle$

$\langle Negation \rangle ::= \langle not \rangle \langle \Sigma - Formula \rangle$

$\langle not \rangle ::= \neg$

$\langle Implication \rangle ::= \langle \Sigma - Formula \rangle \langle Implies \rangle \langle \Sigma - Formula \rangle$

$\langle Implies \rangle ::= \implies$

$\langle Equality \rangle ::= \langle \Sigma - Formula \rangle \langle Equals \rangle \langle \Sigma - Formula \rangle$

$\langle Equals \rangle ::= \ =$

$\langle Boole \rangle ::= true \mid false$

$\langle B - \Sigma - Term \rangle ::= \langle \alpha - term \rangle \mid \langle \beta - term \rangle$

$\langle F - \Sigma - Function \rangle ::= \langle f \rangle \mid \langle g \rangle$

$\langle f \rangle ::= \alpha \times \beta \langle Implies \rangle \alpha$

$\langle g \rangle ::= \beta \langle Implies \rangle \beta$

$\langle F - \Sigma - Predicate \rangle ::= \langle p \rangle \mid \langle q \rangle$

$\langle p \rangle ::= \alpha \langle Implies \rangle \langle Boole \rangle$

$\langle q \rangle ::= \beta \times \beta \langle Implies \rangle \langle Boole \rangle$

$\langle Variable - V \rangle ::= u \mid v \mid w \mid x \mid y \mid z$