MATHEMATICS 1LS3   TEST 3

Day Class   F. Font, M. Lovrić, D. Lozinski
Duration of Examination: 60 minutes
McMaster University, 14 November 2016

First name (PLEASE PRINT): _____

Family name (PLEASE PRINT): _____

Student No.: _____

THIS TEST HAS 8 PAGES AND 6 QUESTIONS. YOU ARE RESPONSIBLE FOR EN-
SURING THAT YOUR COPY OF THE PAPER IS COMPLETE.

Total number of points is 40. Marks are indicated next to the problem number. Any
non-graphing calculator is allowed.

**EXCEPT ON QUESTIONS 1 AND 2, YOU MUST SHOW CORRECT WORK
TO EARN CREDIT.**

USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL YOUR TEST WILL NOT
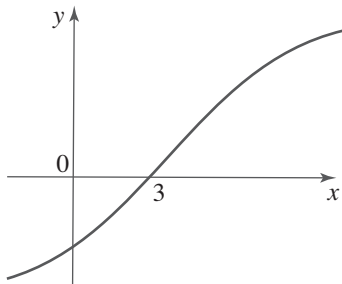BE ACCEPTED FOR REMARKING (IF NEEDED).

You must show work to receive full credit.

Problem	Points	Mark
1	4	
2	6	
3	5	
4	8	
5	8	
6	9	
TOTAL	40	

1. **Multiple choice questions: circle ONE answer. No justification is needed.**

(a)[2] Identify all correct statements about the **antiderivative of the function** shown below.

(I) It is increasing on $(-\infty, \infty)$

(II) It is increasing on $(3, \infty)$

(III) It is concave down on $(3, \infty)$

(A) none   (B) I only   (C) II only   (D) III only

(E) I and II   (F) I and III   (G) II and III   (H) all three

(b)[2] Identify all positive numbers (positive means grater than zero).

$$\text{(I) } \int_{-1}^{2} \arctan x \, dx \qquad \text{(II) } \int_{-2}^{1} \arctan x \, dx \qquad \text{(III) } \int_{1}^{1} \arctan x \, dx$$

(A) none   (B) I only   (C) II only   (D) III only

(E) I and II   (F) I and III   (G) II and III   (H) all three

2. **Identify each statement as true or false (circle your choice). You do not need to justify your answer.**

(a)[2] $\int \dfrac{1}{x^3}\,dx = \ln|x^3| + C.$

<p style="text-align:right">TRUE   FALSE</p>

(b)[2] The function $f(x) = 2xe^{x^2}$ is an antiderivative of the function $g(x) = e^{x^2}$.

<p style="text-align:right">TRUE   FALSE</p>

(c)[2] A function $f(x)$ for which $\int_a^b f(x)\,dx > 0$ must satisfy $f(x) > 0$ for all x in $[a, b]$.

<p style="text-align:right">TRUE   FALSE</p>

**Questions 3-6: You must show CORRECT work to receive full credit.**

3. (a)[3] Find an approximation of the area of the region below the graph of $y = e^{-x}$ and over the interval $[-2, 2]$, using L_4 (i.e., left sum with four rectangles). Round off to three decimal places. Sketch the function and the four rectangles involved.

(b)[2] Suppose that you computed R_{50} (i.e., right sum with 50 rectangles) for the function in (a). Which of the two sums, L_4 or R_{50}, is larger? Give a reason for your answer.

4. Consider the initial value problem $y' = 4xy - 2$, $y(1) = 0$, for an unknown function $y(x)$.

(a)[3] Write down all necessary information to start Euler's method; i.e., state the values of x_0, y_0, and the formulas for x_{n+1} and y_{n+1}. The step size is Δx.

(b)[3] Explain how the formula for y_{n+1} in terms of y_n is obtained. Show both x_{n+1} and y_{n+1} in the coordinate system.
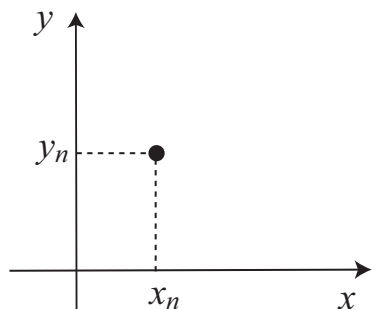
(c)[2] Compute the first two steps of Euler's Method with step size $\Delta x = 0.4$.

5. (a)[3] Find the most general antiderivative of the function $f(x) = \dfrac{x^2 + 1}{\sqrt{x}}$.

(b)[2] Find the numeric value of the definite integral $\displaystyle\int_0^1 \frac{4}{1 + x^2}\, dx$.

(b)[3] Find the indefinite integral $\displaystyle\int \left(a\cos(\pi t) + b\sec^2 t \right)\, dt$.

6. (a)[3] A sample of bacteria, initially at the temperature of 16.7^OC, is put into a -45^OC refrigerator. Let $T(t)$ be the temperature of the sample at time t. The temperature of the sample changes proportionally to the square root of the difference between the temperature of the sample and the temperature of the refrigerator. Describe this event as an initial value problem (i.e., write down a differential equation and an initial condition). **Do not solve the equation.**

(b)[3] Describe the following event as initial value problem: an amoeba cell starts at a volume of $600\,\mu\text{m}^3$ and **loses** volume at the rate of $1.4e^{0.02t}\,\mu\text{m}^3/s$.

(c)[3] Find the solution of the initial value problem in (b).