4.1 Series (Chapter 11.2)

"adding elements of sequence"

An **infinite series**, or short **series**, is given by $\sum_{n=1}^{\infty} a_n$.

We define the **partial sum** S_n by $\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \dots + a_n$.

sequence $\{S_1, S_2, S_3, \dots\}$ with $S_1 = a_1$, $S_2 = a_1 + a_2$, $S_3 = a_1 + a_2 + a_3$

Relation: $\lim_{n\to\infty} \sum_{i=1}^{n} a_i = \sum_{i=1}^{\infty} a_i$. limit of $\boxed{\text{partial sums}}$ is series.

We say that the series is **convergent** if limit of S_n exists as a finite nr, else, the series is divergent.

Any number can be expressed as a series. How?

- 10 $= \sum_{i=1}^{\infty} a_i = 10 + 0 + 0 + 0 + \dots$ $\quad a_1 = 10 \quad a_2 = a_3 = \dots = 0$

  $= \sum_{i=1}^{\infty} b_i \cdot 10^{i-1} = b_1 \cdot 10^0 + b_2 \cdot 10 + b_3 \cdot 10^2 + b_4 \cdot 10^3 + \dots$

  $0 + 0 + \frac{1}{10} \cdot 100 + 0 + \dots = 10$ ✓

  $b_1 = 0 \quad b_2 = 0 \quad b_3 = \frac{1}{10} \quad b_4 = b_5 = \dots = 0$

  $b_1 = 0 \quad b_2 = 1 \quad b_3 = b_4 = \dots = 0$

  $b_1 = 10 \quad b_2 = 0 \quad b_3 = 0 = b_4 = \dots$

- 0.34

  series converges

  $0.34 = \sum_{i=1}^{\infty} b_i \cdot 10^{1-i} = b_1 + b_2 \cdot \frac{1}{10} + \frac{b_3}{100} + \frac{b_4}{1000} + \dots$ (b_1, b_2, b_3 integer)

  $\boxed{b_1 = 0 \quad b_2 = 3 \quad b_3 = 4 \quad b_4 = b_5 = b_6 = \dots = 0}$

- 4.12345678

- π

Let $a \neq 0$ and $r \in \mathbb{R}$, then the geometric series is

Can we calculate the value of this series?

Hint: Look at $S_n - rS_n$.

general rule:

Example: Find

1. $\sum\limits_{n=1}^{\infty} \frac{1}{5^n}$

2. $\sum\limits_{n=0}^{\infty} 2^{n+1}3^{-n}$

A **telescoping series** is a series, where the terms can be written as $a_n = c_n - c_{n+1}$ for some c_n.

Example: $\sum\limits_{n=1}^{\infty} \frac{1}{n(n+1)}$

Result: Assume $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n =$, because

$\Rightarrow$ **Test for Divergence**

Example: $\sum_{n=1}^{\infty} \frac{n}{n+1}$

Example: $\sum_{n=1}^{\infty} (-1)^n$

If $\lim_{n\to\infty} a_n = 0$, can we conclude that $\sum_{n=1}^{\infty} a_n$ converges?

Example: $\sum_{n=1}^{\infty} \frac{1}{n}$

Conclusion/Rule:

Limit Rules for Series: Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be convergent. Then,

a) $\sum_{n=1}^{\infty} (a_n + b_n) =$

b) $\sum_{n=1}^{\infty} (a_n + b_n) =$

c) $\sum_{n=1}^{\infty} ca_n =$

Example: $\sum_{n=1}^{\infty} \frac{2}{3^n} - \frac{1}{2^{n+1}}$