MATHEMATICS 1LS3 TEST 1

Day Class
Duration of Examination: 60 minutes
McMaster University, 3 October 2016

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First name (PLEASE PRINT): SOLUTIONS

Family name (PLEASE PRINT): ________________

Student No.: ________________

THIS TEST HAS 8 PAGES AND 7 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE.

Total number of points is 40. Marks are indicated next to the problem number. Any non-graphing calculator is allowed.

USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

You must show work to receive full credit.

Problem	Points	Mark
1	4	
2	6	
3	4	
4	8	
5	6	
6	4	
7	8	
TOTAL	40	

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1. **Multiple choice questions: circle ONE answer. No justification is needed.**

(a)[2] What is the domain of the function $f(x) = \ln(5-x) + \log_{10}(x+2)$?

(A) $x < 0$ (B) $x < -2$ (C) $x < 5$ (D) $x < -2$ and $x > 5$
(E) $-2 < x < 0$ (F) $-2 < x < 5$ (G) $0 < x < 5$ (H) $x < -2$ and $x > 0$

$5 - x > 0 \rightarrow x < 5$
$x + 2 > 0 \rightarrow x > -2$

(b)[2] The body mass index BMI is given by the formula BMI $= m/h^2$, where m is the mass in kg and h is the height in m. Identify **all** correct statements.

(I) If m doubles, then BMI doubles as well ✓
(II) If h increases by 10%, then BMI decreases by 10% ✗
(III) If h increases by 10%, then BMI increases by 10% ✗

(A) none (B) I only (C) II only (D) III only
(E) I and II (F) I and III (G) II and III (H) all three

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2. Identify each statement as true or false (circle your choice). You do not need to justify your answer.

(a)[2] A substance started exponentially decaying in year 2000, and reached 30% of its orginal amount in 2010. By 2020, it will decay to 15% of its orginal amount.

TRUE FALSE

2000 $\longrightarrow$ 2010 $\longrightarrow$ 2020

100% 30%

half-life is less than 10 years

will reach 15% here

(b)[2] An oscillatory input (intensity function of a group of spiking neurons) is given by the formula $\hat{\lambda}_1(t) = v_0 + a\cos(2\pi f_m(t+d))$. The period of $\hat{\lambda}_1(t)$ is f_m.

TRUE FALSE

$$\frac{2\pi}{2\pi f_m} = \frac{1}{f_m}$$

(c)[2] If $f(x)$ is continuous at $x = 3$ and $f(3) = 0.7$, then the function $g(x) = \sqrt{f(x)} + 5$ is continuous at $x = 3$.

TRUE FALSE

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Questions 3-7: You must show work to receive full credit.

3. The visibility index tells us how clearly we can see an object submerged in water. In saltwater, the visibility index is given by the function

$$v(d) = \frac{1.75}{0.79 + \ln(8.3d + 2.61)}$$

where d is the depth in metres, $0 \le d \le 50$ (~~in which case~~ so $d = 0$ labels the surface, and $d = 3$ is 3 m below the surface).

(a)[1] State (in one sentence) what question related to the visibility index is answered by finding the inverse function.

if we know the visibility index, what is the depth?

(b)[3] Find the inverse function of $v(d)$.

$$v = \frac{1,75}{0,79 + \ln(8,3d + 2,61)}$$

$$0,79v + v\ln(8,3d + 2,61) = 1,75$$

$$\ln(8,3d + 2,61) = \frac{1,75 - 0,79v}{v}$$

$$8,3d + 2,61 = e^{\frac{1,75 - 0,79v}{v}}$$

$$\boxed{d = \underbrace{\frac{1}{8,3}}_{\approx 0,1205}\left(e^{\frac{1,75 - 0,79v}{v}} - 2,61\right)}$$

can be simplified: $e^{\frac{1,75}{v} - 0,79}$

not needed

$$d \approx 0,1205\, e^{\frac{1,75}{v} - 0,79} - 0,3145$$

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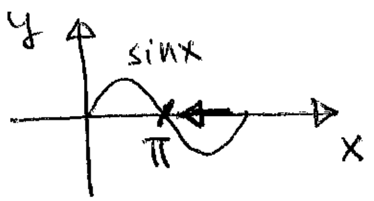
4. Find each limit (or else say that the limit does not exist).

(a)[3] $\lim_{x\to\infty} \frac{e^{-x}+3e^{x}}{1+2e^{x}}$ $\frac{\div e^x}{\div e^x}$

$$= \lim_{x\to\infty} \frac{e^{-2x}+3}{e^{-x}+2} = \boxed{\frac{3}{2}}$$ since $e^{-x}, e^{-2x} \to 0$ as $x \to \infty$

(b)[2] $\lim_{x\to\pi^+} \frac{13x+1}{\sin x} = \frac{13\pi+1}{0} = \boxed{-\infty}$



as $x \to \pi^+$, $\sin x \to 0^-$

(c)[3] $\lim_{x\to\infty} \left(\ln(2x^3+4) - \ln(x^2+x+2)\right)$

$$= \lim_{x\to\infty}\left(\ln \frac{2x^3+4}{x^2+x+2}\right) = \ln \infty = \boxed{\infty}$$

since $\lim_{x\to\infty} \frac{2x^3+4}{x^2+x+2} = \lim_{x\to\infty} \frac{2x^3}{x^2} = \lim_{x\to\infty} 2x = \infty$

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5. The survival rate (i.e., percent) $S(D)$ of clonogenic cells (cancer cells) exposed to a radiation treatment can be modelled by

$$S(D) = e^{-0.5D^2-D}$$

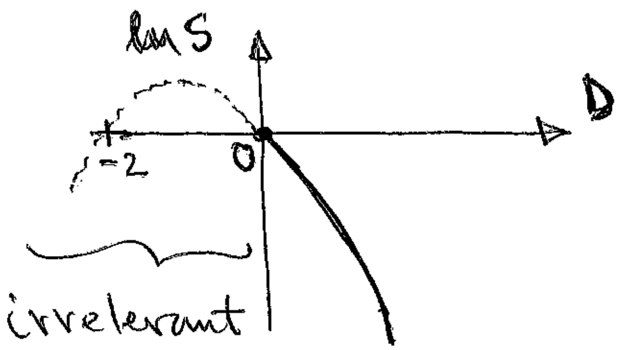
where $D \geq 0$ represents the applied radiation dose (measured in grays, Gy).

(a)[2] What is $S(0)$? Does it make sense?

$S(0) = e^0 = \underline{\underline{1}}$ ... makes sense: if radiation dose is zero (ie, no radiation), all cells will survive ($1 = 100\%$)

(b)[2] Sketch the semilog graph (use ln) of the survival rate for $D \geq 0$. Label the axes.

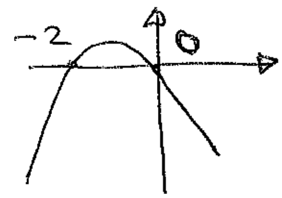
$\ln S = -0.5D^2 - D$



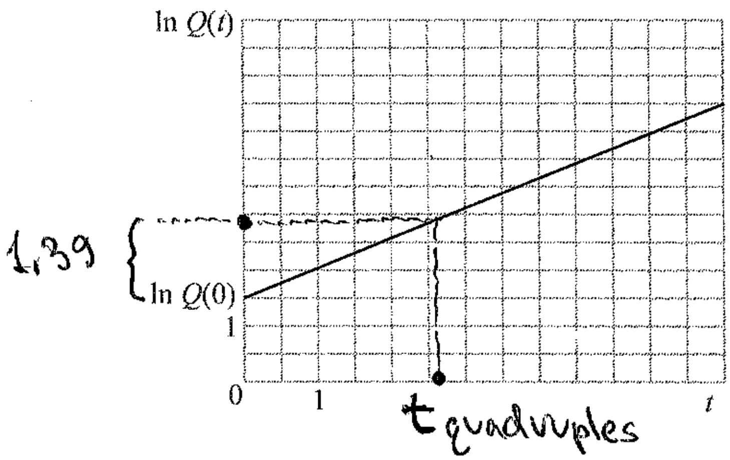
$-0.5D^2 - D = 0$

$D(\underbrace{-0.5D-1}) = 0$

$D = -2$



(c)[2] The semilog graph below shows an exponentially increasing quantity $Q(t)$. Identify the point on the t axis which represents the time when the quantity quadruples (i.e., is four times larger than initially). Justify your answer.



$\ln(4Q(0)) = \ln 4 + \ln Q(0)$

$= \ln Q(0) + 1.39$

$(\ln 4 \approx 1.39)$

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6. The following excerpt is taken from *The laminar cortex model: a new continuum cortex model incorporating laminar architecture.* J. Du, V. Vegh, and D.C. Reutens. PLoS Computational Biology. 8.10 (Oct. 2012).

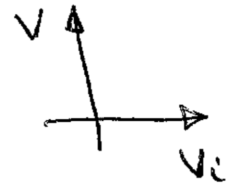
the average of membrane potentials of neurons in the element, that is

$$V = \frac{N_e V_e + N_i V_i}{N_e + N_i}$$

where N_e, N_i are the numbers of excitatory and inhibitory neurons and V_e and V_i are the (average) membrane potentials of excitatory and inhibitory neuron populations respectively.

(a)[2] View V as a function of V_i. Describe its graph in words.

$$V = \frac{NeVe}{Ne+Ni} + \frac{Ni}{Ne+Ni} Vi$$



$\boxed{\text{line}}$, slope $= \frac{Ni}{Ne+Ni}$

vertical (V) intercept $= \frac{NeVe}{Ne+Ni}$

(b)[2] View V as a function of N_e. What is the limit of V as N_e increases beyond any bounds (i.e., as it approaches ∞)?

$$\lim_{Ne \to \infty} \frac{NeVe + NiVi}{Ne + Ni} \quad \begin{matrix} \div Ne \\ \div Ne \end{matrix}$$

$$= \lim_{Ne \to \infty} \frac{Ve + NiVi/Ne}{1 + Ni/Ne} = \underline{\underline{Ve}}$$

or: $$\lim_{Ne \to \infty} \frac{NeVe + NiVi}{Ne+Ni} = \lim_{Ne \to \infty} \frac{\cancel{Ne}Ve}{\cancel{Ne}} = \underline{\underline{Ve}}$$

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7. (a)[2] What is the domain of the function $f(x) = 3 - \arcsin(3x - 7)$?

$$-1 \leq 3x - 7 \leq 1$$
$$6 \leq 3x \leq 8$$
$$\boxed{2 \leq x \leq \frac{8}{3}} \quad \text{or} \quad \left[2, \frac{8}{3}\right]$$

(b)[3] What is the range of the function $P(t) = 2\left(1 + \arctan\frac{t - 12}{7}\right)$?

not relevant for range

$$-\frac{\pi}{2} < \arctan\frac{t-12}{7} < \frac{\pi}{2} \quad \Big| \text{ add } 1$$
$$1 - \frac{\pi}{2} < 1 + \arctan\frac{t-12}{7} < 1 + \frac{\pi}{2} \quad \Big| \cdot 2$$
$$\boxed{2 - \pi < 2\left(1 + \arctan\frac{t-12}{7}\right) < 2 + \pi}$$
$$\text{or} \quad (2-\pi, 2+\pi)$$

(c)[3] Identify all x for which the function $f(x) = \sqrt{e^{1-x} - 4}$ is continuous.

$$e^{1-x} - 4 \geq 0$$
$$e^{1-x} \geq 4$$
$$1 - x \geq \ln 4$$
$$\boxed{x \leq 1 - \ln 4} \approx -0.386$$

THE END