Math 1LT3 — Assignment 23

Continuous random variables (Section 13)

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1. Explain why $f(x) = 1 + x - x^2$, $x \in [1, 2]$, cannot be a probability density function of any random variable.

2. Explain why $f(x) = 1/4$, $x \in [1, 4]$, cannot be a probability density function of any random variable.

3. Find the value of the constant a so that $f(x) = ax(1 - x)$, $0 \leq x \leq 1$, satisfies the properties of the probability density function.

4. Check that the function $f(x) = \dfrac{1}{2\sqrt{x}}$, $0 < x \leq 1$, can be a probability density function. Find its mean.

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5. (a) The function $f(x) = 0.5 - 0.125x$, $0 \leq x \leq 4$, is a probability density function of a random variable X. Find $P(2 \leq X \leq 3)$.

(b) The function $f(x) = 6x(1-x)$, $0 \leq x \leq 1$, is a probability density function of a random variable X. Find $P(0.2 \leq X \leq 0.5)$.

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6. In each case:
(i) Check that $f(x)$ satisfies properties (1) and (2) in Definition 36.
(ii) X be a continuous random variable whose probability density function is $f(x)$. Find the expected value μ of X.
(iii) Find the probability $P(X \leq \mu)$.

(a) $f(x) = 4x^3$, $0 \leq x \leq 1$

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(b) $f(x) = \frac{3}{4}x(2-x)$, $0 \leq x \leq 2$

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7. Consider the continuous random variable X given by the probability density function $f(x) = 0.3 + 0.2\,x$, $0 \leq x \leq 2$. Find $P(0.5 \leq X \leq 2)$. Find the probability that the values of X are at least one standard deviation above the mean.

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