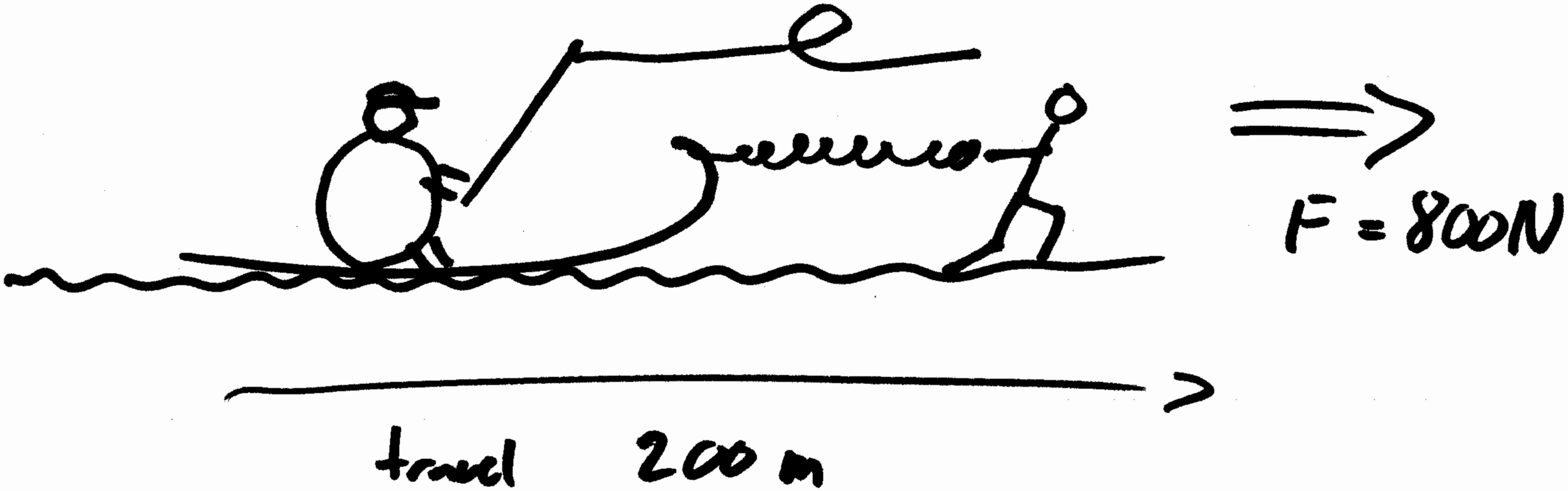
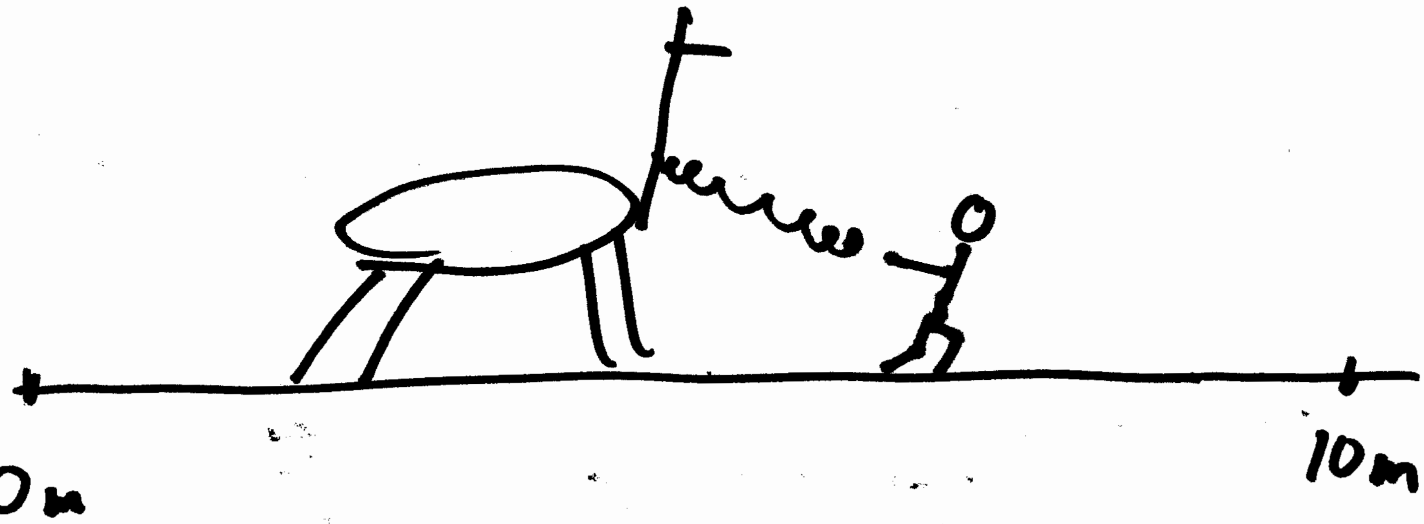
An Application of Integration: Work



$$W = F \cdot d = (800\,N)(200\,m)$$
$$= 160{,}000\ J = \underline{160\,kJ}$$

$$\text{“}J\text{”} = \text{“}N\text{”} \cdot \text{“}m\text{”}$$

eg.



$F(x)$ = Force as a func. of position

$= 8x - x^2$

How can I compute work in pulling the llama from $x=0$ to $x=10m$?

Too hard! ⇒ re-imagine as pulling llama a sequence of small distances (Δx) with const. force!

Net Work $\doteq \sum$ bits of work $\doteq \sum \begin{pmatrix}\text{const force}\\ \text{on interval}\end{pmatrix} \begin{pmatrix}\text{width}\\ \text{interval}\end{pmatrix}$

$$\doteq \sum_{i=1}^{n} F(x_i) \cdot \Delta x$$

by defn. of definite integral

$$\text{Actual Net Work} = \int_0^{10} F(x)\,dx$$

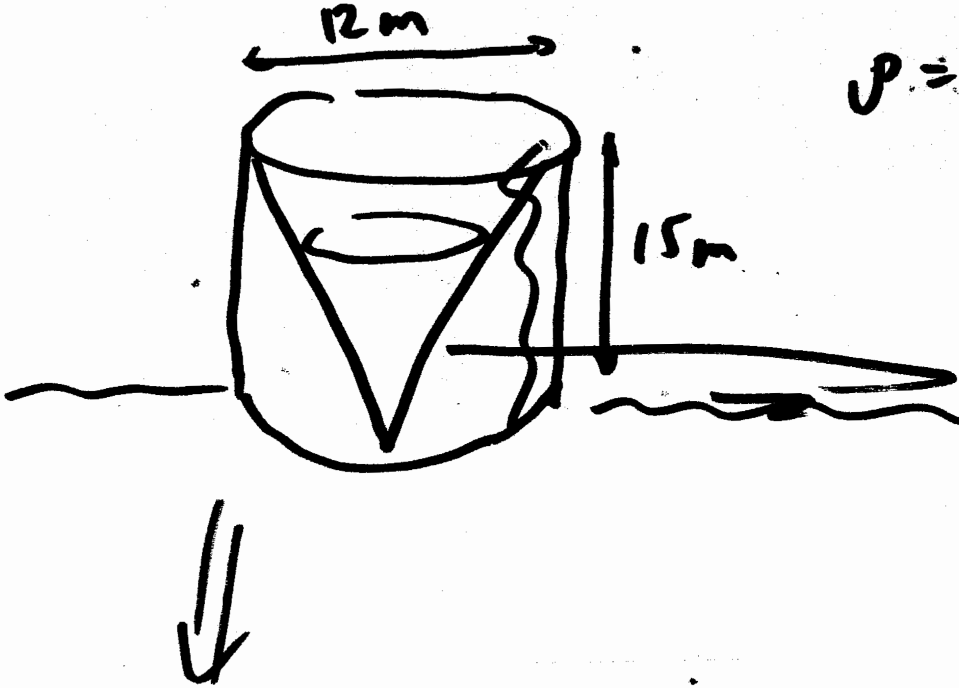
Here $W = \int_0^{10} 8x - x^2\,dx$

$$= 4x^2 - \frac{1}{3}x^3 \Big|_0^{10}$$

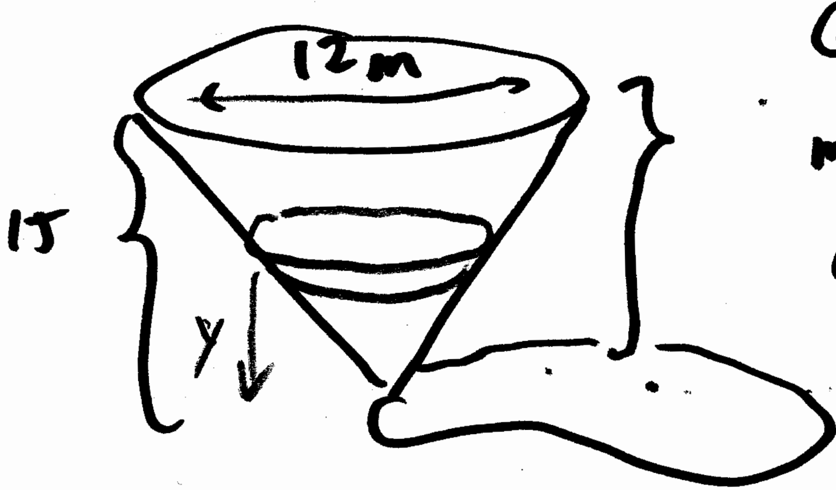
$$= 400 - \frac{1}{3}(1000) - 0$$

$\doteq$ [redacted] $\doteq 66.7$ J

The Boston Molasses Disaster



ρ = density = 1400 kg/m^3

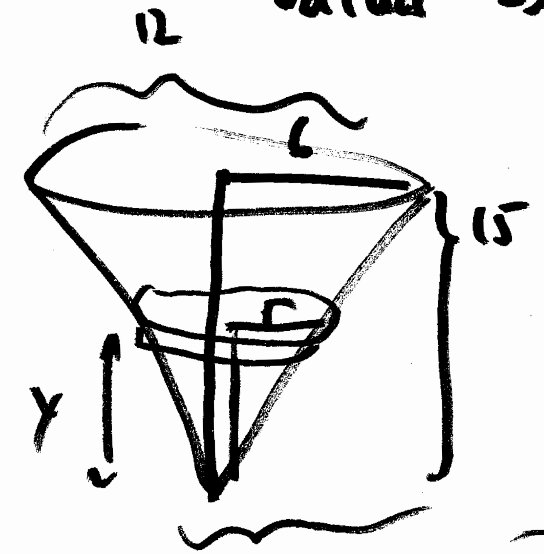


Given a cone full of molasses breaks open & dumps its contents, how much Energy is released?

(How much work is done by gravity?)

$\rho = 1400 \, kg/m^3$, $g = 10 \, m/s^2$

Solution To simplify into a sum of $F \cdot d$ constant values $\Rightarrow$ cut into slices that fall fixed distance.



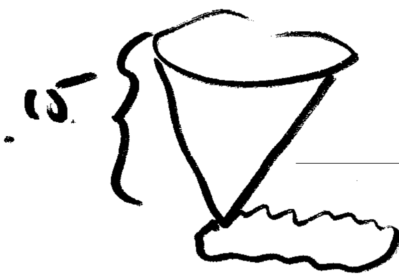
$$F_{slice} = mg = \rho V_{slice} g$$
$$= \rho g A_{slice} \cdot (\text{thick})$$
$$= \rho g \pi r^2 \cdot \Delta y$$
$$= \rho g \pi \frac{4y^2}{25} \Delta y$$

$$\frac{r}{y} = \frac{6}{15} = \frac{2}{5}$$
$$r = \frac{2}{5} y$$

Net W $\doteq \sum W_{slice} = \sum_{i=1}^{n} F_{slice} \cdot dist.$

$$= \sum_{i=1}^{n} \left(\frac{4\pi\rho g}{25} y_i^2 \Delta y \right) y_i$$

$$= \sum_{i=1}^{n} \frac{4}{25} \pi \rho g \, y_i^3 \Delta y$$



$$\text{Net work} = \int_0^{15} \underline{\frac{4}{25} \pi \rho g} \, y^3 \, dy$$

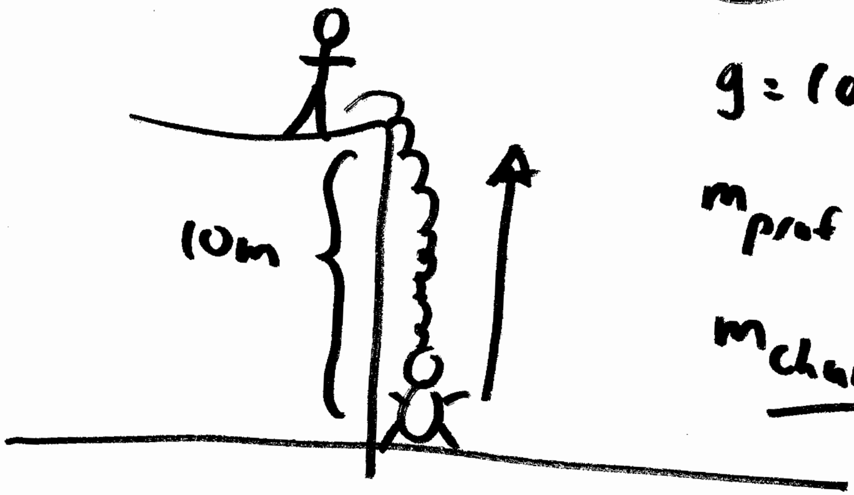
$$= \frac{4}{25} \pi \rho g \cdot \frac{1}{4} y^4 \Big|_0^{15}$$

$$= \frac{\cancel{4}}{25}\cdot \pi \cdot (1400)(10)\cdot \frac{1}{\cancel{4}}\cdot 15^4 - 0$$

$$= \underline{\text{Big}}\ \# \quad \text{in units of } \underline{MJ}$$

$$= \underline{10^6\ J}$$

eg



$g = 10\,m/s^2$

$m_{prof} = 200kg$

$m_{\underline{chain}} = \underline{10kg}$, & $\underline{10m}$ long

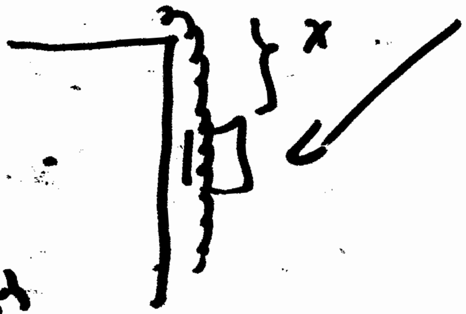
Find total work in raising chain & prof to
<u>top of roof!</u>

Solution Pt. 1 Prof

10m


$$W = F \cdot \text{dist.}$$
$$= mg \cdot (10\,m)$$
$$= (200)(10)(10)$$
$$= 2 \times 10^4\ J$$
$$= 20\ kJ$$

Pt. 2 Chain



$m_{chain} = 10$ kg

Tot. length $= 10$ m

Work on 1 "chunk"

$$= F_{chunk} \cdot \text{distance}$$
$$= m_{chunk} \cdot g \cdot x$$
$$\Delta x$$
$$= (m_{chain}) \cdot \left(\frac{\text{chunk length}}{\text{total length}} \right) \cdot g \cdot x$$

10 kg   10 m

$$= \frac{10}{10} \cdot \Delta x \cdot g \cdot x = 10x \, \Delta x$$

$$\text{Net } W \text{ of chain} \approx \underbrace{\sum_{i=1}^{n}}_{\text{add chunks}} W_{\text{chunk } i} = \sum_{i=1}^{n} 10\, x_i \cdot \Delta x$$

$$W = \int_0^{10} 10x \, dx = \frac{10}{2} x^2 \Big|_0^{10} = 500 \text{ J}$$

So $W_{chain} = 500$ J, $W_{prof} = 20{,}000$ kJ

Total $W = 20{,}500$ J $= 20.5$ kJ