

Theme 1

Introductory Material

Module T1M1:
The Predictable Universe

Vectors

- Scalars
 - Answers a question like:
 - How hot?
 - How heavy?
 - How far can you throw the person sitting next to you?
- Vectors
 - Used when a number doesn't give enough info
 - Velocity $v = 20 \text{ m/s}$ [north]

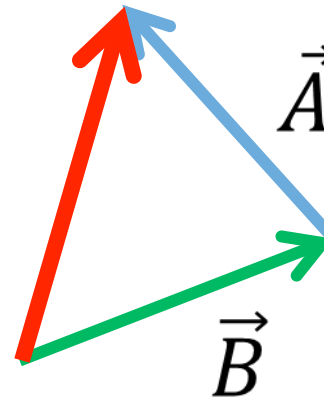
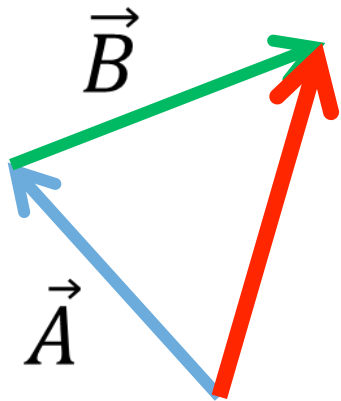
You will want to become comfortable with vector addition & subtraction, and in particular, working with vector components

Vector Addition

- When adding vectors, line them up 'tip-to-tail'



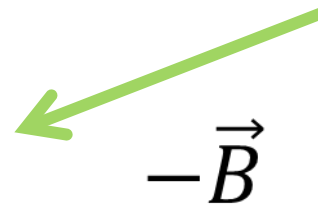
$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$



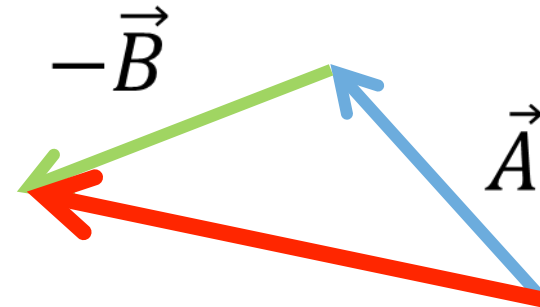
Vector Subtraction



- What about $A - B$?
 - Let's create a vector $-B$
- Now we can simply add:

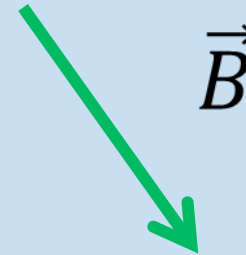
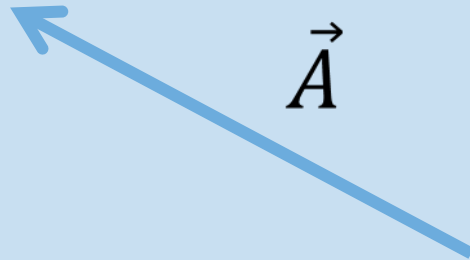


$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$



Clicker Quiz

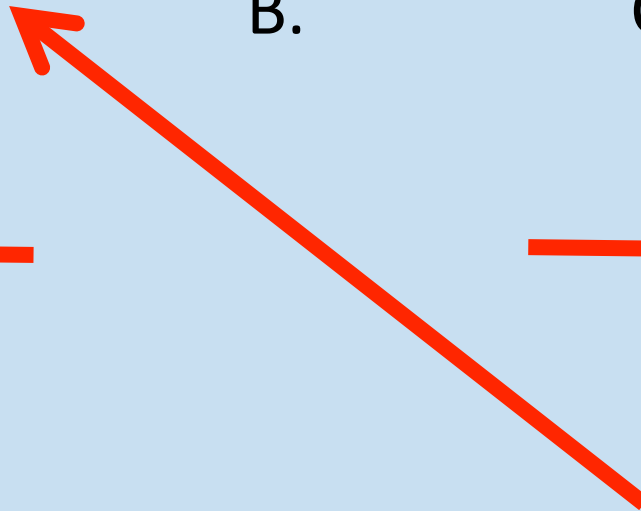
- For the two vectors shown, what is $\vec{A} + \vec{B}$



A.



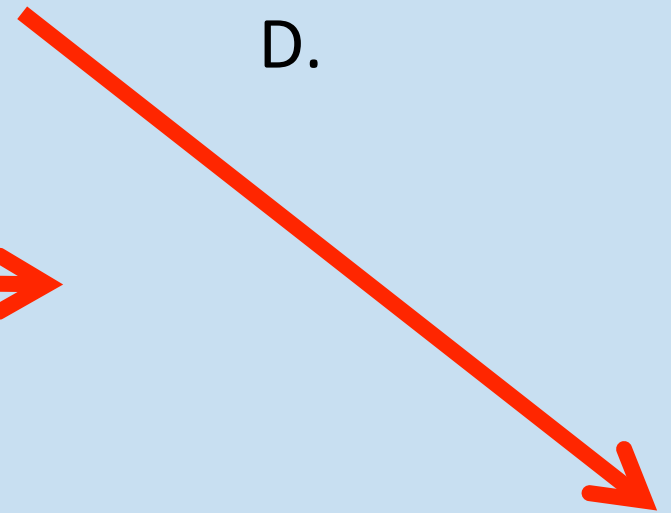
B.



C.

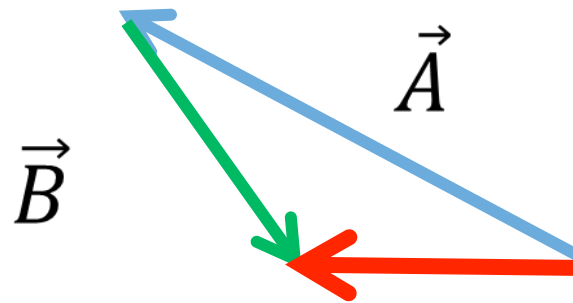


D.



Clicker Quiz

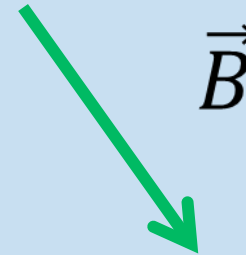
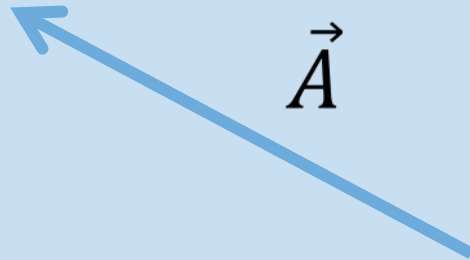
- For the two vectors shown, what is $\vec{A} + \vec{B}$



A. Is the answer

Clicker Quiz

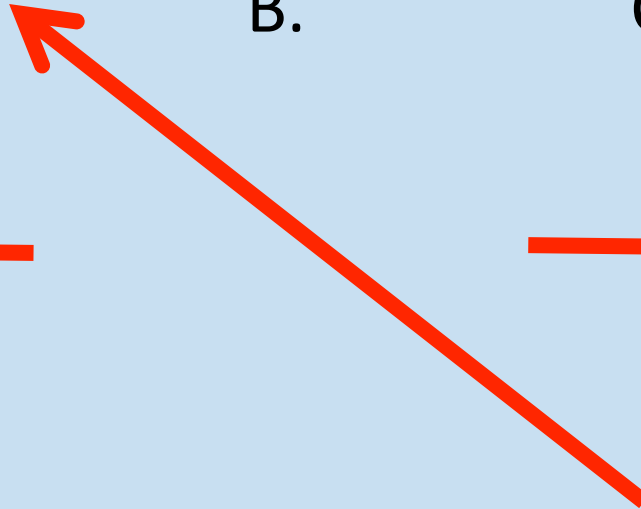
- For the two vectors shown, what is $\vec{A} - \vec{B}$



A.



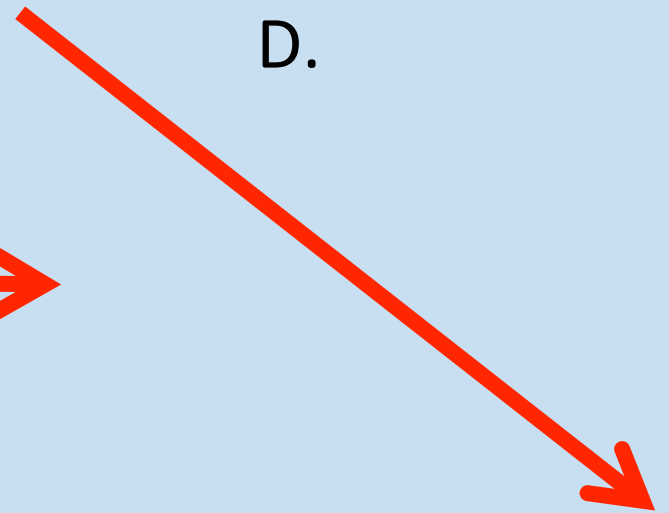
B.



C.

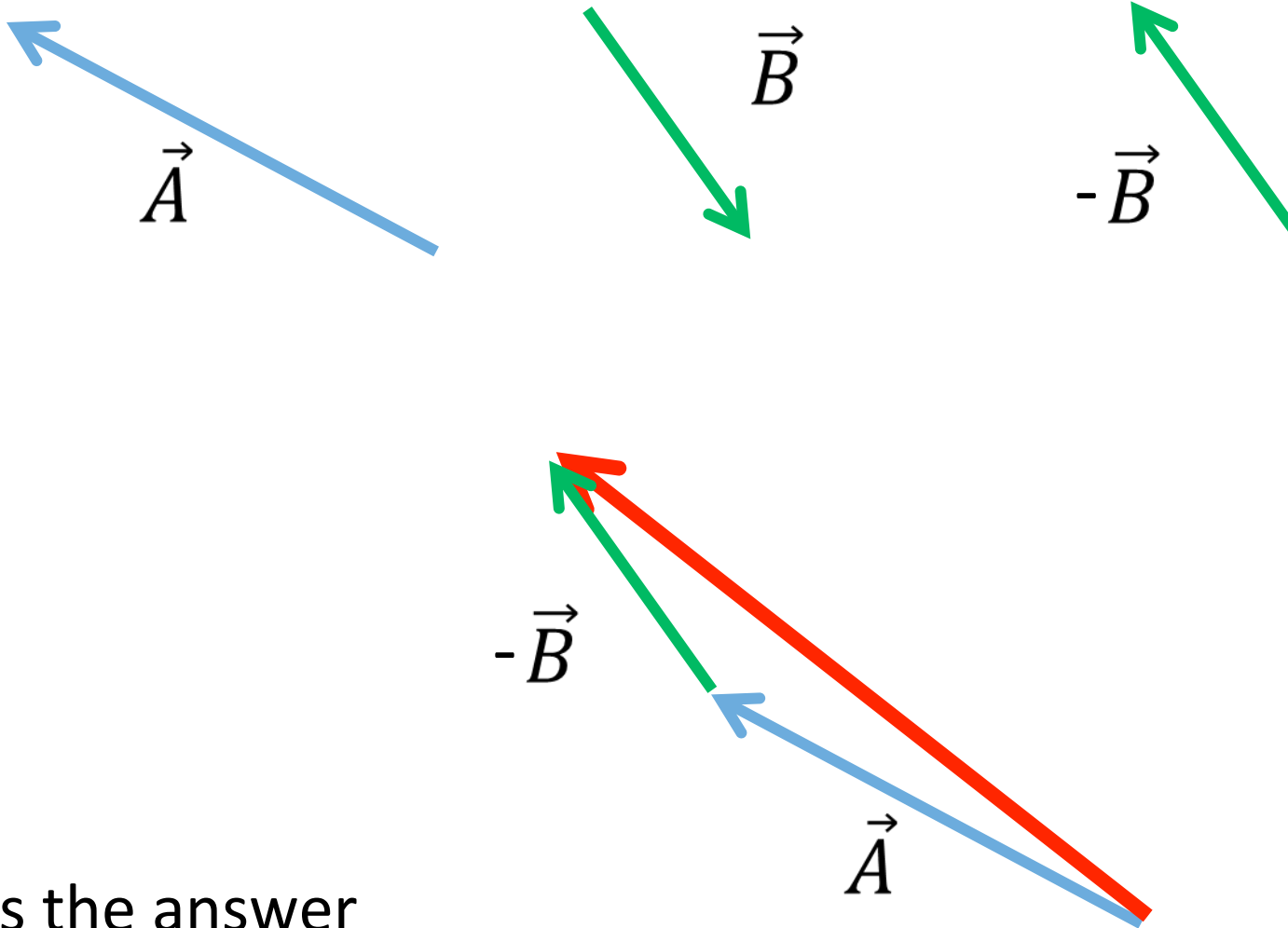


D.



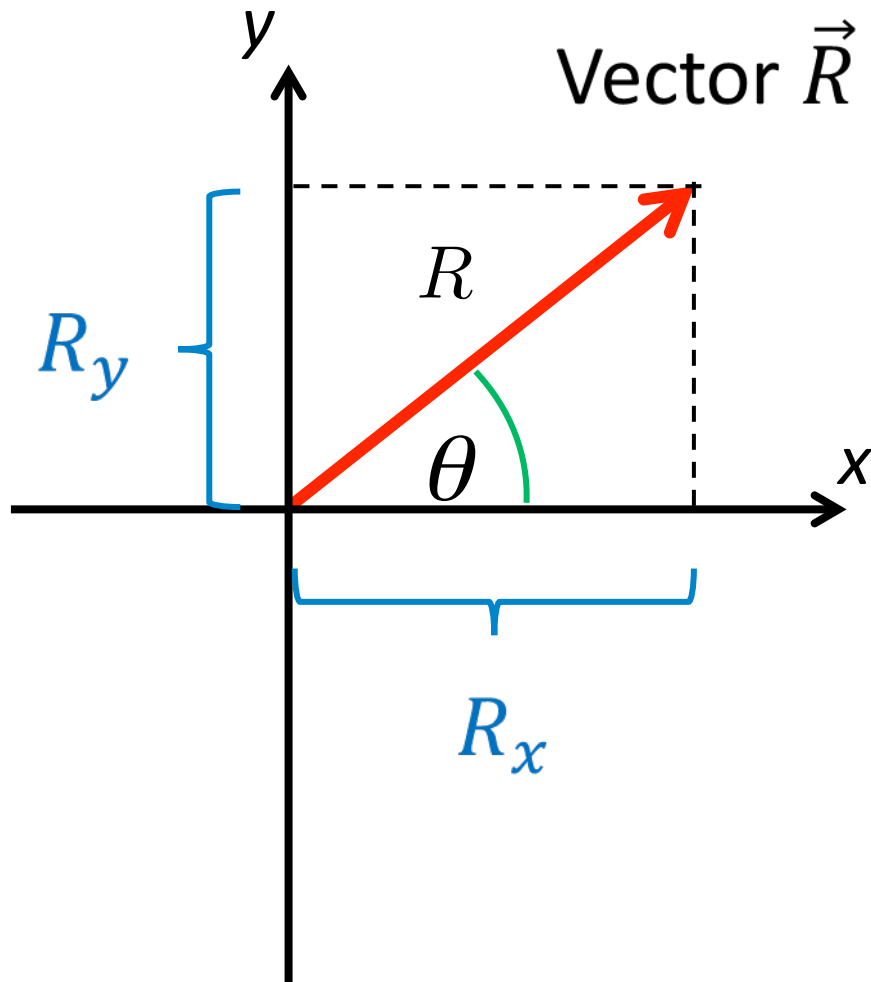
Clicker Quiz

- For the two vectors shown, what is $\vec{A} - \vec{B}$



B. is the answer

Vector Notation



$$1) \vec{R} = (R, \theta)$$

$R = |\vec{R}|$ is the 'magnitude'
 θ is the direction,
relative to the $+x$ axis

$$2) \vec{R} = (R_x, R_y)$$

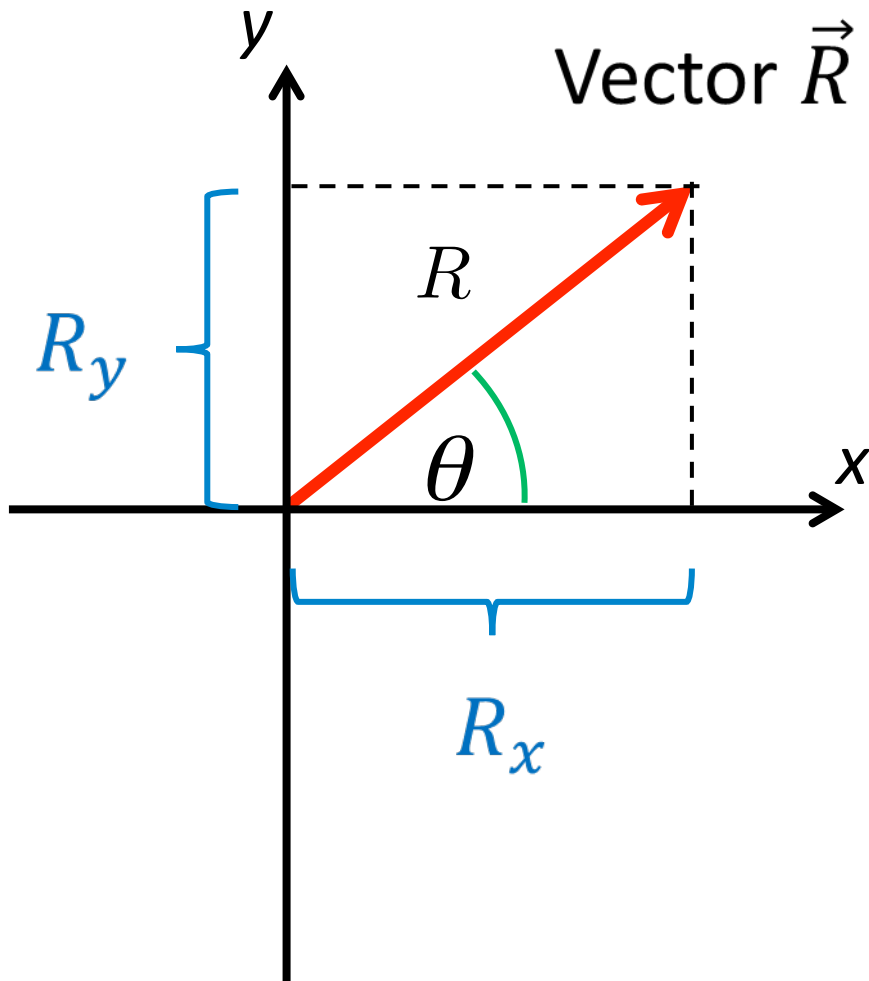
“vector components”

Sometimes also write:

$$\vec{R} = R_x \hat{i} + R_y \hat{j}$$

where \hat{i} and \hat{j} indicate the
 $+x$ and $+y$ directions

Vector Notation



$$1) (R, \theta) \rightarrow (R_x, R_y)$$

$$R_x = R \cos(\theta)$$

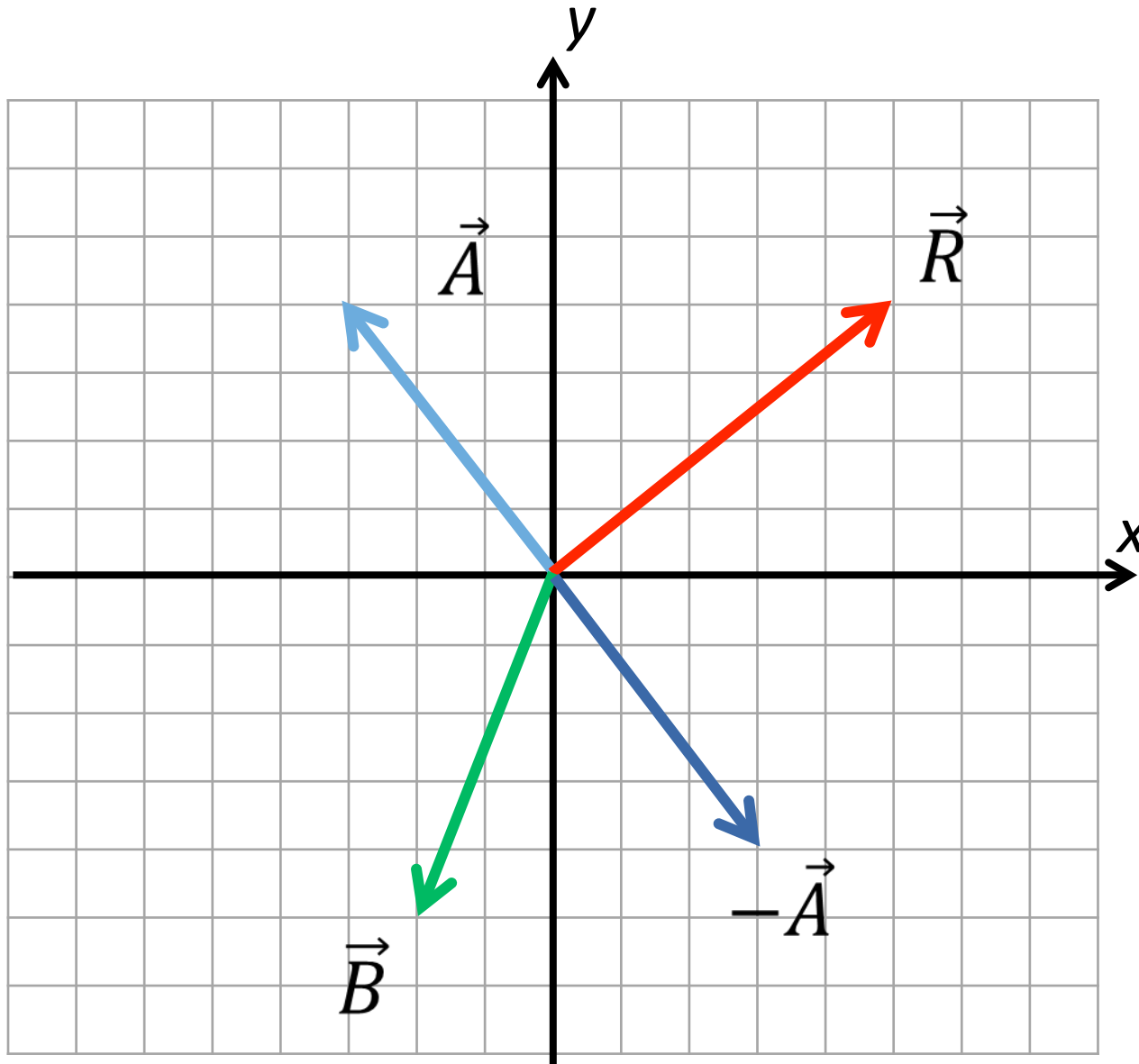
$$R_y = R \sin(\theta)$$

$$2) (R_x, R_y) \rightarrow (R, \theta)$$

$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

$$\tan(\theta) = R_y / R_x$$

Vector Notation

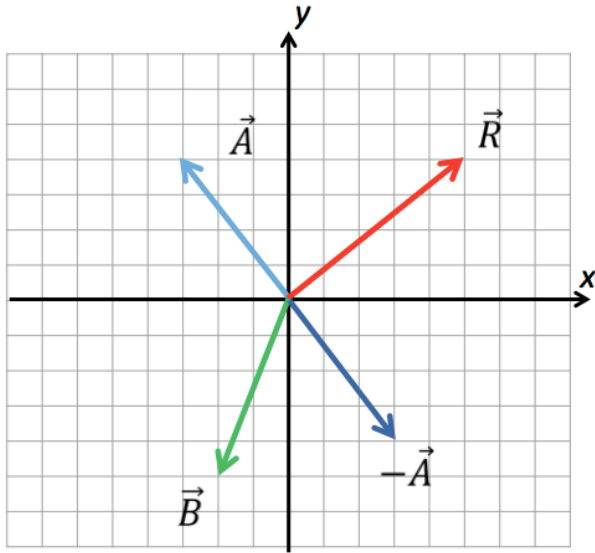


- degrees vs. radians
- +ve vs -ve angles:
(0 to 360) or
(-180 to 180)

Vector B:

$\text{atan}(-5/-2) = 68.2^\circ$
correct angle is:
 $68.2^\circ + 180^\circ = 248.2^\circ$
or
 $68.2^\circ - 180^\circ = -111.8^\circ$

Vector Notation: practice



See if you get the same answers as me:

(x,y), (r,theta)

Vector R:

(5,4), (6.4, 38.7° [0.675 rad])

Vector A:

(-3,4), (5, 126.9° [2.21 rad])

Vector B:

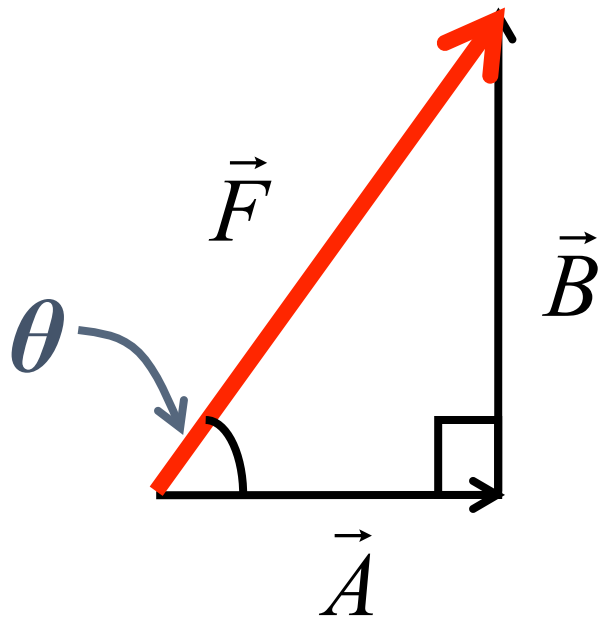
(-2,-5), (5.39, -111.8° [-1.95 rad])

Vector -A:

(3,-4), (5, -53.1° [-0.93 rad])

(same magnitude as A, but different by 180°)

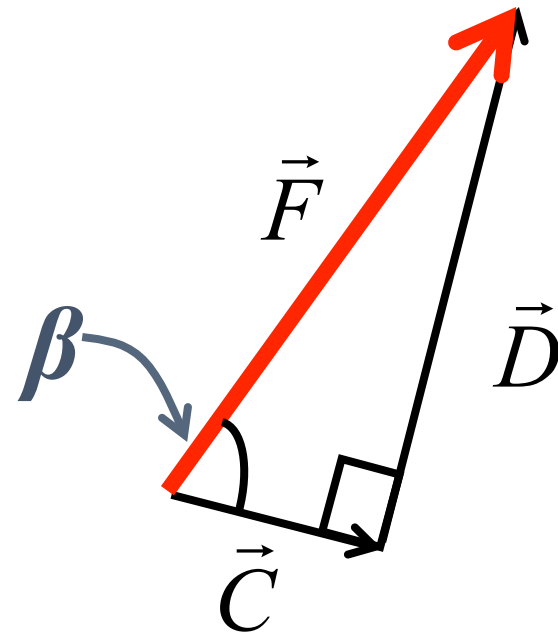
Vector Components



$$\vec{F} = \vec{A} + \vec{B}$$

$$|\vec{A}| = F \cos \theta$$

$$|\vec{B}| = F \sin \theta$$



$$\vec{F} = \vec{C} + \vec{D}$$

$$|\vec{C}| = F \cos \beta$$

$$|\vec{D}| = F \sin \beta$$

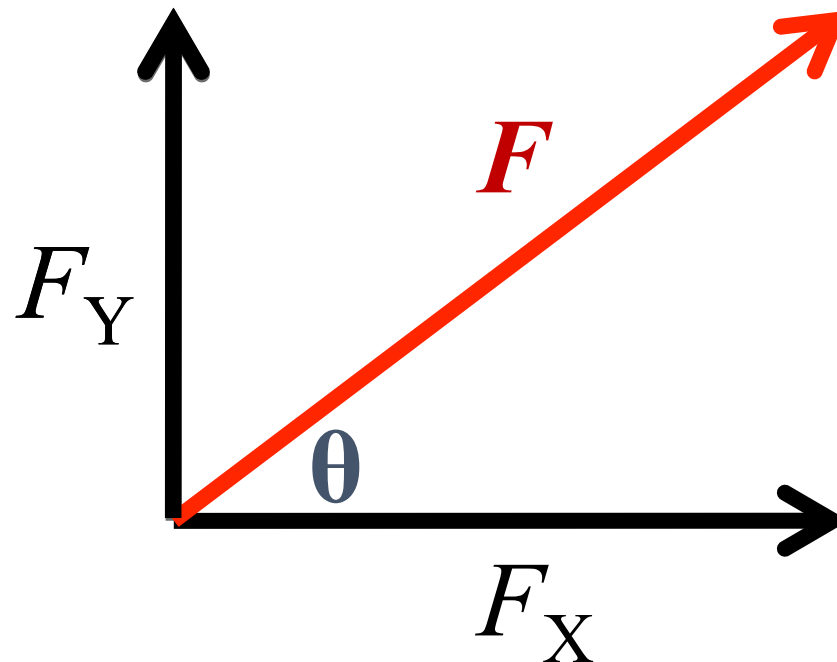
$\sin(\theta)$ or $\cos(\theta)$?

SOH CAH TOA

- **S**in = **O**pposite/**H**ypoteneuse
- **C**os = **A**djacent/**H**ypoteneuse
- **T**an = **O**pposite/**A**djacent

PEN SWIPE

- Swipe over θ gives 'cos'
- Swipe away from θ gives 'sin'



Adding two vectors using components

$$F_{2x} = F_2 \cos \alpha$$

$$F_{2y} = F_2 \sin \alpha$$

$$F_{1x} = F_1 \cos \beta$$

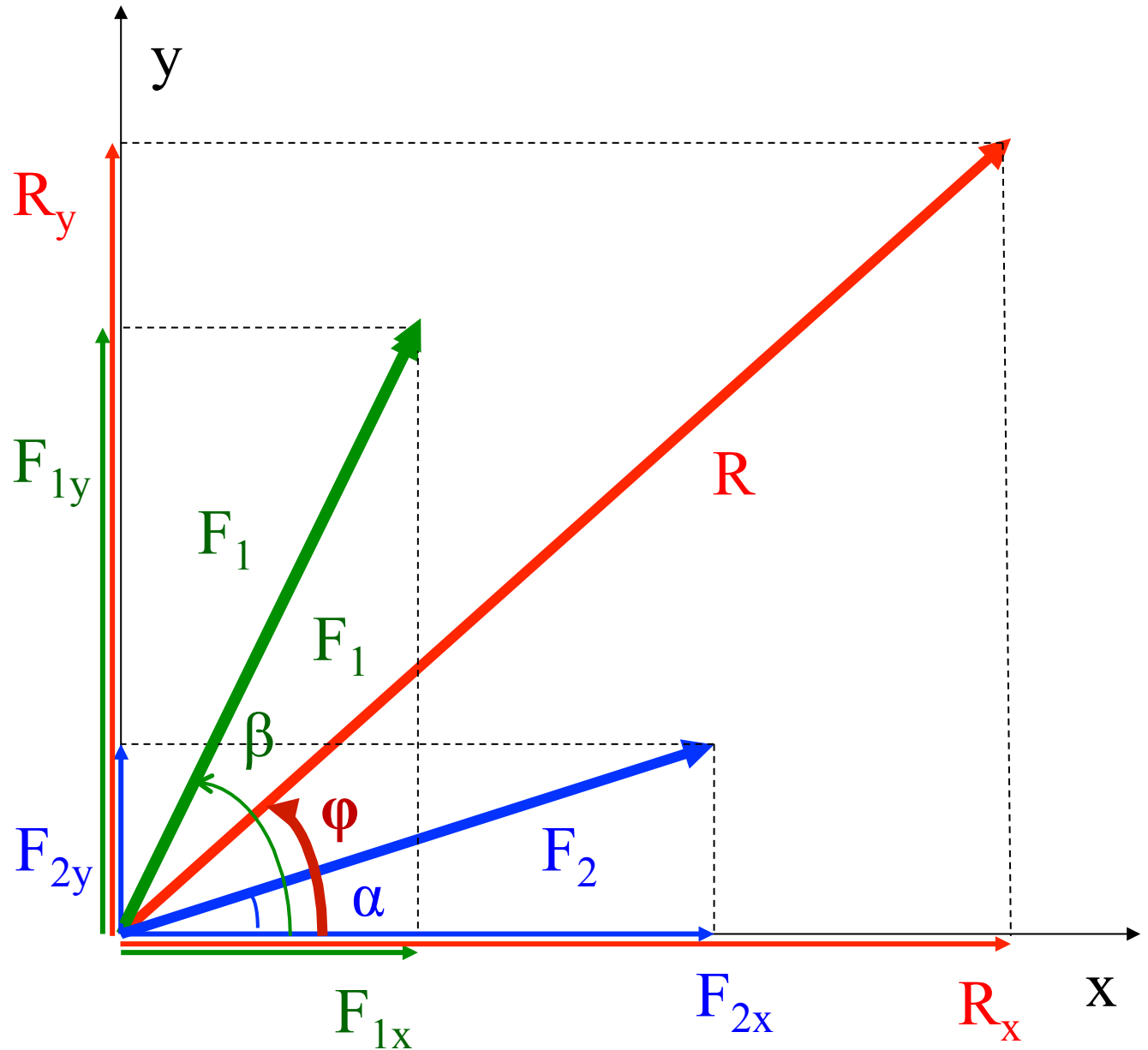
$$F_{1y} = F_1 \sin \beta$$

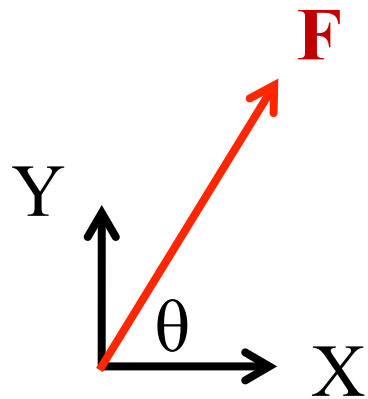
$$R_x = F_{1x} + F_{2x}$$

$$R_y = F_{1y} + F_{2y}$$

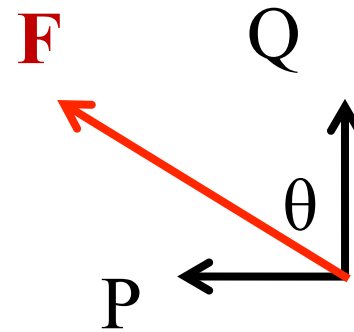
$$R = \sqrt{R_x^2 + R_y^2}$$

$$\tan \varphi = R_y / R_x$$

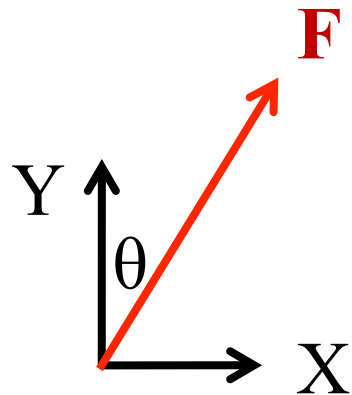




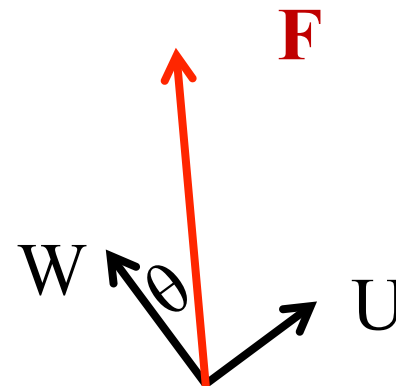
$$F_X = F \cos(\theta)$$



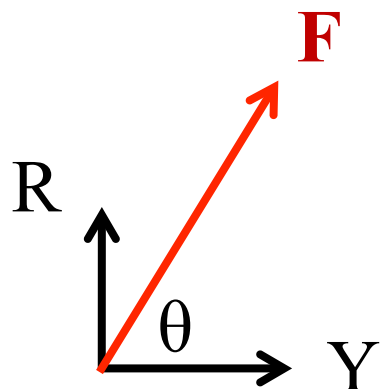
$$F_Q = F \cos(\theta)$$



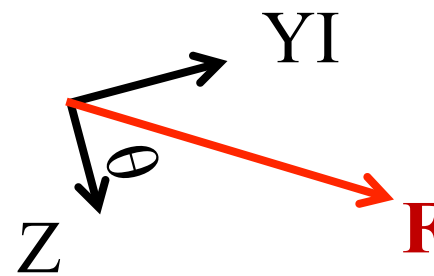
$$F_Y = F \cos(\theta)$$



$$F_U = F \sin(\theta)$$



$$F_R = F \sin(\theta)$$



$$F_{YI} = F \sin(\theta)$$

Theme 1

Introductory Material

Module T1M2:
Precision and Estimation

Learning Objectives

- Recognize that the presentation of a numerical quantity, using **significant figures** and **scientific notation**, reflects the accuracy of a measurement.
- Carry the appropriate significant figures through simple arithmetic calculations.
- Appreciate the importance of **estimating unknown quantities** as a means of understanding a system and predicting outcomes.
- Develop the skill of making an estimate and performing '**order of magnitude**' **approximations**.

Significant Figures

- When expressing a quantity, we want to communicate how precisely we know its value
- Example: My height is 16.705423 TP-sheets
- Is this a trustworthy statement?
- We usually count as “*significant*” all digits up to the first uncertain one
 - Based on how the measurement was made
 - Could be statistically determined (result of variations observed over successive measurements)

Significant Figures

- For a properly written quantity

- 43.84
- 27




Non-zero digits are typically assumed to be significant

- 5.0502
- 2001




Zeros on the interior of a number are significant

- 14.50
- 270.



Zeros on the right side of the number are typically significant

- 0.013
- 0.04710
- 270



The exception: sometimes zeros are used as placeholders, and are not significant.

How many sig figs?

Assuming these quantities have been properly expressed, how many significant figures does each have?

- 26.38 (4 sig figs),
- 27 (2 sig figs),
- 0.00500 (3 sig figs),
- 0.03040 (4 sig figs),
- 3.0880 (5 sig figs),
- 0.00418 (3 sig figs),
- 3.2088×10^6 (5 sig figs).

Sig Figs & Arithmetic

- When **adding/subtracting** quantities which have a specific number of significant figures

the number of decimal places in our answer must match that of the least reliable measurement

- Examples:

a) $2.54 \text{ cm} + 1.2 \text{ cm} = ?$ $= 3.7 \text{ cm}$ (**Not 3.74**)

b) $7.432 \text{ cm} + 2 \text{ cm} = ?$ $= 9 \text{ cm}$ (**Not 9.432**)

- Don't forget to round!

c) $7.632 \text{ cm} + 2 \text{ cm} = ?$ $= 10 \text{ cm}$ (**Not 9.632**)

Sig Figs & Arithmetic

- When **multiplying/dividing** quantities which have a specific number of significant figures

the number of significant figures in our answer must match that of the least reliable measurement

- Example:

a) $56.78 \text{ cm} \times 2.45 \text{ cm} = ?$

$$= 139 \text{ cm}^2 \text{ (Not } 139.111 \text{ cm}^2)$$

b) $813.2 \text{ m} \div 35 \text{ s} = ?$

$$= 23 \text{ m/s} \text{ (Not } 23.234 \text{ m/s)}$$

Sig Figs & Scientific Notation

- What about this one?

a) $8132 \text{ m} \div 35 \text{ s} = ?$

$$= 232 \text{ m/s} ?$$

$$= 230 \text{ m/s} ?$$

Use Scientific Notation: $8132 \text{ m} \div 35 \text{ s} = 2.3 \times 10^2 \text{ m/s}$

Clicker Quiz

- How many significant figures should be written in the sum of:

$$14.65 \text{ g} + 9.023 \text{ g} + 850.0078 \text{ g} + 26540.4390 + 0.80 \text{ g}?$$

- A. 3
- B. 4
- C. 5
- D. 6
- E. 7

Clicker Quiz

- How many significant figures should be written in the sum of:

$$14.65 \text{ g} + 9.023 \text{ g} + 850.0078 \text{ g} + 26540.4390 + 0.80 \text{ g}?$$

A. 3

B. 4

C. 5

D. 6

E. 7

Least reliable, therefore 2 decimal places:

Answer: E 27414.92

Clicker Quiz

- A parking lot is 134.3 m long and 37.66 m wide. The parking lot area is
 - A. $5.05774 \times 10^3 \text{ m}^2$
 - B. $5.0577 \times 10^3 \text{ m}^2$
 - C. $5.058 \times 10^3 \text{ m}^2$
 - D. $5.06 \times 10^3 \text{ m}^2$
 - E. $5.1 \times 10^3 \text{ m}^2$

Clicker Quiz

- A parking lot is 134.3 m long and 37.66 m wide. The parking lot area is

- A. $5.05774 \times 10^3 \text{ m}^2$
- B. $5.0577 \times 10^3 \text{ m}^2$
- C. $5.058 \times 10^3 \text{ m}^2$
- D. $5.06 \times 10^3 \text{ m}^2$
- E. $5.1 \times 10^3 \text{ m}^2$

Answer is C:

$$134.3 \text{ m} \times 37.66 \text{ m} \\ = 5057.738 \text{ m}^2$$

But should use 4 sig figs:

$$5058 \text{ m}^2, \\ \text{or } \mathbf{5.058 \times 10^3 \text{ m}^2}$$