12A3

Last Day: Trig. Integration, $\sin x$ & $\cos x$

Today: $\sec x$, $\tan x$ integration!

$\int \sec^2 x \, dx = \tan x + c$, $\int \sec x \tan x \, dx = \sec x + c$

eg. $\int \tan^2 x \sec^4 x \, dx = \int \tan^2 x \underbrace{\sec^2 x}_{\tan^2 x + 1} \underbrace{\sec^2 x \, dx}_{\frac{d}{dx} \tan x}$

So! Let $u = \tan x$ $du = \sec^2 x \, dx$

$\sec^2 x = \tan^2 x + 1 = 1 + u^2$

$$= \int u^2(1+u^2)\,du = \int u^2 + u^4\,du$$

$$= \frac{1}{3}u^3 + \frac{1}{5}u^5 + C$$

$$= \frac{1}{3}\tan^3 x + \frac{1}{5}\tan^5 x + C$$

In general If I have $\int \tan^m x \sec^n x\,dx$

If n even : let $u = \tan x$, $du = \sec^2 x\,dx$

$\sec^2 x = 1 + \tan^2 x = 1 + u^2$

If m odd let $u = \sec x$, $du = \sec x \tan x\,dx$

$\tan^2 x = \sec^2 x - 1 = u^2 - 1.$

$u = \sec x$

$du = \sec x \tan x \, dx$

eg. $\int \tan^3 x \sec^3 x \, dx$

$= \int \underbrace{\tan^2 x}_{(u^2-1)} \ \underbrace{\sec^2 x}_{u^2} \ \underbrace{\sec x \tan x \, dx}_{du}$

$= \int (u^2-1) \, u^2 \, du = \int u^4 - u^2 \, du$

$= \frac{1}{5} u^5 - \frac{1}{3} u^3 + C$

$= \frac{1}{5} \sec^5 x - \frac{1}{3} \sec^3 x + C$

Note: $\int \tan^m x \sec^n x \, dx$, $\left.\begin{matrix} m \text{ even} \\ n \text{ odd} \end{matrix}\right\} \Rightarrow$ cry!

eg
$$\int \sec x \, dx = \int \sec x \cdot \overbrace{\frac{(\sec x + \tan x)}{(\tan x + \sec x)}}^{=1} dx$$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\tan x + \sec x} \, dx \leftarrow du$$

$$\uparrow u$$

$$= \int \frac{1}{u} \, du = \ln|u| + C.$$

$$\boxed{\int \sec x \, dx = \ln|\sec x + \tan x| + C}$$

$$\int (\tan x)^0 (\sec x)^3 \, dx$$

$m = 0$ even, $n = 3$ odd $\Rightarrow$ Bad!

g $\int \sec^3 x \, dx$

$= \int (\sec x)(\sec^2 x)\, dx$ } try IbyP

$= \int u \, dv = uv - \int v\, du$

$= (\sec x)(\tan x) - \int (\tan x)(\sec x \tan x)\, dx$

$= \sec x \cdot \tan x - \int \sec x \tan^2 x \, dx$

$(\sec^2 x - 1)$

$\underline{\int \sec^3 x \, dx} = \sec x \cdot \tan x - \underline{\int \sec^3 x \, dx} + \int \sec x \, dx$

$2 \int \sec^3 x \, dx = \sec x \tan x + \ln|\sec x + \tan x| + C$

$$\Rightarrow \int \sec^3 x\,dx = \frac{1}{2}\sec x\tan x + \frac{1}{2}\ln|\sec x+\tan x| + C$$

$$\int \csc^2 x\,dx = -\cot x + c$$

& $$\int \csc x\cot x\,dx = -\csc x + c.$$

$$\vdots$$

One more Identity!

$$\int \sin(3x)\cos(5x)\,dx$$

Remember

$\sin(a)\sin(b)$

$= \frac{1}{2}\left(\cos(a-b) - \cos(a+b)\right)$

$\cos(a)\cos(b)$

$= \frac{1}{2}\left(\cos(a-b) + \cos(a+b)\right)$

$$\int \underbrace{\sin(3x)}_{b=3x} \underbrace{\cos(5x)}_{a=5x} \, dx$$

$$\cos(a)\sin(b) = \frac{1}{2}\left(\sin(a+b) - \sin(a-b)\right)$$

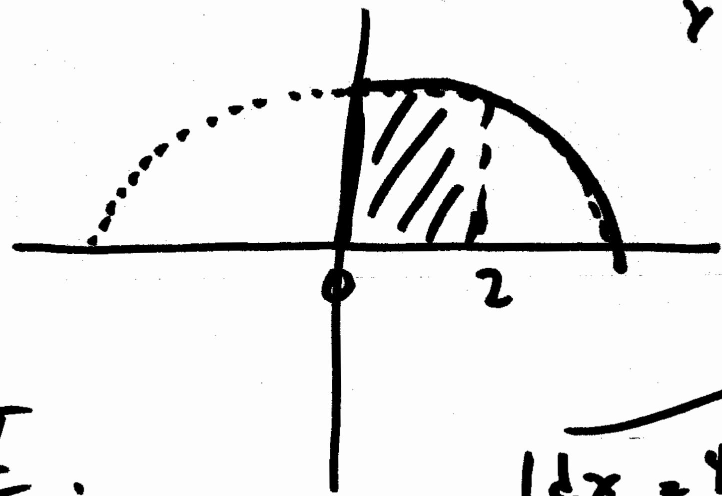
$$= \frac{1}{2} \int \sin(8x) - \sin(2x) \, dx$$

$$= \frac{1}{2}\left[-\frac{1}{8}\cos(8x) + \frac{1}{2}\cos(2x)\right] + C$$

$$= -\frac{1}{16}\cos(8x) + \frac{1}{4}\cos(2x) + C$$

Trig. Substitution

$y = \sqrt{16-x^2} \Rightarrow x^2+y^2 = 4^2$
$y \geq 0$



eg $\displaystyle\int_0^4 \sqrt{16-x^2}\,dx$

$= \frac{1}{4}\pi r^2 = \frac{1}{\cancel{4}}\pi \cdot 4^{\cancel{2}} = 4\pi$

eg $\displaystyle\int_0^2 \sqrt{16-x^2}\,dx$ } Try $x = 4\sin t$

$\frac{dx}{dt} = 4\cos t$

$\boxed{dx = 4\cos t\,dt}$

$16 - x^2 = 16 - 16\sin^2 t$

$= 16(1-\sin^2 t)$

$= 16\cos^2 t$

$= \displaystyle\int_0^{\pi/6} \sqrt{16\cos^2 t}\;\; 4\cos t\,dt$

$$= 16 \int_0^{\pi/6} |\cos t| \cos t \, dt$$

$$|\cos t| = \cos t$$

$$\cos t > 0$$

$$\sin(0) = 0$$

$$4 \sin(0) = 4 \cdot 0 = 0$$

$t = 0$ $\quad$ $x = 0$

$$x = 2 = 4 \sin t$$

$$\sin t = 1/2$$

$$t = \sin^{-1}(\tfrac{1}{2})$$

$$= \pi/6$$