COMPSCI/SFWRENG 2FA3
Discrete Mathematics with Applications II
Winter 2020

Week 07 Exercises

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Exercises

1. Construct deterministic finite automata $M = (Q, \Sigma, \delta, s, F)$ such that:

   a. $\Sigma = \{a, b, \ldots, z\}$ and $L(M)$ contains the single string *calculemus*.
   b. $\Sigma = \{a, b\}$ and $L(M) = \{x \in \Sigma^* \mid |x| \equiv 0 \bmod 3\}$.
   c. $\Sigma = \{0, 1\}$ and $L(M) = \{x \in \Sigma^* \mid x \text{ contains the string } 101\}$.
   d. $\Sigma = \{0, 1\}$ and $L(M)$ is set of strings in Σ^* of the form $0^m 1^n$ where $m, n \geq 1$.
   e. $\Sigma = \{a, b\}$ and $L(M)$ is the set of strings in Σ^* that contain at least three occurrences of *bbb*. Note: Overlapping is permitted so $bbbbb \in L(M)$.
   f. $\Sigma = \{a, b\}$ and $L(M) = \{x_1 a x_2 \mid |x_1| \geq 3 \text{ and } |x_2| \leq 4\}$.

2. Let $M = (Q, \Sigma, \delta, s, F)$ and $M' = (Q, \Sigma, \delta, s, Q \setminus F)$ be DFAs. Prove that $L(M') = {\sim}L(M)$.

3. Let Σ be a finite alphabet and $B \subseteq \Sigma^*$. B is *reflexive* if $\epsilon \in B$ and is *transitive* if $BB \subseteq B$. Prove that, if $A \subseteq \Sigma^*$, then A^* is smallest reflexive and transitive set containing A. That is, show that (1) $A \subseteq A^*$, (2) A^* is reflexive, (3) A^* is transitive, and (4) if B is any other reflexive and transitive set containing A, then $A^* \subseteq B$.

4. Let $M = (Q, \Sigma, \delta, s, F)$ be a DFA. Prove by induction on $|y|$ that, for all $x, y \in \Sigma^*$ and $q \in Q$,

$$\hat{\delta}(q, xy) = \hat{\delta}(\hat{\delta}(q, x), y).$$