COMPSCI/SFWRENG 2FA3 Midterm Test 1
McMaster University

Answer Key: Large arrow ($\Longleftarrow$) for correct, small ($\leftarrow$) for partially correct

Day Class CS 01, CS 02, SE 01, **Version 1** Dr. W. M. Farmer

DURATION: 2 hours February 5, 2020

Please CLEARLY print:

NAME:

Student ID:

In an addition to this examination paper, you will be given two answer sheets for this test. This examination paper includes 15 pages and 30 questions. You are responsible for ensuring that your copy of the examination paper is complete. Bring any discrepancy to the attention of your invigilator.

The examination will be conducted in two stages:

First Stage: You have 90 minutes to answer the questions in the examination paper on the first answer sheet working **by yourself**. Getting any help in any form from your fellow students and anyone else will be treated as academic dishonesty. You must submit your first answer sheet to your invigilator by the end of the 90-minute period. Your performance on the answer sheet counts for 85% of the Midterm Test 1 mark. You may want to fill out the second answer sheet as you fill out the first leaving blank those questions that you want to work on during the second stage.

Second Stage: You have 30 minutes to answer the questions in the examination paper on the second answer sheet working **with the other students in the test room**. You may walk around the test room, but you may not leave the test room. You must submit your second answer sheet and your examination paper to your invigilator by the end of the 30-minute period. Your performance on the answer sheet counts for 15% of the Midterm Test 1 mark.

Special Instructions:

1. It is your responsibility to ensure that the two answer sheets are properly completed. Your examination result depends upon proper attention to these instructions:

   - A heavy mark must be made, completely filling the circular bubble, with an HB pencil.
   - Print your name, student number, course name, course number and the date in the space provided on the top of Side 1 and **fill in the corresponding bubbles underneath**.
   - **Fill in the bubble corresponding to your version number.**
   - Mark only **ONE** choice from the alternatives (1, 2, 3, 4, 5 or A, B, C, D, E) provided for each question. If there is a True/False question, mark 1 (or A) for True, and 2 (or B) for False. The question number is to the left of the bubbles. Make sure that the number of the question on the scan sheet is the same as the number on the examination paper.

- Pay particular attention to the “Marking Directions” given on the scan sheet.
- Begin answering the questions using the first set of bubbles, marked “1.” Answer all questions.

2. The use of notes and textbooks is **not** permitted in both stages of the test.
3. Calculators, computers, cell phones, and all other electronic devices are **not** to be utilized in both stages of the test.
4. Read each question carefully.
5. Try to allocate your time sensibly and divide it appropriately between the questions.
6. Select the **best** answer for each question.

Question 1 [1 mark]
The best way of proving a universal statement

$$\forall x \in S \, . \, A$$

is almost always by some form of induction. Is this statement true or false?

A. True.

B. False. $\Longleftarrow$

ANSWER:
Proof by induction only works if S is $\mathbb{N}$, an inductive set, a well-ordered set, or a well-founded set.

Question 2 [1 mark]
If $S \subseteq \mathbb{R}$ has a least upper bound x with respect to the usual weak total order $\leq$ on $\mathbb{R}$, then $x \in S$. Is this statement true or false?

A. True.

B. False. $\Longleftarrow$

ANSWER:
The least upper bound of $S = \{r \in \mathbb{R} \mid r < 1\}$ is 1, but $1 \notin S$.

Question 3 [1 mark]
Traditional proofs are usually better for communicating mathematical ideas than formal proofs. Is this statement true or false?

A. True. $\Longleftarrow$

B. False.

ANSWER:
Traditional proofs are best for communication, while formal proofs are best for certification.

Question 4 [1 mark]
Let $R \subseteq U \times U$. If (U, R) is Noetherian, then it is a well-order. Is this statement true or false?

A. True.

B. False. $\Longleftarrow$

ANSWER:
(U, R) is Noetherian iff it is well-founded, but (U, R) could be well-founded without being a well-order.

Question 5 [1 mark]
It is easy to prove that the Ackermann function is total (i.e., defined on all inputs) using weak induction. Is this statement true or false?

A. True.

B. False. $\Longleftarrow$

ANSWER:
The Ackermann can be easily proved to be total by induction for $(\mathbb{N} \times \mathbb{N}, <_{\mathsf{lex}})$. It is much harder to prove by weak or strong induction.

Question 6 [1 mark]
In MSFOL, terms denote nonboolean values, while formulas denote boolean values. Is this statement true or false?

A. True. $\Longleftarrow$

B. False.

ANSWER:
Terms are for referring to or describing values, while formulas are for making true-or-false assertions about values.

Question 7 [1 mark]
If Σ is a signature $(\mathcal{B}, \mathcal{C}, \mathcal{F}, \mathcal{P}, \tau)$ of MSFOL and t is a Σ-term of type α, then $\alpha \in \mathcal{B}$. Is this statement true or false?

A. True. $\Longleftarrow$

B. False.

ANSWER:
The type of a Σ-term must be a base type of Σ.

Question 8 [1 mark]
Both weak and strong induction are special cases of well-founded induction. Is this statement true or false?

A. True. $\Longleftarrow$

B. False.

ANSWER:
The weak and strong induction principles are essentially the well-founded induction principles for $(\mathbb{N}, R_{\mathsf{suc}})$ and $(\mathbb{N}, <)$, respectively.

Question 9 [1 mark]
Strong induction is *not* a special case of

A. Structural induction. $\Longleftarrow$

B. Ordinal induction.

C. Well-founded induction.

D. Any of the above.

ANSWER:
Strong induction is not a special case of structural induction, while it is a special case of both ordinal induction and well-founded induction since $(\mathbb{N}, <)$ is both a well-order and well-founded.

Question 10 [1 mark]
If U is nonempty and (U, R) is both an equivalence relation and a weak total order, then

A. U contains a single element. $\Longleftarrow$

B. U contains at least two elements.

C. (U, R) has neither a maximum nor a minimum element.

D. (U, R) is well-founded.

ANSWER:
Suppose $x, y \in U$. Then $x \; R \; y$ or $y \; R \; x$ by the total axiom. This implies $x \; R \; y$ and $y \; R \; x$ by the symmetry axiom. And this implies $x = y$ by antisymmetry axiom.

Question 11 [1 mark]
Let $\mathsf{BinTree}$ be the inductive set defined by the following constructors:

$\mathsf{Leaf} : \mathbb{N} \to \mathsf{BinTree}.$

$\mathsf{Branch} : \mathsf{BinTree} \times \mathbb{N} \times \mathsf{BinTree} \to \mathsf{BinTree}.$

Then $\mathsf{sum} : \mathsf{BinTree} \to \mathbb{N}$ is defined by structural recursion as:

$\mathsf{sum}(\mathsf{Leaf}(n)) = n.$

$\mathsf{sum}(\mathsf{Branch}(t_1, n, t_2)) = n + \mathsf{sum}(t_1) + \mathsf{sum}(t_2).$

We can consider this definition of sum to be by natural number recursion if we choose the complexity function $c : \mathsf{BinTree} \to \mathbb{N}$ to be defined as:

A. $c(t)$ is the number of nodes in the tree represented by t.

B. $c(t)$ is the number of Leaf constructors occurring in t.

C. $c(t)$ is the number of Branch constructors occurring in t.

D. Any of the above. $\Longleftarrow$

ANSWER:
For each definition of c, $c(t_1), c(t_2) < c(\mathsf{Branch}(t_1, n, t_2))$.

Question 12 [1 mark]
Let S be a nonempty set and $R \subseteq S \times S$. Define $X_{\mathsf{eq-rel}}$, $X_{\mathsf{pre-ord}}$, $X_{\mathsf{w-par-ord}}$, and $X_{\mathsf{w-tot-ord}}$ to be the set of all equivalence relations, pre-orders, weak partial orders, and weak total orders of the form (S, R), respectively. What is the relationship between these four sets of structures?

A. $X_{\mathsf{eq-rel}} \subseteq X_{\mathsf{pre-ord}} \subseteq X_{\mathsf{w-par-ord}} \subseteq X_{\mathsf{w-tot-ord}}$.

B. $X_{\mathsf{pre-ord}} \subseteq X_{\mathsf{eq-rel}}$ and $X_{\mathsf{pre-ord}} \subseteq X_{\mathsf{w-par-ord}} \subseteq X_{\mathsf{w-tot-ord}}$.

C. $X_{\mathsf{w-tot-ord}} \subseteq X_{\mathsf{w-par-ord}}$ and $X_{\mathsf{pre-ord}} \subseteq X_{\mathsf{eq-rel}}$.

D. $X_{\mathsf{w-tot-ord}} \subseteq X_{\mathsf{w-par-ord}} \subseteq X_{\mathsf{pre-ord}}$ and $X_{\mathsf{eq-rel}} \subseteq X_{\mathsf{pre-ord}}$. $\Longleftarrow$

ANSWER:
The axioms of a weak total order include the axioms of a weak partial order which includes the axioms of a pre-order, and the axioms of an equivalence relation include the axioms of a pre-order.

Question 13 [1 mark]
Traditional proofs

A. Can be easily checked by a machine.

B. Are usually written in a formal language.

C. May include unstated assumptions. $\Longleftarrow$

D. All of the above.

ANSWER:
Traditional proofs often included unstated assumptions. Formal proofs, not traditional proofs, satisfy A and B.

Question 14 [1 mark]
An implication $A \Rightarrow B$ can be proved by

A. Assuming $\neg B$ and proving $\neg A$. $\Longleftarrow$

B. Assuming $\neg A$ and proving B.

C. Proving A or proving B.

D. None of the above.

ANSWER:
A is correct since $A \Rightarrow B$ is logically equivalent to $\neg B \Rightarrow \neg A$, the contrapositive of $A \Rightarrow B$.

Question 15 [1 mark]
Any proof of

$$\forall\, n \in \mathbb{N} \,.\, P(n)$$

by strong induction can be easily transformed into a proof of

$$\forall\, n \in \mathbb{N} \,.\, Q(n)$$

by weak induction where

A. $Q(n) \equiv P(n) \wedge P(n+1)$.

B. $Q(n) \equiv \forall\, m \in \mathbb{N} \,.\, n \leq m \Rightarrow P(m)$.

C. $Q(n) \equiv \forall\, m \in \mathbb{N} \,.\, m \leq n \Rightarrow P(m)$. $\Longleftarrow$

D. $Q(n) \equiv P(0) \wedge (\forall\, m \in \mathbb{N} \,.\, P(m) \Rightarrow P(m+1))$.

ANSWER:
Only C is correct.

Question 16 [1 mark]
Weak induction is essentially the same induction principle as

A. The structural induction principle for the natural numbers. $\Longleftarrow$

B. The ordinal induction principle for $(\mathbb{N}, <)$.

C. The well-founded induction principle for $(\mathbb{N}, <)$.

D. All of the above.

ANSWER:
The structural induction principle for the inductive set Nat defined in the lectures is essentially weak induction.

Question 17 [1 mark]
Define $\Sigma = (\{\alpha, \beta, \gamma\}, \emptyset, \emptyset, \{p, q\}, \tau)$ be a signature of MSFOL where $\tau(p) = \tau(q) = \alpha \times \beta \times \gamma \to \mathbb{B}$. Let A be the Σ-formula

$$(\exists\, y : \beta \,.\, p(x : \alpha, y : \beta, z : \gamma)) \Rightarrow (\forall\, z : \gamma \,.\, q(x : \alpha, y : \beta, z : \gamma)).$$

What is the values of $\mathsf{fvar}(A)$ and $\mathsf{bvar}(A)$?

A. $\{x : \alpha\}$ and $\{y : \beta, z : \gamma\}$.

B. $\{x : \alpha, z : \gamma\}$ and $\{y : \beta, z : \gamma\}$.

C. $\{x : \alpha, y : \beta, z : \gamma\}$ and $\{y : \beta, z : \gamma\}$. $\Longleftarrow$

D. $\{x : \alpha, y : \beta, z : \gamma\}$ and $\{x : \alpha, y : \beta, z : \gamma\}$.

ANSWER:
Only C is correct.

Question 18 [1 mark]
Define $\Sigma = (\{\alpha\}, \emptyset, \emptyset, \{p, q\}, \tau)$ be a signature of MSFOL where $\tau(p) = \tau(q) = \alpha \times \alpha \times \alpha \to \mathbb{B}$. Let A be the Σ-formula

$$(\exists\, y : \alpha \,.\, p(x : \alpha, y : \alpha, z : \alpha)) \Rightarrow (\forall\, z : \alpha \,.\, q(x : \alpha, y : \alpha, z : \alpha)).$$

Which of the following substitutions results in a variable capture?

A. $A[y \mapsto (x : \alpha)]$.

B. $A[y \mapsto (y : \alpha)]$.

C. $A[y \mapsto (z : \alpha)]$. $\Longleftarrow$

D. None of the above.

ANSWER:
Only C is correct.

Question 19 [1 mark]
Let $\Sigma = (\{\alpha, \beta\}, \{c, d\}, \{f, g\}, \emptyset, \tau)$ be a signature of MSFOL where $\tau(c) = \beta$, $\tau(d) = \alpha$, $\tau(f) = \alpha \times \beta \rightarrow \beta$, and $\tau(g) = \alpha \rightarrow \beta$. Which of the following is a Σ-term?

A. $f(c, d)$.

B. $f(d, g(d))$. $\Longleftarrow$

C. $g(f(d, c))$.

D. $f(f(c, c), c)$.

ANSWER:
Only B is a Σ-term. A, C, and D are not Σ-terms because the types of their subcomponents are mismatched.

Question 20 [1 mark]
A mathematical proof is

A. A plausible argument that a statement is true.

B. A deductive argument that a statement follows from a set of premises. $\Longleftarrow$

C. A preponderance of evidence for the truth of a statement.

D. An evaluation of a boolean-valued expression.

ANSWER:
B is correct. A and C are incorrect. D is only sometimes correct.

Question 21 [1 mark]
Let $\Sigma = (\{\alpha\}, \mathcal{C}, \mathcal{F}, \mathcal{P}, \tau)$ be a signature of MSFOL. If $d \in \mathcal{C} \cup \mathcal{F} \cup \mathcal{P}$, then which of following would not be a legitimate value for $\tau(d)$?

A. α.

B. $\alpha \times \alpha \rightarrow \mathbb{B}$.

C. $\alpha \rightarrow \alpha$

D. $\mathbb{B} \rightarrow \alpha$. $\Longleftarrow$

ANSWER:
$\mathbb{B} \rightarrow \alpha$ is a $\{\alpha\}$-type, but in MSFOL it cannot be the value of $\tau(d)$ for any constant, function, or predicate symbol d.

Question 22 [**1 mark**]
Consider the following proof.
Proof Let $P(\alpha) \equiv a(\alpha) \leq b(\alpha) + 1$. We will prove $P(\alpha)$ for all $\alpha \in \mathsf{Type}$ by structural induction.

Base case: $\alpha = \mathbb{B}$ or $\alpha \in \mathcal{B}$. We need to prove $P(\alpha)$.

$$\begin{aligned} & a(\alpha) \\ \leq\ & 1 \\ =\ & 0 + 1 \\ =\ & b(\alpha) + 1 \end{aligned}$$

So $P(\alpha)$ holds.
Induction step: $\alpha = C(\beta_1, \beta_2)$ where C is $\mathsf{Function}$ or $\mathsf{Product}$ and $\beta_1, \beta_2 \in \mathsf{Type}$. Assume $P(\beta_1)$ and $P(\beta_2)$. We need to prove $P(\alpha)$.

$$\begin{aligned} & a(C(\beta_1, \beta_2)) \\ =\ & a(\beta_1) + a(\beta_2) \\ \leq\ & (b(\beta_1) + 1) + (b(\beta_2) + 1) \\ =\ & (1 + b(\beta_1) + b(\beta_2)) + 1 \\ =\ & b(C(\beta_1, \beta_2)) + 1 \end{aligned}$$

So $P(\alpha)$ holds.

Therefore, $P(\alpha)$ holds for all $\alpha \in \mathsf{Type}$ by structural induction. $\square$

How many times are $P(\beta_1)$ and $P(\beta_2)$ used, respectively, in the induction step?

A. 1 and 0.

B. 0 and 1.

C. 1 and 1. $\Longleftarrow$

D. 2 and 2.

ANSWER:
$P(\beta_1)$ and $P(\beta_2)$ are each used once in the third line of the induction step.

Question 23 [1 mark]
In MSFOL, $\exists\, x : \alpha \,.\, A$ stands for

A. $\forall\, x : \alpha \,.\, \neg A$.

B. $\neg(\forall\, x : \alpha \,.\, A)$.

C. $\neg(\forall\, x : \alpha \,.\, \neg A)$. $\Longleftarrow$

D. None of the above.

ANSWER:
Only C expresses the meaning of the existential quantifier in terms of the universal quantifier.

Question 24 [1 mark]
Let Poly be the inductive type defined by the following constructors:

$\mathsf{X} : \mathsf{Poly}$.

$\mathsf{Coeff} : \mathbb{Z} \to \mathsf{Poly}$.

$\mathsf{Sum} : \mathsf{Poly} \times \mathsf{Poly} \to \mathsf{Poly}$.

$\mathsf{Prod} : \mathsf{Poly} \times \mathsf{Poly} \to \mathsf{Poly}$.

Then $f : \mathsf{Poly} \times \mathsf{Poly} \to \mathsf{Poly}$ is defined by:

$f(\mathsf{X}, q) = q$.

$f(\mathsf{Coeff}(n), q) = \mathsf{Coeff}(n)$.

$f(\mathsf{Sum}(p_1, p_2), q) = \mathsf{Sum}(f(p_1, q), f(p_2, q))$.

$f(\mathsf{Prod}(p_1, p_2), q) = \mathsf{Prod}(f(p_1, q), f(p_2, q))$.

If $r_1 = \mathsf{Prod}(\mathsf{Sum}(\mathsf{X}, \mathsf{Coeff}(1)), \mathsf{Sum}(\mathsf{X}, \mathsf{Coeff}(-1)))$ and $r_2 = \mathsf{Coeff}(2)$, then $f(r_1, r_2)$ is

A. $\mathsf{Coeff}(3)$.

B. $\mathsf{Sum}(\mathsf{Coeff}(2), \mathsf{Coeff}(1))$.

C. $\mathsf{Sum}(\mathsf{Prod}(\mathsf{Coeff}(2), \mathsf{Coeff}(2)), \mathsf{Coeff}(-1))$.

D. $\mathsf{Prod}(\mathsf{Sum}(\mathsf{Coeff}(2), \mathsf{Coeff}(1)), \mathsf{Sum}(\mathsf{Coeff}(2), \mathsf{Coeff}(-1)))$. $\Longleftarrow$

ANSWER:
f formalizes the substitution of one polynomial for the occurrences of x in another polynomial. With that understanding, D is the obvious answer.

Question 25 [1 mark]
Which form of induction is especially suited for formulating complicated induction arguments?

A. Ordinal induction. $\Longleftarrow$

B. Strong induction.

C. Structural induction.

D. Weak induction.

ANSWER:
Yes, ordinal induction is often the best choice for complicated induction arguments.

Question 26 [1 mark]
Both weak and strict partial orders are

A. Reflexive.

B. Asymmetric.

C. Antisymmetric.

D. Transitive. $\Longleftarrow$

ANSWER:
Transitivity is the only axiom that weak and strict partial order have in common.

The next four questions are based on the following two definitions:

Color is the inductive set defined by the following constructors:

$\mathsf{Blue} : \mathsf{Color}.$

$\mathsf{Red} : \mathsf{Color}.$

Tree is the inductive set defined by the following constructors:

$\mathsf{Leaf} : \mathbb{Q} \to \mathsf{Tree}.$

$\mathsf{Branch1} : \mathsf{Color} \times \mathsf{Tree} \to \mathsf{Tree}.$

$\mathsf{Branch2} : \mathsf{Color} \times \mathsf{Tree} \times \mathsf{Tree} \to \mathsf{Tree}.$

$\mathbb{Q}$ is here the set of rational numbers.

Question 27 [**1 mark**]
The sets Color and Tree contain _______ elements and _______ elements, respectively. Which answer fills in the blanks in order correctly?

A. 2 and 3.

B. 2 and 6.

C. 2 and infinitely many. $\Longleftarrow$

D. infinitely many and infinitely many.

ANSWER:
The type Color has just two values, Blue and Red. Since the type Tree is recursive, it has infinitely many members.

Question 28 [**1 mark**]
The structural induction principle induced by the definition of Tree has the form

$$((\forall\, q \in \mathbb{Q} \,.\, P(\mathsf{Leaf}(q)) \wedge X \wedge Y) \Rightarrow (\forall\, t : \mathsf{Tree} \,.\, P(t))$$

where X and Y are _______ and _______ and P is any property of the members of Tree.

A. $\forall\, c \in \mathsf{Color}, t \in \mathsf{Tree} \,.\, P(\mathsf{Branch1}(c, t))$ and
$\forall\, c \in \mathsf{Color}, t_1, t_2 \in \mathsf{Tree} \,.\, P(\mathsf{Branch2}(c, t_1, t_2))$.

B. $\forall\, c \in \mathsf{Color}, t \in \mathsf{Tree} \,.\, P(t) \wedge P(\mathsf{Branch1}(c, t))$ and
$\forall\, c \in \mathsf{Color}, t_1, t_2 \in \mathsf{Tree} \,.\, P(t_1) \wedge P(t_2) \wedge P(\mathsf{Branch2}(c, t_1, t_2))$.

C. $\forall\, t \in \mathsf{Tree} \,.\, P(t) \Rightarrow \forall\, c \in \mathsf{Color} \,.\, P(\mathsf{Branch1}(c, t))$ and
$\forall\, t_1, t_2 \in \mathsf{Tree} \,.\, (P(t_1) \wedge P(t_2)) \Rightarrow \forall\, c \in \mathsf{Color} \,.\, P(\mathsf{Branch2}(c, t_1, t_2))$. $\Longleftarrow$

D. $\forall\, t \in \mathsf{Tree} \,.\, (\forall\, c \in \mathsf{Color} \,.\, P(\mathsf{Branch1}(c, t))) \Rightarrow P(t)$ and
$\forall\, c \in \mathsf{Color}, t_1, t_2 \in \mathsf{Tree} \,.\, (\forall\, c \in \mathsf{Color} \,.\, P(\mathsf{Branch2}(c, t_1, t_2))) \Rightarrow (P(t_1) \wedge P(t_2))$.

ANSWER:
Only C is correct. A, B, and C do not assume that P holds for subtrees.

Question 29 [1 mark]
Consider the function $\mathsf{val} : \mathsf{Tree} \to \mathbb{Q}$ defined by pattern matching as follows:

$$\mathsf{val}(\mathsf{Leaf}(q)) = q.$$

$$\mathsf{val}(\mathsf{Branch1}(\mathsf{Blue}, t)) = -\mathsf{val}(t).$$

$$\mathsf{val}(\mathsf{Branch1}(\mathsf{Red}, t)) = \frac{1}{\mathsf{val}(t)}.$$

$$\mathsf{val}(\mathsf{Branch2}(\mathsf{Blue}, t_1, t_2)) = \mathsf{val}(t_1) + \mathsf{val}(t_2).$$

$$\mathsf{val}(\mathsf{Branch2}(\mathsf{Red}, t_1, t_2)) = \mathsf{val}(t_1) * \mathsf{val}(t_2).$$

What is the value of $\mathsf{val}(t)$ where t is

$$\begin{aligned}&\mathsf{Branch1}(\mathsf{Red},\\ &\qquad\mathsf{Branch2}(\mathsf{Blue},\\ &\qquad\qquad\mathsf{Branch1}(\mathsf{Blue}, \mathsf{Leaf}(2)),\\ &\qquad\qquad\mathsf{Branch2}(\mathsf{Red}, \mathsf{Leaf}(3), \mathsf{Leaf}(2))))?\end{aligned}$$

A. -2.

B. $-\frac{1}{2}$.

C. $\frac{1}{4}$. $\Longleftarrow$

D. 4.

ANSWER:
Just use the defining equations of val to compute $\mathsf{val}(t)$.

Question 30 [1 mark]
The definition of val shows that members of Tree represent mathematical expressions constructed from the rational numbers using additive inverse, multiplicative inverse, addition, and multiplication operators. Consider following the mathematical expression:

$$2 * (2 + 2)^{-1}$$

(which can also be written as $\frac{2}{2+2}$). Which member of Tree represents it?

A. $\mathsf{Branch2}(\mathsf{Blue}, \mathsf{Leaf}(2), \mathsf{Branch1}(\mathsf{Blue}, \mathsf{Branch2}(\mathsf{Red}, \mathsf{Leaf}(2), \mathsf{Leaf}(2))))$.

B. $\mathsf{Branch2}(\mathsf{Red}, \mathsf{Leaf}(2), \mathsf{Branch2}(\mathsf{Blue}, \mathsf{Leaf}(2), \mathsf{Leaf}(2)))$.

C. $\mathsf{Branch2}(\mathsf{Red}, \mathsf{Leaf}(2), \mathsf{Branch1}(\mathsf{Red}, \mathsf{Branch2}(\mathsf{Blue}, \mathsf{Leaf}(2), \mathsf{Leaf}(2))))$. $\Longleftarrow$

D. $\mathsf{Branch1}(\mathsf{Red}, \mathsf{Branch2}(\mathsf{Red}, \mathsf{Leaf}(2), \mathsf{Branch2}(\mathsf{Blue}, \mathsf{Leaf}(2), \mathsf{Leaf}(2))))$.

ANSWER:
C represents $2 * (2 + 2)^{-1}$. A, B, and D represent $2 + (-(2 * 2))$, $2 * (2 + 2)$, and $(2 * (2 + 2))^{-1}$, respectively.

Please bubble in your **student number** and **version number** on your scan sheet!

THE END.