Math 1AA3/1ZB3

Sample Test 3, Version #1

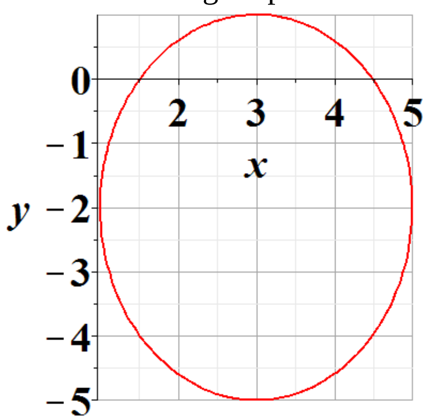
Name:____

(Last Name) (First Name)

Student Number:____________________ **Tutorial Number:**___________________

This test consists of 16 multiple choice questions worth 1 mark each (no part marks), and 1 question worth 1 mark (no part marks) on proper computer card filling. All questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Calculators are NOT allowed.

1. Find the foci of the following ellipse.



(a) $(\sqrt{3},\, \pm 5)$ **(b)** $(3,\, \pm\sqrt{5}-2)$ **(c)** $(0,\, \pm\sqrt{5})$ **(d)** $(\sqrt{5},\, \pm 3)$
(e) $(\pm\sqrt{5}-3,\, 2)$

2. Find the vertices of the following conic section.

$$25x^2 - 4y^2 + 50x + 16y = 91.$$

(a) $(-3,2),(1,2)$ **(b)** $(0,\, \pm 5)$ **(c)** $(-1,7),(-1,-3)$ **(d)** $(-1,-2),(3,-2)$
(e) $(1,3),(1,-7)$

3. Evaluate the following limit,

$$\lim_{(x,y)\to(0,0)} \frac{xye^y}{x^4+4y^2}$$

(a) 0 **(b)** d.n.e. **(c)** 1 **(d)** $\frac{1}{2}$ **(e)** $\frac{1}{4}$

4. Let $w(s,t) = F(u(s,t), v(s,t))$, where F, u, and v are differentiable, $u(1,0) = 2$, $u_s(1,0) = -2,\ u_t(1,0) = 6,\ v(1,0) = 3,\ v_s(1,0) = 5,\ v_t(1,0) = 4,\ F_u(2,3) = -1,$ and $F_v(2,3) = 10$. Find $w_t(1,0)$.
(a) 41 **(b)** 5 **(c)** 16 **(d)** 28 **(e)** 34

5. Which of the following differential equations is satisfied by the function $u = \ln\sqrt{x^2 + y^2}$?
(a) $u_{xy} - u_{xx} = 0$ **(b)** $2u_{xy} - u_{xx} = 0$ **(c)** $u_{xx} + u_{yy} = 0$
(d) $u_{xx} - u_{yy} = 0$ **(e)** $2u_{xx} - u_{yy} = 0$

6. Find the linearization of $f(x,y) = y \ln x$ at $(2,1)$.
(a) $\frac{1}{2}x + (\ln 2)y - 1$ **(b)** $\frac{1}{2}x - (\ln 2)y - 1$ **(c)** $\frac{1}{2}x + (\ln 2)y + 1$
(d) $2x - 1$ **(e)** $2x + (\ln 2)y - 1$

7. Find the equation of the tangent plane to the surface $z = 5 - x^2 + y^2$ at the point $(-2, 1, 2)$.
(a) $z = 4x + 2y + 8$ **(b)** $z = 4x + 2y - 8$ **(c)** $z = 4x - 2y + 8$
(d) $z = 4x - 2y - 8$ **(e)** $z = 4x - 2y + 6$

8. Estimate the volume of the solid that lies below the surface $z = xy$ and above the rectangle

$$R = \{(x,y)\,|\,0 \le x \le 6,\ 0 \le y \le 4\}$$

Use a Riemann sum with $m = 3,\ n = 2$, and take the sample point to be the lower left corner of each square.
(a) 32 **(b)** 24 **(c)** 16 **(d)** 48 **(e)** 96

9. Set up a double integral which represents the volume under the function $z = 2xy$ and above the triangular region with vertices $(0,0), (1,2),$ and $(0,3)$.
(a) $\displaystyle\int_0^1 \int_x^{-x+3} 2xy\,dy\,dx$ **(b)** $\displaystyle\int_0^1 \int_{2x}^{3} 2xy\,dy\,dx$ (c) $\displaystyle\int_0^1 \int_0^{-x+3} 2xy\,dy\,dx$
(d) $\displaystyle\int_0^1 \int_{2x}^{-x} 2xy\,dy\,dx$ **(e)** $\displaystyle\int_0^1 \int_{2x}^{-x+3} 2xy\,dy\,dx$

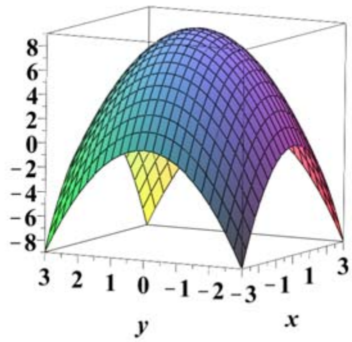
10. Evaluate

$$\int_0^2 \int_0^1 y e^{xy} dy dx$$

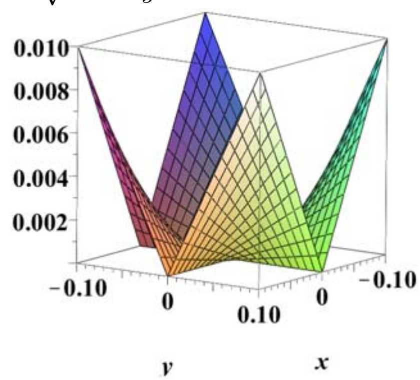
(a) $\frac{1}{2}(e^2 - 3)$ **(b)** $\frac{1}{4}(e^2 + 2)$ **(c)** $e^2 - 1$ **(d)** $e^2 + 3$ **(e)** $\frac{1}{3}(e^2 - 1)$

11. Sketch the graph of the function $f(x, y) = \sqrt{x^2 + y^2}$

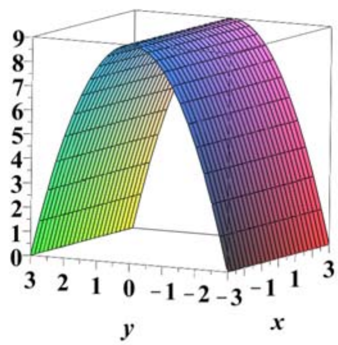
(a)



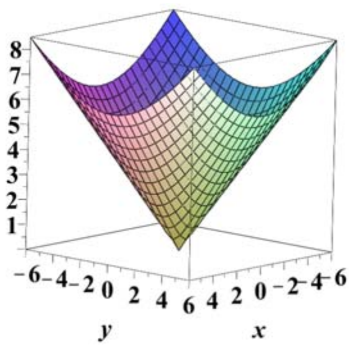
(b)



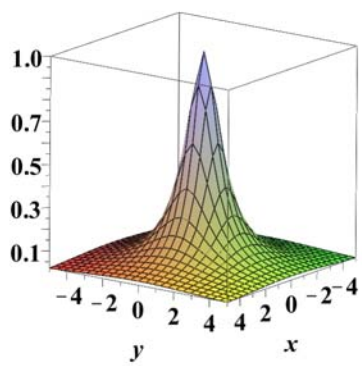
(c)



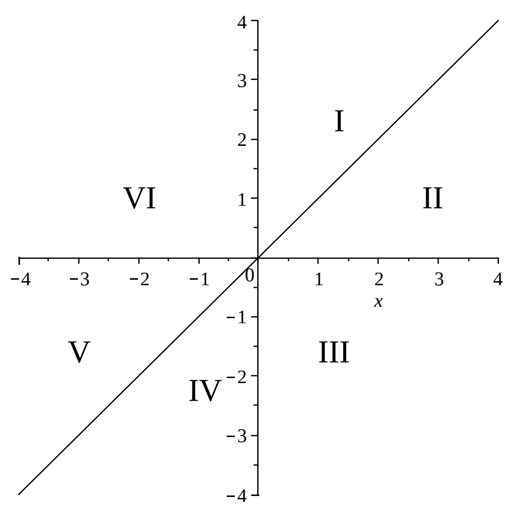
(d)



(e)



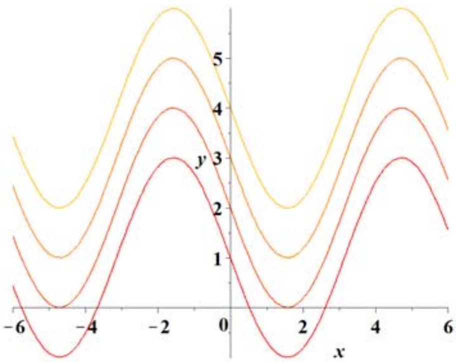
12. Which of the regions in the figure to the right represents the domain of $f(x, y) = \ln(xy) + \sqrt{x - y}$?



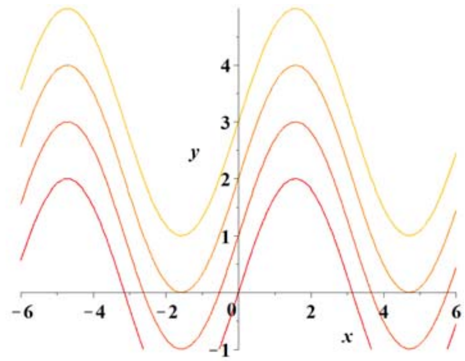
(a) Regions II, III, and IV
(b) Regions II and IV
(c) Reions II and V
(d) Regions I and IV
(e) Regions I, VI, and V

13. Draw a contour plot of $f(x, y) = \ln(y - 1 - 2\sin x)$.

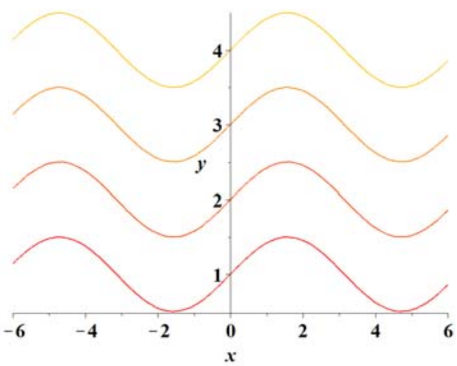
(a)



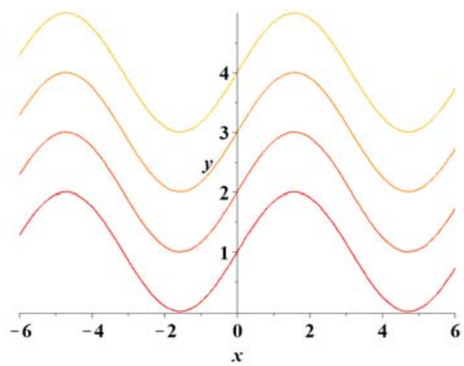
(b)



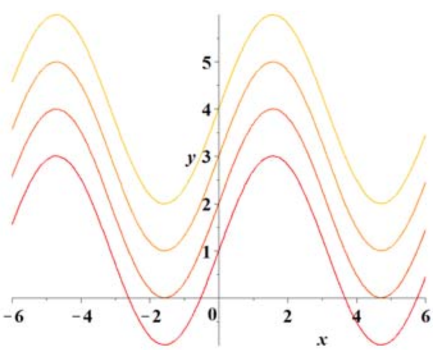
(c)



(d)



(e)



14. If $z = x^2 - xy + 3y^2$ and (x, y) changes from $(3, -1)$ to $(2.96, -0.95)$, find the value of the differential dz.

(a) $-\frac{73}{100}$ **(b)** $\frac{17}{100}$ **(c)** $\frac{73}{100}$ **(d)** $-\frac{17}{100}$ **(e)** $\frac{17}{10}$

15. Find the directional derivative of $f(x, y) = x\sin(xy)$ at the point $(2, 0)$ in the direction indicated by the angle $\pi/3$.

(a) $\frac{\sqrt{3}}{2} + \frac{1}{2}$ **(b)** $2\sqrt{3} + \frac{1}{2}$ **(c)** $2\sqrt{3}$ **(d)** $\frac{\sqrt{3}}{2}$ **(e)** $\frac{1}{2}$

16. Which of the below integrals is equal to

$$\int_1^2 \int_0^{\ln x} f(x,y)dydx$$

(a) $\displaystyle\int_e^{e^2} \int_1^y f(x,y)dxdy$ **(b)** $\displaystyle\int_0^{\ln 2} \int_{e^y}^2 f(x,y)dxdy$ **(c)** $\displaystyle\int_0^{\ln x} \int_1^2 f(x,y)dxdy$

(d) $\displaystyle\int_0^1 \int_{e^y}^1 f(x,y)dxdy$ **(e)** $\displaystyle\int_0^1 \int_0^{\ln y} f(x,y)dxdy$

Math 1AA3/1ZB3

Sample Test 3, Version #1

Name:____

(Last Name) (First Name)

Student Number:______________________ **Tutorial Number:**_____________________

This test consists of 16 multiple choice questions worth 1 mark each (no part marks), and 1 question worth 1 mark (no part marks) on proper computer card filling. All questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Calculators are NOT allowed.

1. Find the vertices of the following ellipse.

$$5x^2 + 20x + 3y^2 - 6y = -8$$

(a) $(0, -2 \pm \sqrt{5})$ **(b)** $(2 \pm \sqrt{5}, 0)$ **(c)** $(2 \pm \sqrt{3}\,, 0)$ **(d)** $(\pm\sqrt{3}, 0)$ **(e)** $(-2, 1 \pm \sqrt{5})$

2. Find an equation of the hyperbola with vertices $(2, -1)$, $(2, 7)$, and foci $(2, -2)$ and $(2, 8)$.

(a) $\dfrac{(y-3)^2}{16} - \dfrac{(x-2)^2}{5} = 1$ **(b)** $\dfrac{(x-2)^2}{5} - \dfrac{(y-3)^2}{9} = 1$

(c) $\dfrac{(y-3)^2}{16} - \dfrac{(x-2)^2}{9} = 1$ **(d)** $\dfrac{(y-2)^2}{5} - \dfrac{(x-1)^2}{4} = 1$ **(e)** $\dfrac{(x-1)^2}{16} - \dfrac{(y-3)^2}{9} = 1$

3. Which of the regions in the figure below represent the domain of $f(x, y) = \ln(1 - xy)$?

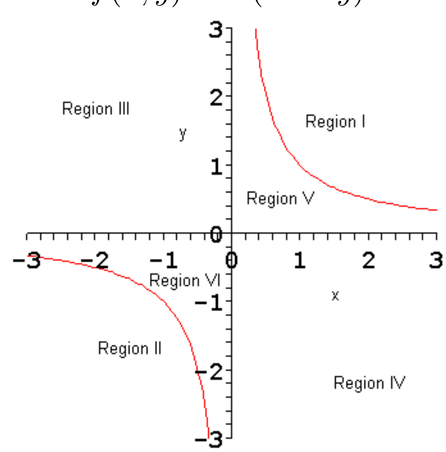
(a) Regions I and II not including the curve

(b) Regions II, IV, and V not including the curve

(c) Regions III, IV, V, and VI not including the curve

(d) Regions I, III, and VI not including the curve

(e) Regions II, III, IV, V, and VI not including the curve



4. Let

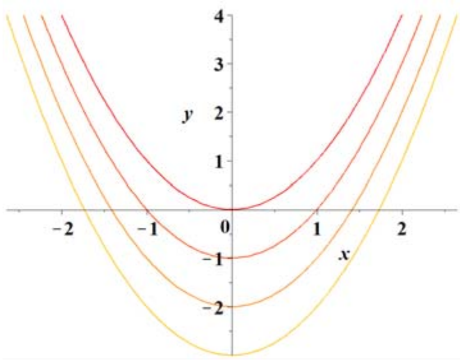
$$f(x,y)=\begin{cases}\frac{2xy}{x^2+2y^2} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases},$$

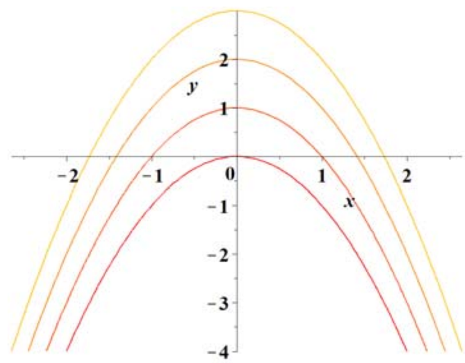
and let

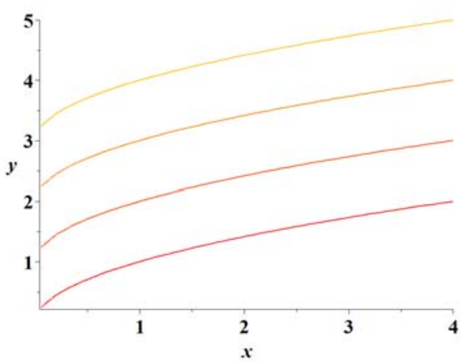
$$g(x,y)=\begin{cases}\frac{3xy}{\sqrt{2x^2+y^2}} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}.$$

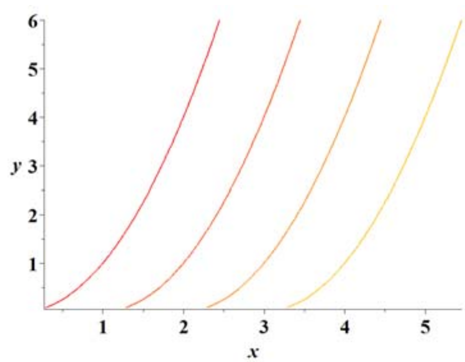
Which of the above functions are continuous at $(0,0)$?
(a) f only **(b)** g only **(c)** f and g **(d)** neither

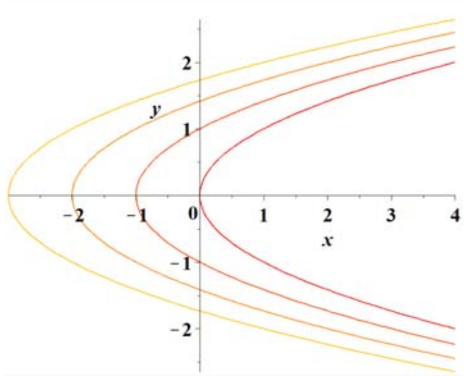
5. Draw a contour plot of $f(x,y)=\ln(x-\sqrt{y})$.

(a)


(b)


(c)


(d)


(e)


6. Matilda wants to use a Riemman sum to approximate the integral

$$\iint_D f(x,y)\,dA$$

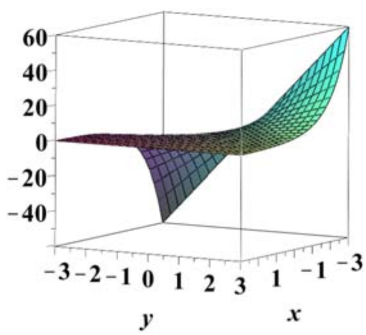
where $D = \{(x,y)\,|\,0 \le x \le 4,\ 0 \le y \le 4\}$. To do so, she divided D into four equal squares and chose a sample point from the middle of each square. The sum of the four pieces was $60+12+12+28=112$. This approximation is appropriate if her function $f(x,y)$ was

(a) $4e^{3x^2}y$ **(b)** $6+\dfrac{9\sin(\pi x/2)\sin(\pi y/2)}{xy}$ **(c)** $12x^y y^x + 12$

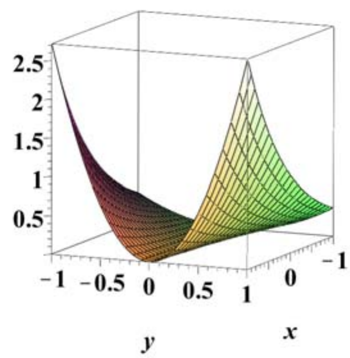
(d) $\ln(x^2+y^2+9)-4$ **(e)** $6\sqrt{1+x^3}(y^4+3)$

7. Sketch the graph of $f(x,y)=\frac{y}{e^x}$.

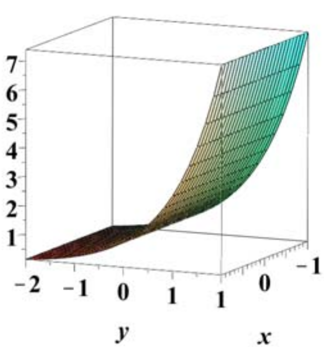
(a)



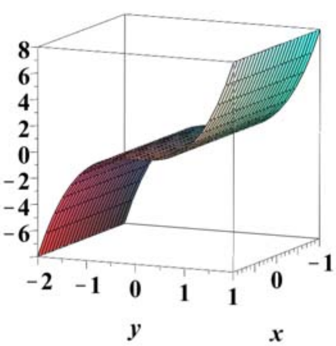
(b)



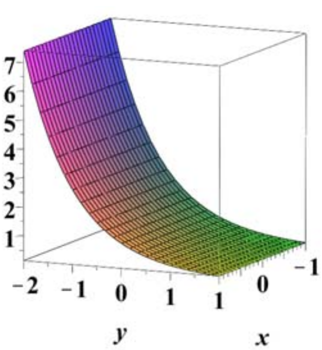
(c)



(a)



(e)



8. Let $z = \frac{x^2}{y}$, $x = rse^t$, $y = re^{st}$. Find $\frac{\partial z}{\partial r}$ when $r = 1,\ s = 1,\ t = 0$.

(a) 0 **(b)** -2 **(c)** -1 **(d)** 2 **(e)** 1

9. Find the direction in which $f(x, y, z) = ze^{xy}$ increases most rapidly at the point $(0, 1, 2)$.

(a) $(1, 1, 2)$ **(b)** $(2, 1, 0)$ **(c)** $(1, 2, 1)$ **(d)** $(1, 0, 2)$ **(e)** $(2, 0, 1)$

10. Find the linear approximation of $M(x, y) = (\ln 4x^2) + y^2$ at $(2, 4)$, and use it to approximate $M(2.1, 3.8)$.

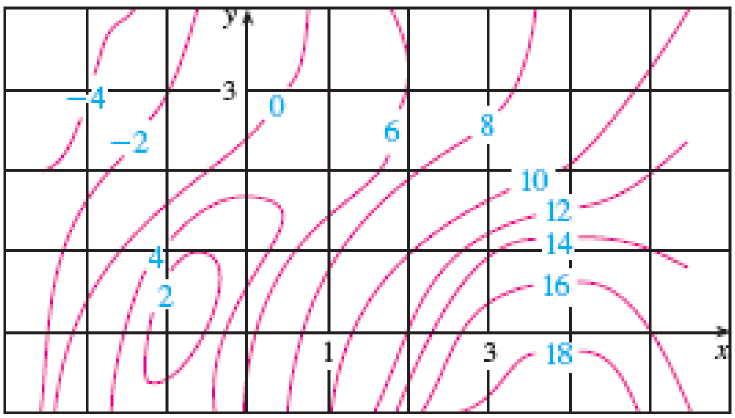
(a) $3\ln 2 + \frac{27}{2}$ **(b)** $4\ln 2 + \frac{29}{2}$ **(c)** $4\ln 2 + \frac{31}{2}$ **(d)** $3\ln 2 + \frac{29}{2}$ **(e)** $2\ln 4 + \frac{27}{2}$

11. Evaluate

$$\int_0^2 \int_0^1 \frac{9x^2y^2}{1 + x^3}\, dy\, dx$$

(a) $\ln 3$ **(b)** $\ln 6$ **(c)** $\ln 4$ **(d)** $\ln 9$ **(e)** $\ln 2$

12. A contour map is given for a function $f(x, y)$.



Based on the contour map, what are the signs of f_x and f_y at the point $(-1, 3)$?

(a) $f_x > 0;\ f_y < 0$ **(b)** $f_x > 0;\ f_y > 0$ **(c)** $f_x < 0;\ f_y > 0$
(d) $f_x < 0;\ f_y < 0$ **(e)** $f_x = 0;\ f_y < 0$

13. Find the equation of the tangent plane to

$$z = \frac{x}{1+xy}$$

at the point $(1, 2, \frac{1}{3})$.

(a) $3x + y - 9z = 1$ **(b)** $x + 3y - z = 0$ **(c)** $x + y - 9z = 0$
(d) $x + y - z = 1$ **(e)** $x - y - 9z = -4$

14. If

$$e^{xyz} = x + y + z$$

then $\frac{\partial z}{\partial x}$ is equal to

(a) $\dfrac{-1}{1-e^{-xyz}}$ **(b)** $\dfrac{yze^{xyz}-1}{1-xye^{xyz}}$ **(c)** $\dfrac{yze^{xyz}-1-y}{1-xye^{xyz}}$
(d) $yze^{xyz}-1-y$ **(e)** $\dfrac{-xye^{xyz}/z^2-1-y}{1-e^{-xyz}}$

15. Let $g(t) = f(t\sin 2t, \cos 2t)$. Suppose that $f_x(0,1) = 2$ and $f_y(0,1) = 3$. Find the value of $g'(0)$.

(a) 0 **(b)** 1 **(c)** 2 **(d)** 3 **(e)** 4

16. The two legs of a right triangle are measured as 5 m and 12 m with a possible error in measurement of at most 0.1 cm in each. Use differentials to estimate the maximum error in the calculated value of the length of the hypotenuse.
(a) $\frac{25}{17000}$ **(b)** $\frac{25}{3000}$ **(c)** $\frac{17}{13000}$ **(d)** $\frac{13}{25000}$ **(e)** $\frac{3}{12500}$

Answers for 1st Version of Sample Test #3

1. b **2.** a **3.** b **4.** e **5.** c **6.** a **7.** a **8.** d **9.** e **10.** a
11. d **12.** b **13.** e **14.** a **15.** c **16.** b

Answers for 2nd Version of Sample Test #3

1. e **2.** c **3.** c **4.** b **5.** d **6.** b **7.** a **8.** e **9.** e **10.** b
11. d **12.** a **13.** e **14.** b **15.** a **16.** c