Name: _____ Student Number:_____

MATH 1AA3, 1ZB3

Winter Session, 2019

Test #1, Seating #1, Version #1

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TERM TEST
DAY & EVENING CLASS
DURATION OF TEST: 90 **min (**1.5 **hrs.**)
MCMASTER UNIVERSITY TERM TEST

Wednesday, Feb. 13, 2019

THIS TEST INCLUDES 6 PAGES AND 19 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING YOUR COPY OF THE TEST IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

Instructions:

1. **NO** calculator is allowed to be used on this test.

2. Make sure your name and student number are at the ***top of each page.***

3. For all questions, mark the answer in pencil on the OMR answer sheets according to the OMR instructions on page 2. Only solutions on the scan card will be graded.

4. Each correct answer is worth one mark.

5. A blank answer is an automatic zero for any question, even if the correct solution is circled on the question itself.

6. Incorrect or multiple answers are also worth zero marks. No negative marks or part marks will be assigned.

7. Scrap paper for rough work has been provided. *All* rough work, and this question sheet must be handed in with the test, but any solutions written either here or on the rough paper will ***NOT*** be graded.

8. *Good Luck!*

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Name: _____ Student Number:_____

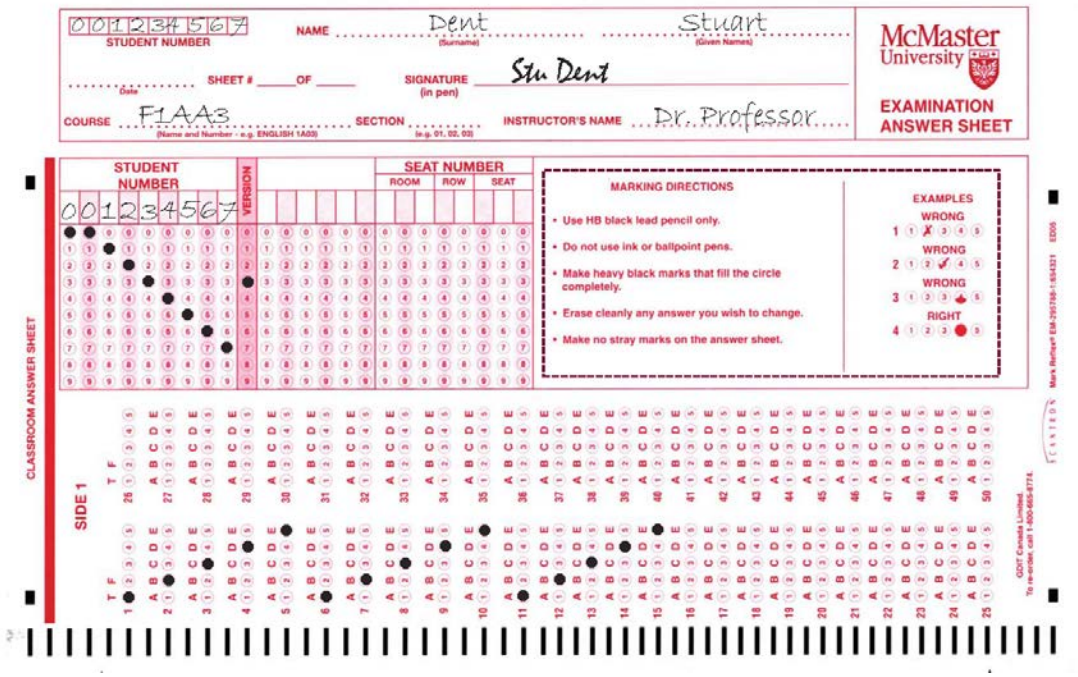
PLEASE READ THE OMR INSTRUCTIONS ON PAGE #2
OMR EXAMINATION INSTRUCTIONS

NOTE: IT IS YOUR RESPONSIBILITY TO ENSURE THAT THE ANSWER SHEET IS PROPERLY COMPLETED: YOUR EXAMINATION RESULT DEPENDS ON PROPER ATTENTION TO THESE INSTRUCTIONS.

The scanner which reads the sheets senses the bubble-shaded areas by their non-reflection of light. A heavy mark must be made, completely filling the circular bubble, with an **HB pencil**. Marks made with a pen or a felt–tip marker will **NOT** be sensed. Erasures must be thorough or the scanner may still sense a mark. Do **NOT** use correction fluid on the sheets. Do **NOT** put any unnecessary marks or writing on the sheet.

1. On side 1 (red side) of the form, in the top box, in pencil, print your student number **(NOTE: 9 digits)**, name, course name, section number, instructor name and date in the spaces provided. Then you **MUST** sign in the space marked SIGNATURE**.**
2. In the second box, with a pencil, mark your student number, exam version number and course section number in the space provided and fill in the corresponding bubble numbers underneath.
3. To indicate your answers, mark only **ONE** choice from the alternatives (1, 2, 3, 4, 5, or A, B, C, D, E) provided for each question. The question number is to the left of the bubbles. Make sure that the number of the question on the scan sheet is the same as the question number on the test paper.
4. Pay particular attention to the Marking Directions on the form.
5. Begin answering the questions using the first set of bubbles, marked "1".

SAMPLE OMR CARD ONLY: DO NOT USE



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1. Find the interval of convergence of the power series, $\sum_{n=1}^{\infty} \frac{x^n}{2^n\sqrt{n+1}}$.

a) $(-2, 2]$ **b)** $(-2, 2)$ **c)** $\left[-\frac{1}{2}, \frac{1}{2}\right]$ **d)** $[-2, 2)$ **e)** $\left[-\frac{1}{2}, \frac{1}{2}\right)$

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2. If the sum of the series, $\sum_{n=1}^{\infty} \frac{(-1)^n}{4n^3}$ is approximated by $S_3 = \sum_{n=1}^{3} \frac{(-1)^n}{4n^3}$, which of the following numbers does the **alternating series error estimate** give as a bound of the magnitude of the remainder, $|S - S_3|$?

a) $\frac{1}{108}$ **b)** $\frac{1}{128}$ **c)** $\frac{1}{256}$ **d)** $\frac{1}{28}$ **e)** $\frac{1}{72}$

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3. For what values of k does the integral $\int_{-1}^{5} (5 - x)^{k/2}\, dx$ converge?

a) $k > -2$ **b)** $k < -1$ **c)** $k < 1$ **d)** $k > -1$ **e)** *No such k exists*

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4. Given the sequence: $b_{n+1} = \frac{2}{5 - b_n}$, $b_1 = 3$, we wish to show that it is monotonic using mathematical induction. Which of the following statements corresponds to a possible induction step?

a) Show $b_1 < b_2 < b_3 < b_4$

b) Assume $b_k < b_{k+1}$, and show $\frac{2}{5 - b_{k+1}} < \frac{2}{5 - b_{k+2}}$

c) Assume $b_k > b_{k+1}$, and show $\frac{2}{5 - b_k} > \frac{2}{5 - b_{k+1}}$

d) Assume $b_k > b_{k+1}$, and show $\frac{2}{5 - b_{k+1}} > \frac{2}{5 - b_{k+2}}$

e) Assume $b_k < b_{k+1}$, and show $\frac{2}{5 - b_k} < \frac{2}{5 - b_{k+1}}$

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5. For the series $\sum_{n=1}^{\infty} b_n$ the m^{th} partial sum is given by: $S_m = \frac{m+2}{3m}$. Find the value of $b_2 + b_3$.

a) 2/3 **b)** 31/4 **c)** –4/9 **d)** 11/9 **e)** *Cannot be determined*

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6. If the sum of the series, $\sum_{n=1}^{\infty} \frac{1}{4n^3}$ is approximated by $S_m = \sum_{n=1}^{m} \frac{1}{4n^3}$, find the smallest possible m such that the **integral error estimate** says $S - S_m < 0.01$?

a) 6 **b)** 5 **c)** 4 **d)** 3 **e)** 2

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7. Which of the following series converge?

I) $\sum_{n=1}^{\infty} \frac{5}{n^{1.1}}$ II) $\sum_{n=1}^{\infty} \frac{3n + n^3}{n^4 + n}$

a) I only **b)** II only **c)** Both I and II **d)** Neither I nor II **e)** Cannot be determined

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8. Which of the following improper integrals converge?

I) $\int_1^{\infty} \arctan(x) e^{-x}\, dx$ II) $\int_1^{\infty} \frac{1 + e^{-x}}{\sqrt{x}} dx$

a) I only **b)** II only **c)** Both I and II **d)** Neither I nor II **e)** Cannot be determined

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9. Which of the following sequences converge?

I) $a_n = \frac{3}{n}$ II) $b_n = (-1)^n \left(\frac{4n}{\sqrt{n^2 + 3}} \right)$

a) I only **b)** II only **c)** Both I and II **d)** Neither I nor II **e)** Cannot be determined

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10. Which of the following converges absolutely: I) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ II) $\sum_{n=1}^{\infty} \frac{(-1)^n}{5^n}$

a) I only **b)** II only **c)** Both I and II **d)** Neither **e)** Cannot be determined

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11. If the series defined by $a_{n+1} = \frac{a_n^2 + 3}{4}$, $a_1 = 2$ is convergent, what is the limit?

a) 5 **b)** 1 **c)** 4 **d)** 3 **e)** –3

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12. Which of the following series converge?

I) $\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$ II) $\sum_{n=2}^{\infty} \left(\frac{1}{2n} - \frac{1}{2n-2} \right)$

a) I only **b)** II only **c)** Both I and II **d)** Neither I nor II **e)** Cannot be determined

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13. The series, $\sum_{n=1}^{\infty} (-1)^n b_n$ converges and $b_n > 0$, consider:

I) The sequence given by the terms, b_n II) The series $\sum_{n=1}^{\infty} b_n$

Which of the following statements must be true?

a) I must converge, II may or may not converge.
b) II must converge, I may or may not converge.
c) Both I and II must converge
d) Both I and II may or may not converge
e) Both must diverge

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14. The power series, $\sum_{n=1}^{\infty} b_n (x-1)^n$ converges at $x = 2$ and diverges at $x = -3$.

What are the minimum and maximum possible values of its radius of convergence?

a) min 2, max 3 **b)** min 2, max 4 **c)** min 1, max 4 **d)** min 1 max 3 **e)** min 3, max 3

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15. Which of the following represents all real values of r such that the series $\sum_{n=0}^{\infty} 2^{nr}$ converges?

a) $r < 1$ **b)** $r < \frac{1}{\ln 2}$ **c)** $r < 0$ **d)** $r < -1$ **e)** Insufficient Information

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16. Which of the following series converge?

I) $\sum_{n=1}^{\infty} n^{-n/5}$ II) $\sum_{n=1}^{\infty} \frac{(2n)!}{3^n n!}$

a) I only **b)** II only **c)** Both I and II **d)** Neither I nor II **e)** Cannot be determined

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17. Find the sum of the series, $\sum_{n=1}^{\infty} \frac{2^{2n}}{5^{n+1}}$ or state it is divergent.

a) $\frac{4}{5}$ **b)** 1 **c)** $\frac{5}{2}$ **d)** Divergent **e)** Cannot be determined

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18. Evaluate the improper integral: $\int_{2}^{\infty} \frac{\ln(x)}{x^2} dx$ or state it is divergent.

a) $8\ln(2) - 12$ **b)** $(3\ln(2) + 1)/4$ **c)** $4\ln(2) - 1$ **d)** $(\ln(2) + 1)/2$ **e)** *Divergent*

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19. Which of the following series converge?

I) $\sum_{n=1}^{\infty} \cos\left(\frac{1}{\sqrt{n}}\right)$ II) $\sum_{n=1}^{\infty} \frac{\sin^4(n)}{n^{4/3}}$

a) I only **b)** II only **c)** Both I and II **d)** Neither I nor II **e)** Cannot be determined

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THE END