12A3

Don't Forget! Test #1 on Oct. 15

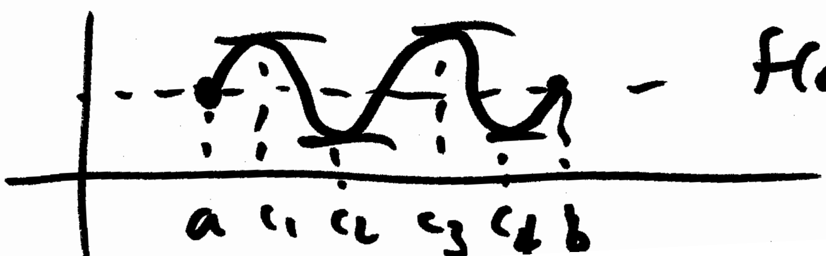
(the Monday Back!)

Last Day Rolle's Theorem

If $f(x)$ cont. on $[a,b]$ & diff. (a,b)

& if $f(a) = f(b)$ then there exists

(at least one) $c \in (a,b)$ such that $f'(c)=0$



– $f(a) = f(b)$

proof

by Extreme value theorem,

$f(x)$ cont. on $[a,b] \Rightarrow f(x)$ attains abs. max & abs min value!

If $f(x)$ constant $\Rightarrow f'(x)=0$ for all $x \in (a,b)$ ✓

Otherwise one of abs. max or abs. min occur on (a,b)

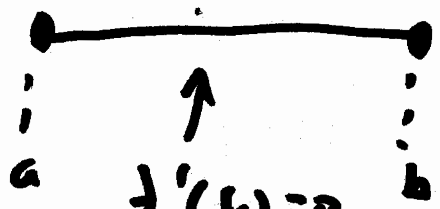
$\Rightarrow$ it occurs at a local max/min point

$\Rightarrow$ occurs at a C.n. on (a,b)

& by Fermat $f'(x)=0$ (since $f(x)$ diff.).

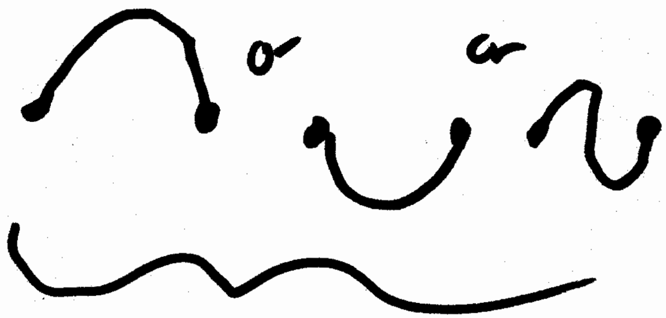
at a flat point.

i.e.



$f'(x)=0$ always.

or



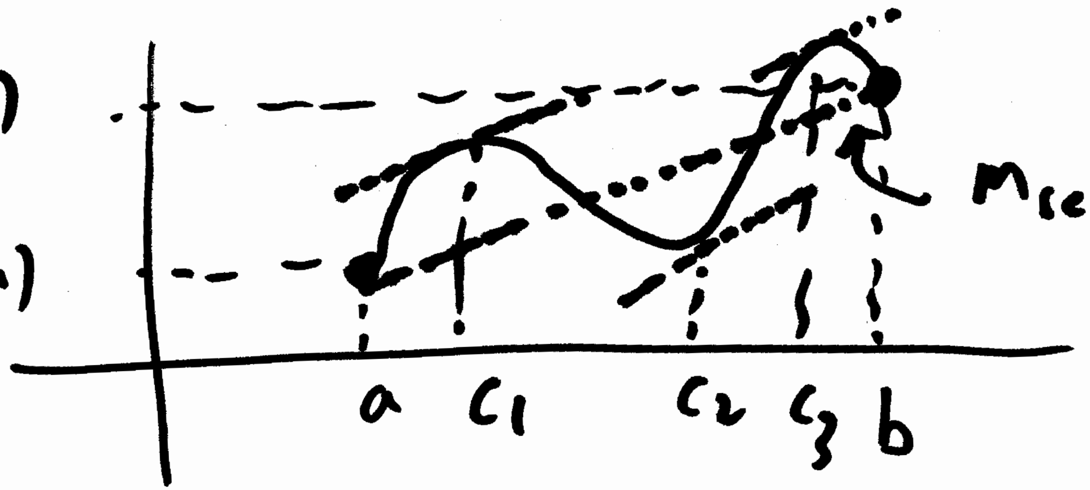
flat spot $f'(c)=0$ for c at abs. max or min. on (a,b).

MVT Mean Value Theorem

If $f(x)$ cont. $[a,b]$ & diff. on (a,b) then there exists $c \in (a,b)$ such that

$$f'(c) = \left.\frac{f(b)-f(a)}{b-a}\right\} = m_{secant}$$

between endpoints



$$m_{secant} = \frac{f(b) - f(a)}{b - a}$$

$$f'(c_1) = f'(c_2) = f'(c_3) = m_{secant}.$$

Proof Given $f(x)$ cont $[a,b]$ diff (a,b)

Let $g(x) = f(x) - m_{sec.} \cdot x$

$$= f(x) - \underbrace{\left(\frac{f(b) - f(a)}{b - a}\right)}_{\text{const.}} x$$

$g(x)$ is cont. on $[a,b]$ & diff. on (a,b) <u>like $f(x)$</u>

$$g(a) = f(a) - ma = f(a) - \left(\frac{f(b)-f(a)}{b-a}\right) a$$

$$g(b) = f(b) - mb = f(b) - \left(\frac{f(b)-f(a)}{b-a}\right) b$$

$$g(a) - g(b) = f(a) - f(b) - \left(\frac{f(b)-f(a)}{\cancel{b-a}}\right)\cancel{(a-b)}^{-1}$$

$$= f(a) - f(b) + f(b) - f(a) = 0$$

$\Rightarrow g(a) = g(b) \Rightarrow$ by <u>Rolle's</u>

$g'(c) = 0$ for $c \in (a,b)$

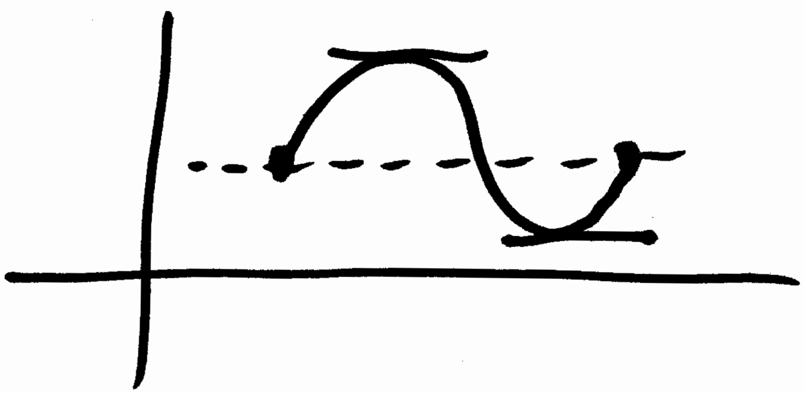
$\Rightarrow \quad f'(c) - m_{sec} = 0$

$\Rightarrow \quad f'(c) = m_{sec} = \dfrac{f(b)-f(a)}{b-a}$ for some $c \in (a,b)$

QED $\square$

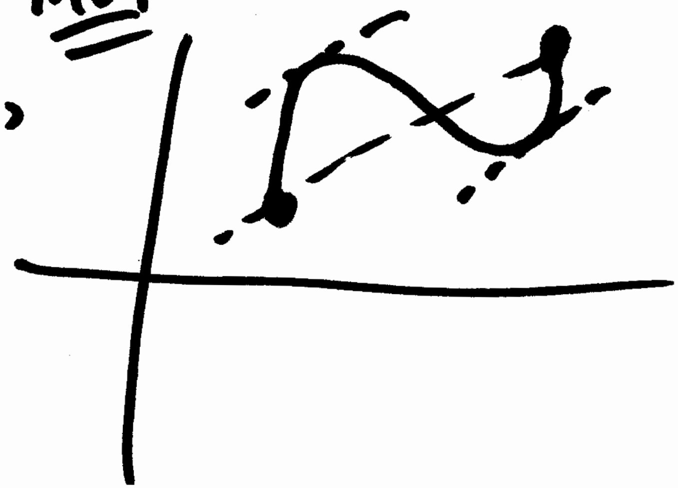
what happened!?

Rolle's



$\Leftrightarrow$

MVT



eg. Base Question Type

Given $f(x) = x^2$ on $[0,4]$ find all values of x that satisfy the conclusion of MVT.

Solution

MVT says: If $f(x)$ is cont. on $[a,b]$
(yes! x^2 cont. on $[0,4]$ ✓)

and $f(x)$ diff. on (a,b)
(yes! $\frac{d}{dx} x^2 = 2x$ exists on $(0,4)$ ✓)

then there exists $c \in (a,b)$ such that
$f'(c) = (f(b) - f(a))/(b-a)$

Let's get $\underline{\underline{c}}$

$$m_{sec} = \frac{f(b) - f(a)}{b - a} = \frac{4^2 - 0^2}{4 - 0} = \frac{16}{4} = \boxed{4}$$

$$= f'(c) = \underline{\underline{2c}} = \underline{\underline{4}}$$

$$\boxed{c = 2}$$

Check! $\underline{\underline{2 \in (0,4)}}$ ✓ $\Rightarrow$ $\underline{\underline{c = 2 \text{ is our value!}}}$

eg. Sneakier Problem

$f(1) = 10$ & $f(x)$ is cont. & diff. for all x

& $|f'(x)| \le 5$. Find the largest possible value

of $f(7)$.

Solution $f(x)$ cont. & diff on all x

$\Rightarrow$ MVT holds for any $x = a, b$

So consider $[1, 7]$

$$f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{f(7) - f(1)}{7 - 1}$$

but

$$\left| \frac{f(7) - \overset{10}{\cancel{f(1)}}}{6} \right| = \underbrace{|f'(c)| \le 5}_{\text{given}}$$

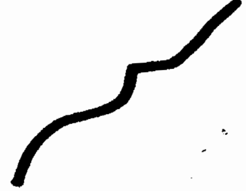
$$\Rightarrow |f(7) - 10| \le 5 \cdot 6 = 30$$

$$\Rightarrow f(7) - 10 \le 30$$

$$\Rightarrow \boxed{f(7) \le 40}$$

4.3 Derivatives & Graph Shape

$f'(x) > 0 \Rightarrow$ $f(x)$ "increasing"



$f'(x) < 0 \Rightarrow$ $F(x)$ is "decreasing"

So let's find intervals of increasing & decreasing!

eg. Find intervals of inc./dec. for

$$y = x^3 - 3x + 6$$

Solution $\quad y' = 3x^2 - 3$

Look for c.n. $\quad y' = 0 \Rightarrow 3x^2 - 3 = 0$

$$x = \pm 1$$

y' DNE $\Rightarrow$? none

between $y' = 0$ & y' DNE $\Rightarrow$ y' has fixed sign!

Chart!

	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
$3(x^2-1)$ $(f'(x))$	$f'(-2)$ $3(4-1) > 0$	$x=0$ $f'(0) = -3$ < 0	$f'(2)$ $= 3(4-1) > 0$
	$+$	$-$	$+$
$f(x)$	inc	dec	inc.
	$\nearrow$	$\searrow$	$\nearrow$