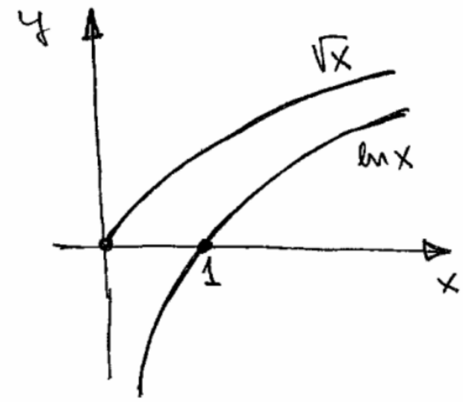
Name:

Student No.:

1. (a)[2] Graph the pair of functions $\sqrt{x}$ and $\ln x$ in the same coordinate system and identify the one that approaches $\infty$ faster as $x$ approaches $\infty$.




| $x$ | $\sqrt{x}$ | $\ln x$ |
|---|---|---|
| 100 | 10 | 4.6 |
| 1000 | 31.6 | 6.9 |
| $10^6$ | 1000 | 13.8 |
| $\vdots$ | $\vdots$ | |
| | $\uparrow$ FASTER | |

(b)[3] Find $\lim_{x\to\infty}\left(\ln(2x^6+1) - 3\ln x^2\right)$ $= \lim_{n\to\infty} \ln \dfrac{2x^6+1}{x^6}$

$$= \underline{\underline{\ln 2}}$$

since $\lim_{x\to\infty} \dfrac{2x^6+1}{x^6} = \lim_{x\to\infty} \dfrac{2x^6}{x^6} = 2$

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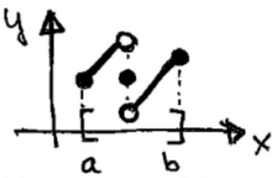
2. True/false questions. Decide whether the statements in questions (a) - (b) are true or false (circle your choice). You must correctly justify your answer to receive credit.

(a)[2] Every function defined on a closed interval $[a, b]$ has an absolute maximum and an absolute minimum.

TRUE FALSE

must be continuous!

or, give an example:




defined on $[a,b]$
but no max, no min

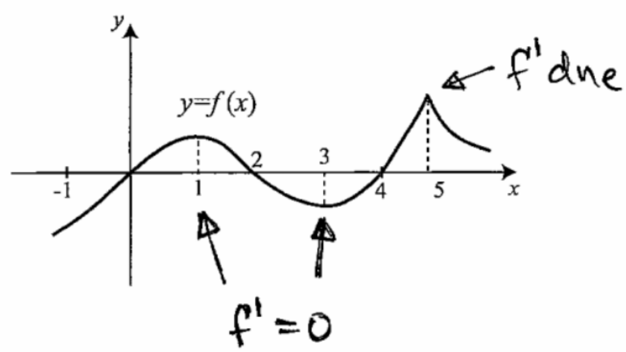
(b)[2] If $f'(c) = 0$, then $f(x)$ must have an extreme value (i.e., either minimum or maximum) at $c$.

TRUE FALSE

$f(x) = x^3$ satisfies $f'(\overset{c}{0}) = 0$
but does not have extreme value at 0

(c)[2] The function below has two critical numbers (critical points).

TRUE FALSE




has three critical points

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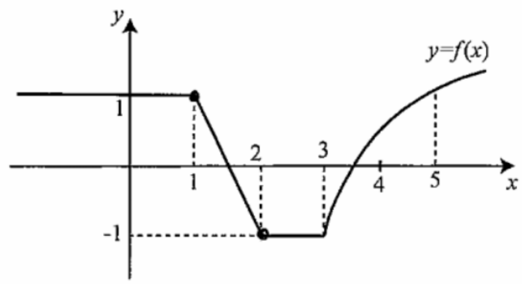
3. Consider the function $f(x)$ given in the graph below.




(a)[2] What is the value of $f'(1.8)$? Explain your answer.

slope of the line $= \frac{-1-1}{2-1} = -2$

(b)[2] The function $f(x)$ has a corner (cusp) at $x = 2$. This means that $f'(2)$ does not exist. Explain why.

as $x \to 2^-$ slopes (are constant, equal to $-2$) approach $-2$

as $x \to 2^+$ slopes approach $0$

so slope at 2 d.n.e.

(c)[3] Sketch the graph of $f'(x)$.

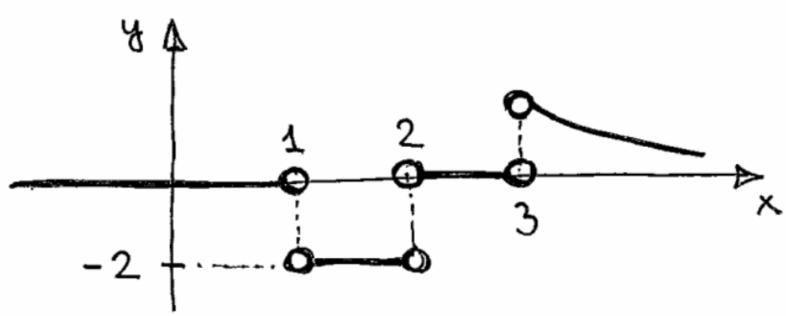
$f'(x) = 0$ on $(-\infty, 1)$

$f'(x) = -2$ on $(1, 2)$

$f'(x) = 0$ on $(2, 3)$

$f'$ positive, decreasing on $(3, \infty)$

$f'$ dne at $1, 2, 3$

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4. Consider the discrete-time dynamical system $p_{t+1} = 1.4p_t(1-p_t)$, where $p_t$ is a population of mice in thousands.

(a)[2] Find all equilibrium points of the system.

$$p^* = 1.4p^*(1-p^*) \rightarrow p^*(1 - 1.4(1-p^*)) = 0$$

so $p^* = 0$ or $1 - 1.4 + 1.4p^* = 0$

$$1.4p^* = 0.4, \quad p^* = \frac{0.4}{1.4} = \frac{2}{7}$$

(b)[3] Determine whether each of the equilibrium points you found in (a) is stable or unstable.

$$f(x) = 1.4x(1-x) = 1.4x - 1.4x^2$$

$$f'(x) = 1.4 - 2.8x$$

$f'(0) = 1.4$ ..... since $|f'(0)| > 1$, $0$ is unstable

$$f'\left(\tfrac{2}{7}\right) = 1.4 - 2.8 \cdot \frac{2}{7} = 0.6 = \frac{3}{5}$$

$\left|f'\left(\tfrac{2}{7}\right)\right| < 1$, $\frac{2}{7}$ is stable

(c)[2] Explain what your answer to (b) means for the population of mice.

populations near 0 (ie small population) will move away from it, ie will increase

populations near $\frac{2}{7}$ thousands will move closer (ie increase or decrease) to $\frac{2}{7}$ thousands

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5. (a)[1] What is the per capita production in the context of population models?

number of offspring produced by a single individual

(b)[1] Identify per capita production in the population discrete system $p_{t+1} = \frac{2p_t}{1+0.005p_t}$.

$$\frac{2}{1+0.005p_t}$$

(c)[3] Find all equilibrium points of the system in (b).

$$p^* = \frac{2p^*}{1+0.005p^*} \rightarrow p^*\left(1 - \frac{2}{1+0.005p^*}\right) = 0$$

so $\underline{\underline{p^* = 0}}$ or $1 + 0.005p^* = 2 \rightarrow p^* = \frac{1}{0.005} = \underline{\underline{200}}$

(d)[2] Consider the largest equilibrium point that you found in (c). Is it stable or not?

$$f(x) = \frac{2x}{1+0.005x}$$

$$f'(x) = \frac{2(1+0.005x) - 2x(0.005)}{(1+0.005x)^2} = \frac{2}{(1+0.005x)^2}$$

$$f'(200) = \frac{2}{(1+(0.005)(200))^2} = \frac{1}{2}$$

stable since $|f'(200)| < 1$

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6. (a) [2] Give the statement of the Extreme Value Theorem.

If $f$ is continuous on $[a,b]$, $[a,b]$ closed

Then $f$ has an abs. max. and an abs. min. in $[a,b]$

(b) [3] Find the absolute maximum and the absolute minimum of the function $f(x) = \frac{\ln x}{x^2}$ on the interval $[1,4]$.

$$f'(x) = \frac{\frac{1}{x}\cdot x^2 - (\ln x)(2x)}{(x^2)^2} = \frac{x(1-2\ln x)}{x^4}$$

$$= \frac{1-2\ln x}{x^3}$$

$f'(x)=0$ ... $1-2\ln x = 0$, $\ln x = \frac{1}{2}$, $x = e^{1/2} = \sqrt{e} \approx 1.65$

$f'(x)$ dne... no such points in $[1,4]$

| $x$ | $f(x) = \frac{\ln x}{x^2}$ |
|---|---|
| $\sqrt{e}$ | $\frac{\ln\sqrt{e}}{(\sqrt{e})^2} = \frac{1/2}{e} = \frac{1}{2e} \approx 0.18$ |
| $1$ | $0$ |
| $4$ | $\frac{\ln 4}{16} \approx 0.09$ |

abs min $= 0$, at $x = 1$

abs max $= \underbrace{\frac{1}{2e}}_{0.18}$, at $x = \underbrace{\sqrt{e}}_{1.65}$

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7. [3] Find all critical numbers (critical points) of the function $f(x) = (x-1)^{2/3} + 1$.

$$f'(x) = \frac{2}{3}(x-1)^{-\frac{1}{3}} = \frac{2}{\sqrt[3]{x-1}}$$

$f'(x) = 0$ .... no $x$

$f'(x)$ dne when $x = 1$

since $x = 1$ is in domain of $f(x)$, it is a c.p.

so $\underline{\underline{x=1}}$ is (the only) c.p.

THE END