Random Variables (Sections 6, 7, 8)

1. Let X be the location of a particle after 100 steps of the random walk. Is X a discrete or a continuous random variable? Is it finite or infinite?

2. Assume that the range of a random variable X is the set $\{0, 1, 2, 3\}$. Define $p(0) = 0$, $p(1) = 0.16$, $p(2) = 0.54$, and $p(3) = 0.3$. Can p be a probability mass function of X? Why or why not?

3. Explain why

$$F(x) = \begin{cases} 0.1 & x < 0 \\ 0.2 & 0 \leq x < 1 \\ 0.6 & 1 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$$

cannot be a cumulative distribution function of a random variable.

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4. Find the probability mass function of each random variable X.

(a) Toss a coin four times. The random variable X counts the number of tails.

(b) Roll two dice. X is the absolute value of the difference of the numbers that show up.

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5. If, at some time, a virus is present in a population, then it will be present the following month with probability 0.75 (thus, it will disappear with probability 0.25). If the virus is absent from the population, then it will be absent the following month with probability 0.8 (i.e., it will (re)appear within the population with probability 0.2). Assume that at this moment the virus is present in the population. Find the probability mass function for the random variable $X =$ "number of virus-free months in the 2-month period from now."

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6. A couple of rhesus monkeys have a baby monkey each year with a chance of 25% that the baby will be dark brown, 35% that it will be light brown, and 40% that it will be grey. Let $R =$ "number of grey baby monkeys born to the couple in a 2-year period." Find the probability mass function for R.

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7. Draw a histogram for the probability mass function given by $p(1) = 0$, $p(2) = 0.2$, $p(3) = 0.1$, $p(4) = 0.2$, $p(5) = 0.15$, $p(6) = 0.1$, $p(7) = 0.25$, $p(8) = 0$, and a pick the word or phrase among "symmetric," "skewed left," "skewed right," and "uniform" that best describes it.

8. Given is the cumulative distribution function of a random variable X. Find the probability mass function of X.

$$F(x) = \begin{cases} 0 & x < -4 \\ 0.5 & -4 \le x < -2 \\ 0.65 & -2 \le x < -1 \\ 0.95 & -1 \le x < 0 \\ 1 & x \ge 0 \end{cases}$$

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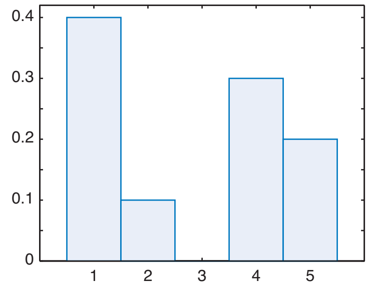
9. Given is the histogram of a random variable X. Find its probability mass function and cumulative distribution function.



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10. Given is the probability mass function of a discrete random variable X.

x	$P(X = x)$
-2	0.25
-1	0.1
0	0.15
1	0.2
2	0.3

(a) Find the cumulative distribution function of X and sketch its graph.

(b) Compute the expected value $\mu = E(X)$.

(c) Consider the random variable $Y = X + 2$. Find $E(Y)$ and compare to (b).

(d) Consider the random variable $Z = 3X$. Find $E(Z)$ and compare to (b).

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(e) Compute $E(X^2)$.

(f) Compute $E(X^4 - 2X)$.

(g) Compute $\mathrm{var}(X)$ using $\mathrm{var}(X) = E[(X-\mu)^2]$.

(h) Compute $\mathrm{var}(X)$ using $\mathrm{var}(X) = E(X^2) - (E(X))^2$ and compare with (g)

(i) Compute the standard deviation of X.

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(j) Consider the random variable $Y = X+2$. Find var(Y) and compare to var(X). Compute the standard deviation of Y.

(k) Consider the random variable $Z = 3X$. Find var(Z) and compare to var(X). Compute the standard deviation of Z.

(l) Compute $E(X^2 - 3)$.

(m) Compute $E(e^X)$.

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11. A random variable X is said to be *uniformly distributed* on the set $S = \{1, 2, 3, \ldots, 8\}$ (where $n \geq 1$) if $P(X = k) = 1/8$ for $k = 1, 2, \ldots, 8$. What is the mean of X? What is the Variance of X?

12. Knowing that $E(X) = 2$ and $E(X^2) = 3$, compute $E(X - X^2 + 7)$.

13. Let X be a random variable with expected value μ and variance σ^2. Define $Y = (X - \mu)/\sigma$, where $\sigma \neq 0$. What is the expected value of Y? What is the standard deviation of Y?

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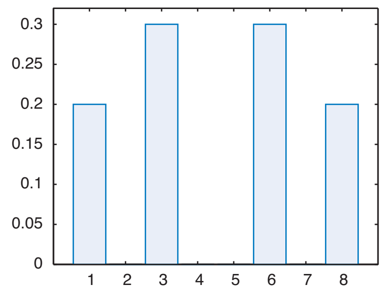
14. Using $\text{var}X = E(X^2) - [E(X)]^2$ and the properties of the expected value, prove that $\text{var}(X+b) = \text{var}X$ for a real number b.

15. Given the histogram of a discrete random variable, find its variance and standard deviation.



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