2.1.2 Case B (lower bound is infinite)

Let $f(x)$ be a function defined on $(-\infty, b]$ and assume that for all $t \le b$, $\int_t^b f(x)\,\mathrm{d}x$ exists.

$$\text{Define} \qquad \int_{-\infty}^{b} f(x)\,\mathrm{d}x =$$

Terminology:

We say that $\int_{-\infty}^{b} f(x)\,\mathrm{d}x$ is **convergent** if ________________________________,

else we say that $\int_{-\infty}^{b} f(x)\,\mathrm{d}x$ is ________________.

Example: $\int_{-\infty}^{b} e^x\,\mathrm{d}x =$

2.1.3 Case C (both bounds are infinite)

Let $f(x)$ be a function defined on $(-\infty, \infty)$ and assume that $\int_{-\infty}^{c} f(x)\,\mathrm{d}x$ and $\int_{c}^{\infty} f(x)\,\mathrm{d}x$ exists for some $c \in \mathbb{R}$.

Define $$\int_{-\infty}^{\infty} f(x)\,\mathrm{d}x =$$

<u>Terminology:</u>

We say that $\int_{-\infty}^{\infty} f(x)\,\mathrm{d}x$ is **convergent** if both, $\int_{-\infty}^{c} f(x)\,\mathrm{d}x$ and $\int_{c}^{\infty} f(x)\,\mathrm{d}x$ are convergent, else we say that $\int_{-\infty}^{\infty} f(x)\,\mathrm{d}x$ is divergent.

Example: $\int_{-\infty}^{\infty} x^3\,\mathrm{d}x =$

Careful: $\lim_{t\to\infty} \int_{-t}^{t} x^3\,dx$

Example: $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,\mathrm{d}x =$

In general:

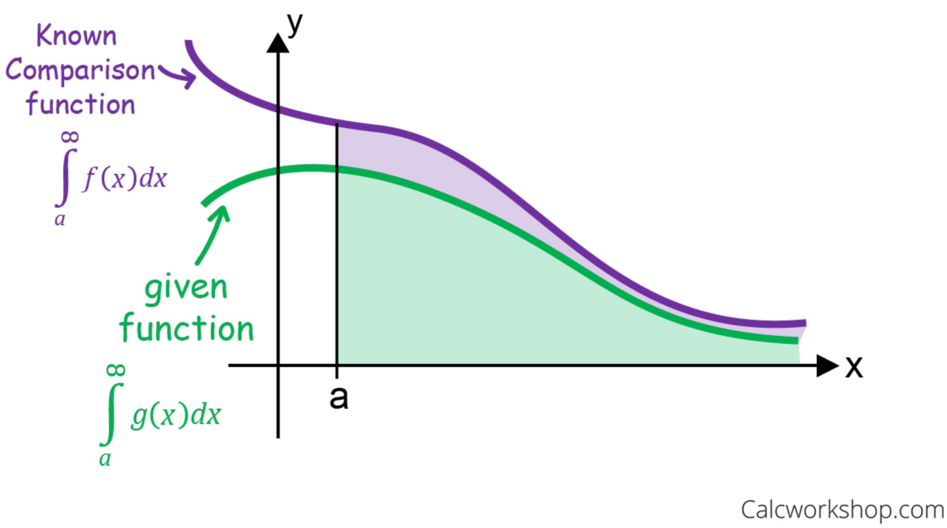
If $f(-x) = f(x)$ and $\int_c^\infty f(x)\,\mathrm{d}x$ is convergent, then $\int_{-\infty}^\infty f(x)\,\mathrm{d}x =$

Comparison Test for Type I for improper integrals:

Assume f, g are continuous functions with $0 \leq g(x) \leq f(x)$ for $a \leq x$.

1. If $\int_a^\infty f(x)\,dx$ is convergent, then $\int_a^\infty g(x)\,dx$ is ________________.

2. If $\int_a^\infty \qquad dx$ is divergent, then $\int_a^\infty \qquad dx$ is also divergent.



Example: $$\int_1^\infty \frac{1}{\sqrt{5+x^3}}\,dx =$$

2.2 Improper Integrals – Type II (discontinuous integrands)

2.2.1 Case A (discontinuous or undefined upper bound)

Assume $f(x)$ is continuous on $[a, b)$ and **discontinuous or undefined** at b.

Define $\displaystyle\int_a^b f(x)\,\mathrm{d}x =$

(The integral is convergent if the limit exists, divergent otherwise.)

Example: $\displaystyle\int_1^2 \frac{1}{\sqrt{2-x}}$

2.2.2 Case B (discontinuous or undefined lower bound)

Assume $f(x)$ is continuous on $(a, b]$ and **discontinuous or undefined** at a.

Define $\displaystyle\int_a^b f(x)\,\mathrm{d}x =$

(The integral is convergent if the limit exists, divergent otherwise.)

Example: $\displaystyle\int_0^1 \frac{1}{x}$

Example: $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}$

General Rule: Let $b > 0$, then

$$\int_0^b \frac{1}{x^p}\,\mathrm{d}x =$$

2.2.3 Case C (discontinuous inbetween bounds)

Assume $f(x)$ is discontinuous at c, where $a < c < b$ and both, $\int_a^c f(x)\,\mathrm{d}x$ and $\int_c^b f(x)\,\mathrm{d}x$ are convergent.

Define $$\int_a^b f(x)\,\mathrm{d}x =$$

(The integral is <u>convergent</u> if both integrals are convergent, else it is <u>divergent</u>.)

Example : $$\int_0^5 \frac{1}{x-1}\,\mathrm{d}x =$$

Q: Why does the Fundamental Theorem of Calculus fail?

Note: **Hybrid Type I & Type II** improper integrals

Example: $\displaystyle\int_0^\infty \frac{e^{-x}}{x}\,\mathrm{d}x =$