McMaster University
Math 1LT3

ASSIGNMENT 5

Sections 4, 5, and 6 in the Red Module

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1. (a) Sketch the graph of the surface $z = x^2 + y^2$.

(b) Explain how to obtain the curve with the property that the slope of its tangent at (2,4) is equal to the partial derivative $f_x(2, 4)$. Add the curve and its tangent to the graph of the surface in part (a).

2. Assume that the function $T(x, y, t)$ models the temperature (in degrees Celsius) at time t in a city located at a longitude of x degrees and a latitude of y degrees. The time t is measured in hours. What is the meaning of the partial derivative $T_t(x, y, t)$? What are its units? What is most likely going to be the sign of $T_y(x, y, t)$ for Winnipeg, Manitoba in January?

3. Below is an excerpt from a table of values of I, the temperature-humidity index, which is the perceived air temperature when the actual temperature is T (degrees fahrenheit), and the relative humidity is h (percent).

T ↓ h →	20	30	40	50	60	70
80	74	76	78	82	83	86
85	81	82	84	86	90	94
90	86	90	93	96	101	106
95	94	94	98	107	111	125
100	99	101	109	122	129	138

(a) Write the definition (equation) of the partial derivative of $I(T, h)$ with respect to h.

(b) Approximate $I_h(95, 40)$ and interpret your answer, i.e., write a statement to explain what this number represents, including units.

4. Compute the indicated partial derivatives.

(a) $f(x,y) = \dfrac{4x - xy}{x^2 + y^2}$; $f_x(x,y)$

(b) $h(x,t) = te^{\sqrt{x-4t^2}}$; $h_t(5,1)$

5. Let $f(x, y) = \ln(3x - y + 1)$.

(a) Compute the partial derivatives of f.

(b) Find and sketch the domains of f_x and f_y. (*Recall*: The domain of a derivative of a function is always a *subset* of the domain of the function).

(c) Is f differentiable at $(1, 0)$? Explain.

(d) Find the equation of the tangent plane to the surface $f(x, y) = \ln(3x - y + 1)$ at the point $(1, 0)$. Is this tangent plane a good approximation of the surface near the point of tangency? Explain.

6. Consider the function $f(x,y) = \sqrt{y + \cos^2 x}$.

(a) Using Theorem 6, show that the function is differentiable at $(0,0)$.

(b) Verify the linear approximation $\sqrt{y + \cos^2 x} \approx 1 + \frac{1}{2}y$ at (0,0).

7. Using Theorem 6, show that the function $f(x,y) = xy(x^2+y^2)^{-1}$ is differentiable at the point $(3,-4)$. What is the largest open disk centred at $(3,-4)$ that you can use?

8. Suppose that $z = x^2 y \sin x$, where $x = 6t$ and $y = e^t$. Use the Chain Rule to find $z'(t)$.

9. Suppose that $z = \dfrac{ab-1}{b^2+1}$, where $a = 3s$ and $b = st$. Use the Chain Rule to find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$ when $s = 1$ and $t = 1$.

10. Wheat production W in a given year depends on the average temperature T and the annual rainfall R. Scientists estimate that the average temperature is rising at a rate of 0.15^oC/year and rainfall is decreasing at a rate of 0.1 cm/year. They also estimate that, at current production levels, $\frac{\partial W}{\partial T} = -2$ and $\frac{\partial W}{\partial R} = 8$.

(a) What is the significance of the signs of these partial derivatives?

(b) Estimate the current rate of change of wheat production, $\frac{dW}{dt}$.

11. Suppose f is a differentiable function of x and y, and $g(r, s) = f(2r - s, s^2 - 4r)$. Use the table of values below to calculate $g_r(1, 2)$ and $g_s(1, 2)$.

	f	g	f_x	f_y
$(0, 0)$	3	6	4	8
$(1, 2)$	6	3	2	5

THE END