Math 1AA3/1ZB3: Test 2 Review

March 13, 2019

1. Find Taylor series for the following functions:
   (a) $f(x) = \frac{2x}{1+x}$, centered at $x = 0$.
   (b) $f(x) = -\frac{1}{x}$, centered at $x = 2$.
2. Compute the sum $\sum_{n=0}^{\infty} \frac{2(ln2)^n}{n!}$.
3. Expand $\binom{1/2}{n}$.
4. Find the surface area produced by rotating the curve $y = x^2$ from $x = 1$ to $x = 3$ about the y-axis.
5. A trapezoid sits upright in 6m of still water. The trapezoid is 4m tall, has width 22m at the bottom and width 10m at the top. Find the hydrostatic force exerted on one surface of the trapezoid.
6. Solve the following differential equations under the condition $y(0) = 1$.
   (a) $\frac{dy}{dx} = 3y - 2x + 6xy - 1$
   (b) $\frac{dy}{dx} + 2xy - e^{x-x^2} = 0$
7. Find the orthogonal trajectories to the family of curves $y^2 = kx^4$.
8. A tank containing 100L of pure water is mixed with brine containing 0.2kg salt per litre at a rate of 10L/min. If the tank is drained at the same rate, how much salt does the tank contain after 4 minutes?
9. Sketch the parametric curve given by $x = \cos(t)$, $y = \sin(2t)$.