Physics 1A03

Assignment 4 solutions

1. a) $m = 0.355\,kg \quad v = 15.5\,m/s$

$$K = \frac{1}{2}mv^2$$
$$= \frac{1}{2}(0.355\,kg)(15.5\,m/s)^2$$
$$= 42.64\,J$$

b) since the velocity term in the equation for kinetic energy is squared, a 2x increase in velocity would lead to a 4x increase in kinetic energy:

$$42.64 \times 4 = 170.58\,J$$

but let's double check:

$$K = \frac{1}{2}mv^2$$
$$= \frac{1}{2}(0.355\,kg)(2 \times 15.5\,m/s)^2$$
$$= 170.58\,J$$

2. $h_a = 5.14\,m \quad h_b = 2.88\,m \quad h_c = 1.96\,m$

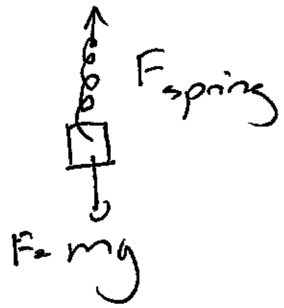
a) $U_a = K_b + U_b$

$$mgh_a = \frac{1}{2}mv_b^2 + mgh_b$$
$$\frac{1}{2}mv_b^2 = mgh_a - mgh_b$$
$$v_b^2 = \frac{2(\cancel{m}gh_a - \cancel{m}gh_b)}{\cancel{m}}$$
$$= 2g(h_a - h_b)$$
$$= 2(9.8\,N/\cancel{kg})(5.14\,m - 2.88\,m)$$
$$= 44.296$$
$$v_b = 6.66\,m/s$$

b) work done by gravity is equal in magnitude to the difference in potential energy between A and C:

$$\Delta U = U_c - U_A$$
$$= mgh_c - mgh_a$$
$$= mg(h_c - h_a)$$
$$= (4.89\,kg)(9.8\,N/kg)(5.14\,m - 1.96\,m)$$
$$\doteq 152.39\,J$$

3. a)



$F = k\Delta d$ $\qquad m = 3.64\,kg$

$F_g = mg$ $\qquad \Delta d = 0.0257\,m$

equilibrium: $F_{spring} = mg$

$$k\Delta d = mg$$
$$k = \frac{mg}{\Delta d}$$
$$= \frac{(3.64\,kg)(9.8\,N/kg)}{0.0257\,m}$$
$$= 1388.016\,N/m$$

now we can find the displacement w/ the new weight:

$$k\Delta d = mg$$
$$\Delta d = \frac{mg}{k}$$
$$= \frac{(1.22\,kg)(9.8\,N/kg)}{1388.016\,N/m}$$
$$= 0.00861\,m$$

b) $W = \frac{1}{2} k \Delta d^2$

$= \frac{1}{2}(1388.016\ N/m)(0.0411\ m)^2$

$= 1.17\ J$

4. remember: $1W = 1J/s$

so:

a 51.0 kg runner dissipating 68.5W is dissipating 68.5 J/s

which means that:

$$\frac{68.5\ \frac{J}{s}}{51.0\ kg} = \frac{1.343\ \frac{J}{s}}{1\ kg}$$

$$= 1.343\ \frac{J \cdot kg}{s}$$

if we know they dissipate 0.539 J per kg per step:

$$0.539\ \frac{J \cdot kg}{step}$$

we can figure out how many steps they take a second by playing with the units:

$$1.343\ \frac{J \cdot kg}{1\ s} \div 0.539\ \frac{J \cdot kg}{1\ step}$$

$$= 1.343\ \frac{\cancel{J \cdot kg}}{s} \times \frac{1\ step}{0.539\ \cancel{J \cdot kg}}$$

$$= 2.492\ \frac{steps}{s}$$

if each step is 1.50 m:

$2.492\ \cancel{steps}/s \times 1.50\ m/\cancel{step} = 3.7379\ m/s$

they're travelling 3.74 m/s.

5.a) $W = mg\Delta y$

$= (1267.0\,kg)(9.8\,N/kg)(63.5\,m)$

$= 78.8 \times 10^4\,J$

b) $P = J/s$

$= \dfrac{78.8 \times 10^4\,J}{68.3\,s}$

$= 11.5 \times 10^3\,W.$

6. $W_1 = 4.95\,J$ $\quad \Delta d_1 = 0.145\,m$

$W_2 = ?$ $\quad \Delta d_2 = 0.145\,m + 0.1\,m$

$\Delta W = ?$ $\quad = 0.245\,m$

① $w_1 = \frac{1}{2} k \Delta d_1^{\,2}$

$4.95\,J = \frac{1}{2} k (0.145\,m)^2$

$k = 470.87\,N/m$

② $w_2 = \frac{1}{2} k \Delta d_2^{\,2}$

now that we have k, we can subtract ① from ② to find ΔW:

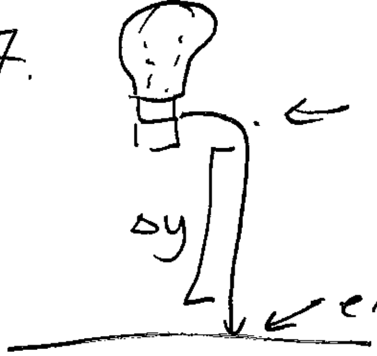
$\Delta W = W_2 - W_1$

$= \left(\frac{1}{2} k \Delta d_2^{\,2}\right) - \left(\frac{1}{2} k \Delta d_1^{\,2}\right)$

$= \frac{1}{2} k (\Delta d_2^{\,2} - \Delta d_1^{\,2})$

$= \frac{1}{2}(470.87\,N/m)(0.245\,m^2 - 0.145\,m^2)$

$= 9.18\,J$

7. 

$\Delta y = 850m \quad m = 50.2kg \quad v_f = 4.79 m/s$

diff btw U and K is the energy lost due to air friction:

$$\Delta E = U - K$$
$$= mgh - \frac{1}{2}mv^2$$
$$= m(gh - \frac{1}{2}v^2)$$
$$= (50.2kg)((9.8N/kg)(850m) - \frac{1}{2}(4.79m/s)^2)$$
$$= 4.18 \times 10^5 J$$

8. $m_o = 8 m_{He}$ (1)

also:

$K_o = K_{He}$

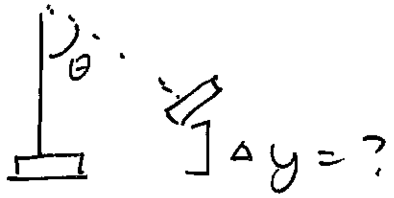
so:

$\frac{1}{2}m_o v_o^2 = \frac{1}{2}m_{He} v_{He}^2$ (2)

sub (1) into (2):

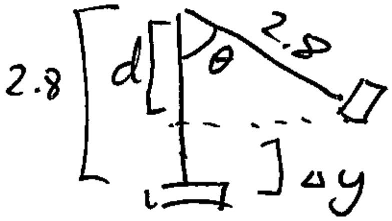
$$\frac{1}{2}(8m_{He})v_o^2 = \frac{1}{2}m_{He}v_{He}^2$$
$$\frac{\cancel{\frac{1}{2}}(8\cancel{m_{He}})}{\cancel{\frac{1}{2}}(\cancel{m_{He}})} = \frac{v_{He}^2}{v_o^2}$$
$$8 = \left(\frac{v_{He}}{v_o}\right)^2$$
$$\frac{v_{He}}{v_o} = \sqrt{8}$$
$$= 2.828$$

9.



$\theta = 47.3°$

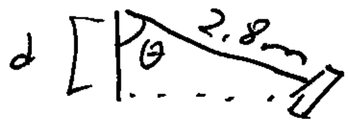
speed on a swing comes from how high you swing → U is converted to K
so how high does she go?



$\theta = 47.3°$

$2.80m = d + \Delta y$

we can find d by looking at:



$\cos\theta = \frac{d}{2.8m}$

$d = 1.8988m$

$\Delta y = 2.80m - d$
$= 0.90m$

now:

$mgh = U = K = \frac{1}{2}mv^2$

$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2$

$(9.8 N/kg)(0.9m) = \frac{1}{2}v^2$

$v = 4.2 m/s$

10. if the two masses were the same, the system would be in equilibrium. since it isn't, the total potential energy due to gravity comes from the extra weight of m_2:

$$U_{net} = (m_2 - m_1)gh$$
$$= (0.292\,kg - 0.186\,kg)(9.8\,N/kg)(0.416\,m)$$
$$= 0.432\,J$$

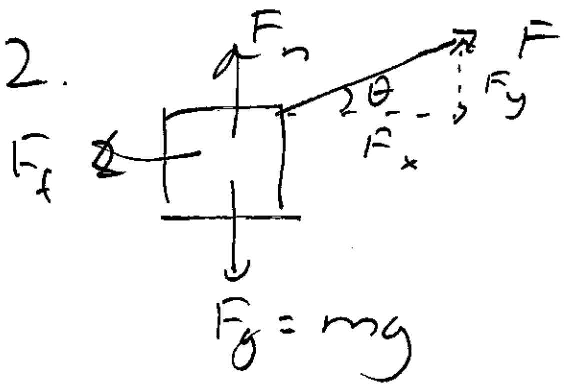
however, this potential energy is converted to kinetic energy for the whole system:

$$K = \frac{1}{2}mv^2$$
$$= \frac{1}{2}(m_1 + m_2)v^2$$
$$0.432\,J = \frac{1}{2}(0.186 + 0.292)$$
$$v^2 = 1.808$$
$$v = 1.34\,m/s$$

11. the plane's kinetic energy is all converted to spring energy:

$$K = E_{spring}$$
$$\cancel{\tfrac{1}{2}}mv^2 = \cancel{\tfrac{1}{2}}k\Delta d^2$$
$$v^2 = \frac{k\Delta d^2}{m}$$
$$= \frac{(6.04\times10^4\,N/m)(32.0\,m)^2}{1.53\times10^4\,kg}$$
$$= 4042.458$$
$$v = 63.58\,m/s$$

12.



$m = 14.8 kg$
$F = 68.5 N$
$\theta = 21.7°$
$\mu_k = 0.267$
$\Delta d = 4.73 m$

a) work is the force done in the direction of displacement times displacement

so:

$W = F_x \cdot \Delta d$
$= F \cos\theta \cdot \Delta d$
$= (68.5 N)(\cos 21.7)(4.73 m)$
$= 301.0 J$

b) no displacement in direction of normal force:

$W = F_n \cdot \Delta d$
$= F_n \cdot 0$
$= 0 J$

c) same as b)

d) energy lost $\rightarrow \Delta E$

$\Delta E = W$

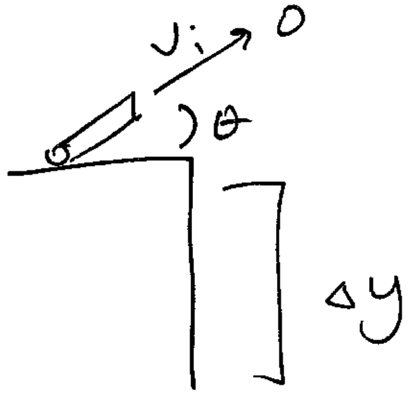
so what's the work done by the force of friction?

$W = F_f \cdot \Delta d$ $\longrightarrow$ F_n opposes <u>net</u> downward force: $= mg - F\sin\theta$
$= F_n \cdot \mu_k \cdot \Delta d$
$= (mg - F\sin\theta) \cdot \mu_k \cdot \Delta d$
$= ((14.8 kg)(9.8 N/kg) - (68.5 N)(\sin 21.7)) \cdot (0.267) \cdot (4.73 m)$

$= 151.2 N$

e) $\Delta K = W_{force} - W_{friction}$
$= 301.0 J - 151.2 J$
$= 149.8 J$

13.



$v_i = 75.6\ m/s$

$\theta = 20.3°$

$\Delta y = 19.4\ m$

$$E_i = K_i + U_i$$
$$= \frac{1}{2} m v_i^2 + mgh$$

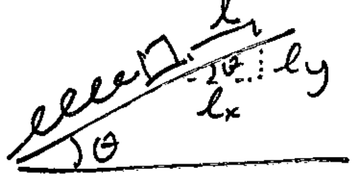
$$E_f = K_f + U_f$$
$$= \frac{1}{2} m v_f^2 + 0$$

conservation of energy: $E_i = E_f$

so:

$$\frac{1}{2} \cancel{m} v_i^2 + \cancel{m} gh = \frac{1}{2} \cancel{m} v_f^2 + 0$$
$$\frac{1}{2} v_i^2 + gh = \frac{1}{2} v_f^2$$
$$v_f^2 = 2\left(\frac{1}{2} v_i^2 + gh\right)$$
$$= v_i^2 + 2gh$$
$$= (75.6\ m/s)^2 + 2(9.8\ m/s^2)(19.4\ m)$$
$$= 78.07\ m/s$$

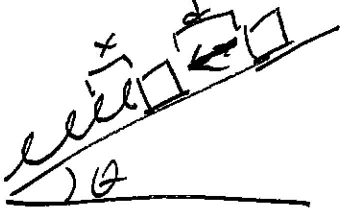
14.



$$m = 0.164\,kg$$
$$k = 1140\,N/m$$
$$\theta = 59.1^\circ$$
$$\Delta d = 0.141\,m$$

a)
$$E_{spring} = U$$
$$\frac{1}{2}k\Delta d^2 = mgh$$
$$= mg\,l_y$$
$$\frac{1}{2}k\Delta d^2 = mg \cdot l \sin\theta$$
$$l = \frac{\frac{1}{2}k\Delta d^2}{mg\sin\theta}$$
$$= \frac{(0.5)(1140\,N/m)(0.141\,m)^2}{(0.164\,kg)(9.8\,N/kg)(\sin 59.1^\circ)}$$
$$= 8.22\,m$$

b)
$$E_{spring} = W_{friction} + U \qquad \mu_k = 0.430$$
$$\frac{1}{2}k\Delta d^2 = F_f \cdot l + mgh$$
$$= F_n \cdot \mu \cdot l + mg\,l_y$$
$$= mg\cdot\cos\theta \cdot \mu \cdot l + mg \cdot l \cdot \sin\theta$$
$$\frac{1}{2}k\Delta d^2 = l\,mg(\cos\theta\,\mu + \sin\theta)$$
$$l = \frac{\frac{1}{2}k\Delta d^2}{mg(\cos\theta\cdot\mu - \sin\theta)}$$
$$= \frac{(0.5)(1140\,N/m)(0.141\,m)^2}{(0.164\,kg)(9.8\,N/kg)(\cos 59.1 \cdot 0.930 + \sin 55.1)}$$
$$= 6.54\,m$$

15. 

$m = 3.34\,kg$
$\theta = 3.42°$
$k = 399\,N/m$
$x = 0.159\,m$
$d = ?$

potential energy due to gravity converted to spring energy:

$$E_{spring} = U$$
$$\tfrac{1}{2}kx^2 = mg(d+x)_y$$
$$= mg(d+x)\sin\theta$$
$$d+x = \frac{\tfrac{1}{2}kx^2}{mg\sin\theta}$$
$$= \frac{(0.5)(399\,N/m)(0.159\,m)^2}{(3.34\,kg)(9.8\,N/kg)(\sin 3.42°)}$$
$$d+x = 0.274\,m$$
$$d = 0.27\,m - x$$
$$= 0.27 - 0.159\,m$$
$$= 0.115\,m$$

16. a) force exerted on the spring is the force due to gravity on the rocket:

$$F_{spring} = F_g$$
$$kx = mg$$
$$x = \frac{mg}{k}$$
$$= \frac{(38.8\,kg)(9.8\,N/kg)}{669.0\,N/m}$$
$$= 0.568\,m$$

b) initial energy: work done by thrust and initial potential spring energy

final energy: U, K, final spring energy

$x_1 = 0.568\,m$ (from a)

$x_2 = 0.384\,m$

$F = 1161.0\,N$

$w = ?$

$$W = F\Delta x$$
$$= F(x_1 + x_2)$$
$$= 1161.0N(0.568m + 0.384m)$$
$$= 1105.272\,J$$

now:

$$W + E_i = E_f + U + K$$
$$W + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}kx_2^2 + mg\Delta x + \tfrac{1}{2}mv^2$$
$$\tfrac{1}{2}mv^2 = W + \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx^2 - mg\Delta x$$
$$v^2 = \frac{2(W + \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2 - mg\Delta x)}{m}$$
$$= \frac{2W + k(x_1^2 - x_2^2)}{m} - 2mg\Delta x \quad \leftarrow x_1 + x_2$$
$$v = 6.42\,m/s$$

c) not tied down → no second spring energy

$$W + E_i = U + K$$
$$W + \tfrac{1}{2}kx_1^2 = mg\Delta x + \tfrac{1}{2}mv^2$$
$$v^2 = \frac{2W + kx_1^2 - 2mg\Delta x}{m}$$
$$v = 6.62\,m/s$$

] this should be larger than your answer for part b)!