12A3

Don't Forget! Test #2 is Nigh!

Test #2 Reviews on Monday (see website)

$$(f(g(x)))' = f'(g(x))\,g'(x) \quad \text{but if } \underline{g(x)} = u$$

$$= f'(u)\frac{du}{dx}$$

$$\boxed{\int f'(g(x))\,g'(x)\,dx} = \boxed{\int f'(u)\,\frac{du}{\cancel{dx}}\,\cancel{dx}}$$

$$= \int (f(g(x)))'\,dx = \boxed{\int f'(u)\,\underline{du}}$$

$$= f(g(x)) + C = \boxed{f(u) + C}$$

Let's create a method of "Integration by Substitution"

Step 1 Given your integral, set (Try?) $u = g(x)$

for some $g(x)$ to make integral pretty

Step 2 Re-express integral in terms of u

$u = g(x)$ & $\frac{du}{dx} = g'(x) \rightsquigarrow dx = \frac{du}{g'(x)}$

"looks like"

Step 3 Integrate in u

Step 4 turn all u back to x's for final answer eg. $\int e^{x} \cos(e^{x})\,dx$

$e^x \to du$, $e^x \to u$

Try $e^{x} = u$

$du/dx = e^{x} \Rightarrow e^{x}dx = du$

$= \int \cos(u)\,du$

$= \sin(u) + c$

$= \sin(e^{x}) + c.$

all u, no x

eg. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

Try $u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

"dx" $= 2\sqrt{x}\,du$

$= \int \frac{e^{u}}{\cancel{\sqrt{x}}} 2\cancel{\sqrt{x}}\,du$

$= 2\int e^{u}\,du = 2e^{u} + c = 2e^{\sqrt{x}} + c$

eg

$$\int \frac{x^2}{x^3+6}\,dx$$

$$= \int \frac{\cancel{x^2}}{u+6} \cdot \frac{1}{3\cancel{x^2}} \cdot du$$

$$= \frac{1}{3} \int \frac{1}{u+6}\,du$$

$$= \frac{1}{3} \int \frac{1}{w}\,dw$$

$$= \frac{1}{3} \ln|w| + c$$

$$= \frac{1}{3} \ln(u+6) + c = \frac{1}{3} \ln|x^3+6| + c$$

Try $u = x^3$

$$du/dx = 3x^2$$

$$dx = \frac{du}{3x^2}$$

Try $w = u+6$

$$\frac{dw}{du} = 1$$

$$du = dw$$

Or let $w = x^3+6$

eg. $\displaystyle\int_e^{e^2} [\ln(x)]^2 / x \, dx$

$$= \int \frac{(\ln(x))^2}{x}\,dx \Big|_e^{e^2}$$ by FTC pt 2 (don't forget!).

let $u = \ln x$ $\quad \frac{du}{dx} = \frac{1}{x} \quad du = \frac{1}{x}\,dx$

$$= \int u^2 \cdot \cancel{\frac{1}{x}\,dx}\,du \Big|_{x=e}^{x=e^2} = \int u^2\,du \Big|_{x=e}^{x=e^2}.$$

$$= \frac{1}{3}u^3 \Big|_{x=e}^{x=e^2} = \frac{1}{3}(\ln x)^3 \Big|_e^{e^2}$$

$$= \frac{1}{3}(8-1) = \underline{\underline{7/3}}.$$

<u>or</u>

$$\int_e^{e^2} \frac{(\ln x)^2}{x}\,dx \qquad u = \ln x \quad \frac{du}{dx} = \frac{1}{x}$$

$$du = \frac{1}{x}\,dx$$

$$= \underline{\underline{\int_1^2 u^2\,du}}$$

$x = e \Rightarrow u = \ln e = 1$

$x = e^2 \Rightarrow u = \ln e^2 = 2$

$$= \frac{1}{3}u^3\Big|_1^2 = \frac{1}{3}(8-1) = \underline{\underline{7/3}}. \quad \checkmark$$

Faster! & we can forget our x!

(no <u>back - substitution!</u>)

eg. $\int_0^1 x\sqrt{1-x}\,dx$

let $u = 1 - x \rightarrow x = 1 - u$

$du/dx = -1$

$-du = dx$

$x = 1 \Rightarrow u = 0$

$x = 0 \Rightarrow u = 1$

$$= \int_1^0 (1-u)\,u^{\frac{1}{2}}(-)du$$

$$= \int_1^0 u^{3/2} - u^{\frac{1}{2}}\,du$$

$$= \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\Big|_1^0$$

$$= 0 - \left(\frac{2}{5} - \frac{2}{3}\right) = 2\left(\frac{1}{3} - \frac{1}{5}\right) = \boxed{4/15}.$$

eg. $\int e^{x^2} dx$

Try $u = x^2$ $\quad \frac{du}{dx} = 2x$

$dx = du/2x$

$x = \pm\sqrt{u}$?

$$= \int e^u \frac{du}{2x} ?$$

$$= \pm \frac{1}{2} \int \frac{e^u}{\sqrt{u}} du$$

no! Not do-able!

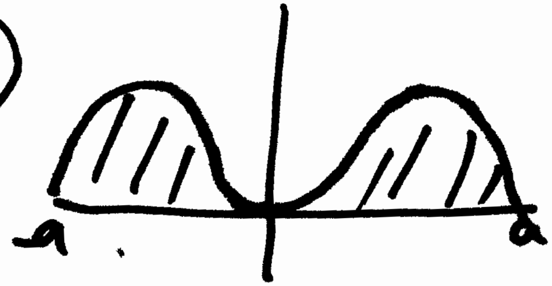
By any method!

Too bad!

If $f(x)$ even $\quad f(-u) = f(u)$

$$\int_{-a}^{a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(u)\,du$$

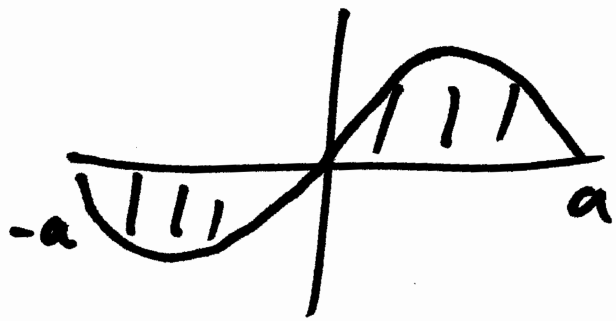
$$= 2\int_0^a f(x)\,dx$$



If $f(x)$ odd $\quad f(-u) = -f(u)$

$$\int_{-a}^{a} f(x)\,dx = \int_0^a f(x)\,dx - \int_0^a f(u)\,du$$

$$= 0$$



Special Case : Symmetry!

Say I have $\int_{-a}^{a} f(x)\,dx$ ~ note symmetric interval abt. $x=0$

$$= \int_0^a f(x)\,dx + \underbrace{\int_{-a}^{0} f(x)\,dx}$$

let $u = -x$. $du = -dx$

$$= \int_0^a f(x)\,dx + \int_a^0 f(-u)(-1)\,du \qquad \begin{cases} x=-a \Rightarrow u=+a \\ x=0 \Rightarrow u=0 \end{cases}$$

$$= \int_0^a f(x)\,dx + \int_0^a f(-u)\,du = \int_{-a}^{a} f(x)\,dx.$$

e)

$$\int_{-59}^{59} \frac{(\sin x)^3 \cdot \cos^{48} x}{\underbrace{100 + x^{98} + x^2 + \cosh(x)}_{}}\, dx$$

$= 0$

"odd"

On symmetric region