Math 1A03/1ZA3: Test #2 Study Sheet

*Note: The existence/absence of any content on this study sheet does NOT guarantee its existence/absence on the test. This sheet is strictly a compact review of the major topics we've covered. You **cannot** use this document as an aid during any tests and/or exams.*

Important Equations:

From the Mean Value Theorem

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Vertical Asymptotes

Given a function of the form $\frac{f(x)}{g(x)}$, set $g(x) = 0$ to determine vertical asymptotes.

Horizontal Asymptotes

A function $f(x)$ has a 'left' horizontal asymptote if

$$\lim_{x \to -\infty} f(x) = L$$

where L is some real number. Similarly, $f(x)$ has a 'right' horizontal asymptote if

$$\lim_{x \to +\infty} f(x) = R$$

where R is some real number. Note that l'Hospital's rule may be needed to compute these limits.

Antiderivatives of Polynomials

If $n \neq -1$, then

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

where C is an arbitrary constant.

Sum Formulas

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \qquad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Sums and Definite Integrals

$$\int_a^b f(x) \, dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x$$

where

$$\Delta x = \frac{b-a}{n}, \qquad \text{and} \qquad x_i = a + i\Delta x$$

Theorems and Concepts:

You should know and understand the following...

(1) Be really comfortable with your derivatives. This will be useful when trying to find antiderivatives.

(2) The Mean Value Theorem. Make sure you understand what the main formula in this theorem means geometrically (tangent line being parallel to a secant line). Be able to solve for the value c for which the Mean Value Theorem holds, if asked.

(3) Understand the definitions of critical points and inflection points. Know the derivative tests and how derivatives can help you know the shape of a graph.

(4) l'Hospital's rule. Remember that we can apply l'Hospital's rule *ONLY* when the limit has the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$. Also, it will be very useful to remember the trick where we can rewrite a product of functions as:

$$f(x)g(x) = \frac{f(x)}{\frac{1}{g(x)}} = \frac{g(x)}{\frac{1}{f(x)}}.$$

Using $\ln(x)$ to get rid of a tricky power is a good thing to recall as well. However, just remember to 'do' e^x when you finish to get the right limit.

(5) Practice more optimization problems!

(6) The Fundamental Theorem of Calculus. This one is VERY important. Know and completely understand both parts of this theorem.

(7) How to find the graph of an antiderivative using the graph of its derivative.

(8) It will be useful for you to be comfortable with converting back and forth between definite integrals and its Riemann sum definition.

(9) Understand that definite integrals represent the *net* area under the graph of a function.

(10) Understand how to do integration by substitution. The trick is *usually* to find a u such that when we 'sub in' for dx, we get a cancelation. Here is an example:

$$\int xe^{x^2}\ dx$$

Let $u = x^2$. Then

$$\frac{du}{dx} = 2x \implies du = 2x\ dx \implies dx = \frac{du}{2x}.$$

Therefore, we have

$$\int xe^{x^2}\ dx = \int \cancel{x}e^u\ \frac{du}{2\cancel{x}} = \frac{1}{2}\int e^u\ du = \frac{e^u}{2} + C = \frac{e^{x^2}}{2} + C.$$

(11) Understand <u>all</u> solutions to your assignment problems.

As I suggested in my last study tip document, try to be confident going into your test (whether you feel ready or not). Writing your test with a positive attitude will help! Study as best you can, that's all anyone can ask.

Good Luck!