COMPSCI/SFWRENG 2FA3

Discrete Mathematics with Applications II

Winter 2020

Week 09 Exercises

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Exercises

1. Write regular expressions that match the following languages over $\{a, b\}$:

   a. The set of all strings that contain an even number of a's.

   Solution:

   $b^* + (b^*ab^*ab^*)^*$.

   b. The set of all strings that contain an odd number of b's.

   c. The set of all strings that contain an even number of a's or an odd number of b's.

   d. The set of all strings that contain an even number of a's and an odd number of b's.

2. Describe in English the sets denoted by the following regular expressions:

   a. $(11 + 0)^*(00 + 1)^*$.

   b. $(1 + 01 + 001)^*(\epsilon + 0 + 00)$.

   Solution:

   The set of strings in $\{0, 1\}^*$ having at most two consecutive 0s.

3. Construct NFAs that are equivalent to the following regular expressions:

   a. $(000^* + 111^*)^*$.

b. $(01 + 10)(01 + 10)(01 + 10)$.

4. Let $L = \{a^m b^n \mid m \leq 4 \text{ and } n \geq 3\}$.

   a. Write a regular expression that matches L.
   b. Write a regular expression that matches $\sim L$.

5. Prove the following equations are valid for regular expressions. Here $\alpha = \beta$ means $L(\alpha) = L(\beta)$.

   a. $\alpha + \alpha = \alpha$.

   b. $\alpha + \beta = \beta + \alpha$.

   c. $\alpha(\beta + \gamma) = \alpha\beta + \alpha\gamma$.

   d. $(\alpha^*)^* = \alpha^*$.

   e. $(\alpha^*\beta^*)^* = (\alpha + \beta)^*$.