MATHEMATICS 1LS3E TEST 1

Evening Class
Duration of Examination: 75 minutes
McMaster University, 9 May 2012

E. Clements

FIRST NAME (please print): SOL^{NS}

FAMILY NAME (please print): ______

Student No.: ______

THIS TEST HAS 8 PAGES AND 10 QUESTIONS. QUESTIONS BEGIN ON PAGE 2. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE.

Total number of points is 40. Marks are indicated next to the problem number. Any non-graphing calculator is allowed.

USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

You need to show work to receive full credit.

Page	Points	Mark
2	5	
3	8	
4	8	
5	6	
6	4	
7	5	
8	4	
TOTAL	40	

Continued on next page

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Student No.: ____________________

1. [2] Determine the domain of each function.

(a) $f(x) = \dfrac{2x-3}{x^2-4x}$

$x^2 - 4x \neq 0$

$x(x-4) \neq 0$

$\boxed{x \neq 0, 4}$

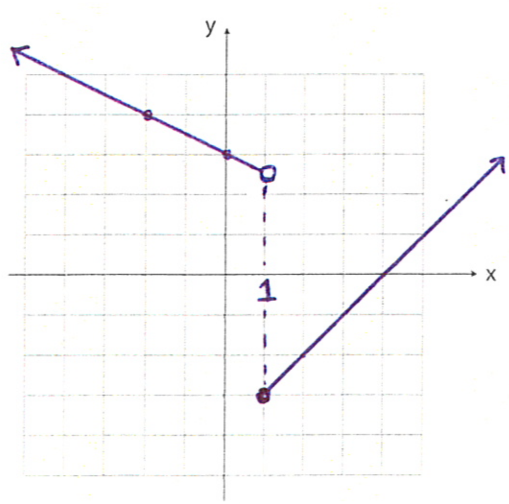
(b) $g(x) = \ln(5-2x)$

$5 - 2x > 0$

$-2x > -5$

$\boxed{x < \frac{5}{2}}$

2. (a) [2] Sketch the graph of $f(x) = \begin{cases} -\frac{1}{2}x + 3 & \text{if} \quad x < 1 \\ x - 4 & \text{if} \quad x \geq 1 \end{cases}$.



(b) [1] State the range of f.

$y \geqslant -3$

3. [3] A table of values for f and g is given below.

x	$f(x)$	$g(x)$
1	3	2
2	1	8
3	7	2

Compute the following:

(a) $(f+g)(1)$

$= f(1) + g(1)$

$= 3 + 2$

$= 5$

(b) $(fg)(2)$

$= f(2) \cdot g(2)$

$= (1)(8)$

$= 8$

(c) $(f \circ g)(3)$

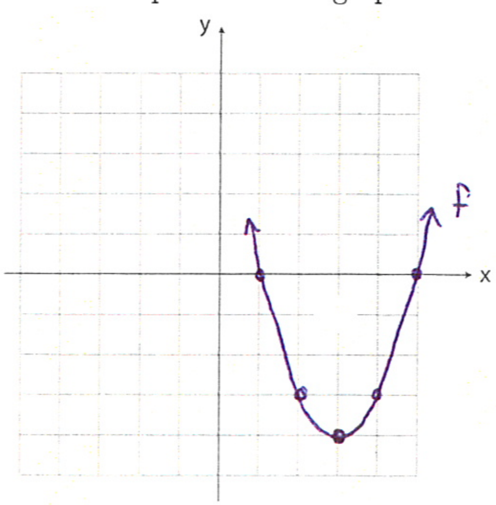
$= f(g(3))$

$= f(2)$

$= 1$

4. Consider the function $f(x) = x^2 - 6x + 5$.

(a) [3] Complete the square to find the vertex. Sketch the graph of f, clearly labelling at least three points on the graph.



$f(x) = x^2 - 6x + 9 - 9 + 5$

$\left(-\frac{6}{2}\right)^2 = (-3)^2 = 9$

$= (x-3)^2 - 4$

Vertex: $V(3,-4)$

(b) [2] Solve $x^2 - 6x + 5 = 0$. Explain how this shows that the function f is not one-to-one.

$(x-5)(x-1) = 0$

$x = 5, 1$

when $y=0$, $x=5$ OR $x=1$
So, for this y-value, there are TWO possible x-values ... violates definition of a 1-1 f^n.

5. (a) [3] The function $f(x) = \frac{1}{2x+3} - 4$ is one-to-one. Find f^{-1}.

$$y = \frac{1}{2x+3} - 4$$

$$y + 4 = \frac{1}{2x+3}$$

$$2x + 3 = \frac{1}{y+4}$$

$$x = \frac{\frac{1}{y+4} - 3}{2}$$

$$\therefore\ f^{-1}(x) = \frac{\frac{1}{x+4} - 3}{2}$$

extra simplifying:

$$f^{-1}(x) = \frac{1 - 3(x+4)}{2(x+4)} = \frac{-3x - 11}{2(x+4)}$$

(b) [2] State the domain and range of $f(x)$.

$$x \neq -\frac{3}{2}$$

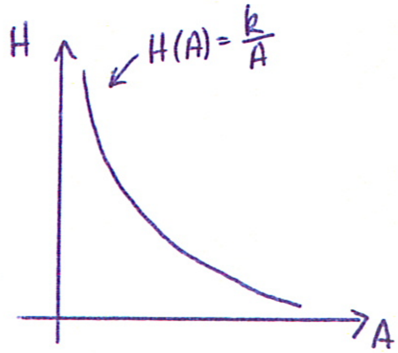
$$y \neq -4$$

6. Suppose that the level of a certain hormone, H, in the human body is inversely proportional to age, A.

[2] (a) Write an equation for $H(A)$, hormone level as a function of age. Sketch a graph of this relationship.

$$H \propto \frac{1}{A} \Rightarrow H(A) = k \cdot \frac{1}{A}$$

$k > 0$



[1] (c) Compare the level of this hormone in a 10 year old to a 40 year old.

$$A_{10} = \frac{1}{4} A_{40}$$

$$H_{10} = k \cdot \frac{1}{A_{10}} = k \cdot \frac{1}{\frac{1}{4} A_{40}} = 4 \cdot k \cdot \frac{1}{A_{40}} = 4 \cdot H_{40}$$

∴ A 10-year old has 4 times the amount of this hormone compared to a 40 year old.

7. The ambient temperature and humidity affect the lifespan of an organism. Assume that the humidity is 40 % and that the lifespan is a linear function of the air temperature. It has been determined that a mosquito lives 28 days at 26°C and 18 days at 46°C.

[2] (a) Find a formula for the lifespan of a mosquito as a function of ambient temperature.

Let L = lifespan in days and T = temp. in °C.

$$m = \frac{\Delta L}{\Delta T} = \frac{18-28}{46-26} = -0.5$$

$$L - 28 = -0.5(T-26) \Rightarrow L(T) = -0.5T + 41$$

[1] (b) What is the lifespan of a mosquito which lives at 30°C?

$L(30) = -0.5(30) + 41 = 26$ days.

[2] (c) What is the slope of the line from (a) (including units)? Interpret the slope in terms of the drop in lifespan.

Slope $= m = \frac{-1}{2}$ days/°C

This means that the lifespan will decrease by half a day for each 1°C increase in temperature.

[1] (d) What quantity is the parameter?

humidity

8. A person develops a small liver tumour which grows exponentially according to the formula

$$S(t) = S(0)e^{kt}$$

where $S(0) = 1g$ and $k = 0.1/\text{day}$.

[2] (a) Suppose that the tumour can only be detected when it has grown to at least 25 grams. What is the earliest this tumour can be detected?

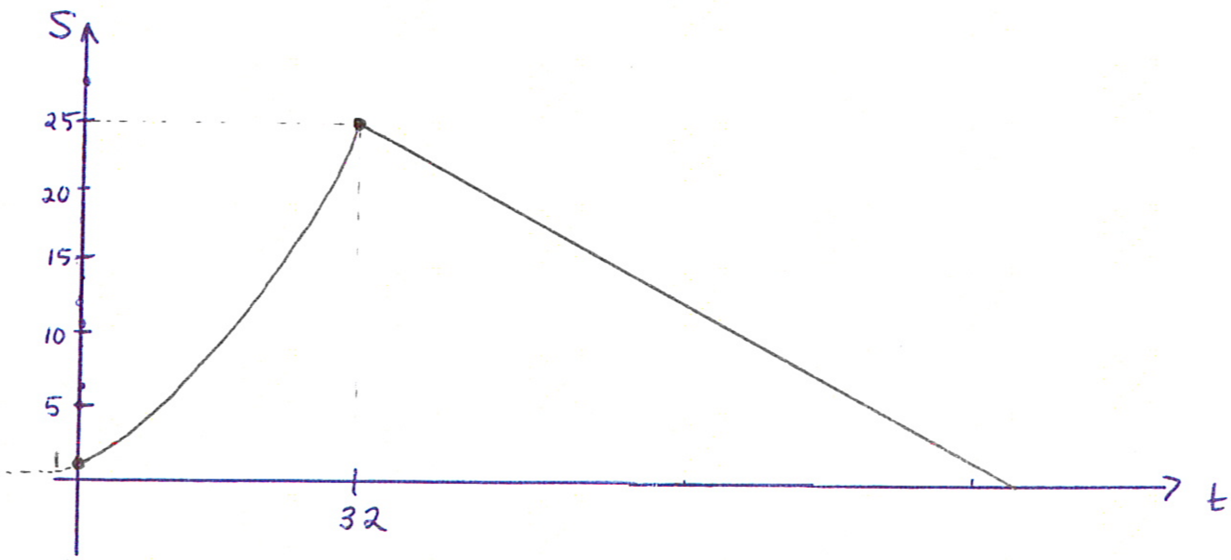
$$S(t) = 1 \cdot e^{0.1t}$$

$$e^{0.1t} = 25$$

$$0.1t = \ln 25$$

$$t = \frac{\ln 25}{0.1} \approx 32 \text{ days}$$

[2] (b) As soon as the tumour is detected, the person begins treatment and the tumour size decreases linearly with slope of $-0.4g/\text{day}$. Sketch a graph to represent the size of the tumour over time.



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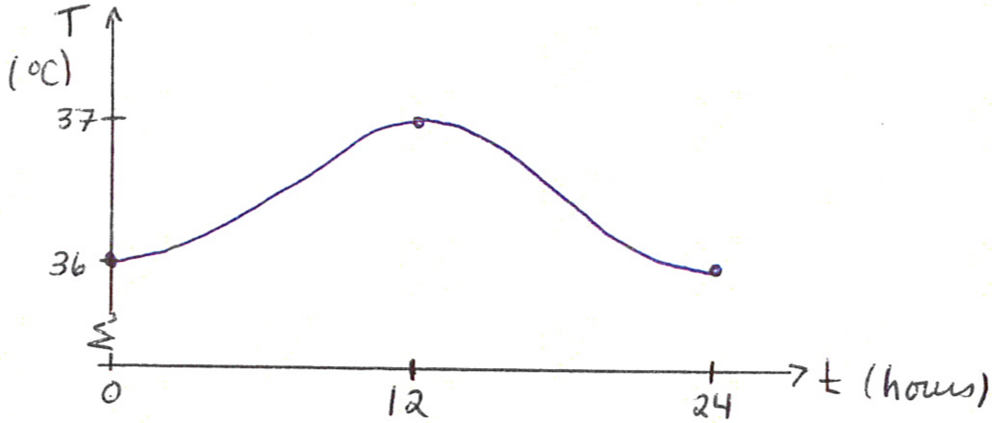
9. Body temperature oscillates between high values during the day and low values at night. Suppose that an individual's body temperature reaches a maximum of 37°C at noon (12:00) and a minimum of 36°C at midnight (24:00).

[2] Sketch a graph to illustrate how body temperature depends on the time of day. Label the axes.



[3] (b) Write a trigonometric function to describe your graph in part (a). (Hint: Start with the graph of $T(t) = \cos t$, where T is temperature in °C and t is time in hours, and determine transformations required to obtain your graph in part (a).)

average value $= \frac{36+37}{2} = 36.5$

amplitude $=$ max − avg. value $= 37 - 36.5 = 0.5$

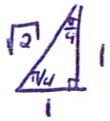
period : $\frac{2\pi}{B} = 24 \rightarrow B = \frac{2\pi}{24} = \frac{\pi}{12}$

$\therefore \quad T(t) = -0.5 \cos \frac{\pi}{12} t + 36.5$

10. (a) [2] Find the exact value of the following. Show your work!

(i) $\arctan(1) = \theta$

$\Leftrightarrow \tan\theta = 1$ where $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$

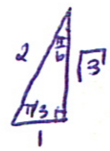


$\theta = \frac{\pi}{4}$

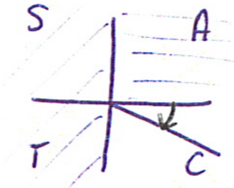
$\therefore \arctan 1 = \frac{\pi}{4}$

(ii) $\arcsin(-\frac{1}{2}) = \theta$

$\Leftrightarrow \sin\theta = -\frac{1}{2}$ where $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$



$\theta' = \frac{\pi}{6}$



$\theta = -\frac{\pi}{6}$

$\therefore \arcsin(-\frac{1}{2}) = -\frac{\pi}{6}$

(b) [2] Is it true that $\arcsin(\sin\pi) = \pi$? Explain.

NO!

$\arcsin(\sin x) = x$ only when $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

since $\pi \notin [-\frac{\pi}{2}, \frac{\pi}{2}]$, we cannot use the cancellation eq^{n}s.

THE END