Math 1AA3/1ZB3: Week 6 Tutorial Problems

February 8, 2019

1. Determine whether the following series converge or diverge. If an alternating series converges, determine whether it converges conditionally or converges absolutely

   (a)
   $$\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$$

   (b)
   $$\sum_{n=0}^{\infty} \frac{(-2)^n}{n+3^n}$$

   (c)
   $$\sum_{n=1}^{\infty} \frac{n^2+3n-1}{\sqrt{n^5+2n^3+3n}}$$

   (d)
   $$\sum_{n=1}^{\infty} \frac{n! \cdot 3^{n+1}}{2 \cdot 4 \cdot \ldots \cdot (2n)}$$

   (e)
   $$\sum_{n=1}^{\infty} \frac{(-1)^n + n}{n^2}$$

   (f)
   $$\sum_{n=1}^{\infty} \frac{(-3)^n \ln n}{(n+1)!}$$

   Chapter 11.7 of the textbook contains a good summary of the various tests for evaluating series, and suggestions for choosing an appropriate strategy.

2. Suppose that $S = \sum_{n=0}^{\infty} a_n$ is a series and the n^{th} partial sum S_n of S is given by $S_n = \frac{3n-1}{4n+2}$.

   (a) Find an expression for the general term a_n.

   (b) Either compute the value of S or show that it diverges.

3. Suppose that $S = \sum_{n=1}^{\infty} a_n$ and $T = \sum_{n=1}^{\infty} b_n$ are strictly positive series and both are convergent. Prove that the following series is also convergent:

$$\sum_{n=1}^{\infty} (a_n + b_n)^2$$