McMaster University
Math 1LT3

2nd Edition, Geese:
Sections 8.2, 8.3, and 8.4

ASSIGNMENT 2

Sections 7.2, 7.3, 7.4

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1. A population of sharks is described by the logistic differental equation

$$\frac{dS}{dt} = 0.8S\left(1 - \frac{S}{400}\right)$$

(a) Find the equilibria of this equation. What do these numbers represent?

(b) Draw a phase-line diagram for this differential equation.

(c) Sketch the equilibrium solutions and solution curves corresponding to the initial conditions $\underset{S}{\cancel{P}}(0) = 20$, $\underset{S}{\cancel{P}}(0) = 300$, and $\underset{S}{\cancel{P}}(0) = 500$.

(d) Suppose it is known that the population will die out if it ever falls below 35 sharks. Write a modified logistic differential equation to illustrate this.

(e) Draw a phase-line diagram for this modified differential equation.

(f) For the equation in part (d), sketch the equilibrium solutions and solution curves corresponding to the initial conditions $\underset{S}{\cancel{P}}(0) = 20$, $\underset{S}{\cancel{P}}(0) = 300$, and $\underset{S}{\cancel{P}}(0) = 500$.

2. Assuming that there is competition within a population of bacteria for resources, a model for limited bacterial growth is given by

$$\frac{db}{dt} = \lambda(b)b$$

where the per capita production rate, λ, is a decreasing function of the population size, b. Suppose that the per capita production rate is a linear function of population size with a maximum of $\lambda(0) = 1$ and a slope of -0.002.

(a) Find $\lambda(b)$ and write the differential equation for b.

(b) Determine the equilibrium solutions.

(c) Graph the rate of change, $\frac{db}{dt}$, as a function of b.

(d) Describe, in words, the dynamics of a population of bacteria modeled by the differential equation found in part (a).

3. Consider the differential equation $\dfrac{dy}{dt} = ye^{-\beta y} - ay$, where a and β are parameters.

(a) Find the equilibria.

(b) Use the stability theorem to determine the stability of the equilibria. (Note: The stability depends on the parameter a so you will need to consider different cases.)

(c) Suppose that $a = 0.5$ and $\overset{\beta}{\cancel{b}} = 1$. Draw the phase-line diagram for $\dfrac{dy}{dt} = ye^{-\beta y} - ay$.

(d) Suppose that $a = e$ and $\overset{\beta}{\cancel{b}} = 1$. Draw the phase-line diagram for $\dfrac{dy}{dt} = ye^{-\beta y} - ay$.

4. Exercise 42 on page 543 in your text.

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#42 on p. 619

5. First, determine the equilibrium solutions of each differential equation. Then, use the separation of variables technique to find remaining solutions for each differential equation.

(a) $\dfrac{dy}{dx} = \dfrac{y^2 \cos x}{1 + y^2}$

(b) $(x^2 + 1)\dfrac{dy}{dx} = xy$

6. Use separation of variables to find the solution of the differential equation that satisfies the given initial condition.

(a) $P'(t) = P(t) + tP(t) + t + 1, \;\; P(0) = 50$

(b) $(2y + e^{3y})y' = x \cos x, \;\; y(0) = 0$

7. Consider a special case of the "selection equation", $\frac{dp}{dt} = p(1-p)$.

2nd Edition, Geese: #51(a)-(f) on p. 629

(a) Solve the selection equation using separation of variables and integration by partial fractions as outlined in exercises 45-50 on pages 551 and 552 in your textbook.

(b) Exercise 52 on page 552. Using your solution from part (a) and the initial condition $p(0) = 0.5$, find the value of the constant. Evaluate the limit of the solution as t approaches infinity.

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THE END