McMaster University
Math 1LT3E

ASSIGNMENT 5

Sections 3, 4, and 5 in the Red Module

---

1. (a) In your own words, explain what is meant by $\lim_{(x,y)\to(a,b)} f(x,y) = L$.

(b) Explain how you would show that $\lim_{(x,y)\to(a,b)} f(x,y)$ does not exist.

2. Show that the following limits do not exist. Sketch the domains and paths involved.

(a) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{(x-y)^2}{x^2+y^2}$

(b) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{2xy^2}{x^2+y^4}$

3. (a) Explain what you would have to show in order to prove that a function $f(x, y)$ is continuous at (a, b).

(b) Find a function g such that $\lim_{(x,y)\to(5,4)} g(x, y)$ exists but g is not continuous at $(5, 4)$.

(c) Find and sketch the largest domain on which $z = \ln(y - x) + \sqrt{y + x}$ is continuous.

4. Use the definition of continuity to show that

$$h(x,y) = \begin{cases} 4 - e^{-x-y+2} & \text{if} \quad (x,y) \neq (1,1) \\ 3 & \text{if} \quad (x,y) = (1,1) \end{cases}$$

is continuous at $(1,1)$.

5. Assume that the function $T(x,y,t)$ models the temperature (in degrees Celsius) at time t in a city located at a longitude of x degrees and a latitude of y degrees. The time t is measured in hours. What is the meaning of the partial derivative $T_t(x,y,t)$? What are its units? What is most likely going to be the sign of $T_y(x,y,t)$ for Winnipeg, Manitoba in January?

6. Below is an excerpt from a table of values of I, the temperature-humidity index, which is the perceived air temperature when the actual temperature is T (degrees fahrenheit), and the relative humidity is h (percent).

$T\downarrow$ $h\rightarrow$	20	30	40	50	60	70
80	74	76	78	82	83	86
85	81	82	84	86	90	94
90	86	90	93	96	101	106
95	94	94	98	107	111	125
100	99	101	109	122	129	138

(a) Write the definition (equation) of the partial derivative of $I(T, h)$ with respect to h.

(b) Approximate $I_h(95, 40)$ and interpret your answer, i.e., write a statement to explain what this number represents.

7. Compute the indicated partial derivatives.

(a) $f(x,y) = \dfrac{4x - xy}{x^2 + y^2}$; $f_x(x,y)$

(b) $h(x,t) = e^{\sqrt{x-4t^2}}$; $h_t(5,1)$

8. A hiker is standing at the point $(2, 1, 21)$ on a hill whose shape is given by the graph of the function $f(x, y) = 24 - (x - 3)^2 - 2(y - 2)^2$.

(a) In which of the two directions (x-direction or y-direction) is the hill steeper?

(b) Sketch a graph of the function $f(x, y) = 24 - (x - 3)^2 - 2(y - 2)^2$. On the graph, draw the curves $z = f(2, y)$ and $z = f(x, 1)$. Draw the tangent line to each curve at the point $(2, 1, 21)$.

(c) For what x- and y-coordinates will the hiker reach the top of the hill? What are the values of f_x and f_y at this point?

9. Let $f(x, y) = \ln(3x - y + 1)$.

(a) Compute the partial derivatives of f.

(b) Find and sketch the domains of f_x and f_y.

(c) Is f differentiable at $(1, 0)$? Explain.

10. Consider the function $f(x, y) = \sqrt{y + \cos^2 x}$.

(a) Show that the function is differentiable at $(0, 0)$.

(b) Verify the linear approximation $\sqrt{y + \cos^2 x} \approx 1 + \dfrac{1}{2}y$ at (0,0).

THE END