MATHEMATICS 1LS3 TEST 2

Day Class
Duration of Examination: 60 minutes
McMaster University, 15 October 2014

E. Clements, M. Lovrić, O. Sanchez

FIRST NAME (please print): SOLUTIONS

FAMILY NAME (please print): ______________

Student No.: ______________

THIS TEST HAS 8 PAGES AND 7 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE.

Total number of points is 40. Marks are indicated next to the problem number. Any non-graphing calculator is allowed.

USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

You must show work to receive full credit.

Problem	Points	Mark
1	6	
2	6	
3	6	
4	6	
5	3	
6	7	
7	6	
TOTAL	40	

Continued on next page

Name:
Student No.:

1. **Multiple choice questions: circle ONE answer. No justification is needed.**

(a)[3] Which of the functions approach(es) ∞ more quickly than $x^{2.3}$ as $x \to \infty$?

(I) $f(x) = x^{2.2}$ X (II) $f(x) = x^{2.4}$ ✓ (III) $f(x) = \ln x$ X

(A) none	(B) I only	(C) II only	(D) III only
(E) I and II	(F) I and III	(G) II and III	(H) all three

(b)[3] Assume that $f(x)$ is continuous at $x = 3$ and $f(3) = -2$. Which of the following statements is/are true for every function which satisfies these assumptions?

(I) $y = \sqrt{f(x)}$ is continuous at $x = 3$ X

(II) $\lim_{x \to 3} f(x) = -2$ ✓

(III) $y = \dfrac{1}{f(x)}$ is continuous at $x = 3$ ✓

(A) none	(B) I only	(C) II only	(D) III only
(E) I and II	(F) I and III	(G) II and III	(H) all three

Continued on next page

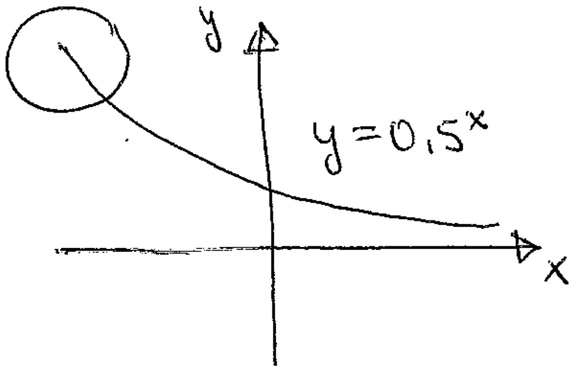
Name:________________

Student No.: ________________

2. Identify each statement as true or false, or yes or no (circle your choice). You do not need to justify your answer.

(a)[2] $\lim_{x\to-\infty} 0.5^x = 0$. ← it is $+\infty$

TRUE FALSE



(b)[2] The average rate of change of $f(x) = 3x - 17$ on the interval $[1.5, 1.5064]$ is 3.0064.

TRUE FALSE

line, so slope = 3!

slope = average rate of change

(c)[2] The line $y = \pi$ is a horizontal asymptote of the graph of $f(x) = 2\arctan(x^3 + 1)$.

TRUE FALSE

$$\lim_{x\to\infty} 2\cdot\arctan(x^3+1)$$

$$= 2\cdot\underbrace{\arctan(\infty)}_{\frac{\pi}{2}} = \pi$$

Continued on next page

Name:

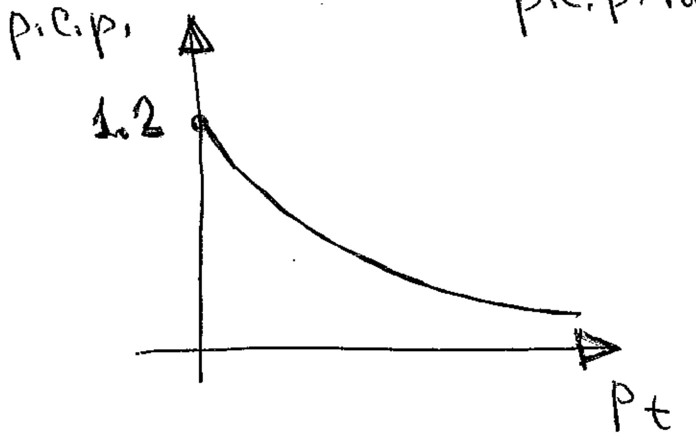
Student No.:

Questions 3-7: You must show work to receive full credit.

3. The dynamical system $p_{t+1} = \dfrac{12p_t}{10 + 0.002p_t}$ models the population of caribou in southern regions of Nunavut (p_t is the number of caribou and t is time in years).

(a)[2] Identify the per capita production rate and make a rough sketch of it.

$$\text{p.c.p. rate} = \frac{12}{10 + 0.002p_t}$$



(b)[1] Does the dependence of the per capita production on the population size make sense? Explain why or why not.

YES ... with an increase in the number of caribou the p.c.p. (i.e., the number of offspring per individual) will decrease (due to competition for resources)

(c)[3] Find all equilibrium points of the given system.

$$p^* = \frac{12p^*}{10 + 0.002p^*}$$

$$p^*\left(1 - \frac{12}{10 + 0.002p^*}\right) = 0$$

$$\underline{\underline{p^* = 0}}$$

$$\frac{12}{10 + 0.002p^*} = 1$$

$$10 + 0.002p^* = 12$$

$$p^* = \frac{2}{0.002}$$

$$\underline{\underline{p^* = 1000}}$$

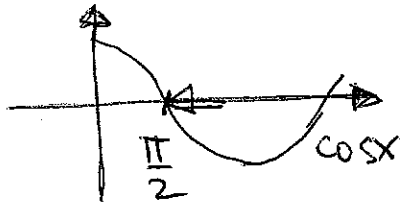
Continued on next page

Name:____________
Student No.: ____________

4. Find each limit.

(a)[2] Find $\lim_{x\to(\pi/2)^+} x^2 \sec x$ $= \lim_{x\to(\frac{\pi}{2})^+} x^2 \cdot \frac{1}{\cos x} = \left(\frac{\pi}{2}\right)^2 \cdot \frac{1}{0} = -\infty$

$\ominus$



(b)[2] $\lim_{x\to 2} \frac{\frac{1}{4}-\frac{1}{x^2}}{x-2}$ $= \frac{0}{0} = \lim_{x\to 2} \frac{\frac{x^2-4}{4x^2}}{x-2}$

$= \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{4x^2} \cdot \frac{1}{\cancel{x-2}} = \frac{4}{4(2)^2} = \frac{1}{4}$

(c)[2] $\lim_{x\to\infty} e^{-x^2-5x-12}$ $= \lim_{x\to\infty} e^{-x^2} = e^{-\infty} = 0$

Continued on next page

Name:_______________

Student No.: _______________

5. [3] The following excerpt is taken from *The laminar cortex model: a new continuum cortex model incorporating laminar architecture.* J. Du, V. Vegh, and D.C. Reutens. PLoS Computational Biology. 8.10 (Oct. 2012).

> the average of membrane potentials of neurons in the element, that is
>
> $$V = \frac{N_e V_e + N_i V_i}{N_e + N_i}$$
>
> where N_e, N_i are the numbers of excitatory and inhibitory neurons and V_e and V_i are the (average) membrane potentials of excitatory and inhibitory neuron populations respectively.

We view V as a function of N_i (thus, all remaning symbols on the right side are parameters). What is the limit of V as N_i increases beyond any bounds (i.e., as it approaches ∞)?

$$\lim_{N_i \to \infty} V = \lim_{N_i \to \infty} \frac{N_e V_e + V_i N_i}{N_e + N_i}$$

like $\frac{3+2x}{4+x}$

$$= \lim_{N_i \to \infty} \frac{V_i N_i}{N_i} = \underline{\underline{V_i}}$$

or: $$\lim_{N_i \to \infty} \frac{N_e V_e + N_i V_i}{N_e + N_i} = \lim_{N_i \to \infty} \frac{\overbrace{\frac{N_e V_e}{N_i}}^{0} + V_i}{\underbrace{\frac{N_e}{N_i}}_{0} + 1} = \underline{\underline{V_i}}$$

divide numerator and denominator by N_i

6. Consider the function

$$f(x) = \begin{cases} \dfrac{x-1}{x^3-x} & \text{if} \quad x < 1 \\ \dfrac{x}{2} & \text{if} \quad x \geq 1 \end{cases}$$

(a)[3] Find $\lim_{x\to 1} f(x)$.

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{x}{2} = \frac{1}{2}$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \frac{x-1}{\underbrace{x^3-x}_{x(x^2-1)}} = \lim_{x\to 1^-} \frac{\cancel{x-1}}{x(\cancel{x-1})(x+1)} = \frac{1}{2}$$

so $\lim_{x\to 1} f(x) = \frac{1}{2}$

(b)[2] Is $f(x)$ continuous at $x = 1$? Explain why or why not.

$\underbrace{\lim f(x)}_{=\frac{1}{2} \text{ from (a)}} = f(1)$ = using the bottom piece $= \frac{x}{2} = \frac{1}{2}$

equal, so f(x) is cont. at $x=1$

(c)[2] Is $f(x)$ continuous at $x = -1$? Explain why or why not.

near $x = -1$, $f(x) = \dfrac{x-1}{x^3-x}$

$x^3 - x = (-1)^3 - (-1) = -1+1 = 0$

f(x) is a rational function, denominator $= 0$ at -1 $\Rightarrow$ not cont. at $x=-1$

Continued on next page

Name:__________________
Student No.: __________________

7. Consider the alcohol consumption dynamical system $a_{t+1} = a_t - \dfrac{10.5a_t}{4.5 + a_t} + d$, where a_t is the amount of alcohol (in grams) at time t (measured in hours).

(a)[1] What is the meaning of the parameter d?

Constant amount of alcohol consumed/added every hour

(b)[2] What is the meaning of the term $\dfrac{10.5a_t}{4.5 + a_t}$ in the formula for a_{t+1}? What are its units?

decrease in the amount of alcohol due to absorption ie elimination

or: amount of alcohol absorbed

grams

(c)[3] For which values of d does the given system have a meaningful equilibrium?

$$\cancel{a^*} = \cancel{a^*} - \frac{10.5a^*}{4.5+a^*} + d$$

$$\frac{10.5a^*}{4.5+a^*} = d$$

$$10.5a^* = 4.5d + a^*d$$

$$a^*(10.5 - d) = 4.5d$$

$$a^* = \frac{4.5d}{10.5-d}$$

meaningful: $a^* > 0$

so $10.5 - d > 0$

$d < 10.5$

($d = 10.5$ would make $a^* = \infty$)

THE END