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COMP SCI 2GA3
Sheet 1 — 2018-09-07
Practice Problems

Computer Architecture

Exercise 1.1

Consider two different implementations, M_1 and M_2, of the same instruction set. There are four classes (A, B, C and D) of instructions in the instruction set.

M_1 has a clock rate of 2500 MHz, and M_2 has a clock rate of 3000 MHz. The average number of cycles for each instruction class on the two machines are as follows:

Instruction Class	CPI on M_1	CPI on M_2
A	3	4
B	4	4
C	6	7
D	9	10

(a) If the number of instructions executed in a certain program is divided equally among the classes of instructions in Exercise 1, what is the speedup of M_2 over M_1?

(b) Assuming the instruction distribution from (a), at what clock rate would M_1 have the same performance as the 3000 MHz version of M_2?

Exercise 1.2

We are interested in two implementations of a machine, one with and one without special floating-point hardware.

- Machine M_{FP} (Machine with Floating Point) has floating-point hardware and can therefore implement the floating-point operations directly.

- Machine M_{NFP} (Machine with No Floating Point) has no floating-point hardware and so must emulate the floating-point operations using integer instructions. (The integer instructions all take 4 clock cycles.)

We consider a program P, containing a mix of operations as indicated in the following table, which also lists the number of *clock cycles* required by for each instruction class on M_{FP}, and the *number of integer instructions* needed on M_{NFP} to implement each of the floating-point operations:

Instruction Class	Frequency in P	CPI on M_{FP}	# of integer instr. on M_{NFP}
floating-point add	20%	6	16
floating-point multiply	15%	8	28
floating-point divide	5%	24	52
integer instructions	60%	4	

Both machines have a clock rate of 1000 MHz.

(a) Find the native MIPS ratings for execution of program P on both machines.

(b) If the machine M_{FP} needs 240 millon instructions for program P, how many integer instructions does the machine M_{NFP} require for the same program?

(c) Assuming the instruction counts from (b), what is the execution time (in seconds) for the program P run on M_{FP} respectively M_{NFP}?

Exercise 1.3

You are the lead designer of a new processor. The processor design and compiler are complete, and now you must decide whether to produce the current design as it stands or spend additional time to improve it. You discuss this problem with your hardware engineering team and arrive at the following options:

- *Leave the design as it stands.* Call this base machine M_{base}. It has a clock rate of 2800 MHz.
- *Optimize the hardware.* The hardware team claims that it can improve the processor design to give it a clock rate of 3200 MHz. Call this machine M_{opt}.

The following table shows the frequency of instructions of the different classes in the test suite used for evaluating the designs, and the CPI values for both machines:

Instruction class	Frequency	CPI on M_{base}	CPI on M_{opt}
A	40%	3	3
B	25%	4	3
C	20%	4	4
D	15%	6	5

(a) What is the CPI for each machine?

(b) What are the native MIPS ratings for M_{base} and M_{opt}?

(c) What is the speedup of M_{opt} over M_{base}?

(d) The compiler team has heard about the discussion to enhance the machine discussed in (a)–(c). The compiler team proposes to improve the compiler for the machine to further enhance performance. Call this combination of the improved compiler and the base machine M_{comp}. The instruction improvements from this enhanced compiler have been estimated as follows:

Instruction class	Percentage of instructions executed vs. base machine
A	90%
B	90%
C	85%
D	95%

For example, if the base machine executed 500 class A instructions, M_{comp} would execute $0.9 \cdot 500 = 450$ class A instructions for the same program.

What is the CPI for M_{comp}?

(e) What is the speedup of M_{comp} over M_{base}?

(f) The compiler group points out that it is possible to implement both the hardware improvements of (a) and compiler enhancements described in (d). If *both* the hardware and compiler improvements are implemented, yielding machine M_{both}, how much faster is M_{both} than M_{base}?

(g) You must decide whether to incorporate the hardware enhancements suggested in (a) or the compiler enhancements of (d) (or both) to the base machine. You estimate that the following time would be required to implement these optimizations:

Optimization	Time to implement	Machine name
Compiler	4 months	M_{comp}
Hardware	6 months	M_{opt}
Both	8 months	M_{both}

Assuming that CPU performance improves by approximately 50% per year, or about 3.4% per month. and also assuming that the base machine has performance equal to that of its competitors, which optimizations (if any) would you choose to implement?

Exercise 1.4 — Binary Numbers

(a) Read the Wikipedia article on binary numbers.

(b) Practice binary counting from 0 to 31 using the fingers of one hand.

(c) The algorithms you use for "manual" addition, subtraction, multiplication, and division of decimal numbers are actually just instances of general algorithms for arithmetic with numbers in positional notations, instantiated with base 10.

If you need to remind yourself of these algorithms, try the following calculations in decimal first:

3752	2465	4269	45632	61583
+ 4426	+ 7538	− 2835	− 27879	− 73266
=	=	=	=	=

$$2345 * 47 = \qquad 357 * 1408 = \qquad 10741 / 23 =$$

Perform the following calculations with base 2, that is, in the binary number system:

10010	101010	1010101	1010011	11010110
+ 10101	+ 1010101	+ 101011	+ 111010	+ 111111
=	=	=	=	=

101011	1010101	1010011	11010110	101010
− 10101	− 101011	− 111010	− 111111	− 1010101
=	=	=	=	=

$$1101011 * 101 = \qquad 11001011 * 101101 = \qquad 11101001101 / 10101 =$$

(d) Check your work by conversion to decimal.

Exercise 1.5 — Hexadecimal Numbers

(a) Read the Wikipedia article on hexadecimal numbers, especially the paragraph on binary conversion

(b) Re-check your work in 1.4(c) by conversion to hexadecimal.

(c) Perform the following calculations in the hexadecimal number system, that is, with base 16:

ABC	ABCD	3E4D	B361A8F4	29DB47F8
+ 123	+ 1234	+ DDD5	+ 2D9E751B	+ 84C49EDB
=	=	=	=	=

ABCD	4A5D	BA9E	B361A8F4	2D9E751B
− 7654	− 3F26	− 6DAF	− 84C49EDB	− 29DB47F8
=	=	=	=	=

$$FA4D * 235 = \qquad DEED * CAB = \qquad 8E1F529 / BEAD =$$