Name: ____________________

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1. (a)[2] Solve the equation $2e^{3-4x} = 5$.

$$e^{3-4x} = 2.5$$

$$3 - 4x = \ln 2.5$$

$$x = \frac{3 - \ln 2.5}{4} \approx 0.52$$

(b)[2] Find all x such that $2\ln(x-4) + \ln 5 = \ln 10$.

$$\ln(x-4)^2 + \ln 5 = \ln 10$$

$$\ln\left((x-4)^2 \cdot 5\right) = \ln 10$$

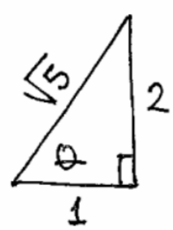
$$5(x-4)^2 = 10 \rightarrow (x-4)^2 = 2$$

so $x - 4 = \pm\sqrt{2}$ and $x = 4 \pm \sqrt{2}$

$x = 4 + \sqrt{2} \approx 5.41$ is a solution

$x = 4 - \sqrt{2} \approx 2.59$ is not a solution

(c)[2] Given that $\tan\theta = 2$ (where $0 < \theta < \pi/2$), find $\sec\theta$.



$$\sec\theta = \frac{1}{\cos\theta} = \sqrt{5}$$

(d)[2] Express $f(x) = 3.7^x$ in the form $f(x) = e^{ax}$; i.e., find a.

$$3.7^x = e^{\ln 3.7^x} = e^{x(\ln 3.7)} = e^{1.31x}$$

$$a = 1.31$$

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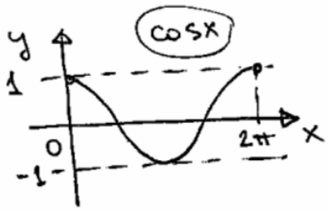
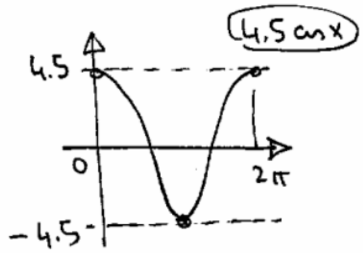
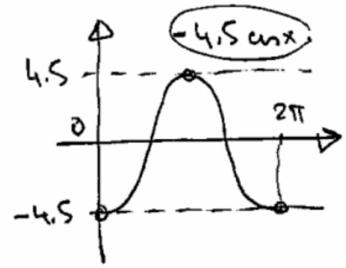
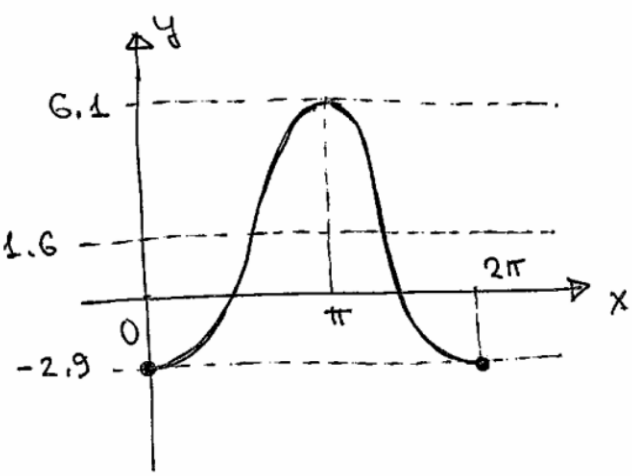
2. (a)[3] Sketch the graph of the function $f(x) = -4.5\cos x + 1.6$. It suffices that you show the main period.









(b)[2] What is the range of the function $f(x)$ from (a)?

$[-2.9, 6.1]$

(c)[2] Identify the maximum value and the average value of $f(x)$.

maximum = 6.1

average = 1.6

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3. Consider the discrete-time dynamical system $N_{t+1} = \dfrac{N_t}{N_t+2}$.

(a)[1] Write the updating function associated with this system.

$$f(N_t) = \frac{N_t}{N_t+2} \quad \text{or} \quad f(x) = \frac{x}{x+2}$$

(b)[2] Find the backwards discrete-time dynamical system for the given system.

solve for N_t (or solve for x)

$$N_{t+1}N_t + 2N_{t+1} = N_t$$

$$N_t(N_{t+1}-1) = -2N_{t+1}$$

$$N_t = \frac{-2N_{t+1}}{N_{t+1}-1} = \frac{2N_{t+1}}{1-N_{t+1}}$$

(c)[2] Given that $N_0 = 1$, find N_3. Express all values as fractions.

$$N_0 = 1$$

$$N_1 = \frac{1}{1+2} = \frac{1}{3}$$

$$N_2 = \frac{1/3}{1/3+2} = \frac{1/3}{7/3} = \frac{1}{7}$$

$$N_3 = \frac{1/7}{1/7+2} = \frac{1/7}{15/7} = \frac{1}{15}$$

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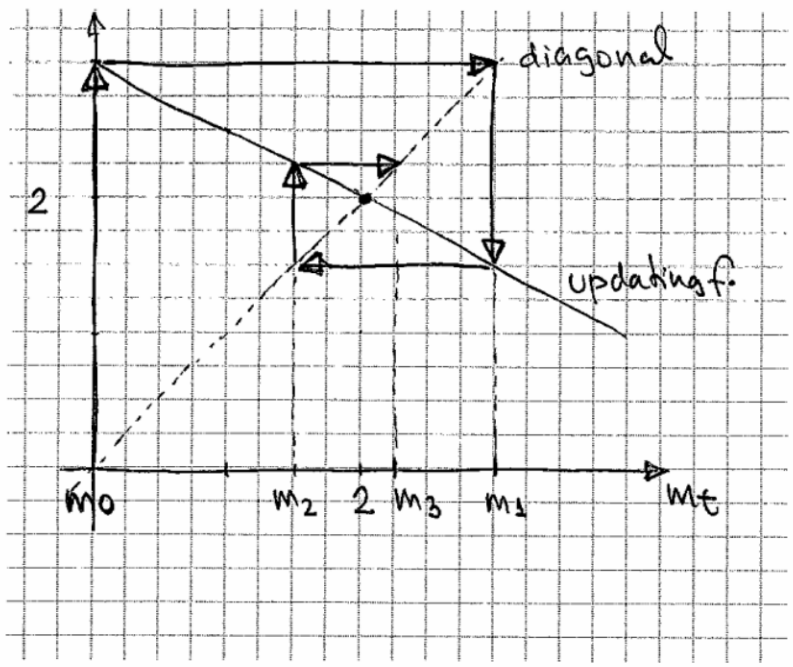
4. Consider the system $m_{t+1} = -0.5m_t + 3$, where $m_0 = 0$.
(a)[1] Find the equilibrium point(s) of the system.

$$m^* = -0.5m^* + 3 \rightarrow 1.5m^* = 3, \quad m^* = 2$$

(b)[3] Staring with $m_0 = 0$, cobweb for three steps; i.e., in your diagram, show m_3. Also, indicate the equilibrium point(s) that you calculated in (a).



(c)[1] Calculate the value of m_3 algebraically and compare with your diagram in (b).

$m_0 = 0$

$m_1 = -0.5(0) + 3 = 3$

$m_2 = -0.5(3) + 3 = 1.5$

$m_3 = -0.5(1.5) + 3 = 2.25$

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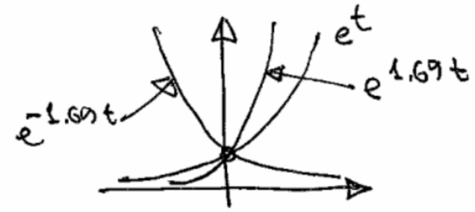
5. (a)[3] Find the half-life for the population that behaves according to $P(t) = 750e^{-1.69t}$, where t is time in hours.

$$0.5 \cdot 7\cancel{50} = 7\cancel{50}\, e^{-1.69t}$$

$$\ln 0.5 = -1.69t$$

$$t = \frac{\ln 0.5}{-1.69} \approx 0.41 \text{ hours}$$

(b) [2] Sketch the graph of the function from (a). Explain in words how you obtained it from the graph of e^t.



$e^t \to$ horizontal compression by factor of 1.69

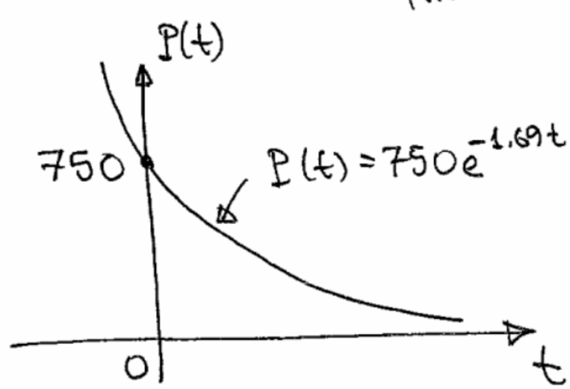
can switch the order

$\downarrow$

reflection about y-axis

$\downarrow$

vertical stretch by factor of 750



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6. Population of bacteria doubles every 6 hours. Initial count is 300 bacteria. Consider the continuous-time model.

(a)[2] What will the population be in 24 hours? [Hint: think! You don't need part (b) for this]

$$300 \xrightarrow{6} 600 \xrightarrow{6} 1200 \xrightarrow{6} 2400 \xrightarrow{6} 4800$$

$$\downarrow$$

4800 in 24 hours

(b)[2] Find the formula for the population $P(t)$ of bacteria as a function of t.

$$P(t) = 300e^{rt} \qquad r = ?$$

$$600 = 300e^{6r} \rightarrow e^{6r} = 2, \quad 6r = \ln 2$$

$$r = \frac{\ln 2}{6} = 0.12$$

$$P(t) = 300e^{0.12t}$$

(c)[2] When will the population reach 11000?

$$3\cancel{0}0\, e^{0.12t} = 11\,0\cancel{00}$$

$$e^{0.12t} = \frac{110}{3} \rightarrow 0.12t = \ln(110/3)$$

$$t = \frac{\ln(110/3)}{0.12} = 30.02 \text{ hours}$$

depends on roundingoff!!

if $r = 0.12$ then $t = 30.02$

if $r = 0.116$ then $t = 31.05$

~~if $r = 0.1155$ then $t = 31.18$~~

if $r = 0.115525$ then $t = 31.18$

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7. (a)[2] Define: equilibrium point for the dynamical system $m_{t+1} = f(m_t)$.

point where $m^* = f(m^*)$

or, value that does not get changed by the dynamical system

(b)[2] It is known that m^* is an equilibrium point of the dynamical system $m_{t+1} = f(m_t)$. Find an equilibrium point of the backward-time dynamical system $m_t = f^{-1}(m_{t+1})$. Explain your reasoning.

$$m^* \underset{f^{-1}}{\overset{f}{\rightleftarrows}} m^*$$

m^* is also equilibrium for f^{-1}

m^* is equilibrium for $f \longrightarrow m^* = f(m^*)$ | apply f^{-1}

$$f^{-1}(m^*) = \underbrace{f^{-1}(f(m^*))}_{m^*}$$

so m^* is equilibrium for f^{-1}

THE END