# Hypebolic Functions

$$\cosh(x) = \frac{e^{x} + e^{-x}}{2} \qquad \sinh(x) = \frac{e^{x} - e^{-x}}{2} \qquad \tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

$$\operatorname{sech}(x) = \frac{1}{\cosh(x)} \qquad \operatorname{csch}(x) = \frac{1}{\sinh(x)}$$

$$\coth(x) = \frac{1}{\tanh(x)} = \frac{\cosh(x)}{\sinh(x)}$$

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$$\boxed{\frac{d}{dx}\sinh(x)} = \frac{d}{dx}\frac{e^{x} - e^{-x}}{2} = \frac{e^{x} - (-e^{-x})}{2} = \frac{e^{x} + e^{-x}}{2}$$

$$\boxed{= \cosh(x)}$$

$$\boxed{\frac{d}{dx}\cosh(x) = \frac{d}{dx}\frac{e^x + e^{-x}}{2} = \frac{e^x - e^{-x}}{2} = \sinh(x)}$$

← watch the sign

$$\frac{d}{dx}\tanh(x) = \frac{d}{dx}\frac{\sinh(x)}{\cosh(x)} = \frac{\left(\frac{d}{dx}\sinh x\right)\cosh x - \left(\frac{d}{dx}\cosh(x)\right)\sinh(x)}{(\cosh(x))^2}$$

$$= \frac{\cosh^2(x) - \sinh^2(x)}{\cosh^2(x)}$$

**Note** $\cosh^2(x) - \sinh^2(x)$

$$= \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2$$

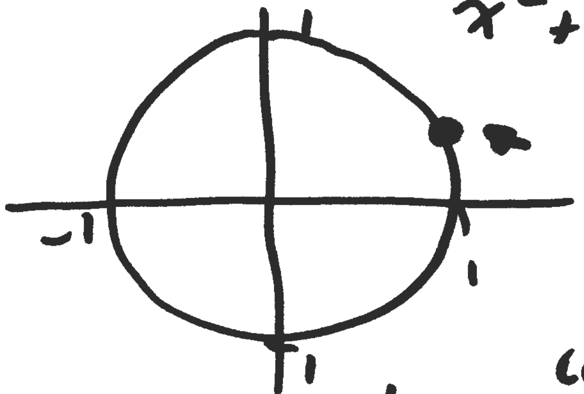
$$= \frac{1}{4}\left[ \cancel{e^{2x}} + \cancel{e^{-2x}} + 2\cancel{e^{x}}e^{-x} - \cancel{e^{2x}} - \cancel{e^{-2x}} + 2\cancel{e^{x}}e^{-x} \right] = \frac{2+2}{4}$$

$$= 1$$

So $\boxed{\cosh^2 x - \sinh^2 x = 1}$

$$\Rightarrow \boxed{\frac{d}{dx}\tanh(x) = \frac{1}{\cosh^2 x} = \operatorname{sech}^2(x)}$$

**Notice**



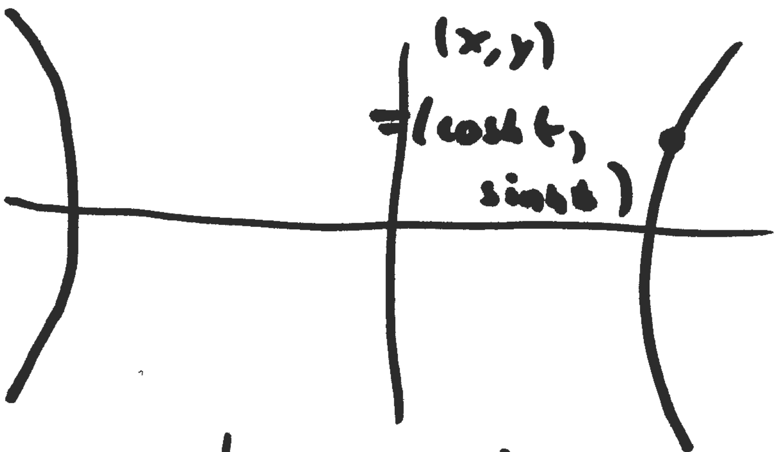
**Circle!**

$$x^2+y^2=1$$

$$(x,y) = (\cos t, \sin t)$$

$$\cos^2 t + \sin^2 t = 1$$

**Trig!**




**hyperbola!**

$$x^2 - y^2 = 1$$

$$\cosh^2 t - \sinh^2 = 1$$

**Hyperbolic!**

$$F = ma = -kx$$

$$a + \frac{k}{m} x = 0$$

$$\boxed{x'' + \omega^2 x = 0}$$

Simple Harmonic Oscillator (SHO)

**Solution**

$\cos(\omega t)$

$\sin(\omega t)$

**but**

$x'' - \omega^2 x = 0 \Rightarrow$ Solutions

$\cosh(\omega t)$

$\sinh(\omega t)$

$i = \sqrt{-1}$

**Extra!**

**Euler**

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\Rightarrow \cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2} = \cosh(i\theta)$$

$$i\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2} = \sinh(i\theta)$$

$$e^{z} \rightarrow \quad z = a + ib \text{ complex!}$$

Cover in detail in 1ZC3

Ok back to hyperbolics

$$\overset{\text{not}}{=} \frac{1}{\cosh(x)}$$

If $f(x) = \cosh(x)$, $x > 0$ then $f^{-1}(x) = \cosh^{-1}(x)$

$$= \operatorname{arcosh}(x)$$

if $f(x) = \sinh(x)$ $\quad f^{-1}(x) = \operatorname{arsinh}(x)$

$= \sinh^{-1}(x)$

if $f(x) = \tanh(x)$ $\quad f^{-1}(x) = \operatorname{artanh}(x)$

$= \tanh^{-1}(x)$

Note explicit inverses are do-able but ugly (avoid)

& $\frac{d}{dx} \tanh^{-1}(x) = \frac{1}{1-x^2}$ etc. are pretty!

but nearly useless! $\Rightarrow$ skip it!

# Chap. 4 Graph Sketching!

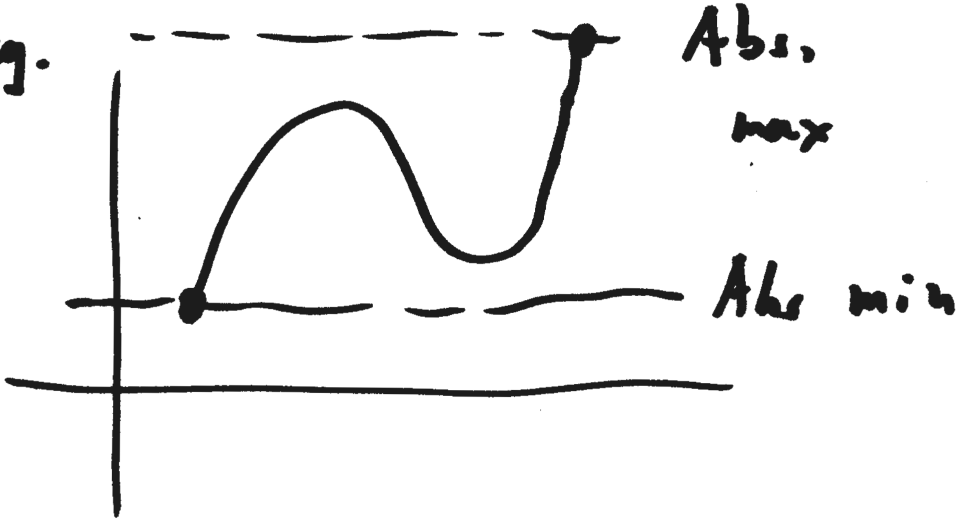
## 4.1 Abs. max & Abs. min.

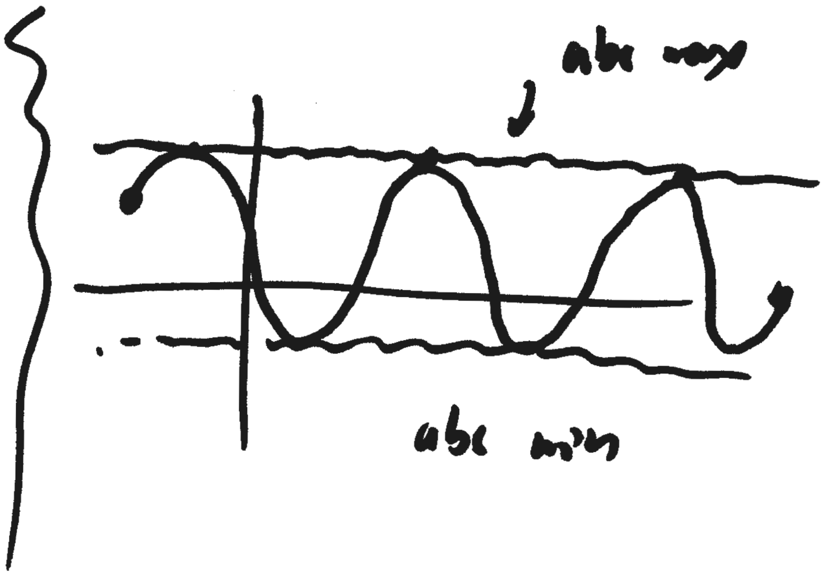
**Define** Absolute Maximum value of $f(x)$
is the highest y-value $y = f(x)$ attains
on given domain

most + ve

Absolute Minimum value of $f(x)$.
is lowest (most -ve) y-value attained
on given domain!

eg.

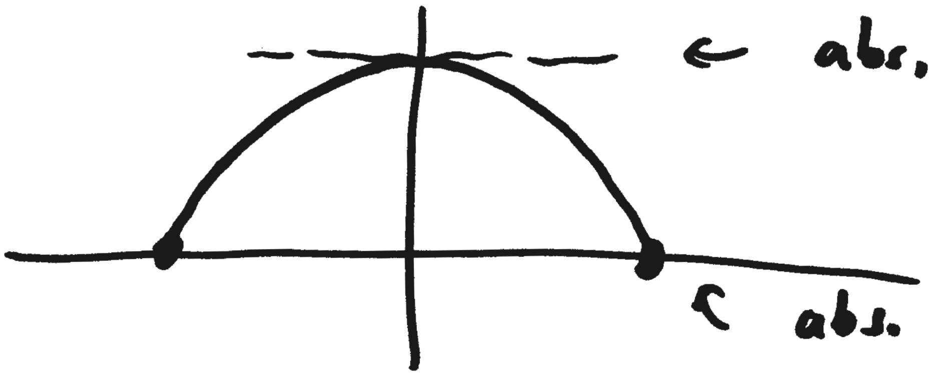



Abs. max/min y-values may be attained more than once!

but there can be at most one abs. max value on the domain

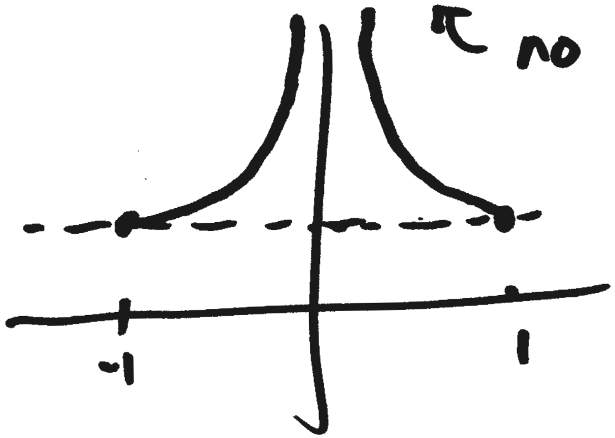
eg. $y = \cos(x)$ on $[-\frac{\pi}{2}, \frac{\pi}{2}]$




**But** what can go wrong!?

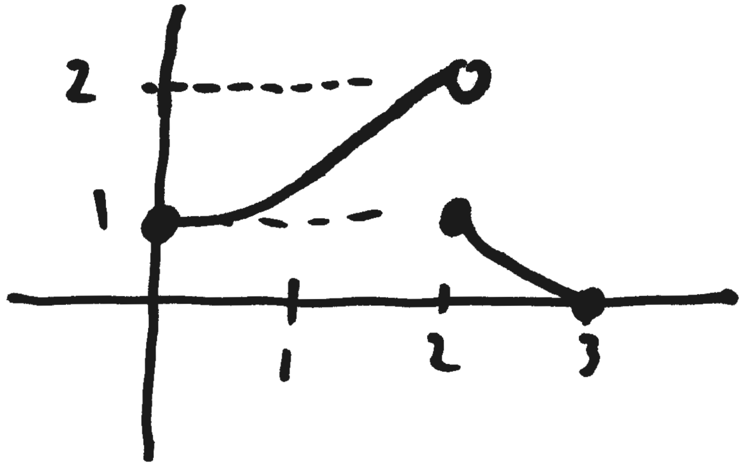
e.g. Find the abs. max & abs. min value of
$y = \frac{1}{x^2}$ on $[-1, 1]$

Solution




$\boxed{\text{abs. min value } y = 1}$

eg. Find abs. max & min of following graph:




Soln. Abs. min $y=0$

Abs max

Values Approach 2, never get to 2

No Highest! Value!

From now on: Denial!

Focus on closed intervals & cont. functions only!