Math 1LT3	Assignment 24

Normal distribution (Section 14)

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1. Consider the function $f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$.

(a) Show that $f(x)$ is an even function (i.e., symmetric with respect to the y-axis).

(b) Show that $\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} f(x) = 0$.

(c) Show that $f(x)$ is increasing for $x < 0$ and decreasing for $x > 0$. It has a local maximum at $x = 0$.

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(d) Show that the inflection points of $f(x)$ are at $x = \pm 1$.

(e) Sketch the graph of $f(x)$.

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2. What is the z-score? Explain how to calculate the probability $P(0 \leq X \leq 5)$ if $X \sim N(2, 9)$.

3. Sketch the graph of the *standard* normal distribution. Shade the region whose area corresponds to the probability $P(1 \leq X \leq 4)$, if $X \sim N(3, 1^2)$.

4. Find each probability using the table on page 182.

(a) Let $Z \sim N(0, 1^2)$; find $P(Z > 1.4)$.

(b) Let $Z \sim N(0, 1^2)$; find $P(Z < 0.4)$.

(c) Let $Z \sim N(0, 1^2)$; find $P(-1 < Z < 2)$.

(d) Let $Z \sim N(0, 1^2)$; find $P(Z < -1)$.

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(e) X is normally distributed with mean 3 and variance 4. Find the probability that X is less than 4.1.

(f) Let $X \sim N(-1, 4)$; find $P(X > 1)$.

(g) Let $X \sim N(-1, 4)$; find $P(X < -2)$.

(h) Let $X \sim N(-2, 4)$; find $P(-3 \leq X \leq 1)$.

5. The wingspan of a blue jay is normally distributed with a mean of 39 cm and a standard deviation of 3 cm. What is the probability that a randomly chosen blue jay has a wingspan wider than 42 cm?

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6. Assume that the random variable $S \sim N(70, 10^2)$ describes the grades on a math test. What is the probability that a student scored more than 85 points?

7. Suppose that the weight of an animal is normally distributed with a mean of 4.5 kg and a standard deviation of 2.5 kg. What is the probability that a randomly chosen animal weighs between 6 kg and 8 kg?

8. Assume that a population is normally distributed with mean μ and variance σ^2. Find the fraction of the population that falls within the interval $(\mu, \mu + 3\sigma)$.

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9. Assume that a population is normally distributed with mean μ and variance σ^2. Find the fraction of the population that falls within the interval $(\mu + \sigma, \infty)$.

10. Assume that a population is normally distributed with mean μ and variance σ^2. Find the fraction of the population that falls within the interval $(\mu - \sigma, \mu + 2\sigma)$.

11. The full running speed (km/h) of a moose is normally distributed, $S \sim N(44, 5^2)$. What percent of moose can run faster than 50 km/h?

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12. Suppose that $X \sim N(2, 12^2)$. Use the table on page 182 to find an x that satisfies each condition (if you cannot find an exact match, use the nearest approximation).

(a) $P(X \leq x) = 0.56$

(b) $P(X \leq x) = 0.95$

(c) $P(X > x) = 0.3$

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13. The grades on a math test are normally distributed with a mean grade of 72 out of 100 and a standard deviation of 8.

(a) What ratio of students scored more than 90% on the test?

(b) What ratio of students scored between 60 and 80 points?

(c) What is the maximum score of the lowest 10% of the tests scores?

THE END