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COMPSCI 3MI3 : Assignment 7
Fall 2021
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Assignments submitted as LaTeXgenerated pdfs are eligible for 1 bonus mark, so long as the source file is provided.

1. (4 points) **Well-Typedness of the Omega Function**
   In Topic 6, we examined the Ω-function:

   $$\Omega = (\lambda x.x\ x)\ (\lambda x.x\ x) \tag{1}$$

   Could this function be well-typed? Why or why not?

2. **Typing Derivation Proofs**
   Show, by means of *formal proof*, that the following expressions have the indicated types:
   (a) (2 points) $f : Bool \Rightarrow Bool \vdash f\ (\texttt{if false then true else false}) : Bool$
   (b) (3 points) $f : Bool \Rightarrow Bool \vdash \lambda x : Bool.f(\texttt{if } x \texttt{ then false else } x) : Bool \Rightarrow Bool$

3. (4 points) **Permutation Lemma** In topic 8, slide 40, we describe the *Permutation Lemma*:

   LEMMA [PERMUTATION]
   If $\Gamma \vdash t : T$ and Δ is a permutation of Γ, then $\Delta \vdash t : T$. Moreover, the latter derivation has the same depth as the former.

   Provide a traditional proof of the above lemma.

4. (4 points) **Weakening Lemma** In topic 8, slide 41, we describe the *Weakening Lemma*:

   LEMMA [Weakening]
   If $\Gamma \vdash t : T$ and $x \notin dom(\Gamma)$, then $\Gamma, x : S \vdash t : T$. The latter derivation also has the same depth as the former.

   Provide a traditional proof of the above lemma.