Theme 1
Introductory Material

Module T1M1:

The Predictable Universe

T1M1 – Learning Objectives

- Identify the approach taken by physicists to understanding complex phenomena.
- Recognize that measurements are really comparisons with a standard ***unit*** of measure, and that different standard units can be related to each other.
- Distinguish between the specific units of a measured quantity, and the more general statement of the ***dimensions*** of the quantity.
- Recognize that the dimensions of a quantity are helpful at predicting the relationships that govern a system.
- Understand the idea of ***proportionality*** to describe the specific way in which quantities are related.

Module Clicker Quiz!

Now that you have had a chance to review the entire first module, T1M1, here is your first

module quiz!

You will need calculator

You will get 120s per question

Module Clicker Quiz!

<u>Unit conversion (120 seconds)</u>

- If Einswine was thrown with a speed of 10 m/s, what is that speed in km/h?

A. 36 km/s
B. 2.8 km/h
C. 16 km/h
D. 52 km/h
E. I don't know

Module Clicker Quiz!

Dimensional analysis (120 seconds)

- In the following formula, what are the dimensions of the variable ***m***?

$$\frac{U}{2} = Pm^2$$

U – dimensions [L/K]

P – dimensions [M/K]

A. $\left[\frac{MK}{L}\right]$

B. $\left[\sqrt{\frac{L}{M}}\right]$

C. $\left[\sqrt{\frac{M}{L}}\right]$

D. $\left[\frac{L^2}{M^2}\right]$

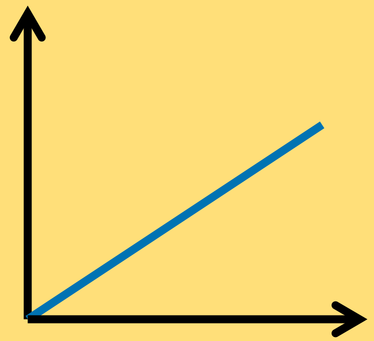
E. I don't know

Module Clicker Quiz!

Proportionality (120 seconds)

- For the formula given, which relationships, when plotted on a graph, would yield a straight line, as shown:

$$\frac{U}{2} = Pm^2$$



A. U vs. ***1/P*** (***m*** held constant)

B. U vs. ***m*** (***P*** held constant)

C. P vs. ***m*** (***U*** held constant)

D. $\boldsymbol{m}^2$ vs. ***U*** (***P*** held constant)

E. I don't know

Proportionality

- The natural world rarely provides us with access to all the details to put together a complete picture!
- The trends and relationships that we observe allow us to infer general rules that we incorporate into a model

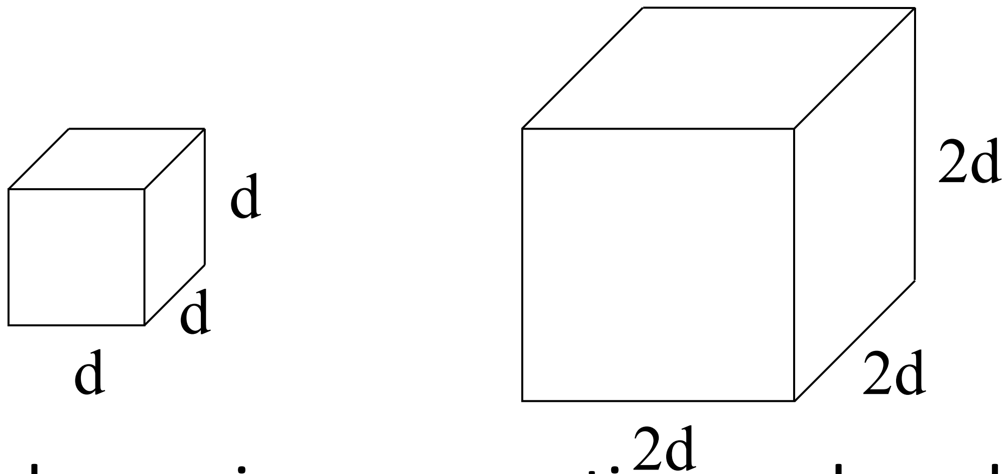
- Proportionality: $x \propto y$ implies that
  - If we double x, then y also doubles
  - If y is reduced by 1/7th of its value, then so is x

- We would incorporate into our model

  $x = a\,y$, where a is a constant of proportionality, independent of the actual values of x and y

Geometric proportionalities

- <u>Isometric</u> = same geometry, different size



- For a simple cube, various properties can be related to the side length:
  - If you double every dimension, how do area and volume change?

  - Area $\propto$ d^2
  - Volume $\propto d^3$

- Does this change for a sphere (radius *r*)?

Using ratios to solve problems

- From previous slide: How many times more volume of large cube compared to small cube?

$$\frac{V_{big}}{V_{small}} = \frac{(2d)^3}{d^3} = 2^3 = 8$$

- What about two spheres of radii *r* and 2*r*?

$$\frac{V_{big}}{V_{small}} = \frac{\frac{4}{3}\pi(2r)^3}{\frac{4}{3}\pi r^3} = \frac{(2r)^3}{r^3} = 2^3 = 8$$

- In both cases, when length scale increases 2x, the volume increases by factor $(2)^3 = 8$!
- *****The details of the shape don't matter (i.e. the 4/3π pre-factors cancel when you take a ratio)**

How do other quantities scale?

- Let’s define a general length scale, **L**.
  - Could be side length, radius, diameter – any linear measurement of object’s size

- We know that:
  - Area $\propto \mathbf{L}^2$
  - Volume $\propto \mathbf{L}^3$
- **What about Mass? $\propto$ Volume $\propto$ $\mathbf{L}^3$**
  - Assume that two objects have the same density

Practical application: Clothing your clone

Mini me weighs exactly 1/8th of Dr. Evil's mass. How much more material is needed for Dr. Evil's suit than for mini me's?

$$\frac{M_{DE}}{M_{mm}} = 8 = \left(\frac{L_{DE}}{L_{mm}}\right)^3$$

$$\frac{L_{DE}}{L_{mm}} = 8^{\frac{1}{3}} = 2$$

$$\frac{A_{DE}}{A_{mm}} = \left(\frac{L_{DE}}{L_{mm}}\right)^2 = 2^2 = 4$$

So, four times more material is needed for Dr. Evil's suit than for mini me's.

How do other quantities vary with size?

- Start by defining a length scale, **L**.
- We know that:
  - Area $\propto L^2$
  - Volume $\propto L^3$
- What about Mass? $\propto$ Volume $\propto L^3$
- **What about:**
  - **flow into/out of an object – for example, heat flow?**
  - **Production of heat/energy/waste by an object?**

Clicker Quiz

How would you expect the amount of ***body heat generated (G)*** to scale with the linear dimension***, L***, of an organism?

A. $G \propto L^{2/3}$

B. $G \propto L^{2}$

C. $G \propto L^{3/2}$

D. $G \propto L^{3}$

E. I have no idea

Clicker Quiz

How would you expect the amount of ***body heat lost (H)*** to scale with the linear dimension, ***L***, of an organism?

A. $H \propto L^{2/3}$

B. $H \propto L^{2}$

C. $H \propto L^{3/2}$

D. $H \propto L^{3}$

E. I have no idea!!!

Example: Rate of heat loss

- A grown adult weighs 70 kg while a newborn baby weighs 5 kg.

  Compare the

  (a) rate of heat loss, and

  (b) ratio of heat generated

ratio of masses

$$\frac{M_a}{M_b} = 14 \rightarrow \frac{L_a}{L_b} = 14^{1/3} = 2.41$$

$$\text{Heat loss} \rightarrow \left(\frac{L_a}{L_b}\right)^2 = 2.41^2 = 5.81$$

$$\text{Heat generated} \rightarrow \left(\frac{L_a}{L_b}\right)^3 = 14$$

a) Rate of heat loss (H):

- The 3-step process to solving scaling problems:
  1. Start with a known ratio: $$\frac{M_{Big}}{M_{Small}} = \frac{70}{5} = 14$$
  2. Get ratio of length scales: $$\frac{M_B}{M_S} = \left(\frac{L_B}{L_S}\right)^3 \rightarrow \frac{L_B}{L_S} = \left(\frac{M_B}{M_S}\right)^{\frac{1}{3}} = 14^{\frac{1}{3}} = 2.41$$
  3. Use this to find ratio of new quantity (H): $$\frac{H_B}{H_S} = \left(\frac{L_B}{L_S}\right)^2 = 2.41^2 = 5.81$$

So, the big guy loses heat 5.81 times faster

b) Heat Generated:

1. This depends on mass $$\frac{G_B}{G_S} = \cdot \frac{M_B}{M_S} = 14$$

c) Heat Loss per kg (or heat loss per heat generated):

$$H \propto L^2, \quad M \propto L^3 \quad \Rightarrow \quad \frac{H}{M} = \frac{L^2}{L^3} = \frac{1}{L} \rightarrow \frac{(H/M)_B}{(H/M)_S} = \frac{1/L_B}{1/L_S} = \frac{L_S}{L_B} = \frac{1}{2.41} = 0.415$$

- The adult loses more heat, but the rate of heat loss **per kilogram** is greater for the baby! Remember the heat generation is proportional to mass (L^3), so this is exactly what we talked about in class.

Are we blowing this out of proportion?

- Where do we see this in the 'real world'?

Cell size (from module) – surface area to volume

- there is a practical limit to the size of a cell

Allometry: The study of the relationship of body size to other characteristics of organisms (shape, anatomy, behaviour…)

- Metabolic rate $\propto$ $(\text{Mass})^{3/4}$
- Breathing, heart rate $\propto$ $(\text{Mass})^{1/4}$
- Optimal cruising speed (flight) $\propto$ $(\text{Mass})^{1/6}$

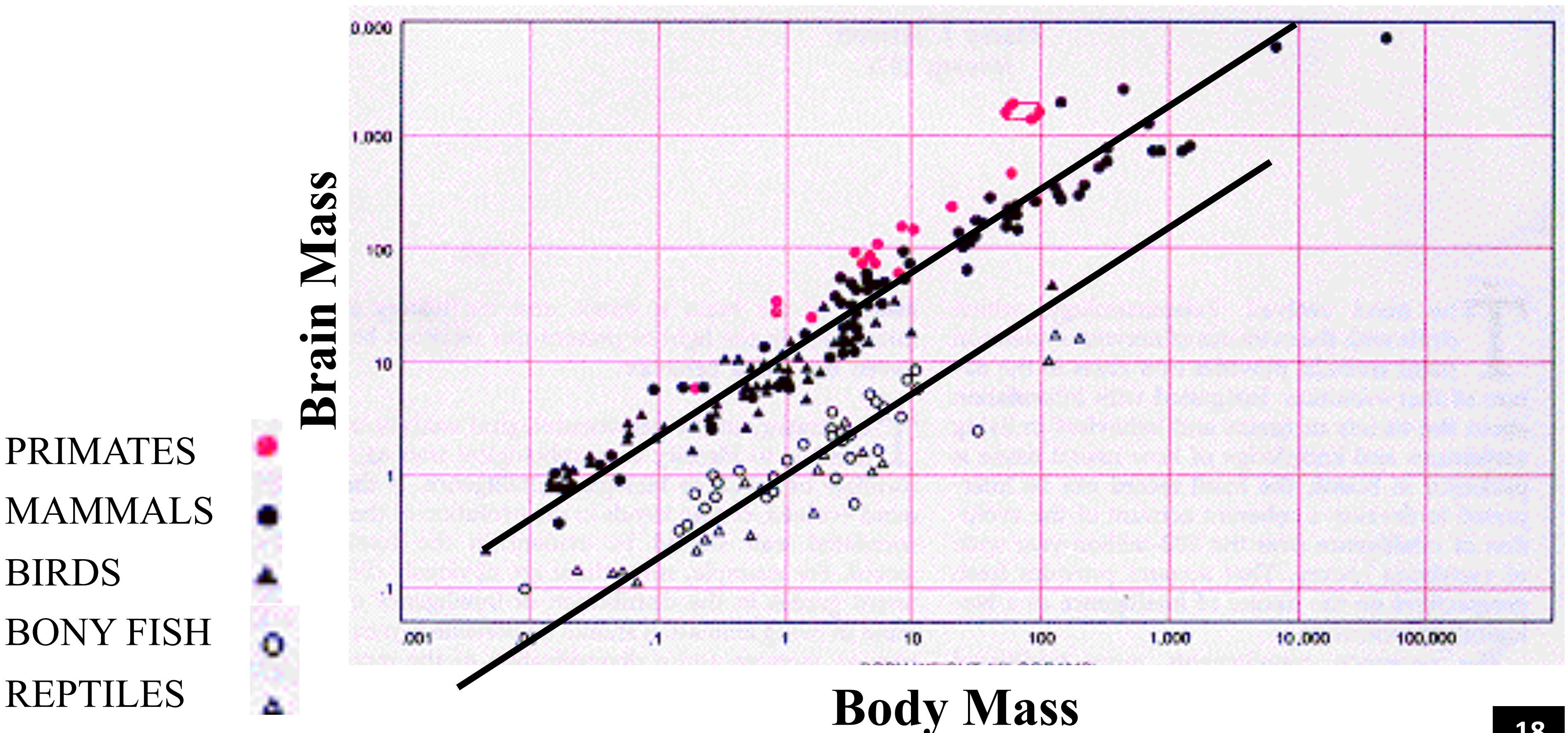
Societal implications

- Density of gas stations vs. city size
- Types of employment, wages vs. city size

What can we infer from these relationships?

- Understanding how two quantities are connected helps us to understand the nature of the relationship!

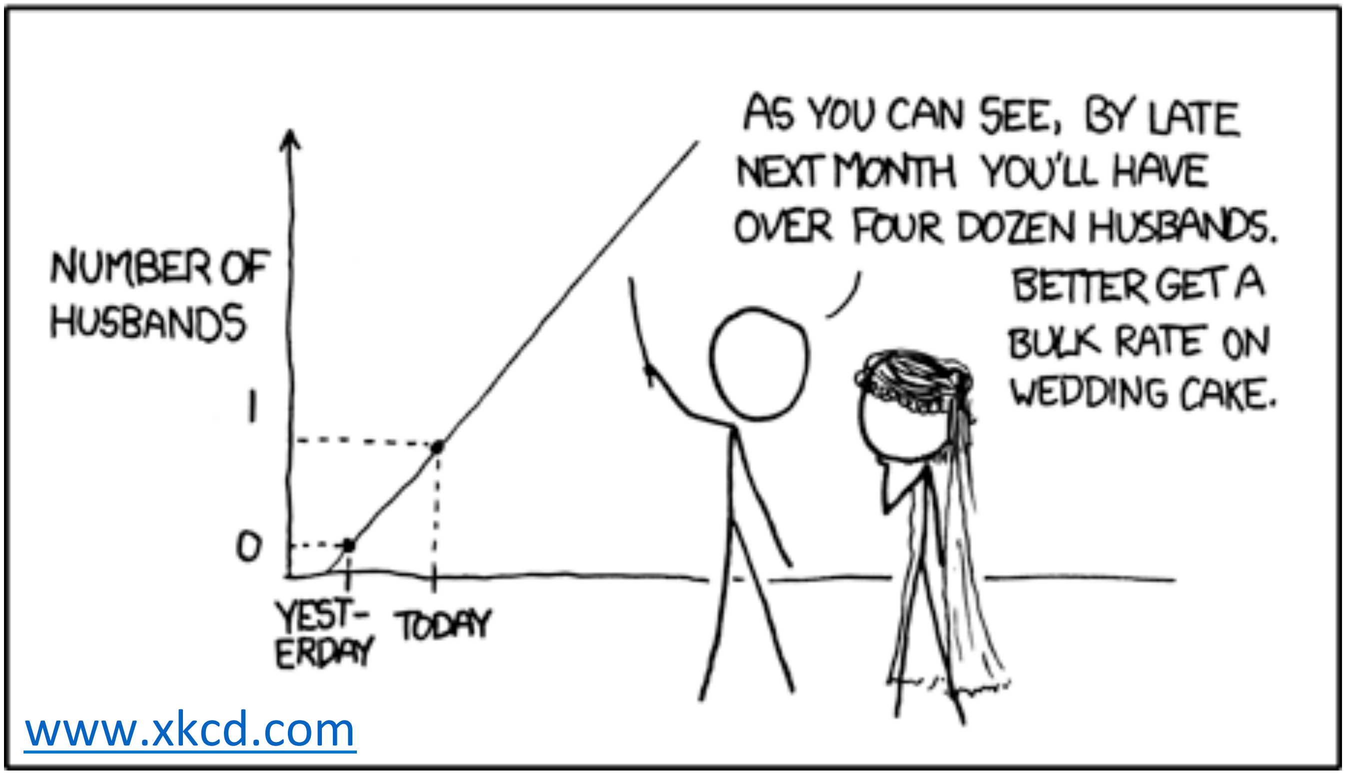
What can this tell us about brain mass and intelligence?



Inferences using proportionality

- Of course, it's not enough to just have a proportionality; there's more to modeling than that!!

MY HOBBY: EXTRAPOLATING



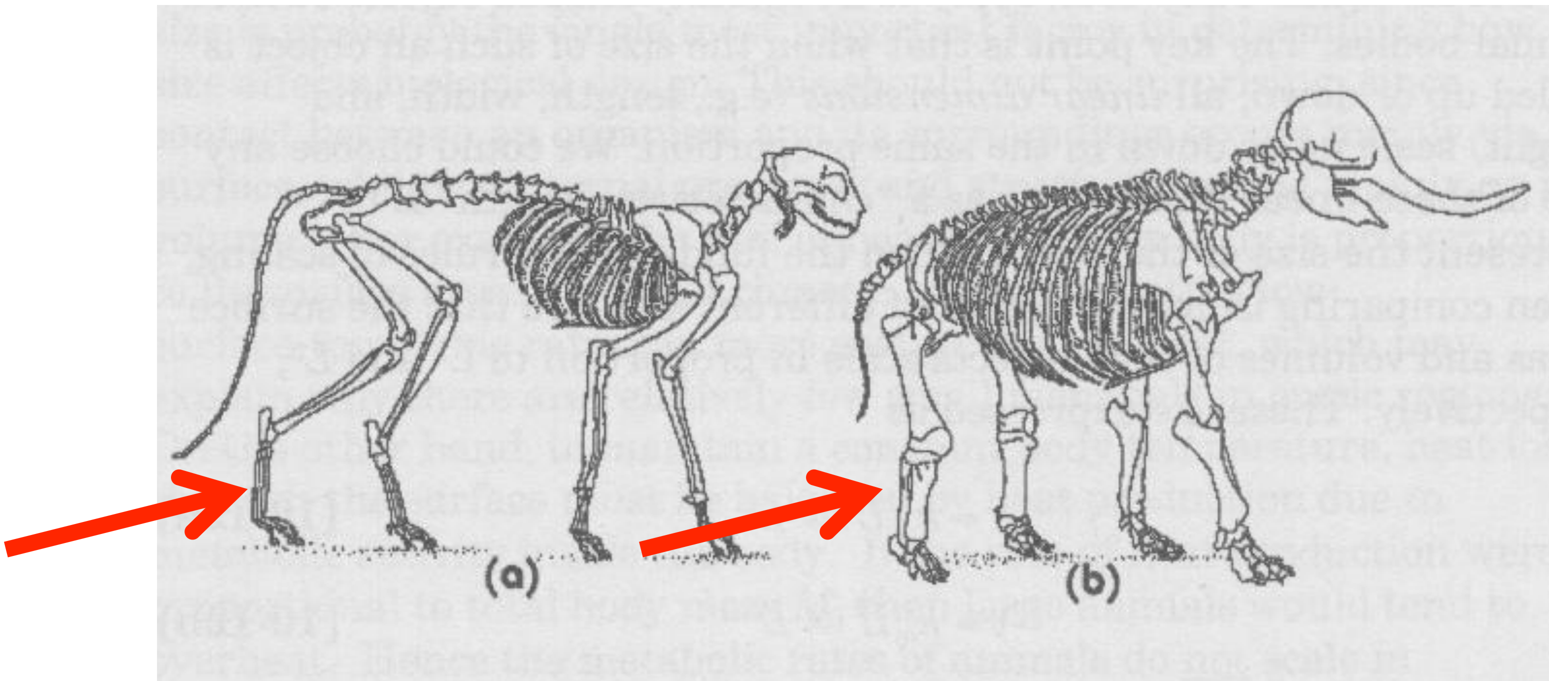
How do other quantities vary with size?

- Start by defining a length scale, **L**.
- We know that:
  - Area $\propto L^2$
  - Volume $\propto L^3$
  - Mass? $\propto$ Volume $\propto L^3$
- Flow (heat, chemical, electrical) $\propto L^2$
- Heat production $\propto L^3$

- **What about strength?**

Limitations on animal size, and mobility

- Skeleton of a house cat and an elephant



- What does this illustration tell you?

Example: Upper limit to size

Could King Kong exist as shown in the movies?

Gorilla: 180 kg, 1.7 m tall, (eats ~25 kg food/day)

King Kong – 7x scaled up version

Closing Remarks

Next class:

- Finish T1M1 – Vector review
- Begin T1M2
  - Significant figures (T1M2)
  - Scientific notation

- New to vectors? (…or, it's been a while)
  - Check out the Vectors Primer Module in Avenue
- What's your comfort with vectors?
  - Check out the review notes