McMaster University
Math 1LT3

ASSIGNMENT 6

Sections 7, 9, and 10 in the Red Module

---

1. (a) Compute the quadratic approximation of the function $f(x, y) = x^2 \arctan(y)$ at (1,0).

(b) Use your formula in part (a) to approximate (i) $1.69 \arctan 0.1$ and (ii) $\arctan 0.5$. Which estimate would you expect to be closer to the actual value? Explain.

(c) Compare each estimate to the actual value. Does this support your claim?

2. (a) Using the following table of values for the function $f(x, y)$, determine whether each partial derivative is positive, negative, or zero.

	$x = 3$	$x = 4$	$x = 5$	$x = 6$
$y = 0$	2.3	2.2	2.0	1.7
$y = 1$	2.4	2.5	2.7	3.0
$y = 2$	2.5	2.7	2.9	3.2
$y = 3$	2.6	3.0	3.0	3.3

(i) $f_y(3, 1)$ (ii) $f_{yy}(3, 1)$ (iii) $f_x(4, 0)$ (iv) $f_{xx}(4, 0)$

(v) $f_y(6, 1)$ (vi) $f_{yy}(6, 1)$ (vii) $f_x(4, 1)$ (viii) $f_{xx}(4, 1)$

(b) Sketch a contour diagram of a functon for which $f_x > 0$ and $f_{xx} < 0$ at all points in the plane.

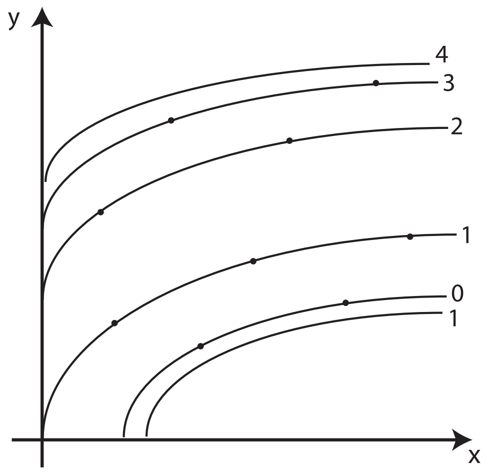
(c) Determine a formula for a functon for which $f_y < 0$ and $f_{yy} > 0$ at all points in the plane.

3. (a) Find the directional derivative of the function $f(x, y) = x \ln y^2 + \frac{x}{y}$ at the point (2,1) in the direction of the vector $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j}$.

(b) What does this number tell us about the function f at the point (2,1)?

(c) Is it possible that (in some direction other than that specified by the vector $\mathbf{v}$ in part (a)) the directional derivative of f at (2,1) is equal to 3? Explain.

4. On the contour diagram for $f(x, y)$ below, draw gradient vectors at the indicated points.



5. Find the maximum rate of change of the function $f(x, y) = 2ye^x + e^{-x}$ at the point $(0,\ 0)$ and the direction in which it occurs.

6. Draw a contour diagram of a function that has a minimum at (-1,0) and a saddle point at (1,1).

7. Reason geometrically (i.e., without the second derivatives test) to show that the function $f(x, y) = y^3 - 4x^2y$ has a saddle point at $(0, 0)$.

8. Find the local minimum and maximum values and saddle points (if any) of each function.

(a) $f(x, y) = x^3 - 2y^2 + 3xy + 4$

(b) $f(x, y) = xye^{-x-y}$

THE END