Solutions - Assignment 2

Assignment Question 1:

Using a table similar to that shown in Figure 3.6, calculate the product of the octal unsigned 6-bit integers 62 and 12 using the hardware described in Figure 3.3.You should show the contents of each register on each step.

Solution Hints:

Iteration	Step	Multiplier	Multiplicand	Product
0	init	001010	000000110010	000000000000
1	If 0 $\rightarrow$ no operation	001010	000000110010	000000000000
	shift multiplicand left	001010	000001100100	000000000000
	shift multiplier right	000101	000001100100	000000000000
2	If 1 $\rightarrow$ prod = prod + mcand	000101	000001100100	000001100100
	shift multiplicand left	000101	000011001000	000001100100
	shift multiplier right	000010	000011001000	000001100100
3	If 0 $\rightarrow$ no operation	000010	000011001000	000001100100
	shift multiplicand left	000010	000110010000	000001100100
	shift multiplier right	000001	000110010000	000001100100
4	If 1 $\rightarrow$ prod = prod + mcand	000001	000110010000	000111110100
	shift multiplicand left	000001	001100100000	000111110100
	shift multiplier right	000000	001100100000	000111110100
5	If 0 $\rightarrow$ no operation	000000	001100100000	000111110100
	shift multiplicand left	000000	011001000000	000111110100
	shift multiplier right	000000	011001000000	000111110100
6	If 0 $\rightarrow$ no operation	000000	011001000000	000111110100
	shift multiplicand left	000000	110010000000	000111110100
	shift multiplier right	000000	110010000000	000111110100

Assignment Question 2:

Using a table similar to that shown in Figure 3.6, calculate the product of the hexadecimal unsigned 8-bit integers 62 and 12 using the hardware described in Figure 3.5. You should show the contents of each register on each step.

Solution Hints:

Iteration	Step	Multiplicand	Product/ Multiplier
0	init val	0110 0010	0000 0000 0001 0010
1	lsb=0, no op	0110 0010	0000 0000 0001 0010
	R shift product	0110 0010	0000 0000 0000 1001
2	prod=prod+Mcand	0110 0010	0110 0010 0000 1001
	R shift product	0110 0010	0011 0001 0000 0100
3	lsb=0, no op	0110 0010	0011 0001 0000 0100
	R shift product	0110 0010	0001 1000 1000 0010
4	lsb=0, no op	0110 0010	0001 1000 1000 0010
	R shift product	0110 0010	0000 1100 0100 0001
5	prod=prod+Mcand	0110 0010	0110 1110 0100 0001
	R shift product	0110 0010	0011 0111 0010 0000
6	lsb=0, no op	0110 0010	0011 0111 0010 0000
	R shift product	0110 0010	0001 1011 1001 0000
7	lsb=0, no op	0110 0010	0001 1011 1001 0000
	R shift product	0110 0010	0000 1101 1100 1000
8	lsb=0, no op	0110 0010	0000 1101 1100 1000
	R shift product	0110 0010	0000 0110 1110 0100

Assignment Question 3:

Using a table similar to that shown in Figure 3.10, calculate 74 divided by 21 using the hardware described in Figure 3.8. You should show the contents of each register on each step. Assume both inputs are unsigned 6-bit integers.

Solution Hints:

Iteration	Step	Quotient	Divisor	Remainder
0	Initial values	0000	010001 000000	000000 111100
1	1:Rem=Rem-Div	0000	010001 000000	101111 111100
	2b:Rem$< 0 \implies +Div, sllQ, Q0 = 0$	0000	010001 000000	000000 111100
	3:shift Div right	0000	001000 100000	000000 111100
2	1:Rem=Rem-Div	0000	001000 100000	111000 011100
	2b:Rem$< 0 \implies +Div, sllQ, Q0 = 0$	0000	001000 100000	000000 111100
	3:shift Div right	0000	000100 010000	000000 111100
3	1:Rem=Rem-Div	0000	000100 010000	111100 101100
	2b:Rem$< 0 \implies +Div, sllQ, Q0 = 0$	0000	000100 010000	000000 111100
	3:shift Div right	0000	000010 001000	000000 111100
4	1:Rem=Rem-Div	0000	000010 001000	111110 110100
	2b:Rem$< 0 \implies +Div, sllQ, Q0 = 0$	0000	00010 001000	000000 111100
	3:shift Div right	0000	00001 000100	000000 111100
5	1:Rem=Rem-Div	0000	00001 000100	111111 111000
	2b:Rem$< 0 \implies +Div, sllQ, Q0 = 0$	0000	00001 000100	000000 111100
	3:shift Div right	0000	00000 100010	000000 111100
6	1:Rem=Rem-Div	0000	00000 100010	000000 011010
	2b:Rem$>= 0 \implies, sllQ, Q0 = 1$	0001	00000 100010	000000 011010
	3:shift Div right	0001	00000 010001	000000 011010
7	1:Rem=Rem-Div	0001	00000 010001	000000001001
	2b:Rem$>= 0 \implies, sllQ, Q0 = 1$	0011	00000 010001	000000001001
	3:shift Div right	0011	00000 001000	000000001001

Assignment Question 4:

IEEE 754-2008 contains a half precision that is only 16 bits wide. The left most bit is still the sign bit, the exponent is 5 bits wide and has a bias of 15, and the mantissa is 10 bits long. A hidden 1 is assumed. Write down the bit pattern to represent $-1.5625\ X\ 10^{-1}$assuming a version of this format. Comment on how the rangeand accuracy of this 16-bit floating point format compares to the single precision IEEE 754 standard.

Solution Hints:

$-1.5625 \times 10^{-1} = 0.1562510$

$0.1562510 \times 2 = 0.3125$

$0.3125 \times 2 = 0.626$ (0)

$0.625 \times 2 = 1.25$ (1)

$0.25 \times 2 = 0.5$ (0)

$0.5 \times 2 = 1$

Hence, = -0.1562510 = -0.00101 x 2^0

$= -1.01 \times 2^{-3} < Hidden1 => 1.XXXXX >$

Sign =1

Exponent = 15-3 = 12 =01100_2 Mantissa = $0100, 0000, 00_2$

IEEE Representation: 1 01100 0100,0000,00

Assignment Question 5:

Calculate the product of -8.0546875 X100and -1.79931640625X 10^{-1} by hand, assuming A and B are stored in the 16-bit half precision format described in Question 4. Assume 1 guard, 1 round bit, and 1 sticky bit, and round to the nearest even. Show all the steps. Indicate if there is overflow or underflow. Write your answer in both the 16-bit floating point format described in Question 4and also as a decimal number. How accurate is your result? How does it compare to the number you get if you do the multiplication on a calculator?

Solution Hints:

$8.0546875 \times 1.7993164062510^{-1}$
$8.0546875 = 1.0000000111 \times 2^3$
$1.79931640625 \times 10^{-1} = 1.0111000010 \times 2^{-3}$
$Exp: -3+3=0, 0+16=16(10000)$
Signs: both negative, result positive
$1.000000111 \times 1.0111000010 = 1.01110011000001001110$
1.0111001100 00 01001110 Guard = 0, Round =0, Sticky 1:NoRnd
1.0111001100 x 2^0= 0100000111001100 (1.0111001100 = 1.44921875)
$-8.0546875 \times -0.179931640625 = 1.4492931365966796875$
Some information was lost because the result did not fit into the available 10-bit field. Answer (only) off by .0000743865966796875

Assignment Question 6:

Calculate by hand 8.625 X 101divided by -4.875 X 10^0. Show all the steps necessary to achieve your answer. Assume there is a guard, a round bit, and a sticky bit, and use them if necessary. Write the final answer in both the 16-bit floating point format described in Question 4and in decimal and compare the decimal result to that which you get if you use a calculator?

Solution Hints:

$8.625 \times 10^1 / 4.875 \times 10^0$
$8.625 \times 10^1 = 1.0101100100 \times 2^6$
$4.875 = 1.0011100000 \times 2^2$
Exponent :6-2 = 4, 4+15 = 19 (10011)
Signs: one positive, one negative, result negative
Fraction: 10011100000 / 10101100100=1.00011011000100111
1.00011011000100111 Guard=0, Round=1, Sticky1=: No Round, fix sign
1.0001101100x2^4 = 1101000001101100 = 10001.101100 = 17.6875
$86.25/4.875 = 17.692307692307$
Some information was lost because the result did not fit into the available 10-bit field. Answer off by 0.00480769230