Theme 2
Mechanics

Module T2M1:
Kinematics

Mid Term on Friday

- **You must write in your designated room:**

  - Mac ID: a___ to f____ write in BSB 147
  - Mac ID: g___ to kh___ write in ITB 137
  - Mac ID: ki___ to n___ write in JHE 376
  - Mac ID: o___ to sh___ write in MDCL 1102
  - Mac ID: si___ to z___ write in MDCL 1105

- **Check avenue carefully**

Free Fall Equations

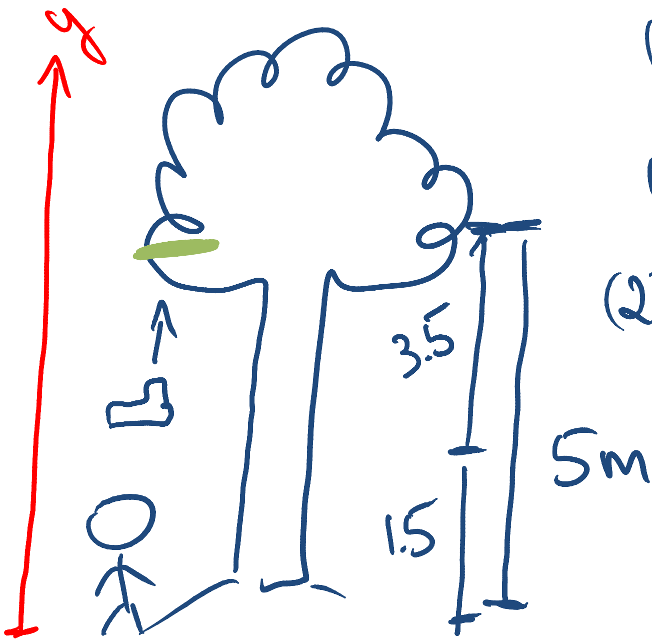
$$g = \text{constant}$$

$$v(t) = gt + v_0$$

$$x(t) = \tfrac{1}{2} g t^2 + v_0 t + x_0$$

Example

Your frisbee is stuck in a branch that is 5.0 m above ground. You throw your shoe straight up to try to knock it down, but your shoe just reaches the frisbee before falling back down. What initial velocity did you give the shoe if it started at 1.5 m above ground?



$$(1)\quad y(t) = -\frac{1}{2} g t^2 + v_0 \cdot t + y_0$$

$$(2)\quad v(t) = -g \cdot t + v_0$$

$$(2) \Rightarrow 0 = -gt + v_0 \Rightarrow t = \frac{v_0}{g}$$

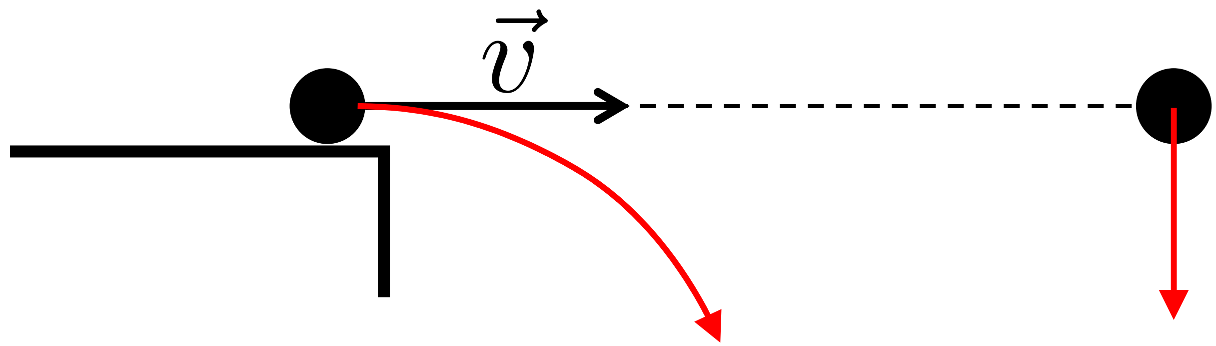
$$(1):\quad 5 = -\frac{1}{2} g t^2 + v_0 \cdot t + 1.5$$

$$5 = -\frac{1}{2} \frac{v_0^2}{g} + \frac{v_0^2}{g} + 1.5$$

$$\Rightarrow v_0 = \sqrt{2 \cdot 9.8 \cdot 3.5} = \sqrt{69} = 83 \frac{m}{s}$$

Concept Question

A ball is pushed off a table with some initial horizontal velocity, an other ball is released from rest from the same height at the same time. Which one hits the floor first?



A. The dropped ball
B. The pushed ball
C. They land at the same time

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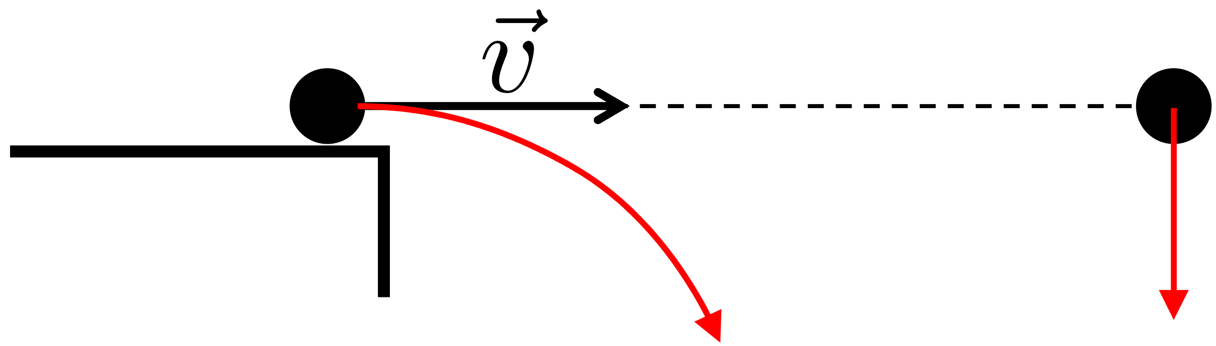

A. The dropped ball
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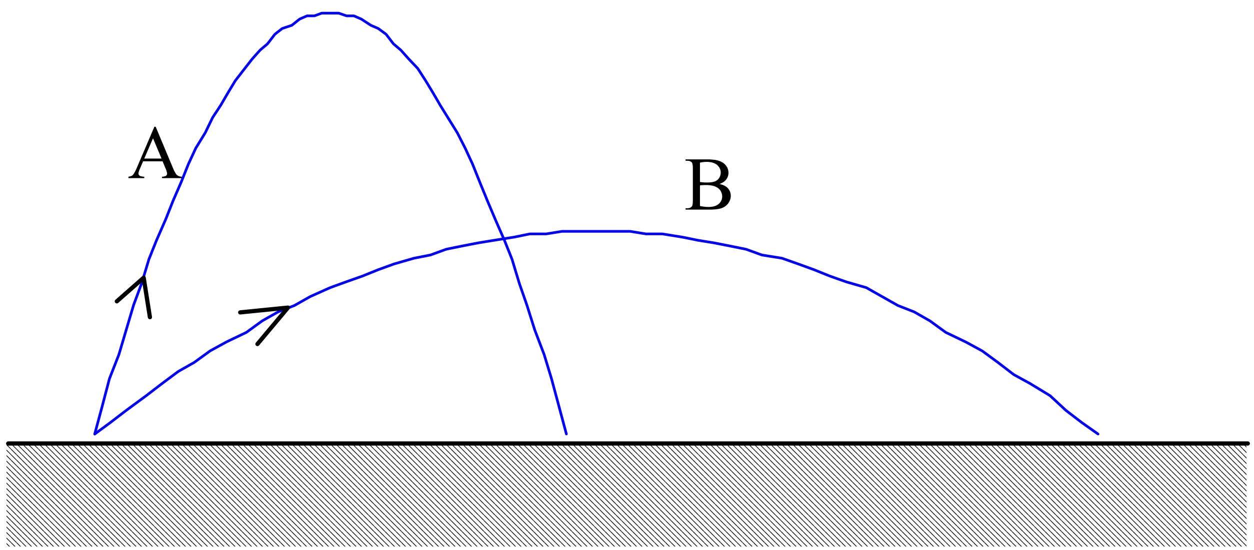
- What does this mean????
  - x and y motion are independent of each other!!

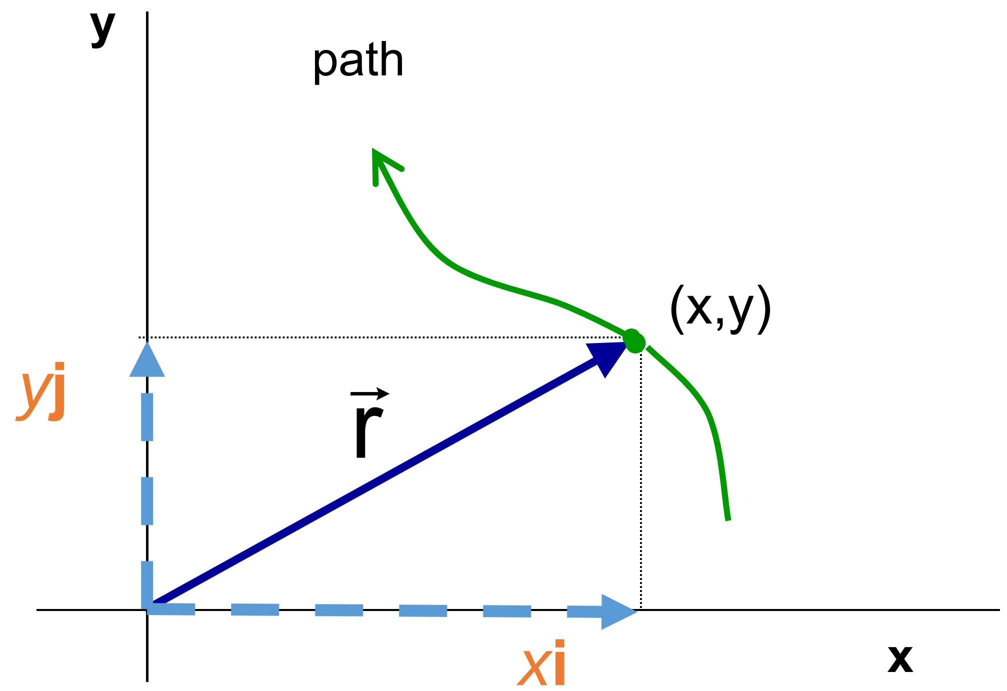
CT3-2. Two projectiles are fired from a cannon. For projectile A, the cannon is tilted upward at an angle twice that of projectile B. (As usual, neglect air resistance.)



Which projectile was in the air longer?

A) A B) B
C) A and B were in the air the same length of time.
D) Not enough information to answer the question.

The **Position vector** $\vec{\mathbf{r}}$ *points from the origin to the particle.*



The components of $\vec{\mathbf{r}}$ are the coordinates (x,y) of the particle: $\vec{r} = x\,\mathbf{i} + y\,\mathbf{j}$

For a moving particle, $\vec{\mathbf{r}}(t)$, x(t), y(t) are functions of time.

Components: Each vector relation implies 2 separate relations for the 2 Cartesian components.

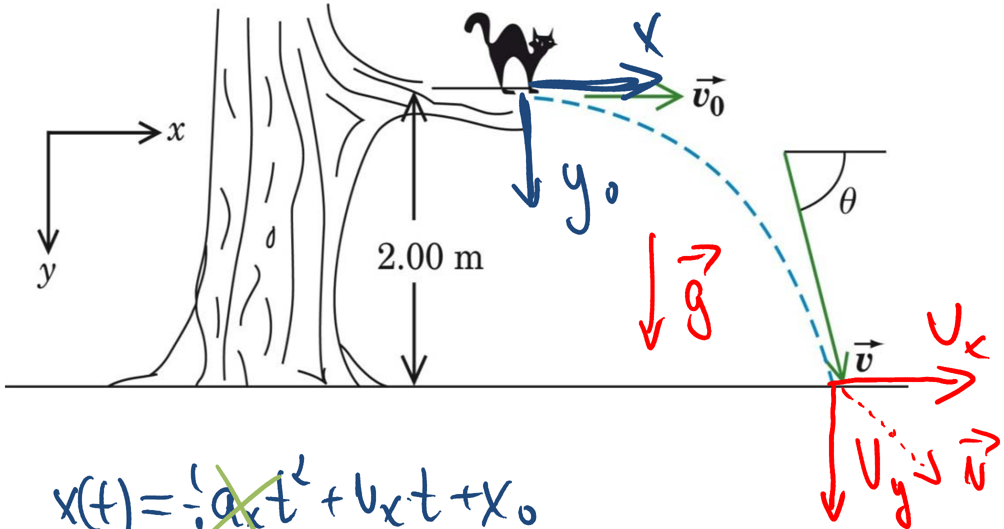
$$\vec{r} = x \cdot \hat{i} + y \, \hat{j} \qquad (\hat{i}, \hat{j} \text{ are unit vectors})$$

$$= x \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} + y \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

velocity $$\vec{v} = \frac{d\vec{r}}{dt} = \left(\frac{dx}{dt}\right) \cdot \hat{i} + \left(\frac{dy}{dt}\right) \hat{j} = v_x \cdot \hat{i} + v_y \cdot \hat{j}$$

$$\vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

acceleration ...



A cat leaps horizontally with a velocity of 4.00 m/s from a tree branch 2.00 m above the ground. What is its velocity when it strikes the ground?

$$x(t) = \cancel{\tfrac{1}{2}a_x t^2} + v_x t + x_0$$

$$y(t) = \tfrac{1}{2}a_y t^2 + v_y t + y_0$$

$$x(t) = v_0 \cdot t$$

$$y(t) = \tfrac{1}{2} g t^2$$

$$v_f^2 - v_i^2 = 2 \cdot a \cdot d \iff v_y^2 - \cancel{v_{y_0}^2} = 2 \cdot g \cdot 2\,m$$

$$v_y = \sqrt{2 \cdot 9.8 \cdot 2} = 6.26\ m/s$$

$$v = \sqrt{v_x^2 + v_y^2} = 7.43\ m/s$$

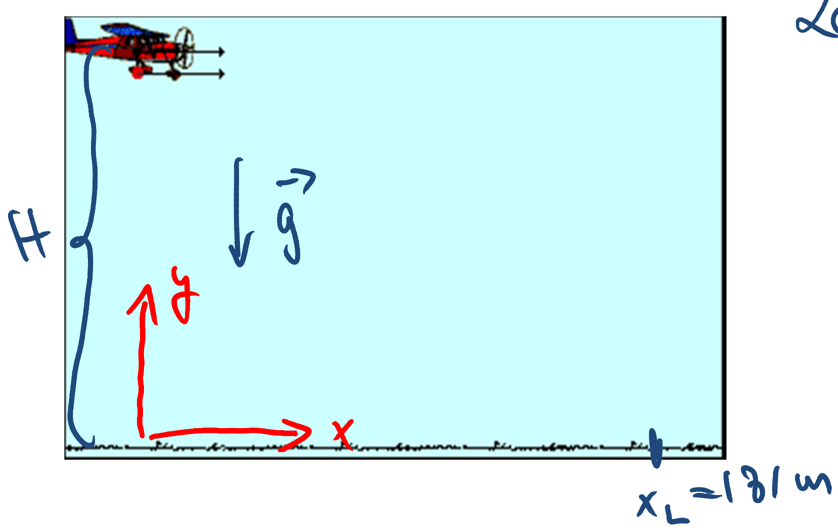
Airplane Drop

- A plane if flying horizontally at 40m/s when it drops a package. If the package lands 181m ahead of the point above which it was dropped, when was it dropped and from what height ?

$$\begin{cases} x(t) = \frac{1}{2}a_x t^2 + v_x \cdot t + x_0 \\ y(t) = \frac{1}{2}a_y t^2 + v_y \cdot t + y_0 \end{cases} \begin{aligned} &= 40 \cdot t \\ &= -\frac{1}{2}gt^2 + H \end{aligned}$$

Landing: $\begin{cases} 181 = 40 \cdot t \Rightarrow t = \frac{181}{40} \\ 0 = -\frac{1}{2}gt^2 + H \end{cases}$

$$\Rightarrow H = \frac{1}{2}gt^2 = \frac{1}{2} \cdot g \cdot \left(\frac{181}{40}\right)^2 = 100\,m$$



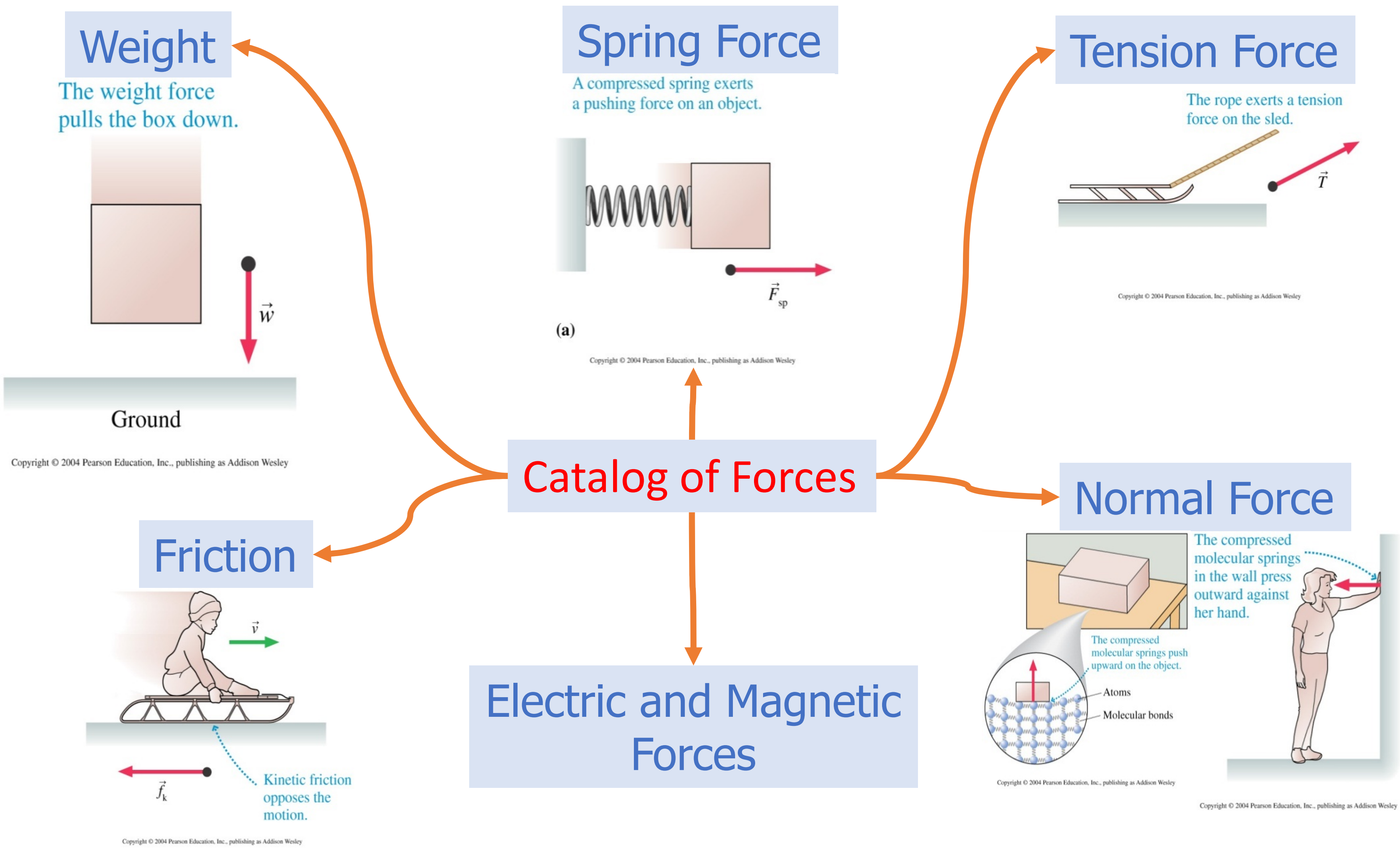
Theme 2
Mechanics

Module T2M2:
Kinematics

What is a force?

http://youtu.be/GmlMV7bA0TM

Catalog of Forces



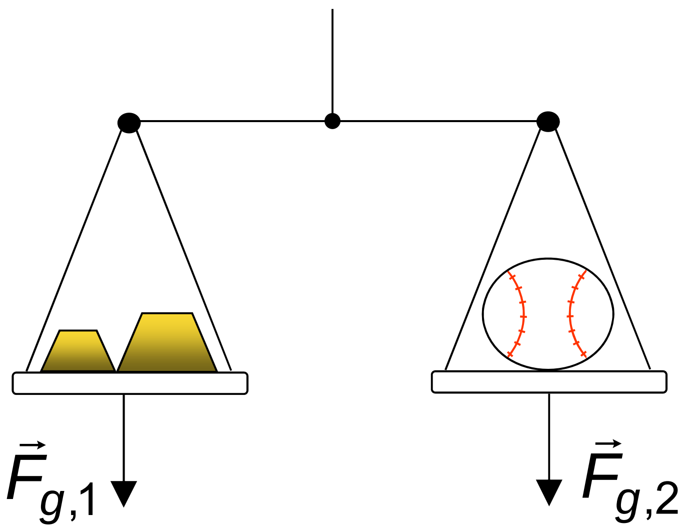
Weight and Mass

Weight is a force; it can be measured using a spring scale

$$\vec{f}_g = \vec{g} \cdot m$$

$$\text{weight} \propto \text{mass}$$

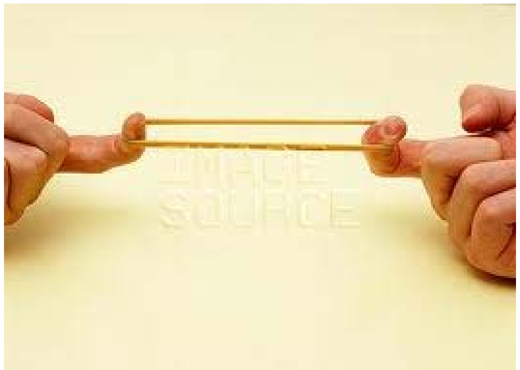
Weights are equal when masses are equal



Hooke's Law

Spring Force ($\vec{F}_S$)

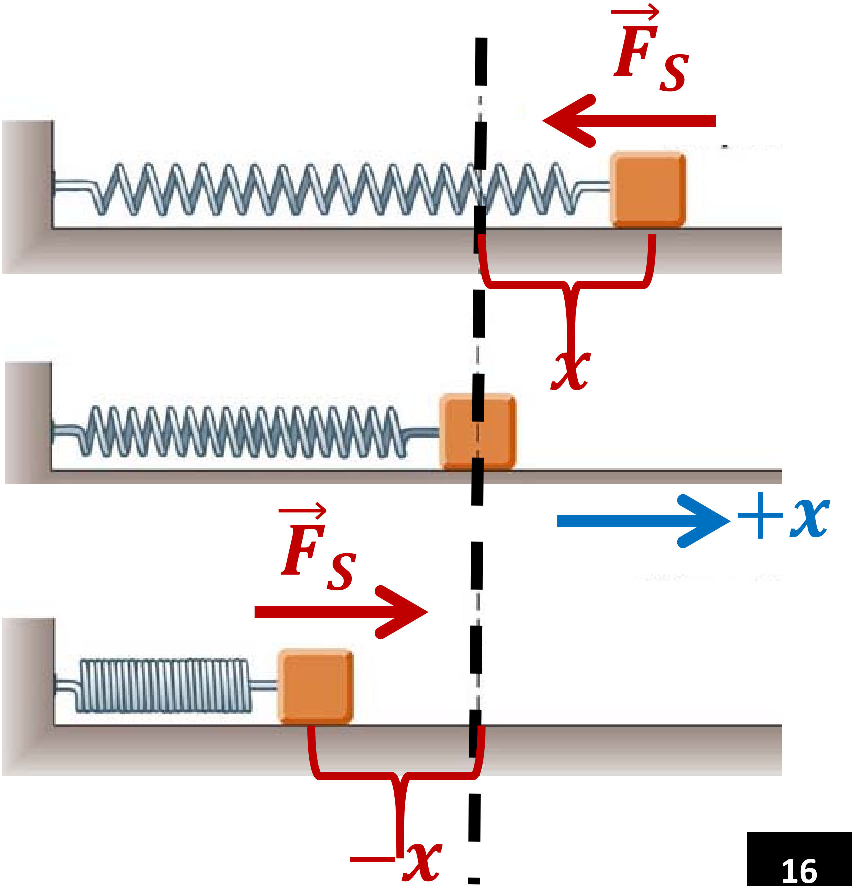
- A ***restoring force***: the elastic tries to return to its equilibrium configuration.
- Such materials are called "elastic"



- Magnitude of the restoring force is proportional to ***how much it has been stretched!***

$$\boxed{\vec{F}_S = -k\vec{x}}$$

- $\boldsymbol{k}$ is called the "spring constant".
  - It is a measure of the ***stiffness*** of the spring.

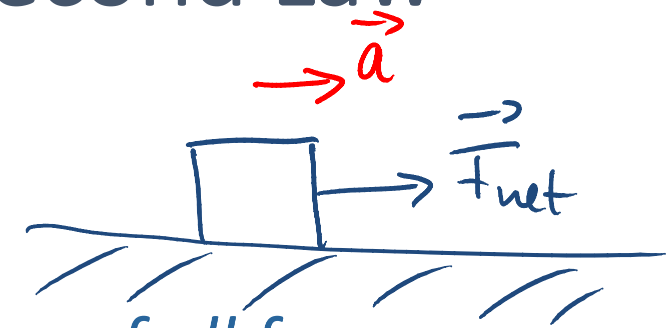


Newton’s First Law (Law of Inertia)

An isolated object, free from external forces, will continue moving at constant velocity, or remain at rest.

Newton's Second Law

$$\vec{F}_{net} = m \cdot \vec{a}$$

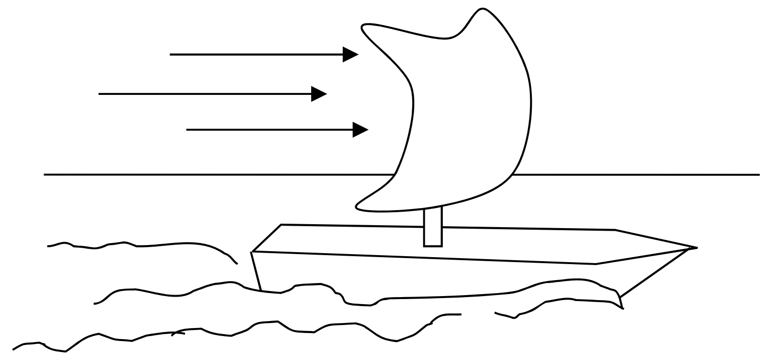


F$_{net}$ (or **F**$_{total}$) *is the* ***vector sum*** *of all forces acting on the particle of mass m:*

$$\vec{F}_{net} = \sum_i \vec{F}_i = \sum_i m_i \vec{a}_i$$

The acceleration **a** *is parallel to the total force, and proportional to it. The proportionality constant is the particle's* ***mass****. Newton defines mass as a measure of an object's* ***inertia****.*

A sailboat is being blown across the sea at a constant velocity.
What is the direction of the net force on the boat?



A) Left ←

B) Right →

C) Net force is zero

D) Down ↓

E) Up ↑