12A3

The "Lost Topics"

If $f(x)$ = position
& x = time

Higher Derivatives

Remember $f'(x) = \frac{d}{dx} f(x)$ = slope of tangent to $f(x)$ at point x

velocity

= rate of change of $f(x)$ at x.

2nd derivative: $\frac{d}{dx}\left(\frac{d}{dx} f(x)\right) = \boxed{\frac{d^2}{dx^2} f(x)}$

or $(f'(x))' = \boxed{f''(x)}$

~~acceleration~~
acceleration

3rd derivative

$$\left.\begin{array}{l} \dfrac{d}{dx}\left(\dfrac{d^2}{dx^2} f(x)\right) = \boxed{\dfrac{d^3}{dx^3} f(x)} \\[2ex] \text{or} \quad \left(f''(x)\right)' = \boxed{f'''(x)} \end{array}\right\} \text{"Jerk"}$$

Higher Derivatives

4th derivative

$$\frac{d^4}{dx^4} f(x) \quad \text{or} \quad f^{(4)}(x)$$

go further!

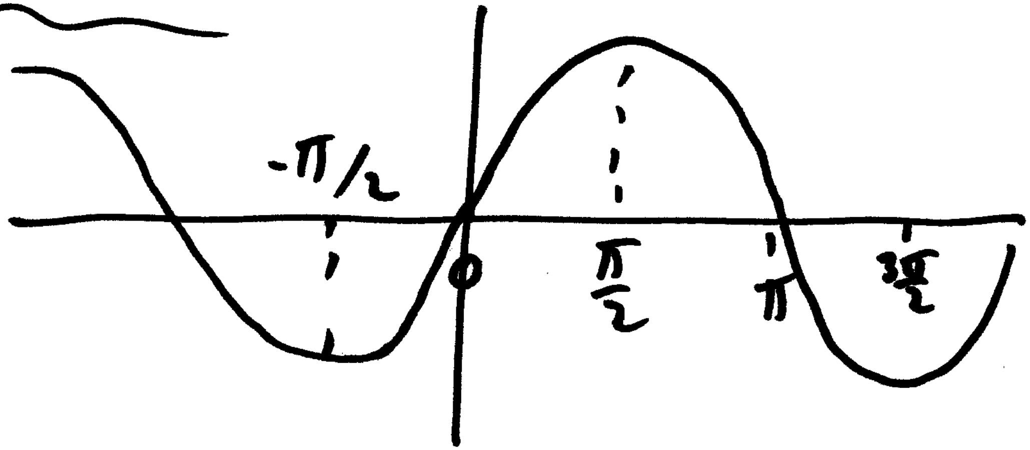
$$\frac{d^{108}}{dx^{108}} f(x), \quad f^{(57)}(x)$$

In general

$$\underbrace{\frac{d^n}{dx^n} f(x) = f^{(n)}(x)}_{\text{"nth" derivative!}}$$

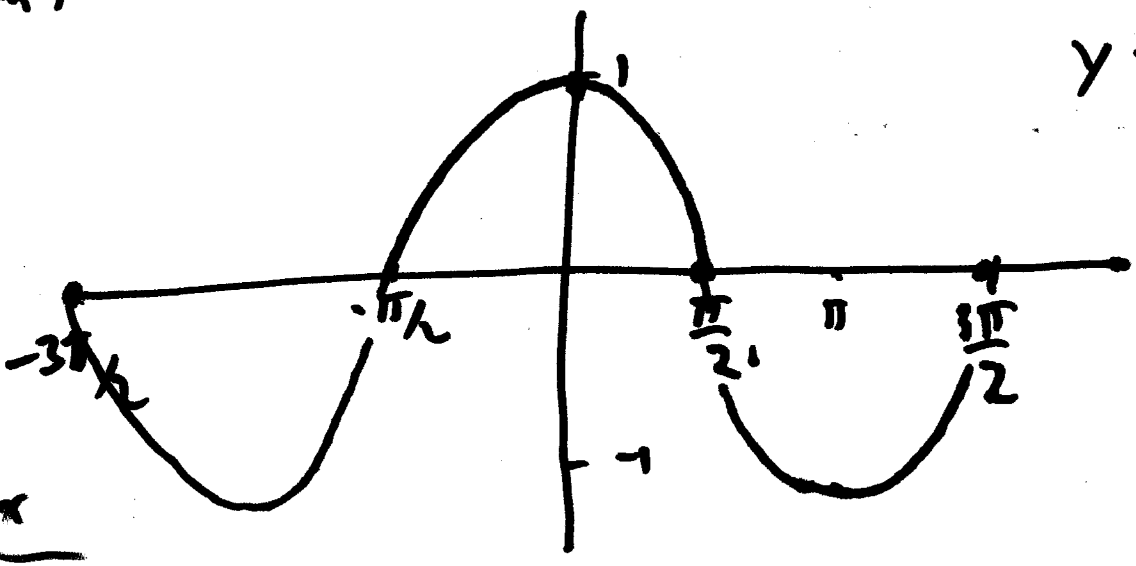
Trig Derivatives (Formal proof in text) (Skip!)

Instead, pictures!



$y = \sin x = f(x)$

$f'(x)$

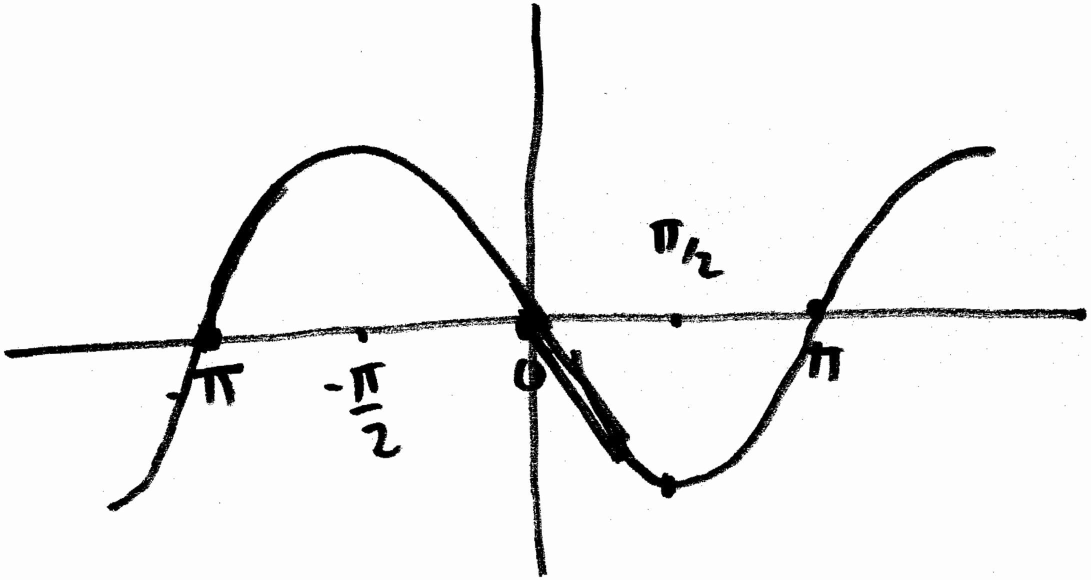


$y = \cos(x) = f'(x)$

$$\boxed{\frac{d}{dx} \sin x = \cos x}$$

$f'(x) = \cos x$

$f'(x)$



$$y = -\sin x = f'(x)$$

$$\boxed{\frac{d}{dx}\cos x = -\sin x}$$

$$\frac{d}{dx}\tan x = \frac{d}{dx}\frac{\sin x}{\cos x} = \frac{\overbrace{\left(\frac{d}{dx}\sin x\right)}^{\cos x}\cos x - \overbrace{\left(\frac{d}{dx}\cos x\right)}^{-\sin x}\sin x}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\boxed{\frac{d}{dx}\tan x = \sec^2 x}$$

Only oth one you <u>must</u> memorize: $\boxed{\frac{d}{dx}\sec(x) = \sec x \tan x}$

"Fun" Examples!

e.g. $$\frac{d}{dx}\left(x^2 \sin x\right) = \cancelto{2x}{\left(\frac{d}{dx}x^2\right)} \cdot \sin x + x^2 \cancelto{\cos x}{\left(\frac{d}{dx}\sin x\right)}$$

$$= 2x\sin x + x^2\cos x$$

e.g. $$\frac{d}{dx}\left(\frac{\tan x}{e^x}\right) = \frac{\cancelto{\sec^2 x}{\left(\frac{d}{dx}\tan x\right)} e^x - \cancelto{e^x}{\left(\frac{d}{dx}e^x\right)} \tan x}{\left(e^x\right)^2}$$

$$= \left(\cancel{e^x}\sec^2 x - \cancel{e^x}\tan x\right) / e^{2x}$$

$$= \left(\sec^2 x - \tan x\right)e^{-x}$$

eg $\frac{d}{dx} \cos(x^2) = -\sin(x^2) \cdot 2x$ by chain

Chain ?!

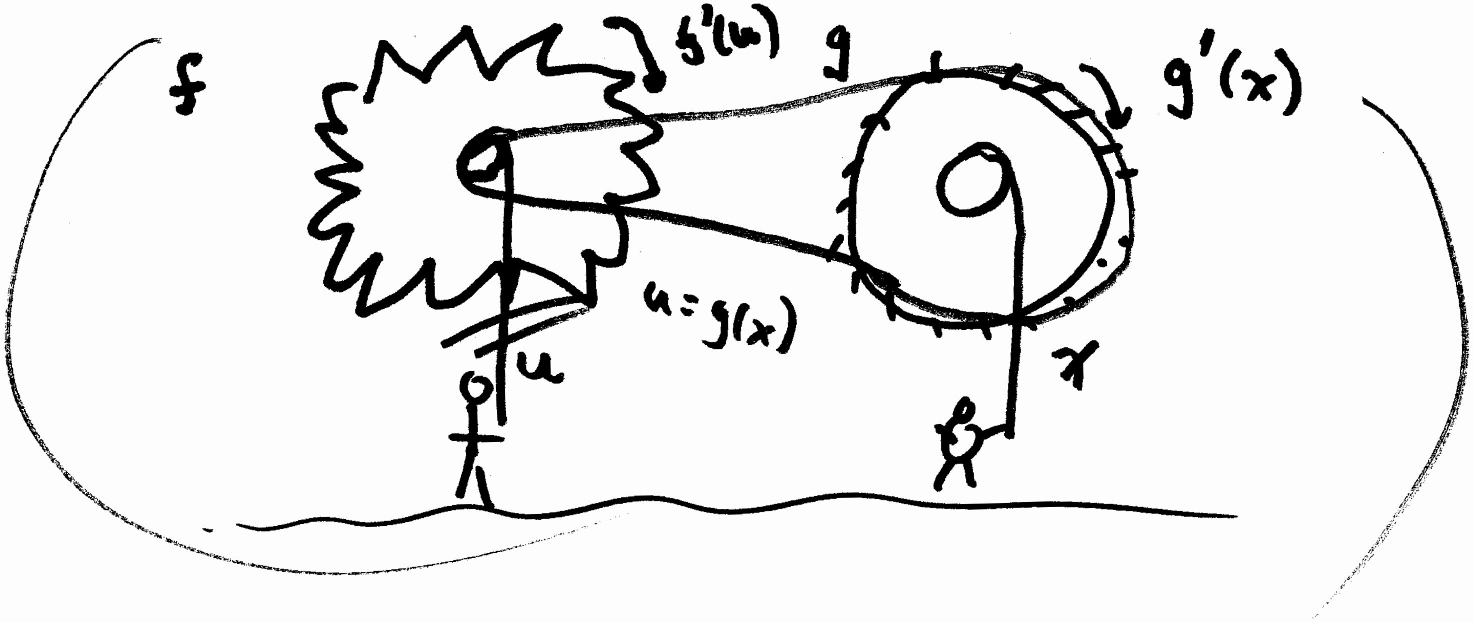
Let's talk chain rule!

Chain Rule:

$$\left(f(g(x))\right)' = \underbrace{f'(g(x))}_{\text{outside derivative}} \cdot \underbrace{g'(x)}_{\text{inside derivative}}$$

or $$\frac{d}{dx} f(g(x)) = \left(\frac{d}{du} f(u)\right)\left(\frac{du}{dx}\right)$$

where $u = g(x)$



eg. $\dfrac{d}{dx}\sin(\underbrace{e^x}_{u=e^x}) = \dfrac{d}{du}\sin(u)\cdot\dfrac{d}{dx}e^x$

$= \cos(u)\cdot e^x = \underline{\underline{e^x\cos(e^x)}}$.

eg $\dfrac{d}{dx}\cos(\underbrace{\tan x}_{u}) = \left(\dfrac{d}{du}\cos(u)\right)\left(\dfrac{d}{dx}\tan x\right)$

$= -\sin(\tan(x))\cdot\sec^2 x$

eg. $\frac{d}{dx} \tan^2 x = \left(\frac{d}{du} u^2\right) \cdot \left(\frac{d}{dx} \tan x\right)$

$u = \tan x$

$f(u) = \tan^2 x = u^2$

$= 2u \cdot \sec^2 x$

$= 2 \tan x \sec^2 x$

$= 2 \sin x / \cos^3 x$

$$\frac{d}{dx}(f(x))^p = p(f(x))^{p-1} \cdot f'(x)$$

"Generalized Power Rule"

eg $\frac{d}{dx} \sqrt{x^2 - 4} = \frac{1}{2}(x^2-4)^{-\frac{1}{2}} \cdot (2x - 0)$

$= \frac{x}{\sqrt{x^2-4}}$

u, $f(u) = \sqrt{u}$

or

$(x^2-4)^p$, $p = \frac{1}{2}$

eg. $\frac{d}{dx} \sin(\underset{u}{\underline{7x}}) = \cos(7x) \cdot 7 = \underline{7} \cos(\underline{7x})$ $\Big\}$ $f'(g(x))\underline{g'(x)}$

note $\boxed{\frac{d}{dx} f(\underline{kx}) = k f'(kx)}$

eg $\frac{d}{dx} \cos(\underline{x+12}) = -\sin(x+12) \cdot (1+0) = -\sin(x+12)$

note $\boxed{\frac{d}{dx} f(x+k) = f'(x+k)}$

eg $\frac{d}{dx} e^{\sin x \leftarrow u} = \left(\cancelto{e^u}{\frac{d}{du} e^u}\right) \cdot \left(\cancelto{\cos x}{\frac{d}{dx} \sin x}\right)$

$= e^{\sin x} \cdot \cos x$

note

$$\frac{d}{dx} e^{f(x)} = \underline{\underline{\left(e^{f(x)}\right) \cdot f'(x)}}$$

eg

$$\frac{d}{dx} e^{\left[e^{e^x}\right] \leftarrow u}$$

$$= \left(\frac{d}{du} e^u\right) \cdot \frac{d}{dx} e^{e^x}$$

$$= e^{e^{e^x}} \cdot \left[\frac{d}{dx} e^{e^x}\right] \leftarrow u$$

$$= e^{e^{e^x}} \cdot \left[\frac{d}{du} e^u\right] \cdot \frac{d}{dx} e^x \quad \text{\small{new}}\ u$$

$$= e^{e^{e^x}} \cdot e^{e^x} \cdot e^x$$

$u = e^{e^x}$

$$\frac{d}{dx} e^{e^{e^x}} = \frac{d}{dx} e^{u} = \left.\frac{d}{du} e^{u} \cdot \frac{du}{dx}\right\} \text{ by chain!}$$

$$= e^{u} \cdot \left[\frac{d}{dx} e^{e^x}\right] = e^{e^{e^x}} \underline{\underline{\left[\frac{d}{dx} e^{e^x}\right]}}$$

$$\frac{d}{dx}\left[e^{e^x}\right] = \frac{d}{dx} e^{u} \leftarrow \text{new } u = e^x$$

$$= \frac{d}{du} e^{u} \cdot \frac{du}{dx} = \frac{d}{du} e^{u} \cdot \frac{d}{dx} e^{x}$$

$$= e^{u} e^{x} = \left[e^{e^x} e^{x}\right]$$

$$\frac{d}{dx} e^{e^{e^x}} = e^{e^{e^x}} e^{e^x} e^{x}$$