Theme 2
Mechanics

Module T2M1:
Kinematics

Mid Term on Friday

- **You must write in your designated room:**
  - Mac ID: a___ to f____ write in BSB 147
  - Mac ID: g___ to kh___ write in ITB 137
  - Mac ID: ki___ to n___ write in JHE 376
  - Mac ID: o___ to sh___ write in MDCL 1102
  - Mac ID: si___ to z___ write in MDCL 1105
- **Check avenue carefully!**
  - News Section also contains the formula sheet

HOMEWORK!

LONCAPA Quiz 1

Bicyclists in the Tour de France reach speeds of 35.3 miles per hour (mi/h) on flat sections of the road.

a) What is the speed in kilometers per hour (km/h)?

b) What is the speed in meters per second (m/s)?

NOTE: 1 Mile = 1609 m

Module Clicker Quiz!

Now that you have had a chance to review the entire first module, T2M1, here is your first

module quiz!

Module Clicker Quiz!

Direction of acceleration (120 seconds)

Person X

At rest then starts running

Person Y

Running, but comes to rest

A. $a_X > 0$, $a_Y > 0$

B. $a_X < 0$, $a_Y > 0$

C. $a_X > 0$, $a_Y < 0$

D. $a_X < 0$, $a_Y < 0$

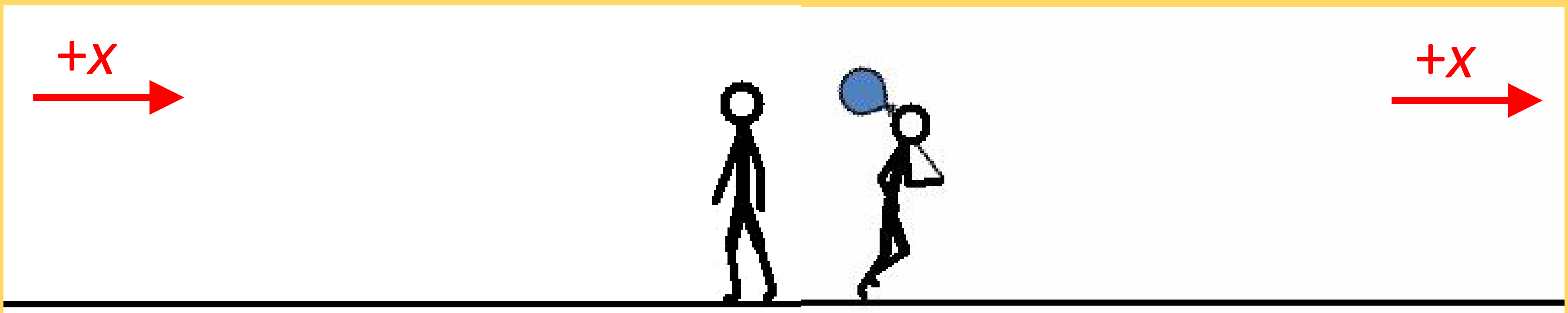
E. I don’t know

Module Clicker Quiz!

Direction of acceleration (120 seconds)

Person X
At rest then starts running

Person Y
Running, but comes to rest



A. $a_X > 0$, $a_Y > 0$

B. $a_X < 0$, $a_Y > 0$

C. $a_X > 0$, $a_Y < 0$

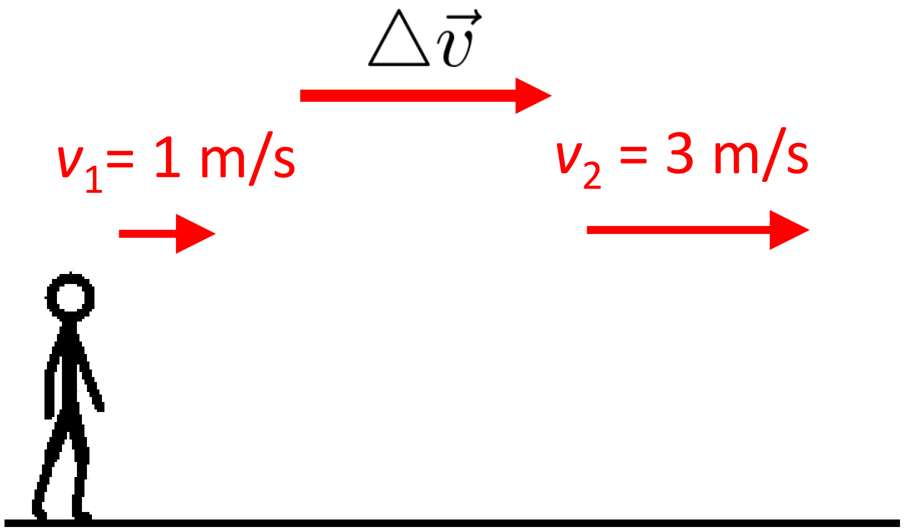
D. $a_X < 0$, $a_Y < 0$

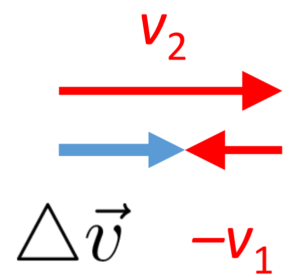
E. I don't know

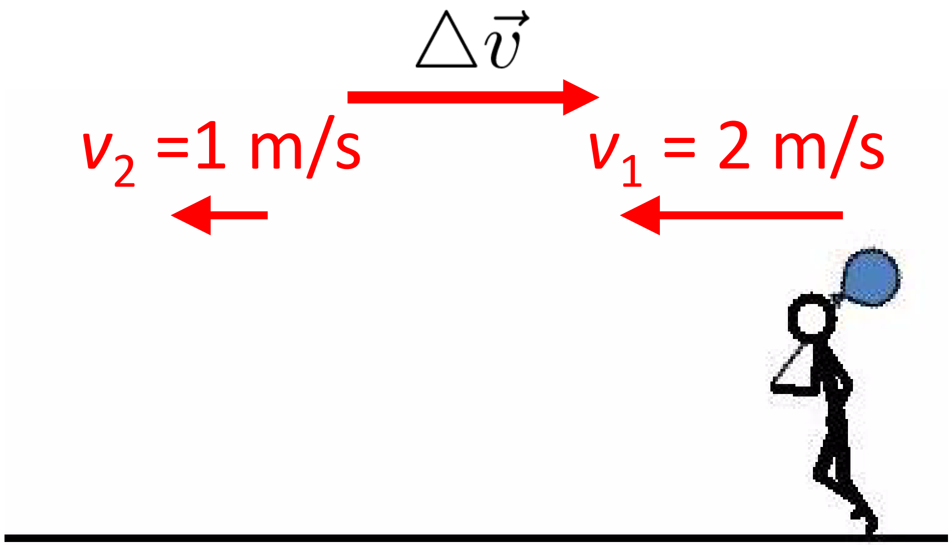
$+x$

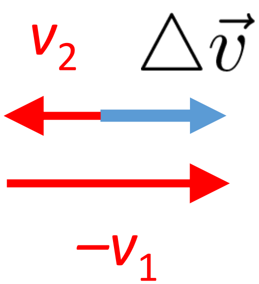
$\Delta t = 1$ s

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

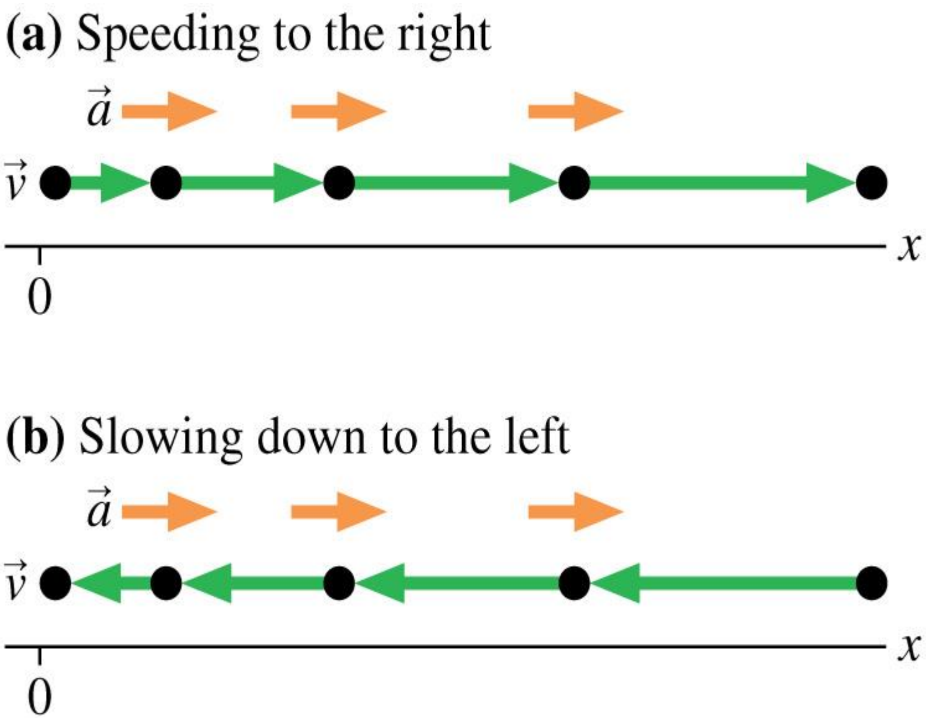






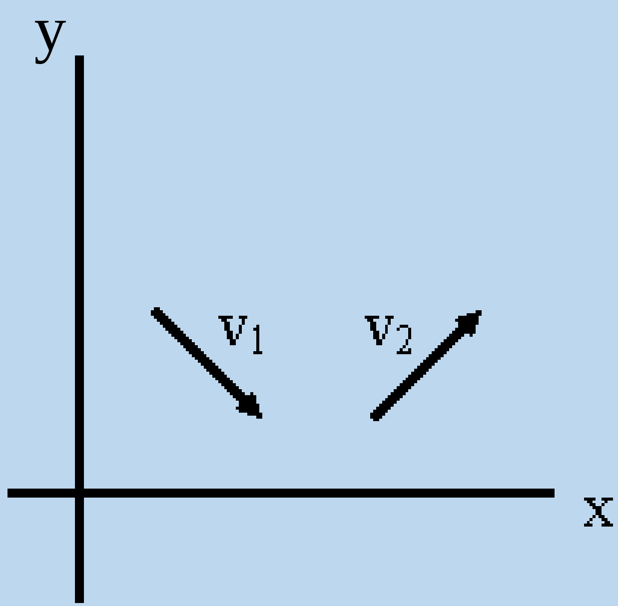


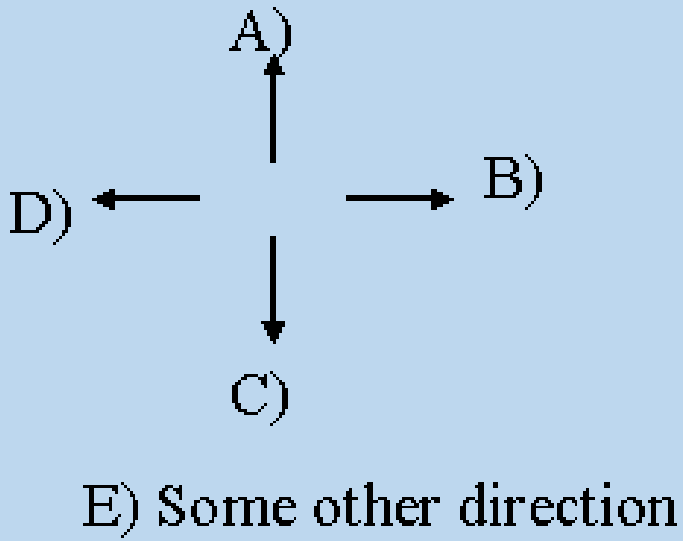
Signs of position, velocity, acceleration



Clicker Quiz: 2D velocity

A particle is moving with constant acceleration. Its velocity vector at two different times is shown below. What is the direction of the acceleration?





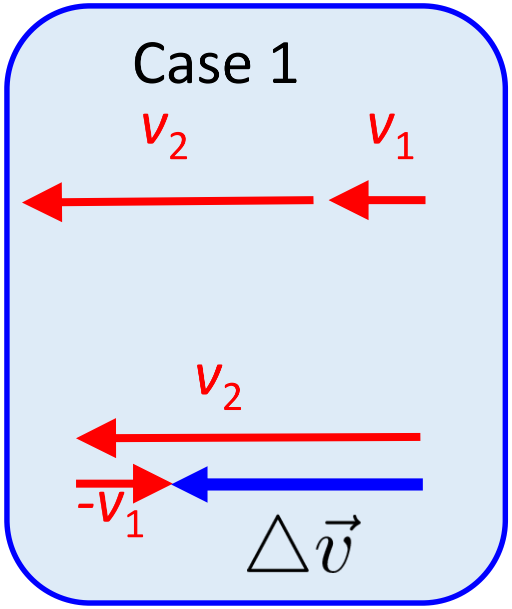
Misconceptions

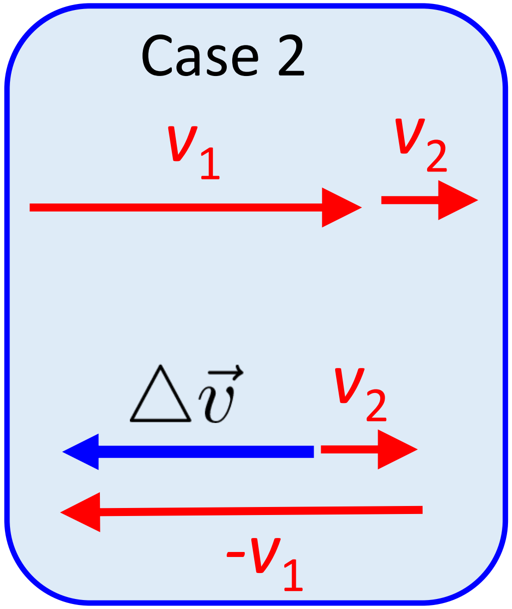
1 velocity is constant then acceleration is also constant.

2 If an object moves with a high speed then its acceleration is high and if an object moves with a low velocity then the acceleration is low.

3 If the acceleration is positive than the object speeds up, and if the acceleration is negative then the object is slowing down.

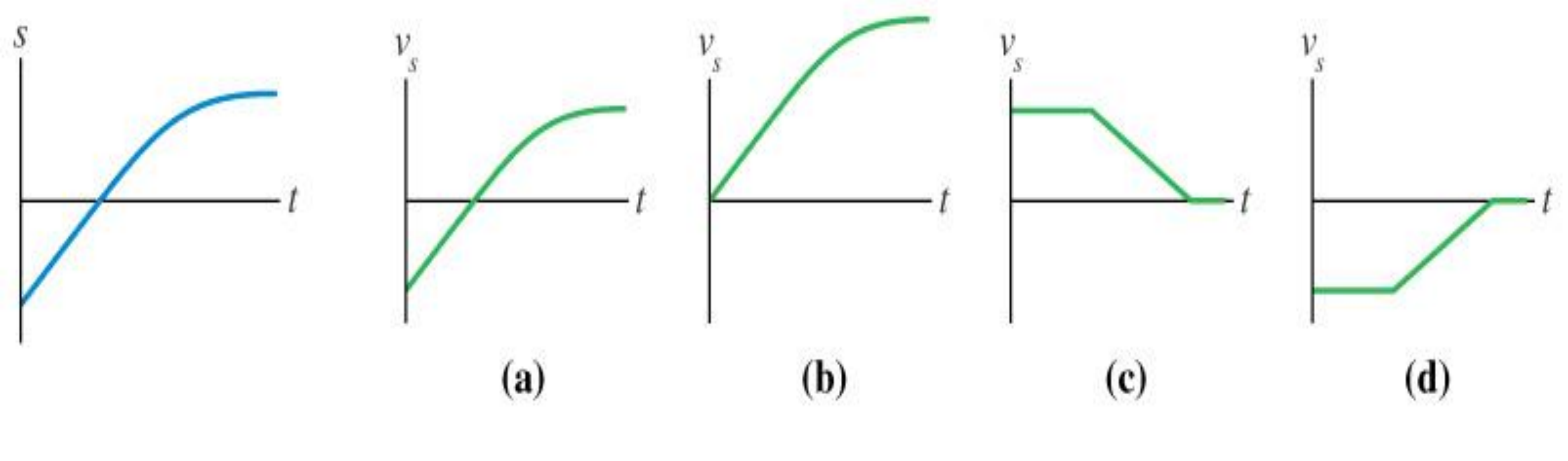
4 What is negative acceleration?





Clicker Quiz: Recap x-t graphs

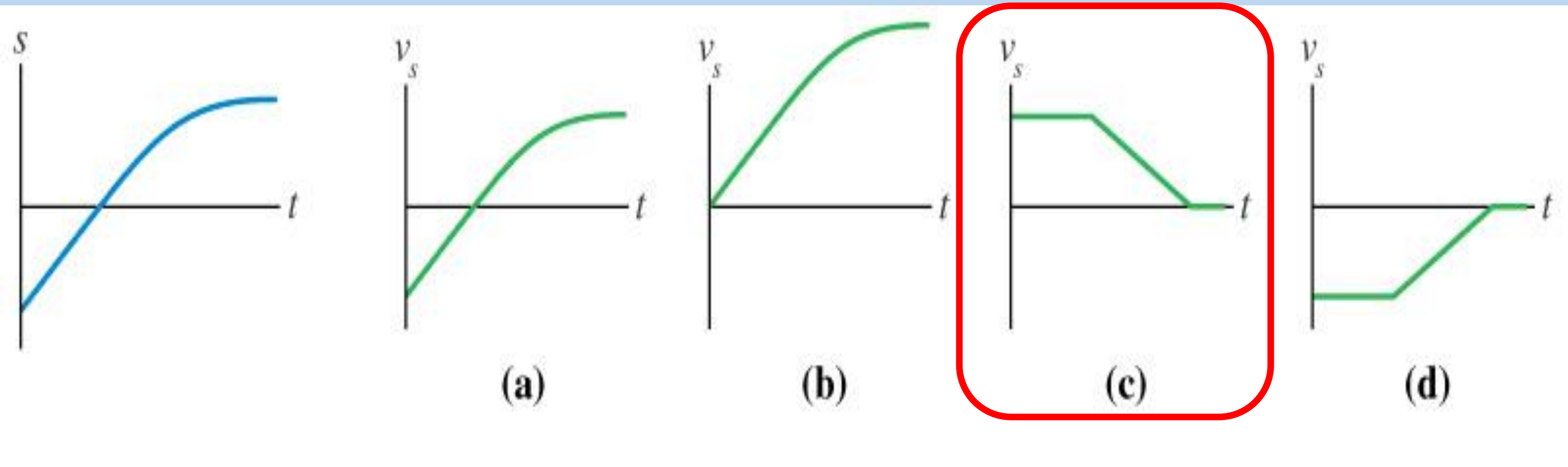
Which velocity-versus-time graph goes with this position-versus-time graph on the left?



Hint: do the pen test!

Clicker Quiz: Recap x-t graphs

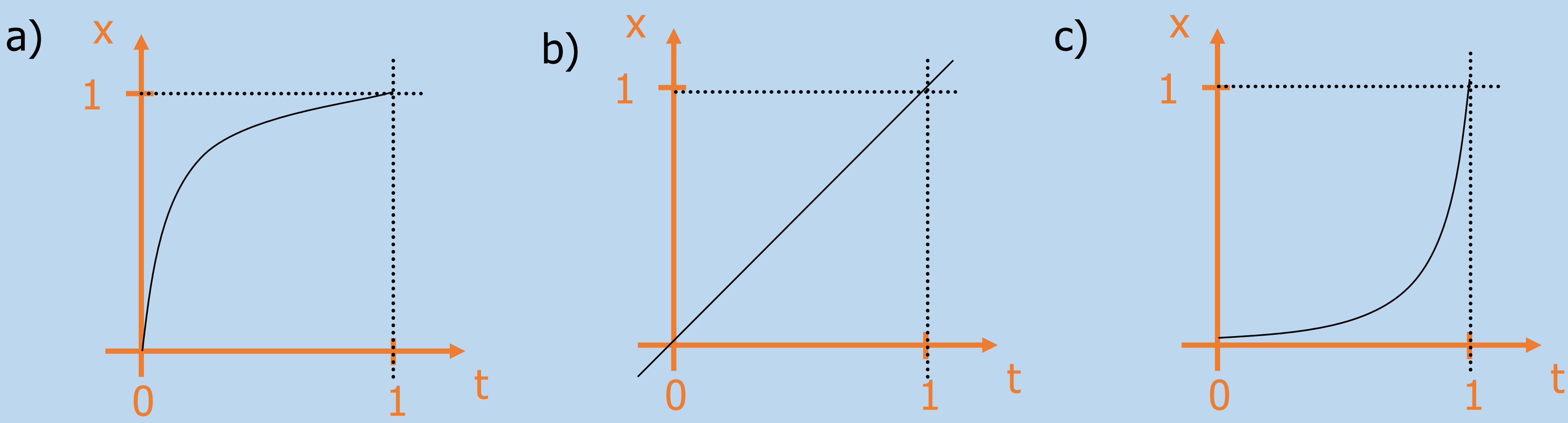
Which velocity-versus-time graph goes with this position-versus-time graph on the left?



Hint: do the pen test!

Clicker Quiz: Recap x-t graphs

Which of the motions described has the larger **average** velocity in the interval 0 s < t < 1 s?

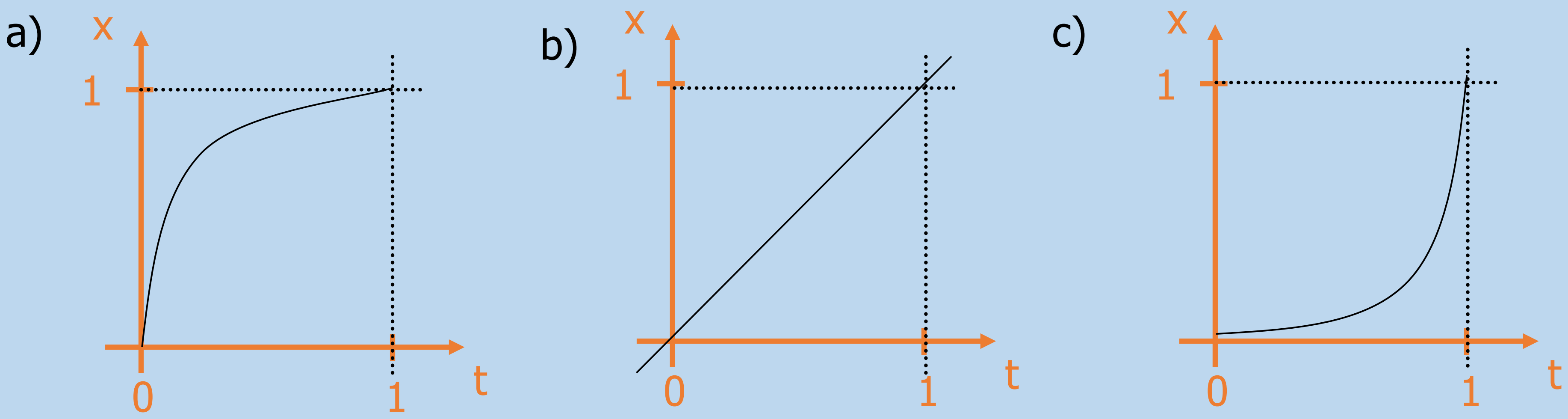


A) $v_a > v_b > v_c$.
B) $v_c > v_b > v_a$.
C) $v_b > v_a > v_c$.
D) $v_b > v_c > v_a$.
E) $v_a = v_b = v_c$.

Hint: $$v_{ave} = \frac{x_2 - x_1}{t_2 - t_1}$$

Clicker Quiz: Recap x-t graphs

Which of the motions described has the larger **average** velocity in the interval 0 s < t < 1 s?

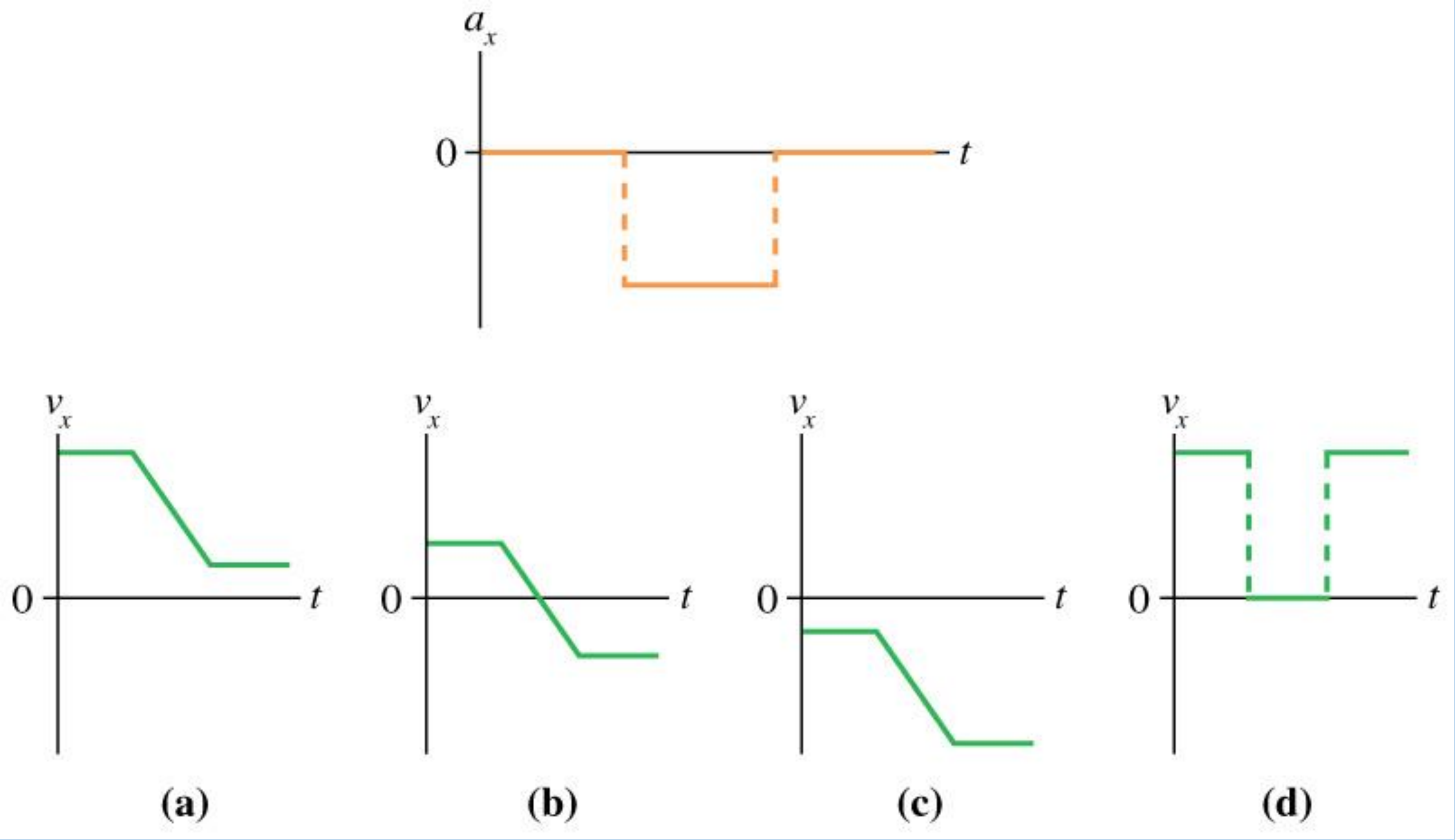


A) $v_a > v_b > v_c$.
B) $v_c > v_b > v_a$.
C) $v_b > v_a > v_c$.
D) $v_b > v_c > v_a$.
E) $\boxed{v_a = v_b = v_c}$.

Hint: $$v_{ave} = \frac{x_2 - x_1}{t_2 - t_1}$$

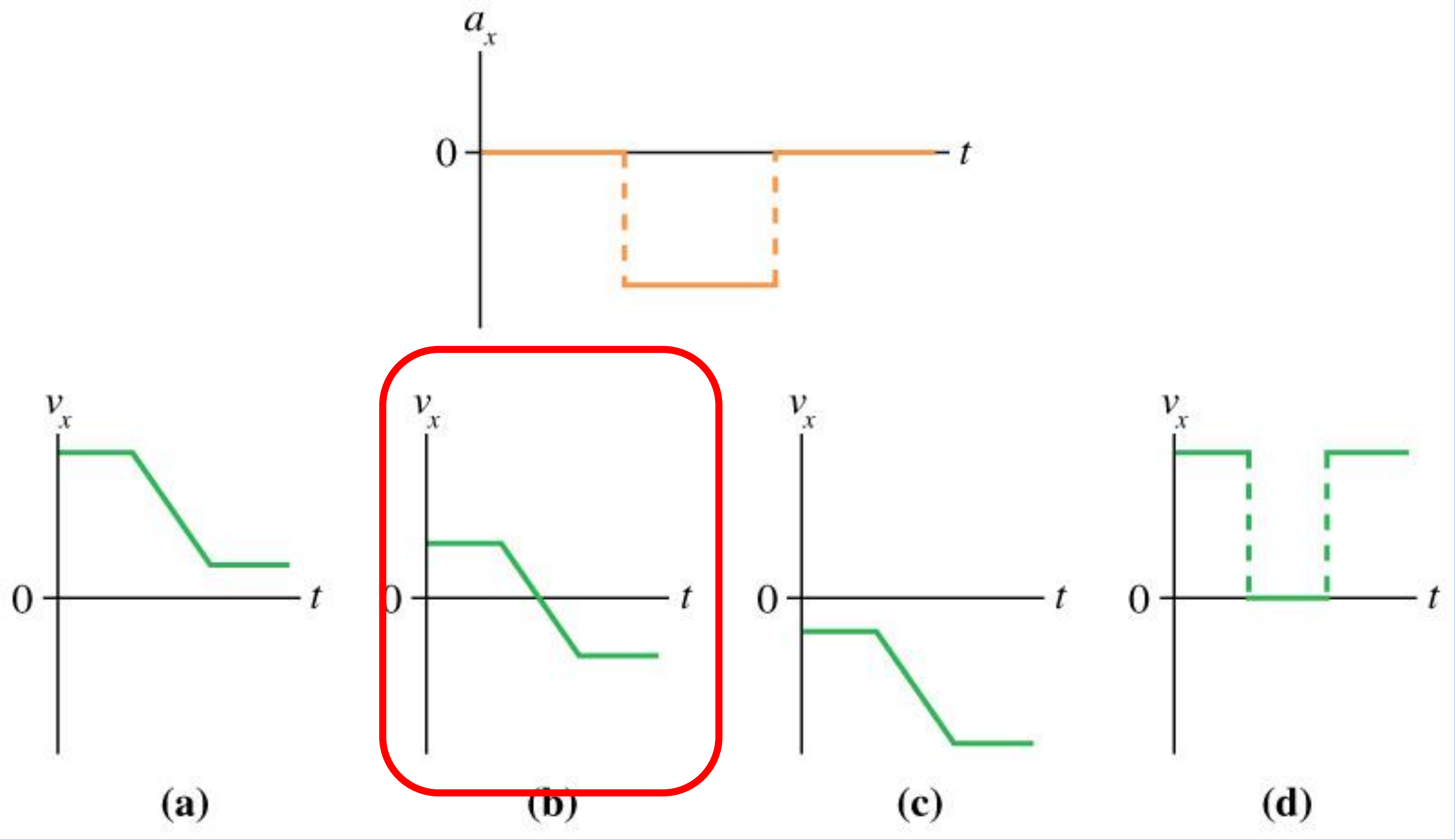
Clicker Quiz: a-t and v-t graphs

Which velocity-versus-time graph or graphs goes with this acceleration-versus-time graph? The particle is initially moving to the right and eventually to the left.



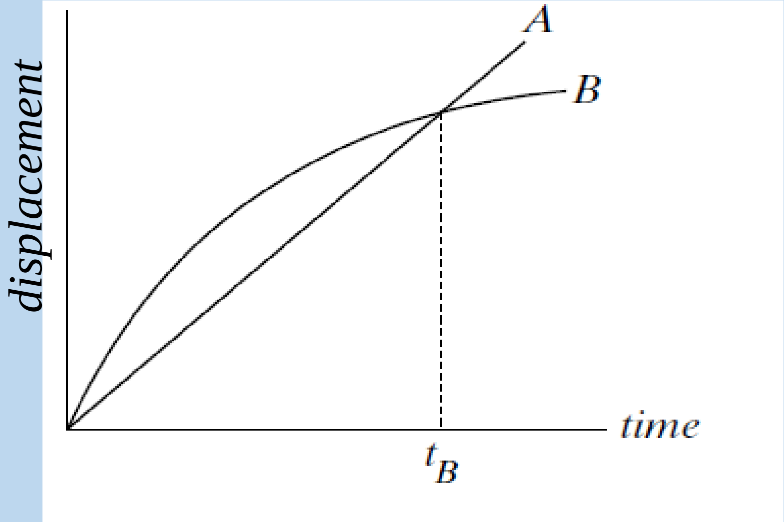
Clicker Quiz: a-t and v-t graphs

Which velocity-versus-time graph or graphs goes with this acceleration-versus-time graph? The particle is initially moving to the right and eventually to the left.



Clicker Quiz: Recap x-t graphs

The graph shows displacement as a function of time for two trains running on parallel tracks. Which of the following is true?



a) At time t_B, both trains have the same velocity.

b) Both trains speed up all the time.

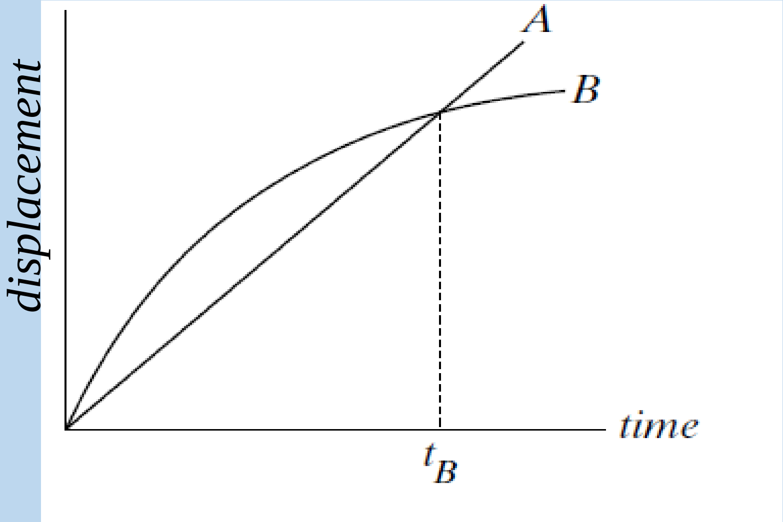
c) Both trains have the same velocity at some time before t_B.

d) Somewhere on the graph, both trains have the same acceleration.

e) None of the above statements is true.

Clicker Quiz: Recap x-t graphs

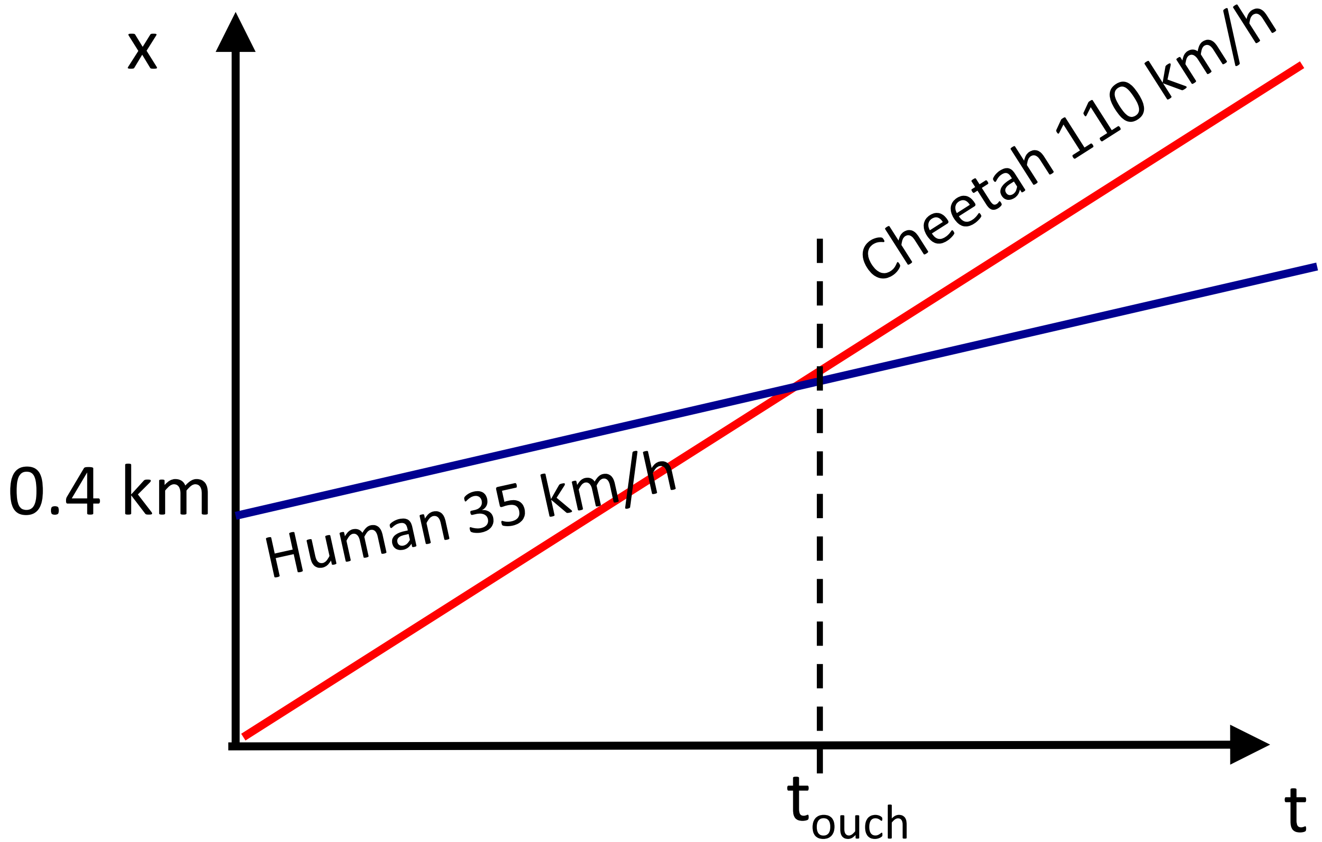
The graph shows displacement as a function of time for two trains running on parallel tracks. Which of the following is true?



a) At time t_B, both trains have the same velocity.
b) Both trains speed up all the time.
c) Both trains have the same velocity at some time before t_B.
d) Somewhere on the graph, both trains have the same acceleration.
e) None of the above statements is true.

Cheetah Sprint

A cheetah can sprint at a speed of 110 km/h. The best a human is capable of is a speed of 35 km/h. A man and a cheetah are initially 0.400 km apart. Assuming that both man and cheetah are running at their top speed, how long does it take the cheetah to overtake the man?



Handling Kinematic Questions

- Start with a picture (a graph or a sketch/motion diagram)

- When dealing with a kinematics problem there are only 5 quantities involved:

1. You can start a problem by writing down these down, just like this:
2. Define the positive direction!

   (some might end up being negative)
3. Now read the problem, filling in the numbers as you go.

   (Put a "?" if no number is given)

$$\Delta x =$$

$$v_i =$$

$$v_f =$$

$$a =$$

$$\Delta t =$$

Now look at your 4 kinematic equations, and choose one which solves for the "?" you want, using the numbers you found, but avoiding any other "?"s in your list!

Kinematic equations of motion

<u>constant *a*</u>

$$v_f = v_i + at$$

$$x_f - x_i = \frac{1}{2}(v_i + v_f)t$$

$$x_f - x_i = v_i t + \frac{1}{2}at^2$$

$$v_f^{\,2} = v_i^{\,2} + 2a(x_f - x_i)$$

$$\Delta x = x_f - x_i$$

<u>*a* = 0</u>

$$v_f = v_i$$

$$x_f = x_i + vt$$

What you need for every kinematics problem

$a =$

$v_i =$

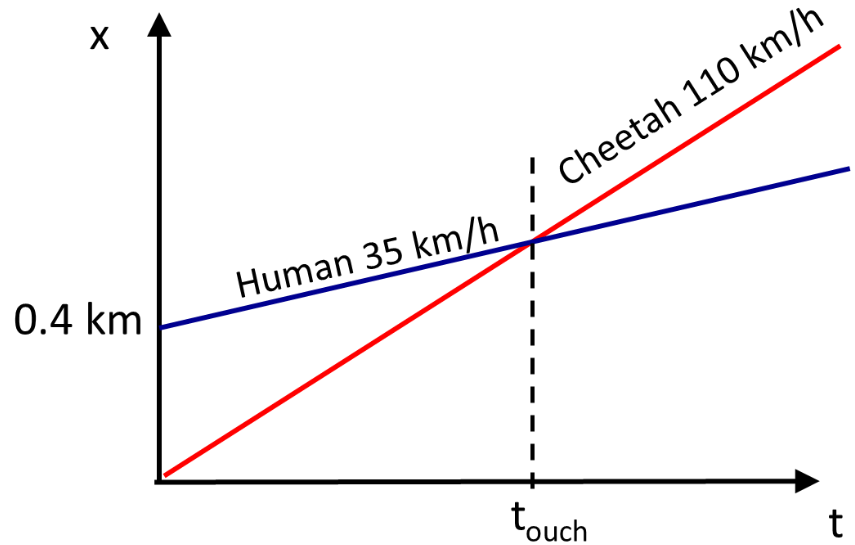
$v_f =$

$\triangle x =$

$\triangle t =$

Cheetah Sprint: exact sc

A cheetah can sp
human is capable
cheetah are initia
man and cheetah
long does it take



Cheetah: $x_i = 0$

$x_f = ?$

$v = 110$

$a = 0$

$t = ?$

Snack: $x_i = 0.4$

$x_f = ?$

$v = 35$

$a = 0$

$t = ?$

Cheetah Sprint: exact solution

A cheetah can sprint at a speed of 110 km/h. The best a human is capable of is a speed of 35 km/h. A man and a cheetah are initially 0.400 km apart. Assuming that both man and cheetah are running at their top speed, how long does it take the cheetah to overtake the man?

Human:
(all units in km & h)

$$a = 0$$
$$v = 35$$
$$x_i = 0.4$$
$$x_f = ?$$
$$t = ?$$

Cheetah:
(all units in km & h)

$$a = 0$$
$$v = 110$$
$$x_i = 0$$
$$x_f = ?$$
$$t = ?$$

Use: $x_f = x_i + vt$

Cheetah: $x_f = 110t$

Human: $x_f = 0.4 + 35t$

$$110t = 0.4 + 35t$$
$$75t = 0.4$$
$$t = 5.33 \times 10^{-3} \text{ h}$$
$$t = 19.2 \text{ s}$$

Example

The human body can survive the trauma due to a sudden stop if the magnitude of acceleration is less than **245 m/s²**. Suppose you are in a car accident with an initial speed of **85 km/h** (**23.6 m/s**) and are stopped by an inflating air bag.

a) How long does it take to come to rest?

b) Over what distance must the air bag stop you if you are to survive the crash?

y information

$\Delta x = ?$
$v_o = 23.6 \text{ m/s}$
$v_f = 0$
$a = -245 \text{ m/s}^2$
$\Delta t = ?$

$$v_f = v_i + a\Delta t$$

$$x_f = x_i + v_i\Delta t + \frac{1}{2}a\Delta t^2$$

$$v_f^2 = v_i^2 + 2a \cdot (x_f - x_i)$$

$$(x_f - x_i) = \frac{v_i + v_f}{2} \cdot \Delta t$$

Part a):

$$v_f = v_i + a\Delta t$$

$$0 = 23.6 - 245\Delta t$$

$$\boxed{\Delta t = 0.096\ s}$$

Part b):

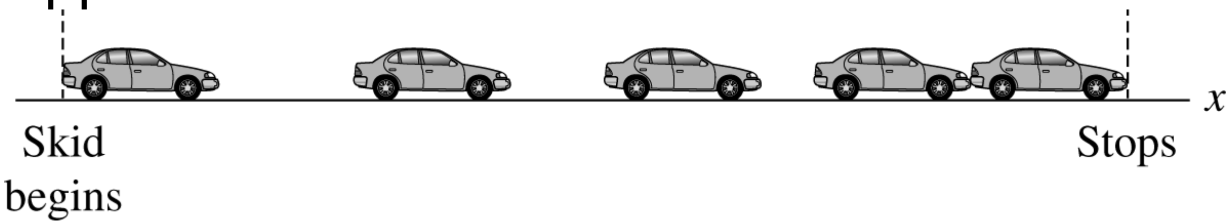
$$v_f{}^2 = v_o{}^2 + 2a\Delta x$$

$$0 = (23.6)^2 + 2(-245)\Delta x$$

$$\boxed{\Delta x = 1.14\ m}$$

Try this on your own!

- A car brakes to avoid hitting a can of tuna. The brakes apply an acceleration of **4.5 m/s²**, and the car comes to rest over a distance of **27 m**. What was the speed of the car at the instant the brakes were applied?



$$v_f = v_i + a\Delta t$$

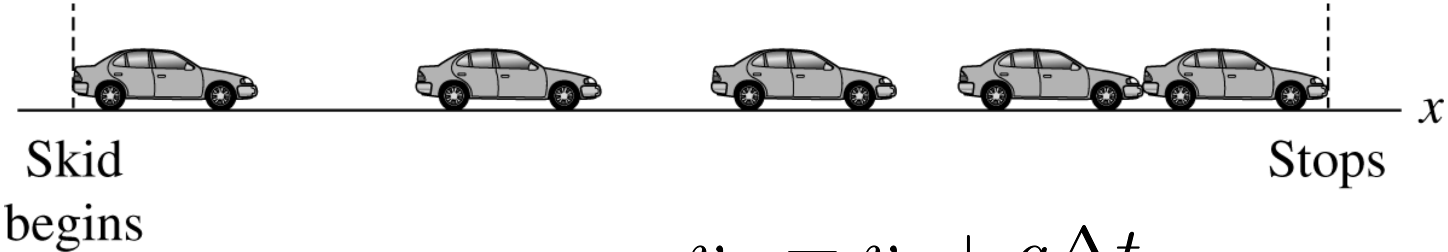
$$x_f = x_i + v_i\Delta t + \frac{1}{2}a\Delta t^2$$

$$v_f^2 = v_i^2 + 2a\cdot(x_f - x_i)$$

$$(x_f - x_i) = \frac{v_i + v_f}{2}\cdot\Delta t$$

Try this on your own!

- A car brakes to avoid hitting a can of tuna. The brakes apply an acceleration of **4.5 m/s²**, and the car comes to rest over a distance of **27 m**. What was the speed of the car at the instant the brakes were applied?



$\Delta x = 27$ m

$v_1 = ?$

$v_2 = 0$

$a = -4.5$ m/s^2

$t = ?$

$$v_f = v_i + a\Delta t$$

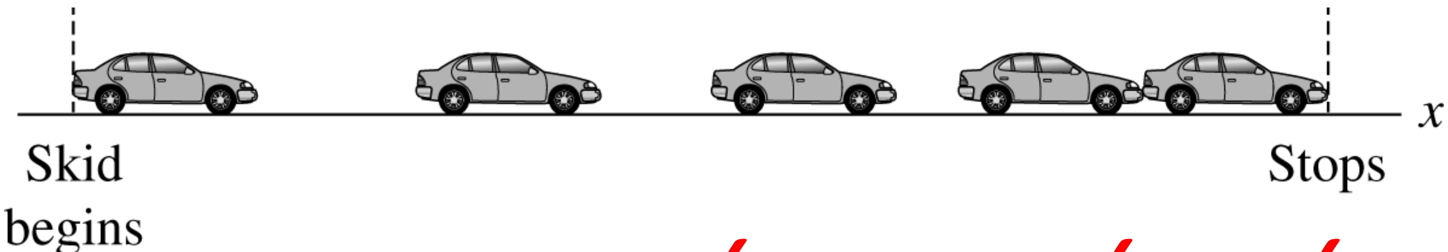
$$x_f = x_i + v_i\Delta t + \frac{1}{2}a\Delta t^2$$

$$v_f^2 = v_i^2 + 2a \cdot (x_f - x_i)$$

$$(x_f - x_i) = \frac{v_i + v_f}{2} \cdot \Delta t$$

Try this on your own!

- A car brakes to avoid hitting a can of tuna. The brakes apply an acceleration of **4.5 m/s²**, and the car comes to rest over a distance of **27 m**. What was the speed of the car at the instant the brakes were applied?



$$v_f^2 = v_i^2 + 2a \cdot (x_f - x_i)$$

$$\Delta x = 27\text{ m}$$
$$v_1 = ?$$
$$v_2 = 0$$
$$a = -4.5\text{ m/s}^2$$
$$t = ?$$

$$0 = v_o^{\,2} + 2(-4.5)(+27)$$
$$v_o^{\,2} = 243$$
$$v_o = +15.6\text{ m/s}$$

The quadratic equation – it happens to the best of us

- You may encounter a kinematics problem in which you are asked to solve for the time, using the equation:

$$\Delta y = v_{yo}t + \frac{1}{2}a_y t^2$$

where $\Delta y, v_{yo}$ and a_y are known.

- You need to use the quadratic equation to solve for t:

$$0 = at^2 + bt + c$$

$$\mathrm{t} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

All objects fall with the same acceleration

1971, Apollo 15 astronaut David Scott dropped both a hammer and a feather together toward the surface of the Moon

Free fall

- All objects fall with the same acceleration:

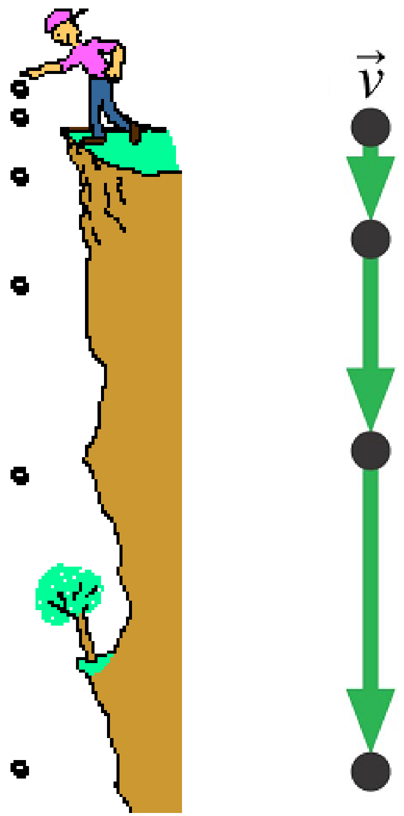
$$\vec{a} = \vec{g} = 9.8\ \mathrm{m/s^2}\ \ [down]$$

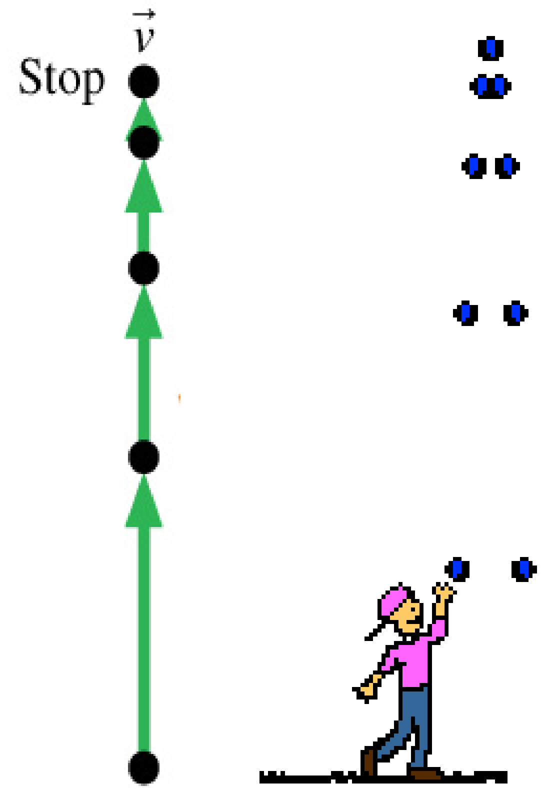
  - Valid near Earth's surface
  - Regardless of object's mass
  - Neglecting air resistance

- "Free fall" (i.e. motion with $\vec{g}$) does not **ONLY** mean downward motion
  - Throw something up in the air – the moment it leaves your hand, it is in free fall (while moving upward as well as downward)

Free falling upward?

- "free fall just refers to acceleration $= \vec{g}$





Example

Your frisbee is stuck in a branch that is 5.0 m above ground. You throw your shoe straight up to try to knock it down, but your shoe just reaches the frisbee before falling back down. What initial velocity did you give the shoe if it started at 1.5 m above ground?

$$v_f = v_i + a\Delta t$$

$$x_f = x_i + v_i\Delta t + \frac{1}{2}a\Delta t^2$$

$$v_f^2 = v_i^2 + 2a\cdot(x_f - x_i)$$

$$(x_f - x_i) = \frac{v_i + v_f}{2}\cdot\Delta t$$

Example

Your frisbee is stuck in a branch that is 5.0 m above ground. You throw your shoe straight up to try to knock it down, but your shoe just reaches the frisbee before falling back down. What initial velocity did you give the shoe if it started at 1.5 m above ground?

Define up as the positive direction

What is Δy?

$\Delta y = 3.5 \text{ m}$

$v_i = ? \text{ (must be +ve)}$

$v_f = 0 \text{ (at the top)}$

$a = -9.8 \text{ m/s}^2$

$\Delta t = ?$

✓ ✓ ✓

$$v_f^2 = v_i^2 + 2a\Delta y$$

$$0 = v_i^2 + 2(-9.8)(3.5)$$

$$v_i = \sqrt{2(9.8)(3.5)} = 8.3 \text{ m/s}$$

Keep track of the units!!!

Would it matter if we had chosen down as the positive direction?

Clicker quiz

You are throwing a ball straight up in the air. At the highest point, the ball’s

a) velocity and acceleration are zero.

b) velocity is nonzero but its acceleration is zero.

c) acceleration is nonzero, but its velocity is zero.

d) velocity and acceleration are both nonzero.

Clicker quiz

You are throwing a ball straight up in the air. At the highest point, the ball's

a) velocity and acceleration are zero.

b) velocity is nonzero but its acceleration is zero.

c) acceleration is nonzero, but its velocity is zero.

d) velocity and acceleration are both nonzero.

Try this on your own!

- You are standing on 20 m tall stilts, and you throw a hotdog straight up into the air. It takes 5.0 seconds for the hotdog to hit the ground below you.

What was the initial velocity of the hotdog?

$$v_f = v_i + a\Delta t$$

$$x_f = x_i + v_i\Delta t + \frac{1}{2}a\Delta t^2$$

$$v_f^2 = v_i^2 + 2a \cdot (x_f - x_i)$$

$$(x_f - x_i) = \frac{v_i + v_f}{2} \cdot \Delta t$$

Try this on your own!

- You are standing on 20 m tall stilts, and you throw a hotdog straight up into the air. It takes 5.0 seconds for the hotdog to hit the ground below you.

What was the initial velocity of the hotdog?

Define up as the positive direction (all units are in m and s)

$$\Delta y = -20$$
$$v_i = ?$$
$$v_f = ?$$
$$a = -9.8$$
$$\Delta t = 5.0$$

$$\Delta y = v_i t + \frac{1}{2} a t^2$$
$$-20 = v_i 5.0 + \frac{1}{2}(-9.8)5^2$$
$$v_i = 20.5 \text{ m/s}$$

Note: we did not have to break this problem into two parts (up and down), the math and proper signs takes care of it!!!

A note on the kinematic equations

- Our equations are valid for motion in which the acceleration is **constant**
- Of course, real motion can have changing acceleration
  - Consider this case of driving: Traffic light turns green, you start from rest, accelerate to 50 km/h, then continue on at that speed (acceleration becomes zero)

- When dealing with this motion, we can break it up into two parts (each with it's own constant acceleration

a. Accelerating up to 50 km/h

b. Continuing on at 50 km/h (with $a = 0$)

Example: 1D motion with two parts

Sitting beside an old lady in the park, you grab her purse and start running. Over the first 10.0 m, you accelerate at 1.20 m/s^2 up to your top running speed, and then continue to sprint at this speed for 15.0 s more before being tackled from behind by the old lady.

(a) How long did it take the old lady to catch you?

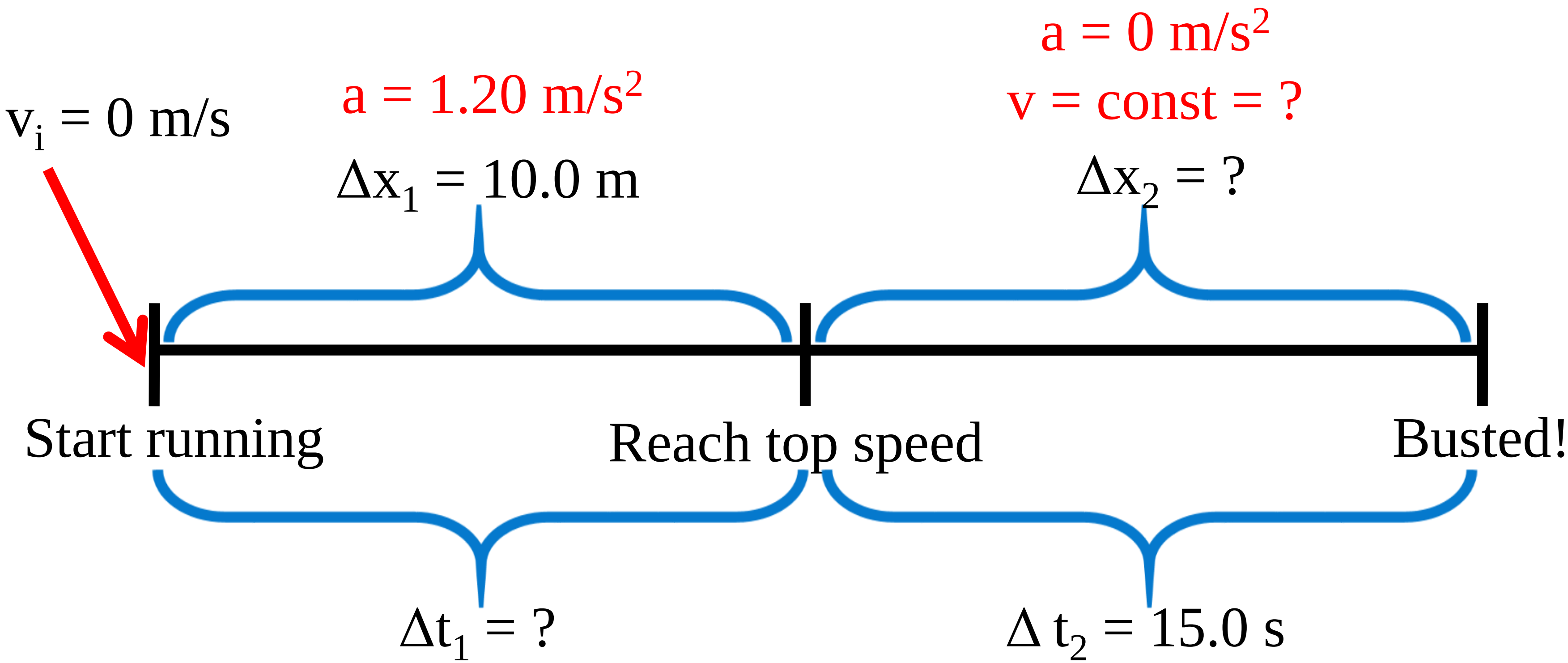
(b) How far from the bench did you get before being caught?

Example: 1D motion with two parts

Sitting beside an old lady in the park, you grab her purse and start running. Over the first 10.0 m, you accelerate at 1.20 m/s^2 up to your top running speed, and then continue to sprint at this speed for 15.0 s more before being tackled from behind by the old lady.

(a) How long did it take the old lady to catch you?

(b) How far from the bench did you get before being caught?



What do we know?

- **First Part**

$\Delta x_1 = 10.0$ m

$v_{1i} = 0$ m/s

$v_{1f} = ?$

$a_1 = 1.2$ m/s^2

$\Delta t_1 = ?$

- **Second Part**

$\Delta x_2 = ?$

$v_{2i} = ?$

$v_{2f} = ?$

$a_2 = 0$ m/s^2

$\Delta t_2 = 15.0$ s

same
(v constant)

Start Solving

- **First Part**

$$\Delta x_1 = v_{1i}\Delta t_1 + \frac{1}{2}a_1\Delta t_1{}^2$$

$$\Rightarrow \quad 10.0 = (0)\Delta t_1 + \tfrac{1}{2}(1.20)\Delta t_1^2$$

$$\Rightarrow \quad 10 = 0.6\Delta t_1^2$$

$$\Rightarrow \Delta t_1 = \sqrt{\frac{10}{0.6}} = 4.10\ s$$

$$v_{1f} = v_{1i} + a_1\Delta t_1$$

$$\Rightarrow v_{1f} = (0) + (1.2)(4.1) = 4.92\ m/s$$

- **Second Part**

$$\Delta x_2 = v_{2i}\Delta t_2 + \frac{1}{2}a_2\Delta t_2{}^2$$

$$\Rightarrow x_2 = (4.92)(15) + \tfrac{1}{2}(0)(15)^2$$

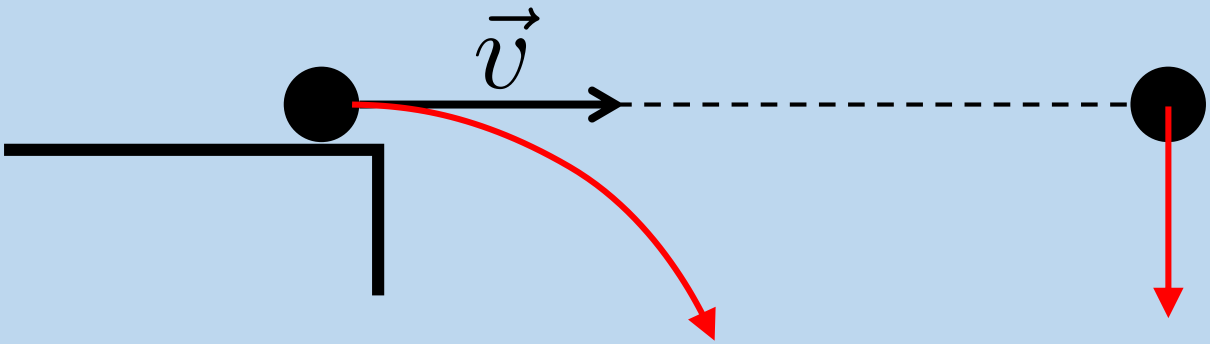
$$\Rightarrow x_2 = 73.8\ m$$

$$\Delta x_{TOT} = \Delta x_1 + \Delta x_2$$

$$\boxed{\Rightarrow x_t = 10.0 + 73.8 = 83.8\ m}$$

Concept Question

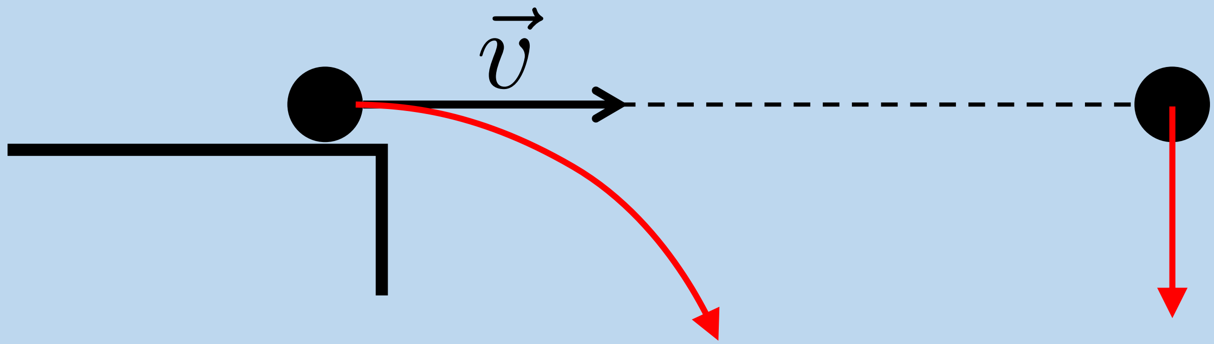
A ball is pushed off a table with some initial horizontal velocity, an other ball is released from rest from the same height at the same time. Which one hits the floor first?



A. The dropped ball
B. The pushed ball
C. They land at the same time

Concept Question

A ball is pushed off a table with some initial horizontal velocity, an other ball is released from rest from the same height at the same time. Which one hits the floor first?



A. The dropped ball
B. The pushed ball
C. They land at the same time

Components make life easy!

- Si
  se

With constant acceleration in the y-direction ($a_x = 0$) life gets even easier!

“projectile motion”

$$v_x = v_{xo} + \cancel{a_x} t$$

$$x - x_o = \frac{1}{2}(v_{xo} + v_x)t$$

$$\boxed{x - x_o = v_x t}$$

$$x - x_o = v_{xo} t + \frac{1}{2}\cancel{a_x} t^2$$

$$v_x^{\,2} = v_{xo}^{\,2} + 2\cancel{a_x}(x - x_o)$$

$$v_y = v_{yo} + a_y t$$

$$y - y_o = \frac{1}{2}(v_{yo} + v_y)t$$

$$y - y_o = v_{yo} t + \frac{1}{2} a_y t^2$$

$$v_y^{\,2} = v_{yo}^{\,2} + 2a_y(y - y_o)$$

Example 1

Taylor Swift jumps swiftly from a balcony. She jumps with a horizontal velocity of 12 m/s, and lands on the ground 22 m below.

a) How long is Taylor in the air?

b) How far from the balcony does she land?

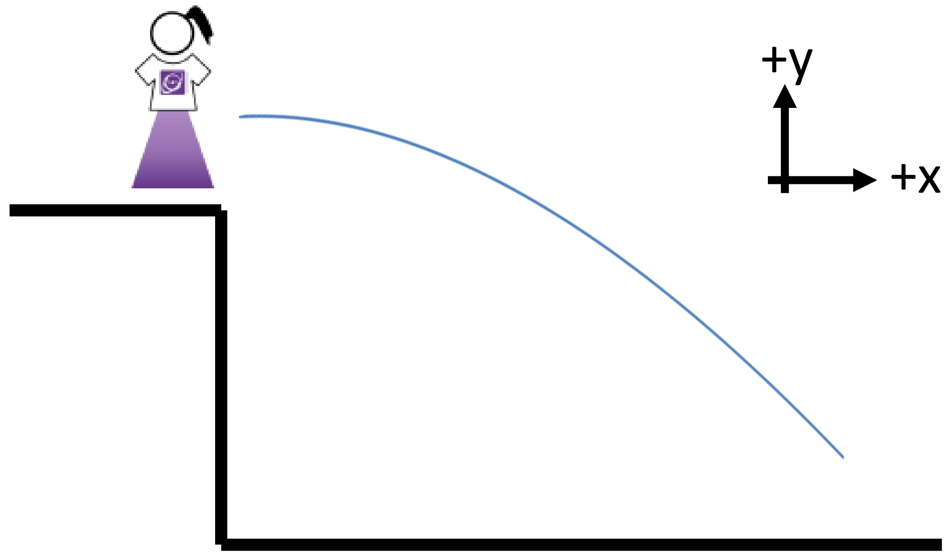
c) Does she get hurt?

x information

$$\Delta x = ?$$
$$v_x = +12$$
$$\Delta t = ?$$

y information

$$\Delta y = -22$$
$$v_{oy} = 0$$
$$v_y = ?$$
$$a = -9.8$$
$$\Delta t = ?$$



Part a):

$$\Delta y = v_{oy}\Delta t + \frac{1}{2}a\Delta t^2$$

$$-22 = 0 + \frac{1}{2}(-9.8)\Delta t^2$$

$$\frac{-22}{\frac{1}{2}(-9.8)} = \Delta t^2$$

$$\boxed{2.1\ s = \Delta t}$$

Part b):

$$\Delta x = v_x\Delta t$$

$$\Delta x = (+12)(2.1)$$

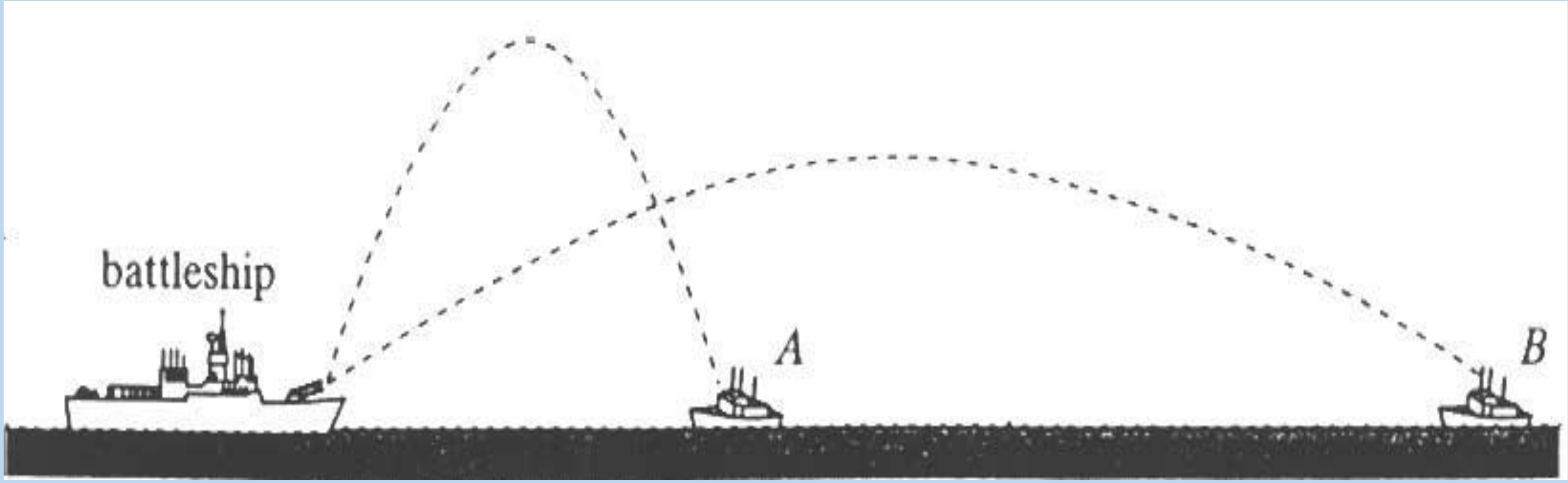
$$\boxed{\Delta x = 25\ m}$$

Part c):

I don’t know if she gets hurt – but if she does, I’m sure she will shake it off…

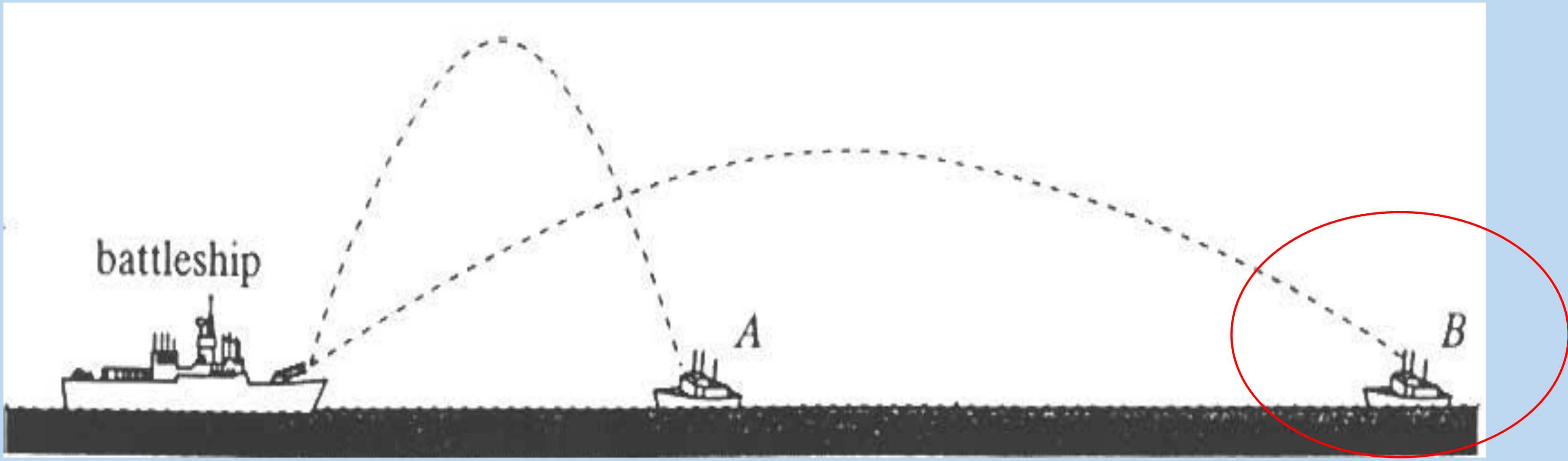
Concept Question

- A battleship simultaneously fires two hot dogs at neighbouring ships. If the hotdogs follow the parabolic trajectories shown, which ship gets their street-meat first?



Concept Question

- A battleship simultaneously fires two hot dogs at neighbouring ships. If the hotdogs follow the parabolic trajectories shown, which ship gets their street-meat first?



Example

You're playing a game of catch with your pet hamster. You throw your hamster to a friend who is standing 10.0 m away from you. The hamster's initial velocity is 11.0 m/s, directed at an angle of 39^o above horizontal. Assuming that the hamster leaves your hand at a height of 2.0 m above the ground, at what height does it reach your friend?

x information

$\Delta x = 10.0$

$v_x = 11\cos 39^o$

$= 8.55$

$\Delta t = ?$

y information

$\Delta y = ?$

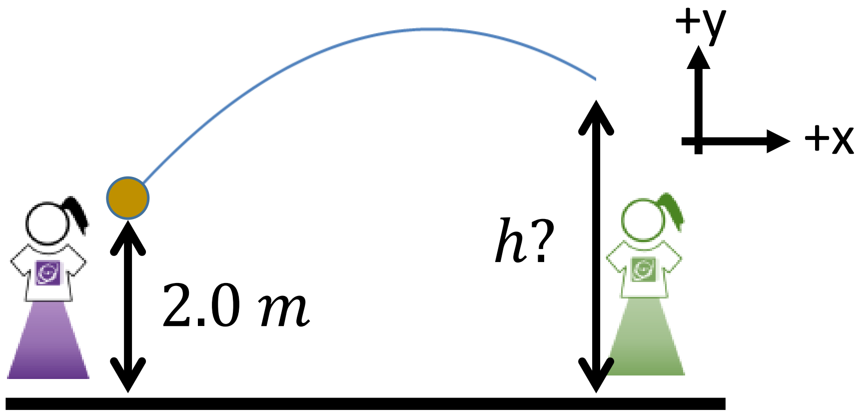
$v_{oy} = 11\sin 39^o$

$= 6.92$

$v_y = ?$

$a = -9.8$

$\Delta t = ?$



Step 1:

$$\Delta x = v_x \Delta t$$

$$10.0 = (8.55)\Delta t$$

$$\boxed{\Delta t = 1.17\ s}$$

Part b):

$$\Delta y = v_{oy}\Delta t + \frac{1}{2}a\Delta t^2$$

$$\Delta y = (6.92)(1.17) + \frac{1}{2}(-9.8)(1.17)^2$$

$$\boxed{\Delta y = 1.39\ m}$$

Step 3: Height above ground when reaches friend?

$$\boxed{h = 2.0 + 1.39 = 3.4\ m}$$