True & False statement at beginning of class:

$$\sum_{n=5}^{\infty}\left(\frac{2}{n}+\frac{6}{n+1}\right)=\sum_{n=5}^{\infty}\frac{2}{n}+\sum_{n=5}^{\infty}\frac{6}{n+1}$$

TRUE OR FALSE

because linearity properties of series only apply if each series converges BUT here, $\sum \frac{2}{n}$ diverges [harmonic series]

Note: $\sum_{n=5}^{\infty}\frac{6}{n+1}$ also a harmonic series

because $\sum_{n=5}^{\infty}\frac{6}{n+1} = \sum_{k=6}^{\infty}\frac{6}{k}$

$k=n+1$

Why is it true that

$$\sum_{n=1}^{\infty}\left(\frac{2}{n^2}+\frac{6}{n^2+1}\right)=2\sum_{n=1}^{\infty}\frac{1}{n^2}+6\sum_{n=1}^{\infty}\frac{1}{n^2+1}\ ?$$

Because now, $\sum_{n=1}^{\infty}\frac{1}{n^2}$ and $\sum_{n=1}^{\infty}\frac{1}{n^2+1}$ both converge

$\sum \frac{1}{n^2}$ converges by p-test

$p = 2 > 1$

$\sum \frac{1}{n^2+1}$ converges by Comparison test + p-test

because $0 \le \frac{1}{n^2+1} \le \frac{1}{n^2}$ $\Rightarrow$ Since $\sum \frac{1}{n^2}$ converges

so does $\sum \frac{1}{n^2+1}$!! ◡