COMPSCI 3MI3 : Assignment 8
Fall 2021
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1. **Proof of Sequencing as a Derived Form**
   In topic 9, slides 26-32, we discuss the sequencing operator ; in two ways, as a separate term of the language, and as a term derived from an inner language. In these slides, we stated the following theorem.

   THEOREM [Sequencing is a Derived Form]
   Define $\lambda^{\mathcal{E}}$ as the **external calculus**. This language will be composed of simply typed λ-Calculus, enriched with the Unit type and term, and with the term ;, E-Seq, E-SeqNext, and T-Seq.

   Define $\lambda^{\mathcal{I}}$ as the **internal calculus**. This language will be composed of the simply typed λ-Calculus and Unit type and term *only*.

   Define $e \in \lambda^{\mathcal{E}} \to \lambda^{\mathcal{I}}$ as an **elaboration function**, which translates from the external language to the internal language. It does so by replacing all instances of $t_1; t_2$ with $(\lambda x : Unit.t_2)\, t_1$. For each term t of $\lambda^{\mathcal{E}}$, we have:

$$t \xrightarrow{\mathcal{E}} t' \iff e(t) \xrightarrow{\mathcal{I}} e(t') \tag{1}$$

$$\Gamma \vdash^{\mathcal{E}} t : T \iff \Gamma \vdash^{\mathcal{I}} e(t) : T \tag{2}$$

   The proof of these statements proceeds by structural induction over t.

   Because these are "if and only if" statements, *both* directions must be proven independently.

   (a) (6 points) Prove $t \xrightarrow{\mathcal{E}} t' \implies e(t) \xrightarrow{\mathcal{I}} e(t')$

   (b) (6 points) Prove $e(t) \xrightarrow{\mathcal{I}} e(t') \implies t \xrightarrow{\mathcal{E}} t'$

   (c) (6 points) Prove $\Gamma \vdash^{\mathcal{E}} t : T \implies \Gamma \vdash^{\mathcal{I}} e(t) : T$

   (d) (6 points) Prove $\Gamma \vdash^{\mathcal{I}} e(t) : T \implies \Gamma \vdash^{\mathcal{E}} t : T$