Math 1LT3 — Assignment 10

Extra practice with equilibria, stability and phase-line diagrams

For each autonomous differential equation (1)-(9):
(a) Find all equilibrium solutions
(b) Draw a phase-line diagram; label each equilibrium as stable or unstable
(c) Determine the stability of each equilibrium point using the Stability Theorem, thus checking your claim about stability in (b).

(1) $\dfrac{dy}{dt} = 4$

(2) $\dfrac{dP(t)}{dt} = 2.4P(t)$

(3) $\dfrac{dP(t)}{dt} = -1.6P(t)$

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(4) $\dfrac{dT(t)}{dt} = 2(45 - T(t))$

(5) $\dfrac{dT(t)}{dt} = 1.7(T(t) - 25)$

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(6) $\dfrac{dP(t)}{dt} = 2.17P(t)\left(1 - \dfrac{P(t)}{60}\right)$

(7) $\dfrac{dP(t)}{dt} = 1.4P(t)\left(1 - \dfrac{25}{P(t)}\right)$

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(8) Assume that the bacterial population $a(t)$ grows according to $a' = 2.1a$, and the bacterial population $b(t)$ grows according to $b' = 1.8b$. Derive the differential equation for the fraction $p(t)$ of bacteria $a(t)$ in the total population (look at Example 7.1.6). Answer questions (a)-(c) from page 1 for $p(t)$.

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(9) Assume that the bacterial population $a(t)$ grows according to $a' = 2.1a$, and the bacterial population $b(t)$ grows according to $b' = 1.8b$. Derive the differential equation for the fraction $p(t)$ of bacteria $b(t)$ in the total population (look at Example 7.1.6). Answer questions (a)-(c) from page 1 for $p(t)$.

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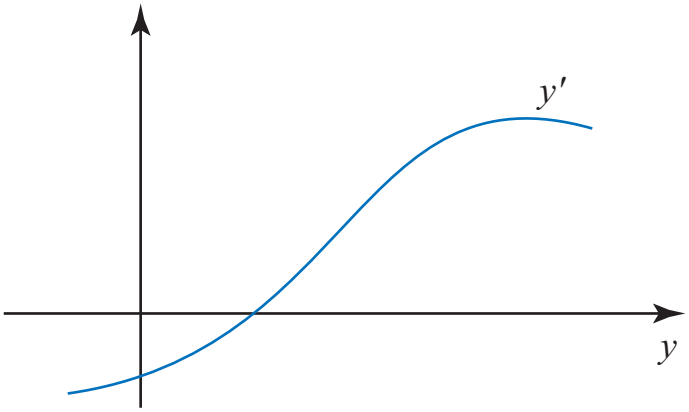
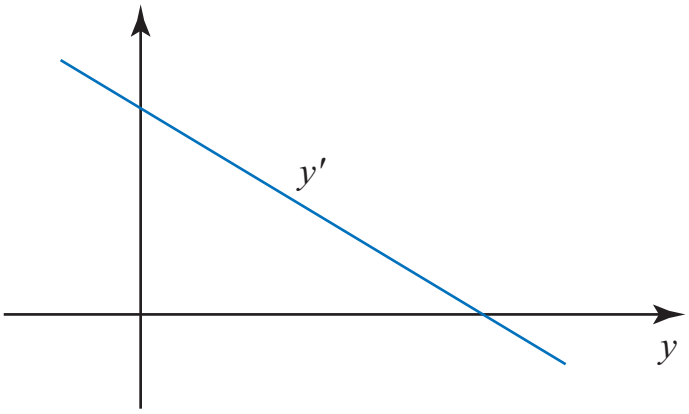
For each autonomous differential equation (10)-(14):
(a) Find all equilibrium solutions
(b) Draw a phase-line diagram; label each equilibrium as stable or unstable.

(10) The rate of change is given in the diagram below.



(11) The rate of change is given in the diagram below.



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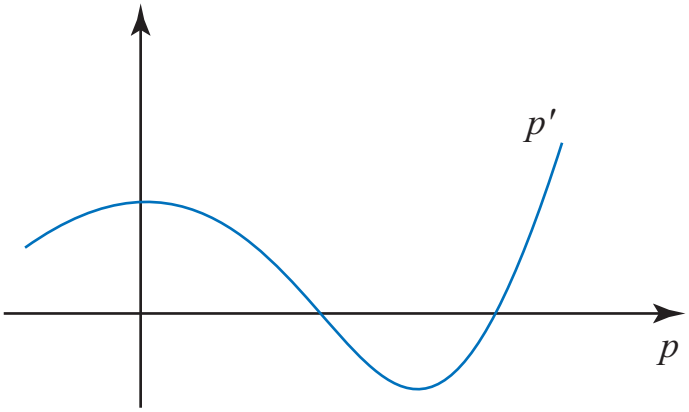
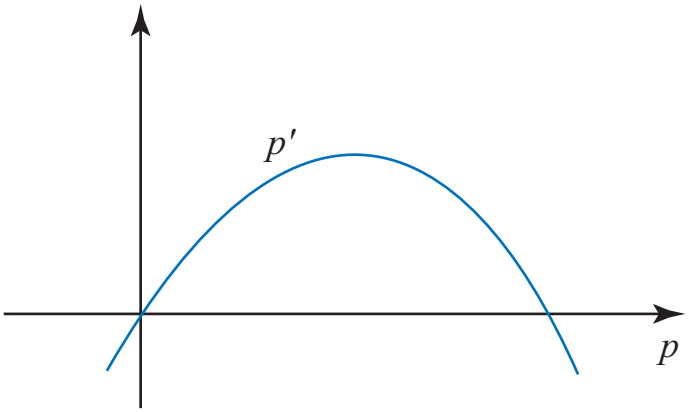
(12) The rate of change is given in the diagram below.



(13) The rate of change is given in the diagram below.



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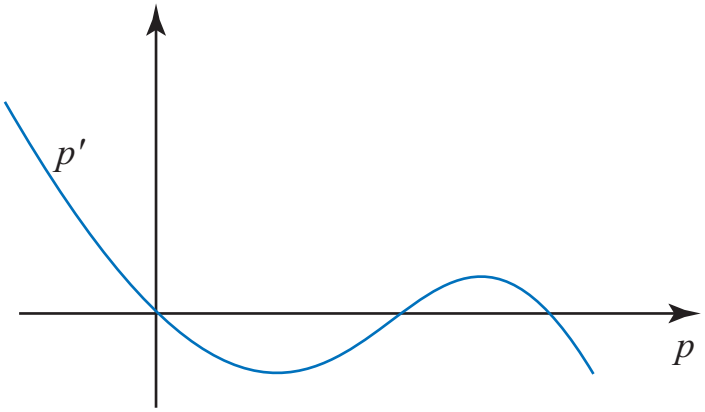
(14) The rate of change is given in the diagram below.



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