Extra l'Hôpital Examples

eg. $\lim\limits_{x \to 0^+} (\sin(x))^x = \text{"}0^0\text{"}$

$\Rightarrow$ let $y = \lim\limits_{x \to 0^+} (\sin x)^x$

$$\underline{\ln y} = \lim_{x \to 0^+} x \ln(\sin x) = \text{"}0 \cdot \ln(0)\text{"} = \text{"}0 \cdot (-\infty)\text{"}$$

$$= \lim_{x \to 0^+} \frac{\ln(\sin x)}{1/x} = \text{"}\frac{-\infty}{\infty}\text{"}$$

$$\overset{(H)}{=} \lim_{x \to 0^+} \frac{\frac{1}{\sin x} \cdot \cos(x)}{-1/x^2} = \lim_{x \to 0^+} -\frac{x^2 \cos(x)}{\sin x} = \text{"}\frac{0}{0}\text{"}$$

$$= \underbrace{\left(\lim_{x \to 0^+} \frac{x^2}{\sin x}\right)}_{\text{"}\frac{0}{0}\text{"}} \underbrace{\left(\lim_{x \to 0^+} \cos(x)\right)}_{1}$$

$$\overset{(H)}{=} \left(\lim_{x \to 0^+} \frac{2x}{\cos x}\right) \cdot 1 = \frac{2 \cdot 0}{1} \cdot 1 = \underline{0}$$

$$\ln y = 0 \Rightarrow y = e^0 = \boxed{1 = \lim_{x \to 0^+} (\sin x)^x}$$

eg. $\lim_{x \to 0^+} (\sin(x))^{1/x} = 0^{\left(\frac{1}{0^+}\right)} = 0^{\infty} = \underline{\underline{0}}$.