MATH 1AA3/1ZB3 Test #2, Seating #1 Full Solutions
Versions #1-4, Alternate & SAS #5
(*Questions sorted by course topic order*)

1. $\frac{1}{5+x^2} = \frac{1}{5}\frac{1}{1-\left(\frac{-x^2}{5}\right)} = \frac{1}{5}\frac{1}{1-y}$ where $y = \frac{-x^2}{5}$. And since $\frac{1}{1-y} = \sum_{n=0}^{\infty} y^n$, we get:

$$\frac{1}{5+x^2} = \frac{1}{5}\sum_{n=0}^{\infty} y^n = \frac{1}{5}\sum_{n=0}^{\infty}\left(\frac{-x^2}{5}\right) = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{5^{n+1}}$$

Answer: $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{5^{n+1}}$

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2. We know that $e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!}$, so: $\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}5^n = \sum_{n=0}^{\infty}\frac{(-5)^n}{n!} = e^{-5}$.

Answer: e^{-5}

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3. Given the function, $f(x) = \ln(4+x)$, we can examine the derivatives:

n	0	1	2	3	4
$f^{(n)}(x)$	$\ln(4+x)$	$1/(4+x)$	$(-1)/(4+x)^2$	$(-1)(-2)/(4+x)^3$	$(-1)(-2)(-3)/(4+x)^4$
$f^{(n)}(-1)$	$\ln(3)$	$1/3$	$-1/3^2$	$(-1)^2\, 2!/3^3$	$(-1)^3\, 3!/3^4$

So $f^{(n)}(-1) = (-1)^{n-1}(n-1)!/3^n$, for $n > 0$, and our series is:

$$\ln(4+x) = \sum_{n=0}^{\infty}\frac{f^{(n)}(-1)}{n!}(x-(-1))^n = \ln 3 + \sum_{n=1}^{\infty}\frac{(-1)^{n-1}(n-1)!}{n!3^n}(x+1)^n = \ln 3 + \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n3^n}(x+1)^n$$

Answer: $\ln(4+x) = \ln 3 + \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n3^n}(x+1)^n$

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4. We know that for Taylor series centred at $x = 2$, that: $f(x) = \sum_{n=0}^{\infty}\frac{f^{(n)}(2)}{n!}(x-2)^n = \sum_{n=0}^{\infty}a_n(x-2)^n$

So if $a_3 = 3$, then $a_3 = \frac{f^{(3)}(2)}{3!} = 3$, and $f^{(3)}(2) = 3(3!) = 3(6) = 18$

Answer: $f^{(3)}(2) = 18$

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5. Taylor polynomial, centred at $x = a$ has the form: $T_m(x) = \sum_{n=0}^{m} \frac{f^{(n)}(a)}{n!}(x-a)^n$

So for the function $f(x) = \sin(x)$ at $a = \pi/6$, we get:

$$T_2(x) = \sum_{n=0}^{2} \frac{f^{(n)}(a)}{n!}(x-a)^n = \frac{\sin(\pi/6)}{0!} + \frac{\cos(\pi/6)}{1!}(x-\pi/6) - \frac{\sin(\pi/6)}{2!}(x-\pi/6)^2$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2}(x-\pi/6) - \frac{1}{4}(x-\pi/6)^2$$

Answer: $\frac{1}{2} + \frac{\sqrt{3}}{2}(x-\pi/6) - \frac{1}{4}(x-\pi/6)^2$

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6. $\binom{1/3}{4} = \frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)\left(\frac{1}{3}-3\right)}{4!} = \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)\left(-\frac{8}{3}\right)}{4!} = \frac{(-1)^3(2\cdot5\cdot8)}{3^4(1\cdot2\cdot3\cdot4)} = -\frac{10}{3^5}$

Answer: $-\frac{10}{3^5}$

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7. For our Taylor error: $|f(x)-T_n(x)| \le M\frac{|x-a|^{n+1}}{(n+1)!}$, where $M \ge \max|f^{(n+1)}(x)|$ on our given interval.

Here, we are given that $n = 4$, $a = 3$ and $f(x) = e^x$, and we are approximating the function on the interval [1,5]. So $|x-a| = |x-3| \le 2$, and $M \ge \max|f^{(n+1)}(x)| = \max(e^x) = e^5$ for $x \in [1,5]$, and we get: $|f(x)-T_n(x)| \le M\frac{|x-a|^{n+1}}{(n+1)!} = e^5\left(\frac{2^5}{5!}\right) = \frac{2^5}{120}e^5 = \frac{\cancel{2}\cdot\cancel{4}\cdot4}{1\cdot\cancel{2}\cdot3\cdot\cancel{4}\cdot5}e^5 = \frac{4}{15}e^5$

Answer: $\frac{4}{15}e^5$

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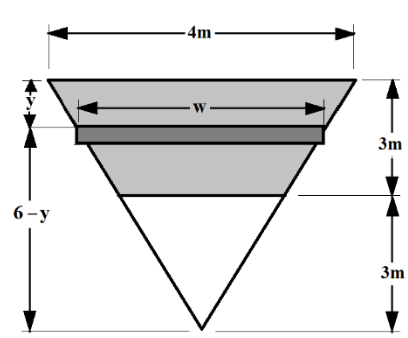
8. Any horizontal slice across the submerged part of our triangle is 4m wide at the surface, and by similar triangles, width given by w satisfies the equation:

$w/(6-y)=4/6$. Or $w = 4 - 2y/3$

So if y is the depth of water, then each thin slice at depth y has a force acting upon it of:

$F_{slice} = (\rho g y)w\Delta y = \rho g(4y - 2y^2/3)\Delta y$



And the total force is:

$$F_{net} = \lim_{n\to\infty}\sum_{i=1}^{n}\rho g(4y_i - 2y_i^2/3)\Delta y = \int_0^3 \rho g(4y - 2y^2/3)dy = \rho g(2y^2 - 2y^3/9)\Big|_0^3 = (18-6)\cdot 10^4$$

$$= 120\text{kN}$$

Answer: 120kN

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9. If $f(x) = x^{1/3}$, $0 \le x \le 1$ is rotated about the y axis:

$$\text{Surface Area} = \int_{Curve} 2\pi r\, ds = \int_0^1 2\pi x\sqrt{1+\left(\frac{d}{dx}x^{1/3}\right)^2}\, dx = \int_0^1 2\pi x\sqrt{1+\frac{1}{9}x^{-4/3}}\, dx$$

$$= \int_0^1 2\pi x\frac{x^{-2/3}}{3}\sqrt{9x^{4/3}+1}\, dx = \int_0^1 \frac{2}{3}\pi x^{1/3}\sqrt{9x^{4/3}+1}\, dx$$

Now, let $u = 9x^{4/3}+1$, $du = 12x^{1/3}\, dx$ and we get:

$$S.A. = \int_1^{10}\frac{2}{3}\cdot\frac{1}{12}\pi u^{1/2}\, du = \frac{\pi}{18}\cdot\frac{2}{3}u^{3/2}\Big|_1^{10} = \frac{\pi}{27}\left(10^{3/2}-1\right)$$

Answer: $S.A. = \dfrac{\pi}{27}\left(10^{3/2}-1\right)$

Equivalently, $f(x) = x^{1/3}$, $0 \le x \le 1$ is rotated about the y axis can be re-expressed as a function of y, and we get: $g(y) = y^3$, $0 \le y \le 1$, rotated about the y axis.

$$\text{Surface Area} = \int_{Curve} 2\pi r\, ds = \int_0^1 2\pi y^3\sqrt{1+\left(\frac{d}{dy}y^3\right)^2}\, dy = \int_0^1 2\pi y^3\sqrt{1+9y^4}\, dy$$

Now, let $u = 9y^4+1$, $du = 36y^3\, dy$ and we get:

$$\text{S.A.} = \int_1^{10}\frac{1}{18}\pi u^{1/2}\, d = \frac{1}{18}\cdot\frac{2}{3}\pi u^{3/2}\Big|_1^{10} = \frac{\pi}{27}\left(10^{3/2}-1\right)$$

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10. If e^{kx} satisfies the differential equation $3y'' + 2y' - y = 0$, then plugging in e^{kx} as y we get:

$3k^2e^{kx} + 2ke^{kx} - e^{kx} = (3k^2 + 2k - 1)e^{kx} = 0$, so $(3k^2 + 2k - 1) = (3k-1)(k+1) = 0$

Answer: $k = -1$ or $1/3$

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11. Given: $y' = \dfrac{(4-y^2)^9}{(x^2+1)^7}$, we can check each of the listed statements:

Passing through the y-axis means that $x = 0$, so $y' = \frac{(4-y^2)^9}{(0+1)^7} = (4-y^2)^9$ which can be positive or negative, depending on y.

Passing through (–1, –2) means $y' = \frac{(4-2^2)^9}{(1^2+1)^7} = \frac{0}{2^7} = 0$ So the tangent line must be horizontal.

Having the value of y between – 2 and 2 means $y^2 < 2$ so $4 - y^2 > 0$. Thus $y' = \frac{(4-y^2)^9}{(x^2+1)^7} > 0$.
So the graph must be increasing.

Answer: "e) None of the above" (ie. all of the listed statements a) through d) are false.)

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12. $y' = 2x^3 e^{-y}$ is a separable differential equation, so let's separate:

$e^y y' = 2x^3$ so $\int e^y \frac{dy}{dx} dx = \int e^y \, dy = \int 2x^3 \, dx$ and $e^y = \frac{1}{2}x^4 + C$.

And since we have the condition $(x,y) = (1,\ln(3))$, then $e^{\ln 3} = 3 = \frac{1}{2}1^4 + C$, so $C = 3 - \frac{1}{2} = \frac{5}{2}$

And $e^y = \frac{1}{2}x^4 + \frac{5}{2}$ so $y = \ln\left(\frac{1}{2}x^4 + \frac{5}{2}\right) = \ln(5/2)$ when $x = 0$

Answer: $\ln(5/2)$

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13. $y^2 = \frac{k}{x^3}$ means $2yy' = -3\frac{k}{x^4}$ and $\frac{y^2}{x} = \frac{k}{x^4}$ so $2yy' = -3\frac{y^2}{x}$ and $y' = -\frac{3y}{2x}$

Now for the orthogonal family, the slopes of the tangents are the negative reciprocals, so:

$y' = \frac{2x}{3y}$, and $3\int y\,dy = 2\int x\,dx$ so $\frac{3}{2}y^2 = x^2 + D$ and $3y^2 - 2x^2 = C$

Answer: $3y^2 - 2x^2 = C$

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14. Since our sample decays exponentially, if $f(t)$ represents the current mass in mg of the radioactive component of the ink, then $f(t) = Ae^{kt}$ for some $k < 0$, and t representing time in years.

We are told that $f(0) = 15$, and $f(17) = 7.5$ (ie. our half-life is 17 years) , so $A = 15$, and $7.5 = 15e^{17k}$ so $k = \frac{1}{17}\ln(1/2)$ (i.e. $k = -\ln(2) / \lambda$).

So to get the amount at t = 17+8 = 25 years, we compute :

$$f(25) = 15e^{\left(-\frac{1}{17}\ln(2)\right)25} = 15e^{-\frac{25}{17}\ln(2)} = 15\left(2^{-25/17}\right) = \frac{15}{2^{25/17}}.$$

Answer: $\frac{15}{2^{25/17}}$

Equivalently, since for half-life problems, $f(t) = f(0)\left(\frac{1}{2}\right)^{t/\lambda}$ then since we start with 15mg, and the half-life is 17 years: $f(25) = \frac{f(0)}{2^{25/\lambda}} = \frac{15}{2^{25/17}}$

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15. We can put our differential equation into standard $y'' + P(x)y = Q(x)$ form for linear differential equations by dividing through by the coefficient of y''.

So $\frac{2}{\cos(x)}y' + \frac{1}{x\cos(x)}y = 1$ becomes $y' + \frac{1}{2x}y = \frac{1}{2}\cos(x)$, and we get $P(x) = \frac{1}{2}\cdot\frac{1}{x}$.

Then the standard form for the integration factor is $I(x) = e^{\int P(x)\,dx} = e^{\int \frac{1}{2}\cdot\frac{1}{x}dx} = e^{\frac{1}{2}\ln(x)} = e^{\ln(\sqrt{x})} = \sqrt{x}$

Answer: $I(x) = \sqrt{x}$

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16. Given $x(t) = t^5 + 1$, then $t = (x-1)^{1/5}$, so $y = e^t = e^{(x-1)^{1/5}}$.

Answer: $y = e^{(x-1)^{1/5}}$

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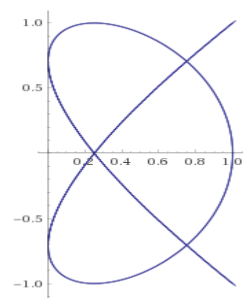
17. If $(x,y) = (\cos^2(2t),\sin(3t))$, we know that $x \geq 0$ for all t, and at $t = 0$, the curve passes through the point $(x,y) = (\cos^2(0),\sin(0)) = (1,0)$.
Of the given graph options in the question, only the graph to the right satisfies both properties.



Answer: The graph to the right with $x \geq 0$ and $(x,y) = (1,0)$ at $t = 0$.

MATH 1AA3/1ZB3 Test #2, Seating #2 Full Solutions

Versions #1(6) - 4(9)

(*Questions sorted by course topic order*)

1. $\dfrac{1}{2+x^3} = \dfrac{1}{2}\dfrac{1}{1-\left(\frac{-x^3}{2}\right)} = \dfrac{1}{2}\dfrac{1}{1-y}$ where $y = \dfrac{-x^3}{2}$. And since $\dfrac{1}{1-y} = \sum_{n=0}^{\infty} y^n$, we get:

$$\frac{1}{2+x^3} = \frac{1}{2}\sum_{n=0}^{\infty} y^n = \frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{-x^3}{2}\right) = \sum_{n=0}^{\infty}\frac{(-1)^n x^{3n}}{2^{n+1}}$$

Answer: $\sum_{n=0}^{\infty}\dfrac{(-1)^n x^{3n}}{2^{n+1}}$

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2. We know that $e^x = \sum_{n=0}^{\infty}\dfrac{x^n}{n!}$, so: $\sum_{n=0}^{\infty}\dfrac{(-1)^n}{3^n n!} = \sum_{n=0}^{\infty}\dfrac{(-1/3)^n}{n!} = e^{-1/3}$.

Answer: $e^{-1/3}$

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3. Given the function, $f(x) = \ln(2 + x)$, we can examine the derivatives:

n	0	1	2	3	4
$f^{(n)}(x)$	$\ln(2+x)$	$1/(2+x)$	$(-1)/(2+x)^2$	$(-1)(-2)/(2+x)^3$	$(-1)(-2)(-3)/(2+x)^4$
$f^{(n)}(1)$	$\ln(3)$	$1/3$	$-1/3^2$	$(-1)^2\, 2!/3^3$	$(-1)^3\, 3!/3^4$

So $f^{(n)}(1) = (-1)^{n-1}(n-1)!/3^n$, for $n > 0$, and our series is:

$$\ln(2+x) = \sum_{n=0}^{\infty}\frac{f^{(n)}(1)}{n!}(x-1)^n = \ln 3 + \sum_{n=1}^{\infty}\frac{(-1)^{n-1}(n-1)!}{n!3^n}(x-1)^n = \ln 3 + \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n3^n}(x-1)^n$$

Answer: $\ln(2+x) = \ln 3 + \sum_{n=1}^{\infty}\dfrac{(-1)^{n-1}}{n3^n}(x-1)^n$

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4. We know that for Taylor series centred at $x = 7$, that: $f(x) = \sum_{n=0}^{\infty}\dfrac{f^{(n)}(7)}{n!}(x-7)^n = \sum_{n=0}^{\infty} b_n (x-7)^n$

So if $b_4 = 2$, then $b_4 = \dfrac{f^{(4)}(7)}{4!} = 2$, and $f^{(4)}(7) = 2(4!) = 2(24) = 48$

Answer: $f^{(4)}(7) = 48$

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5. Taylor polynomial, centred at $x = a$ has the form: $T_m(x) = \sum_{n=0}^{m} \frac{f^{(n)}(a)}{n!}(x-a)^n$

So for the function $f(x) = \cos(x)$ at $a = \pi/3$, we get:

$$T_2(x) = \sum_{n=0}^{2} \frac{f^{(n)}(a)}{n!}(x-a)^n = \frac{\cos(\pi/3)}{0!} - \frac{\sin(\pi/3)}{1!}(x-\pi/3) - \frac{\cos(\pi/3)}{2!}(x-\pi/3)^2$$

$$= \frac{1}{2} - \frac{\sqrt{3}}{2}(x-\pi/3) - \frac{1}{4}(x-\pi/3)^2$$

Answer: $\frac{1}{2} + \frac{\sqrt{3}}{2}(x-\pi/3) - \frac{1}{4}(x-\pi/3)^2$

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6. $\binom{1/4}{3} = \frac{\left(\frac{1}{4}\right)\left(\frac{1}{4}-1\right)\left(\frac{1}{4}-2\right)}{3!} = \frac{\left(\frac{1}{4}\right)\left(-\frac{3}{4}\right)\left(-\frac{7}{4}\right)}{3!} = \frac{(-1)^2\left(1\cdot\cancel{3}\cdot 7\right)}{4^3\left(1\cdot 2\cdot\cancel{3}\right)} = \frac{7}{2^7}$

Answer: $\frac{7}{2^7}$

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7. For our Taylor error: $|f(x)-T_n(x)| \le M\frac{|x-a|^{n+1}}{(n+1)!}$, where $M \ge \max|f^{(n+1)}(x)|$ on our given interval.

Here, we are given that $n = 3$, $a = 4$ and $f(x) = e^x$, and we are approximating the function on the interval [2,6]. So $|x-a| = |x-4| \le 2$, and $M \ge \max|f^{(n+1)}(x)| = \max(e^x) = e^6$ for $x \in [2,6]$, and we get: $|f(x)-T_n(x)| \le M\frac{|x-a|^{n+1}}{(n+1)!} = e^6\left(\frac{2^4}{4!}\right) = \frac{2^4}{24}e^6 = \frac{\cancel{2}\cdot\cancel{4}\cdot 2}{1\cdot\cancel{2}\cdot 3\cdot\cancel{4}}e^6 = \frac{2}{3}e^6$

Answer: $\frac{2}{3}e^6$

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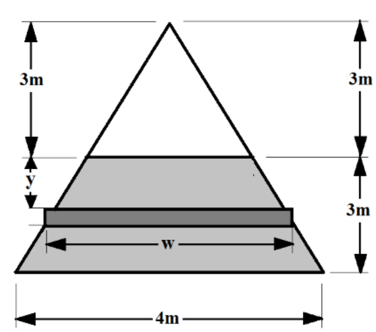
8. Any horizontal slice across the submerged part of our triangle is 4m wide at the surface, and by similar triangles, width given by w satisfies the equation:

$w/(3 + y) = 4/6$. Or $w = 2 + 2y/3$

So if y is the depth of water, then each thin slice at depth y has a force acting upon it of:

$F_{slice} = (\rho g y)w\Delta y = \rho g(2y + 2y^2/3)\Delta y$

And the total force is:



$$F_{net} = \lim_{n\to\infty}\sum_{i=1}^{n}\rho g(2y_i + 2y_i^2/3)\Delta y = \int_0^3 \rho g(2y + 2y^2/3)dy = \rho g(y^2 + 2y^3/9)\Big|_0^3 = (9+6)\cdot 10^4$$

$= 150\text{kN}$

Answer: 150kN

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9. If $f(x) = x^{1/3}$, $0 \le x \le 1$ is rotated about the y axis:

$$\text{Surface Area} = \int_{Curve} 2\pi r\, ds = \int_0^1 2\pi x\sqrt{1+\left(\frac{d}{dx}x^{1/3}\right)^2}\, dx = \int_0^1 2\pi x\sqrt{1+\frac{1}{9}x^{-4/3}}\, dx$$

$$= \int_0^1 2\pi x\frac{x^{-2/3}}{3}\sqrt{9x^{4/3}+1}\, dx = \int_0^1 \frac{2}{3}\pi x^{1/3}\sqrt{9x^{4/3}+1}\, dx$$

Now, let $u = 9x^{4/3}+1$, $du = 12x^{1/3}\, dx$ and we get:

$$S.A. = \int_1^{10}\frac{2}{3}\cdot\frac{1}{12}\pi u^{1/2}\, du = \frac{\pi}{18}\cdot\frac{2}{3}u^{3/2}\Big|_1^{10} = \frac{\pi}{27}\left(10^{3/2}-1\right)$$

Answer: $S.A. = \frac{\pi}{27}\left(10^{3/2}-1\right)$

Equivalently, $f(x) = x^{1/3}$, $0 \le x \le 1$ is rotated about the y axis can be re-expressed as a function of y, and we get: $g(y) = y^3$, $0 \le y \le 1$, rotated about the y axis.

$$\text{Surface Area} = \int_{Curve} 2\pi r\, ds = \int_0^1 2\pi y^3\sqrt{1+\left(\frac{d}{dy}y^3\right)^2}\, dy = \int_0^1 2\pi y^3\sqrt{1+9y^4}\, dy$$

Now, let $u = 9y^4+1$, $du = 36y^3\, dy$ and we get:

$$\text{S.A.} = \int_1^{10}\frac{1}{18}\pi u^{1/2}\, d = \frac{1}{18}\cdot\frac{2}{3}\pi u^{3/2}\Big|_1^{10} = \frac{\pi}{27}\left(10^{3/2}-1\right)$$

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10. If e^{kx} satisfies the differential equation $3y'' - 5y' - 2y = 0$, then plugging in e^{kx} as y we get:

$3k^2e^{kx} - 5ke^{kx} - 2e^{kx} = (3k^2 - 5k - 2)e^{kx} = 0$, so $(3k^2 - 5k - 2) = (3k+1)(k-2) = 0$

Answer: $k = -1/3$ or 2

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11. Given: $y' = \dfrac{(4-x^2)^5}{(y^2+1)^7}$, we can check each of the listed statements:

Passing through the x-axis means that $y = 0$, so $y' = \dfrac{(4-x^2)^5}{(0+1)^7} = (4-x^2)^5$ which can be positive or negative, depending on x.

Passing through (–2, –2) means $y' = \dfrac{(4-2^2)^5}{(2^2+1)^7} = \dfrac{0}{5^7} = 0$ So the tangent line must be horizontal.

Having the value of x between – 2 and 2 means $x^2 < 2$ so $4 - x^2 > 0$. Thus $y' = \dfrac{(4-x^2)^5}{(y^2+1)^7} > 0$.
So the graph must be increasing.

Answer: "e) None of the above" (ie. all of the listed statements a) through d) are false.)

---

12. $y' = x^4 e^{-y}$ is a separable differential equation, so let's separate:

$e^y y' = x^4$ so $\int e^y \dfrac{dy}{dx} dx = \int e^y\, dy = \int x^4\, dx$ and $e^y = \dfrac{1}{5}x^5 + C$.

And since we have the condition $(x,y) = (1,\ln(3))$, then $e^{\ln 3} = 3 = \dfrac{1}{5}1^5 + C$, so $C = 3 - \dfrac{1}{5} = \dfrac{14}{5}$

And $e^y = \dfrac{1}{5}x^5 + \dfrac{14}{5}$ so $y = \ln\left(\dfrac{1}{5}x^5 + \dfrac{14}{5}\right) = \ln(14/5)$ when $x = 0$

Answer: $\ln(14/5)$

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13. $y^3 = \dfrac{k}{x}$ means $3y^2 y' = -\dfrac{k}{x^2}$ and $\dfrac{y^3}{x} = \dfrac{k}{x^2}$ so $3y^2 y' = -\dfrac{y^3}{x}$ and $y' = -\dfrac{y}{3x}$

Now for the orthogonal family, the slopes of the tangents are the negative reciprocals, so:

$y' = \dfrac{3x}{y}$, and $\int y\, dy = 3\int x\, dx$ so $\dfrac{1}{2}y^2 = \dfrac{3}{2}x^2 + D$ and $y^2 - 3x^2 = C$

Answer: $y^2 - 3x^2 = C$

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14. Since our sample decays exponentially, if $f(t)$ represents the current mass in g of the radioactive component of the dye, then $f(t) = Ae^{kt}$ for some $k < 0$, and t representing time in minutes.

We are told that $f(0) = 20$, and $f(7) = 10$ (ie. our half-life is 7 minutes) , so $A = 20$, and $10 = 20e^{7k}$ so $k = \dfrac{1}{7}\ln(1/2)$ (i.e. $k = -\ln(2) / \lambda$).

So to get the amount at t = 7+23 = 30 minutes, we compute :

$$f(30) = 20e^{(-\frac{1}{7}\ln(2))30} = 20e^{-\frac{30}{7}\ln(2)} = 20\left(2^{-30/7}\right) = \frac{20}{2^{30/7}}.$$

Answer: $\frac{20}{2^{30/7}}$

Equivalently, since for half-life problems, $f(t) = f(0)\left(\frac{1}{2}\right)^{t/\lambda}$ then since we start with 20g, and the half-life is 7 minutes: $f(25) = \frac{f(0)}{2^{30/\lambda}} = \frac{20}{2^{30/7}}$

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15. We can put our differential equation into standard $y'' + P(x)y = Q(x)$ form for linear differential equations by dividing through by the coefficient of y''.

So $\frac{1}{2\sin(x)}y' + \frac{1}{x\sin(x)}y = 1$ becomes $y' + \frac{2}{x}y = 2\sin(x)$, and we get $P(x) = 2\cdot\frac{1}{x}$.

Then the standard form for the integration factor is $I(x) = e^{\int P(x)\,dx} = e^{\int 2\cdot\frac{1}{x}dx} = e^{2\ln(x)} = e^{\ln(x^2)} = x^2$

Answer: $I(x) = x^2$

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16. Given $x(t) = e^t$ then $t = \ln(x)$, so $y = t^5 + 1 = (\ln(x))^5 + 1$.

Answer: $y = (\ln(x))^5 + 1$

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17. If $(x,y) = (\sin^2(2t),\cos(3t))$, we know that $x \geq 0$ for all t, and at $t = 0$, the curve passes through the point $(x,y) = (\sin^2(0),\cos(0)) = (0,1)$.
Of the given graph options in the question, only the graph to the right satisfies both properties.

Answer: The graph to the right with $x \geq 0$ and $(x,y) = (0,1)$ at $t = 0$.