

Data Structures and Algorithms – (COMP SCI 2C03)  
Winter 2021  
Tutorial-II

January Feb 1, 2021

- Question 1 Consider the Weighted Quick Union-Find approach mentioned in Section 1.5 of the textbook. Prove that the depth of any node  $x$  is at most  $\log_2 N$  in the trees constructed using this approach.
- Question 2 Prove the basic combinatorial property of binary trees that a tree of height  $h$  has no more than  $2^h$  leaves.
- Question 3 If a full binary tree has  $N$  leaves. Then how many internal nodes does the tree have.
- Question 4 Prove the recurrence  $T(n) = T(n - 1) + T(0) + cn$  is in  $O((n^2))$ . Since the recursive call on an array of size 0 just returns,  $T(0) = \Theta(1)$ .
- Question 5 Give traces, in the style of the trace given with Algorithm 2.3, showing how the sequence  $\langle 5, 21, 3, 4, 8, 0, 1, 12, 15, 8, 10, 17, 3 \rangle$  is sorted with Shellsort.
- Question 6 Give traces, in the style of the trace given with Algorithm 2.3, showing how the sequence  $\langle 5, 21, 3, 4, 8, 0, 1, 12, 15, 8, 10, 17, 3 \rangle$  is sorted with bottom-up approach of Mergesort.
- Question 7 How does Mergesort fare when there are duplicate values in the array?