MATHEMATICS 1LT3 TEST 2

Day Class
Duration of Test: 60 minutes
McMaster University

E. Clements

27 February 2013

FIRST NAME (please print): ______________________

FAMILY NAME (please print): ______________________

Student No.: ______________________

THIS TEST HAS 8 PAGES AND 9 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE.

Total number of points is 40. Marks are indicated next to the problem number in square brackets. Any Casio fx991 (or lower, non-graphing) calculator is allowed.

USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL, YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

You need to show work to receive full credit, except for question 3 (matching).

Page	Points	Mark
2	5	
3	7	
4	7	
5	8	
6	8	
7	5	
TOTAL	40	

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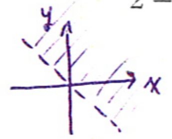
1. (a) [3] Find and sketch the domain of $f(x,y) = \dfrac{\ln(x+y)}{2-|x|}$.

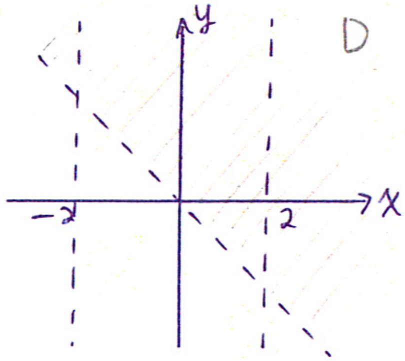
① $x+y>0 \Rightarrow y>-x$



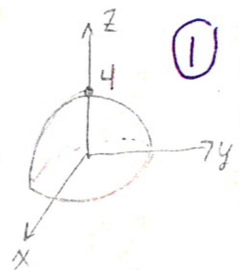
AND

② $2-|x| \neq 0 \Rightarrow |x| \neq 2 \Rightarrow x \neq \pm 2$

SO $D = \{(x,y) \in \mathbb{R}^2 \mid y>-x \text{ and } x \neq \pm 2\}$



(b) [2] Determine the range of $g(x,y) = \sqrt{16-x^2-y^2}$. (Note: You do not have to provide a formal proof like we did in class, just explain your reasoning.)



① graph is top half of a sphere centred at $(0,0,0)$ with radius 4 so range is $0 \le z \le 4$.

or

②

① $g(x,y) \geq 0$ since $+\sqrt{}$

AND

② $x^2+y^2 \geq 0$

$0 \geq -x^2-y^2$

$16 \geq 16-x^2-y^2$

$\sqrt{16} \geq \sqrt{16-x^2-y^2}$

$4 \geq g(x,y)$

① + ② $\Rightarrow$ $0 \le g(x,y) \le 4$

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2. [3] Consider the function $h(x,y) = ye^x$. Create a contour map for h including level curves for $k = -2, -1, 0, 1, 2$.

$ye^x = k \Rightarrow y = ke^{-x}$

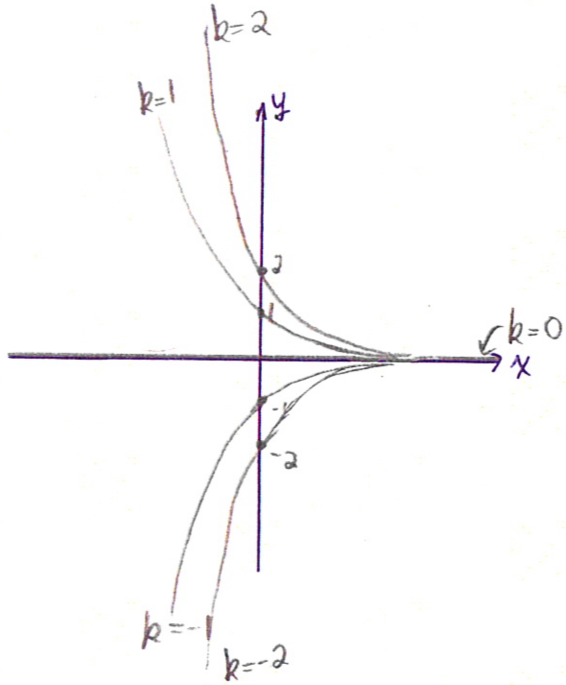
$k=-2 \Rightarrow y = -2e^{-x}$

$k=-1 \Rightarrow y = -e^{-x}$

$k=0 \Rightarrow y = 0$

$k=1 \Rightarrow y = e^{-x}$

$k=2 \Rightarrow y = 2e^{-x}$



3. [4] Match the equation of each function with its graph (labeled I-IV).

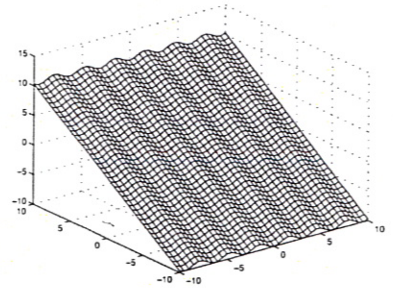
(a) $z = x - y^2 + 4xy^2$ IV

(b) $z = \sin^2 x + y$ I

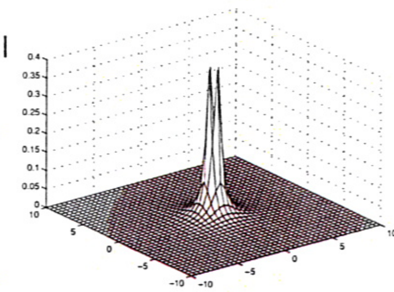
(c) $z = \dfrac{1}{x^2+y^2}$ II

(d) $z = \sqrt{xy}$ III

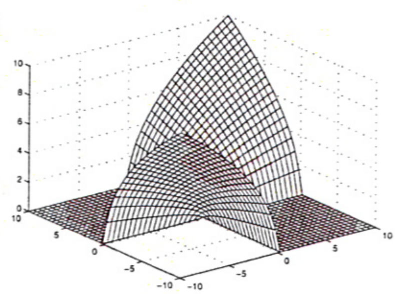
I



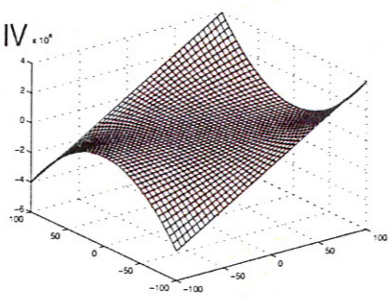
II



III



IV



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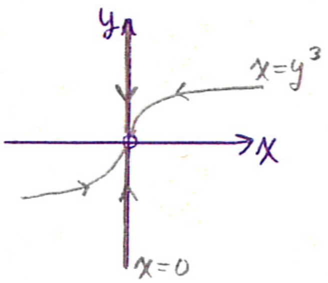
4. (a) [2] Explain why we cannot show that $\lim_{(x,y)\to(0,0)} f(x,y)$ exists by computing limits along various paths to $(0,0)$.

We would have to check along ALL paths to $(0,0)$ and there are infinitely many.

(b) [3] Show that $\lim_{(x,y)\to(0,0)} \dfrac{2xy^3}{3x^2+y^6}$ does not exist. Sketch the domain of the function and the paths involved.



Approach $(0,0)$ along $x=0$:

$f(0,y) = \frac{0}{y^6} = 0$

$f(x,y) \to 0$ as $(x,y)\to(0,0)$ along $x=0$

Approach $(0,0)$ along $x=y^3$:

$f(y^3,y) = \frac{2y^3y^3}{2y^6+y^6} = \frac{2}{3}$

$f(x,y) \to \frac{2}{3}$ as $(x,y)\to(0,0)$ along $x=y^3$

$\therefore \lim_{(x,y)\to(0,0)} f(x,y)$ D.N.E.

5. [2] Use the definition of continuity to determine whether or not

$$f(x,y) = \begin{cases} 2xye^{x-y^2} & \text{if } (x,y)\neq(1,1) \\ 2 & \text{if } (x,y)=(1,1) \end{cases}$$

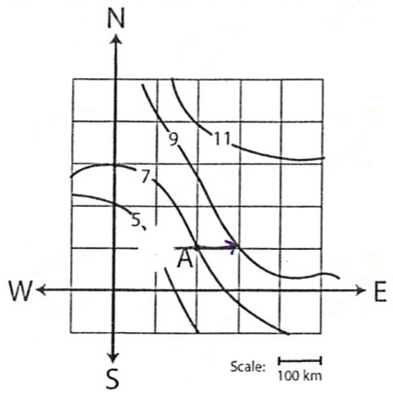
is continuous at $(1,1)$.

① $\lim_{(x,y)\to(1,1)} f(x,y) = \lim_{(x,y)\to(1,1)} 2xye^{x-y^2} = 2e^0 = 2$

② $f(1,1) = 2$

③ $\because \lim_{(x,y)\to(1,1)} f(x,y) = f(1,1) \therefore f$ is continuous at $(1,1)$

6. (a) [2] Given the contour map for the temperature function, T, in degrees Celsius, for a certain area, approximate the rate of change in temperature at the point A as we travel east. Remember to include units!



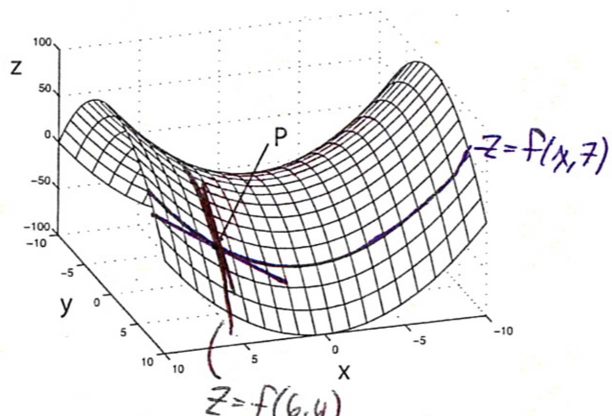
avg rate of change $= \frac{9-7}{100} = 0.02 \frac{°C}{km}$

At the point A, the temperature is increasing at a rate of about $0.02°C$ per km as we move east.

(b) [2] In which direction, north or east, is the rate of change in temperature the greatest at A? Explain.

In both directions, the temperature is increasing but the next level curve is closer when we move to the east. So the rate of change in temperature is greater when we move east.

7. Consider the graph of $z = x^2 - y^2$ below.



(a) [2] Draw and label the curves $z = f(x, 7)$ and $z = f(6, y)$ and the tangent lines to these curves at $P(6, 7, -13)$.

(b) [2] Determine the signs of f_x and f_y at (6,7).

$f_x(6,7) > 0$ $\qquad$ $f_y(6,7) < 0$

8. (a) [2] True or False: If $f_x(x,y)$ and $f_y(x,y)$ exist at (a,b), then $f(x,y)$ is differentiable at (a,b). Explain.

FALSE!
f_x and f_y must be continuous on some open disk centred at (a,b) in order to conclude that f is differentiable at (a,b).

(b) [2] Compute the partial derivatives of $f(x,y) = \arctan(x\sqrt{y})$. ← $y \geq 0$

$$f_x(x,y) = \frac{1}{1+(x\sqrt{y})^2} \cdot \sqrt{y} = \frac{\sqrt{y}}{1+x^2y} \quad (y \geq 0 \text{ and } 1+x^2y \neq 0)$$

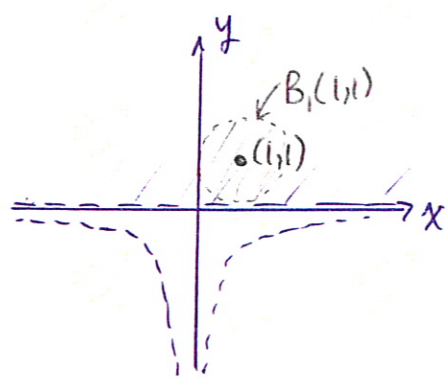
$$f_y(x,y) = \frac{1}{1+(x\sqrt{y})^2} \cdot \frac{x}{2\sqrt{y}} = \frac{x}{2\sqrt{y}(1+x^2y)} \quad (y>0 \text{ and } 1+x^2y \neq 0)$$

(c) [2] Find and sketch the domain of $f_x(x,y)$ and $f_y(x,y)$.

① because of f_y, $y > 0$

② from both, $1+x^2y \neq 0 \Rightarrow y \neq \frac{-1}{x^2}$

① + ② ⇒ domain $= \{(x,y) \in \mathbb{R}^2 \mid y > 0\}$



(d) [2] Explain why the function $f(x,y) = \arctan(x\sqrt{y})$ is differentiable at (1,1). What is the largest open disk centred at (1,1) that you can use?

f_x and f_y are continuous on $B_1(1,1)$
⇒ f is differentiable at $(1,1)$ (Theorem)

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9. The number of wolves W in a certain fixed region depends on both the availability of food F and the distance L from urban areas. Suppose that $\frac{\partial W}{\partial F} = 0.4$ and $\frac{\partial W}{\partial L} = 0.6$.

(a) [2] What is the significance of the signs of these partial derivatives?

$\frac{\partial W}{\partial F} = 0.4 > 0 \Rightarrow$ as food availability increases, # of wolves increases.

$\frac{\partial W}{\partial L} = 0.6 > 0 \Rightarrow$ as distance from urban areas increases, # of wolves increases

(b) [3] Suppose that over time, the availability of food decreases at a rate of 2 units per year, and the urban areas grow, shortening the distance to the wolves' habitat by 0.4 km per year. Estimate the current change $\frac{dW}{dt}$ in the population of wolves. Remember to include units!

$\frac{dF}{dt} = -2 \qquad \frac{dL}{dt} = -0.4.$

$$\frac{dW}{dt} = \frac{\partial W}{\partial F} \cdot \frac{dF}{dt} + \frac{\partial W}{\partial L} \cdot \frac{dL}{dt}$$

$$= (0.4)(-2) + (0.6)(-0.4)$$

$$= -1.04 \text{ wolves/year.}$$

$\therefore$ The wolf pop^n is decreasing by about one wolf per year.