

Independence (Section 5)

1. Define the sample space $S = \{1, 2, 3, 4, 5\}$, and assume that $P(1) = 0.2$, $P(2) = 0.1$, $P(3) = 0.2$, $P(4) = 0.4$, and $P(5) = 0.1$. Let $A = \{1\}$ and $B = \{1, 2, 3, 4, 5\}$. Calculate $P(B|A)$ and $P(B)$. Are A and B independent?

2. The sample space consists of four elements, $S = \{1, 2, 3, 4\}$. Assume that $P(1) = P(2) = P(3) = 0.2$, and $P(4) = 0.4$. Are the events $A = \{1, 4\}$ and $B = \{2, 3, 4\}$ independent?

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3. The sample space consists of four elements, $S = \{1, 2, 3, 4\}$. Assume that $A = \{2, 3\}$ and $B = \{3, 4\}$ are independent and $P(A) = 0.3$ and $P(B) = 0.2$. Find $P(3)$ and $P(1)$.

4. The sample space consists of four elements, $S = \{1, 2, 3, 4\}$. Assume that $P(1) = 0.1$, $P(2) = 0.4$, $P(3) = 0.1$, and $P(4) = 0.4$, and let $A = \{1, 3\}$. Find an event B that contains two elements and is independent of A .

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5. A quiz has five multiple-choice questions, each with three choices. Without reading them, a student randomly answers all questions.

(a) What is the probability that the student will answer at least one question correctly?

(b) What is the probability that the student will answer all questions correctly?

6. The probability that a couple has a female child is 0.54 and the probability that they have a male child is 0.46. Assume that births are independent events.

(a) A couple has three children. What is the probability that exactly two of their children are girls?

(b) A couple has four children. What is the probability that at least two children are boys?

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7. A medical test for a certain disease gives a false-positive result with a probability of 0.002. (A false positive describes the situation where the test turns out positive although the person tested does not have the disease.) What is the probability that in a group of 100 people, at least one false positive will occur?

8. The average efficacy of an oral contraceptive (birth control pill) is about 97.5% per year. This means that, within a year, 2.5% of sexually active women who are taking the pill will get pregnant. What is the probability that a sexually active woman who takes birth control pills will get pregnant at least once in a 5-year period?