Math 1AA3/1ZB3

1st Sample Test #2

- Updated March 2

Name:____

(Last Name) (First Name)

Student Number:______________________ **Tutorial Number:**______________________

This test consists of 19 multiple choice questions worth 1 mark each (no part marks), and 1 question worth 1 mark (no part marks) on proper computer card filling. All questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator.

1. Find a power series representation of

$$f(x) = x \tan^{-1} x$$

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} x^{n+2}$ **(b)** $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$ **(c)** $\sum_{n=0}^{\infty} (-1)^n 2n\, x^{2n}$

(d) $\sum_{n=0}^{\infty} \frac{1}{2n+1} x^{2n+2}$ **(e)** $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+2}$

2. Scientists can determine the age of ancient objects by a method called *Radiocarbon dating*. The bombardment of the upper atmosphere by cosmic rays converts nitrogen to a radioactive isotope of carbon, carbon-14, with a half-life of about 5730 years. Vegetation absorbs carbon dioxide through the atmosphere and animal life assimilates carbon-14 through food chains. When a plant or animal dies, it stops replacing its carbon and the amount of carbon-14 begins to decrease through radioactive decay. Therefore, the level of radioactivity must also decay exponentially. A parchment fragment was discovered that had about 74% as much carbon-14 radioactivity as does plant material on earth today. Estimate the age of the parchment.

(a) $\left(\frac{1}{2}\right)^{37/(50\cdot 5730)}$ years **(b)** $-5730\frac{\ln 2}{\ln\left(\frac{37}{50}\right)}$ years **(c)** $-\frac{\ln 2}{5730\ln\left(\frac{37}{50}\right)}$ years

(d) $5730\frac{\ln\left(\frac{37}{50}\right)}{\ln 2}$ years **(e)** $-5730\frac{\ln\left(\frac{37}{50}\right)}{\ln 2}$ years

3. Suppose that a function f has the following Maclaurin series:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3^n n}(x-3)^n$$

Find the radius of convergence.

(a) 4 **(b)** 0 **(c)** 3 **(d)** 1 **(e)** ∞

4. There is considerable evidence to support the theory that for some species there is a minimum population size m with the property that the species will become extinct if the population falls below m. If $m = 100$ then such a population can be modelled by the following differential equation.

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{300}\right)\left(1 - \frac{100}{P}\right)$$

If $k > 0$, for what values of P is the population increasing?

(a) $100 < P < 300$ **(b)** $P > 300$ **(c)** $P < 100$ **(d)** $P > 100$
(e) $P < 100$ and $P > 300$

5. Find the Taylor series for

$$f(x) = \frac{1}{\sqrt{x}}$$

centered at $a = 4$.

(a) $\frac{1}{2}\left[1 + \sum_{n=1}^{\infty}(-1)^n \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{n!\, 2^{3n}}(x-4)^n\right]$

(b) $\frac{1}{2}\left[1 + \sum_{n=1}^{\infty}(-1)^n \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n+1)}{n!\, 2^{3n-1}}(x-4)^n\right]$

(c) $\frac{1}{2}\left[2 + \sum_{n=1}^{\infty}(-1)^{n+1} \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{n!\, 2^{3n}}(x-4)^n\right]$

(d) $\frac{1}{2}\left[1 + \sum_{n=1}^{\infty}(-1)^n \frac{2 \cdot 5 \cdot 8 \cdot \cdots \cdot (3n-1)}{n!\, 2^{3n-1}}(x-4)^n\right]$

(e) $\frac{1}{2}\left[2 + \sum_{n=1}^{\infty}(-1)^n \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n+1)}{n!\, 2^{3n}}(x-4)^n\right]$

6. Evaluate the following indefinite integral as an infinite series.

$$\int \frac{\sin x - x}{x^3}\,dx$$

(a) $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+3)!}$ **(b)** $\displaystyle\sum_{n=0}^{\infty}(-1)^{n+1} \frac{x^{2n+4}}{(2n+4)(2n+3)!}$

(c) $\displaystyle\sum_{n=0}^{\infty}(-1)^{n+1} \frac{x^{2n+1}}{(2n+1)(2n+3)!}$ **(d)** $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)(2n+1)!}$

(e) $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$

7. Consider the Taylor polynomial $T_3(x)$ for $f(x) = \sqrt{x}$ at $a = 4$. Use Taylor's inequality to estimate the accuracy of the approximation $f(x) \approx T_2(x)$ when x lies in the interval $2 \leq x \leq 6$.

(a) $\dfrac{1}{8\sqrt{2}}$ **(b)** $\dfrac{2}{9\sqrt{2}}$ **(c)** $\dfrac{5}{3 \cdot 2^{5/2}}$ **(d)** $\dfrac{1}{3!\, 2^{5/2}}$ **(e)** $\dfrac{3}{8 \cdot 2^{3/2}}$

8. On a warm day, the temperature of a room is 23°C. A biological sample is taken from cold storage at −33°C at noon, and warms to −5°C at 1:00pm. What is its temperature at 2:00pm?

(a) 5°C **(b)** 9°C **(c)** 12°C **(d)** 0°C **(e)** 15°C

9. Which of the following is equal to $\binom{1/3}{n}$?

(a) $(-1)^n \dfrac{3 \cdot 5 \cdot 7 \cdot \cdots \cdot (2n+1)}{3^n n!}$ **(b)** $(-1)^{n-1} \dfrac{2 \cdot 5 \cdot 8 \cdot \cdots \cdot (3n-4)}{3^n n!}$, $n \geq 2$

(c) $(-1)^n \dfrac{2 \cdot 5 \cdot 8 \cdot \cdots \cdot (3n-1)}{3^n n!}$ **(d)** $(-1)^n \dfrac{2 \cdot 5 \cdot 8 \cdot \cdots \cdot (3n-4)}{n!}$, $n \geq 2$

(e) $(-1)^{n-1} \dfrac{3 \cdot 5 \cdot 7 \cdot \cdots \cdot (3n-2)}{3^n n!}$, $n \geq 1$

10. Find the area of the surface obtained by rotating the curve $y = \frac{x^3}{6} + \frac{1}{2x}$, $1 \leq x \leq 2$ about the y-axis.

(a) $\pi(\frac{15}{4} + \ln 2)$ **(b)** $\frac{43}{16}\pi$ **(c)** $\pi(\frac{15}{2} + \ln 2)$ **(d)** $\frac{47}{16}\pi$ **(e)** $\pi(\frac{13}{2} + \ln 3)$

11. A gate in an irrigation canal is constructed in the form of a trapezoid 3 ft wide at the bottom, 5 ft wide at the top, and 2 ft high. It is placed vertically in the canal so that the water just covers the gate. Find the hydrostatic force on one side of the gate.
(a) $\frac{23}{3} \cdot 62.5\,\text{lb}$ **(b)** $8 \cdot 62.5\,\text{lb}$ **(c)** $\frac{25}{3} \cdot 62.5\,\text{lb}$ **(d)** $\frac{26}{3} \cdot 62.5\,\text{lb}$ **(e)** $\frac{22}{3} \cdot 62.5\,\text{lb}$

12. Solve the initial value problem

$$(x\ln x)y' + y = x^2e^x \qquad y(e) = 1$$

(a) $y = \dfrac{xe^x + 1 - e^{e+1}}{\ln x}$ **(b)** $y = \dfrac{e^x(x+1) + 1 - e^e(e+1)}{x}$
(c) $y = \dfrac{xe^x(x-1) + 1 - e^{e+1}(e-1)}{\ln x}$ **(d)** $y = \dfrac{e^x(x-1) + 1 - e^e(e-1)}{\ln x}$
(e) $y = \dfrac{e^x(x^2-1) + 1 - e^e(e^2-1)}{\ln x}$

13. Find an equation of the curve that passes through the point $(1, 1)$ and whose slope at any point (x, y) on the curve is $3y - 2x + 6xy - 1$.
(a) $y = \dfrac{1}{3}(2e^{3(x+x^2)-6} + 1)$ **(b)** $y = \dfrac{1}{5}(2e^{3(x+x^2)-6} + 3)$
(c) $y = \dfrac{1}{4}(3e^{3(x+x^2)-6} + 1)$ **(d)** $y = \dfrac{1}{3}(2e^{3(x-x^2)} + 1)$
(e) $y = \dfrac{1}{3}(2e^{(x+x^2)-2} + 1)$

14. Find the orthogonal trajectories of the family of curves

$$y = \frac{1}{(x+k)^3}$$

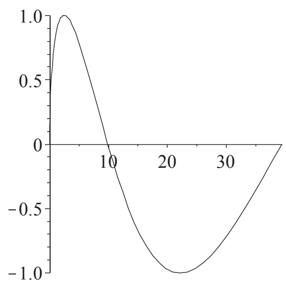
(a) $y = \left(\frac{7}{3}x + C\right)^{9/7}$ **(b)** $y = \left(\frac{9}{7}x + C\right)^{7/3}$ **(c)** $y = \left(\frac{7}{9}x + C\right)^{7/3}$
(d) $y = \left(\frac{9}{7}x + C\right)^{3/7}$ **(e)** $y = \left(\frac{7}{9}x + C\right)^{3/7}$

15. A tank contains 750 L of pure water. Brine that contains .12 kg of salt per liter of water enters the tanke at a rate of 5 L/min. Brine that contains .04 kg of salt per liter of water enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at a rate of 15 L/min. How much salt is in the tank after 1 hour?

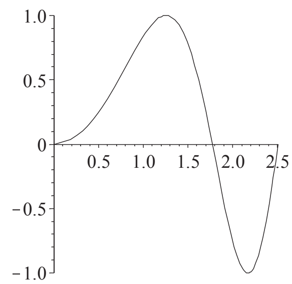
(a) $50(1 - e^{-6/5})$ **(b)** $50(1 + e^{-5/7})$ **(c)** $50(1 - e^{-1/15})$ **(d)** $50(1 + e^{-7/15})$
(e) $50(1 - e^{12/5})$

16. Sketch the parametric curve $x = \sqrt{t},\ y = \sin t,\ 0 \le t \le 2\pi$.

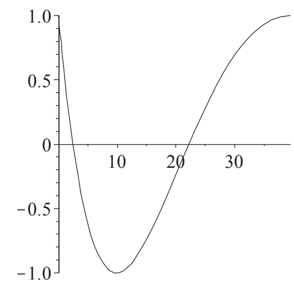
(a)



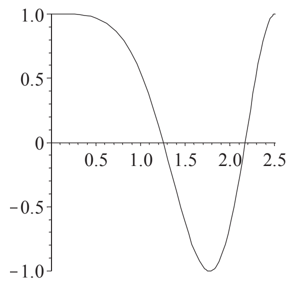
(b)



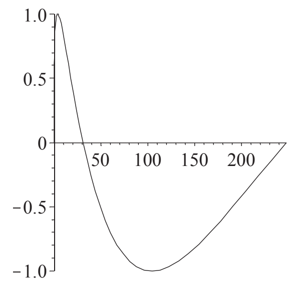
(c)



(d)



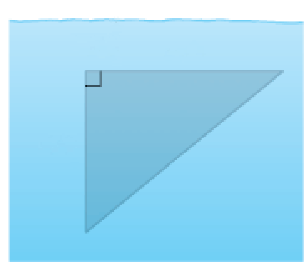
(e)



17. Find the cartesian equation of the parametric curve $x = 1 + e^{2t},\ y = e^t$.
(a) $x^2 = y^2 - 1$ **(b)** $y^2 = x^2 - 1$ **(c)** $y^2 = \sqrt{x} - 1$ **(d)** $x = y^2 + 1$ **(e)** $y = \sqrt{x} - 1$

18. Suppose that we were to approximate $f(x) = \cos x$ using the 3rd degree Taylor polynomial $T_3(x) = 1 - \frac{x^2}{2!}$. What is the largest range of x for which the approximation is accurate to within .0001?
(a) $-\frac{\sqrt[3]{24}}{100} \le x \le \frac{\sqrt[3]{24}}{100}$ **(b)** $-\frac{\sqrt[4]{8}}{100} \le x \le \frac{\sqrt[4]{8}}{100}$ **(c)** $-\frac{\sqrt{10}}{24} \le x \le \frac{\sqrt{10}}{24}$
(d) $-\frac{\sqrt[4]{10}}{24} \le x \le \frac{\sqrt[4]{10}}{24}$ **(e)** $-\frac{\sqrt[4]{24}}{10} \le x \le \frac{\sqrt[4]{24}}{10}$

19. A triangular plate with height 5 m and a base of 7 m is submerged vertically in water (whose density is ρ) so that the top is 2 m below the surface. Find the hydrostatic force against one side of the plate.



(a) $\frac{45}{6}\rho g$ **(b)** $\frac{385}{6}\rho g$ **(c)** $\frac{381}{6}\rho g$ **(d)** $\frac{163}{3}\rho g$ **(e)** $50\rho g$

Math 1AA3/1ZB3

2nd Sample Test #2

- Updated March 2

Name:____

(Last Name) (First Name)

Student Number:______________________ **Tutorial Number:**_____________________

This test consists of 18 multiple choice questions worth 1 mark each (no part marks), and 1 question worth 1 mark (no part marks) on proper computer card filling. All questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator.

1. Find a power series representation of

$$f(x) = \frac{x^2}{27 + x^3}$$

(a) $\sum_{n=0}^{\infty} \frac{(-1)^{3n} x^{3n+2}}{27^n}$ **(b)** $\sum_{n=0}^{\infty} \frac{(-1)^{n} x^{3n+2}}{27^{n+1}}$ **(c)** $\sum_{n=0}^{\infty} \frac{(-1)^{n} x^{3n}}{27^n}$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^{3n} x^{3n+1}}{27^{n+1}}$ **(e)** $\frac{1}{27}\sum_{n=0}^{\infty} (-1)^n x^{n+2}$

2. Find the sum of the series

$$\sum_{n=2}^{\infty} n(n-1)x^{n-2}$$

(a) $\frac{6}{(1-x)^4}$ **(b)** $\frac{1}{(1-x)^2}$ **(c)** $\frac{-2}{(1-x)^3}$ **(d)** $\frac{2}{(1-x)^3}$ **(e)** $-\frac{6}{(1-x)^4}$

3. Zylex-7, an unstable medicine, decays exponentially into an inert substance over time. If only 10% remains from an original pure sample after 381 days, what is its half life?

(a) $381\frac{\ln 2}{\ln 10}$ days **(b)** $-381\frac{\ln 2}{\ln\left(\frac{9}{10}\right)}$ days **(c)** $\frac{\ln 2}{381 \ln 10}$ days **(d)** $\frac{\ln 10}{381 \ln 2}$ days

(e) $-381\frac{\ln\left(\frac{9}{10}\right)}{\ln 2}$ days

4. Use the binomial series (or any other method) to find the Maclaurin series for

$$f(x) = \frac{1}{(2+x^2)^2}$$

(a) $\sum_{n=0}^{\infty}(-1)^n \frac{(n+1)}{2^{n+2}} x^{2n}$ **(b)** $\sum_{n=0}^{\infty}(-1)^n \frac{1}{n!\,2^{n+1}} x^{2n}$ **(c)** $\sum_{n=0}^{\infty}(-1)^{n-1} \frac{n}{2^n} x^{2n+1}$
(d) $\sum_{n=0}^{\infty}(-1)^{n-1} \frac{(n+1)}{2^{n+1}} x^{2n+1}$ **(e)** $\sum_{n=0}^{\infty}(-1)^n \frac{n}{2^{n+1}} x^{2n}$

5. Find the Taylor series for $f(x) = \ln x$ centered at $a = 3$.

(a) $\ln 3 + \sum_{n=1}^{\infty} \frac{(-1)^n}{3^n n}(x-3)^n$ **(b)** $\ln 3 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3^n n}(x-3)^n$
(c) $\ln 3 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3^n (n+1)}(x-3)^n$ **(d)** $\ln 3 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3^{n+1} n}(x-3)^n$
(e) $\ln 3 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3^n n!}(x-3)^n$

6. Find the sum of the series

$$3 - 3^3 + \frac{3^5}{2!} - \frac{3^7}{3!} + \frac{3^9}{4!} - \cdots$$

(a) $3e^3$ **(b)** e^{-3} **(c)** e^{-9} **(d)** $3e^{-9}$ **(e)** $3e^{-3}$

7. To which of the following equations is $y = e^{2x}$ a solution?

I) $y'' - 3y' + 2y = 0$ *II)* $y'' + 4y = 5e^{2x}$ *III)* $y'' - 2y' + y = e^{2x}$

(a) I and III **(b)** I only **(c)** II only **(d)** III only **(e)** II and III

8. A Trapezoid shaped panel 3m across the top, 5 meters across the base and 4 meters tall is used to hold water inside a tank. If the top of the water is only half way up the side of the panel, what is the total hydrostatic force on the plate? (For simplicity, assume g = 10m/s^2.)

(a) 93.3 kN **(b)** 226.5 kN **(c)** 90.0 kN **(d)** 75.1 kN **(e)** 67.8 kN

9. Which of the following differential equation(s) is/are both separable and linear?

I) $\frac{dy}{dx} + (e^{2x})y^2 = 0$ II) $\frac{dy}{dx} + (e^{2x})y = e^{2x}$ III) $\frac{dy}{dx} + (e^{3x})y = 9\ln(x)$

(a) I only **(b)** II only **(c)** III only **(d)** I and II only **(e)** I and III only

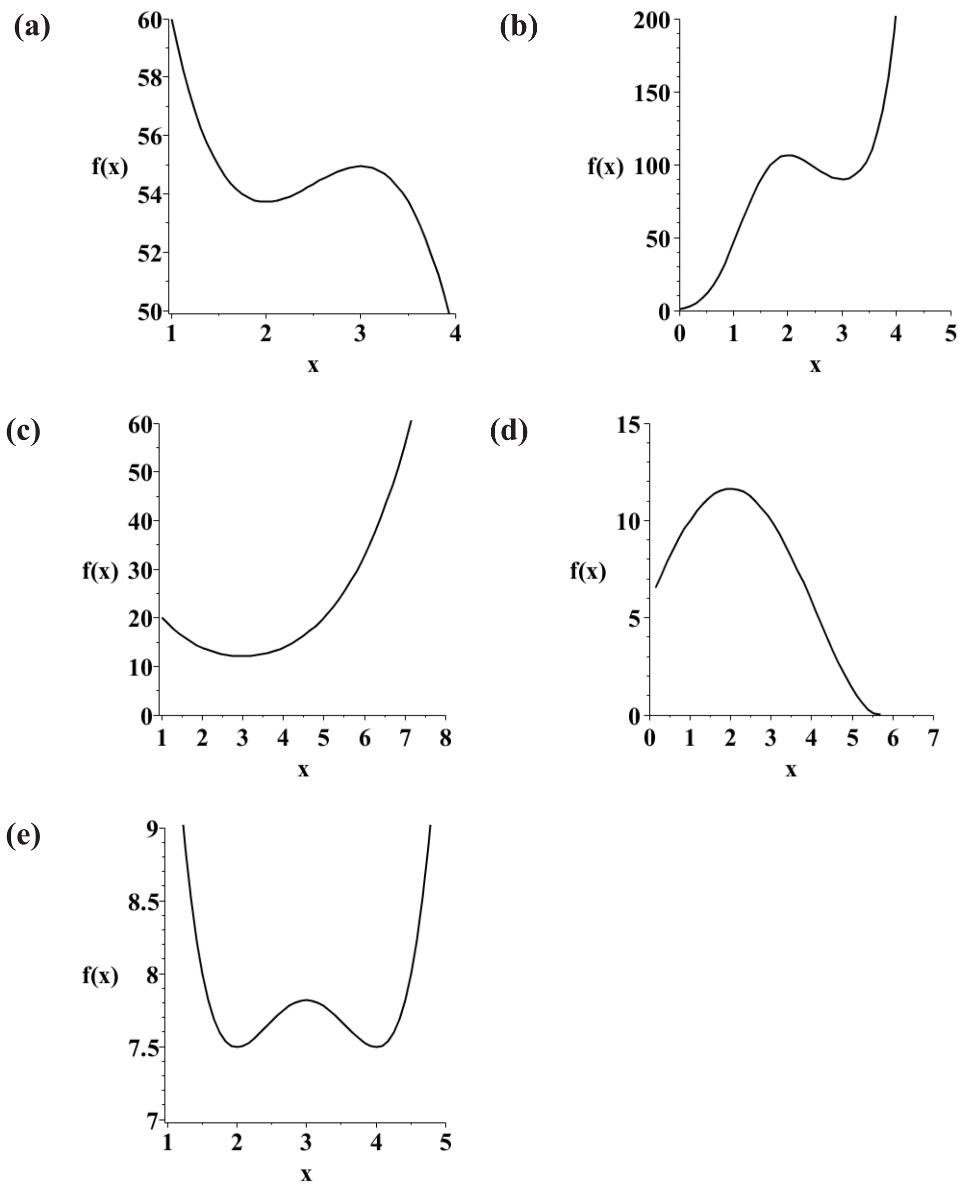
10. A function, $g(x)$ with continuous first, second and third derivatives, has $g(5) = 1$, $g'(5) = 4$, $g''(5) = 6$. Which of the following is its Taylor polynomial, $T_2(x)$, centred about $x = 5$?

(a) $1+4(x-5)$ **(b)** $1+4(x-5)+3(x-5)^2$ **(c)** $1+4(x-5)+6(x-5)^2$

(d) $1+5(x-4)+25(x-6)^2$ **(e)** $1+5(x-4)-(5/2)(x-6)^2$

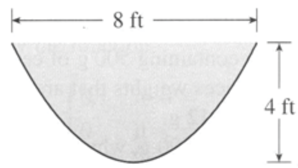
11. Which of the below graphs could be the graph of a function that satisfies the following differential equation?

$$y' = y(x-2)(x-3)$$



12. Use Taylor's inequality to find the minimum value of n needed so that the nth degree Taylor polynomial $T_n(x)$ (with $a = 0$) is an approximation to $f(x) = \sin x$, $-\frac{1}{2} \le x \le \frac{1}{2}$, with a maximum error of .001.
(a) 6 **(b)** 5 **(c)** 7 **(d)** 4 **(e)** 3

13. A trough is filled with water and its vertical ends have the shape of the parabolic region in the figure to the right. Find the hydrostatic force on one end of the trough.
(a) $\frac{511}{12} \cdot 62.5$ lb **(b)** $\frac{512}{15} \cdot 62.5$ lb **(c)** $\frac{513}{12} \cdot 62.5$ lb
(d) $\frac{512}{13} \cdot 62.5$ lb **(e)** $\frac{511}{15} \cdot 62.5$ lb



14. Solve the initial value problem

$$\frac{dy}{dt} = 2y\left(\frac{y}{5} - 1\right), \qquad y > 5, \qquad y(0) = 6$$

(a) $y = \frac{30}{6-5e^{2t}}$ **(b)** $y = \frac{6}{5-4e^{2t}}$ **(c)** $y = \frac{30}{6-e^{2t}}$
(d) $y = \frac{24}{5-e^{2t}}$ **(e)** $y = \frac{42}{9-2e^{2t}}$

15. Suppose that $y(x)$ is the solution of the initial value problem

$$y' = y(1-x), \qquad y(1) = e.$$

Find $y(2)$.
(a) $\frac{1}{2}e$ **(b)** 1 **(c)** $2e$ **(d)** e^2 **(e)** $e^{1/2}$

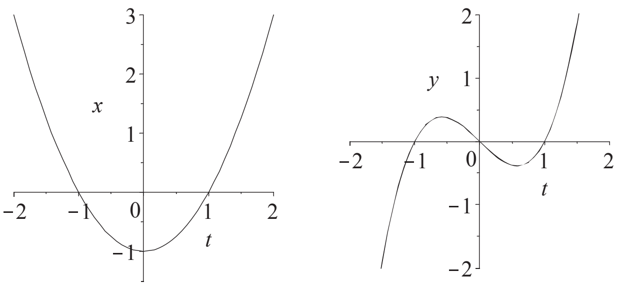
16. Solve the initial value problem

$$x^2\frac{dy}{dx} + 2xy = \cos x, \qquad y(\pi) = 0.$$

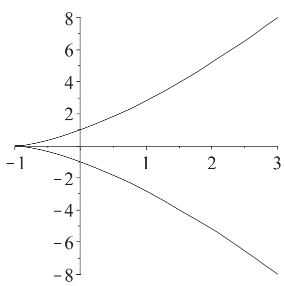
(a) $y = 3\dfrac{(\cos x) + 1}{x^3}$ **(b)** $y = \dfrac{3\sin x}{x^3}$ **(c)** $y = \dfrac{(\cos x) + 1}{x^2}$
(d) $y = \dfrac{2\sin x}{x^2}$ **(e)** $y = \dfrac{\sin x}{x^2}$

17. Find a cartesian equation of the parametric curve $x = 2\cos t$, $y = 1 + \sin t$.
(a) $\frac{x^2}{4} + (y-1)^2 = 1$ **(b)** $\frac{x}{2} + y = 2$ **(c)** $\frac{x^2}{4} + y^2 = 2$ **(d)** $\frac{x^2}{4} + y^2 = 1$
(e) $\frac{x}{2} + y = 1$

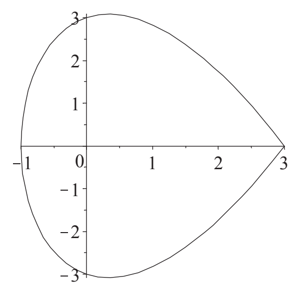
18. Use the graphs of $x = f(t)$ and $y = g(t)$ below to sketch the parametric curve $x = f(t)$, $y = g(t)$.



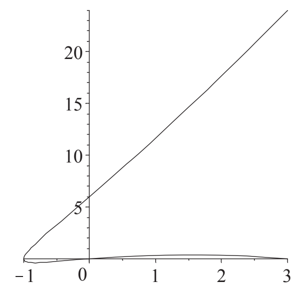
(a)



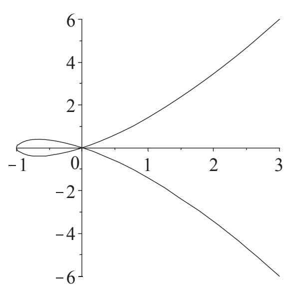
(b)



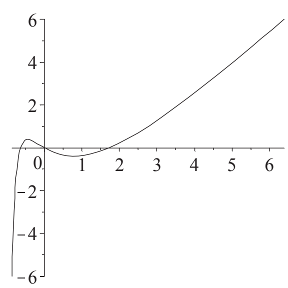
(c)



(d)



(e)



Answers for 1st Sample Test #2

1. e **2.** e **3.** c **4.** a **5.** a **6.** c **7.** a **8.** b **9.** b **10.** a
11. e **12.** d **13.** a **14.** e **15.** a **16.** b **17.** d **18.** e **19.** b

Answers for 2nd Sample Test #2

1. b **2.** d **3.** a **4.** a **5.** b **6.** d **7.** b **8.** a **9.** b **10.** b
11. b **12.** d **13.** b **14.** c **15.** e **16.** e **17.** a **18.**d