Name: _____ Student Number:_____

MATH 1AA3, 1ZB3

Winter Session, 2019

Test #2, Seating #1, Version #1

Announced Corrections in Red

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TERM TEST
DAY & EVENING CLASS
DURATION OF TEST: 90 **min** **(**1.5 **hrs.**)
MCMASTER UNIVERSITY TERM TEST

Monday, Mar. 18, 2019

THIS TEST INCLUDES 6 PAGES AND 17 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING YOUR COPY OF THE TEST IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

Instructions:

1. **NO** calculator is allowed to be used on this test.

2. Make sure your name and student number are at the ***top of each page.***

3. For all questions, mark the answer in pencil on the OMR answer sheets according to the OMR instructions on page 2. Only solutions on the scan card will be graded.

4. Each correct answer is worth one mark.

5. A blank answer is an automatic zero for any question, even if the correct solution is circled on the question itself.

6. Incorrect or multiple answers are also worth zero marks. No negative marks or part marks will be assigned.

7. Scrap paper for rough work has been provided. *All* rough work, and this question sheet must be handed in with the test, but any solutions written either here or on the rough paper will ***NOT*** be graded.

8. *Good Luck!*

Continued on page #2

Name: _____ Student Number:_____

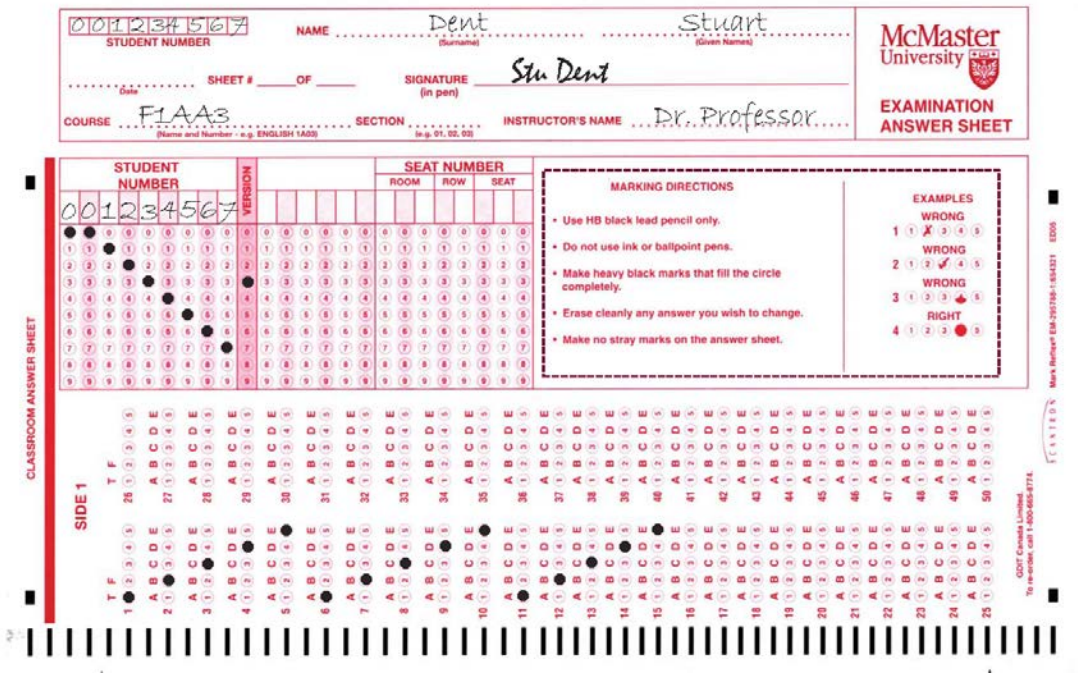
PLEASE READ THE OMR INSTRUCTIONS ON PAGE #2
OMR EXAMINATION INSTRUCTIONS

NOTE: IT IS YOUR RESPONSIBILITY TO ENSURE THAT THE ANSWER SHEET IS PROPERLY COMPLETED: YOUR EXAMINATION RESULT DEPENDS ON PROPER ATTENTION TO THESE INSTRUCTIONS.

The scanner which reads the sheets senses the bubble-shaded areas by their non-reflection of light. A heavy mark must be made, completely filling the circular bubble, with an **HB pencil**. Marks made with a pen or a felt–tip marker will **NOT** be sensed. Erasures must be thorough or the scanner may still sense a mark. Do **NOT** use correction fluid on the sheets. Do **NOT** put any unnecessary marks or writing on the sheet.

1. On side 1 (red side) of the form, in the top box, in pencil, print your student number **(NOTE: 9 digits)**, name, course name, section number, instructor name and date in the spaces provided. Then you **MUST** sign in the space marked SIGNATURE**.**
2. In the second box, with a pencil, mark your student number, exam version number and course section number in the space provided and fill in the corresponding bubble numbers underneath.
3. To indicate your answers, mark only **ONE** choice from the alternatives (1, 2, 3, 4, 5, or A, B, C, D, E) provided for each question. The question number is to the left of the bubbles. Make sure that the number of the question on the scan sheet is the same as the question number on the test paper.
4. Pay particular attention to the Marking Directions on the form.
5. Begin answering the questions using the first set of bubbles, marked "1".

SAMPLE OMR CARD ONLY: DO NOT USE



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Name: _____ Student Number:_____

1. Which of the following is a power series representation of the function: $\dfrac{1}{5+x^2}$

a) $\displaystyle\sum_{n=0}^{\infty}\frac{x^{2n}}{5^{n+1}n!}$ **b)** $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{5^{2n}}$ **c)** $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^{n+1}x^{2n+1}}{5^n}$ **d)** $\displaystyle\sum_{n=0}^{\infty}\frac{x^{2n}}{5^n(2n)!}$ **e)** $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{5^{n+1}}$

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2. Find the value at $x = 0$ of the function passing through the point $(x,y) = (1,\ln(3))$ whose slope is given by $y' = 2x^3e^{-y}$.

a) $\ln(2/3)$ **b)** $\ln(e^2+2)$ **c)** $\left(\ln\left(5/2\right)\right)^{1/3}$ **d)** $\ln(5/2)$ **e)** 5

_____

3. Evaluate $\binom{1/3}{4}$.

a) $\dfrac{80}{3^4}$ **b)** $\dfrac{50}{3^5}$ **c)** $-\dfrac{7}{3^4}$ **d)** $-\dfrac{10}{3^5}$ **e)** $-\dfrac{16}{3^3}$

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4. Find the family of orthogonal trajectories to the family of curves given by $y^2 = k/x^3$, where k is an arbitrary real constant.

a) $3y^2 - 2x^2 = C$ **b)** $y^2 + \dfrac{3}{2}x^2 = C$ **c)** $4y^3 + \dfrac{3}{x^4} = C$ **d)** $y = \dfrac{1}{C - x^2}$

e) $y = 3x - \ln|x| + C$

_____

5. Find the Taylor series for $f(x) = \ln(4 + x)$ centred about $x = -1$.

a) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{4^{n+1}}(x+1)^n$ **b)** $\ln 3 + \displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n3^n}(x+1)^n$ **c)** $\ln 3 - \displaystyle\sum_{n=1}^{\infty}\frac{1}{n3^n}(x+1)^n$

d) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n3^n}(x+1)^n$ **e)** $\ln(1/2) + \displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{4^{n+1}}(x+1)^n$

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6. Find the surface area of revolution of the curve $f(x) = x^{1/3}$, $0 \le x \le 1$ rotated about the y axis.

a) $\frac{\pi}{8}(10^{3/2}-1)$ **b)** $\frac{\pi}{36}(10^{5/2}-1)$ **c)** $\frac{6\pi}{10}$ **d)** $\frac{7\pi}{27}$ **e)** $\frac{\pi}{27}(10^{3/2}-1)$

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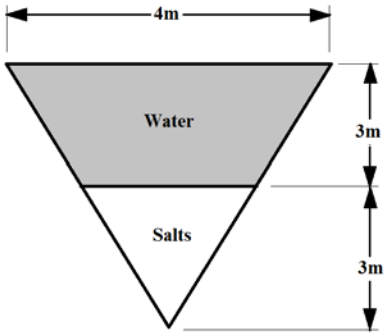
7. A wall with a triangular cross section is 6m tall and 4m wide at its top. A layer of dense insoluble salts has built up behind this wall to a depth of 3m, and the remainder of the space behind the wall has filled with water.



Assuming the salt is solid and waterproof (and so applies no pressure), how much hydrostatic force is on this wall? For simplicity, consider the density of water as 1000kg/m^3 and g = 10m/s^2.

a) 200kN **b)** 150kN **c)** 120kN **d)** 60kN **e)** 270kN

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8. Find the Taylor polynomial, $T_2(x)$, centred at $x = \pi/6$ for $f(x) = \sin(x)$.

a) $\frac{1}{2}+\frac{\sqrt{3}}{2}(x-\frac{\pi}{6})-\frac{1}{4}(x-\frac{\pi}{6})^2$ **b)** $1+\frac{\sqrt{3}}{2}(x-\frac{\pi}{6})$ **c)** $\frac{1}{2}+\frac{\sqrt{3}}{2}(x-\frac{\pi}{6})$

d) $\frac{\sqrt{3}}{2}(x-\frac{\pi}{6})$ **e)** $\frac{1}{2}+\frac{\sqrt{3}}{2}(x-\frac{\pi}{6})-\frac{1}{2}(x-\frac{\pi}{6})^2$

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9. Given the differential equation $y' = \frac{(4-y^2)^9}{(x^2+1)^7}$, which of the following statements must be true?

a) Any solution passing through the y-axis is increasing as it does so.

b) Any solution passing through (–1, –2) has a vertical tangent.

c) All solutions are decreasing when the value of y between – 2 and 2.

d) All of the above

e) None of the above

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10. Evaluate: $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} 5^n$, or state it is divergent.

a) $\cos(-5)$ **b)** $\frac{1}{6}$ **c)** $\sin(5)$ **d)** e^{-5} **e)** *Divergent*

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11. Given $f(x)$ has a Taylor series centred at $x = 2$ given by $\sum_{n=0}^{\infty} a_n (x-2)^n$ and if we know $a_3 = 3$, which of the following must be true?

a) $f^{(4)}(2) = 24$ **b)** $f^{(3)}(2) = 18$ **c)** $f^{(3)}(2) = 9$ **d)** $f^{(3)}(0) = 24$ **e)** $f^{(4)}(2) = 6$

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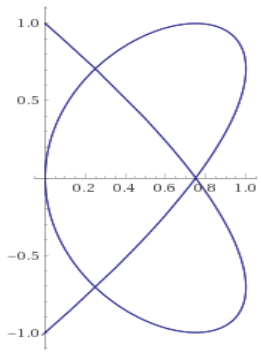
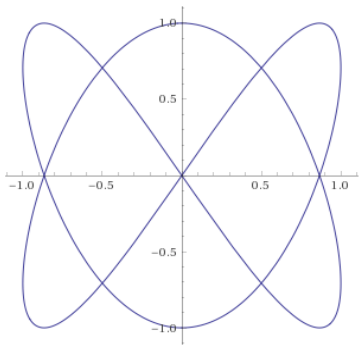
12. Which of the following is the plot of the parametric curve given by $(x,y) = (\cos^2(2t), \sin(3t))$?

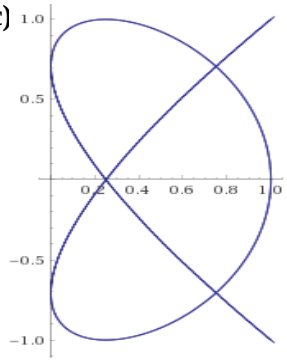
a)



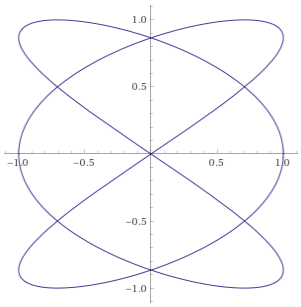
b)



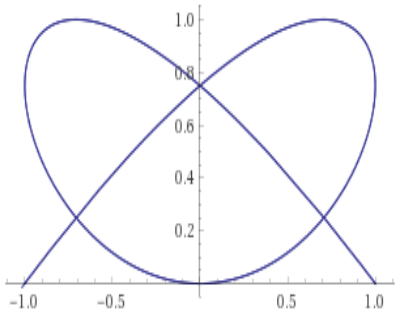
c)



d)



e)



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13. Find the value(s) of the real constant k such that e^{kx} is a solution of the differential equation given by: $3y'' + 2y' - y = 0$.

a) 3, 1 **b)** – 1 only **c)** –1, 1/3 **d)** 0 only **e)** No such real k exists

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14. Express the curve given by $x(t) = t^5 + 1$, $y(t) = e^t$, $t > 0$, as a function, $y = f(x)$.

a) $y = \frac{1}{5}\ln(x-1)$ **b)** $y = e^{(x-1)^{1/5}}$ **c)** $y = (\ln(x))^5 + 1$ **d)** $y = (\ln(x))^{1/5} - 1$ **e)** $y = \frac{e^x}{x^5+1}$

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15. In order to solve the first order linear differential equation: $\frac{2}{\cos(x)}y' + \frac{1}{x\cos(x)}y = 1$, which of the following is an appropriate integrating factor?

a) $\frac{1}{2}e^{1/x}$ **b)** $e^{\int \frac{1}{x\cos(x)}dx}$ **c)** $\sqrt{x}$ **d)** $\frac{x}{2}$ **e)** $e^{1/x}$

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16. A sample of radioactive ink found in a museum is measured to contain 15mg of a rare radioactive substance. After 17 years the same sample is discovered to only contain 7.5mg. How many mg of ink remain in the sample after 8 additional years?

a) $\frac{15}{2^{25/17}}$ **b)** $15(2^{17/25})$ **c)** $15(2^{25/17})$ **d)** $17(15^{1/25})$ **e)** $17\frac{\ln(25)}{\ln(2)}$

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17. Find the Taylor error estimate for approximating $f(x) = e^x$ on [1,5] by the Taylor polynomial $T_4(x)$ centred on $a = 3$.

a) $\frac{4}{15}e$ **b)** $\frac{4}{15}e^5$ **c)** $\frac{2}{3}e^5$ **d)** $\frac{3}{4}e^3$ **e)** $\frac{2}{3}e$

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THE END