Remember, $\int_1^\infty \frac{1}{x^p}$ converges for $p > 1$ and diverges for $p \leq 1$.

So what can we conclude for series?

Suppose $a_n = f(n)$ with f positive, continuous, and decreasing.

Let $S_n = \sum_{i=1}^n a_i$ converge to S^*. We ask:

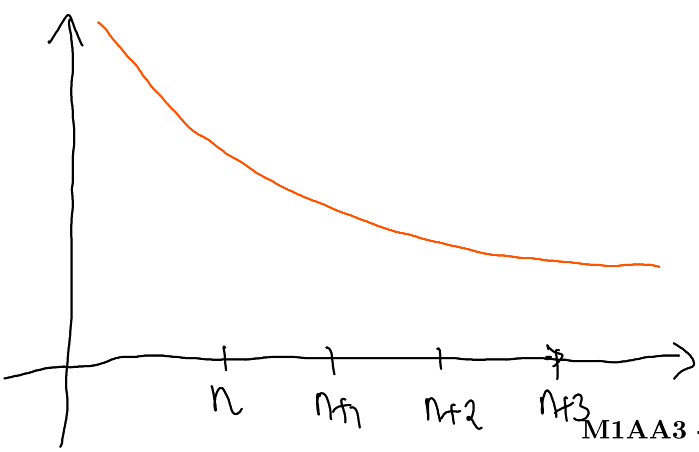
How fast is the convergence?

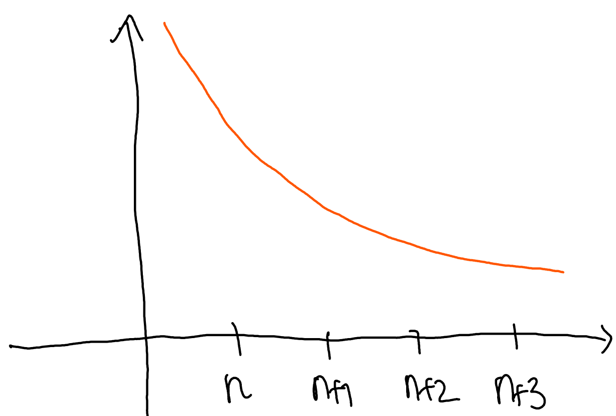
Define $R_n := S^* - S_n =$ ______________________________

We call R_n the **remainder**.

Remainder estimate for the integral test

______________________ $\leq R_n =$ ____________ $\leq$ ______________________





Example: Let $S = \sum_{n=1}^{\infty} \frac{1}{n^3}$. How many terms are necessary to approximate S within 0.01 using partial sums?

Example: Which is approximately the upper bound for the difference between $\sum_{n=2}^{\infty} \frac{\ln(n)}{n^3}$ and $\sum_{n=2}^{9} \frac{\ln(n)}{n^3}$?

4.3 The Comparison Tests (Chapter 11.4)

We focus on series now with non-negative terms, i.e., $S = \sum_{n=1}^{\infty} a_n$ with $a_n \geq 0$.

$\Rightarrow \qquad S_{n+1} \qquad\qquad S_n$

Thus, if S_n is ________________, then $\{S_n\}$ converges, i.e., $\sum_{n=1}^{\infty} a_n$ ________________.

Comparison Test:

Consider series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ with $0 \leq a_n \leq b_n$ for all $n = 1, 2, 3, \ldots$.

$$\sum_{n=1}^{\infty} a_n \qquad\qquad \Longrightarrow \quad \sum_{n=1}^{\infty} b_n$$

and

$$\sum_{n=1}^{\infty} b_n \qquad\qquad \Longrightarrow \quad \sum_{n=1}^{\infty} a_n$$

Relaxing of conditions are possible:

Example:

$A)$ $$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^5 + n + 1}}$$

$B)$ $$\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$$