Math 1LT3	Assignment 18

Basics of probability (Sections 2,3,4)

1. Consider the stochastic dynamical system $m_{t+1} = r_t m_t$, where $m_0 = 1$ and $r_t = 2$ with a 50% chance and $r_t = 1/2$ with a 50% chance ($t = 0, 1, 2, \ldots$).

(a) What is the chance that $m_2 = 1$?

(b) What is the sample space for m_3 (i.e., the set of all possible values for m_3)?

(c) What is the sample space for m_4?

2. A population of leopards p_t, $t = 0, 1, 2 \ldots$, is modelled by $p_{t+1} = p_t + I_t$. The immigration term is equal to $I_t = 10$ with a 90% chance and $I_t = -100$ with a 10% chance. What is more likely to happen to the number of leopards—an increase or a decrease? Or will the population remain at about the same size? Explain why.

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3. The trait of the dominant allele A is brown hair and the trait of the recessive allele B is blond hair.

(a) What is the chance that an offspring of AB and AB parents has brown hair?

(b) What is the chance that an offspring of AA and AB parents has brown hair?

4. Assume that the molecule in Example 2.6 has a 5% chance of leaving the region during any 1-hour time interval. Write the dynamical system for the chance p_t that the molecule is still inside the region after t hours. After how many hours will the chance p_t fall below 50%?

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5. Consider the random walk described in Example 2.8.

(a) Add two more steps to the diagram in Figure 2.7; i.e., add all possible locations of the particle for $t = 4$ and $t = 5$.

(b) Identify the number of different ways a particle can arrive at the locations you listed in (a).

(c) Look at the numbers in the squares in Figure 2.7 and your answer to (b). Does the pattern look familiar?

(d) Create a table of probabilities (like Table 2.6) for the steps $t = 4$ and $t = 5$.

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6. We toss a fair coin ten times and calculate the following difference: the number of heads minus the number of tails. List all elements of the sample space S for this experiment. What is $|S|$ (i.e., the number of elements in S)?

7. We toss a coin n times, $n \geq 1$. How many elements does the sample space have?

8. S is the set of all non-negative integers, $S = \{0, 1, 2, 3, \ldots\}$; A is the set of even numbers (take 0 to be even) and B is the set of all numbers divisible by 3. Find $A \cup B$, $A \cap B$, A^{c}, and $A \cap B^{\mathrm{c}}$.

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9. Given that $P(A \cap B) = 0.2$ and $P(A \cap B^{\mathrm{c}}) = 0.45$, find $P(A)$.

10. Given that $P(A) = 0.3$, $P(B) = 0.2$, and $P(A \cup B) = 0.4$, find $P(A \cap B)$ and $P(A^{\mathrm{c}} \cap B^{\mathrm{c}})$.

11. Explain why it is not possible to assign probabilities to A and B in the following way: $P(A) = 0.1$, $P(B) = 0.2$, and $P(A \cap B) = 0.4$.

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12. Given are the sample set S and the assignment of probabilities for all but one simple event. Find the requested probabilities and answer the questions.

$S = \{1, 2, 3, 4, 5\}$; $P(1) = 0.1$, $P(2) = 0.1$, $P(4) = 0.1$, $P(5) = 0.2$

(a) Find $P(3)$.

(b) Let $A = \{1, 2, 3\}$ and $B = \{4, 5\}$. Find $P(A)$, $P(A^{\mathrm{c}})$, $P(B)$, and $P(A \cup B)$.

(c) Is $P(A \cup B)$ equal to $P(A) + P(B)$? Why or why not?

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13. We roll two fair dice. What is the probability that the sum of the numbers that come up is odd?

14. Assume that female and male children are equally likely to be born. A family has five children. Find the probability that all five are girls.

15. Assume that female and male children are equally likely to be born. A family has five children. Find the probability that at least one child is a girl.

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16. Consider the following modification of the random walk routine: a particle is released from $x = 0$ at $t = 0$; during each time interval, with a probability of $1/2$, it moves left for 1 unit, or right for 2 units.

(a) Find the sample space S when $t = 2$ (i.e., after two steps of random motion), and assign probabilities to each simple event in S.

(b) Find the sample space when $t = 3$ and $t = 4$ and assign probabilities to each simple event.

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