MATHEMATICS 1LS3 TEST 2

Day Class
Duration of Examination: 60 minutes
McMaster University, 28 October 2015

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First name (PLEASE PRINT): ____________________

Family name (PLEASE PRINT): ____________________

Student No.: ____________________

THIS TEST HAS 8 PAGES AND 7 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE.

Total number of points is 40. Marks are indicated next to the problem number. Any non-graphing calculator is allowed.

USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

You must show work to receive full credit.

Problem	Points	Mark
1	4	
2	6	
3	6	
4	7	
5	6	
6	6	
7	5	
TOTAL	40	

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1. **Multiple choice questions: circle ONE answer. No justification is needed.**

(a)[2] If $f(x) = Ax\ln(B+x)$, then $f'(0)$ is equal to

(A) A	(B) B	(C) AB	(D) B ln B
(E) AB ln B	(F) AB ln A	(G) A ln B	(H) B ln A

(b)[2] If $f(x) = \arctan\left(\frac{x}{3}+1\right)$, then $f'(1)$ is equal to

(A) 9/75	(B) 9/15	(C) 1/75	(D) 1/25
(E) 1/15	(F) 3/25	(G) 9/5	(H) 3/17

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2. Identify each statement as true or false, or yes or no (circle your choice). You do not need to justify your answer.

(a)[2] Let $m(t)$ represent the mass of melting snow in kilograms, where t is time in days. The units of $m'(t)$ are kilograms.

TRUE OR FALSE

(b)[2] Knowing that $g''(x) = (x-5)\arctan x$, we conclude that the function $g(x)$ is concave up on $(0, 5)$.

TRUE OR FALSE

(c)[2] The function $g(x) = x\sin(\pi x)$ has a horizontal tangent at $x = 1$.

TRUE OR FALSE

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Questions 3-7: You must show work to receive full credit.

3. (a)[3] Find $f'(1)$, if $f(x) = 3^{\ln x} + \sqrt{1 + \ln x} + 3^5$.

(b)[3] Find $y'(x)$, if $\cos(x^2 y) = \sin y + \tan x$.

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4. (a)[3] In the article *Migration behaviour of grizzly bears in Northern British Columbia: contribution to a modelling approach.* G. Brown et al, Bear Science 4 (June 2012), we find the formula

$$P(t) = \arctan(1.7t) + 4.7$$

where t represents time.

Next, we read "initially, $P(t) \approx 1.7t + 4.7$, which gives a linear relationship." Explain why this statement is correct. [Hint: Think in terms of the linear approximation at $t = 0$.]

(b)[4] A simple model of diffusion states that the concentration of a substance diffusing in air is given by

$$c(x) = e^{-ax^2+b}$$

where a and b are constants and x is the distance from the source. This formula is sometimes simplified using a quadratic approximation near $x = 0$. Find that approximation.

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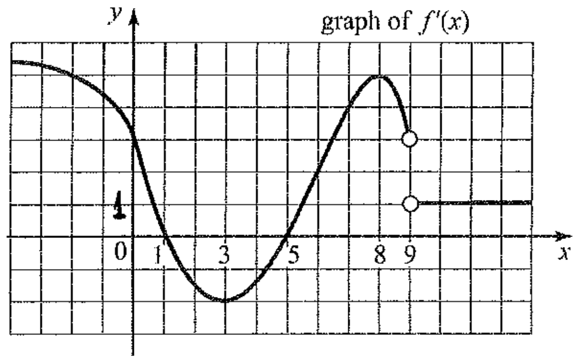
5. Drawn below is the derivative of a function $f(x)$.



(a)[2] State all intervals where $f(x)$ is increasing. Justify your answer.

(b)[2] Find all intervals where $f(x)$ is concave down. Justify your answer.

(c)[2] Describe in words the graph of $f(x)$ on the interval $(9, \infty)$.

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6. The quadratic model for the percent S of cancer cells surviving radiation treatment states that

$$S(d) = e^{-d^2-0.2}$$

where $d \geqslant 0$ is the dose (in Gray) per treatment of radiation.

(a)[1] Show that S is a decreasing function of d when $d \geqslant 0$.

(b)[2] Find (if any) inflection points of $S(d)$. for $d \geqslant 0$

(c)[3] Based on information in (a) and (b), make a sketch of the function $S(d)$ for $d \geqslant 0$. Label intercepts, if any.

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7. The following excerpt is taken from *The laminar cortex model: a new continuum cortex model incorporating laminar architecture.* J. Du, V. Vegh, and D.C. Reutens. PLoS Computational Biology. 8.10 (Oct. 2012).

> the average of membrane potentials of neurons in the element, that is
>
> $$V = \frac{N_e V_e + N_i V_i}{N_e + N_i}$$
>
> where N_e, N_i are the numbers of excitatory and inhibitory neurons and V_e and V_i are the (average) membrane potentials of excitatory and inhibitory neuron populations respectively.

You know that the numbers N_e and N_i are positive, and the membrane potentials V_e and V_i are negative.

(a)[3] Assume that V is a function of N_e. Find the derivative of V and simplify.

(b)[2] Assume that $V_e > V_i$. What does your answer in (a) say about the dependence of V on N_e? Justify your answer.

THE END