

MATHEMATICS 1LS3 TEST 2

Day Class

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Duration of Examination: 60 minutes

McMaster University, 15 October 2014

FIRST NAME (please print): _____

FAMILY NAME (please print): _____

Student No.: _____

THIS TEST HAS 8 PAGES AND 7 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE.

Total number of points is 40. Marks are indicated next to the problem number. Any non-graphing calculator is allowed.

USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

You must show work to receive full credit.

Problem	Points	Mark
1	6	
2	6	
3	6	
4	6	
5	3	
6	7	
7	6	
TOTAL	40	

1. Multiple choice questions: circle ONE answer. No justification is needed.

(a)[3] Which of the functions approach(es) ∞ more quickly than $x^{2.3}$ as $x \rightarrow \infty$?

(I) $f(x) = x^{2.2}$ (II) $f(x) = x^{2.4}$ (III) $f(x) = \ln x$

- (A) none (B) I only (C) II only (D) III only
(E) I and II (F) I and III (G) II and III (H) all three

(b)[3] Assume that $f(x)$ is continuous at $x = 3$ and $f(3) = -2$. Which of the following statements is/are true for every function which satisfies these assumptions?

(I) $y = \sqrt{f(x)}$ is continuous at $x = 3$

(II) $\lim_{x \rightarrow 3} f(x) = -2$

(III) $y = \frac{1}{f(x)}$ is continuous at $x = 3$

- (A) none (B) I only (C) II only (D) III only
(E) I and II (F) I and III (G) II and III (H) all three

2. Identify each statement as true or false, or yes or no (circle your choice). You do not need to justify your answer.

(a)[2] $\lim_{x \rightarrow -\infty} 0.5^x = 0$.

TRUE FALSE

(b)[2] The average rate of change of $f(x) = 3x - 17$ on the interval $[1.5, 1.5064]$ is 3.0064.

TRUE FALSE

(c)[2] The line $y = \pi$ is a horizontal asymptote of the graph of $f(x) = 2 \arctan(x^3 + 1)$.

TRUE FALSE

Questions 3-7: You must show work to receive full credit.

3. The dynamical system $p_{t+1} = \frac{12p_t}{10 + 0.002p_t}$ models the population of caribou in southern regions of Nunavut (p_t is the number of caribou and t is time in years).

(a)[2] Identify the per capita production rate (i.e., give a formula for it) and make a rough sketch of it (you need to show a correct shape and identify the vertical intercept).

(b)[1] Does the dependence of the per capita production on the population size make sense? Answer by using an appropriate biological argument.

(c)[3] Find all equilibrium points of the given system.

4. Find each limit.

(a)[2] Find $\lim_{x \rightarrow (\pi/2)^+} x^2 \sec x$

(b)[2] $\lim_{x \rightarrow 2} \frac{\frac{1}{4} - \frac{1}{x^2}}{x - 2}$

(c)[2] $\lim_{x \rightarrow \infty} e^{-x^2 - 5x - 12}$

5. [3] The following excerpt is taken from *The laminar cortex model: a new continuum cortex model incorporating laminar architecture*. J. Du, V. Vegh, and D.C. Reutens. PLoS Computational Biology. 8.10 (Oct. 2012).

the average of membrane potentials of neurons in the element, that is

$$V = \frac{N_e V_e + N_i V_i}{N_e + N_i}$$

where N_e , N_i are the numbers of excitatory and inhibitory neurons and V_e and V_i are the (average) membrane potentials of excitatory and inhibitory neuron populations respectively.

We view V as a function of N_i (thus, all remaining symbols on the right side are parameters). What is the limit of V as N_i increases beyond any bounds (i.e., as it approaches ∞)?

6. Consider the function

$$f(x) = \begin{cases} \frac{x-1}{x^3-x} & \text{if } x < 1 \\ \frac{x}{2} & \text{if } x \geq 1 \end{cases}$$

(a)[3] Find $\lim_{x \rightarrow 1} f(x)$.

(b)[2] Is $f(x)$ continuous at $x = 1$? Explain why or why not.

(c)[2] Is $f(x)$ continuous at $x = -1$? Explain why or why not.

7. Consider the alcohol consumption dynamical system $a_{t+1} = a_t - \frac{10.5a_t}{4.5 + a_t} + d$, where a_t is the amount of alcohol (in grams) at time t (measured in hours).

(a)[1] What is the meaning of the parameter d ?

(b)[2] What is the meaning of the term $\frac{10.5a_t}{4.5 + a_t}$ in the formula for a_{t+1} ? What are its units?

(c)[3] For which values of d does the given system have a biologically meaningful equilibrium?