1ZA3

Last Day

$$\boxed{\frac{d}{dx} f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}}$$

$$\frac{d}{dx} \arcsin(x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx} \arctan x = \frac{1}{1+x^2}$$

$$\frac{d}{dx} \arccos(x) = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx} \ln x = 1/x$$

Note

$$\boxed{\frac{d}{dx} \ln(\underbrace{f(x)}_{\text{"}u\text{"}})} = \left(\frac{d}{du} \ln u\right) \cdot \frac{d}{dx} u = \frac{1}{u} \cdot f'(x)$$

$$= \boxed{\frac{f'(x)}{f(x)}}$$

Solution

Step 1: take "ln" of $y = f(x)$

$$\ln(y) = \ln\left[(x^2+1)^{57}(x+2)^{102}/(x-3)^{47}\right]$$

Step 2 Expand using "ln" properties

① $\ln(ab) = \ln a + \ln b$
② $\ln(a/b) = \ln a - \ln b$
③ $\ln(a^b) = b \ln a$

$$\ln(y) = \ln\left((x^2+1)^{57}\right) + \ln\left((x+2)^{102}\right) - \ln\left((x-3)^{47}\right)$$

$$= 57\ln(x^2+1) + 102\ln(x+2) - 47\ln(x-3$$

$$\frac{f'(x)}{f(x)} = \text{relative rate of change}$$

$= \text{rate per unit of "stuff"}$

eg <u>if</u> P = pop, $P(t)$, t = time, $P'(t)$ rate of change of pop

& $\frac{P'(t)}{P(t)}$ = rel. rate of change.

= "bunny per hour per bunny"

"Logarithmic Differentiation"

eg. Let's use "Log. Diff." to get $f'(x)$, if

$$f(x) = (x^2+1)^{57}(x+2)^{102} / (x-3)^{41}$$

Step 3 differentiate : don't forget $\frac{d}{dx} \ln(f(x)) = f'(x)/f(x)$

$$\underline{\underline{\frac{1}{y} y'}} = 57 \cdot \frac{2x}{x^2+1} + 102 \cdot \frac{1}{x+2} - 47 \cdot \frac{1}{x-3}$$

implicit! be careful! will always get this!

Step 4 Mult. by y & substitute original $\underline{\underline{f(x)}}$

$$y' = \underline{\underline{y}} \left[\frac{114x}{x^2+1} + \frac{102}{x+2} - \frac{47}{x-3} \right]$$

$$= \left(\frac{(x^2+1)^{57} (x+2)^{102}}{(x-3)^{47}} \right) \left(\frac{114x}{x^2+1} + \frac{102}{x+2} - \frac{47}{x-3} \right)$$

TAH DAH

eg. If $f(x) = x^{\sin x}$

Solution!

Must Use Log. Diff

Step 1 $\ln y = \ln\left(x^{\sin x}\right)$

Step 2

$\ln y = (\sin x)(\ln x)$

Step 3 take derivative!

$$y'/y = \left(\cancel{\frac{d}{dx}\sin x}\right)\ln x + \sin x\left(\cancel{\frac{d}{dx}\ln x}\right)$$

$\cos x$ $\quad$ $1/x$

$$y'/y = \cos x \ln x + \sin x/x$$

These don't

Recap!

$$\frac{d}{dx} x^p = p x^{p-1}$$

p const!

$$\frac{d}{dx} a^{f(x)} = a^{f(x)} \cdot \ln a \cdot f'(x)$$

a constant

Apply Here!

Step 4

$$y' = y\left(\cos x \ln x + \sin x / x\right)$$
$$= x^{\sin x}\left(\cos x \ln x + \sin x / x\right)$$

Hyperbolic Functions

Define Hyperbolic Cosine

$$\cosh(x) = \frac{e^{x} + e^{-x}}{2}$$

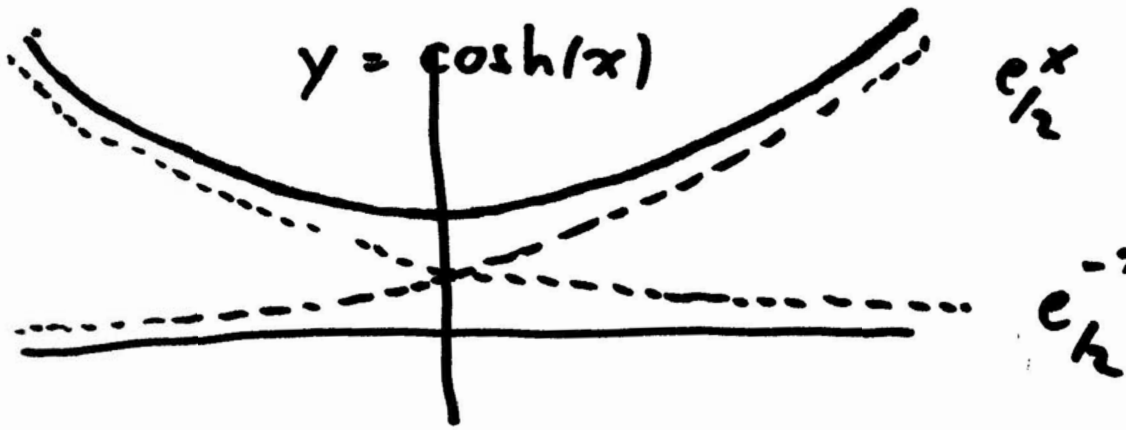
note: $\cosh(0) = (e^{0} + e^{-0})/2 = (1+1)/2 = \underline{\underline{1}}$ } Similar

$$\underline{\underline{\cosh(-x)}} = \frac{e^{-x} + e^{+x}}{2} = \underline{\underline{\cosh(x)}}$$

<u>It's even</u> $\left(f(-x) = f(x) \right)$

to cosine!

Graph!



Define <u>Hyperbolic sine</u>:

$$\sinh(x) = \frac{e^{x} - e^{-x}}{2}$$

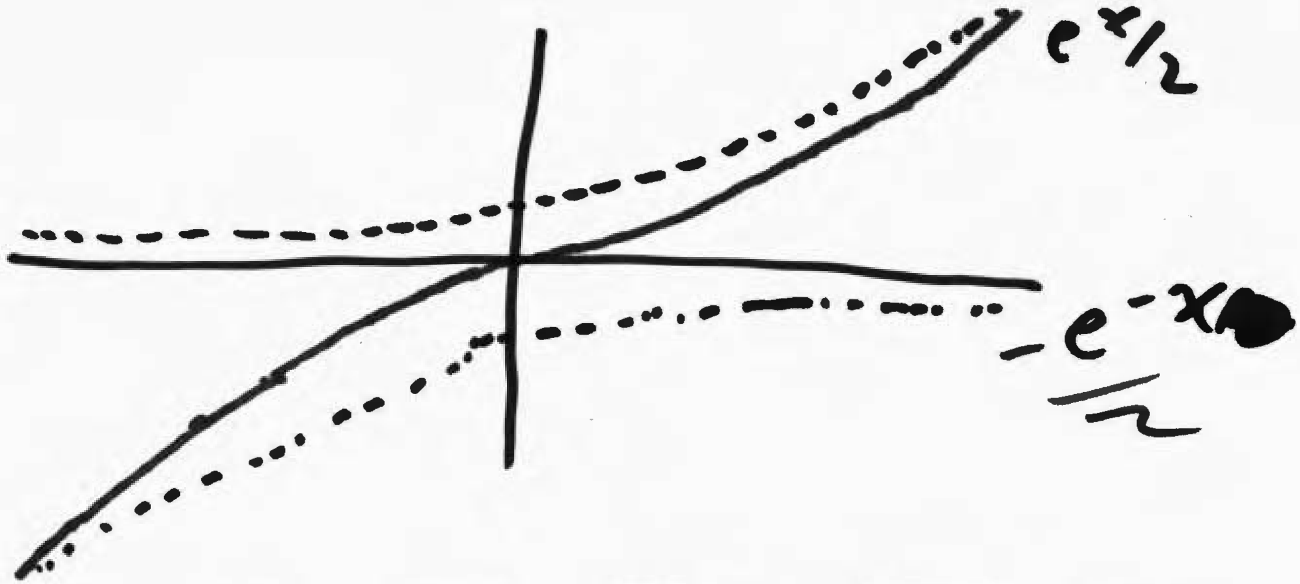
Just $\left\{ \sinh(0) = \frac{e^0 - e^0}{2} = \frac{1-1}{2} = \underline{\underline{0}} \right.$

like $\sin(x)$

$$\sinh(-x) = \frac{e^{-x} - e^{-(-x)}}{2} = \frac{e^{-x} - e^{+x}}{2} = -\sinh(x)$$

$\sinh(x)$ is odd $\left(f(-x) = -f(x) \right)$

Graph



Define Hyperbolic Tangent: $\tanh(x) = \dfrac{\sinh x}{\cosh(x)}$

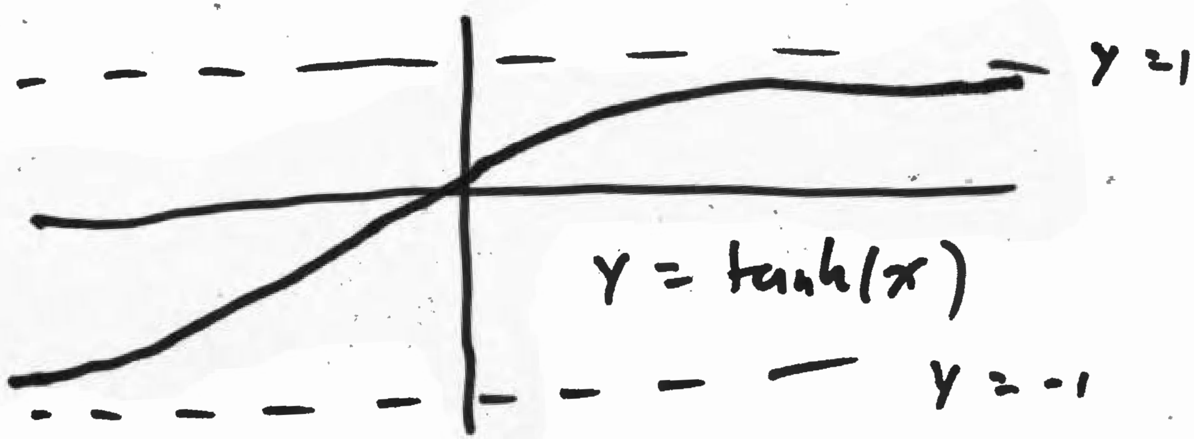
Notice $\tanh(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

$\tanh(0) = \sinh(0)/\cosh(0) = 0/1 = 0$

$\tanh(-x) = \sinh(-x)/\cosh(-x) = -\dfrac{\sinh(x)}{\cosh(x)}$

$= -\tanh(x)$ is odd

graph



$$\lim_{x \to \infty} \tanh(x) = \lim_{x \to \infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} = 1$$

$$\sim \frac{e^x}{e^x} \to 1$$

$$\lim_{x \to \infty} \tanh(x) = \lim_{x \to -\infty} \frac{e^x - \boxed{e^{-x}}}{e^x + \boxed{e^{-x}}} = -1$$

$$\sim \frac{-e^{-x}}{e^{-x}} = -1$$