Math 1A03/1ZA3 Test #2 (Version 1)
November 13th, 2018

Name:_____
(Last Name) (First Name)

Student Number:_____

This test consists of 20 multiple choice questions worth 1 mark each (no part marks), and 1 question worth 1 mark (no part marks) on proper computer card filling. Questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Only the McMaster standard calculator **Casio fx-991 MS or MS Plus** is allowed.

1. Suppose that $2 \leq f'(x) \leq 6$ for all x. Using the Mean Value Theorem, which of the following must be true?

(a) $1 \leq f(5) - f(1) \leq 4$ **(b)** $6 \leq f(7) - f(4) \leq 18$ **(c)** $2 \leq f(5) - f(1) \leq 8$
(d) $3 \leq f(7) - f(4) \leq 8$ **(e)** $2 \leq f(5) - f(1) \leq 12$

2. Let $f(x) = 2\sin x + \sin^2 x$, $0 \leq x \leq 2\pi$. Find the interval(s) on which f is increasing.
(a) $(\frac{\pi}{2}, 2\pi)$ only **(b)** $(0, \frac{3\pi}{2})$ only **(c)** $(\frac{\pi}{2}, \frac{3\pi}{2})$ only **(d)** $(0, \frac{\pi}{2}), (\frac{3\pi}{2}, 2\pi)$ **(e)** $(\frac{3\pi}{2}, 2\pi)$ only

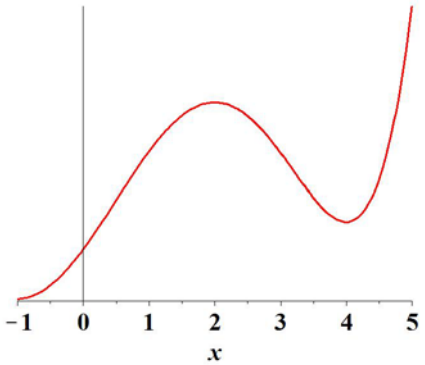
3. Evaluate the following limit,

$$\lim_{x \to \infty} x^{3/2} \sin(1/x)$$

(a) 1 **(b)** $-\frac{2}{3}$ **(c)** ∞ **(d)** 0 **(e)** $-\frac{3}{2}$

4. Suppose that the *derivative* f' of a function f is given to the right. On what interval(s) is f concave up?

(a) $(-1, 2), (4, 5)$ **(b)** $(-1, 1), (3, 5)$
(c) $(2, 4)$ **(d)** $(1, 3)$ **(e)** $(-1, 1), (4, 5)$



5. Suppose that

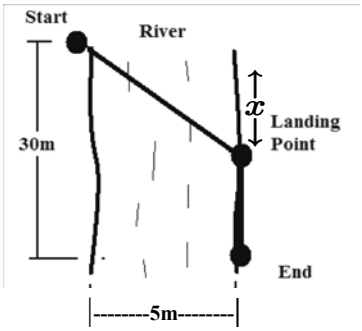
$$\lim_{x\to a} f(x) = 0, \qquad \lim_{x\to a} g(x) = 0, \qquad \lim_{x\to a} h(x) = 1, \qquad \lim_{x\to a} p(x) = \infty$$

Which of the following limits are indeterminate?

(i) $\lim_{x\to a} f(x)^{p(x)}$
(ii) $\lim_{x\to a} f(x)^{g(x)}$
(iii) $\lim_{x\to a} h(x)^{p(x)}$

(a) all of them **(b)** (ii) and (iii) only **(c)** (i) and (iii) only **(d)** (i) and (ii) only **(e)** (ii) only

6. A deer sees a tasty bush of berries across a 5 m wide river, but 30 m downstream. He'd better hurry to get these berries before some other deer sees them. He could swim across the river, then run along the shore to the berries, swim directly to the berries, or swim to a point part way along and run the rest of the way along the shore. If the deer swims at 8m/s and runs on land at 10m/s, what landing point x on the shore will minimize the time to get to the berries?



(a) 15 **(b)** $\sqrt{5/7}$ **(c)** 4 **(d)** $20/\sqrt{21}$ **(e)** $20/3$

7. Consider the set of all points on the circle about the origin with radius $\sqrt{8}$. What is the largest possible value of the product, xy, for a point (x, y) on that circle?

(a) 4 **(b)** 2 **(c)** 1 **(d)** $\frac{1}{2}$ **(e)** $\sqrt{2}$

8. Evaluate the following limit,

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(\frac{i}{n}\right)^3$$

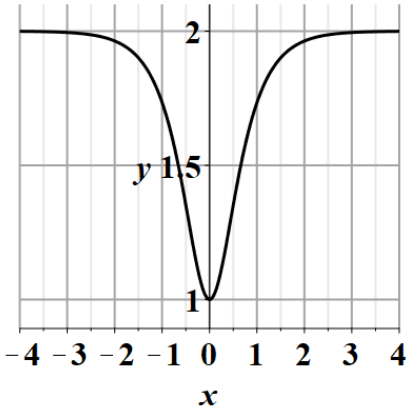
(a) 2 **(b)** 0 **(c)** $\frac{1}{4}$ **(d)** $\frac{1}{3}$ **(e)** $\frac{1}{2}$

9. Estimate the area under the graph of $y = x^2 + 2$ on the interval $[1, 3]$ using 4 rectangles and midpoints.
(a) $\frac{101}{8}$ **(b)** $\frac{43}{4}$ **(c)** $\frac{59}{4}$ **(d)** $\frac{101}{4}$ **(e)** $\frac{59}{2}$

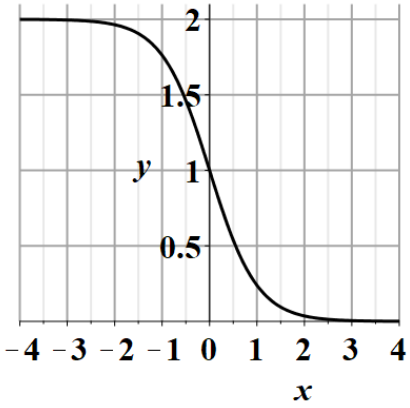
10. Which of the following is the graph of the function

$$f(x) = \frac{2}{1 + e^{-2x}} \, ?$$

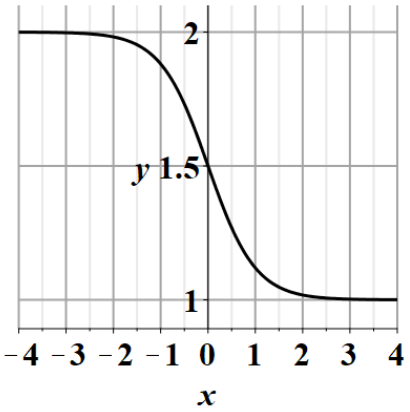
(a)



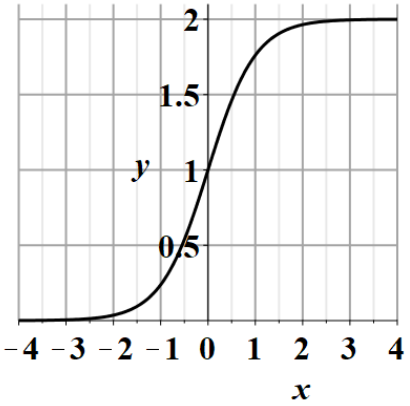
(b)



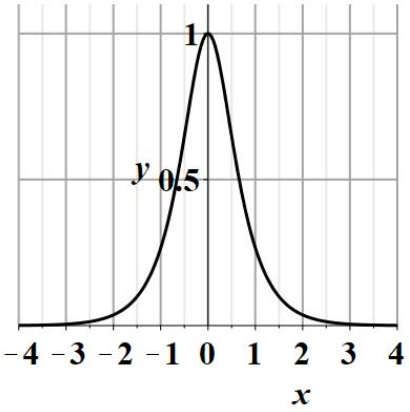
(c)



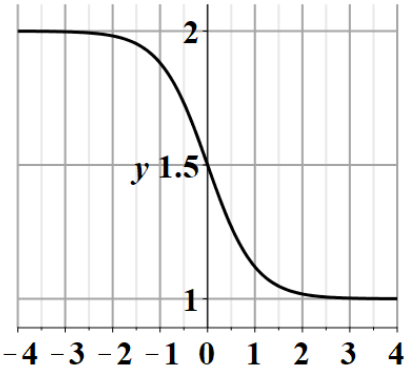
(d)



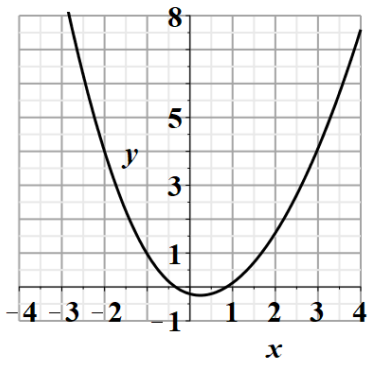
(e)



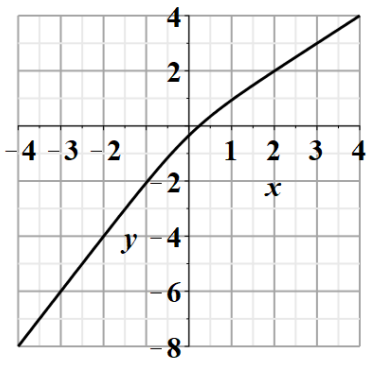
11. Consider the function f whose graph is given to the right. Which of the following is the graph of an *antiderivative* of f?



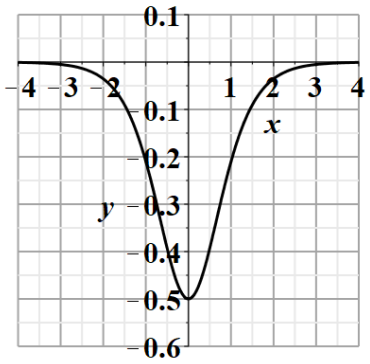
(a)



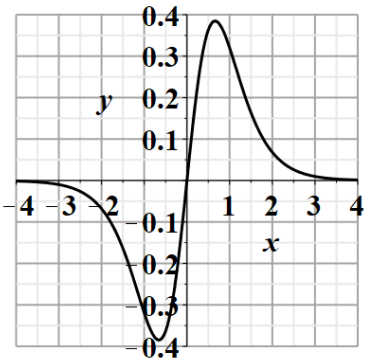
(b)



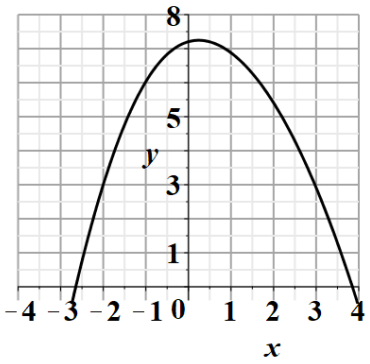
(c)



(d)



(e)



12. Evaluate the following sum,

$$\sum_{i=1}^{n}[i^5-(i-1)^5]$$

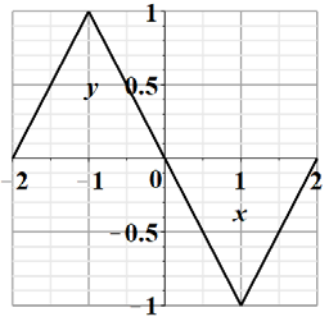
(a) n^5-1 **(b)** $\dfrac{n^5(n^5+1)}{2^5}$ **(c)** $\dfrac{n^5(n^5+1)(2n^5+1)}{6^5}$ **(d)** n^5 **(e)** n^5+1

13. Find an expression for the area under the graph of $f(x)=\sqrt{x},\ 1\le x\le 4$ as a limit.

(a) $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\frac{4}{n}\sqrt{\frac{4i}{n}}$ **(b)** $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\frac{3}{n}\sqrt{\frac{3i}{n}}$ **(c)** $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\frac{1}{n}\sqrt{1+\frac{4i}{n}}$

(d) $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\frac{1}{n}\sqrt{\frac{4i}{n}}$ **(e)** $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\frac{3}{n}\sqrt{1+\frac{3i}{n}}$

14. Let f be the function whose graph is shown to the right. Evaluate $\int_{-1}^{2}[f(x)+3]\,dx$.



(a) 0 **(b)** $\frac{17}{2}$ **(c)** $\frac{19}{2}$ **(d)** $\frac{5}{2}$ **(e)** $\frac{7}{2}$

15. Let

$$g(x)=\int_{2x}^{3}\sqrt{x^3+2x^2+9}\,dx$$

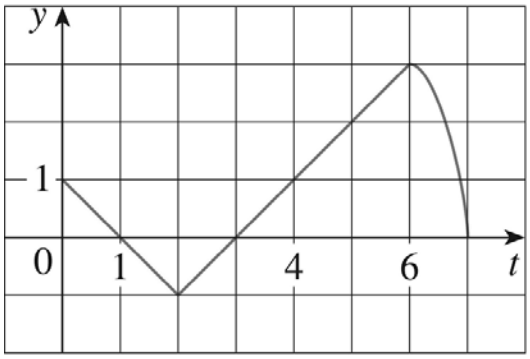
Find $g'(1)$.

(a) -5 **(b)** 10 **(c)** -10 **(d)** 5 **(e)** 3

16. Suppose that $f'(x)=e^{\sqrt{x}}/\sqrt{x}$ and that $f(1)=0$. Find $f(4)$.

(a) $e(e-1)$ **(b)** e^2 **(c)** $2e^2$ **(d)** $2e(e-1)$ **(e)** $\frac{1}{2}e^2$

17. Let $f(t)$ be the function whose graph is shown to the right. Let $g(x) = \int_0^x f(t)\,dt$.
On what intervals is g concave up?



(a) $(0,1),(3,7)$ **(b)** $(2,6)$ **(c)** $(1,3)$
(d) $(0,2),(6,7)$ **(e)** $(0,7)$

18. Evaluate the following integral,

$$\int \frac{\sqrt{x^3}+x}{\sqrt{x}}\,dx$$

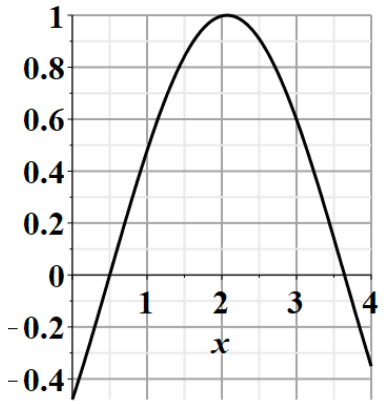
(a) $1+\frac{1}{2}x^{-1/2}+C$ **(b)** $\frac{1}{3}x^3+\frac{2}{3}x^{3/2}+C$ **(c)** $\dfrac{\frac{2}{5}x^{5/2}+\frac{1}{2}x^2}{\frac{2}{3}x^{3/2}}+C$
(d) $2x^{1/2}+\frac{1}{2}x^{-1/2}+C$ **(e)** $\frac{1}{2}x^2+\frac{2}{3}x^{3/2}+C$

19. Let $f(x)$ be a continuous function with $f(x)>0$ for all x, $f(1)=2$, and $f(3)=5$. Which of the following integrals is equal to

$$\int_1^3 f'(x)\ln[f(x)]\,dx\ ?$$

(a) $\int_2^5 \ln u\,du$ **(b)** $\int_2^5 \frac{1}{u}\ln u\,du$ **(c)** $\int_1^3 \frac{1}{u}\ln u\,du$ **(d)** $\int_1^3 \ln u\,du$ **(e)** $\int_1^5 u^2\ln u\,du$

20. Let f be the function whose graph is shown to the right. Find all values of c that satisfy the conclusion of the Mean Value Theorem on the interval $[0,4]$.



(a) 1 and 3 **(b)** 1 only **(c)** 1.9 only
(d) 3 only **(e)** 0.5 and 3.7

21. Correctly fill out the bubbles corresponding to all 9 digits of your student number, as well as the version number of your test in the correct places on the computer card.
Note: You are writing **Version 1**.

Answers (Version 1)**:**

1. b **2.** d **3.** c **4.** a **5.** b **6.** e **7.** a **8.** c **9.** a **10.** d
11. b **12.** d **13.** e **14.** b **15.** c **16.** d **17.** b **18.** e **19.** a **20.** c