Math 1A03/1ZA3 Test #1 (Version 1)
October 17th, 2017

Name:____
(Last Name) (First Name)

Student Number:______________________

This test consists of 20 multiple choice questions worth 1 mark each (no part marks), and 1 question worth 1 mark (no part marks) on proper computer card filling. Questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Only the McMaster standard calculator Casio fx-991 MS or MS Plus is allowed.

1. Find a simplified expression for $\cos(\tan^{-1} x)$.

(a) $\frac{x}{\sqrt{x^2-1}}$ **(b)** $\frac{1}{\sqrt{x^2-1}}$ **(c)** $\frac{1}{\sqrt{1+x^2}}$ **(d)** $\frac{x}{\sqrt{1+x^2}}$ **(e)** $\frac{1}{\sqrt{1-x^2}}$

2. Consider the equation $\ln x = x - 2$. Use of the Intermediate Value Theorem on which one of the following intervals would lead to the conclusion that the equation has at least one root on the given interval?

(a) $(1, e^2)$ **(b)** $(1, e)$ **(c)** (e^2, e^3) **(d)** $(2, 3)$ **(e)** $(1, 2)$

3. Find a formula for the inverse function of

$$y = e^{1-e^x}$$

(a) $1 + \ln(\ln x)$ **(b)** $\ln(1 - \ln x)$ **(c)** $\ln(\ln x)$ **(d)** $\ln(1 + \ln x)$ **(e)** $1 - \ln(\ln x)$

4. Find the equation of the tangent line to the following curve at the point $(0, \frac{1}{2})$.

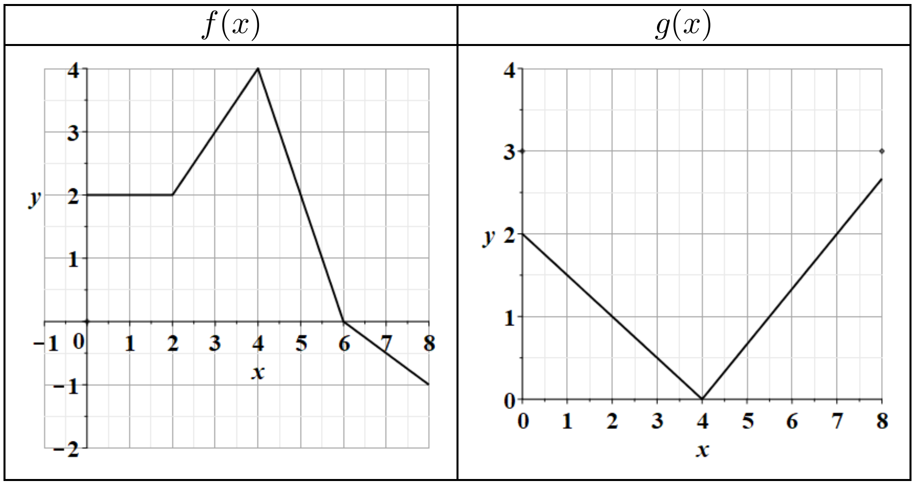
$$y = \frac{e^x}{x + 2e^x}$$

(a) $y = -\frac{1}{2}x + \frac{1}{2}$ **(b)** $y = -\frac{1}{4}x + 1$ **(c)** $y = x + \frac{1}{2}$ **(d)** $y = -\frac{1}{4}x + \frac{1}{2}$ **(e)** $y = \frac{1}{2}x + \frac{1}{2}$

5. Find the slope of the tangent line to the curve $xy + e^{2y} = e$ at the point where $x = 0$.
(a) $-\frac{1}{2e}$ **(b)** $\frac{1}{2e}$ **(c)** $-\frac{1}{4e}$ **(d)** $\frac{1}{2}$ **(e)** $-\frac{1}{2}$

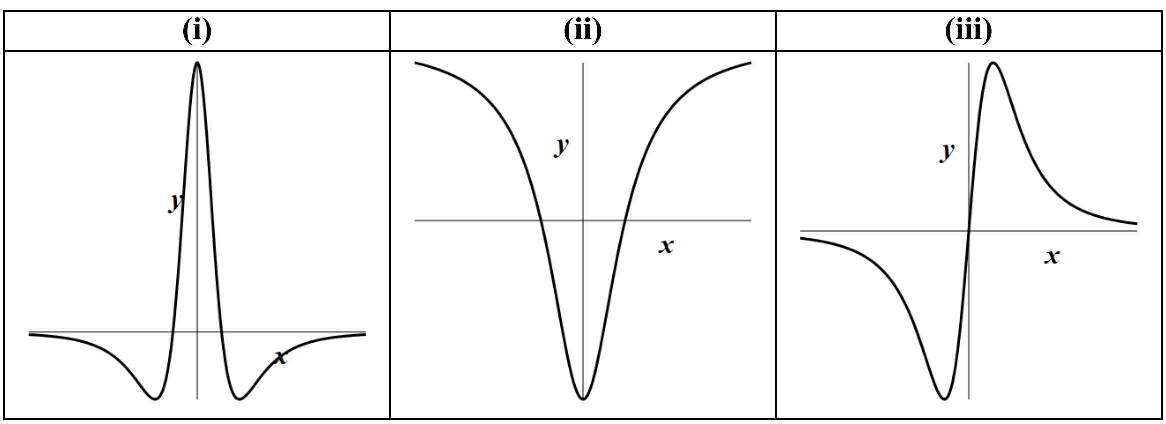
6. If Newton's method is used to find a root of the equation $2^x = 2 - x^2$ with initial approximation $x_1 = 0$, find the second approximation x_2.
(a) 1 **(b)** $-\ln 2$ **(c)** $\ln 2$ **(d)** -1 **(e)** $\frac{1}{\ln 2}$

7. Let f and g be the functions whose graphs are shown below, and let $h(x) = g(x) + x^2 f(x)$. Find $h'(5)$



(a) $\frac{85}{3}$ **(b)** 29 **(c)** -29 **(d)** $-\frac{85}{3}$ **(e)** $-\frac{88}{3}$

8. The following figures are the graphs of $f(x)$, $f'(x)$, and $f''(x)$, in SCRAMBLED order. Identify each curve.



(a) (i) $f'(x)$, (ii) $f(x)$, (iii) $f''(x)$ **(b)** (i) $f(x)$, (ii) $f''(x)$, (iii) $f'(x)$
(c) (i) $f''(x)$, (ii) $f(x)$, (iii) $f'(x)$ **(d)** (i) $f'(x)$, (ii) $f''(x)$, (iii) $f(x)$
(e) (i) $f''(x)$, (ii) $f'(x)$, (iii) $f(x)$

9. Let $f(x) = \sec x$. Find $f''(\pi/4)$.
(a) $3\sqrt{2}$ **(b)** $2\sqrt{2}$ **(c)** $\frac{3}{\sqrt{2}}$ **(d)** $2\sqrt{3}$ **(e)** $\frac{2}{\sqrt{3}}$

10. Find the point on the graph of the function $f(x) = \ln x - \ln(2 + \ln x)$ where the tangent line is horizontal.
(a) e^2 **(b)** e **(c)** e^{-3} **(d)** e^{-2} **(e)** e^{-1}

11. Let $f(x) = x^{x^2}$. Find $f'(2)$.

(a) $32(1 + 2\ln 2)$ **(b)** $128 \ln 2$ **(c)** $128(1 + \ln 2)$ **(d)** 128 **(e)** $64(1 + 2\ln 2)$

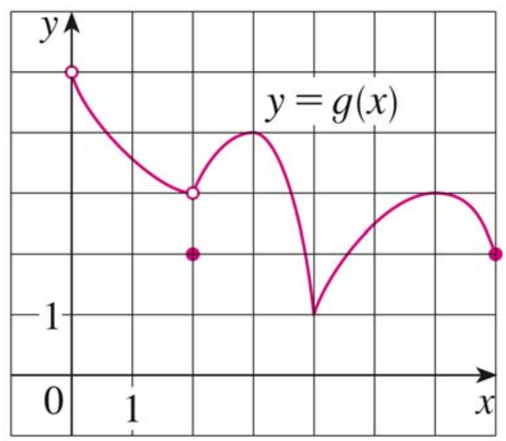
12. Let $f(x) = \tan^{-1}\sqrt{1 + x^2}$. Find $f'(1)$.
(a) $\frac{1}{\sqrt{2}(1+\sqrt{2})}$ **(b)** $\frac{\sqrt{2}}{3}$ **(c)** $\frac{1}{2\sqrt{2}}$ **(d)** $\frac{1}{3\sqrt{2}}$ **(e)** $\frac{1}{2\sqrt{2}(1+\sqrt{2})}$

13. Let $g(x)$ be the function defined on the interval $(0, 7]$ whose graph is given to the right. Consider the following statements

(i) g has no absolute maximum
(ii) $\lim_{x\to 2} g(x)$ does not exist
(iii) g has no absolute minimum

Which of the above statements are true?

(a) (i) and (ii) only **(b)** (i) only **(c)** none of them
(d) (i) and (iii) only **(e)** all of them



14. Find the absolute maximum and absolute minimum of the function $f(x) = e^x + e^{-2x}$ on the interval $[0, \ln 2]$.
(a) $5 \cdot 2^{1/3}$, 2 **(b)** $3 \cdot 2^{1/3}$, 2 **(c)** $\frac{9}{4}$, $3 \cdot 2^{-2/3}$ **(d)** $\frac{9}{4}$, 2 **(e)** $\frac{9}{4}$, $2^{-2/3}$

15. Which of the following is equal to $\cosh^2 x + \sinh^2 x$?

(a) $\cosh 2x$ **(b)** 1 **(c)** $\sinh 2x$ **(d)** e^{2x} **(e)** e^{-2x}

16. Let $h(x) = f(xf(x))$ where $f(2) = 3$, $f'(2) = 4$, and $f'(6) = 2$. Find $h'(2)$.

(a) 21 **(b)** 22 **(c)** 20 **(d)** 19 **(e)** 18

17. Suppose that Newton's method is used to locate a root of the equation $f(x) = 0$ with initial approximation is $x_1 = 2$. If the second approximation is $x_2 = 5$, find the equation of the tangent line to $y = f(x)$ at the point $(2, 6)$.

(a) $y = x + 4$ **(b)** $y = \frac{1}{2}x + 5$ **(c)** $y = 2x + 2$ **(d)** $y = -2x + 10$ **(e)** $y = -x + 5$

18. In Maple, what command could we use to convert the expression $4x^3$ into a function f?

(a) `>f=convert(4*x^3,function);`
(b) `>f:=unapply(4*x^3,x);`
(c) `>f:=unapply(4x^3,x);`
(d) `>f:=convert(4x^3,function);`
(e) `>f:=convert(4*x^3,function);`

19. Consider the following function

$$f(x) = \begin{cases} \frac{x^2-2x}{x^2-4} & x < 1 \\ a & x = 1 \\ \frac{\sqrt{x}-1}{x-1} & x > 1 \end{cases}$$

For what value of a is $f(x)$ right continuous at $x = 1$?

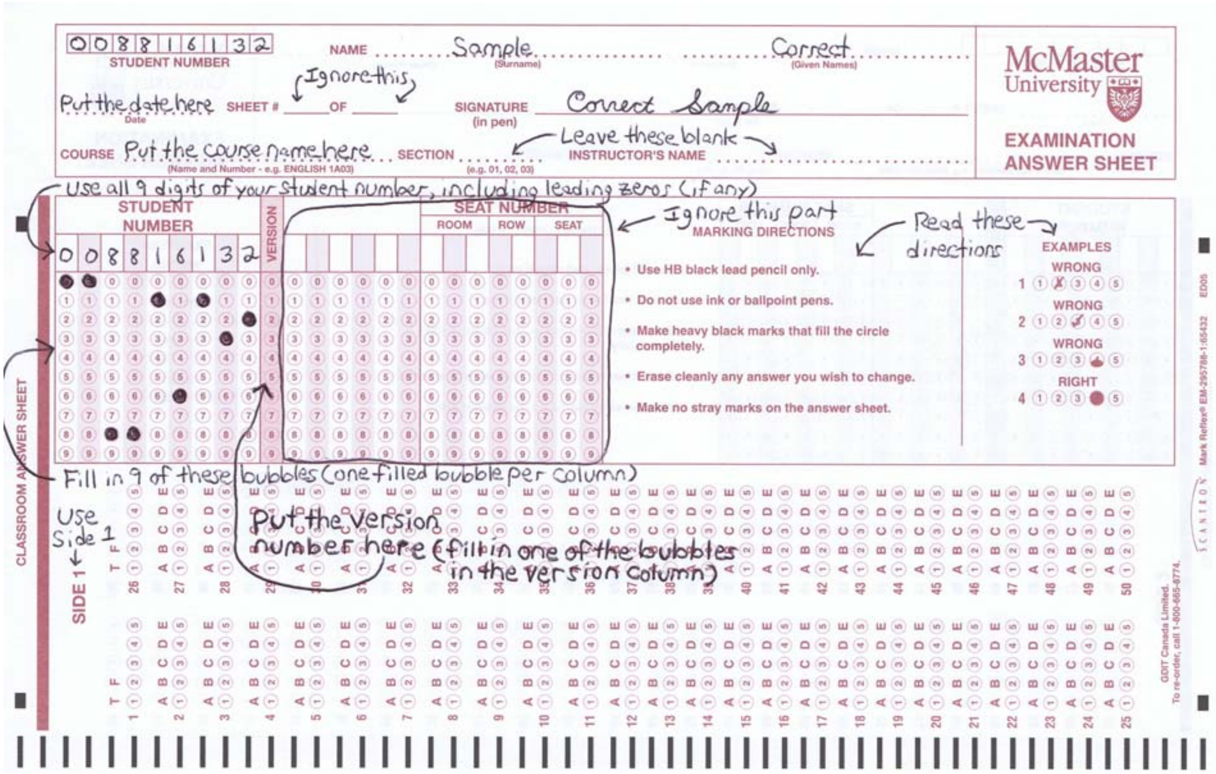
(a) 2 **(b)** $\frac{1}{\sqrt{2}}$ **(c)** $\frac{1}{4}$ **(d)** $\frac{1}{2}$ **(e)** $\frac{1}{3}$

20. Which of the following limits represents the slope of the tangent line to the following function at $x = 0$?

$$f(x) = \frac{2}{1-3x}$$

(a) $\lim\limits_{x\to 0} \dfrac{\frac{2}{1-3x} + 1}{x}$ **(b)** $\lim\limits_{x\to 0} \dfrac{\frac{2}{1-3x} - 2}{x}$ **(c)** $\lim\limits_{h\to 0} \dfrac{\frac{2}{1-3+h} - \frac{2}{1-3x}}{h}$ **(d)** $\lim\limits_{h\to 0} \dfrac{\frac{2}{1-3-h} - \frac{2}{1-3x}}{h}$
(e) $\lim\limits_{x\to 0} \dfrac{\frac{2}{1-3x} - 1}{x}$

21. Correctly fill out the bubbles corresponding to all 9 digits of your student number, as well as the version number of your test in the correct places on the computer card.
Note: You are writing **Version 1**.
Hint:



Answers (Version 1)**:**

1. c **2.** a **3.** b **4.** d **5.** c **6.** e **7.** e **8.** c **9.** a **10.** e
11. a **12.** d **13.** b **14.** c **15.** a **16.** b **17.** d **18.** b **19.** d **20.** b