COMPSCI 3MI3 : Assignment 1
Fall 2021
Instructor: Nicholas Moore
Maximum Grade 25/24

General Instructions

- Your answers should take the form of well-reasoned arguments, proving the result the question asks for. For every step in your solution, you should be able to say, x is true, so y *must* be the case. Not every step needs to be written in equations, but every step needs to be *mathematical* and *detailed*.
- Submit your answers as a PDF document. Solutions not in PDF format will receive a 2 mark penalty. Solutions typeset using LaTeXwill receive a 1 mark bonus, but you must also submit your source file to be eligible. Handwritten solutions will not be accepted.
- The length of the text comprising a question has no correlation to the length of time required to complete it.
- This assignment is to be submitted through the corresponding assignment dropbox on Avenue.

Questions

1. (5 points) **Question 1: Reflexive Closures**
   Suppose we are given a relation R on a set S. Define the relation R' as follows:

   $$R' = R \cup (s, s)|s \in S$$

   That is to say, R' contains all the pairs in R, plus all pairs of the form (s, s). Show that R' is the reflexive closure of R.

2. (5 points) **Question 2: Preservation**
   Suppose that R is a binary relation on a set S, and P is a predicate on S that preserves R. Show that P also preserves P^*, where P^* is the *reflexive and transitive closure* of R.

3. (6 points) **Question 3: Transitive Closure**
   The following is a more constructive definition of the transitive closure on a relation R. First, we define the following sequence of sets of pairs.

   $$R_0 = R \tag{1}$$

   $$R_{i+1} = R_i \cup \{(s, u) \mid \text{for some}\, t, (s, t) \in R_i and (t, u) \in R_i\} \tag{2}$$

   Another way to say this, is that we construct each $R_i + 1$ by adding to R_i all the pairs that can be obtained by "one step of transitivity" from pairs already in R. Finally, we define the relation R^+ as the union of all the R_i:

   $$R^+ = \bigcup_i R_i \tag{3}$$

   Show that R^+ is the transitive closure of R.

4. (3 points) **Question 4: Ordinary Induction**
   Demonstrate, using the principle of ordinary induction covered in lecuture, that each element in the Fibbonaci sequence above the 2^{nd} is greater than the preceeding number.

5. (2 points) **Question 5: Complete Induction**
   Modify the above proof to use complete induction rather than ordinary induction.

6. (3 points) **Question 6: Structural Induction**
   Demonstrate, using the principle of structural induction, that, for search operations over binary search trees (`https://en.wikipedia.org/wiki/Binary_search_tree`), only one branch of the tree needs to be searched.