

Note: Hybrid Type I & Type II improper integrals

Example: $\int_0^{\infty} \frac{1}{\sqrt{x}(1+x)} dx =$

3 Mathematical Induction (Appendix E)

Let S_n a statement involving the positive integer n .

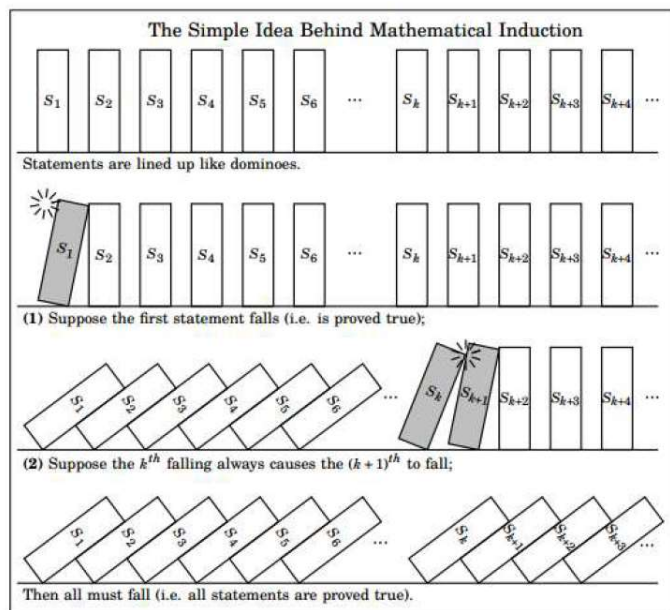
Assume that

1. S_1 is true and
2. if S_k is true, then S_{k+1} is true.

Then, S_n is true **for all** positive integers.

For example: If you know that

1. the first domino falls and
 2. if any specific domino falls, then the next domino falls,
- then you can conclude that **all** dominos fall.



Example: Show that $1 + 2 + \dots + n = \frac{n(n+1)}{2}$ for all $n \in \{1, 2, 3, \dots\}$.

The statement to check is: $S_n =$ _____

1. Let us check that S_1 is true, i.e.,

2. Suppose that S_k is true, i.e.,

_____,

show that S_{k+1} is true, i.e.,

_____.

Then, the induction axiom implies that

Example: Show that $3n \leq 3^n$ for all $n \in \{0, 1, 2, 3, \dots\}$.

The statement to check is: $S_n =$ _____

1. Let us check that S_1 is true, i.e.,

2. Suppose that S_k is true, i.e.,

_____,

show that S_{k+1} is true, i.e.,

Then, the induction axiom implies that

_____.

Example: Decide if $2 + 3n < 2^n$ for all $n \in \{3, 4, 5, \dots\}$.

The statement to check is: $S_n =$ _____

1. Let us check that S_1 is true, i.e.,

2. Suppose that S_k is true, i.e.,

_____,

show that S_{k+1} is true, i.e.,

Then, the induction axiom implies that

4 Infinite Sequences and Series (Chapter 11)

A **sequence** is an _____ of real numbers written in a particular order, e.g.

$a_1, a_2, a_3, \dots, a_n, \dots$, denoted by _____, or short _____.

a_n is the ____ term of the sequence and n is called _____.

Typically, a sequence is given by a formula.

Write down the first couple of terms of this sequence

- $a_n = (-1)^{n+1}$

- $a_n = \frac{n}{n+1}$

- $a_n = \sqrt{n-5}$, for _____

- $a_n = a_{n-1} + a_{n-2}$ with $a_1 = a_2 = 1$.

If we can make a_n as close as possible to L as we like by taking n sufficiently large, then we write _____. We say that $\{a_n\}$ **converges** if _____, else, we say that $\{a_n\}$ _____.

Examples: Find

- $\lim_{n \rightarrow \infty} (-1)^n$

- $\lim_{n \rightarrow \infty} \frac{n}{n+1}$

- $\lim_{n \rightarrow \infty} \frac{1}{n^k}$

- $\lim_{n \rightarrow \infty} \ln(n)$