McMaster University
Math 1LT3

ASSIGNMENT 8

Sections 6, 7, 8, 10, 12, 13, and 14 in the Grey Module

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1. Consider a population of tigers that grows according to $p_{t+1} = p_t + I_t$ where $I_t = 35$ with a 40% chance and $I_t = -20$ with a 60% chance, and t is measured in years. Suppose that initially there are 100 tigers. Define X to be the number of tigers after three years.

(a) Determine the probability mass function, $p(x)$, for X. Draw a histogram for $p(x)$.

(b) Compute the expected value and standard deviation for X.

(c) What is the probability that the number of tigers will be within one standard deviation of the mean after three years?

2. A shipment of 2000 containers has arrived at the port of Vancouver. Suppose that each container has a 0.4% chance of containing some form of contraband (e.g. illegal drugs). As part of the customs inspection, 25 containers are selected at random and checked.

(a) Out of the 25 containers checked, what is the expected number of containers containing contraband? How many containers should be checked if you want the expected number to be exactly 1?

(b) If only 25 are checked, what is the probability that at least one will contain contraband?

2. continued...

(c) Express the number of ways that 25 containers can be chosen from 2000 in the form of a binomial coefficient, then write this in terms of factorials.

(d) Suppose that customs decided to increase the number of inspected containers to 200. Find the probability that there will be at least one container containing contraband in two ways: using the binomial distribution, and then using the Poisson approximation. Is it valid to use the Poisson approximation in this case? In the case in part (b)?

3. Consider the function $f(x) = \dfrac{3}{x^4}$ on the interval $[1, \infty)$.

(a) Verify that $f(x)$ is a probability density function for a continuous random variable X.

(b) Determine the corresponding cumulative distribution function, $F(x)$.

(c) Graph both $f(x)$ and $F(x)$.

3. continued...

(d) Calculate $P(X > 2)$. Illustrate on both graphs in (c) what this number represents geometrically.

(e) Compute the expected value and standard deviation of X.

(f) What is the probability that a value X will be within one standard deviation of its mean?

4. Find the following using Table 14.4.

(a) Given that X is normally distributed with mean 3 and variance 4, find the probability that X is less than 4.1.

(b) Let $X \sim N(-2, 9)$; find $P(1 \leq X \leq 5)$.

(c) Suppose that $X \sim N(2, 144)$. Find an approximate value of x such that (i) $P(X \leq x) = 0.95$ and (ii) $P(X > x) = 0.3$.

5. Let H represent the height of a university student. Assume that heights are approximately normally distributed with a mean of 168 cm and a standard deviation of 6 cm.

(a) Write the formula for the probability density function for H and sketch its graph, labelling the mean, maximum, and location of inflection points. Approximately what percentage of students are between 162 cm and 174 cm tall?

(b) What is the probability that a randomly chosen student is taller than 180 cm? Shade the area representing this probability on your sketch in part (a). Sketch the probability density function for the standard normal random variable Z and shade in the area representing the equivalent probability.

6. (a) Determine the Taylor polynomial of degree 6 for $f(x) = e^{-\frac{x^2}{2}}$ near $x = 0$. (Hint: Find the Taylor polynomial of degree 3 for $g(x) = e^x$ then replace x by $-\frac{x^2}{2}$ in the formula).

(b) Use your approximation in part (a) to estimate $P(0 \leq Z \leq 0.5)$. Compare this to the value obtained using Table 14.4.

THE END