Math 1A03/1ZA3

Sample Test Questions for Test #2

Name:____

(Last Name) (First Name)

Student Number:_____________________ **Tutorial Number:**_____________________

This test consists of 85 multiple choice questions worth 1 mark each (no part marks), and 1 question worth 1 mark (no part marks) on proper computer card filling. All questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Only the McMaster standard calculator Casio fx-991 is allowed.

1. Given the following functions on the interval $[-1, 1]$, for which can we conclude using the Mean Value Theorem that $f'(x) = \frac{f(1)-f(-1)}{1-(-1)}$ for some c on the interval $(-1, 1)$?
(a) $f(x) = x^{1/3}$ **(b)** $f(x) = \tan x$ **(c)** $f(x) = 1/x$ **(d)** $f(x) = \frac{4x^2-1}{2x+1}$ **(e)** $f(x) = |x|$

2. Find all points that satisfy the conclusion of the Mean Value Theorem for the function $y = x^2 - 2x + 1$ on the interval $[-2, 2]$
(a) $x = 0$ **(b)** $x = -1, 1$ **(c)** $x = 1$ only **(d)** $x = 2$ **(e)** No such point

3. A function $f(x)$ is a continuous on $[2, 5]$ and differentiable on $(2, 5)$. Furthermore, the derivative is bounded such that $|f'(x)| \leq \frac{1}{x}$. Using Mean Value Theorem, what can we conclude about $f(5) - f(2)$?
(a) $0.2 \leq f(5) - f(2) \leq 0.5$ **(b)** $0.6 \leq f(5) - f(2) \leq 1.5$
(c) $-1.5 \leq f(5) - f(2) \leq 1.5$ **(d)** $-0.6 \leq f(5) - f(2) \leq 0.6$
(e) $-0.5 \leq f(5) - f(2) \leq 0.5$

4. Find the horizontal asymptotes of

$$f(x) = \frac{e^x}{e^x + x}$$

(a) $y = 1$ **(b)** $y = 0$ **(c)** $y = -1, 0$ **(d)** $y = 0, 1$ **(e)** none

5. Find the value of the limit $\lim\limits_{x\to 1^-} \frac{\ln(2x)}{\ln x}$.
(a) $\frac{1}{2}$ **(b)** 1 **(c)** 2 **(d)** $-\infty$ **(e)** ∞

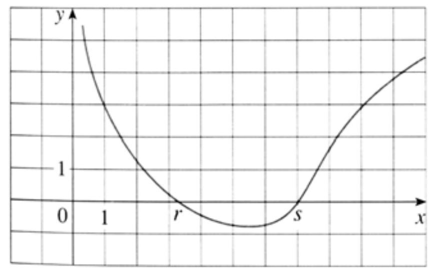
6. Let $f(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 + x - 2$. Find the intervals (if any) where f is concave up.
(a) $(-\infty, 0)$ **(b)** $(-\frac{2}{3}, \infty)$ **(c)** $(-\frac{2}{3}, \infty), (\frac{2}{3}, \infty)$
(d) $(-\infty, 0), (\frac{2}{3}, \infty)$ **(e)** $(-\infty, -\frac{2}{3}), (0, \infty)$

7. The function $f(x) = x + \frac{1}{x}$ is:

(a) Increasing on $(-1, 0)$ **(b)** Decreasing on $(0, 1)$
(c) Increasing on $(0, 1)$ **(d)** Decreasing on $(1, \infty)$
(e) Increasing on $(0, \infty)$

8. The graph of the *derivative* f' of a function f is shown. On what intervals is f increasing?

(a) $(0, 7.5)$ **(b)** $(0, r), (7, 11)$ **(c)** $(5.5, 11)$
(d) $(r, 7)$ **(e)** $(r, 11)$



9. The graph of the *derivative* f' of a function f is On what intervals is f concave upward?

(a) $(0, 7.5)$ **(b)** $(0, r), (7, 11)$ **(c)** $(5.5, 11)$ **(d)** $(r, 7)$ **(e)** $(r, 11)$

10. Find $\lim_{x\to\infty} \left(\frac{3x}{1+2x^2}\right)^x$

(a) 1 **(b)** 0 **(c)** ∞ **(d)** $e^{3/2}$ **(e)** $-\infty$

11. Find $\lim_{x\to(\pi/2)^-} (\tan x)^{\cos x}$

(a) 1 **(b)** 0 **(c)** ∞ **(d)** $-\infty$ **(e)** $e^{\pi/2}$

12. Let $f(x) = xe^{1/x}$. Find the intervals on which f is concave up.

(a) $(0, \infty)$ **(b)** $(-\infty, 0)$ **(c)** (e, ∞) **(d)** $(-\infty, e)$ **(e)** $(1, \infty)$

13. Find $\lim_{x\to 0^+} (1 - 2x)^{1/x}$

(a) 0 **(b)** 1 **(c)** ∞ **(d)** $-\infty$ **(e)** e^{-2}

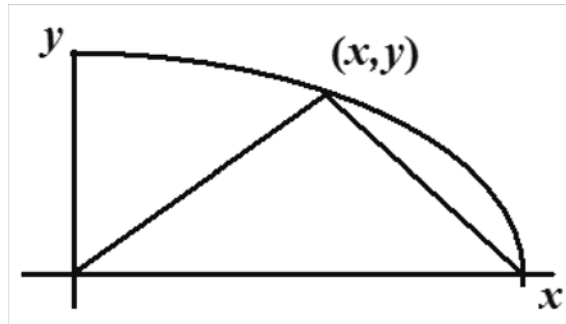
14. Find the area of the largest rectangle that can be inscribed in a semicircle of radius 6.

(a) 36 **(b)** 6 **(c)** 6π **(d)** 36π **(e)** 30

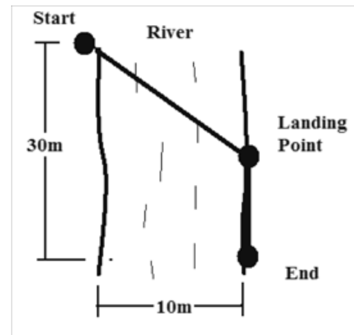
15. A cylindrical container is made with a volume of 2000π m^3. What is the diameter of the cylinder with the minimum surface area?

(a) 10 **(b)** 20 **(c)** $\frac{4}{\pi}$ **(d)** $\frac{2\sqrt{3}}{\pi}$ **(e)** $2\left(\frac{50}{\pi}\right)^{1/3}$

16. A triangle is inscribed on the quarter-ellipse given by $x^2 + 4y^2 = 16, \ x, y \geq 0$
Find the greatest possible area of such a triangle.
(a) 0 **(b)** 2 **(c)** 4 **(d)** $2\sqrt{3}$ **(e)** $2\sqrt{5}$



17. A deer sees a tasty bush of berries across a 10 m wide river, but 30 m downstream. He better hurry to get these berries before some other deer sees them. He could swim across the river, then run along the shore to the berries, swim directly to the berries, or swim to a point part way along and run the rest of the way along the shore. If the deer swims at 4m/s and runs on land at 10m/s, what point on the shore will the landing point minimize the time to get to the berries?
(a) 15 **(b)** $\sqrt{5/7}$ **(c)** 4 **(d)** $20/\sqrt{21}$ **(e)** $20\sqrt{3}$



18. A closed tin box is to be made with a square base. After it is assembled the bottom is to be lined with a layer of velvet. At the time of it's production, we are told that tin costs 2 cents/cm^2, and velvet is 10 cents/cm^2. If the total cost of the box is to be fixed at \$2.40, what is the cost of velvet that will be needed for the box that has the maximum enclosed volume?
(a) \$0.32 **(b)** \$0.72 **(c)** \$0.57 **(d)** \$1.32 **(e)** \$1.73

19. What is the y-value of the point on the parabola $y = x^2$ that is closest to the point $(1, \frac{1}{2})$?
(a) $2^{-1/3}$ **(b)** $2^{-2/3}$ **(c)** $\frac{1}{2}$ **(d)** 1 **(e)** $1 + 2^{-2/3}$

20. A rocket accelerates on its journey according to the following equation:

$$a(t) = te^{2t^2-4t}, \ 0 \leq t \leq 10$$

where t is measured in seconds. Assuming positive speed, what time does it reach its maximum velocity?
(a) 0 seconds **(b)** 0.5 seconds **(c)** 2 seconds **(d)** 10 seconds **(e)** Never

21. Consider the set of all points on the radius two circle about the origin. What is the largest possible value of the product, xy, for a point (x, y) on that circle?
(a) 1 **(b)** 2 **(c)** $\sqrt{2}$ **(d)** 4 **(e)** $\frac{1}{2}$

22. Let $f(x) = \frac{\ln x}{x}$. Find the intervals on which f is increasing.
(a) $(0, 1)$ **(b)** $(0, e)$ **(c)** (e, ∞) **(d)** $(\frac{1}{e}, \infty)$ **(e)** $(1, \infty)$

23. Find $f(1)$ if $f''(t) = 3/\sqrt{t}$, $\quad f(4) = 20$, $\quad f'(4) = 7$.
(a) 5 **(b)** 6 **(c)** 7 **(d)** 8 **(e)** 9

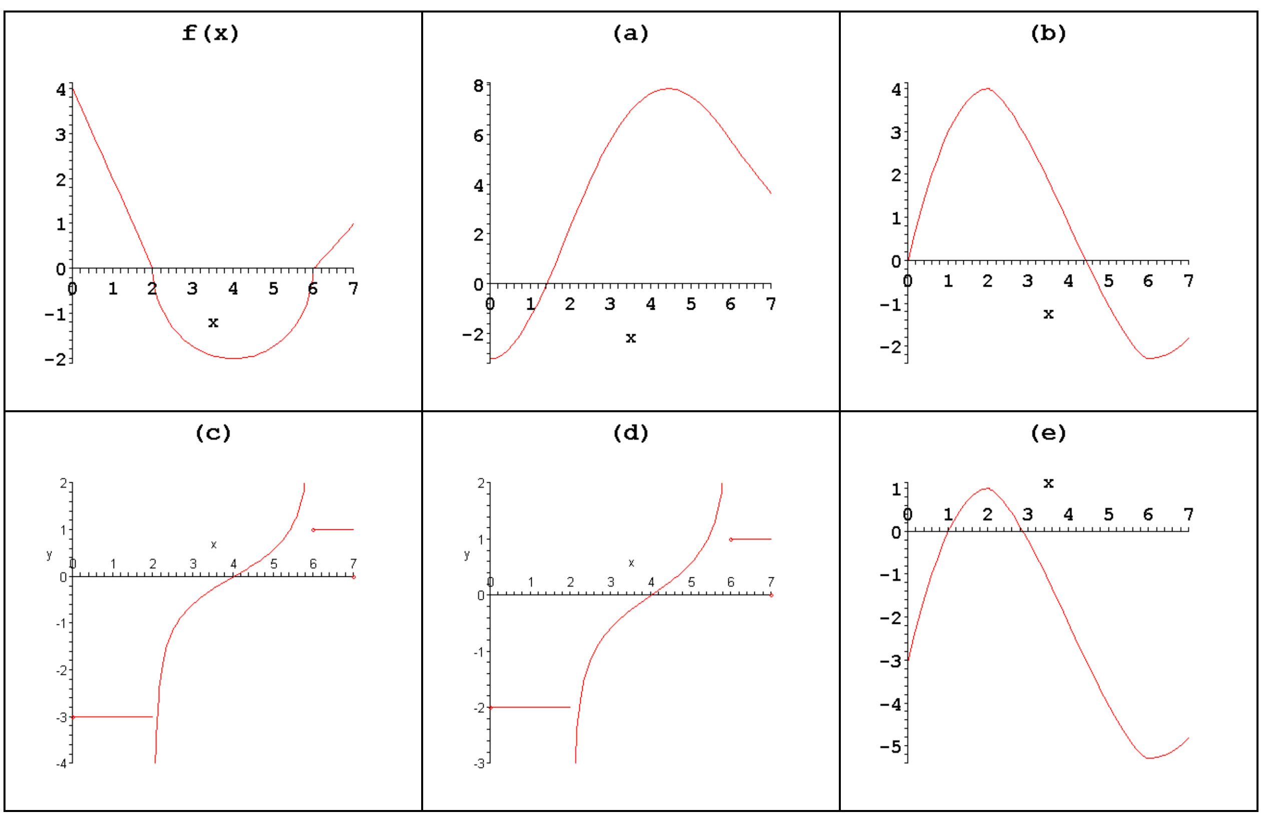
24. Find $f(\frac{\pi}{3})$ if $f'(x) = \sec x \tan x + \sec^2 x$, $f(0) = 2$.
(a) $2+\sqrt{2}$ **(b)** $1+\sqrt{2}$ **(c)** $3+\sqrt{3}$ **(d)** $2+\sqrt{3}$ **(e)** $3+\frac{1}{\sqrt{3}}$

25. Evaluate $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{2}{n}\left[\left(\frac{3i}{n}\right)^3 + \frac{5i}{n} + 2\right]$
(a) 45 **(b)** 5 **(c)** $\frac{45}{4}$ **(d)** 15 **(e)** $\frac{45}{2}$

26. Write the following sum in sigma notation: $\frac{3}{4} + \frac{5}{9} + \frac{7}{16} + \cdots + \frac{17}{81}$.
(a) $\sum\limits_{i=1}^{7} \frac{(2i+1)}{(i+1)^2}$ **(b)** $\sum\limits_{i=1}^{8} \frac{(2i+1)}{(i+1)^2}$ **(c)** $\sum\limits_{i=0}^{8} \frac{(2i+1)}{(i+1)^2}$ **(d)** $\sum\limits_{i=1}^{8} \frac{2i}{(i+1)^2}$ **(e)** $\sum\limits_{i=1}^{n} \frac{(2i+1)}{(i+1)^2}$

27. Evaluate $\int x(x+3)^2 dx$
(a) $\frac{7}{2}x^2 + 2x^3 + \frac{1}{4}x^4 + C$ **(b)** $\frac{9}{2}x^2 + 2x^3 + \frac{1}{4}x^4 + C$ **(c)** $\frac{1}{6}x^2(x+3)^3 + C$
(d) $\frac{9}{2}x^2 + 2x^3 + \frac{3}{4}x^4 + C$ **(e)** $\frac{1}{2}x^2 + 2x^3 + \frac{1}{4}x^4 + C$

28. The graph of a function $f(x)$ is shown in the figure. Make a rough sketch of an antiderivative F, given that $F(0) = -3$.



29. Use the definition of area to find an expression for the area under the graph of f as a limit. Do not evaluate the limit. $f(x) = \sin x\sqrt{e^x + 3}, \quad 3 \le x \le 8$

(a) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left(\sin(\frac{5i}{n})\sqrt{e^{\frac{5i}{n}} + 3}\right)\frac{5}{n}$

(b) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left(\sin(1 + \frac{8i}{n})\sqrt{e^{(1+\frac{8i}{n})} + 3}\right)\frac{8}{n}$

(c) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left(\sin(3 + \frac{5i}{n})\sqrt{e^{(3+\frac{5i}{n})} + 3}\right)\frac{5}{n}$

(d) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left(\sin(\frac{8i}{n})\sqrt{e^{\frac{8i}{n}} + 3}\right)\frac{8}{n}$

(e) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left(\sin(1 + \frac{5i}{n})\sqrt{e^{(1+\frac{5i}{n})} + 3}\right)\frac{5}{n}$

30. Write as a single integral in the form $\int_a^b f(x)dx$:

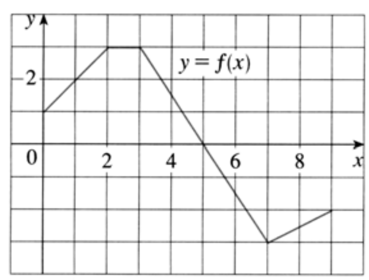
$$\int_0^2 f(x)dx - \int_{-2}^2 f(x)dx - \int_{-4}^{-2} f(x)dx$$

(a) $\int_{-4}^2 f(x)dx$ **(b)** $-\int_{-4}^{-2} f(x)dx$ **(c)** $-\int_{-4}^2 f(x)dx$ **(d)** $-\int_{-4}^0 f(x)dx$
(e) $\int_{-4}^{-2} f(x)dx$

31. Estimate the area under the graph of $f(x) = 4 - x^2$ from $x = -1$ to $x = 2$ using three rectangles and left endpoints.
(a) 10 **(b)** 7 **(c)** $\frac{37}{4}$ **(d)** $\frac{17}{2}$ **(e)** 9

32. If $F(x) = \int_{x^2+x}^1 e^{-t^2}\, dt$, find the value of $F'(1)$.
(a) $-\frac{3}{e}$ **(b)** $-\frac{3}{e^4}$ **(c)** $\frac{3}{e^4}$ **(d)** $-\frac{1}{e^4}$ **(e)** $\frac{1}{e^4}$

33. Let $f(x)$ be the function whose graph is given to the right. Evaluate $\int_3^9 f(x)dx$
(a) -8 **(b)** 8 **(c)** 11 **(d)** -5 **(e)** 3



34. Let $g(x) = \int_0^x f(t)dt$, where f is the function whose graph is given in #33 above. find the intervals (if any) on which g is concave up.
(a) $(3, 7)$ **(b)** $(0, 2), (7, 9)$ **(c)** $(3, 9)$ **(d)** $(0, 5)$ **(e)** none

35. Evaluate

$$\int_{-\pi/2}^{\pi/2} \sin x \sqrt{x^2+9}\, dx$$

(a) $\frac{\pi}{2}\sqrt{\left(\frac{\pi}{4}\right)^2+9}$ **(b)** $\pi\sqrt{\left(\frac{\pi}{4}\right)^2+9}$ **(c)** 0 **(d)** $2\sqrt{\left(\frac{\pi}{4}\right)^2+9}$ **(e)** 1

36. Evaluate $\lim_{n\to\infty}\left[\sum_{i=1}^{n} e^{(1+5i/n)}\left(\frac{5}{n}\right)\right]$ by first expressing the given limit as a definite integral.

(a) $\frac{e^7}{7}-\frac{e^2}{2}$ **(b)** $\ln e^6 - \ln e$ **(c)** e^5-1 **(d)** e^6-e **(e)** $e^{5/n}$

37. Evaluate $\int_{-2}^{2}\left(3\sqrt{4-x^2}+2|x|\right)dx$ by using the properties of the integral and relating it to areas. (i.e., do NOT try to find antiderivatives).

(a) $12\pi+4$ **(b)** $12\pi+16$ **(c)** $6\pi+16$ **(d)** $12\pi+8$ **(e)** $6\pi+8$

38. Evaluate $\int_{\pi/6}^{\pi/4} \sec x \tan x\, dx$

(a) $\frac{\sqrt{3}}{2}-2$ **(b)** $\sqrt{2}-\frac{2}{\sqrt{3}}$ **(c)** $\frac{1}{\sqrt{2}}-\frac{2}{\sqrt{3}}$ **(d)** $\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}$ **(e)** $2-\frac{1}{\sqrt{2}}$

39. Evaluate $\displaystyle\int_{1/\sqrt{2}}^{1} \frac{6}{\sqrt{1-t^2}} dt$

(a) 2π **(b)** 4π **(c)** π **(d)** $\frac{9}{2}\pi$ **(e)** $\frac{3}{2}\pi$

40. Write as a single integral in the form $\int_a^b f(x)dx$:

$$\int_{-2}^{2} f(x)dx + \int_{2}^{5} f(x)dx - \int_{-2}^{1} f(x)dx$$

(a) $\int_1^2 f(x)dx$ **(b)** $\int_{-2}^{1} f(x)dx$ **(c)** $\int_{-2}^{5} f(x)dx$ **(d)** $\int_1^5 f(x)dx$ **(e)** $\int_{-2}^{2} f(x)dx$

41. Let $F(x) = \int_1^x f(t)dt$, where $f(t) = \int_1^{t^3} \frac{\sqrt{1+u}}{u} du$. Find $F''(2)$.

(a) $\frac{9}{2}$ **(b)** $\frac{3}{8}$ **(c)** $\frac{9}{2}-\sqrt{2}$ **(d)** $\frac{3}{8}-\sqrt{2}$ **(e)** $\frac{9}{2}-\frac{3}{8}$

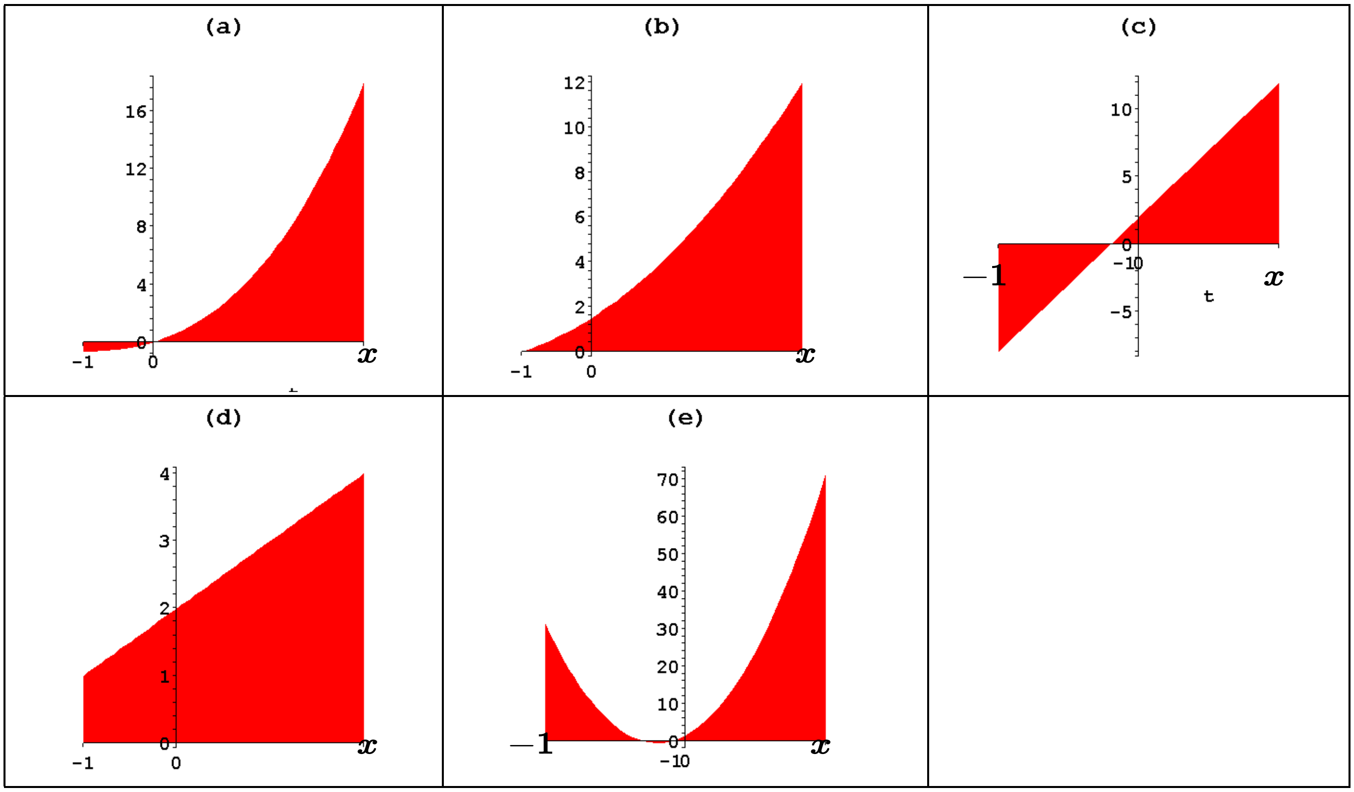
42. Evaluate $\int_1^3\left(10^x + \frac{\sqrt[3]{x^2}+x^4}{x^5}\right)dx$

(a) $990 - \frac{3}{10\sqrt[3]{3^{10}}} + \ln 3 + \frac{3}{10}$ **(b)** $\frac{990}{\ln 10} - \frac{3}{10\sqrt[10]{3^3}} + \ln 3 + \frac{3}{10}$

(c) $\frac{990}{\ln 10} - \frac{3}{100\sqrt[3]{3^{10}}} + \ln 3 + \frac{3}{10}$ **(d)** $\frac{990}{\ln 10} - \frac{3}{10\sqrt[3]{3^{10}}} + \ln 3 + \frac{3}{100}$

(e) $\frac{990}{\ln 10} - \frac{3}{10\sqrt[3]{3^{10}}} + \ln 3 + \frac{3}{10}$

43. Let $g(x) = \displaystyle\int_{-1}^{x} (t+2)dt,\ x > -1$. Sketch the area represented by $g(x)$.



44. Evaluate the following integral. $\int_0^1 \frac{3x^2}{\sqrt[4]{16-15x^3}}dx$

(a) $\frac{28}{45}$ **(b)** $\frac{4}{45}$ **(c)** $\frac{32}{45}$ **(d)** $-\frac{28}{45}$ **(e)** $-\frac{32}{45}$

45. Evaluate the following integral. $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$

(a) e **(b)** 1 **(c)** 2 **(d)** 4 **(e)** 3

46. Evaluate the following integral. $\int_0^1 \sqrt[3]{8-7x}\,dx$

(a) $\frac{16}{28}$ **(b)** $-\frac{45}{28}$ **(c)** $-\frac{3}{28}$ **(d)** $\frac{3}{28}$ **(e)** $\frac{45}{28}$

47. Evaluate the following integral. $\int_0^{\pi^2/4} \frac{\cos\sqrt{x}}{\sqrt{x}}dx$

(a) $\frac{1}{\sqrt{2}}$ **(b)** 1 **(c)** 0 **(d)** $\sqrt{2}$ **(e)** 2

48. Evaluate the following integral. $\int_0^3 \frac{dx}{2x+3}$

(a) $\frac{8}{81}$ **(b)** $\frac{1}{2}\ln 3$ **(c)** $\frac{1}{2}\ln 6$ **(d)** $\ln 3$ **(e)** $\ln 6$

49. Find the vertical asymptotes of $f(x) = e^{-1/(x+1)^2}$
(a) $x = 0$ **(b)** $x = 0, -1$ **(c)** $x = -1$ **(d)** $x = 1$ **(e)** none

50. How many inflection points does $f(x) = x^4 + 3x + 2$ have?
(a) 0 **(b)** 1 **(c)** 2 **(d)** 3 **(e)** 4

51. The function $f(x) = |x^2 - 1|$ is:
(a) Increasing on $(0, \infty)$ **(b)** Decreasing on $(-1, 0), (1, \infty)$
(c) Increasing on $(0, 1)$ **(d)** Increasing on $(-1, 0), (1, \infty)$
(e) Decreasing on $(0, \infty)$

52. Let $f(x) = x(\ln x)^2$. Find the intervals on which f is increasing.
(a) $(\frac{1}{e^2}, 1)$ **(b)** $(0, 1), (e, \infty)$ **(c)** $(0, \frac{1}{e^2}), (1, \infty)$ **(d)** $(1, e)$ **(e)** (e^2, ∞)

53. Let $f(x) = x(\ln x)^2$. Find the intervals on which f is concave up.
(a) $(0, \frac{1}{e})$ **(b)** $(0, \frac{1}{e^2})$ **(c)** $(\frac{1}{e^2}, \infty)$ **(d)** $(\frac{1}{e}, \infty)$ **(e)** $(1, e)$

54. Find $\lim_{x \to 0^+} x(\ln x)^2$.
(a) 0 **(b)** 1 **(c)** ∞ **(d)** $-\infty$ **(e)** e^2

55. Find $\lim_{x \to \infty} \frac{e^x}{\ln(1 - \frac{2}{x})}$
(a) 0 **(b)** 1 **(c)** ∞ **(d)** $-\infty$ **(e)** $\ln 2$

56. Find $\lim_{x \to 0^+} xe^{1/x}$
(a) 0 **(b)** 1 **(c)** ∞ **(d)** $-\infty$ **(e)** e

57. Find the shortest distance from the line $y = 3x + 5$ to the origin.
(a) $\frac{3}{2}$ **(b)** $\frac{\sqrt{370}}{2}$ **(c)** 5 **(d)** $\frac{\sqrt{10}}{2}$ **(e)** $\frac{\sqrt{5}}{3}$

58. Let $f(x) = xe^{-1/x^2}$. Find all of the asymptotes of $f(x)$.
(a) $x = 0, y = 1$ **(b)** $x = 0, y = 0$ **(c)** $x = 0$ **(d)** $x = 0, y = e$ **(e)** none

59. Let $f(x) = xe^{-1/x^2}$. Find the largest interval on which f is increasing.
(a) $(1, \infty)$ **(b)** (e, ∞) **(c)** $(-\infty, \infty)$ **(d)** $(0, \infty)$ **(e)** $(-\infty, 0)$

60. Let $f(x) = xe^{-1/x^2}$. Find the intervals on which f is concave up.
(a) $(-\infty, -\sqrt{2}), (0, \sqrt{2})$ **(b)** $(-\sqrt{2}, 0)$ **(c)** $(-\infty, \sqrt{2})$ **(d)** $(-\sqrt{2}, \infty)$
(e) $(-\sqrt{2}, \sqrt{2})$

61. Find $f(e)$ if $f'(x) = 10^x + \frac{1}{2x}$, $f(1) = 2$.
(a) $\frac{10^{e+1}}{e+1} - \frac{95}{2}$ **(b)** $\frac{10^e - 10}{\ln 10} + \frac{5}{2}$ **(c)** $\frac{10^e - 10}{\ln 10} + 4$ **(d)** $10^e - \frac{15}{2}$ **(e)** $10^e - 8$

62. Find $f(\frac{1}{\sqrt{2}})$ if $f'(x) = \frac{6}{\sqrt{1-x^2}}$, $f(0) = 1$.
(a) $\pi + 1$ **(b)** 2π **(c)** $\frac{3\pi}{2}$ **(d)** $2\pi + 1$ **(e)** $\frac{3\pi}{2} + 1$

63. Find $f(8)$ if $f'(x) = \frac{\sqrt[3]{x^2} + x^2}{x^3}$, $f(1) = \frac{5}{4}$
(a) $\frac{125}{64} + 3\ln 2$ **(b)** $\frac{75}{64} + 3\ln 2$ **(c)** $\frac{125}{64} + 2\ln 2$ **(d)** $\frac{75}{64} + 2\ln 2$ **(e)** $\frac{75}{64} + \ln 2$

64. Evaluate the following limit

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} \left(\frac{i}{n}\right)^2$$

(a) $\frac{1}{3}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{1}{6}$ **(d)** $\frac{1}{8}$ **(e)** $\frac{1}{5}$

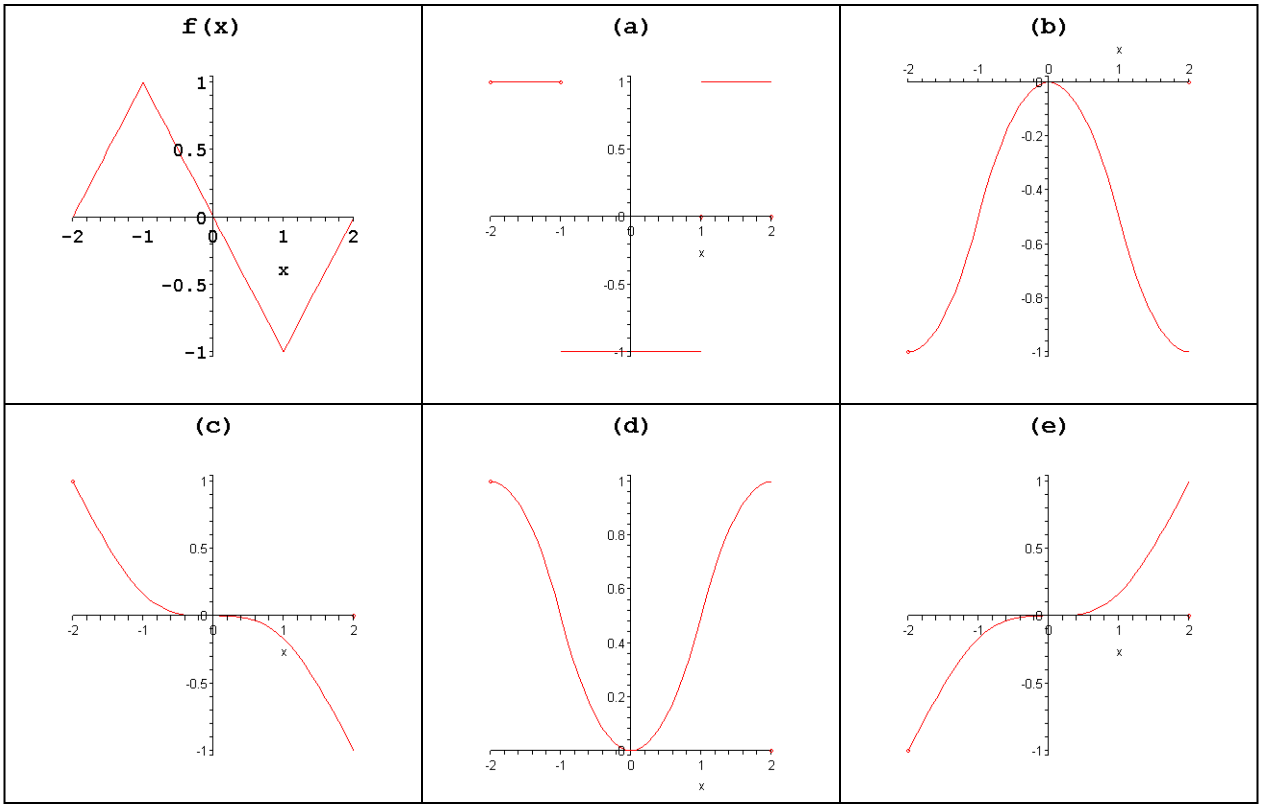
65. Evaluate the following telescoping sum:

$$\sum_{i=4}^{9{,}999} \left(\frac{1}{i} - \frac{1}{i+1}\right)$$

(a) $\frac{39}{10{,}000}$ **(b)** $\frac{297}{40{,}000}$ **(c)** $\frac{397}{40{,}000}$ **(d)** $\frac{2{,}499}{10{,}000}$ **(e)** $\frac{97}{40{,}000}$

66. Evaluate $\int \tan^3 x \, dx$
(a) $\frac{1}{4}\tan^4 x + C$ **(b)** $\frac{1}{4\sec^2 x}\tan^4 x + C$ **(c)** $\cot x - \frac{1}{3}\cot^3 x + C$
(d) $\frac{1}{2}\tan^2 x + \ln|\cos x| + C$ **(e)** $3\tan^2 x \sec^2 x + C$

67. The graph of a function f is shown in the figure below. Make a rough sketch of an antiderivative F, given that $F(0) = 0$.



68. Evaluate $\int_1^{e^2} \frac{1}{2x} dx$

(a) 0 **(b)** 1 **(c)** $\frac{1}{2}\ln(e^2-1)$ **(d)** $\frac{1}{2}(1-e^{-4})$ **(e)** 2

69. Evaluate $\int_0^2 f(x)dx$ where $f(x) = \begin{cases} x^2 & \text{if } 0 \le x < 1 \\ \sqrt{x} & \text{if } 1 \le x \le 2 \end{cases}$

(a) $\frac{1}{3} + \frac{2}{3}(2\sqrt{2}-1)$ **(b)** $\frac{1}{3} - \frac{2}{3}(2\sqrt{2}-1)$ **(c)** $\frac{8}{3} + \frac{4}{3}\sqrt{2}$
(d) $\frac{8}{3} + \frac{2}{3}(2\sqrt{2}-1)$ **(e)** $\frac{2}{9}(2\sqrt{2}-1)$

70. Evaluate $\int_0^{\pi/3} \sec^2 t \, dt$

(a) 0 **(b)** $\frac{7}{3}$ **(c)** 3 **(d)** $\sqrt{3}$ **(e)** π

71. Evaluate $\int_{\sqrt{3}}^{1} \frac{2}{1+t^2} \, dt$

(a) $-2\ln 2$ **(b)** $\frac{\pi}{6}$ **(c)** $2\ln 2$ **(d)** $-\frac{4}{3}\sqrt{3}$ **(e)** $-\frac{\pi}{6}$

72. Evaluate $\int_{-2}^{1} |x+1| dx$

(a) $\frac{3}{2}$ **(b)** $\frac{5}{2}$ **(c)** 2 **(d)** 1 **(e)** 3

73. Use the midpoint rule with $n = 4$ to approximate $\int_1^3 (x^3+1)\, dx$.

(a) 22 **(b)** $\frac{45}{2}$ **(c)** $\frac{87}{4}$ **(d)** 16 **(e)** 29

74. Using the definition of integral,

$$\int_1^3 \sin x\, dx$$

is equal to

(a) $\lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(\frac{2i}{n}\right)\frac{2}{n}$ **(b)** $\lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(1+\frac{i}{n}\right)\frac{1}{n}$ **(c)** $\lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(\frac{2i}{n}\right)\frac{1}{n}$

(d) $\lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(1+\frac{2i}{n}\right)\frac{1}{n}$ **(e)** $\lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(1+\frac{2i}{n}\right)\frac{2}{n}$

75. Evaluate $\lim_{x\to 0} \frac{1}{x^3}\int_0^x \ln(1+t^2)dt$

(a) 0 **(b)** 1 **(c)** ∞ **(d)** $\frac{1}{3}$ **(e)** $\frac{1}{2}$

76. Estimate the area under the graph of $f(x) = 2 - \sqrt{x}$ from $x = 0$ to $x = 4$ using 5 rectangles and midpoints.

(a) $0.8(2-\sqrt{.4}) + 0.8(2-\sqrt{1.2}) + 0.8(2-\sqrt{2.0}) + 0.8(2-\sqrt{2.8})$
$+\,0.8(2-\sqrt{3.6})$

(b) $0.8(2-\sqrt{.8}) + 0.8(2-\sqrt{1.6}) + 0.8(2-\sqrt{2.4}) + 0.8(2-\sqrt{3.2})$
$+\,0.8(2-\sqrt{4.0})$

(c) $0.8(2-0) + 0.8(2-\sqrt{.8}) + 0.8(2-\sqrt{1.6}) + 0.8(2-\sqrt{2.4})$
$+\,0.8(2-\sqrt{3.2})$

(d) $(2-\sqrt{.4}) + (2-\sqrt{1.2}) + (2-\sqrt{2.0}) + (2-\sqrt{2.8}) + (2-\sqrt{3.6})$

(e) $(2-\sqrt{.8}) + (2-\sqrt{1.6}) + (2-\sqrt{2.4}) + (2-\sqrt{3.2}) + (2-\sqrt{4.0})$

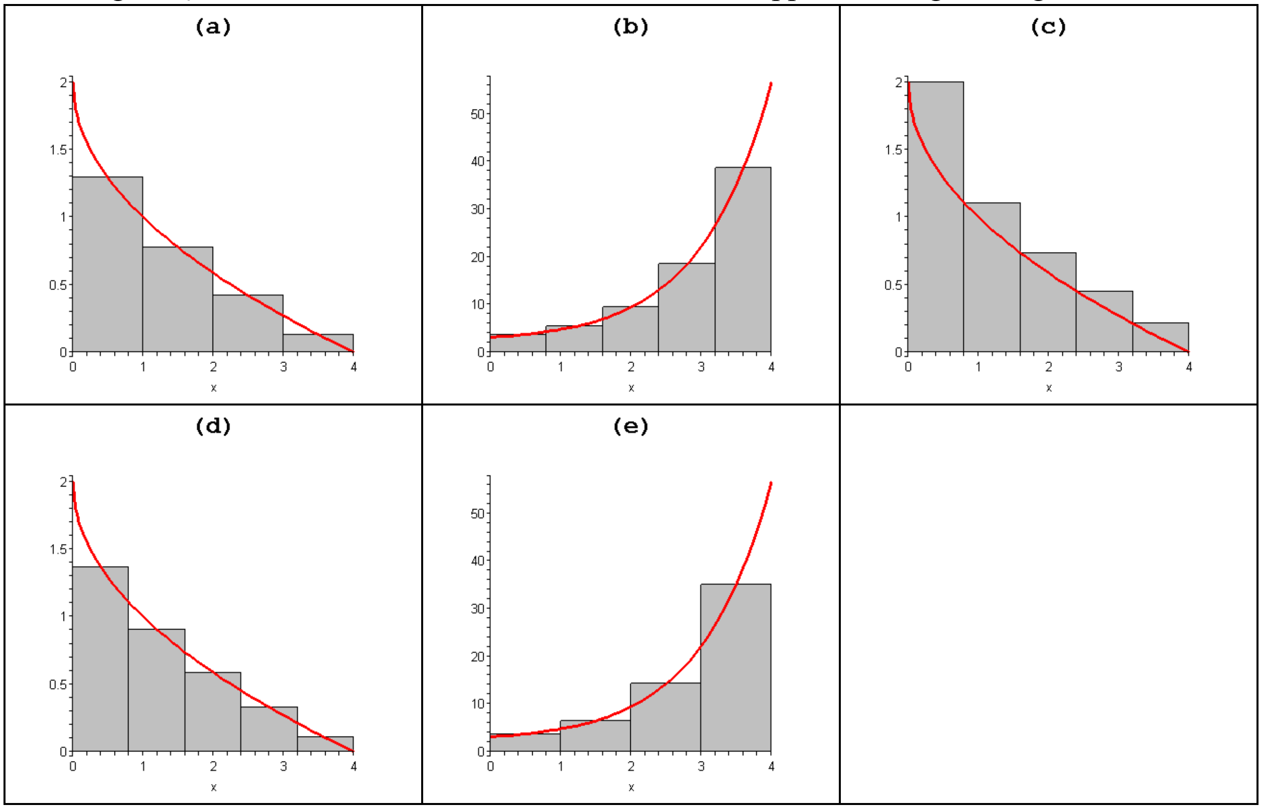
77. Express the limit as a definite integral on the given interval.

$\lim_{n\to\infty} \sum_{i=1}^{n} x_i \sin x_i \Delta x_i, \quad [0,\pi]$

(a) $\int_0^\pi x\sin x\, dx$ **(b)** $\int_0^\pi (\sin x + x\cos x)\, dx.$ **(c)** $\int_0^\pi \sin x\, dx$

(d) $\int_0^\pi x\cos x\, dx$ **(e)** $\int_0^\pi \sqrt{x}\sin x\, dx$

78. Referring to Question #76 above, sketch the curve and the approximating rectangles.



79. If $f(5) = 3$, f' is continuous, and $\int_2^5 f'(x)dx = 17$, what is the value of $f(2)$?
(a) -14 **(b)** 14 **(c)** 20 **(d)** -20 **(e)** 3

80. Find the derivative of

$$f(x) = \int_{\sqrt{x}}^{\cos x} t\cos\sqrt{t}\,dt$$

(a) $\cos x \cos\sqrt{\cos x} - \sqrt{x}\cos\sqrt{\sqrt{x}}$

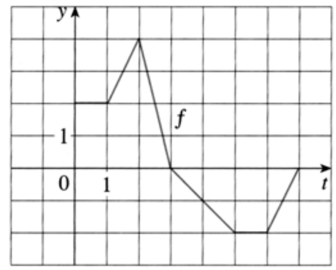
(b) $\cos x \cos\sqrt{\cos x}(-\sin x) - \sqrt{x}\cos\sqrt{\sqrt{x}} \cdot \frac{1}{2}x^{-1/2}$

(c) $\cos x \cos\sqrt{\cos x}(-\sin x)$

(d) $\sqrt{\cos x}(-\sin x) - \cos\sqrt{\sqrt{x}} \cdot \frac{1}{2}x^{-1/2}$

(e) $\cos\sqrt{\cos x} - \cos\sqrt{\sqrt{x}}$

81. Let $g(x) = \int_0^x f(t)dt$, where f is the function whose graph is shown. Find the intervals (if any) on which g is increasing.
(a) $(2,5)$ **(b)** $(0,3)$ **(c)** none
(d) $(1,2),(6,7)$ **(e)** $(3,7)$



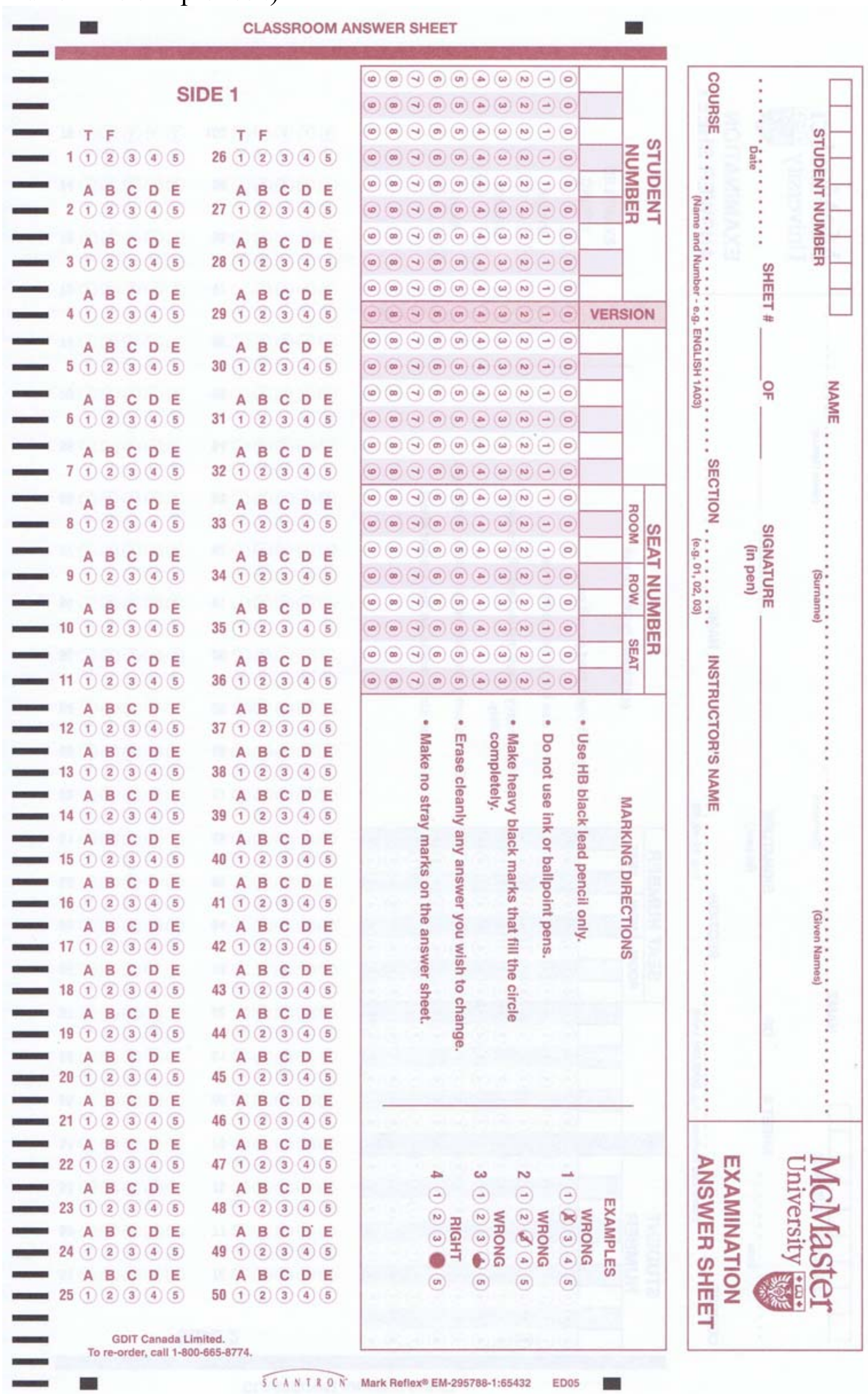
82. Evaluate the following integral. $\int_0^1 \frac{3x^2+3x}{\sqrt{2x^3+3x^2+4}}dx$
(a) $\frac{1}{3}$ **(b)** 3 **(c)** $\frac{1}{2}$ **(d)** 1 **(e)** 2

83. Evaluate the following integral. $\int_0^1 \frac{\sin(\arctan(x))}{1+x^2}dx$
(a) $\frac{1}{\sqrt{2}}+1$ **(b)** 0 **(c)** $-\frac{1}{\sqrt{2}}$ **(d)** 1 **(e)** $1-\frac{1}{\sqrt{2}}$

84. Evaluate the following integral. $\int_0^1 \frac{6}{(3-2x)^4}dx$
(a) 1 **(b)** $\frac{1}{27}$ **(c)** $\frac{26}{27}$ **(d)** $\frac{28}{27}$ **(e)** 2

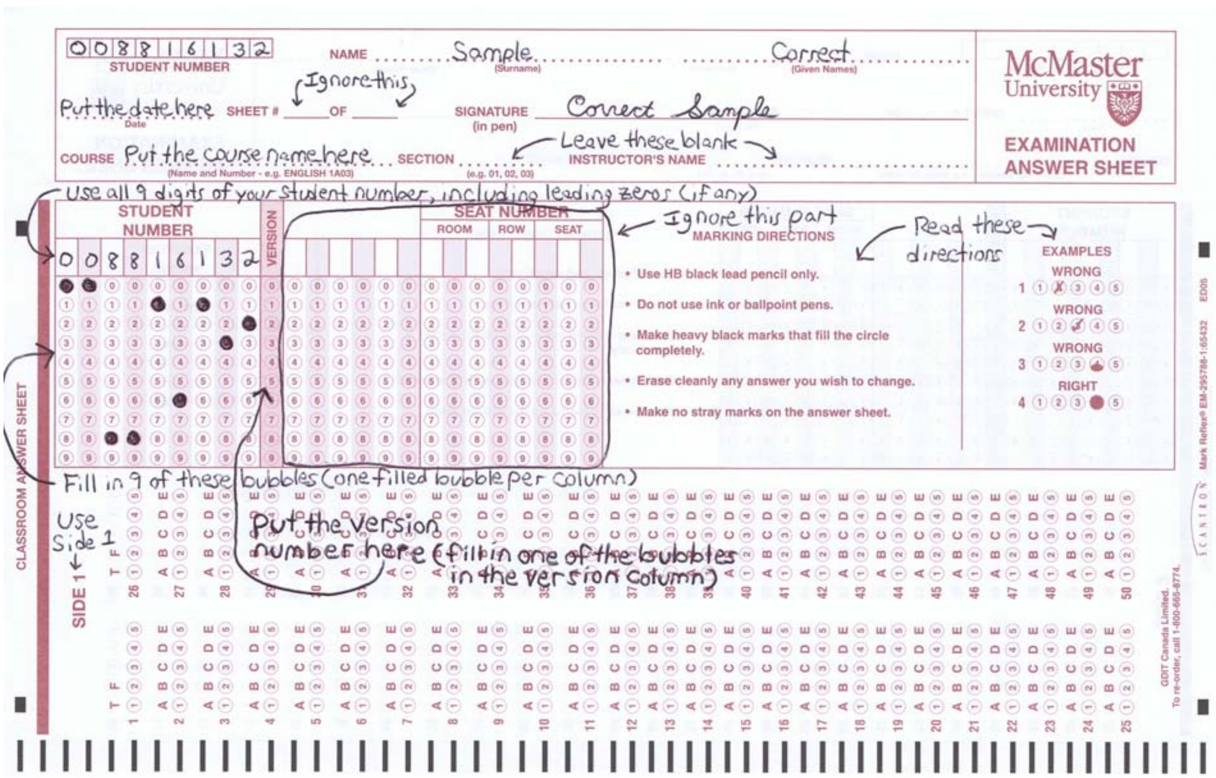
85. Evaluate the following integral. $\int \frac{x}{\sqrt[4]{x+2}}dx$
(a) $\frac{4}{7}(x+2)^{7/4}-\frac{5}{3}(x+2)^{3/4}$ **(b)** $\frac{4}{7}(x+2)^{7/4}-\frac{8}{3}(x+2)^{3/4}$
(c) $\frac{3}{7}(x+2)^{7/4}-\frac{8}{3}(x+2)^{3/4}$ **(d)** $\frac{4}{7}(x+2)^{7/4}-\frac{4}{3}(x+2)^{3/4}$
(e) $\frac{12}{7}(x+2)^{7/4}-\frac{8}{3}(x+2)^{3/4}$

86. Correctly fill out the bubbles corresponding to all 9 digits of your student number, as well as the version number of your test in the correct places on the computer card. (Use the below computer card for this sample test.)



Answers

1. b **2.** a **3.** c **4.** d **5.** d **6.** e **7.** b **8.** b **9.** c **10.** b
11. a **12.** a **13.** e **14.** a **15.** b **16.** c **17.** d **18.** c **19.** b **20.** d
21. b **22.** b **23.** c **24.** c **25.** e **26.** b **27.** b **28.** e **29.** c **30.** d
31. a **32.** b **33.** d **34.** b **35.** c **36.** d **37.** e **38.** b **39.** e **40.** d
41. a **42.** e **43.** d **44.** a **45.** c **46.** e **47.** e **48.** b **49.** e **50.** a
51. d **52.** c **53.** d **54.** a **55.** d **56.** c **57.** d **58.** e **59.** c **60.** a
61. b **62.** e **63.** a **64.** a **65.** d **66.** d **67.** b **68.** b **69.** a **70.** d
71. e **72.** b **73.** c **74.** e **75.** d **76.** a **77.** a **78.** d **79.** a **80.** b
81. b **82.** d **83.** e **84.** c **85.** b
86.



NOTE: On the sample tests, a version number is not given. On the actual tests, it will say "Version X" at the top, where X is the version number that you will have to fill in on the computer card. The sample above assumes that your student number is 008816132. On the actual test, you will have to fill in the bubbles corresponding to YOUR student number (not 008816132).