Math 1A03/1ZA3 Test #2 (Version 1)
November 14th, 2017

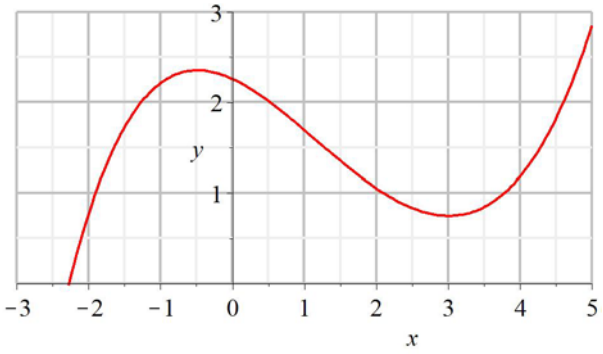
Name:_____
(Last Name) (First Name)

Student Number:_____

This test consists of 19 multiple choice questions worth 1 mark each (no part marks), and 1 question worth 1 mark (no part marks) on proper computer card filling. Questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Only the McMaster standard calculator Casio fx-991 MS or MS Plus is allowed.

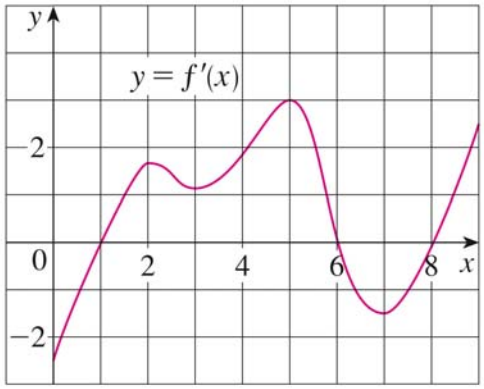
1. Let $f(x)$ be the function whose graph is given to the right. Find the *smallest* value of c that satisfies the conclusion of the Mean Value Theorem on the interval $[-2, 4]$

(a) $-\frac{3}{2}$ **(b)** $-\frac{3}{4}$ **(c)** $-\frac{7}{4}$ **(d)** 0 **(e)** $\frac{1}{2}$



2. The graph of the first derivative f' of a function f is shown to the right. On what inverval(s) is f concave upward?

(a) $(1, 6), (8, 9)$ **(b)** $(0, 1), (6, 8)$
(c) $(3, 4.5), (5.5, 9)$ **(d)** $(0, 2), (3, 5), (7, 9)$
(e) $(3, 5), (6, 8)$



3. Suppose that $f'(x) = \frac{x+1}{\sqrt{x}}$, and $f(1) = 5$. Find $f(4)$.

(a) $\frac{35}{3}$ **(b)** $\frac{34}{3}$ **(c)** 11 **(d)** $\frac{32}{3}$ **(e)** $\frac{31}{3}$

4. What is the minimum vertical distance between the parabolas $y = x^2 + 1$ and $y = x - x^2$?

(a) $\frac{3}{4}$ **(b)** $\frac{5}{4}$ **(c)** $\frac{9}{8}$ **(d)** 1 **(e)** $\frac{7}{8}$

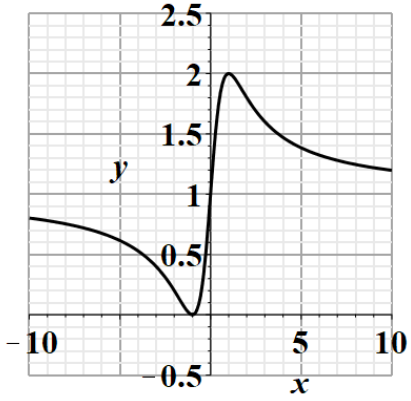
5. Let $f(x) = \sin x + \cos x,\ 0 \le x \le 2\pi$. Find the largest interval(s) on which f increasing.

(a) $(0, \frac{\pi}{4}), (\frac{5\pi}{4}, 2\pi)$ **(b)** $(0, \frac{\pi}{4}), (\frac{5\pi}{4}, \frac{7\pi}{4})$ **(c)** $(\frac{\pi}{4}, \frac{5\pi}{4})$ **(d)** $(\frac{\pi}{4}, \frac{7\pi}{4})$ **(e)** $(0, \frac{\pi}{4}), (\frac{7\pi}{4}, 2\pi)$

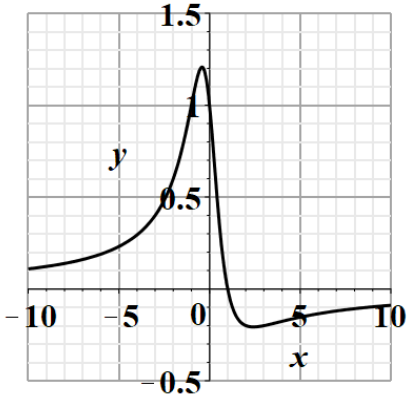
6. Sketch the following function.

$$f(x) = \frac{(x-1)^2}{x^2+1}$$

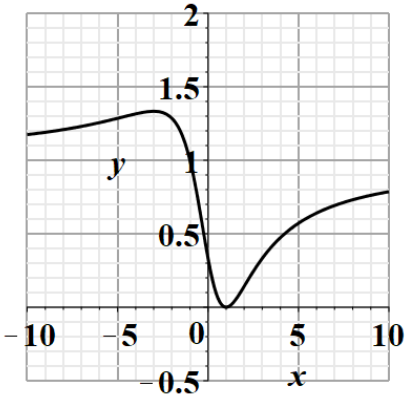
(a)



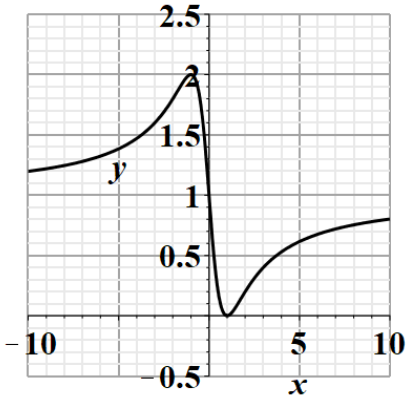
(b)



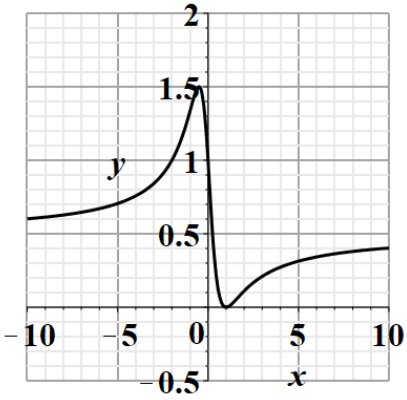
(c)



(d)



(e)



7. Evaluate the following limit.

$$\lim_{x\to\pi}\frac{1-\cos 2x}{1-\cos x}$$

(a) $-\infty$ **(b)** 0 **(c)** 4 **(d)** -4 **(e)** ∞

8. Evaluate the following limit.

$$\lim_{x\to\infty} x\ln\left(1+\frac{6}{x}\right)$$

(a) ∞ **(b)** 0 **(c)** 6 **(d)** -6 **(e)** e^6

9. Evaluate the following sum.

$$\sum_{i=1}^{1000}(2^i-2^{i+1})$$

(a) $4-2^{1000}$ **(b)** $2^{1000}-2$ **(c)** $2-2^{1001}$ **(d)** $2-2^{1000}$ **(e)** -2^{1001}

10. Evaluate the following limit.

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\left[3\left(\frac{2i}{n}\right)^2-2\left(\frac{3i}{n}\right)\right]$$

(a) 2 **(b)** ∞ **(c)** 0 **(d)** 1 **(e)** 3

11. Evaluate the following integral.

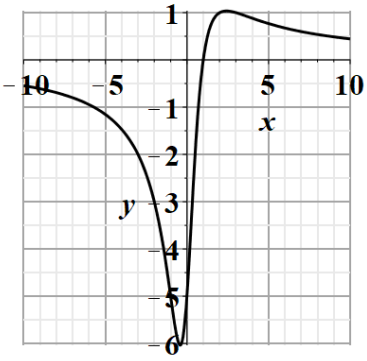
$$\int\frac{x}{\sqrt{1+x^2}}\,dx$$

(a) $-\frac{2}{\sqrt{1+x^2}}+C$ **(b)** $\frac{1}{3}(1+x^2)^{3/2}+C$ **(c)** $(1+x^2)^{3/2}+C$ **(d)** $\sqrt{1+x^2}+C$
(e) $-\frac{1}{\sqrt{1+x^2}}+C$

12. Let $f(x)=\int_0^{2x}\ln(1+t^2)\,dt$. Find $f''(2)$.

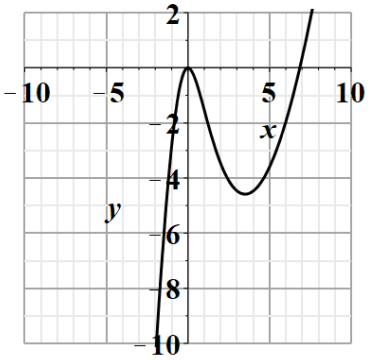
(a) $\frac{5}{17}$ **(b)** $\frac{16}{5}$ **(c)** $\frac{32}{17}$ **(d)** $\frac{16}{17}$ **(e)** $\frac{32}{5}$

13. Consider the function f whose graph is given to the right.

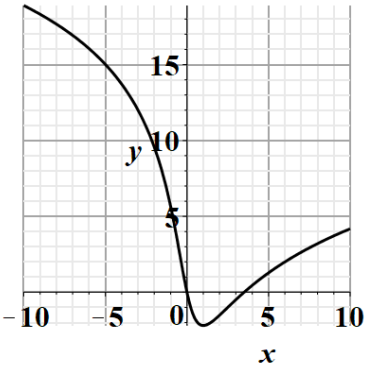


Which of the following graphs is an antiderivative of f?

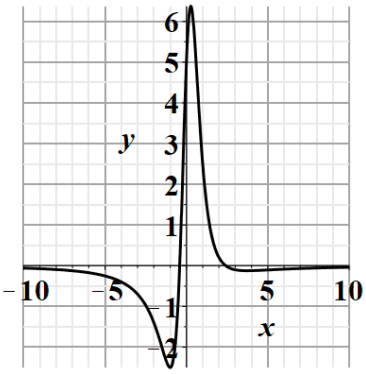
(a)



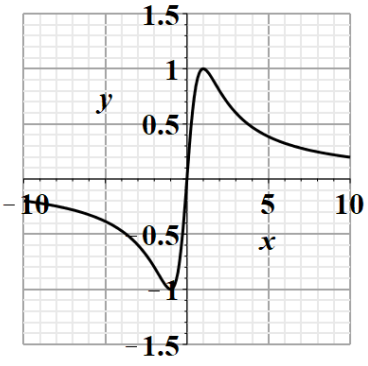
(b)



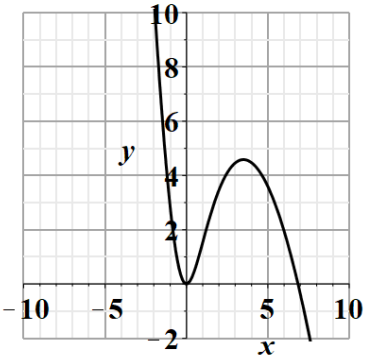
(c)



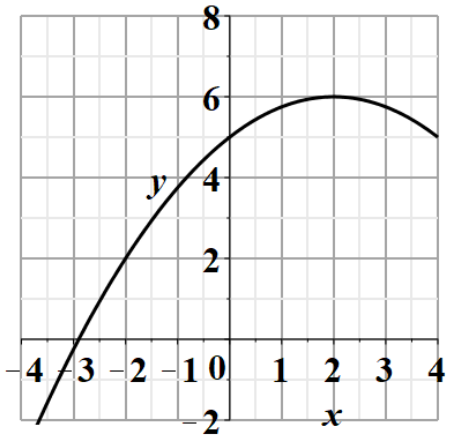
(d)



(e)



14. Let f be the function whose graph is shown to the right. Estimate the area under the graph of f on the interval $[-2, 4]$ using three rectangles and left endpoints.



(a) 29 **(b)** 16 **(c)** 32 **(d)** 13 **(e)** 26

15. Which of the following is equal to

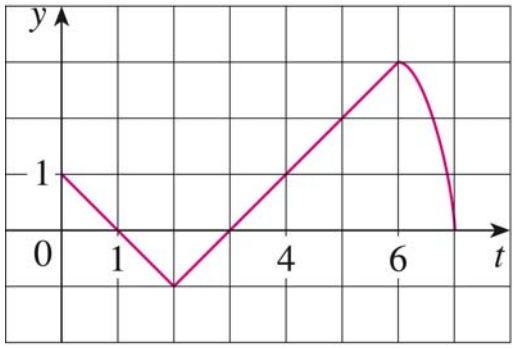
$$\int_1^4 \sin(x^2)\,dx\,?$$

(a) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{3}{n} \sin\left(1 + \frac{9i^2}{n^2}\right)$ **(b)** $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{3}{n} \sin\left(1 + \frac{6i}{n} + \frac{9i^2}{n^2}\right)$

(c) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{3}{n} \sin\left(1 + \frac{3i}{n}\right)$ **(d)** $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{4}{n} \sin\left(1 + \frac{4i}{n}\right)$

(e) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{4}{n} \sin\left(1 + \frac{16i^2}{n^2}\right)$

16. Consider the function f whose graph is given to the right. Which of the following quantities are positive (i.e., > 0)?
(i) $\int_0^2 f(t)\,dt$ (ii) $\int_0^4 f(t)\,dt$ (iii) $\int_1^3 f(t)\,dt$



(a) none of them **(b)** (iii) only **(c)** (i) and (ii) only
(d) all of them **(e)** (i) only

17. Suppose that $f'(x) = \frac{(\ln x)^2}{x}$, and $f(e) = 1$. Find $f(e^2)$.
(a) $\frac{5}{3}$ **(b)** $\frac{1}{3}$ **(c)** $\frac{7}{3}$ **(d)** $\frac{2}{3}$ **(e)** $\frac{10}{3}$

18. Suppose that the following was defined as an expression in Maple.

$$f(x) = \frac{x^3 + 1}{x^3 + x^2 - 2}$$

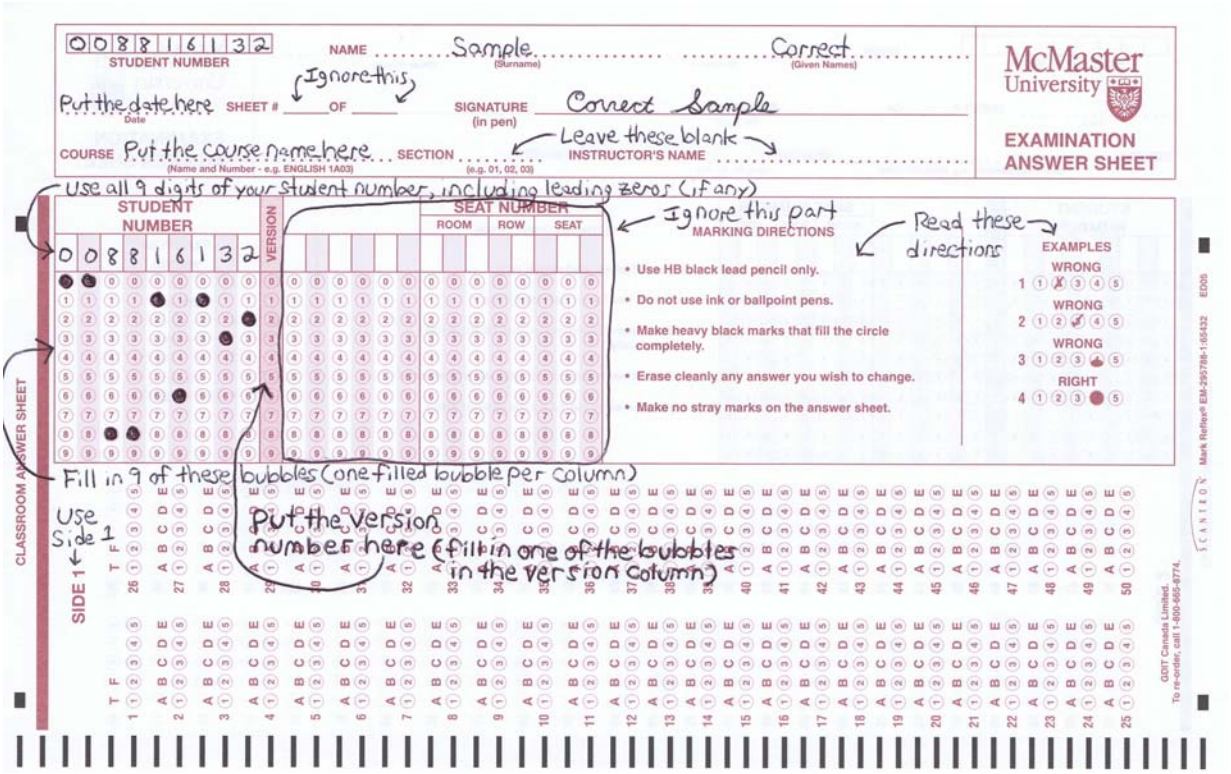
Which of the following commands could be used to find the vertical asymptote of f?

(a) `>asymptotes(f,vertical);`
(b) `>fsolve(denom(f)=0,x=0..2);`
(c) `>limit(f,x=infinity);`
(d) `>vertical_asymptote(f,x);`
(e) `>vertical_asymptote(f,x=0..2);`

19. If $f(1) = 9$ and $f'(x) \geq 8$ for $1 \leq x \leq 7$ then, according to the Mean Value Theorem, what is the smallest possible value for $f(7)$?

(a) 63 **(b)** 56 **(c)** 57 **(d)** 72 **(e)** 71

20. Correctly fill out the bubbles corresponding to all 9 digits of your student number, as well as the version number of your test in the correct places on the computer card.
Note: You are writing **Version 1**.
Hint:



Answers (Version 1)**:**

1. b **2.** d **3.** a **4.** e **5.** a **6.** d **7.** b **8.** c **9.** c **10.** d
11. d **12.** c **13.** b **14.** e **15.** b **16.** a **17.** e **18.** b **19.** c