MATHEMATICS 1LS3   TEST 4

Day Class
Duration of Examination: 60 minutes
McMaster University, 25 November 2015

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First name (PLEASE PRINT): _____

Family name (PLEASE PRINT): _____

Student No.: _____

THIS TEST HAS 8 PAGES AND 7 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE.

Total number of points is 40. Marks are indicated next to the problem number. Any non-graphing calculator is allowed.

USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

You must show work to receive full credit.

Problem	Points	Mark
1	4	
2	6	
3	9	
4	3	
5	6	
6	6	
7	6	
TOTAL	40	

1. **Multiple choice questions: circle ONE answer. No justification is needed.**

(a)[2] It is known that $\left(\dfrac{3x-1}{2x+1}\right)' = \dfrac{5}{(2x+1)^2}$. What is the value of $\displaystyle\int_0^{1/3} \dfrac{1}{(2x+1)^2}\, dx$?

    (A) 0   (B) 1/5   (C) 1/3   (D) 1/2

    (E) 1   (F) 2   (G) 3   (H) 5

(b)[2] Which of the following definite integral(s) is/are positive?

$$\text{(I)} \int_0^2 \cos x\, dx \qquad \text{(II)} \int_0^3 \cos x\, dx \qquad \text{(III)} \int_0^4 \cos x\, dx$$

    (A) none   (B) I only   (C) II only   (D) III only

    (E) I and II   (F) I and II   (G) II and III   (H) all three

2. **Identify each statement as true or false (circle your choice). You do not need to justify your answer.**

(a)[2] The following calculation is correct:

$$\int_{-2}^{1} \frac{1}{x}\, dx = \ln|x| \Big|_{-2}^{1} = \ln|1| - \ln|-2| = -\ln 2$$

TRUE   OR   FALSE

(b)[2] The left and the midpoint Riemann sums of $f(x) = x^{-1/3}$ on $[2, 12]$ satisfy $M_{15} < L_{15}$.

TRUE   OR   FALSE

(c)[2] It is known that $\int_{1}^{6} f(x)\, dx = -10$. Thus, $f(x) < 0$ for all x in $[1, 6]$.

TRUE   OR   FALSE

Questions 3-7: You must show work to receive full credit.

3. (a)[2] Find the Taylor polynomial $T_3(x)$ for $f(x) = \sin x$ at $x = 0$.

(b)[1] Use your answer in (a) to find a polynomial approximation of $\sin(x^2)$.

(c)[3] Use the polynomial from (b) to find an approximation of $\displaystyle\int_0^1 \sin(x^2)\, dx$.

(d)[3] Use L_4 (left sum with four rectangles) to approximate $\int_0^1 \sin(x^2)\, dx$.

4. [3] Find the average value of the function $f(x) = \dfrac{\sqrt{\ln x}}{x}$ on $[1, e]$.

5. The *rate of change* of the number of new individuals infected by a strain H2T1 of influenza A virus is given by the function $p(t) = 120te^{-0.1t}$. The variable t is time in days; the time $t = 0$ represents 1 February 2015.

(a)[4] Find $\int_0^4 120te^{-0.1t} \, dt$

(b)[2] What does the answer you obtained in (a) represent?

6. The rate of change of the length of wild Pacific salmon is given by

$$\frac{dL}{dt} = 13.2e^{-0.1t+0.5}$$

where t is time in years and L is the length in centimetres.

(a)[2] Without evaluating integrals, explain why a wild Pacific salmon grows more (i.e., gains more in length) from year 1 to year 3, than from year 3 to year 5.

(b)[4] How much does a wild Pacific salmon grow in length (in cm) from year 3 to year 5?

7. (a)[3] Consider the region R bounded by the graphs of $y = \arctan x$, $x = 0$, $x = 10$, and $y = \pi/4$. Write a formula for the volume of the solid obtained by rotating the region R about the x-axis. You do NOT need to compute the volume.

(b)[3] Sketch (shade) the region bounded by the graphs of $y = \sin x$ and $y = \cos x$ on $[\pi, 2\pi]$. Write a formula for the area of this region. You do NOT need to compute the integral(s).