12A3

Last Day: Abs. Max & Min Values

(most -ve)
most +ve

Absolute Maximum (Minimum) value of $f(x)$ is the highest (lowest) y-value attained by $f(x)$ on the domain

Notice Discontinuities can be bad

$\Rightarrow$ focus on fcn. that are continuous on closed intervals!

"The Extreme Value Theorem"

If $f(x)$ is continuous on $[a,b]$ then $f(x)$

{ attains an abs. max. & min. value on $[a, b]$

A local (relative) maximum is a point $x = c$ on domain of $f(x)$ such that $f(c) \geq f(x)$ for all x in a neighborhood of c.

(a little open interval around c)

Similarly A local (relative) minimum is a point $x = c$ in domain of $f(x)$ such that $f(c) \leq f(x)$ for all x in a neighbourhood of c.

So clearly abs. maximum & minimum values of $f(x)$ <u>must</u> occur at endpoints of $[a, b]$ or local max/min points.

Fermat's Theorem

If $f(x)$ is differentiable & $x = c$ is a local <u>max</u> or <u>min</u> then $\underline{f'(c) = 0}$

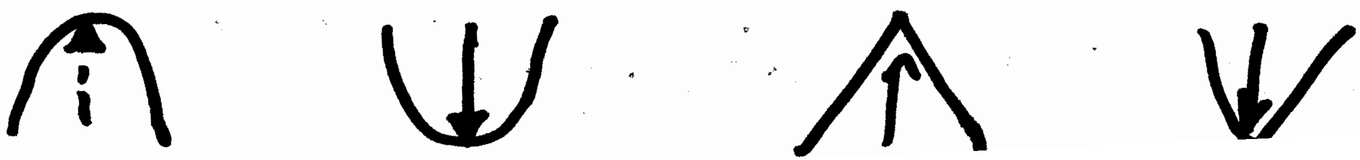
→ <u>At local max or min</u> $f'(c) = 0$

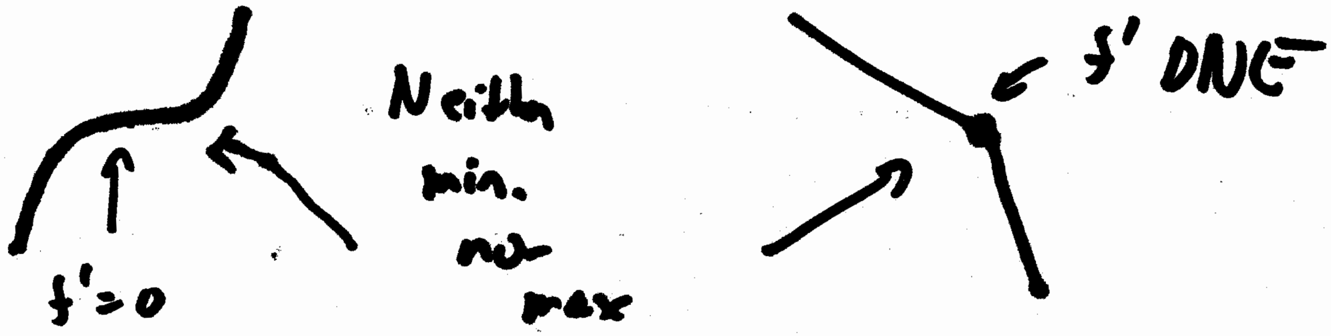
or $f'(c)$ <u>DNE</u>

But

$f' = 0$ — Local Max, Local Min

$f'(x)$ DNE — Local Max, Local Min





So/ Let's call all points $\underline{x=a}$ such that:

1) $f'(a) = 0$

or 2) $f'(a)$ DNE (but $f(a)$ <u>does</u>)

are our <u>critical numbers</u>

Attain abs max/min value $\Big\} \Rightarrow$ At local max/min $\Rightarrow$ at a critical number (c.n.)

$\not\Leftarrow$ or endpoint $\not\Leftarrow$

So let's use c.n. & endpoints as a "short list" to find abs. max & min!

eg. Given $f(x) = x^3 - 24x + 12$ find the abs. max & abs. minimum values on $x \in [0, 10]$.

Solution

Get C.n.

1) $f'(x) = 0 = 3x^2 - 24$

$\Rightarrow x^2 = 8 \Rightarrow x = \pm 2\sqrt{2}$

Only use $x = +2\sqrt{2}$.

$x = -2\sqrt{2}$ not in domain.

2) $f'(x)$ DNE

$\Rightarrow 3x^2 - 24$ DNE $\Big\}$ not going to happen!

$x = \sqrt{8} = 2\sqrt{2}$ only c.n. on $[0, 10]$.

So check c.n. & endpoints! Biggest y-value = max

Smallest y-value = min.

$f(x) = x^3 - 24x + 12$

$f(0) = 0 - 0 + 12 = 12.$

abs min

$f(\sqrt{8}) = \underbrace{(\sqrt{8})^3}_{8\sqrt{8}} - 24\sqrt{8} + 12 = \boxed{-16\sqrt{8} + 12}$

abs max

$f(10) = 10^3 - 24(10) + 12$

$= 1000 - 240 + 12 = \boxed{772}$

Note $y = \frac{1}{x^2}$ has no c.n. at $x = 0$.

why? $\frac{d}{dx} \frac{1}{x^2} = \frac{-2}{x^3}$ DNE at $x=0$

but $x=0$ not in domain of $y = \frac{1}{x^2}$.

But eg Find c.n. of $f(x) = x^{2/3} = (x^2)^{1/3}$

(specifically)

Solution $f(x) = x^{2/3}$

$f'(x) = \frac{2}{3} x^{(2/3-1)} = \frac{2}{3} x^{-\frac{1}{3}} = \frac{2}{3x^{1/3}}$

c.n. 1) $f'(x) = 0 = \frac{2}{3x^{1/3}} \Rightarrow 0 = 2$?

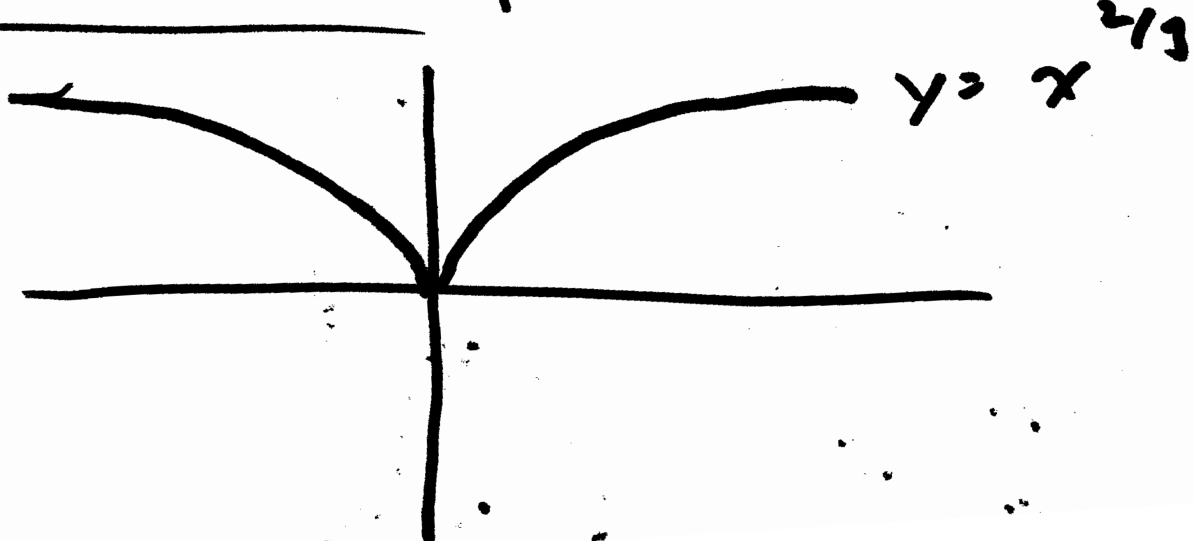
no, not possible.

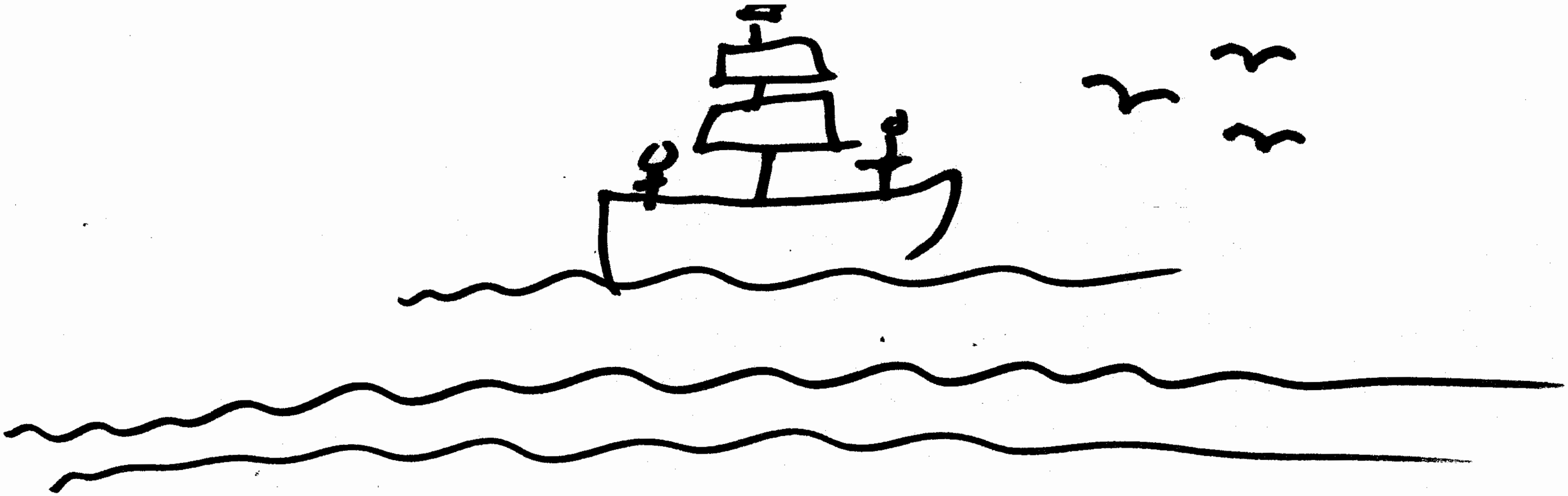
2) $f'(x)$ DNE $\Rightarrow f'(x) = \frac{2}{3x^{1/3}}$ DNE

$\Rightarrow x = 0$ } $f(0) = 0^{2/3} = 0$

so $x = 0$ is in $f(x)$ domain

$\Rightarrow$ $\boxed{x = 0 \text{ only c.n.}}$





Mean Value Theorem (MVT)

Before we do ↑ let's talk about "Rolle's Theorem"

"Rolle's Theorem"

If $f(x)$ is continuous on $[a,b]$

and $f'(x)$ exists (ie. $f(x)$ diff.) on (a,b)

and $f(a) = f(b)$

then there exist $c \in (a,b)$ such that $\underline{\underline{f'(c) = 0}}$