Math 1AA3/1ZB3

Sample Test 1, Version #1

Name:____

(Last Name) (First Name)

Student Number:_____________________ **Tutorial Number:**____________________

This test consists of 20 multiple choice questions worth 1 mark each (no part marks), and 1 question worth 1 mark (no part marks) on proper computer card filling. All questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Only the McMaster standard calculator Casio fx-991 is allowed.

1. Evaluate the following integral,

$$\int_0^{\sqrt[4]{\pi}} x^7 \sin x^4 \, dx$$

(a) $\frac{\pi}{4}$ **(b)** $\frac{\pi}{3}$ **(c)** $\frac{\pi}{2}$ **(d)** π **(e)** $\frac{3\pi}{2}$

2. Which of the following series converge?

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{e^{-\sqrt{n}}}{\sqrt{n}}$ (ii) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$

(a) (i) only **(b)** (ii) only **(c)** (i) and (ii) **(d)** neither

3. What is the minimum number of terms needed in order to estimate the sum

$$\sum_{n=1}^{\infty} \frac{1}{(3n+5)^4}$$

correct to within .001?

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4 **(e)** 5

4. Evaluate the following improper integral.

$$\int_{-\infty}^{0} x^4 e^{x^5} dx$$

(a) 1 **(b)** $\frac{1}{4}$ **(c)** $\frac{1}{5}$ **(d)** 0 **(e)** Divergent

5. Evaluate the following integral.

$$\int \frac{x-9}{x^2+3x-10}dx$$

(a) $\ln|x+5| - 2\ln|x-2|$ **(b)** $2\ln|x+5| - \ln|x-2|$
(c) $\ln|x+5| + 2\ln|x-2|$ **(d)** $-\ln|x+5| - 2\ln|x-2|$
(e) $2\ln|x+5| + \ln|x-2|$

6. Which of the below integrals is equal to

$$\int \frac{\sqrt{x^2-4}}{x^6}dx$$

(with an appropriately defined θ)?

(a) $\int \frac{1}{16}\cos^3\theta\sin^2\theta\,d\theta$ **(b)** $\int \frac{1}{32}\cos^5\theta\sin\theta\,d\theta$ **(c)** $\int \frac{1}{16}\cos^3\theta\sin^3\theta\,d\theta$
(d) $\int \frac{1}{16}\frac{\cos^2\theta}{\sin^6\theta}d\theta$ **(e)** $\int \frac{1}{32}\frac{\cos\theta}{\sin^6\theta}d\theta$

7. Using the comparison theorem, which of the following integrals is convergent?

(i) $\int_1^\infty \frac{x\sin^2 x}{\sqrt[3]{1+x^7}}dx$ (ii) $\int_1^\infty \frac{dx}{x+e^{2x}}$ (iii) $\int_2^\infty \frac{x^2}{\sqrt{x^6-1}}dx$

(a) (i) only **(b)** (ii) only **(c)** (i) and (ii) only **(d)** (i) and (iii) only **(e)** (ii) and (iii) only

8. Consider the sequence defined by $a_1 = 2,\ a_{n+1} = \frac{1}{2}(a_n + 6)$. Which of the following statements is correct?

(a) $\{a_n\}$ is increasing and bounded above by 3
(b) $\{a_n\}$ converges to 5
(c) $\{a_n\}$ is increasing and bounded above by 5
(d) $\{a_n\}$ is increasing and bounded above by 6
(e) $\{a_n\}$ diverges

9. Determine whether the following sequences are convergent or divergent. When convergent, find the limit.

(i) $a_n = \frac{(-1)^n n^3}{n^3+2n^2+1}$
(ii) $a_n = n\sin(n\pi)$

(a) diverges, diverges **(b)** diverges, 0 **(c)** 0, 0 **(d)** 1, diverges **(e)** 1, 0

10. If you were to use use Mathematical Induction to show that

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6},$$

which of the following would be the second step?

(a) Assume $\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$, and show that $\sum_{i=1}^{k} (i+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$

(b) Assume $\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$, and show that $\sum_{i=1}^{k+1} i^2 = \frac{(k-1)k(2k-1)}{6}$

(c) Assume $\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$, and show that $\sum_{i=1}^{k-1} i^2 = \frac{(k-1)k(2k-1)}{6}$

(d) Assume $\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$, and show that $\sum_{i=1}^{k+1} i^2 = \frac{(k+1)(k+2)(2k+3)}{6}$

(e) Assume $\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$, and show that $\sum_{i=1}^{k-1} i^2 = \frac{(k+1)(k+2)(2k+3)}{6}$

11. Find the sum of the following series

$$\sum_{n=1}^{\infty} \frac{\sin^n x}{3^n}$$

(a) $\dfrac{\sin x - 3}{3}$ **(b)** $\dfrac{3}{\sin x}$ **(c)** $\dfrac{\sin x}{3}$ **(d)** $\dfrac{\sin x}{3 - \sin x}$ **(e)** $\dfrac{3}{3 - \sin x}$

12. If the n^{th} partial sum of the series $\sum_{n=0}^{\infty} a_n$ is $s_n = \frac{2n+1}{4n+3} - \frac{n}{\ln n}$, find $\sum_{n=0}^{\infty} a_n$.

(a) $\frac{1}{2}$ **(b)** $\frac{1}{3}$ **(c)** $\frac{1}{4}$ **(d)** $\frac{2}{3}$ **(e)** divergent

13. Which of the following series converge?

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^3 + 2}$ (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3 + 3n^2 + n + 2}{\sqrt{n^7 + 4n^4 + n + 1}}$

(a) (i) only **(b)** (ii) only **(c)** (i) and (ii) **(d)** neither

14. What is the minimum number of terms needed in order to estimate the following sum to within .001?

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(n+1)!}$$

(a) 2 **(b)** 3 **(c)** 4 **(d)** 5 **(e)** 6

15. Which of the following series converge?

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{2^n}$ (ii) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot \cdots \cdot (2n-1)}{(2n+1)!}$

(a) (i) only **(b)** (ii) only **(c)** (i) and (ii) **(d)** neither

16. Which of the following series are absolutely convergent?

(i) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\frac{n^2+2}{n^3+1}\right)^n$ (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n+3}$

(a) (i) only **(b)** (ii) only **(c)** (i) and (ii) **(d)** neither

17. Find the interval of convergence of

$$\sum_{n=1}^{\infty} \frac{n(3x+2)^n}{n^2+1}$$

(a) $(-\infty, \infty)$ **(b)** $[-\frac{1}{3}, -\frac{2}{3}]$ **(c)** $[-\frac{1}{3}, -\frac{2}{3})$ **(d)** $[-1, -\frac{1}{3}]$ **(e)** $[-1, -\frac{1}{3})$

18. If $\sum_{n=0}^{\infty} c_n 2^n$ is convergent, what can you conclude about the convergence of the following series?

(i) $\sum_{n=0}^{\infty} c_n(-2)^n$ (ii) $\sum_{n=0}^{\infty} c_n(-3)^n$

(a) convergent, nothing **(b)** convergent, convergent **(c)** nothing, divergent
(d) nothing, nothing **(e)** convergent, divergent

19. Find the radius of convergence of

$$\sum_{n=1}^{\infty} (n+1)!(3x-1)^n$$

(a) ∞ **(b)** 0 **(c)** 1 **(d)** $\frac{1}{3}$ **(e)** $\frac{2}{3}$

20. Find the radius of convergence of the following power series.

$$\sum_{n=0}^{\infty}(-1)^n \frac{1 \cdot 4 \cdot 7 \cdot \cdots \cdot (3n-2)}{n! 3^n} x^{3n}$$

(a) $\sqrt[3]{3}$ **(b)** $\frac{1}{\sqrt[3]{3}}$ **(c)** 1 **(d)** 0 **(e)** ∞

Math 1AA3/1ZB3

Sample Test 2, Version #2

Name:____

(Last Name) (First Name)

Student Number:______________________ **Tutorial Number:**_____________________

This test consists of 20 multiple choice questions worth 1 mark each (no part marks), and 1 question worth 1 mark (no part marks) on proper computer card filling. All questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Only the McMaster standard calculator Casio fx-991 is allowed.

1. Evaluate the following integral,

$$\int_{\ln(\pi/2)}^{\ln \pi} e^x \sin^2(e^x)\, dx$$

(a) $\frac{\pi}{2}$ **(b)** $\frac{\pi}{3}$ **(c)** $\frac{\pi}{4}$ **(d)** $\frac{\pi}{6}$ **(e)** π

2. Evaluate the following integral.

$$\int \frac{1}{x^2\sqrt{9+x^2}}\, dx$$

(a) $\frac{1}{3}\left[\sec^{-1}x + \frac{\sqrt{9+x^2}}{x}\right] + C$ **(b)** $-\frac{1}{9}\frac{\sqrt{9+x^2}}{x} + C$ **(c)** $\frac{1}{3}\left[\tan^{-1}x + \frac{x}{\sqrt{9+x^2}}\right] + C$
(d) $\frac{1}{3}\left[x + \frac{\sqrt{9+x^2}}{x^2}\right] + C$ **(e)** $\frac{1}{3}\frac{x}{\sqrt{9+x^2}} + C$

3. Consider the sequence defined by $a_1 = 1$, $a_{n+1} = \frac{1}{3-a_n}$. To show that this sequence is monotonic using induction, which of the following would be the second step?
(a) Assume that $a_{k+1} \geq a_k$ and show that $\frac{1}{3-a_{k+2}} \geq \frac{1}{3-a_{k+1}}$
(b) Assume that $a_{k+1} \geq a_k$ and show that $\frac{1}{3-a_{k+1}} \quad \frac{1}{3-a_k}$
(c) Assume that $a_{k+1} \leq a_k$ and show that $\frac{1}{3-a_{k+2}} \leq \frac{1}{3-a_{k+1}}$
(d) Assume that $a_{k+1} \leq a_k$ and show that $\frac{1}{3-a_{k+1}} \leq \frac{1}{3-a_k}$
(e) Assume that $\frac{1}{3-a_{k+2}} \leq \frac{1}{3-a_{k+1}}$ and show that $a_{k+1} \leq a_k$

4. Find the values of p for which the series is convergent.

$$\sum_{n=2}^{\infty} \frac{(\ln n)^{p-1}}{n}$$

(a) $p \geq 1$ **(b)** $p < 0$ **(c)** $p \leq 0$ **(d)** $p < 1$ **(e)** $p \leq 1$

5. If we use the partial sum s_{10} to approximate the sum of the series

$$\sum_{n=1}^{\infty} \frac{1}{n^4}$$

estimate the error involved in the approximation.
(a) $\frac{1}{3000}$ **(b)** $\frac{1}{5000}$ **(c)** $\frac{1}{10000}$ **(d)** $\frac{1}{1000}$ **(e)** $\frac{1}{14641}$

6. Evaluate the following improper integral

$$\int_1^{\infty} \frac{1}{3x-2}\, dx$$

(a) $\ln 3$ **(b)** $\ln 2$ **(c)** 0 **(d)** divergent **(e)** 1

7. Evaluate the following improper integral

$$\int_1^{\infty} \frac{\ln x}{x^2} dx$$

(a) 0 **(b)** 1 **(c)** $\ln 2$ **(d)** $\frac{1}{2}$ **(e)** Divergent

8. Evaluate the following sum:

$$\sum_{n=4}^{\infty} \frac{1}{n(n+1)}$$

(a) divergent **(b)** $\frac{1}{3}$ **(c)** 1 **(d)** $\frac{1}{4}$ **(e)** $\frac{1}{5}$

9. Determine whether the following sequences are convergent or divergent. When convergent, find the limit.
(i) $a_n = \ln(n+1) - \ln(2n)$
(ii) $a_n = n\sin(1/n)$
(a) divergent, divergent **(b)** divergent, 0 **(c)** $\ln\frac{1}{2}$,divergent **(d)** $\ln\frac{1}{2}, 0$
(e) $\ln\frac{1}{2}$, 1

10. A sequence a_n is defined by $a_1 = 1$ and $a_{n+1} = \frac{1}{1+a_n}$. Assuming $\{a_n\}$ is convergent, find its limit.
(a) 2 **(b)** 1 **(c)** 0 **(d)** $\dfrac{-1+\sqrt{5}}{2}$ **(e)** $\dfrac{-1-\sqrt{5}}{2}$

11. Find the sum of the following series

$$\frac{1}{3} - \frac{1}{4} + \frac{3}{16} - \frac{9}{64} + \cdots$$

(a) $\frac{4}{21}$ **(b)** $\frac{4}{3}$ **(c)** $\frac{4}{7}$ **(d)** 4 **(e)** $\frac{1}{12}$

12. Find the values of x for which the series converges

$$\sum_{n=0}^{\infty} \frac{(2x+5)^n}{3^n}$$

(a) $(-\frac{2}{3}, \frac{5}{3})$ **(b)** $(-8,-2)$ **(c)** $(-3,3)$ **(d)** $(-\frac{2}{5}, \frac{2}{5})$ **(e)** $(-1,-4)$

13. Which of the following series converge?
(i) $\dfrac{1}{2} + \dfrac{1}{6} + \dfrac{1}{10} + \dfrac{1}{14} + \dfrac{1}{18} + \cdots$ (ii) $\displaystyle\sum_{n=1}^{\infty} 2^{2n} 3^{1-n}$
(a) (i) only **(b)** (ii) only **(c)** (i) and (ii) **(d)** neither

14. Which of the following series converge?
(i) $\displaystyle\sum_{n=0}^{\infty} \frac{1+5^n}{1+6^n}$ (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{n^2}$
(a) (i) only **(b)** (ii) only **(c)** (i) and (ii) **(d)** neither

15. Which of the following series converge?

(i) $\frac{4}{7} - \frac{5}{8} + \frac{6}{9} - \frac{7}{10} + \frac{8}{11} - \cdots$ (ii) $\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n}$

(a) (i) only **(b)** (ii) only **(c)** (i) and (ii) **(d)** neither

16. Which of the following series are conditionally convergent?

(i) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$ (ii) $\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{3^n+2}$

(a) (i) only **(b)** (ii) only **(c)** (i) and (ii) **(d)** neither

17. Which of the following series converge?

(i) $\sum_{n=1}^{\infty} \frac{4^n n^3}{(n+1)!}$ (ii) $\sum_{n=1}^{\infty} (-1)^n e^{1/n}$

(a) (i) only **(b)** (ii) only **(c)** (i) and (ii) **(d)** neither

18. Find the interval of convergence of

$$\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{n2^n}$$

(a) $[-1, 1]$ **(b)** $(-\frac{1}{2}, \frac{1}{2}]$ **(c)** $[-\frac{1}{2}, \frac{1}{2}]$ **(d)** $(-2, 2]$ **(e)** $[-2, 2]$

19. If $\sum_{n=0}^{\infty} c_n(x-5)^n$ converges when $x = 3$, what can you say about the convergence of the following series?

(i) $\sum_{n=0}^{\infty} c_n(-1)^n$ (ii) $\sum_{n=0}^{\infty} c_n 2^n$

(a) convergent, nothing **(b)** divergent, nothing **(c)** convergent, convergent
(d) nothing, nothing **(e)** convergent, divergent

20. Find the radius of convergence of

$$\sum_{n=1}^{\infty} \frac{(2x+3)^n}{2^n(n+1)}$$

(a) 1 **(b)** 2 **(c)** 3 **(d)** $\frac{2}{3}$ **(e)** $\frac{3}{2}$

Answers for 1^{st} Sample Test #1

1. a **2.** c **3.** a **4.** c **5.** b **6.** a **7.** c **8.** d **9.** b **10.** d
11. d **12.** e **13.** a **14.** d **15.** c **16.** a **17.** e **18.** d **19.** b **20.** c

Answers for 2^{nd} Sample Test #1

1. c **2.** b **3.** d **4.** b **5.** a **6.** d **7.** b **8.** d **9.** e **10.** d
11. a **12.** e **13.** d **14.** c **15.** b **16.** a **17.** a **18.** d **19.** a **20.** a