McMaster University
Math 1LT3E

ASSIGNMENT 3

Sections 7.4 and 7.5

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1. First, determine the equilibrium solutions of each differential equation. Then, use the separation of variables technique to find remaining solutions for each differential equation.

(a) $\dfrac{dy}{dx} = \dfrac{y^2 \cos x}{1 + y^2}$

(b) $(x^2 + 1)\dfrac{dy}{dx} = xy$

2. Use separation of variables to find the solution of the differential equation that satisfies the given initial condition.

(a) $P'(t) = P(t) + tP(t) + t + 1, \;\; P(0) = 50$

(b) $(2y + e^{3y})y' = x \cos x, \;\; y(0) = 0$

3. Consider a special case of the "selection equation", $\dfrac{dp}{dt} = p(1-p)$.

(a) Solve the selection equation using separation of variables and integration by partial fractions as outlined in exercises 45-50 on pages 551 and 552 in your textbook.

3. continued...

(b) Exercise 51. Using your solution from part (a) and the initial condition $p(0) = 0.01$, find the value of the constant. Evaluate the limit of the solution as t approaches infinity.

(c) Exercise 52. Using your solution from part (a) and the initial condition $p(0) = 0.5$, find the value of the constant. Evaluate the limit of the solution as t approaches infinity.

4. An isolated island is populated by cats and mice. Suppose it is known that the two populations change according to the predator-prey equations defined in class and the constants are determined to be $k = 0.1$, $a = 0.005$, $r = 0.05$, and $b = 0.0001$ where time t is measured in months.

(a) Write the predator-prey equations for the population on this island.

(b) Graph the per capita growth rate of each species.

(c) By examining these equations, describe what happens if there are cats on the island but suddenly no mice. What happens if there are mice but no cats?

(d) Determine the equilibrium solutions and interpret your results.

(e) Suppose the initial population is 15 cats and 40 mice. What do you expect to happen in the immediate future for each population? What will happen long term?

(f) By hand, approximate the population of cats and mice in 2 years using Euler's method and a step size of 6 months.

(g) Using your computer (Excel is probably easiest for most but you may use any program you're familiar with), approximate the population of cats and mice in 2 years and then in 7 years using Euler's method and a step size of one month.

5. Each system of differential equations is a model for two species that either compete for the same resources or cooperate for mutual benefit (flowering plants and insect pollinators, for instance). Decide whether the system describes competition or cooperation and explain why it is a reasonable model.

(a) $\dfrac{dx}{dt} = 0.12x - 0.0006x^2 + 0.00001xy, \quad \dfrac{dy}{dt} = 0.08x + 0.00004xy$

(b) $\dfrac{dx}{dt} = 0.15x - 0.0002x^2 - 0.0006xy, \quad \dfrac{dy}{dt} = 0.2y - 0.00008y^2 - 0.0002xy$

6. Question 24 on p. 562

THE END