12A3

## FTC #2

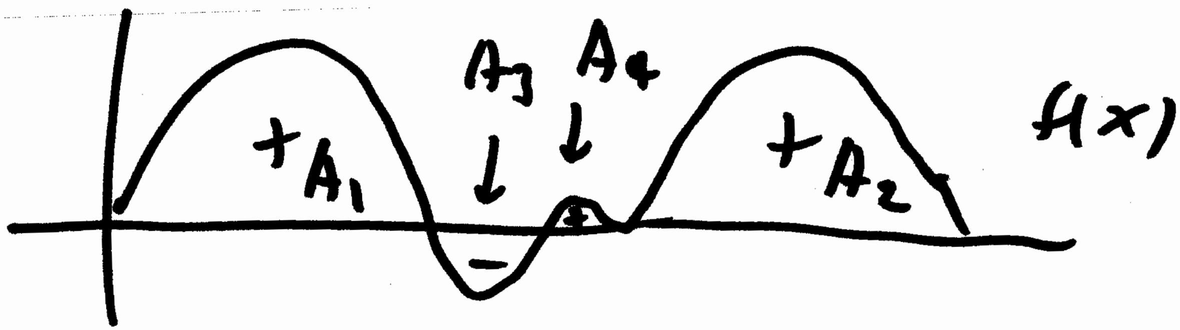
$$\int_a^b f(x)\,dx = F(b) - F(a) \quad \text{where! } F'(x) = f(x)$$

$$= F(x)\Big|_a^b$$

$$= \int f(x)\,dx \Big|_a^b$$

$$= \underline{\text{Area Above}} - \underline{\text{Area below}}$$

eg

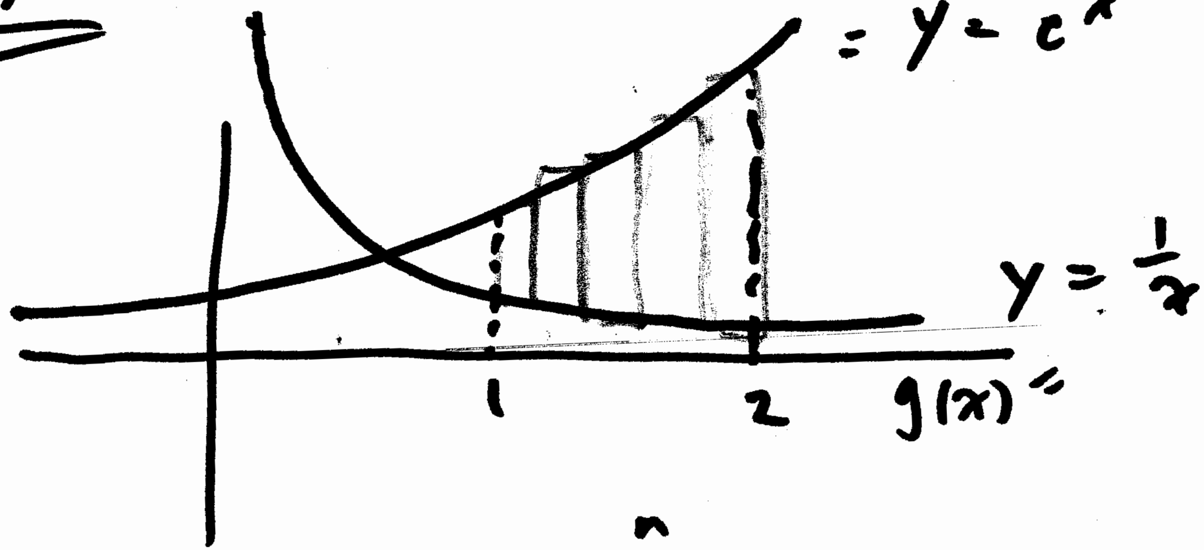


$\int f(x)\,dx = A_1 + A_2 - A_3 + A_4$

Find the area between $y = e^x$, $y = \frac{1}{x}$

on $[1, 2]$

How?




Area $\approx$ $\Sigma$ blocks $= \sum_{i=1}^{n}$ height · width.

$$= \sum_{i=1}^{n} \underbrace{[f(x_i) - g(x_i)]}_{\text{as long as } f(x) \geq g(x)} \Delta x \} \geq \underline{\underline{0}}$$

$\Rightarrow$ height $\geq 0$

$$\underline{\text{Exact}} \text{ Area} = \lim_{n \to \infty} \sum_{i=1}^{n} [f(x_i) - g(x_i)] \Delta x$$

$$= \int_a^b f(x) - g(x)\, dx \geq \underline{\underline{0}}$$

If $f(x) \geq g(x)$ only! & $b > a$

back to the question! We wanted:

Area between $e^x$, $\frac{1}{x}$ on $[1,2]$

$$\Rightarrow \text{Area} = \int_1^2 e^x - \frac{1}{x}\,dx$$

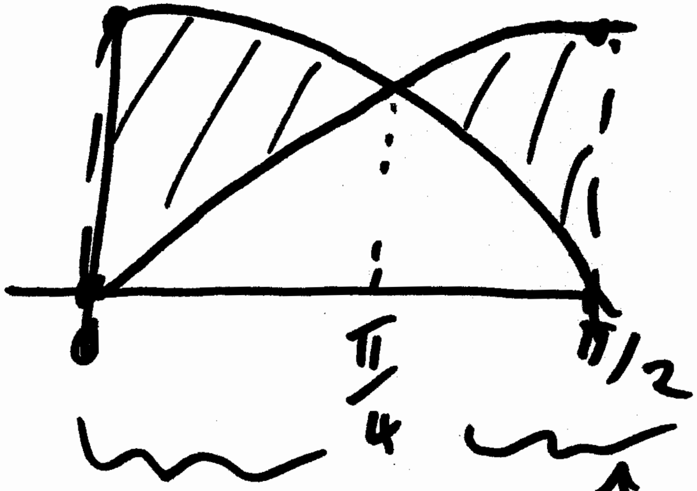
$$= e^x - \ln x \Big|_1^2$$

$$= (e^2 - e) - \ln 2 + 0$$

$$= e^2 - e - \ln 2 > 0$$

eg. Find the area between $\cos x$ & $\sin x$ on $[0, \pi/2]$

Solution




$y = \sin x$

$y = \cos x$

$$\int_0^{\pi/4} \cos x - \sin x \;\; \cancel{\sin x - \cos x} \, dx$$

$$\int_{\pi/4}^{\pi/2} \sin x - \cos x \;\; \cancel{\cos x - \sin x} \, dx$$

$$= \sin x + \cos x \Big|_0^{\pi/4}$$

$$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - (0 + 1)$$

$$= \frac{2}{\sqrt{2}} - 1 = \sqrt{2} - 1$$

$$= -\cos x - \sin x \Big|_{\pi/4}^{\pi/2}$$

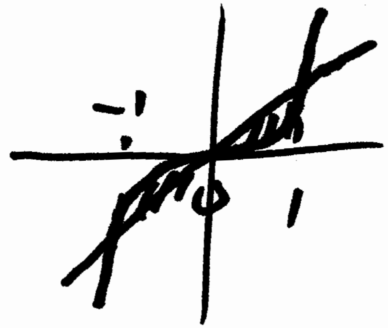
$$= (0 - 1) - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$$

$$= \sqrt{2} - 1$$

All together: $2(\sqrt{2} - 1) = 2\sqrt{2} - 2$

e.g. Find the area enclosed by $y = x$, $y = x^3$

Solution:




Let's look for crossing points!

$$f(x) = y(x) \Rightarrow x = x^3$$

$$x^3 - x = 0$$

$$x(x^2-1) = x(x-1)(x+1) = 0$$

So $x = -1, 0, 1$.

$\Rightarrow$ enclosed intervals are $[-1, 0]$ $[0, 1]$

$$\text{Area} = \int_{-1}^{0} x^3 - x \, dx + \int_0^1 x - x^3 \, dx$$

$$= \frac{1}{4}x^4 - \frac{1}{2}x^2 \Big|_{-1}^{0} + \text{----}$$

$$= 0 - \left[\frac{1}{4}\cdot 1 - \frac{1}{2}\cdot 1\right] + \text{---}$$

$$= 1/4 + \frac{1}{4} = \frac{1}{2}$$

by symetry!

Note Notation!

$$\text{Area} = \int_a^b \text{higher} - \text{lower}\, dx$$

$$= \int_a^b |f(x) - g(x)|\, dx$$

$$= \left|\int_a^{c_1} f(x) - g(x)\, dx\right| + \left|\int_{c_1}^{c_2} f(x) - g(x)\, dx\right| + \cdots$$

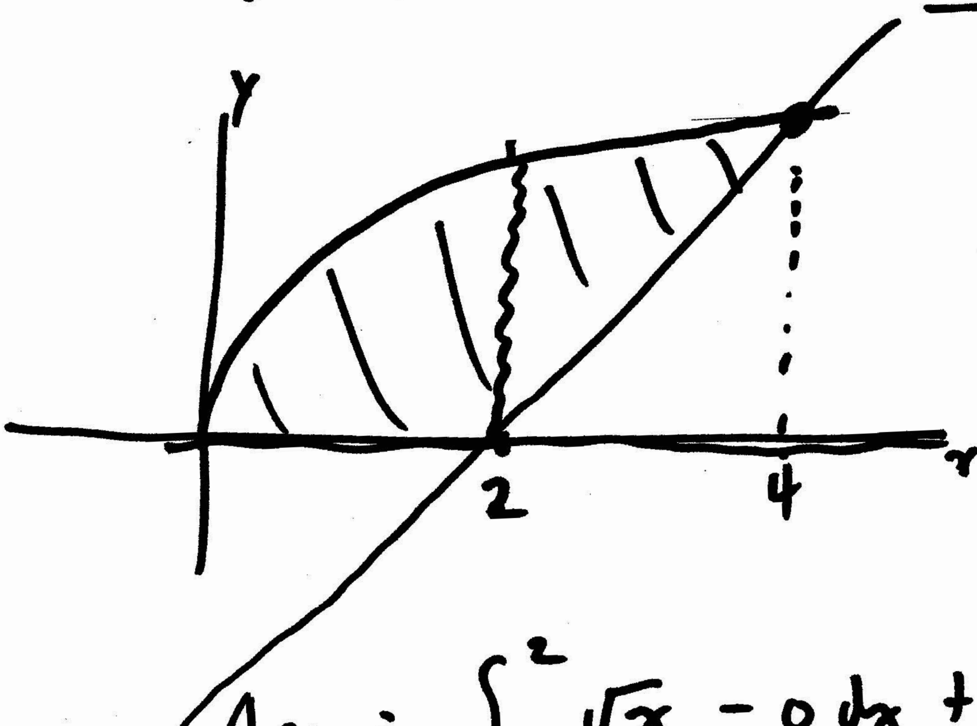
$c_1, c_2$ etc. are intersection of $f(x)$ & $g(x)$.

---

eg. Find the area enclosed by graphs:

$y=0, \quad x=y^2, \quad y=x-2$ in First Quadrant.

Solution




$x=y^2$ or $y=\sqrt{x}$

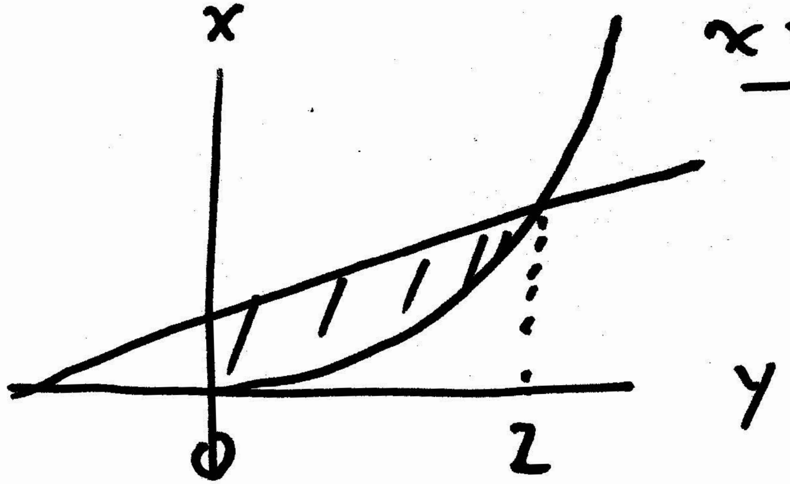
$y \geq 0$ only!

Way #1

$$\text{Area} = \int_0^2 \sqrt{x} - 0 \, dx + \int_2^4 \sqrt{x} - x + 2 \, dx$$

$= \dots\dots$ ✓

~~or~~ Way#2




$x = y^2$

$y = x - 2$

$\Leftrightarrow x = y + 2$

$$\text{Area} = \int_0^2 y + 2 - y^2 \, dy$$

$$= \frac{1}{2} y^2 + 2y - \frac{1}{3} y^3 \Big|_0^2$$

$$= \left( \frac{1}{2} \cdot 2^2 + 2^2 - \frac{1}{3} \cdot 2^3 \right) - 0$$

$$= 6 - 8/3 = 10/3$$