# MATHEMATICS 1LS3 TEST 3

Day Class

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Duration of Examination: 60 minutes

McMaster University, 14 November 2016

First name (PLEASE PRINT): SOLUTIONS

Family name (PLEASE PRINT): ______________

Student No.: ______________

THIS TEST HAS 7 PAGES AND 6 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE.

Total number of points is 40. Marks are indicated next to the problem number. Any non-graphing calculator is allowed.

**EXCEPT ON QUESTIONS 1 AND 2, YOU MUST SHOW CORRECT WORK TO EARN CREDIT.**

USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

**You must show work to receive full credit.**

| Problem | Points | Mark |
|---|---|---|
| 1 | 4 | |
| 2 | 6 | |
| 3 | 5 | |
| 4 | 8 | |
| 5 | 8 | |
| 6 | 9 | |
| TOTAL | 40 | |

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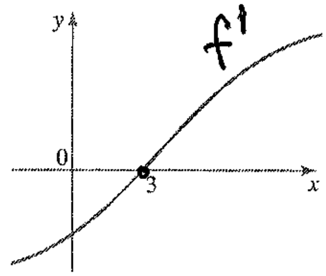
1. **Multiple choice questions: circle ONE answer. No justification is needed.**

(a)[2] Identify all correct statements about the **antiderivative of the function** shown below.




that's the function whose derivative is shown in the picture (call it f')

(I) It is increasing on $(-\infty,\infty)$ X because $f'$ is not $>0$ for all x

(II) It is increasing on $(3,\infty)$ ✓ because $f'>0$ on $(3,\infty)$

(III) It is concave down on $(3,\infty)$ X $f'$ is increasing $\to$ f is CU

(A) none (B) I only (C) II only (D) III only

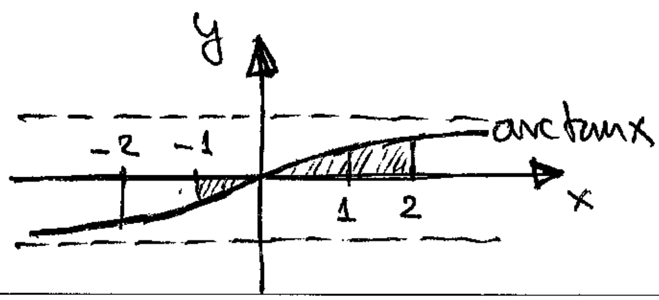
(E) I and II (F) I and III (G) II and III (H) all three

(b)[2] Identify all positive numbers (positive means grater than zero).

(I) $\int_{-1}^{2} \arctan x\,dx$ $\oplus$ (II) $\int_{-2}^{1} \arctan x\,dx$ $\ominus$ (III) $\int_{1}^{1} \arctan x\,dx = 0$

(A) none (B) I only (C) II only (D) III only

(E) I and II (F) I and III (G) II and III (H) all three

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Name:________________
Student No.: ____________

2. **Identify each statement as true or false (circle your choice). You do not need to justify your answer.**

(a)[2] $\int \underbrace{\frac{1}{x^3}} dx = \ln|x^3| + C$.

TRUE FALSE

$$(\ln|x^3|+C)' = \frac{1}{x^3} \cdot 3x^2$$

$$= \frac{3}{x} \neq \underbrace{\frac{1}{x^3}}$$

(b)[2] The function $f(x) = 2xe^{x^2}$ is an antiderivative of the function $g(x) = e^{x^2}$.

TRUE FALSE

recall: $f$ is an antiderivative of $g$

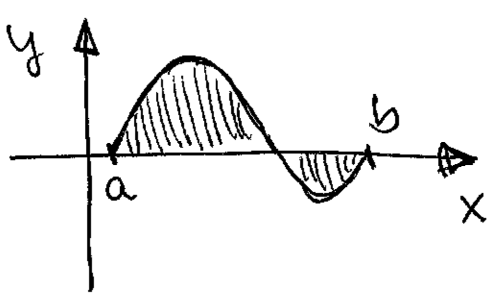
if $f' = g$

but $f' = (2xe^{x^2})' = 2e^{x^2} + 2xe^{x^2}(2x) \neq \underbrace{e^{x^2}}_{g}$

(c)[2] A function $f(x)$ for which $\int_a^b f(x)\,dx > 0$ must satisfy $f(x) > 0$ for all $x$ in $[a, b]$.

TRUE FALSE

for instance:




$\int_a^b f(x)dx$ = area above - area below $>0$

but $f(x)$ is not $>0$ for all $x$ in $[a,b]$

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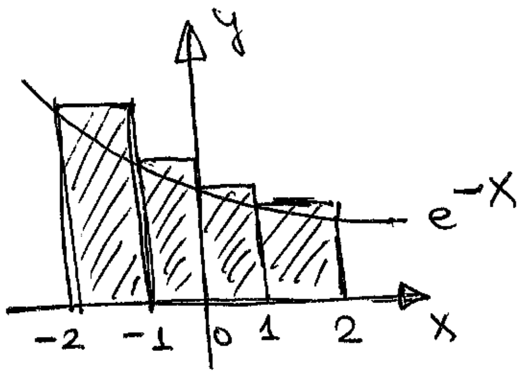
Name:______________

Student No.: ______________

**Questions 3-6: You must show CORRECT work to receive full credit.**

3. (a)[3] Find an approximation of the area of the region below the graph of $y = e^{-x}$ and over the interval $[-2, 2]$, using $L_4$ (i.e., left sum with four rectangles). Sketch the function and the four rectangles involved. Three decimal places.




$$L_4 = 1 \cdot e^{+2} + 1 \cdot e^{+1} + 1 \cdot e^{0} + 1 \cdot e^{-1}$$

$$\approx 7{,}389 + 2{,}718 + 1 + 0.368$$

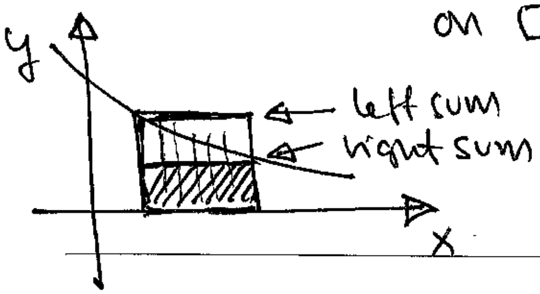
$$= 11{,}475$$

(b)[2] Suppose that you computed $R_{50}$ (i.e., right sum with 50 rectangles) for the function in (a). Which of the two sums, $L_4$ or $R_{50}$, is larger? Give a reason for your answer.

$\underline{L_4 > R_{50}}$

$f(x) = e^{-x}$ is a decreasing function, so

$L_4 >$ area under $e^{-x} > R_{50}$ on $[-2,2]$

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4. Consider the initial value problem $y' = 4xy - 2$, $y(1) = 0$, for an unknown function $y(x)$.

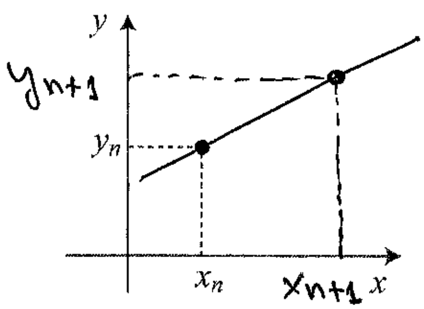
(a)[3] Write down all necessary information to start Euler's method; i.e., state the values of $x_0$, $y_0$, and the formulas for $x_{n+1}$ and $y_{n+1}$.

$x_0 = 1, y_0 = 0$

$x_{n+1} = x_n + \Delta x$

$y_{n+1} = y_n + (4x_n y_n - 2)\Delta x$

(b)[3] Explain how the formula for $y_{n+1}$ in terms of $y_n$ is obtained. Show both $x_{n+1}$ and $y_{n+1}$ in the coordinate system.




draw tangent through $(x_n, y_n)$

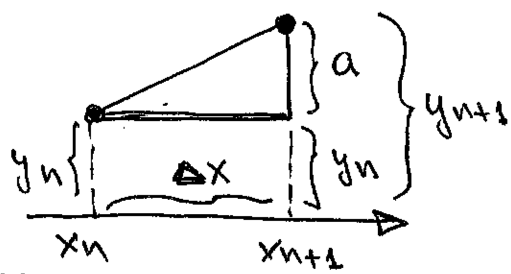
its slope is $y' = 4x_n y_n - 2$

$\rightarrow y - y_n = (4x_n y_n - 2)(x - x_n)$

substitute $x = x_{n+1}$:

$y = y_n + (4x_n y_n - 2)\underbrace{(x_{n+1} - x_n)}_{\Delta x}$

or:




$y_{n+1} = y_n + a$

slope $= \frac{a}{\Delta x}$

$\rightarrow a = \text{slope} \cdot \Delta x = (4x_n y_n - 2)\Delta x$

(c)[2] Compute the first two steps of Euler's Method with step size $\Delta x = 0.4$.

$x_0 = 1$ $\quad \Delta x = 0.4$

$y_0 = 0$

$x_1 = \underline{1.4}$

$y_1 = y_0 + (4x_0 y_0 - 2)(0.4) = 0 + (-2)(0.4) = \underline{-0.8}$

$x_2 = \underline{1.8}$

$y_2 = y_1 + (4x_1 y_1 - 2)(0.4) =$

$= -0.8 + (4(1.4)(-0.8) - 2)(0.4)$

$= \underline{-3.392}$

Name:________________

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5. (a)[3] Find the most general antiderivative of the function $f(x) = \dfrac{x^2+1}{\sqrt{x}}$.

$$f(x) = (x^2+1) \cdot x^{-1/2} = x^{3/2} + x^{-1/2}$$

$$\int f(x)\,dx = \int (x^{3/2} + x^{-1/2})\,dx$$

$$= \frac{x^{5/2}}{5/2} + \frac{x^{1/2}}{1/2} + C = \frac{2}{5}x^{5/2} + 2x^{1/2} + C$$

(b)[2] Find the numeric value of the definite integral $\int_0^1 \frac{4}{1+x^2}\,dx$.

$$= 4\arctan x \Big|_0^1$$

$$= 4\underbrace{\arctan 1}_{\pi/4} - 4\underbrace{\arctan 0}_{0} = \pi$$

(b)[3] Find the indefinite integral $\int \left(a\cos(\pi t) + b\sec^2 t\right)\,dt$.

$$= a \cdot \sin(\pi t) \cdot \frac{1}{\pi} + b \cdot \tan t + C$$

$$= \frac{a}{\pi}\sin(\pi t) + b\tan t + C$$

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6. (a)[3] A sample of bacteria, initially at the temperature of 16.7°C, is put into a −45°C refrigerator. Let $T(t)$ be the temperature of the sample at time $t$. The temperature of the sample changes proportionally to the square root of the difference between the temperature of the sample and the temperature of the refrigerator. Describe this event as an initial value problem (i.e., write down a differential equation and an initial condition). **Do not solve the equation.**

$$T(0) = 16.7$$

$$T'(t) = k \cdot \sqrt{T(t) - (-45)}$$

constant

(b)[3] Describe the following event as initial value problem: an amoeba cell starts at a volume of $600\,\mu\text{m}^3$ and **loses** volume at the rate of $1.4e^{0.02t}\,\mu\text{m}^3/s$.

$$V(0) = 600$$

$$V'(t) = -1.4e^{0.02t}$$

(c)[3] Find the solution of the initial value problem in (b).

$$V(t) = \int (-1.4\, e^{0.02t})\, dt$$

$$= -1.4 \cdot \frac{1}{0.02} e^{0.02t} + C$$

so $V(t) = -70\, e^{0.02t} + C$

$V(0) = 600 \longrightarrow 600 = -70 \cdot 1 + C$

so $C = 670$

ie $V(t) = -70\, e^{0.02t} + 670$

THE END