First Order ODE Basics Summary Sheet

1) Basic Case:

If you have: $\frac{dy}{dx} = f(x)$

Method: *<u>Integrate</u>* - $y = \int f(x)\,dx$

2) Separable DE Case:

If you have: $\frac{dy}{dx} = f(x)g\left(y\right)$

Method: *<u>Separate</u>* - $\frac{1}{g(y)}\frac{dy}{dx} = f(x)$

<u>Integrate</u> - $\int \frac{1}{g(y)}dy = \int f(x)\,dx$

If possible, *<u>Isolate</u>* for y.

2) Linear DE Case:

If you have: $\frac{dy}{dx} + P(x)y = Q(x)$

Method: Construct an

<u>Integration Factor</u> - $I(x) = e^{\int P(x)dx}$

Turn into a

<u>Product Rule</u> - $I(x)\frac{dy}{dx} + I(x)P(x)y = I(x)Q(x)$

$$I(x)\frac{dy}{dx} + I'(x)y = I(x)Q(x)$$

$$(I(x)y)' = I(x)Q(x)$$

To Finish, *<u>Integrate</u>* - $I(x)y = \int I(x)Q(x)dx$

$$y = \frac{1}{I(x)}\left(\int I(x)Q(x)dx + C\right)$$

Note: The final form will always be expressible as the sum of two parts: One with no arbitrary constant, and one that's a multiple of C.

Things to Remember:

1) Not every ODE is solvable by these methods.
2) ODE's can be linear, separable, both, and neither. So watch out!
3) Arbitrary constants are CONSTANTS. You can not eliminate functions multiplying them in your solutions.
4) Don't just "tack on" a "+C" at the end of your calculation. Arbitrary constants should appear as the result of your integration, not just an afterthought at the end of the calculations.
5) The methods above give a family of solutions. You need additional information to get a particular solution, as we see in initial value problems.
6) Although in "most" circumstances our solutions exist and are unique for a given initial condition, there are cases (where we have discontinuities in our terms, or possible domain issues with our functions involved) where no solution or infinite may exist.

   We'll cover the constraints for existence and uniqueness in year 2.

Also, Don't Forget!

1) We can verify a function we already have is a solution to a given ODE by plugging it in for our "y" and checking if LS = RS. Solving in these cases isn't necessary.

2) To identify if a given graph (ie. a picture) represents a solution of a first ODE, we can often tell by checking if the slope of the tangent lines to the graph correspond in sign to regions where $y' = f(x,y) > 0$, or < 0. Or even if we have horizontal tangent lines only when $f(x,y) = 0$.