

## Summary of Theorems about Series (as of 02/07)

**Definition**  $\sum a_n = \lim_{N \rightarrow \infty} \underbrace{\sum_{n=1}^N a_n}_{=S_N}$

$\Rightarrow \sum a_n$  converges if  $\{S_N\}$  converges.

works for any  $a_n$  but often difficult to get value of  $S_n$ .

### Special Series

Geometric Series  $\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r} \quad (|r| < 1)$

Harmonic Series  $\sum_{n=1}^{\infty} \frac{1}{n}$  diverges

### p-Test

$$\sum \frac{1}{n^p} \begin{cases} \text{converges} & p > 1 \\ \text{diverges} & p \leq 1 \end{cases}$$

### Test of Divergence

If  $\lim_{n \rightarrow \infty} a_n \neq 0$ , then  $\sum a_n$  diverges.

works for any  $a_n$  (also negative)  
Always good as a first try.

### Integral Test

If  $a_n = f(n)$ , where  $f$  is positive, continuous, and decreasing.

Then  $\sum_{n=1}^{\infty} a_n$  converges if and only if  $\int_1^{\infty} f(x) dx$  converges.

restriction

### Comparison Test

Let  $0 \leq a_n \leq b_n$ .

If  $\sum b_n$  converges, then  $\sum a_n$  converges.

If  $\sum a_n$  diverges, then  $\sum b_n$  diverges.

If your  $a_n$ 's are positive then check if  $f$  is cont & decr. to use IT else try Comparison test or Limit CT.

### Limit Comparison Test

Let  $0 < a_n, b_n$ .

If  $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c \in (0, \infty)$ , then  $\sum a_n$  and  $\sum b_n$  either both converge or they both diverge.

### Alternating Series Test

Let  $b_n > 0$ .

If  $b_{n+1} \leq b_n$  and  $\lim_{n \rightarrow \infty} b_n = 0$ , then  $\sum (-1)^{n-1} b_n$  converges.

Use this test if your series is alternating.