Math 1B03/1ZC3 Test #2 (Version 1)
March 22nd, 2017

Name:____
(Last Name) (First Name)

Student Number:______________________

This test consists of 19 multiple choice questions worth 1 mark each (no part marks), and 1 question worth 1 mark (no part marks) on proper computer card filling. Questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Calculators are NOT allowed.

1. Find the reduced row-echelon form of the following (complex valued) matrix.

$$A = \begin{bmatrix} 1 & i+2 & i+1 \\ 1 & 2 & i \end{bmatrix}$$

(a) $\begin{bmatrix} 1 & 0 & 2i-1 \\ 0 & 1 & -i \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & 0 & 3i \\ 0 & 1 & -i \end{bmatrix}$ **(c)** $\begin{bmatrix} 1 & 0 & 2i-3 \\ 0 & 1 & i+1 \end{bmatrix}$
(d) $\begin{bmatrix} 1 & 0 & 2i+1 \\ 0 & 0 & 1 \end{bmatrix}$ **(e)** $\begin{bmatrix} 1 & 0 & 1-2i \\ 0 & 1 & i+1 \end{bmatrix}$

2. Find the area of the triangle in 3-space that has the following vertices.

$$P_1(-1,2,0),\ P_2(1,1,1),\ P_3(2,0,1)$$

(a) $\frac{1}{2}\sqrt{3}$ **(b)** $\sqrt{3}$ **(c)** $\frac{1}{2}\sqrt{6}$ **(d)** $\frac{1}{2}\sqrt{2}$ **(e)** $\sqrt{6}$

3. Find all of the roots $(-8)^{1/3}$. (i.e., find all of the cubed roots of -8).
(a) $-2, 1-\frac{1}{2}i, 1+\frac{1}{2}i$ **(b)** $-2, 1-\frac{\sqrt{3}}{2}i,\ 1+\frac{\sqrt{3}}{2}i$ **(c)** $-2,\ 1-\sqrt{3}i,\ 1+\sqrt{3}i$
(d) $-2,\ \frac{1}{2}-\frac{\sqrt{3}}{2}i,\ \frac{1}{2}+\frac{\sqrt{3}}{2}i$ **(e)** $-2, \frac{1}{2}-\frac{1}{2}i,\ \frac{1}{2}+\frac{1}{2}i$

4. On a given day the air quality in a certain city is either good or bad. Records show that when the air quality is good on one day, then there is a $\frac{4}{5}$ chance that it will be good the next day, and when the air quaity is bad on one day there is a $\frac{1}{4}$ chance that it will be bad the next day. If the air quality is good today, what is the probability that it will be good two days from now?

(a) $\frac{31}{100}$ **(b)** $\frac{39}{100}$ **(c)** $\frac{63}{100}$ **(d)** $\frac{79}{100}$ **(e)** $\frac{69}{100}$

5. Let **u** and **v** be vectors in R^3. Which of the following statements are always true?

(i) If $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w}$ then $\mathbf{v} = \mathbf{w}$.
(ii) If $\mathbf{u} \cdot \mathbf{v} = 0$ then either $\mathbf{u} = 0$ or $\mathbf{v} = 0$.
(iii) $\|\mathbf{u} + \mathbf{v}\| = \|\mathbf{u}\| + \|\mathbf{v}\|$.

(a) (i) and (ii) only **(b)** (i) and (iii) only **(c)** (i) only **(d)** (iii) only **(e)** none of them

6. In Matlab which of the following commands could be used to compute the cross product of the vectors **u** and **v**.
(a) `>>u (cross) v` **(b)** `>>u crossproduct v` **(c)** `>>u cross v` **(d)** `>>uxv`
(e) `>>cross(u,v)`

7. Let **u**, **v**, and **w** be vectors in R^3. Which of the following statements are always true?

(i) If **u** is orthogonal to both **v** and **w** then **u** is orthogonal to $k_1\mathbf{v} + k_2\mathbf{w}$ for all scalars k_1 and k_2.
(ii) If **w** is a nonzero vector and $\text{proj}_{\mathbf{w}}\mathbf{u} = \text{proj}_{\mathbf{w}}\mathbf{v}$ then $\mathbf{u} = \mathbf{v}$.
(iii) If **u** and **v** are orthogonal vectors then for all nonzero scalars k_1 and k_2, then $k_1\mathbf{u}$ and $k_2\mathbf{v}$ are orthogonal vectors.

(a) (ii) and (iii) only **(b)** (i) and (iii) only **(c)** (iii) only **(d)** all of them
(e) (i) and (ii) only

8. Suppose that a 2×2 matrix A has eigenvalues $\lambda = -2$ and -1, with corresponding eigenvectors $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 7 \\ -2 \end{bmatrix}$, respectively. Find A^2.

(a) $\begin{bmatrix} 1 & -\frac{4}{3} \\ -\frac{1}{3} & \frac{5}{3} \end{bmatrix}$ **(b)** $\begin{bmatrix} \frac{7}{3} & \frac{1}{3} \\ -1 & \frac{4}{3} \end{bmatrix}$ **(c)** $\begin{bmatrix} \frac{5}{3} & \frac{7}{3} \\ \frac{2}{3} & \frac{10}{3} \end{bmatrix}$ **(d)** $\begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{5}{3} & \frac{4}{3} \end{bmatrix}$ **(e)** $\begin{bmatrix} \frac{4}{3} & -\frac{1}{3} \\ \frac{7}{3} & -\frac{8}{3} \end{bmatrix}$

9. Let P be the point $(1,2,-1)$. If the point $(3,0,2)$ is $\frac{1}{4}$ of the way from P to Q, what is Q?

(a) $(5,1,3)$ **(b)** $(10,-5,6)$ **(c)** $(8,-3,10)$ **(d)** $(9,-6,11)$ **(e)** $(8,8,12)$

10. Find the distance between the point $(1,1)$ and the line $2x+y-1=0$.
(a) $\frac{2}{\sqrt{5}}$ **(b)** $\frac{1}{\sqrt{3}}$ **(c)** $\sqrt{5}$ **(d)** $\sqrt{3}$ **(e)** $\frac{2}{\sqrt{3}}$

11. Consider the following matrix.

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

The eigenvalues of A are $\lambda = 4,\ -1,\ 1$. The eigenvector corresponding to $\lambda = 4$ is $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, and the eigenvector corresponding to $\lambda = -1$ is $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$. Find a matrix P such that $P^{-1}AP = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

(a) $\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & -1 & 1 \\ -2 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ **(c)** $\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & -2 \\ 1 & 1 & 1 \end{bmatrix}$ **(d)** $\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$
(e) $\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 1 \\ -2 & 1 & 1 \end{bmatrix}$

12. Which if the following statements about complex numbers are always true?

(i) Re$(i\,\overline{z})$ = Im(z)
(ii) $z + \overline{z} =$ Re(z)
(iii) If $|z_1| = |z_2|$ then $z_1 = \pm z_2$.

(a) all of them **(b)** (i) and (iii) only **(c)** (i) and (ii) only **(d)** (i) only **(e)** none of them

13. Find the parametric equations of the line in R^3 that passes through the points $(1, -1, -2)$ and $(3, -3, -4)$.

(a) $x = 1 - 2t$, $y = 1 + 2t$, $z = -1 - 2t$ **(b)** $x = 1 + 4t$, $y = 1 - 2t$, $z = -2 - 2t$ **(c)** $x = 1 + 2t$, $y = -1 - 2t$, $z = -2 - 2t$ **(d)** $x = 2 + 2t$, $y = -1 - 2t$, $z = -2 + 2t$
(e) $x = 1 + t$, $y = 1 - 2t$, $z = 2 - 2t$

14. Which of the following statements are always true?

(i) If A and B are similar $n \times n$ matrices, then there is an invertible matrix P such that $PA = PB$
(ii) If A is diagonalizable then there is a unique matrix P such that $P^{-1}AP$ is diagonal.
(iii) If A is diagonalizable then A is invertible.

(a) (i) only **(b)** (i) and (iii) only **(c)** none of them **(d)** (ii) and (iii) only **(e)** (iii) only

15. Solve the following equation for the complex number z. $z(2 + i) = \overline{z} + (2 - 6i)$
(a) $-2i$ **(b)** $1 - 2i$ **(c)** $1 + 2i$ **(d)** $2 - 3i$ **(e)** $3 - 2i$

16. After exposure to certain live pathogens, the body develops long-term immunity. The evolution over time of the associated disease can be modeled as a dynamical system whose state vector at time t consists of the number of people who have not been exposed and are therefore susceptible, the number who are currently sick with the disease, and the number who have recovered and are now immune. Suppose that the associated 3×3 yearly transition matrix A has eigenvalues $\lambda = 1, \frac{1}{2}, 0$, and that the eigenvectors corresponding to the first two eigenvalues are $\mathbf{x}_1 = (60, 20, 30)$ and $\mathbf{x}_2 = (-60, -30, 90)$, respectively. The initial state vector for the population is given by

$$\mathbf{v}_0 = 500\mathbf{x}_1 + 200\mathbf{x}_2 + 100\mathbf{x}_3$$

where the third eigenvector $\mathbf{x}_3$ is not given here. How many people will have not been exposed to the disease 1 year later?

(a) 4,000 **(b)** 27,000 **(c)** 8,500 **(d)** 7,000 **(e)** 24,000

17. If $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = 2$, find the value of $(\mathbf{u} + \mathbf{v}) \cdot (\mathbf{w} \times \mathbf{v})$.

(a) 0 **(b)** -1 **(c)** 1 **(d)** -2 **(e)** 2

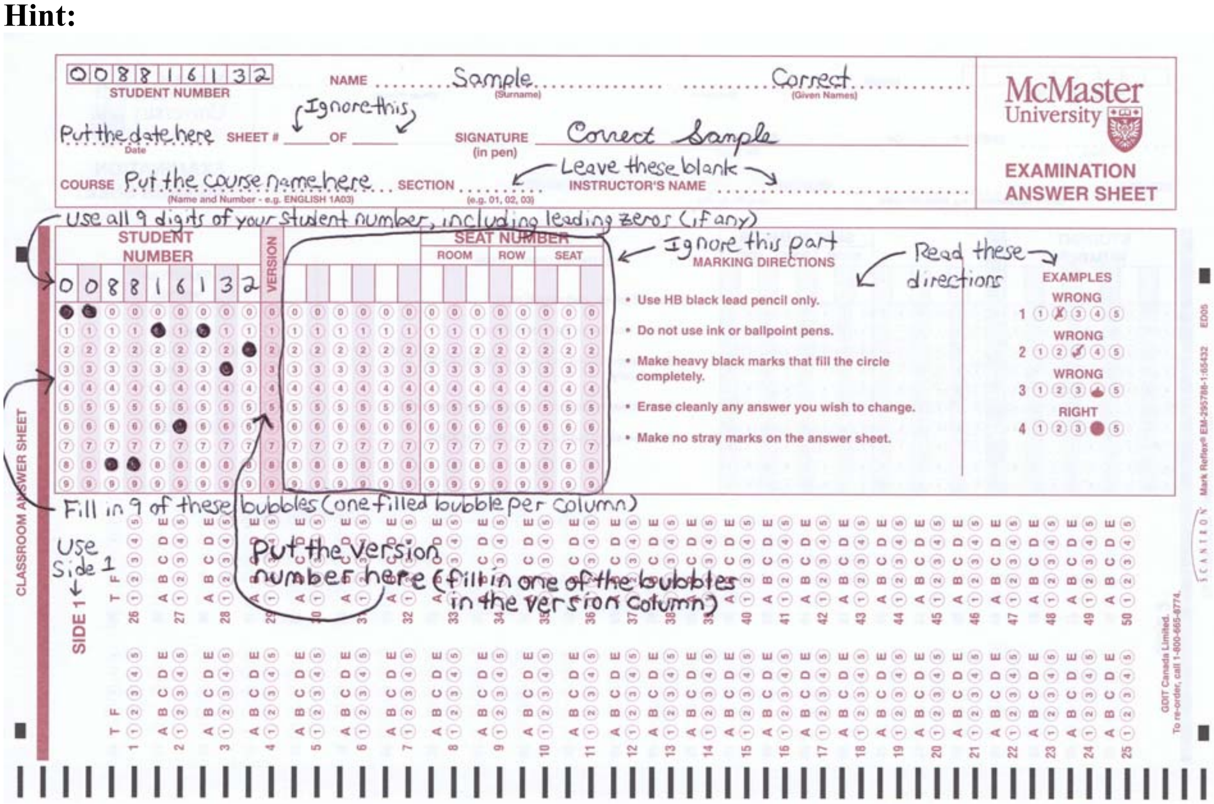
18. Suppose that a Markov chain has the following transition matrix.

$$\begin{bmatrix} 0 & \frac{2}{3} & \frac{1}{2} \\ \frac{2}{3} & 0 & \frac{1}{2} \\ * & * & * \end{bmatrix}$$

Fill in the missing entries and then determine the proportion of time (in the long run) that the system is in State 3.

(a) $\frac{1}{4}$ **(b)** $\frac{3}{8}$ **(c)** $\frac{1}{2}$ **(d)** $\frac{2}{3}$ **(e)** $\frac{5}{8}$

19. Correctly fill out the bubbles corresponding to all 9 digits of your student number, as well as the version number of your test in the correct places on the computer card.
Note: You are writing **Version 1**.
Hint:



Answers (Version 1)**:**

1. b **2.** a **3.** c **4.** d **5.** e **6.** e **7.** b **8.** c **9.** d **10.** a
11. b **12.** d **13.** c **14.** c **15.** a **16.** e **17.** d **18.** a