MATH 1AA3 - Winter 2022

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1 Integration Review

1.1 Basic Definitions and the Fundamental Theorem

The **indefinite integral** of $f(x)$ is denoted by ______________.

We call $F(x)$ an **anti-derivative** of $f(x)$ if ______________________.

Q: Is the anti-derivative of a function $f(x)$ unique?

The **definite integral** of a continuous function $f(x)$ for $x \in [a, b]$ is denoted

by ______________________ and defined by the so-called ______________________,

i.e.,

Fundamental Theorem of Calculus:

$$\boxed{\int_a^b f(x)\,\mathrm{d}x =}$$

1.2 Evaluation of Integrals

1.2.1 Some basic indefinite integrals

$$\int x^n \,\mathrm{d}x =$$

$$\int \frac{1}{x} \,\mathrm{d}x =$$

$$\int e^x \,\mathrm{d}x =$$

$$\int \sin(x) \,\mathrm{d}x =$$

$$\int \cos(x) \,\mathrm{d}x =$$

$$\int \sec^2(x) \,\mathrm{d}x =$$

$$\int \frac{1}{x^2+1} \,\mathrm{d}x =$$

1.2.2 Some techniques of integration

A. ____________________

Example: $\int x^2 e^{x^3}\,\mathrm{d}x$

$\Longrightarrow$ If $\displaystyle\int f(x)\,\mathrm{d}x = F(x) + C,$ then $\displaystyle\int f(kx)\mathrm{d}x =$

B. ____________________

Example: $\int xe^{2x}\,\mathrm{d}x$

C. ____________________

Example: $\int \frac{1}{x^2-x}\,\mathrm{d}x$

D. ____________________

Example: $\int \cos^2(x)\,\mathrm{d}x$

2 Improper Integrals (Ch. 7.8)

2.1 Improper Integrals – Type I

2.1.1 Case A

Let $f(x)$ be a function defined on $[a, \infty)$ and assume that for all $t \geq a$, $\int_a^t f(x)\,\mathrm{d}x$ exists.

Define $$\int_a^\infty f(x)\,\mathrm{d}x =$$

Terminology:

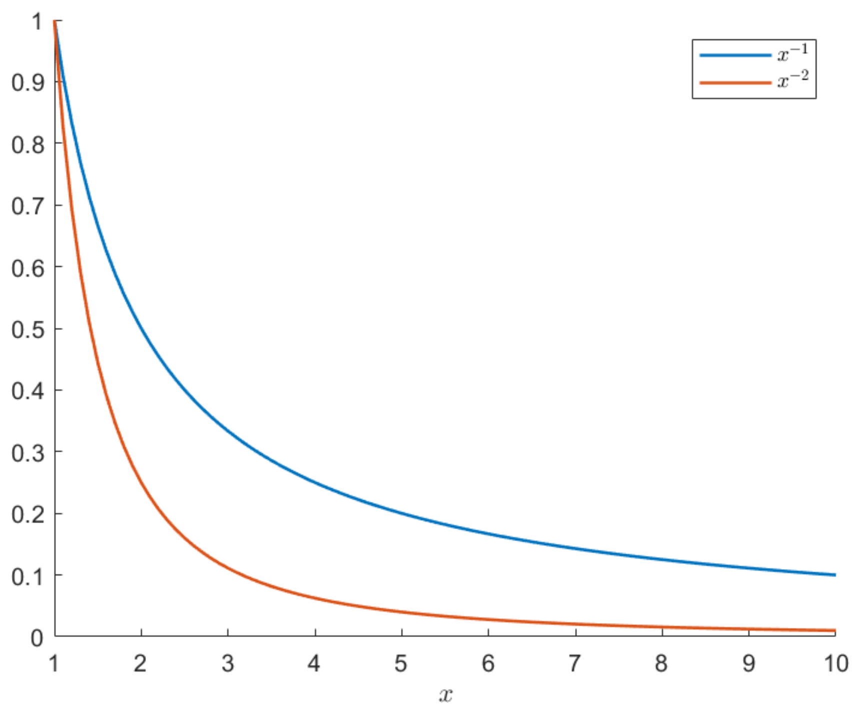
We say that $\int_a^\infty f(x)\,\mathrm{d}x$ is **convergent** if ____________________________,

else we say that $\int_a^\infty f(x)\,\mathrm{d}x$ is ______________.

Example:

1.) $$\int_1^t \frac{1}{x^2}\,\mathrm{d}x =$$

2.) $\displaystyle\int_1^t \frac{1}{x}\,\mathrm{d}x =$



General Rule:

$$\int_1^\infty \frac{1}{x^p}\,\mathrm{d}x =$$

Example $$\int_0^\infty \cos(x)\,\mathrm{d}x =$$

Example $$\int_0^\infty x e^{-x}\,\mathrm{d}x =$$