$SA = \int_{Axis} 2\pi r\, ds$ in general. And in this case, we are rotating $f(x) = \frac{x^3}{6} + \frac{1}{2x},\ \frac{1}{2} \le x \le 1$

about the x-axis.

So, let's integrate in the x variable: $SA = \int_{1/2}^{1} 2\pi f(x)\sqrt{1+(f'(x))^2}\, dx$

Now, $f(x) = \frac{x^3}{6} + \frac{1}{2x}$ so $f'(x) = \frac{x^2}{2} - \frac{1}{2x^2}$ and $\left(f'(x)\right)^2 = \frac{x^4}{4} + \frac{1}{4x^4} - \frac{1}{2}$

so $\sqrt{1+\left(f'(x)\right)^2} = \sqrt{\frac{x^4}{4} + \frac{1}{4x^4} + \frac{1}{2}} = \sqrt{\left(\frac{x^2}{2} + \frac{1}{2x^2}\right)^2} = \frac{x^2}{2} + \frac{1}{2x^2}$

Notice that's the "magic half" trick from term 1 arclength calculations!

Now, let's do the integral:

$$SA = \int_{1/2}^{1} 2\pi\left(\frac{x^3}{6} + \frac{1}{2x}\right)\left(\frac{x^2}{2} + \frac{1}{2x^2}\right)dx = \frac{\pi}{6}\int_{1/2}^{1}\left(x^3 + 3x^{-1}\right)\left(x^2 + x^{-2}\right)dx$$

$$= \frac{\pi}{6}\int_{1/2}^{1}\left(x^5 + x\right) + \left(3x + 3x^{-3}\right)dx = \frac{\pi}{6}\int_{1/2}^{1} x^5 + 4x + 3x^{-3}dx = \frac{\pi}{6}\left(\frac{x^6}{6} + 2x^2 - \frac{3}{2x^2}\right)\Bigg|_{1/2}^{1}$$

$$= \frac{263}{256}\pi$$