Math 1AA3/1ZB3: Test 1 Review

February 12, 2019

1. Compute the following integrals, or show that they diverge.

   (a) $\int_2^\infty \frac{1}{x(\ln x)^2} dx$

   (b) $\int_0^1 \frac{\cos x}{x^2+x} dx$

2. Either find the limits of the following sequences, or show that they diverge:

   (a) $a_n = \cos\left(\frac{n\pi}{2}\right) \cdot \sin\left(\frac{n\pi}{2}\right)$

   (b) $a_{n+1} = \frac{3}{1+a_n}$

3. According to an appropriate error estimate, how many terms are needed to approximate $\sum_{n=1}^\infty \frac{1}{2n^3}$ to within an error of 0.0001

4. Find the sum of the series: $\sum_{n=0}^\infty \frac{1+2^{n+1}}{3^{n-1}}$

5. Find the radius and interval of convergence of the power series: $\sum_{n=0}^\infty \frac{(3x-2)^n}{4^{n+1}}$

6. Suppose that the series $\sum_{n=1}^\infty a_n$ is convergent and $a_n > 0$ for all n. Which of the following statements **must** be true?

   (a) $\sum_{n=1}^\infty a_n^2$ is convergent.

   (b) $\sum_{n=1}^\infty (-1)^n a_n$ is convergent.

   (c) $\sum_{n=1}^\infty \sqrt{a_n}$ is convergent.

7. Determine whether the following series converge or diverge. If an alternating series is convergent, determine whether it is absolutely convergent or conditionally convergent.

   (a) $\sum_{n=2}^\infty \frac{n+\cos n}{\sqrt{2n^3+3n}}$

   (b) $\sum_{n=1}^\infty \frac{(-2)^n \ln n}{(n+1)!}$

   (c) $\sum_{n=2}^\infty \frac{1}{n(\ln n)^2}$

   (d) $\sum_{n=0}^\infty \frac{2^n}{3^n+4^n}$

   (e) $\sum_{n=1}^\infty \left(n \sin(3/n)\right)^{2n}$

   (f) $\sum_{n=1}^\infty \frac{n! \cdot 2^{n+1}}{2 \cdot 5 \cdot 8 \cdot \ldots \cdot (3n-1)}$

   (g) $\sum_{n=1}^\infty (-1)^n \frac{(1+\ln n)}{1+n}$