COMPSCI/SFWRENG 2FA3 Midterm Test 2
McMaster University

Day Class CS 01, CS 02, SE 01, **Version 1** Dr. W. M. Farmer

DURATION: 2 hours March 15, 2019

Please CLEARLY print:

NAME:

Student ID: | | | | | | | | | |

In an addition to this examination paper, you will be given two answer sheets (scan sheets) for this test. This examination paper includes 10 pages and 30 questions. You are responsible for ensuring that your copy of the examination paper is complete. Bring any discrepancy to the attention of your invigilator.

The examination will be conducted in two stages:

First Stage: You have 90 minutes to answer the questions in the examination paper on the first answer sheet working **by yourself**. Getting any help in any form from your fellow students and anyone else will be treated as academic dishonesty. You must submit your first answer sheet to your invigilator by the end of the 90-minute period. Your performance on the answer sheet counts for 85% of the Midterm Test 2 mark. You may want to fill out the second answer sheet as you fill out the first leaving blank those questions that you want to work on during the second stage.

Second Stage: You have 30 minutes to answer the questions in the examination paper on the second answer sheet working **with the other students in the test room**. You may walk around the test room, but you may not leave the test room. You must submit your second answer sheet and your examination paper to your invigilator by the end of the 30-minute period. Your performance on the answer sheet counts for 15% of the Midterm Test 2 mark.

Special Instructions:

1. It is your responsibility to ensure that the two answer sheets are properly completed. Your examination result depends upon proper attention to these instructions:

   - A heavy mark must be made, completely filling the circular bubble, with an HB pencil.
   - Print your name, student number, course name, course number and the date in the space provided on the top of Side 1 and fill in the corresponding bubbles underneath.
   - **Fill in the bubble corresponding to your version number.**
   - Mark only **ONE** choice from the alternatives (1, 2, 3, 4, 5 or A, B, C, D, E) provided for each question. If there is a True/False question, mark 1 (or A) for True, and 2 (or B) for False. The question number is to the left of the bubbles. Make sure that the number of the question on the scan sheet is the same as the number on the examination paper.

- Pay particular attention to the "Marking Directions" given on the scan sheet.
- Begin answering the questions using the first set of bubbles, marked "1." Answer all questions.

Please note that, if either answer sheet is completed with ink or the student number or version number are not bubbled correctly, your mark for the test will be reduced by one question (so, e.g., 20/30 becomes 19/30).

2. The use of notes and textbooks is **not** permitted in both stages of the test.
3. Calculators, computers, cell phones, and all other electronic devices are **not** to be utilized in both stages of the test.
4. Read each question carefully.
5. Try to allocate your time sensibly and divide it appropriately between the questions.
6. Select the **best** answer for each question.

Question 1 [1 mark]
Every well-founded relation is transitive. Is this statement true or false?

A. True.

B. False.

Question 2 [1 mark]
The halting problem is an example of an undecidable decision problem. Is this statement true or false?

A. True.

B. False.

Question 3 [1 mark]
The structural induction principle for regular expressions over an alphabet Σ can be directly expressed in MSFOL. Is this statement true or false?

A. True.

B. False.

Question 4 [1 mark]
Let $(K, 0, 1, +, \cdot, ^*)$ be a Kleene algebra. Then the functions $+$ and $\cdot$ are both associative and commutative. Is this statement true or false?

A. True.

B. False.

Question 5 [1 mark]
A lexical analyzer typically executes a finite automaton to recognize tokens. Is this statement true or false?

A. True.

B. False.

Question 6 [1 mark]
For every NFA there is an equivalent NFA with ϵ-transitions that has exactly one final state. Is this statement true or false?

A. True.

B. False.

Question 7 [1 mark]
If $M = (Q, \Sigma, \delta, s, F)$ is a DFA such that $F = Q$, then $L(M) = \Sigma^*$. Is this statement true or false?

A. True.

B. False.

Question 8 [1 mark]
If L is a regular language, it will usually be much easier to construct a DFA that accepts L than an NFA that accepts L. Is this statement true or false?

A. True.

B. False.

Question 9 [1 mark]
Let $\Sigma = (\{\alpha\}, \emptyset, \emptyset, \{p\}, \tau)$ be a signature of MSFOL where $\tau(p) = \alpha \to \mathbb{B}$ and T_Σ be the set of Σ-terms. Which statement about T_Σ is true?

A. T_Σ is infinite.

B. All of the terms in T_Σ are open.

C. None of the terms in T_Σ contain function symbols.

D. All of the above.

Question 10 [1 mark]
Strong induction is *not* a special case of

A. Structural induction.

B. Ordinal induction.

C. Well-founded induction.

D. All of the above.

Question 11 [1 mark]
Let $\Sigma = (\{\alpha\}, \emptyset, \emptyset, \{\leq\}, \tau)$ be a signature of MSFOL where $\tau(\leq) = \alpha \times \alpha \to \mathbb{B}$. Which statement says that each pair of elements of type α has a least upper bound?

A. $\forall\, x, y : \alpha \,.\, \exists\, z : \alpha \,.\, (x \leq z \wedge y \leq z \wedge \forall\, w : U \,.\, ((w \leq x \wedge w \leq y) \vee z \leq w))$.

B. $\forall\, x, y : \alpha \,.\, \exists\, z : \alpha \,.\, (x \leq z \wedge y \leq z \wedge \forall\, w : U \,.\, (x \leq w \wedge y \leq w \wedge z \leq w))$.

C. $\forall\, x, y : \alpha \,.\, \exists\, z : \alpha \,.\, (x \leq z \wedge y \leq z \wedge \forall\, w : U \,.\, ((x \leq w \wedge y \leq w) \Rightarrow z \leq w))$.

D. $\forall\, x, y : \alpha \,.\, \exists\, z : \alpha \,.\, (x \leq z \wedge y \leq z \wedge \forall\, w : U \,.\, ((w \leq x \wedge w \leq y) \Rightarrow w \leq z))$.

Question 12 [1 mark]
Let Σ be a signature of MSFOL, $\forall\, x : \alpha \,.\, A$ be a closed Σ-formula, and $\mathcal{M}$ be a Σ-structure. If

$$V_{\varphi}^{\mathcal{M}}(\forall\, x : \alpha \,.\, A) = \mathrm{T}$$

for some $\varphi \in \mathsf{assign}(\mathcal{M})$, then which of the following statements about A must be true?

A. A is not satisfiable.

B. A is valid in $\mathcal{M}$.

C. A is universally valid.

D. None of the above.

Question 13 [1 mark]
Let P be a proof system for MSFOL. If, for all signatures Σ of MSFOL, $\Gamma \vdash_P A$ for all sets Γ of Σ-formulas and Σ-formulas A, then

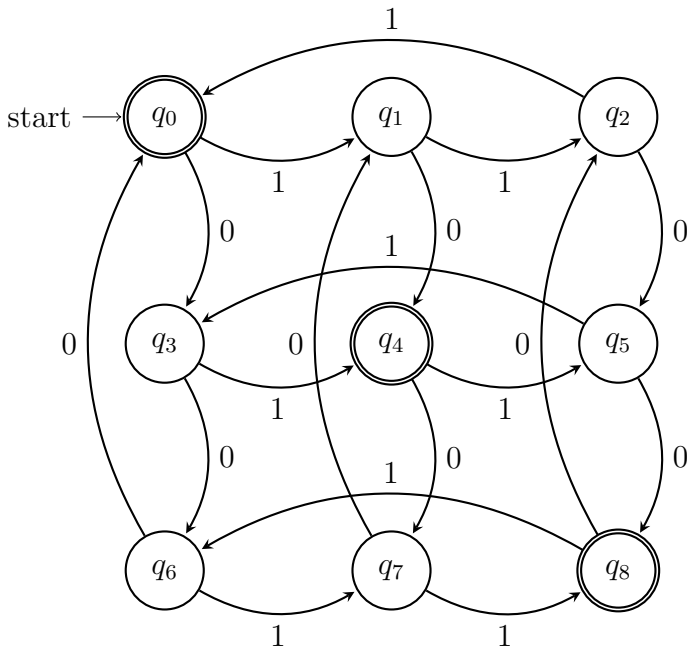
A. P is complete but not sound.

B. P is sound but not complete.

C. P is sound and complete.

D. P is neither sound nor complete.

Question 14 [1 mark]
Consider the DFA M defined by the following transition diagram:



What language over $\Sigma = \{0, 1\}$ does M accept?

A. All strings $x \in \Sigma^*$ for which $\#0(x)$ and $\#1(x)$ are both divisible by 3.

B. All strings $x \in \Sigma^*$ for which $\#0(x) + \#1(x)$ is divisible by 3.

C. All strings $x \in \Sigma^*$ for which $\#0(x) - \#1(x)$ is divisible by 3.

D. All strings $x \in \Sigma^*$ for which $\#0(x) + \#1(x) + 1$ is divisible by 3.

Question 15 [1 mark]
Let $N = (Q, \Sigma, \Delta, S, F)$ be an NFA. Recall that, for a set S, $\mathcal{P}(S)$ is the set of all subsets of S. What is the type of $\hat{\Delta}$ (capital delta hat)?

A. $Q \times \Sigma^* \to Q$.

B. $Q \times \Sigma^* \to \mathcal{P}(Q)$.

C. $\mathcal{P}(Q) \times \Sigma^* \to Q$.

D. $\mathcal{P}(Q) \times \Sigma^* \to \mathcal{P}(Q)$.

Question 16 [1 mark]
Let Σ be a finite alphabet. Which $E \subseteq \Sigma^*$ satisfies the property that

$$EA = AE = A$$

for all $A \subseteq \Sigma^*$.

A. $\emptyset$.

B. ϵ.

C. $\{\epsilon\}$.

D. Σ^*.

Question 17 [1 mark]
Which of the following statements is the easiest to prove?

A. If a DFA accepts a language, then there is an NFA that accepts the same language.

B. If a DFA accepts a language, then there is a regular expression that matches the same language.

C. If an NFA accepts a language, then there is a DFA that accepts the same language.

D. If a regular expression matches a language, then there is an NFA with ϵ-transitions that accepts the same language.

Question 18 [1 mark]
Who introduced the notion of a nondeterministic finite automaton?

A. Alonzo Church and Alan Turing.

B. Stephen Kleene.

C. Dexter Kozen.

D. Michael Rabin and Dana Scott.

Question 19 [1 mark]
Who first showed that there are undecidable decision problems?

A. Alonzo Church and Alan Turing.

B. Stephen Kleene.

C. Gottfied Leibniz.

D. Michael Rabin and Dana Scott.

Question 20 [1 mark]
Let $N = (Q, \Sigma, \Delta, S, F)$ be an NFA such that $F = Q$. Which of the following statements is true?

A. N must accept ϵ.

B. N must accept some nonempty member of Σ^*.

C. N must accept every member of Σ^*.

D. All of the above.

Question 21 [1 mark]
Thompson's construction is used to

A. Produce a DFA that is equivalent to a given regular expression.

B. Produce a DFA that is equivalent to an NFA with ϵ-transitions.

C. Produce an NFA with ϵ-transitions that is equivalent to a given regular expression.

D. Produce a regular expression that is equivalent to a given DFA.

Question 22 [1 mark]
Let $\Sigma = \{0, 1\}$ and L be the set of all strings $x \in \Sigma^*$ such that there are two 0s in x separated by a string of length $3k$ for $k \geq 0$. Which regular expression matches L.

A. $(0 + 1)^*0(0 + 1)^*(0 + 1)^*(0 + 1)^*0(0 + 1)^*$.

B. $(0 + 1)^*0((0 + 1)(0 + 1)(0 + 1))^*0(0 + 1)^*$.

C. $(0 + 1)^*0(((0 + 1)^*)^*)^*0(0 + 1)^*$.

D. All of the above.

Question 23 [1 mark]
Which of the following equations is valid for regular expressions? Here $\alpha = \beta$ means $L(\alpha) = L(\beta)$.

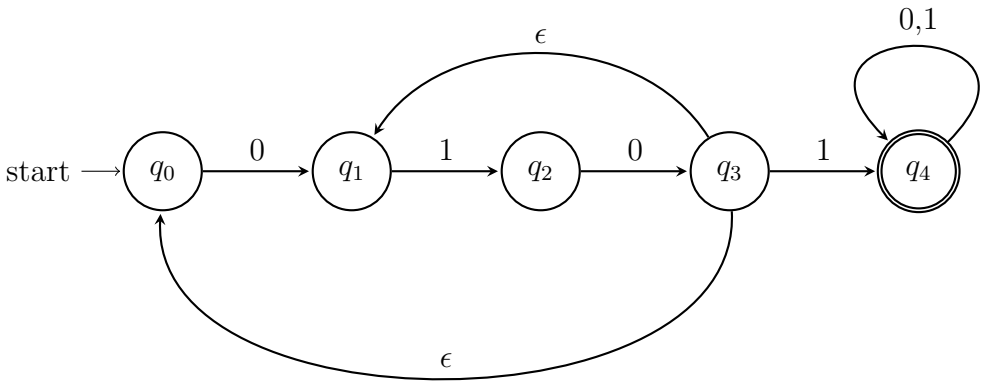
A. $\alpha\emptyset = \alpha$.

B. $\alpha\beta = \beta\alpha$.

C. $(\alpha\beta)^* = (\alpha + \beta)^*$.

D. $\epsilon + \alpha\alpha^* = \alpha^*$.

Question 24 [1 mark]
Consider the NFA N with ϵ-transitions defined by the following transition diagram:



Which regular expression matches $L(N)$?

A. $(0(10)^*)^*1(0+1)^*$.

B. $(010(10)^*)^*1(0+1)^*$.

C. $010(010(10)^*)^*1(0+1)^*$.

D. $010(10)^*(010(10)^*)^*1(0+1)^*$.

Question 25 [1 mark]
Recall that, for a regular expression α over an alphabet Σ, $L(\alpha)$ is the language $L \subseteq \Sigma^*$ matched by α. We defined $L(\alpha)$ by recursion and pattern matching using _____ equations.

A. 3.

B. 4.

C. 6.

D. 8.

Question 26 [1 mark]
Which of the following mathematical structures is a monoid?

A. $(\mathcal{P}(\Sigma^*), \{\epsilon\}, \mathsf{concatenation})$.

B. $(\mathcal{P}(\Sigma^*), \emptyset, \cup)$.

C. $(\mathcal{P}(\Sigma^*), \Sigma^*, \cap)$.

D. All of the above.

The next four questions are based on the following two definitions:

Let $N = (Q_N, \Sigma, \Delta, S_N, F_N)$ be the NFA defined by the following transition table:

	Q \ Σ	0	1
start →	p	$\{q, s\}$	$\{q\}$
final →	q	$\{r\}$	$\{q, r\}$
	r	$\{s\}$	$\{p\}$
final →	s	$\{\}$	$\{p\}$

Let $M = (Q_M, \Sigma, \delta, s_M, F_M)$ be the DFA (including inaccessible states) equivalent to N that is produced by the subset construction.

Question 27 [1 mark]
What is s_M, the start state of M?

A. $\{p\}$.

B. $\{q, s\}$.

C. $\{q, r, s\}$.

D. $\{p, q, r, s\}$.

Question 28 [1 mark]
What is $\delta(\{p, q, r\}, 1)$?

A. $\{q, r\}$.

B. $\{p, q, r\}$.

C. $\{q, r, s\}$.

D. $\{p, q, r, s\}$.

Question 29 [1 mark]
How many states (both accessible and inaccessible) are in F_M?

A. 2.

B. 8.

C. 12.

D. 16.

Question 30 [1 mark]
Which of the following states in Q_M is inaccessible?

A. $\{p, q\}$.

B. $\{q, r\}$.

C. $\{q, s\}$.

D. $\{r, s\}$.

THE END.