Extra practice with limits and continuity

1. (a) You are asked to find out whether $\lim_{(x,y)\to(0,0)} f(x,y)$ exists for some function f. Instead of checking approaches along lines one by one, you decide to calculate the limit of f along the path $y = mx$ and obtain $3m + 4$. What can you say about $\lim_{(x,y)\to(0,0)} f(x,y)$?

(b) You are asked to find out whether $\lim_{(x,y)\to(0,0)} f(x,y)$ exists for some function f. Instead of checking approaches along lines one by one, you decide to calculate the limit of f along the path $y = mx$ and obtain 5. What can you say about $\lim_{(x,y)\to(0,0)} f(x,y)$?

2. Using the properties of limits, calculate each limit.

(a) $\displaystyle\lim_{(x,y)\to(-2,-1)} (x^2 - 3y - 2x^2y)$

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{e^x - e^{2y-1}}{xy+1}$

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3. (a) Show that the limit of $f(x,y) = \dfrac{xy}{x^2 - 5y^2}$ as $(x,y) \to (0,0)$ does not exist by calculating the limit of $f(x,y)$ along the lines $y = -x$ and $y = 2x$.

(b) Show that the limit of $f(x,y) = \dfrac{x^2y}{(x^2+y^2)^2}$ as $(x,y) \to (0,0)$ does not exist by calculating the limit of f along suitably chosen lines.

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4. Explain why each function is continuous near the point where the limit is to be computed. Then use the continuity to calculate each limit.

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \left(e^{-x-y-2} + \sin(xy)\right)$

(b) $\displaystyle\lim_{(x,y)\to(1,-1)} \frac{x^4 - y - xy^3 + 1}{x^2 + y^2}$

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5. Find (sketch) the largest domain on which each function is continuous.

(a) $f(x, y) = \ln(x - y^3)$

(b) $f(x, y) = \dfrac{\sin x}{e^x - 3}$

(c) $f(x, y) = \dfrac{x^3 - 2}{x^2 + y^2 - 10}$

THE END