Math 1A03/1ZA3: Final Exam Study Sheet

Note 1: *The existence/absence of any content on this study sheet does NOT guarantee its existence/absence on the exam. This sheet is strictly a compact review of the major topics covered. You* ***cannot*** *use this document as an aid during the exam.*

Note 2: *You should combine this document with the other two for studying, as the final exam is cumulative.*

Important Equations:

Area Between Two Curves

$$A = \int_a^b |f(x) - g(x)| \, dx$$

where

$$|f(x) - g(x)| = \begin{cases} f(x) - g(x) & \text{when } f(x) \geq g(x) \\ g(x) - f(x) & \text{when } g(x) \geq f(x) \end{cases}.$$

Recall that $f(x) \geq g(x)$ means that the graph of $f(x)$ lies above the graph of $g(x)$.

Volume of Solid Rotated About:

the x-axis

$$V = \int_a^b A(x) \, dx$$

the y-axis

$$V = \int_c^d A(y) \, dy.$$

Work:

$$W = \int_a^b f(x) \, dx$$

where $f(x)$ is a function representing *force* with respect to distance.

Average Value of a Function $f(x)$ Over an Interval $[a, b]$:

$$f_{\text{ave}} = \frac{1}{b-a} \int_a^b f(x) \, dx.$$

Integration by Parts:

$$\int f(x)g'(x) \, dx = f(x)g(x) - \int g(x)f'(x) \, dx$$

or equivalently, if $u = f(x)$ and $v = g(x)$, then

$$\int u \, dv = uv - \int v \, du.$$

Also,

$$\int_a^b f(x)g'(x) \, dx = f(x)g(x)]_a^b - \int_a^b g(x)f'(x) \, dx.$$

Arc Length of a Function $f(x)$ Over an Interval $[a, b]$:

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx.$$

Theorems and Concepts:

You should know and understand the following...

(1) Finding volumes of revolution. This includes finding volumes of solids which are obtained by rotating about a line that is *not* the x-axis or y-axis. For example, $x = 1$, or $y = -10$.

(2) Practice lots of 'work' problems! These questions are best learned through experience with the actual questions themselves. Also, remember that the measurement 'pounds' (lbs) is a unit of force! (This means that if you see units of pounds, you do not need to include 9.8 m/s^2 in your calculations!)

(3) Understand how to use integration by parts. Remember for this technique, we need to pick a 'u' and a 'dv'. Once we do this, we differentiate to get 'du' and integrate to get 'v'. It takes a lot of practice in order to tell what you should pick as your u or your dv!!! (If any of that sounded confusing, read the section on integration by parts in the textbook and look over your notes).

(4) Understand how to use trigonometric substitution. As a note, if you see an integrand with a function of the form...

$a^2 - b^2x^2$, try setting $x = \frac{a}{b}\sin(\theta)$ and use $1 - \sin^2(\theta) = \cos^2(\theta)$ to simplify;

$a^2 + b^2x^2$, try setting $x = \frac{a}{b}\tan(\theta)$ and use $1 + \tan^2(\theta) = \sec^2(\theta)$ to simplify;

$b^2x^2 - a^2$, try setting $x = \frac{a}{b}\sec(\theta)$ and use $\sec^2(\theta) - 1 = \tan^2(\theta)$ to simplify.

(5) Go through and understand *all* cases for integration by partial fractions. Be able to distinguish when we have distinct or repeated linear factors in the denominator.

(6) When calculating arc length, use the '1' underneath the square root to your advantage. Try making a common denominator with it or use it for factoring.

(7) READ SECTION 7.5!!!! There is no set way to determine which way to integrate a function...this comes with practice. Section 7.5 should help you with this issue.

Lastly, and as always, be confident going into the exam (whether you feel ready or not). Writing your exam with a positive attitude will help! So mentally pump yourself up before taking the exam! Walk in with a clear mind and don't let the test-writing environment get the best of you. This exam is just a tiny part in the grand scheme of your academic career here at McMaster. So just study as best you can, that's all anyone can ask.

It was an absolute pleasure working with every one of you this term. I wish you the best of luck!

Lee van Brussel