Recap

$$f'(x) = \frac{d}{dx} f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

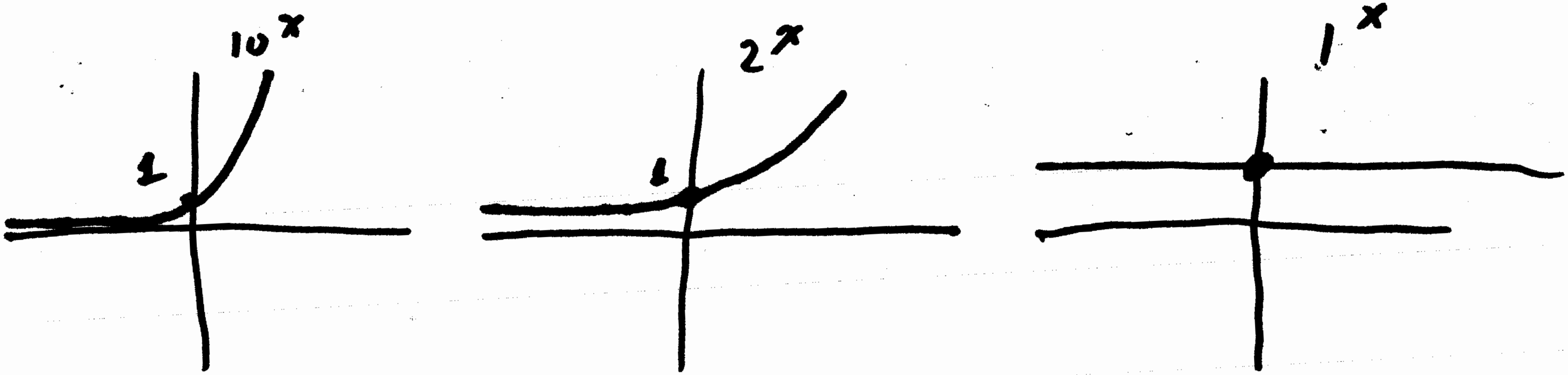
or
$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \to = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

eg. Find the derivative $\frac{d}{dx} e^x$

Let's look at $\frac{d}{dx} a^x$

$$\frac{d}{dx} a^x = \lim_{h \to 0} \frac{\cancelto{a^x \cdot a^h}{a^{x+h}} - a^x}{h} = a^x \left(\lim_{h \to 0} \frac{a^h - 1}{h} \right)$$

$$\Rightarrow \quad \frac{d}{dx} a^x = a^x \cdot (\text{slope of } a^x \text{ at } x = 0)$$



There <u>must</u> exist an "a" for a^x to have

$f'(0) = 1$ — Let's call this "a", <u>e</u>

so $\boxed{\frac{d}{dx} e^x = e^x \cdot 1 = e^x}$ by Definition!

(we'll find $\frac{d}{dx} a^x$, later!)

Derivative Laws (How to avoid limits & principles!)

1) $\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$

2) $\frac{d}{dx}(k f(x)) = k \frac{d}{dx} f(x)$

k constant

3) $\frac{d}{dx} 1 = 0$

4) $\frac{d}{dx} c = 0$, $c = \text{const}$

5) $\frac{d}{dx} x = 1$

6) $\frac{d}{dx} x^2 = 2x$

7) $\frac{d}{dx} x^3 = 3x^2$

8) $\frac{d}{dx} x^p = p x^{p-1}$

note: $\frac{d}{dx} 1/x = \frac{d}{dx} x^{-1} = -x^{-2} = \frac{-1}{x^2}$

$\frac{d}{dx} \sqrt{x} = \frac{d}{dx} x^{\frac{1}{2}} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

9) $\frac{d}{dx} f(x) g(x) = f'(x) g(x) + g'(x) f(x)$

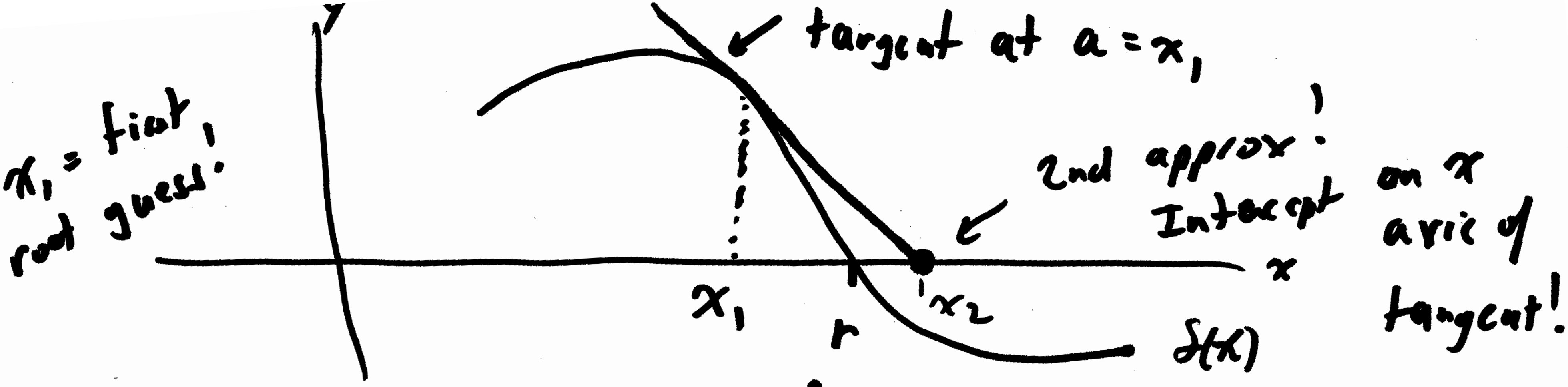
10) $\frac{d}{dx} f(x)/g(x) = \frac{f'(x)\,g(x) - g'(x)\,f(x)}{(g(x))^2}$

eg. $\frac{d}{dx} e^x/(x^2-1) = \frac{e^x \cdot (x^2-1) - (2x) \cdot e^x}{(x^2-1)^2}$

$= \frac{e^x(x^2-2x-1)}{(x^2-1)^2}$

Before we continue! Let's do an application!

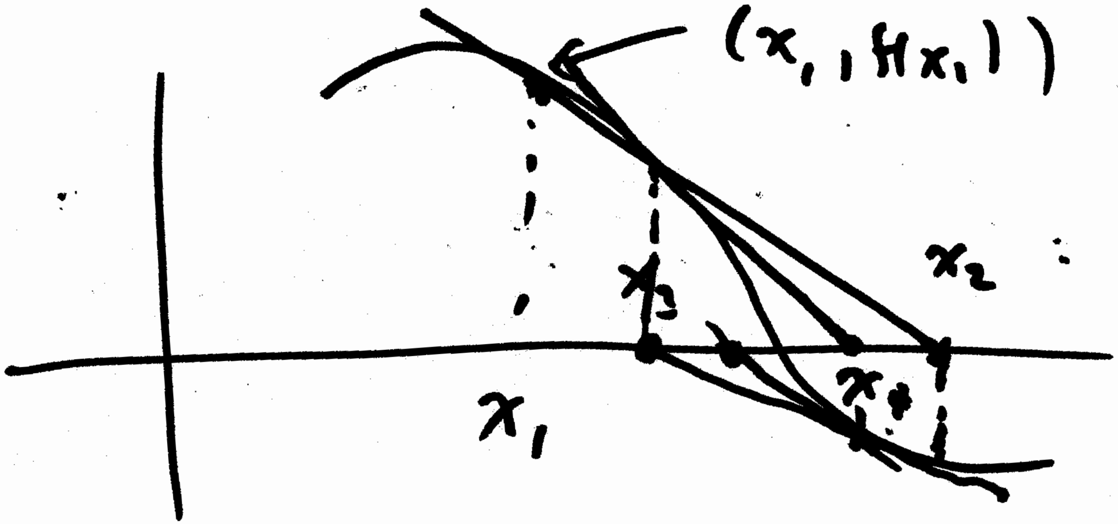
Newton's Method (of finding roots).



$f(r) = 0$, r is root!

x_2 is "generally" closer to r than x_1 is!

$\Rightarrow$ repeat!



$(x_2, 0)$

Newton Proved these approx. must converge to the root, r, if they converge at all!

Let's make an algorithm to make x_2 from x_1

A tangent line at x_1 has slope $m = f'(x_1)$

& passes through: $(x_1, f(x_1))$ & $(x_2, 0)$

so $f'(x_1) = \dfrac{0 - f(x_1)}{x_2 - x_1} \Rightarrow (x_2 - x_1) = -\dfrac{f(x_1)}{f'(x_1)}$

so $\boxed{x_2 = x_1 - f(x_1)/f'(x_1)}$

In general

$$\boxed{x_{n+1} = x_n - f(x_n)/f'(x_n)}$$

e.g. If $f(x) = x^2 - 4$, find where it goes to zero using $x_1 = 1$ & get x_3 using Newton's method!

Solution

$x_1 = 1$

$$x_2 = x_1 - f(x_1)/f'(x_1) = 1 - \frac{(1^2-4)}{2(1)}$$

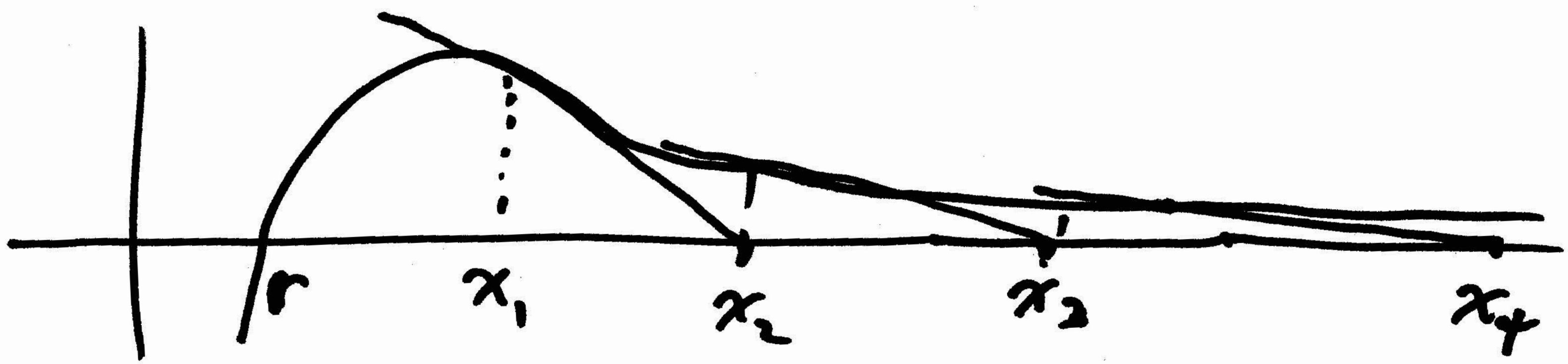
$$= 1 + 3/2 = 5/2$$

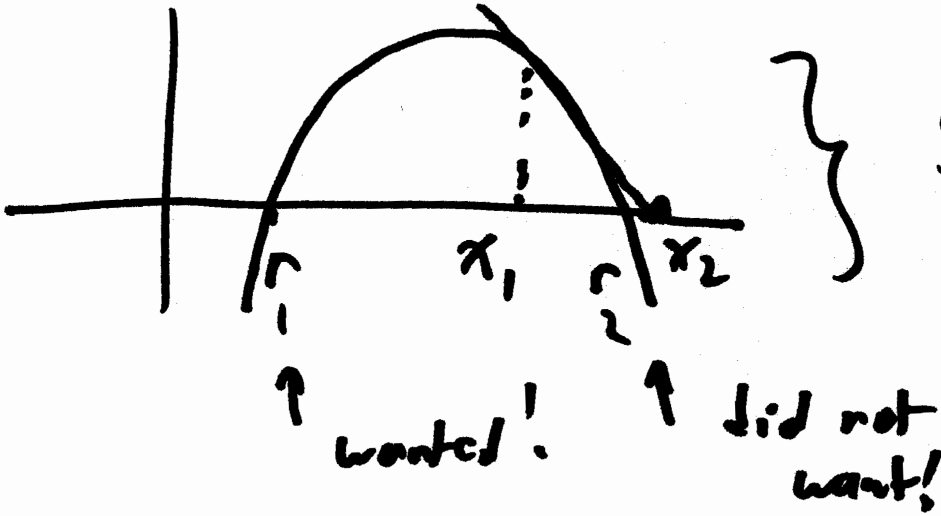
$$x_3 = x_2 - f(x_2)/f'(x_2) = \frac{5}{2} - \frac{\left((5/2)^2 - 4\right)}{2 \cdot 5/2}$$

$$= 5/2 - \left(25/4 - 16/4\right)/5$$

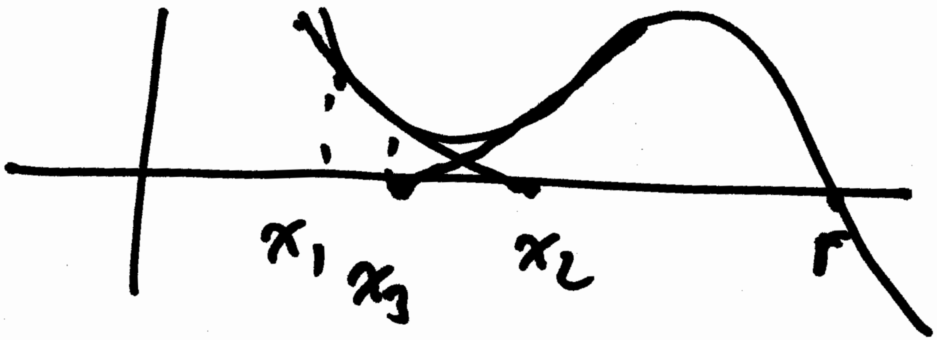
$$= \frac{50}{20} - \frac{9}{20} = \underline{\underline{\frac{41}{20}}}$$

<u>Newton is Terrible</u> (as a Method!)

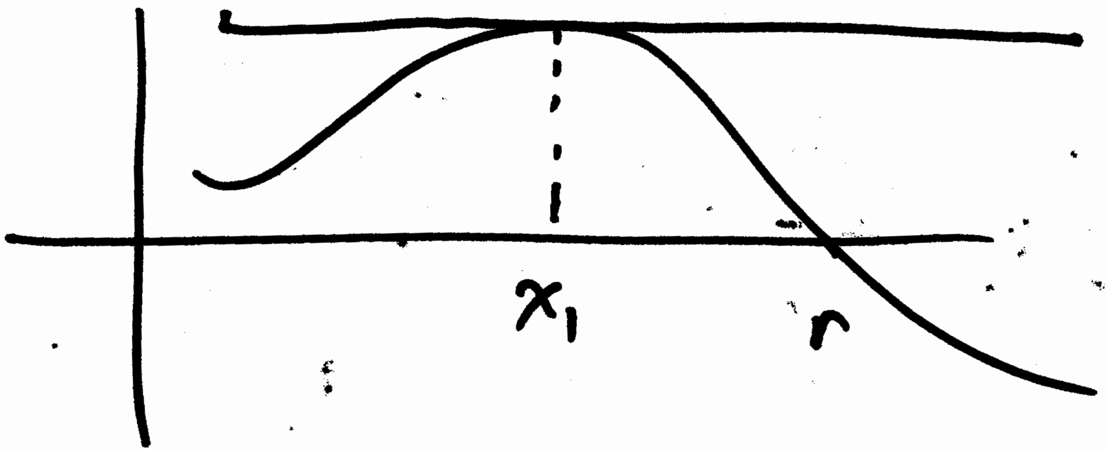




goes to "wrong" root!



gets easily stuck so conv. **very** **slowly**.



x_2?

if $f'(x_1) = 0$

$\Rightarrow$ FAIL //