

Math 1A03/1ZA3
Last Set of Sample Test Questions

Name: _____
(Last Name) (First Name)

Student Number: _____ **Tutorial Number:** _____

This test consists of 33 multiple choice questions worth 1 mark each (no part marks), and 1 question worth 1 mark (no part marks) on proper computer card filling. All questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Only the McMaster standard calculator Casio fx-991 is allowed.

1. Which of the following integrals represents the area of the region enclosed by the curves

$$y = \cos x, \quad y = \sin 2x, \quad x = 0, \quad x = \pi/2$$

- (a) $\int_0^{\pi/2} (\cos x - \sin 2x) dx$
(b) $\int_0^{\pi/4} (\cos x - \sin 2x) dx + \int_{\pi/4}^{\pi/2} (\sin 2x - \cos x) dx$
(c) $\int_0^{\pi/4} (\sin 2x - \cos x) dx + \int_{\pi/4}^{\pi/2} (\cos x - \sin 2x) dx$
(d) $\int_0^{\pi/6} (\cos x - \sin 2x) dx + \int_{\pi/6}^{\pi/2} (\sin 2x - \cos x) dx$
(e) $\int_0^{\pi/6} (\sin 2x - \cos x) dx + \int_{\pi/6}^{\pi/2} (\cos x - \sin 2x) dx$

2. Find the area of the region enclosed by the curves $y = 12 - x^2$ and $y = x^2 - 6$.

- (a) 72 (b) 66 (c) 54 (d) 86 (e) 44

3. Select the answer which represents the area of the region enclosed by the curves $y = x^2$, $y = x$, $x = 0$, $x = \frac{3}{2}$.

- (a) $\int_0^{3/2} (x^2 - x) dx$ (b) $\int_0^{3/2} (x - x^2) dx$ (c) $\int_0^1 (x - x^2) dx + \int_1^{3/2} (x^2 - x) dx$
(d) $\int_0^1 (x^2 - x) dx + \int_1^{3/2} (x - x^2) dx$ (e) $\int_0^1 x dx - \int_1^{3/2} x^2 dx$

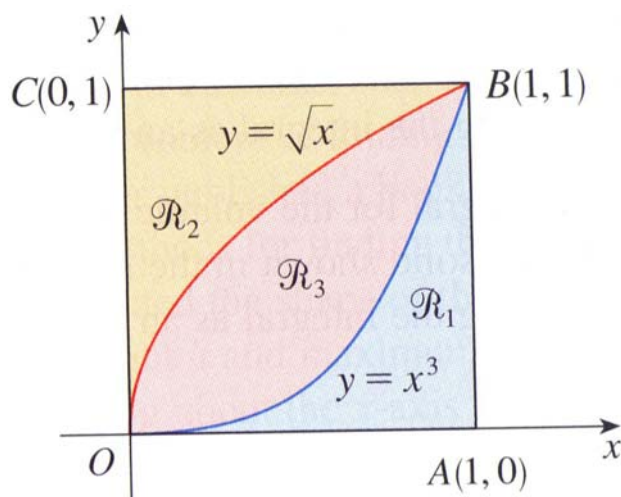
4. The region \mathcal{R} enclosed by the curves $y = x$ and $y = x^2$ is rotated about the x -axis. Find the volume of the resulting solid.

- (a) $\frac{2\pi}{15}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{12}$ (e) $\frac{5\pi}{12}$

5. Refer to the figure given on the right.

Which of the following integrals represents the volume generated by rotating the region \mathcal{R}_2 about AB .

- (a) $\int_0^1 \pi[1 - (1 - \sqrt{x})^2]dx$
 (b) $\int_0^1 \pi[1 - (1 - y^2)^2]dy$
 (c) $\int_0^1 \pi[1 - y^4]dy$
 (d) $\int_0^1 \pi(1 - x)dx$
 (e) $\int_0^1 \pi[(1 - x^2)^2 - (1 - \sqrt{x})^2]dx$



6. A 40-lb cable is 10 ft long and hangs vertically from the top of a tall building. How much work is required to lift the cable to the top of the building?

- (a) 1000 ft-lb (b) 4000 ft-lb (c) 10,000 ft-lb (d) 200 ft-lb (e) 100 ft-lb

7. Find the average value of $f(x) = \sqrt[3]{x}$ on the interval $[1, 8]$.

- (a) $\frac{21}{5}$ (b) $\frac{38}{17}$ (c) $\frac{32}{23}$ (d) $\frac{45}{4}$ (e) $\frac{45}{28}$

8. Evaluate the following integral,

$$\int_0^{\pi} 2^x \sin x \, dx$$

- (a) $\frac{2+\pi}{(\ln 2)^2}$ (b) $\frac{1+2\pi}{1+(\ln 2)^2}$ (c) $\frac{2\pi+1}{2+\ln 2}$ (d) $\frac{2\pi+\ln 2}{1+(\ln 2)^2}$ (e) $\frac{2\pi+(\ln 2)^2}{\pi+(\ln 2)^2}$

9. Evaluate the following integral,

$$\int_0^1 x^5 e^{x^3} \, dx$$

- (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{5}$ (e) $\frac{1}{6}$

10. Find the volume of the solid obtained by rotating the region bounded by $y = e^x$, $y = 0$, $x = 0$, and $x = 1$, about the x -axis.

- (a) $\pi \ln 2$ (b) $\frac{\pi}{2} \ln(1 + e^2)$ (c) $\frac{\pi}{2}(e^2 - 1)$ (d) $\pi(e - 1)$ (e) $\pi(e^2 - 1)$

11. Select the integral which represents the volume of the solid obtained by rotating the region bounded by $y = e^x$, $y = 0$, $x = 0$, and $x = 1$, about the line $y = -1$.

- (a) $\int_0^1 \pi(e^x)^2 dx$ (b) $\int_0^1 [\pi(e^x)^2 - \pi] dx$ (c) $\int_0^1 [\pi(1 + e^x)^2 - \pi(1 + x)^2] dx$
 (d) $\int_0^1 [\pi(1 + e^x)^2 - \pi] dx$ (e) $\int_0^1 \pi e^x dx$

12. A particle is moved along the x -axis by a force that measures $10/(1 + x)^2$ pounds at a point x in feet from the origin. Find the work done in moving the particle from the origin to a distance of 9 ft.

- (a) 8 (b) 9 (c) 7 (d) 10 (e) 6

13. Find the average value of $f(t) = te^{-t^2}$ on the interval $[0, 5]$.

- (a) $\frac{1}{10}(1 - e^{-25})$ (b) $\frac{1}{5}(e^{-5} - 1)$ (c) e^{-25} (d) $5e^{-25}$ (e) $\frac{1}{5}(e^{25} - 1)$

14. Evaluate the following integral,

$$\int_0^{\sqrt[4]{\pi}} x^7 \sin x^4 dx$$

- (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) π (e) $\frac{3\pi}{2}$

15. Evaluate the following integral,

$$\int_0^1 \frac{\sin^{-1} \sqrt{x}}{\sqrt{x}} dx$$

- (a) $\pi - 2$ (b) $\pi - 1$ (c) π (d) $\pi + 1$ (e) $\pi + 2$

16. Which of the below integrals is equal to $\int \frac{\sqrt{x^2-4}}{x^6} dx$?

- (a) $\int \frac{1}{16} \cos^3 \theta \sin^2 \theta d\theta$ (b) $\int \frac{1}{32} \cos^5 \theta \sin \theta d\theta$ (c) $\int \frac{1}{16} \cos^3 \theta \sin^3 \theta d\theta$
 (d) $\int \frac{1}{16} \frac{\cos^2 \theta}{\sin^6 \theta} d\theta$ (e) $\int \frac{1}{32} \frac{\cos \theta}{\sin^6 \theta} d\theta$
 (with an appropriately defined θ)

17. Evaluate the following integral. $\int \tan^5 x \sec^3 x \, dx$
- (a) $\frac{1}{6}\sec^6 x - \frac{2}{5}\sec^5 x + \frac{1}{3}\sec^3 x$ (b) $\frac{1}{7}\sec^7 x - \frac{2}{5}\sec^5 x + \frac{1}{4}\sec^4 x$
 (c) $\frac{1}{6}\sec^6 x \cdot \frac{1}{4}\sec^4 x$ (d) $\frac{1}{7}\sec^7 x - \frac{2}{5}\sec^5 x + \frac{1}{3}\sec^3 x$
 (e) $\frac{1}{4}\sec^4 x + \frac{2}{5}\sec^5 x + \frac{1}{3}\sec^3 x$

18. Write out the form of the partial fraction decomposition of the following function

$$\frac{x^4+5x^3+4x^2+3x+2}{x^3(x-1)(x^2+9)^2}.$$

- (a) $\frac{A}{x^3} + \frac{B}{x-1} + \frac{Cx+D}{(x^2+9)^2}$
 (b) $\frac{A}{x^3} + \frac{B}{x-1} + \frac{Cx+D}{x^2+9} + \frac{Ex+F}{(x^2+9)^2}$
 (c) $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-1} + \frac{E}{x^2+9} + \frac{F}{(x^2+9)^2}$
 (d) $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-1} + \frac{Ex+F}{x^2+9} + \frac{Gx+H}{(x^2+9)^2}$
 (e) $\frac{A}{x^3} + \frac{B}{x-1} + \frac{C}{(x^2+9)^2}$

19. Evaluate the following integral. $\int \frac{x-9}{x^2+3x-10} dx$

- (a) $\ln|x+5| - 2\ln|x-2|$ (b) $2\ln|x+5| - \ln|x-2|$
 (c) $\ln|x+5| + 2\ln|x-2|$ (d) $-\ln|x+5| - 2\ln|x-2|$
 (e) $2\ln|x+5| + \ln|x-2|$

20. Perform the following division: $\frac{x^3-2x+5}{x^2-x-6}$

- (a) $x+1 + \frac{5x+11}{x^2-x-6}$ (b) $5x+11 + \frac{x+1}{x^2-x-6}$ (c) $x^2 - x - 6 + \frac{x+1}{5x+11}$
 (d) $x+1 + \frac{3x-1}{x^2-x-6}$ (e) $3x-1 + \frac{x+1}{x^2-x-6}$

21. Evaluate the following integral. $\int \ln(x^2+3) dx$

- (a) $x\ln(x^2+3) - \left(2x - \frac{6}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}\right)$
 (b) $x\ln(x^2+3) - \left(x - \frac{2}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}\right)$
 (c) $\frac{x}{2}\ln(x^2+3) - \left(2x - \frac{6}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}\right)$
 (d) $x\ln(x^2+3) - (2x - 6\tan^{-1}x)$
 (e) $x\ln(x^2+3) + \left(x - \frac{6}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}\right)$

22. Evaluate the following integral,

$$\int_1^{e^\pi} \frac{\sin^3(\ln x) \cos^4(\ln x)}{x} dx$$

- (a) $\frac{1}{35}$ (b) $\frac{2}{35}$ (c) $\frac{3}{35}$ (d) $\frac{4}{35}$ (e) $\frac{1}{7}$

s23. Evaluate the following integral,

$$\int_0^2 \frac{x^2 + 2}{(x+1)(x-3)} dx$$

- (a) $2 - \frac{7}{2}\ln 3$ (b) $7 - \frac{2}{3}\ln 3$ (c) $3 - \frac{2}{7}\ln 3$ (d) $2 - \frac{3}{2}\ln 3$ (e) $4 - \frac{1}{5}\ln 3$

24. Evaluate the following integral. $\int \cot^5 x \sin^4 x dx$

- (a) $\ln|\cos x| - \sin^2 x + \frac{1}{4}\sin^4 x$ (b) $\frac{1}{6}\sin^2 x + \frac{2}{3}\sin^3 x + \frac{1}{4}\sin^4 x$
 (c) $\frac{1}{2}\ln|\sin x| - \frac{1}{3}\sin^2 x + \frac{1}{4}\sin^4 x$ (d) $\frac{1}{3}\sin^3 x - \sin^2 x + \frac{1}{4}\sin^4 x$
 (e) $\ln|\sin x| - \sin^2 x + \frac{1}{4}\sin^4 x$

25. Evaluate the following integral. $\int \frac{1}{x^3\sqrt{x^2-1}} dx$

- (a) $\frac{1}{2}\left[\sec^{-1}x + \frac{\sqrt{x^2-1}}{x}\right]$ (b) $\frac{1}{2}\left[\sec^{-1}x + \frac{\sqrt{x^2-1}}{x^2}\right]$ (c) $\frac{1}{2}\left[\sin^{-1}x + \frac{\sqrt{x^2-1}}{x^2}\right]$
 (d) $\frac{1}{2}\left[x + \frac{\sqrt{x^2-1}}{x^2}\right]$ (e) $\frac{1}{2}\left[\sin^{-1}x + \frac{x^2}{\sqrt{x^2-1}}\right]$

26. Perform the following division: $\frac{x^3+3x^2+2}{x-5}$

- (a) $x - 5 + \frac{202}{x^2+8x+40}$ (b) $x^2 + 8x + 5 + \frac{27}{x-5}$ (c) $27 + \frac{x-5}{x^2+8x+5}$
 (d) $x^2 + 8x + 40 + \frac{202}{x-5}$ (e) $x - 5 + \frac{27}{x^2+8x+5}$

27. Evaluate the following integral. $\int \frac{x^3-2x^2+x+1}{x^4+5x^2+4} dx$

- (a) $\frac{3}{2}\ln|x^2+4| - \frac{1}{2}\tan^{-1}\frac{x}{2} + \tan^{-1}x$ (b) $\frac{1}{2}\ln|x^2+4| - \frac{3}{2}\tan^{-1}\frac{x}{2} + 2\tan^{-1}x$
 (c) $\frac{1}{2}\ln|x^2+4| - \frac{3}{2}\tan^{-1}\frac{x}{2} + \tan^{-1}x$ (d) $\frac{1}{2}\ln|x^2+4| - \frac{3}{4}\tan^{-1}\frac{x}{2} + \tan^{-1}x$
 (e) $\frac{1}{2}\ln|x^2+4| - 2\tan^{-1}x$

28. Evaluate the following integral. $\int_{-1/2}^{1/2} \frac{1}{4x^2+4x+5} dx$

- (a) $\frac{\pi}{16}$ (b) $\frac{\pi}{8}$ (c) $\frac{\pi}{4}$ (d) 8 (e) 4

29. Evaluate the following integral. $\int_0^{1/2} x\sin^{-1}x dx$

- (a) $-\frac{\pi}{48} + \frac{\sqrt{3}}{4}$ (b) $-\frac{\pi}{4} + \frac{\sqrt{3}}{16}$ (c) $-\frac{\pi}{48} + \frac{\sqrt{3}}{16}$ (d) $-\frac{\pi}{48} + \frac{\sqrt{2}}{16}$ (e) $-\frac{\pi}{12} + \frac{\sqrt{3}}{16}$

30. Evaluate the following integral,

$$\int_{\ln(\pi/2)}^{\ln \pi} e^x \sin^2(e^x) dx$$

- (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$ (e) π

31. Evaluate the following integral,

$$\int \frac{x^3 + 4}{x(x^2 + 2)} dx$$

(a) $2x - \frac{1}{2}\ln(x^2 + 2) - \frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}} + \ln x + C$

(b) $x - \frac{1}{2}\ln(x^2 + 2) - \frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}} + 2\ln x + C$

(c) $x - \frac{1}{2}\ln(x^2 + 2) - \sqrt{2}\tan^{-1}\frac{x}{\sqrt{2}} + \ln x + C$

(d) $x - \ln(x^2 + 2) - \frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}} + 2\ln x + C$

(e) $x - \ln(x^2 + 2) - \sqrt{2}\tan^{-1}\frac{x}{\sqrt{2}} + 2\ln x + C$

32. Find the length of the curve $y = \sin^{-1}x + \sqrt{1 - x^2}$ for $0 \leq x \leq 1$.

(a) $2(2 - \sqrt{2})$ **(b)** $2 + \sqrt{2}$ **(c)** $\sqrt{2} - 1$ **(d)** $1 + \sqrt{2}$ **(e)** $2(\sqrt{2} - 1)$

33. Find the length of the curve $y = \ln(1 - x^2)$ for $0 \leq x \leq \frac{1}{2}$.

(a) $\ln 2 - \frac{1}{3}$ **(b)** $3 - \ln 2$ **(c)** $2 - \ln 3$ **(d)** $2 + \ln 3$ **(e)** $\ln 3 - \frac{1}{2}$

- [illegible]

Answers

1. d 2. a 3. c 4. a 5. b 6. d 7. e 8. b 9. c 10. c
 11. d 12. b 13. a 14. a 15. a 16. a 17. d 18. d 19. b 20. a
 21. a 22. d 23. a 24. e 25. b 26. d 27. c 28. a 29. c 30. c
 31. e 32. a 33. e
 34.

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 COURSE: Put the course name here (Name and Number - e.g. ENGLISH 1A03)
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Use all 9 digits of your student number, including leading zeros (if any)

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 2 1 1 1 1 1
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 3 1 1 1 1 1
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