Extra practice with partial derivatives (section 4)

1. Write the limit definition of the derivative.

(a) N_z for a function $N(z, k)$.

(b) $\dfrac{\partial H}{\partial t}$ for a function $H(a, t, q)$.

(c) $\dfrac{\partial f}{\partial M_o}$ for a function $f(M_i, M_o)$.

2. Compute the partial derivative and interpret your answer.

(a) Recall the body mass index formula $\mathrm{BMI}(m, h) = m/h^2$ (the height h is in metres and the mass m is in kilograms). Compute and interpret $\partial\mathrm{BMI}/\partial m(70, 1.6)$

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(b) Recall the type-2 functional response (start of section 4) $c(N, T_h) = \dfrac{0.1N}{1 + 0.1NT_h}$, where $c(N, T_h)$ is the number of prey captured, N is the density of prey and T_h is the handling time. Compute and interpret $\partial \mathrm{c}/\partial N$ when $N = 30$ (number of prey per square kilometre) and $T_h = 2$ (days).

(c) Same context as in (b). Compute and interpret $\partial \mathrm{c}/\partial T_h$ when $N = 50$ (number of prey per square kilometre) and $T_h = 1.2$ (days).

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(d) The function $F(p, t)$ measures the average total carbon footprint (measured in tons of CO_2) in Canada produced by a human population of size p (number of humans) during time t (in years). Interpret the fact that $\partial F/\partial p(3.1, 1) = 13.5$. (Note: $p = 3.1$ is the average size of a husband-wife family in Canada; Statsitics Canada, 2006 Census.)

(e) Same context as in (d). Interpret the fact that $\partial F/\partial t(2.5, 1) = 33$. (Note: $p = 2.5$ is the average size of a lone-parent family in Canada; Statsitics Canada, 2006 Census.)

(f) The function $P(T, f)$ describes how the population of salmon (measured in millions) depends on the water temperature T (in degrees C) and food availability f. What does $\partial P/\partial T(16, 0.8) > 0$ mean for the population of salmon?

(g) Same context as in (f). What does $\partial P/\partial T(21, 0.8) < 0$ mean for the population of salmon?

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3. (a) Sketch a contour diagram of a function that satisfies $\partial f/\partial x < 0$ and $\partial f/\partial y > 0$ at all points (x, y) in $\mathbb{R}^2$.

(b) Sketch a contour diagram of a function that satisfies $\partial f/\partial x = 0$ and $\partial f/\partial y > 0$ at all points (x, y) in $\mathbb{R}^2$.

(c) Find a formula for a function which satisfies the conditions in (b).

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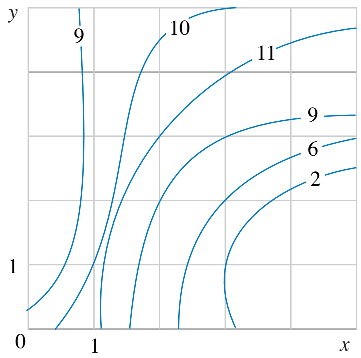
4. Consider a function $f(x, y)$ given by its contour diagram.



(a) Estimate $f_x(2, 3)$.

(b) Estimate $\partial f/\partial y(3, 2)$.

(c) What is the sign of $f_x(1, 1)$?

(d) Which of the two numbers $\partial f/\partial x(2, 2)$ and $\partial f/\partial x(3, 2)$ is a larger negative number?

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5. Find partial derivatives.

(a) $g(m, v) = m(1 - v^2)^{-1}$; $g_v(60, 120)$

(b) $h(x, t) = \sqrt{x^2 + xt + 4}$; $h_x(0, 4)$

(c) $f(x, y) = \arctan(y/x)$; $\partial f/\partial y(x, y)$

(d) $g(x, y, w) = xw\sec(xy)$; $\partial g/\partial y(x, y, w)$

(e) $g(x, y, w) = xw\sec(xy)$; $\partial g/\partial w(x, y, w)$

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6. The humidex $H(T, h)$ is a measure used by meteorologists to describe the combined effects of heat and humidity on an average person's feeling of hotness. In the table below we give values of humidex based on measurements of temperature (in degrees Celsius) and relative humidity h (given as a percent).

	$T = 22$	$T = 26$	$T = 30$	$T = 34$
$h = 70$	27	33	41	49
$h = 60$	25	32	38	46
$h = 50$	24	30	36	43

(a) Estimate $H_T(30, 60)$ and interpret your answer.

(b) Estimate $H_h(30, 60)$ and interpret your answer.

THE END