THEOREM

Rolle's Theorem

Let f be a function that satisfies the following three hypotheses:

1. f is continuous on the closed interval $[a, b]$,
2. f is differentiable on the open interval (a, b),
3. $f(a) = f(b)$.

Then there is a number c in (a, b) such that $f'(c) = 0$.

THEOREM

The Mean Value Theorem (MVT)

Let f be a function that satisfies the following hypotheses:

1. f is continuous on the closed interval $[a, b]$,
2. f is differentiable on the open interval (a, b).

Then there is a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

DEFINITION

Critical Number

A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

TEST

First Derivative Test

Suppose that c is a critical number of a continuous function f.
(a) If f' changes from positive to negative at c, then f has a local maximum at c.
(b) If f' changes from negative to positive at c, then f has a local miniumum at c.
(c) If f' is positive to the left and right of c, or negative to the left and right of c, then f has no local maximum or minimum at c.

Test

Concavity Test

(a) If $f''(x) > 0$ for all x in an interval I, then the graph of f is concave upward on I.

(b) If $f''(x) < 0$ for all x in an interval I, then the graph of f is concave downward on I.

Definition

Inflection Point

A point P on a curve $y = f(x)$ is called an **inflection point** if f is continuous there and the curve changes from concave upward to concave downward or from concave downward to concave upward at P.

Test

Second Derivative Test

Suppose f'' is continuous near c.

(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

Definition

Antiderivative

A function F is called an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I.

Indefinite Integral

$$\int x^n \, dx = ?$$

$$\frac{x^{n+1}}{n+1} + C, \quad \text{(assuming } n \neq -1\text{)}$$

Indefinite Integral

$$\int e^x \, dx = ?$$

$$e^x + C$$

Indefinite Integral

$$\int \sin(x) \, dx = ?$$

$$-\cos(x) + C$$

Indefinite Integral

$$\int b^x \, dx = ?$$

$$\frac{b^x}{\ln(b)} + C$$

Indefinite Integral

$$\int \frac{1}{x}\, dx = ?$$

$$\ln|x| + C$$

Indefinite Integral

$$\int \frac{1}{1+x^2}\, dx = ?$$

$$\tan^{-1}(x) + C$$

Indefinite Integral

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = ?$$

$$\sin^{-1}(x) + C$$

Formula

$$\sum_{i=1}^{n} i = ?$$

$$\frac{n(n+1)}{2}$$

FORMULA

$\sum_{i=1}^{n} i^2 = ?$

$$\frac{n(n+1)(2n+1)}{6}$$

FORMULA

$\sum_{i=1}^{n} i^3 = ?$

$$\left[\frac{n(n+1)}{2}\right]^2$$

THEOREM

The Fundamental Theorem of Calculus (Part I)

If f is continuous on $[a, b]$, then the function g defined by

$$g(x) = \int_a^x f(t)\, dt, \qquad a \leq x \leq b$$

is continuous on $[a, b]$ and differentiable on (a, b) and $g'(x) = f(x)$.

THEOREM

The Fundamental Theorem of Calculus (Part II)

If f is continuous on $[a, b]$, then

$$\int_a^b f(x)\, dx = F(b) - F(a)$$

where F is any antiderivative of f.

Formula

Suppose $F(x) = \int_{g(x)}^{h(x)} f(t)\,dt$, *then* $F'(x) =$?

$f(h(x))h'(x) - f(g(x))g'(x)$

Theorem

The Net Change Theorem

The integral of rate of change is the net change:

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

Rule

The Substitution Rule

If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du$$

Indefinite Integral

$\int \tan(x)\,dx =$?

$\ln|\sec(x)| + C$