1ZA3 / 1A03

Help Centre: HH / 104 (Basement)

2:30 - 8:30 PM (Mon - Thurs)

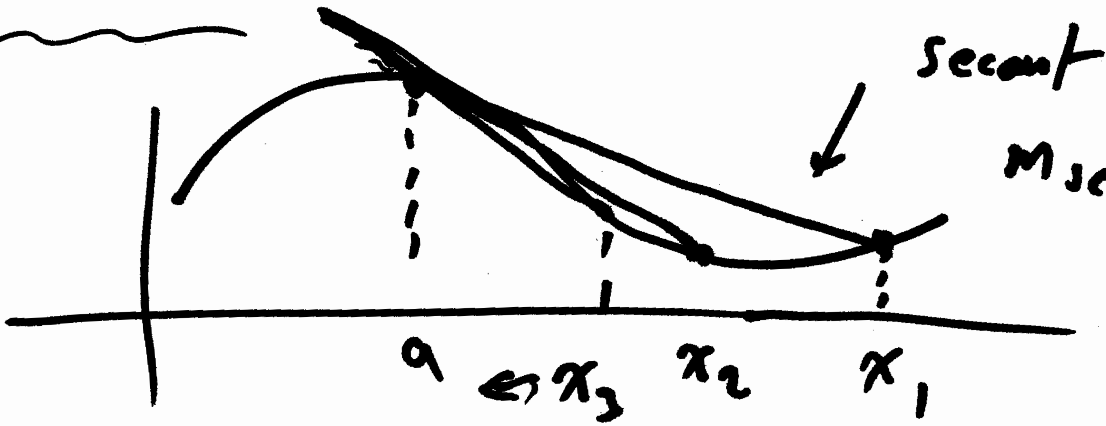
2:30 - 6:30 PM Fri.

Office Hours 12:40 - 1:30 } BSB Basement B124

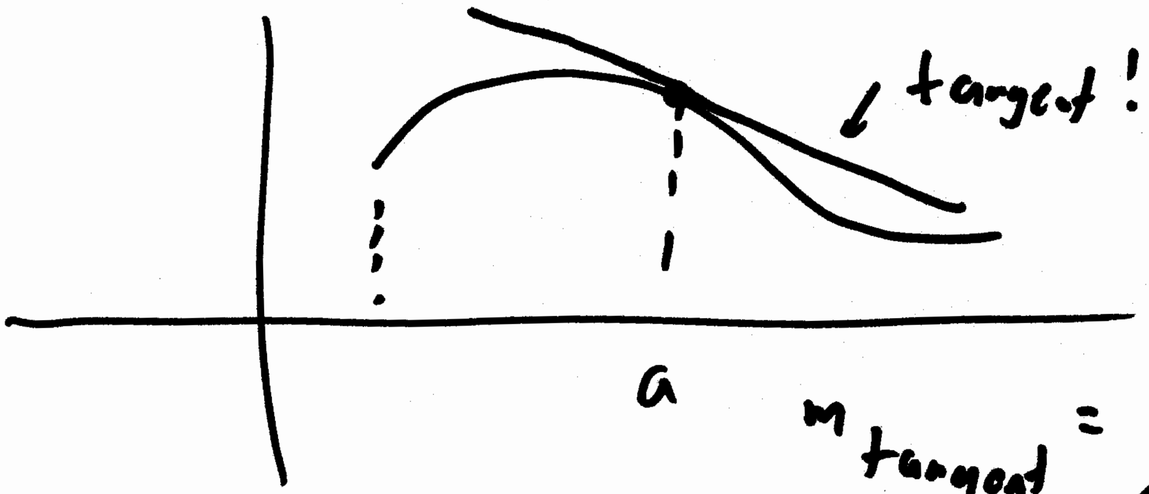
3:40 - 4:30

Tues. Thurs. Fri.

Last Day



$m_{secant} = \frac{\Delta y}{\Delta x}$



$$m_{tangent} = \lim_{\Delta x \to 0} m_{secant}$$

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = f'(a) = \text{derivative of } f(x) \text{ at } \underline{\underline{x = a}}.$$

eg. Find the tangent line to $y = x^2$ $\underline{at}$ $\underline{x = 3}$

Solution Tangent Line: $y = mx + b$

$(x-3)(x+3)$

$$m = \lim_{x \to 3} \frac{f(x) - f(3)}{x - 3} = \lim_{x \to 3} \frac{x^2 - 9}{x - 3}$$

$= 3+3$

$$\boxed{m = 6 = f'(3)}$$

tangent at $x = 3 \Rightarrow$ hits $(3, f(3))$

$= (3, 3^2) = (3, 9)$

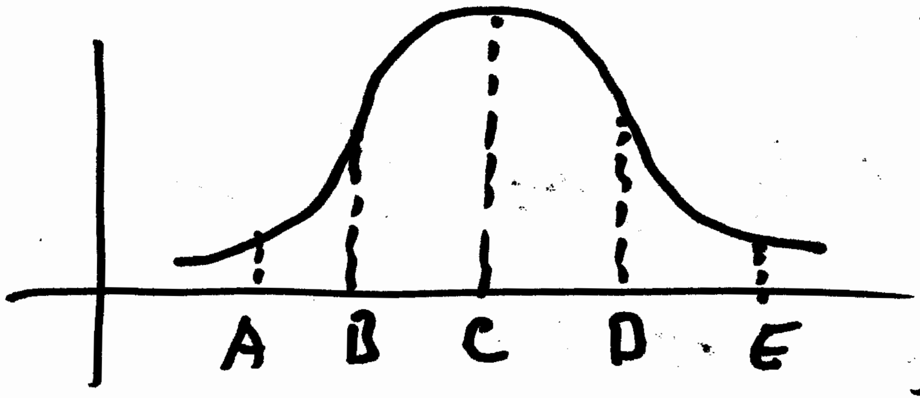
$= (x, y)$

plug in $y = mx + b$ to get b

$9 = 6(3) + b \rightsquigarrow \boxed{b = -9}$

so $\boxed{y = 6x - 9}$ is tangent to $y = x^2$ at $x = 3$.

eg



Sort A - E in order of most neg. to most +ve tangent slope!

Solution

$$D < E < \underset{\approx 0}{C} < A < B$$

notice If $f'(a)$ exists $\Leftrightarrow$ $f(x)$ differentiable at $x=a$

"diff."

Also note: $f'(a)$ exists $\Rightarrow$ $f(x)$ cont. at $x=a$

cont. at $x=a$ $\not\Rightarrow$ $f'(a)$ exists!

eg. Find $f'(0)$ for $f(x) = |x|$

Solution Go back to our defn.

$$f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{|x| - 0}{x - 0}$$

$$= \lim_{x \to 0} \frac{|x|}{x}$$

$-x/x \to -1$

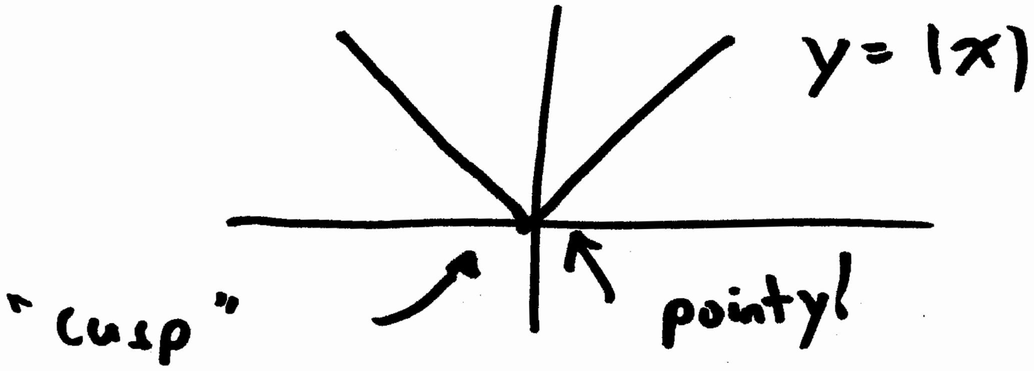
$x/x = 1$

$$\lim_{\underbrace{x \to 0^-}_{x<0}} \cancel{\frac{|x|}{x}} = -1$$

$$\lim_{\underbrace{x \to 0^+}_{x>0}} \cancel{\frac{|x|}{x}} = 1$$

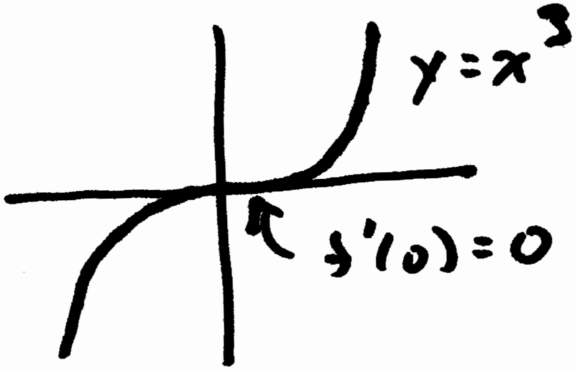
$LS \neq RS \Rightarrow \lim_{x \to 0} \frac{|x|}{x}$ DNE

FAIL.

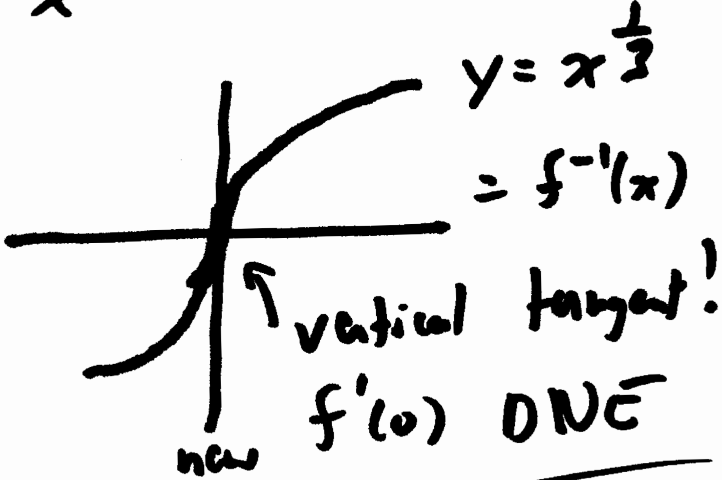


---

e.g. Find $f'(0)$ for $f(x) = x^{1/3}$



$\Longrightarrow$



Proof Go back to definition!

"Leave as an excercise for the student!"

Let's extend our idea of $f'(a)$ to a new function

The Derivative Function

$f'(x)$ = slope of tangent to $f(x)$ at
that value of x

$f'(x)$ diff. on interval (a,b) if $f'(x)$ defined on every $x \in (a,b)$

Re-express our "First principles"! Using "$h = \Delta x$"

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

notice $\quad f'(x) = \lim_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x} = \dfrac{dy}{dx}$

Leibnitz notation $\rightsquigarrow$ $\quad = \dfrac{d}{dx} f(x)$

---

eg $\quad$ Find the derivative function for $f(x) = \dfrac{1}{x}$

ie. find $\dfrac{df(x)}{dx}$

Solution $\quad$ by "first principles"

$$f'(x) = \lim_{h \to 0} \frac{\overset{\frac{1}{x+h}}{f(x+h)} - \overset{\frac{1}{x}}{f(x)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h\to 0} \frac{\frac{x-(x+h)}{x(x+h)}}{h}$$

$$= \lim_{h\to 0} \frac{-\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}} = \frac{-1}{x(x+0)} = \boxed{\frac{-1}{x^2}}$$

<u>Note</u> " $\frac{d}{dx}$ " used as an "operation"

eg $\frac{d}{dx}(x^2+2x+7)$ = derivative of $f(x) = x^2+2x+7$

eg $\left.\frac{d}{dx}x^2\right|_{x=3}$ means if $f(x) = x^2$ find $f'(3)$

"evaluate at"

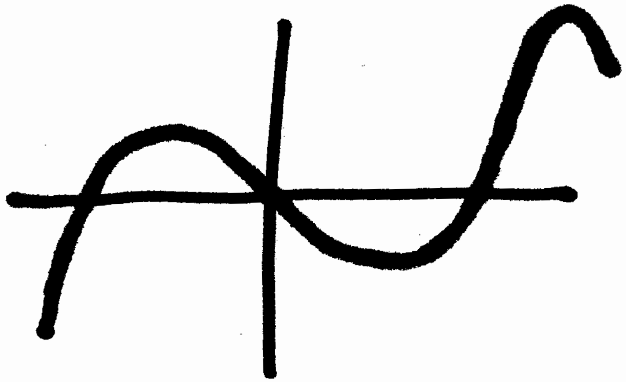
Alternate, uncommon notation

$$\underbrace{f'(x) = \frac{d}{dx}f(x) = \frac{dy}{dx}}_{\text{always!}} = D_x f(x) = D f(x)$$

$$= \underbrace{\mathcal{D} f(x)}_{\text{never!}}$$

---

eg. Given $f(x)$



Roughly sketch the derivative, $\underline{\underline{f'(x)}}$

Solution