Math 1LT3 — Assignment 14

Extra practice with linear approximations (Section 5)

1. (a) Write a formula for a linearization and a linear approximation of a function $M(x, y)$ near a point (p, q).

(b) Verify that $\arctan(y/x) \approx y$ near the point $(1, 0)$.

(c) Imitate the calculation in (b) to obtain a linear approximation of $f(x, y) = \sqrt{x^2 + y^2}$ near the point $(0, 2)$.

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2. (a) State Theorem 6 in your own words.

(b) Using Theorem 6, show that the function $g(x, y) = x^2 y \sin(x - 4) - 2$ is differentiable at all (x, y) in $\mathbb{R}^2$.

(c) Using Theorem 6, show that the function $g(x, y) = \ln(x - y + 4)$ is differentiable for (x, y) near $(2, 0)$. Draw an open disk to which the theorem refers.

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(d) Explain why the function $f(x, y) = xy(x^2 + y^2)^{-1}$ is differentiable at $(1, -1)$. Find the linearization of f at $(1, -1)$.

(e) Using Theorem 6, show that the function $f(x, y) = x \tan y$ is differentiable at $(0, 0)$. What is the largest open disk (refer to the statement of the theorem) centred at $(0, 0)$ that you can use?

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3. Find the equation of the plane tangent to the given surface at the point indicated.

(a) $f(x, y) = \sqrt{xy + 7}$; $(6, 7, 7)$

(b) $f(x, y) = \arctan(2y/x)$; $(2, 1, \pi/4)$

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4. Approximate the value of the given expression and compare with the calculator value.

(a) $\sin 1.5 \cos 0.1$

(b) $1.95^4 e^{0.02}$

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5. Consider $f(x, y) = x^3 + 4x - xy + 2$.

(a) Find the differential df.

(b) Find $f(1, 3)$. Using differentials, estimate the change in the function as the variables change from $(1, 3)$ to $(1.06, 2.97)$. Find an approximation for $f(1.06, 2.97)$.

(c) Compare (b) to the true value of $f(1.06, 2.97)$.

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6. Consider the body surface area $S(m, h) = 0.20247m^{0.425}h^{0.725}$ of a human of height h (metres) and mass m (kilograms). Find the differential dS when $m = 65$ kg, $h = 1.55$ m, $dm = 0.5$ kg, and $dh = 0.03$ m, and interpret your answer.

7. Consider the body surface area $S(m, h) = 0.20247m^{0.425}h^{0.725}$ of a human of height h (metres) and mass m (kilograms). If the mass increases by 3% and the height by 2%, by how much does the surface area increase?

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