Theme 2
Mechanics

Module T2M1:
Kinematics

Equations of Motion

derivative

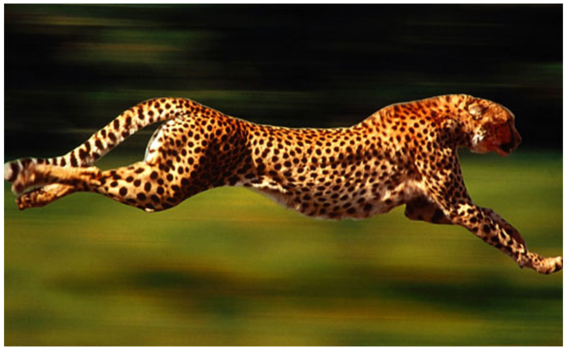
integrate

$$a = constant$$

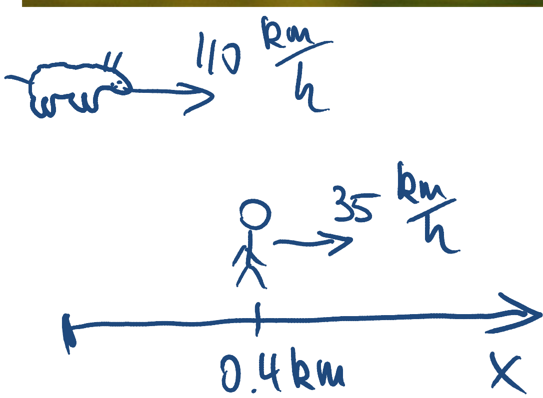
$$v(t) = at + v_0$$

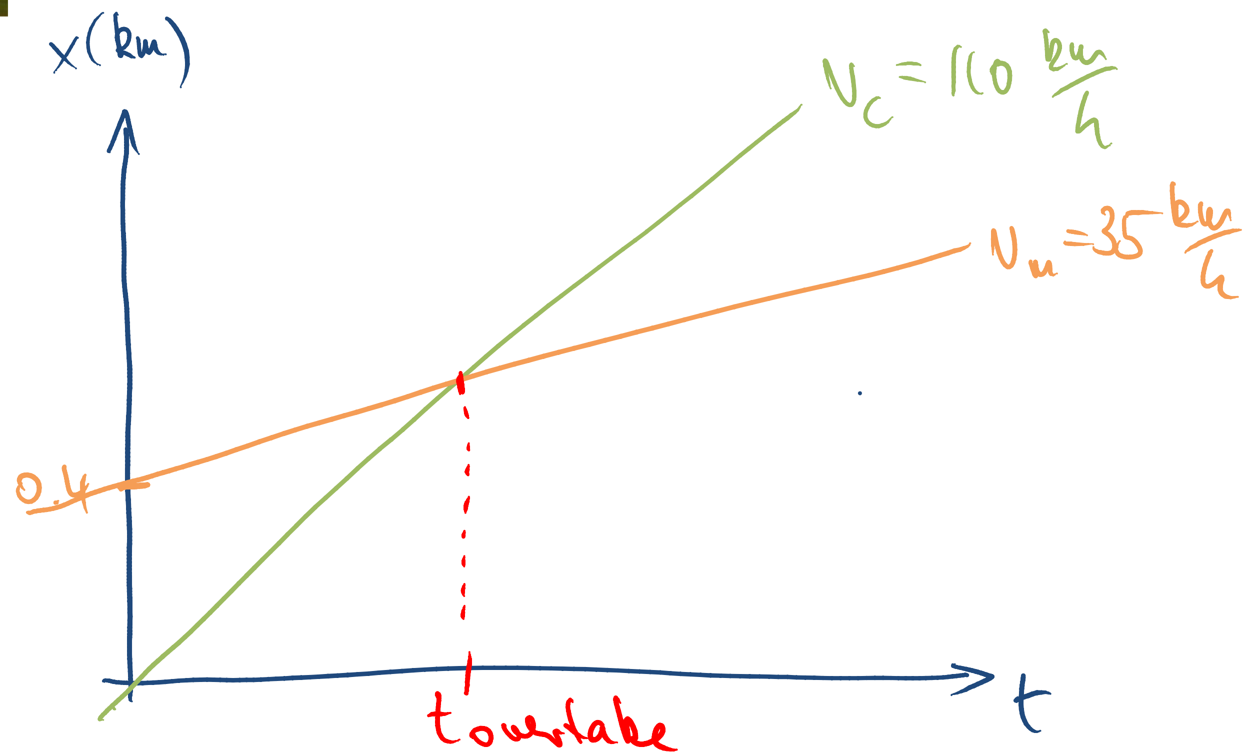
$$x(t) = \frac{1}{2}at^2 + v_0t + x_0$$

Cheetah Sprint – Graphical Solution

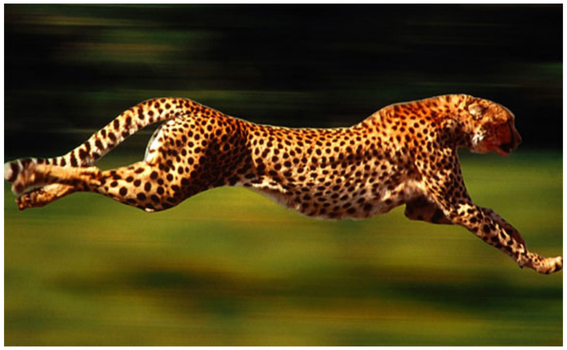


A cheetah can sprint at a speed of 110 km/h. The best a human is capable of is a speed of 35 km/h. A man and a cheetah are initially 0.400 km apart. Assuming that both man and cheetah are running at their top speed, how long does it take the cheetah to overtake the man?





Cheetah Sprint



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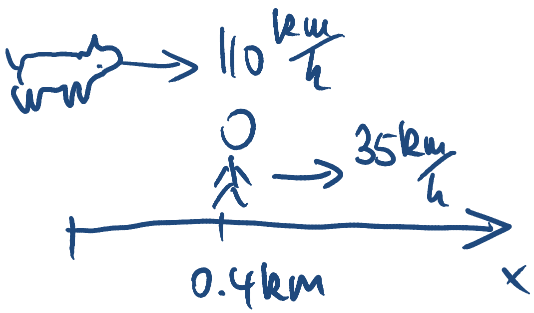


$$x_c(t) = \cancel{\tfrac{1}{2} a_c t^2} + v_c \cdot t + \cancel{x_c}$$

$$x_m(t) = \cancel{\tfrac{1}{2} a_m t^2} + v_m \cdot t + x_m$$

overtake: $x_c(t) = x_m(t)$

$$x_c(t) = v_c \cdot t$$

$$x_m(t) = v_m \cdot t + x_m$$

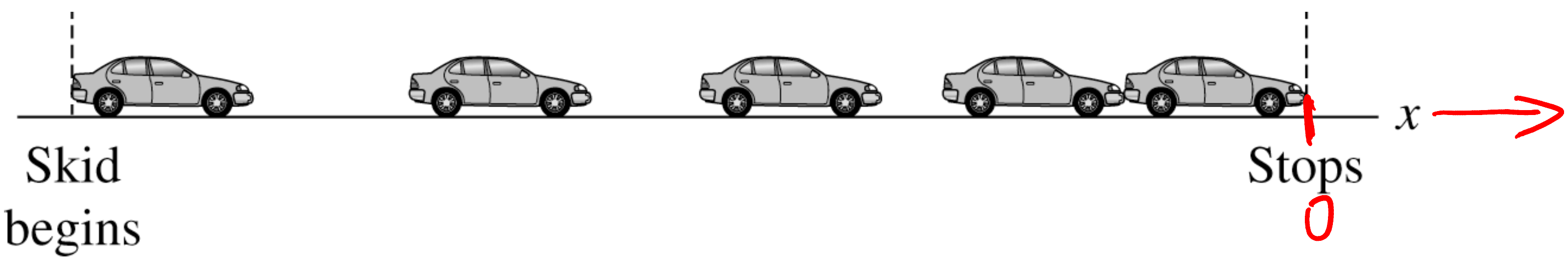
$$v_c \cdot t = v_m \cdot t + x_m$$

$$\Rightarrow t = \frac{x_m}{v_c - v_m} = 0.0053h = 19s$$

$$\Rightarrow x_c(0.0053h) = 0.59 \text{ km}$$

Example

- A car brakes to avoid hitting a can of tuna. The brakes apply an acceleration of **4.5 m/s²**, and the car comes to rest over a distance of **27 m**. What was the speed of the car at the instant the brakes were applied?



$$x(t) = \frac{1}{2}at^2 + v_0 t + x_0$$

$$0 = \frac{1}{2}at^2 + v_0 t + x_0$$

$$v(t) = a\cdot t + v_0$$

$$0 = a\cdot t + v_0$$

$$\Rightarrow t = -\frac{v_0}{a}$$

$$0 = \frac{1}{2}a\frac{v_0^2}{a^2} - \frac{v_0^2}{a} + x_0$$

$$0 = \frac{1}{2}\frac{v_0^2}{a} - \frac{v_0^2}{a} + x_0 = -\frac{1}{2}\frac{v_0^2}{a} + x_0 \Rightarrow x_0 = \frac{1}{2}\frac{v_0^2}{a} \Rightarrow v_0^2 = 2\cdot x_0\cdot a$$

$$v_0 = \sqrt{2x_0\cdot a} = \sqrt{2\cdot 27\cdot 4.5} = 15.6\,\frac{m}{s}$$

Example: 1D motion with two parts

Sitting beside an old lady in the park, you grab her purse and start running. Over the first 10.0 m, you accelerate at 1.20 m/s^2 up to your top running speed, and then continue to sprint at this speed for 15.0 s more before being tackled from behind by the old lady.

(a) How long did it take the old lady to catch you?

(b) How far from the bench did you get before being caught?

$a = 1.2 \frac{m}{s^2}$ $\quad a = 0 \frac{m}{s^2}$

$v_i = 0 \frac{m}{s}$ $\quad \Delta x_1 = 10\,m$ $\quad \Delta x_2 = ?$ $\quad v = ?$

$\Delta t_1 = ?$ $\quad \Delta t_2 = 15\,s$

start running $\quad$ reach top speed $\quad$ Busted

1^{st} part

$$\Delta x_1 = \frac{1}{2} a_1 \cdot \Delta t_1^2 + \cancel{v_1 \Delta t_1} + \cancel{x_0}$$

$$\Rightarrow \quad 10 = \frac{1}{2} \cdot 1.2 \cdot \Delta t_1^2$$

$$\Rightarrow \quad 10 = 0.6 \cdot \Delta t_1^2$$

$$\Rightarrow \quad \Delta t_1 = \sqrt{\frac{10}{0.6}} = 4.1\ s$$

$$v_{1f} = v_{1i} + a \cdot \Delta t_1$$

$$= 0 + \frac{1}{2} \cdot 4.1\ s = 4.92\ \frac{m}{s}$$

2^{nd} part

$$\Delta x_2 = \cancel{\frac{1}{2} a_2 \cdot \Delta t_2^2} + v_2 \cdot \Delta t_2 .$$

$$\Rightarrow \quad \Delta x_2 = 4.92 \cdot 15$$

$$\Rightarrow \quad \Delta x_2 = 73.8\ m$$

$$x_i = 10 + 73.8 = 83.8\ m$$

Free Fall

All objects in free fall move with *constant downward acceleration:*

$$a = g \approx 9.80\ \mathrm{m/s^2}\quad [\mathrm{downwards}]$$

This was demonstrated by Galileo around 1600 A.D.

The constant "*g*" is called the "acceleration due to gravity".

Free Fall

Free Fall Equations

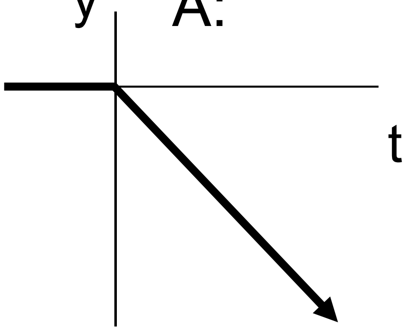
$$g = \text{constant}$$

$$v(t) = gt + v_0$$

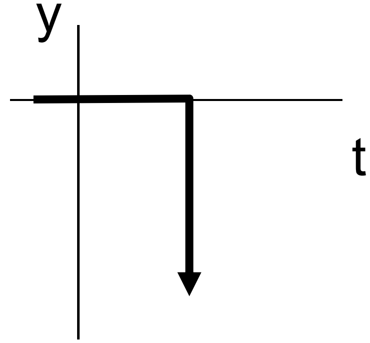
$$x(t) = \tfrac{1}{2} g t^2 + v_0 t + x_0$$

I drop a book at t=0. Which plot below best represents *position* as a function of time?

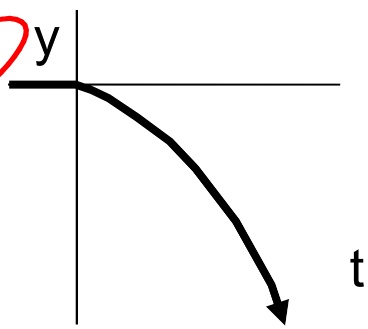
A:



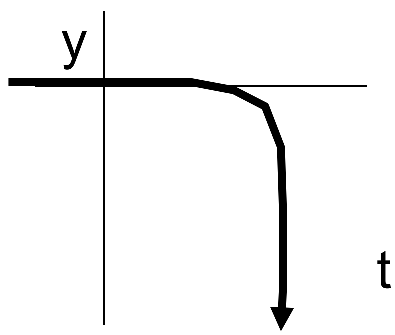
B:



C:

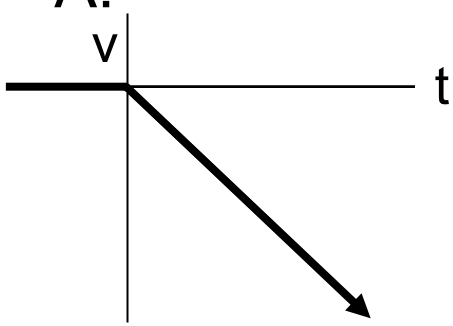


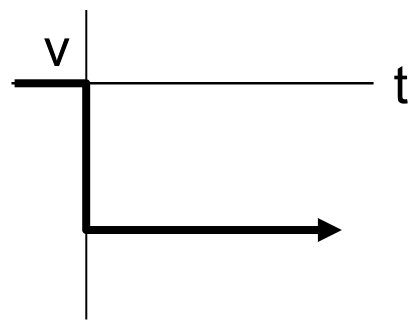
D:



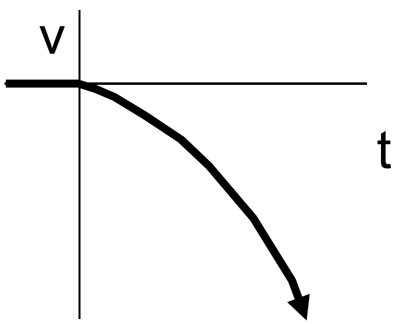
25 I drop a book at t=0. Which plot below best represents *velocity* as a function of time?

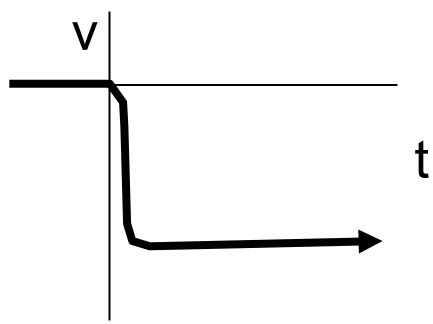
A: B:





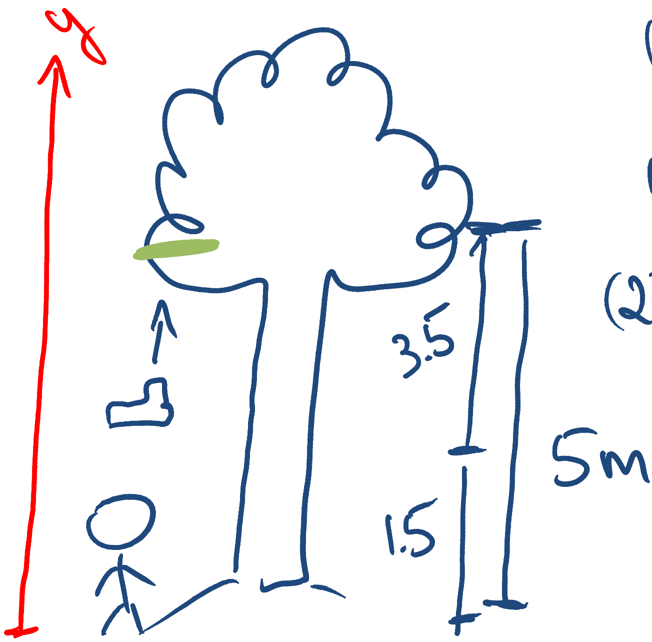
C: D:





Example

Your frisbee is stuck in a branch that is 5.0 m above ground. You throw your shoe straight up to try to knock it down, but your shoe just reaches the frisbee before falling back down. What initial velocity did you give the shoe if it started at 1.5 m above ground?



(1) $y(t) = -\frac{1}{2} g t^2 + v_0 \cdot t + y_0$

(2) $v(t) = -g \cdot t + v_0$

(2) $\Rightarrow 0 = -gt + v_0 \Rightarrow t = \frac{v_0}{g}$

(1): $5 = -\frac{1}{2} g t^2 + v_0 \cdot t + 1.5$

$$5 = -\frac{1}{2} \frac{v_0^2}{g} + \frac{v_0^2}{g} + 1.5$$

$$\Rightarrow v_0 = \sqrt{2 \cdot 9.8 \cdot 3.5} = \sqrt{69} = 83 \frac{m}{s}$$