McMaster University
Math 1LT3E

ASSIGNMENT 4

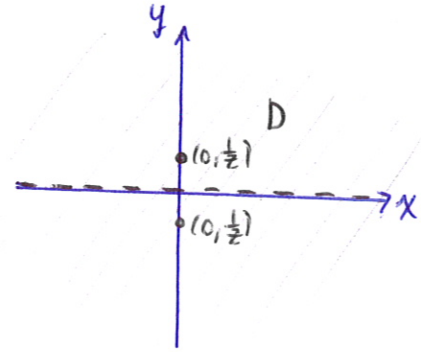
Sections 1 and 2 in the Red Module

1. Consider the function $f(x,y) = \dfrac{e^x}{y}$.

(a) Find and sketch the domain of f.

$y \neq 0$

$D = \{(x,y) \in \mathbb{R}^2 \mid y \neq 0\}$



(b) Determine the range of f.

$\oplus$ / $\oplus$ or $\ominus$ $(y \neq 0)$: $\dfrac{e^x}{y} = z \Rightarrow y = \dfrac{e^x}{z} \quad (z \neq 0)$

Choose $x = 0$ and $y = \frac{1}{z}$ for any $z \in \mathbb{R} \setminus \{0\}$.

<u>Note</u>: $(0, \frac{1}{z}) \in D$.

Then $f(0, \frac{1}{z}) = \dfrac{e^0}{\frac{1}{z}} = z$.

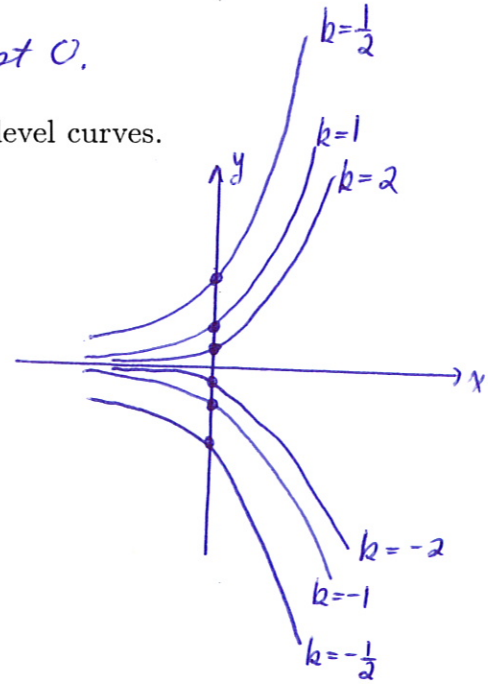
$\therefore$ The range of f is all of $\mathbb{R}$ except 0.

(c) Sketch a contour map of f. Include at least 5 level curves.

$\dfrac{e^x}{y} = k \quad (k \in \mathbb{R} \setminus \{0\})$.

$\Rightarrow y = \dfrac{1}{k} e^x$

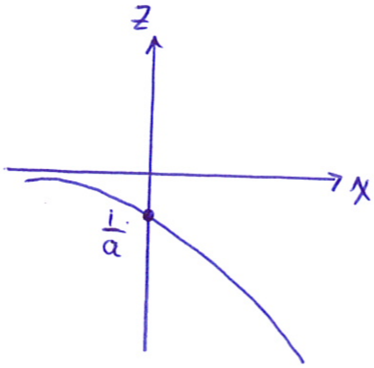
~~Choose $k =$~~



(d) Treat y as a parameter and sketch a graph in two-dimensions to illustrate how f depends on x. (Consider the case when $y < 0$ and then when $y > 0$.)

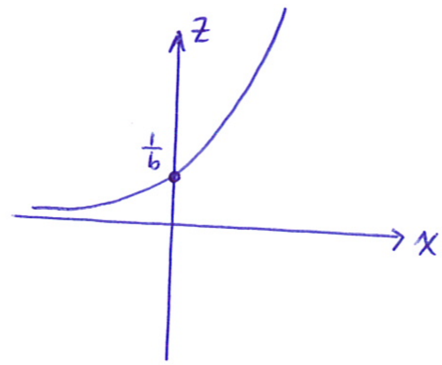
Let $y = a$ where $a < 0$.

$$f(x,a) = \frac{e^x}{a}$$



Let $y = b$ where $b > 0$.

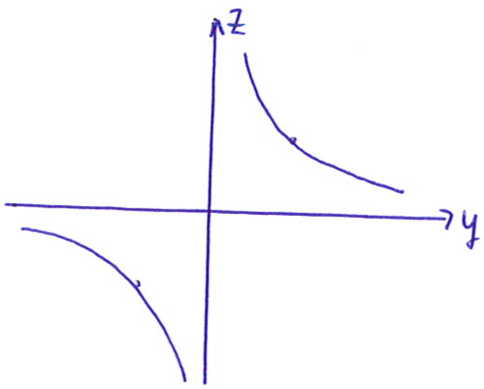
$$f(x,b) = \frac{e^x}{b}$$



(e) Treat x as a parameter and sketch a graph in two-dimensions to illustrate how f depends on y.

Let $x = c$ where $c \in \mathbb{R}$.

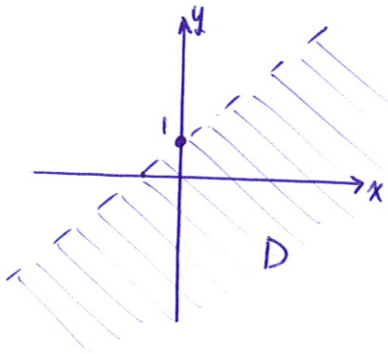
$$f(c,y) = \frac{e^c}{y} \ \} \oplus$$



2. Find and sketch the domain of the following functions.

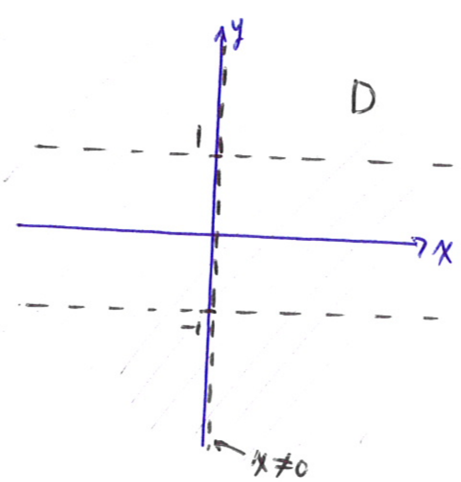
(a) $f(x, y) = \ln(1 + x - y)$

$1 + x - y > 0 \Rightarrow y < x + 1$



(b) $g(x, y) = \dfrac{3x + 1}{xy^2 - x}$

$xy^2 - x \neq 0 \Rightarrow x(y^2 - 1) \neq 0 \Rightarrow x \neq 0, y \neq \pm 1$

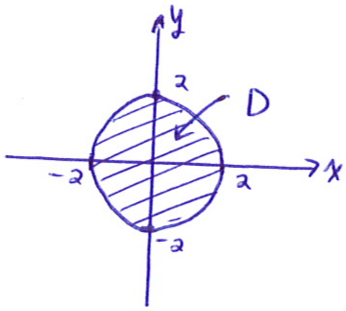


3. Let $f(x, y) = \sqrt{4 - x^2 - y^2}$.

(a) Find and sketch the domain.

$$4 - x^2 - y^2 \geq 0 \Rightarrow x^2 + y^2 \leq 4$$

all points on the edge and inside of a circle centred at $(0,0)$ with radius 2



(b) Determine the range.

$$\underbrace{\sqrt{4 - x^2 - y^2}}_{+} = z \quad \text{where } z \geq 0.$$

$$4 - x^2 - y^2 = z^2 \Rightarrow \underbrace{x^2 + y^2}_{+} = 4 - z^2 \Rightarrow 4 - z^2 \geq 0$$

$$\Rightarrow z^2 \leq 4$$

$$\Rightarrow -2 \leq z \leq 2$$

Since $z \geq 0$ and $-2 \leq z \leq 2$, we have that $0 \leq z \leq 2$

(c) Create a contour map for the function.

$\sqrt{4-x^2-y^2} = k$ where $k \in [0,2]$

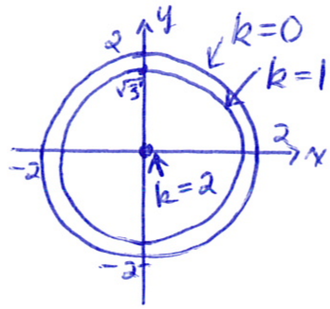
$4-x^2-y^2 = k^2 \Rightarrow x^2+y^2 = 4-k^2$

So, level curves are circles w/ centre $(0,0)$ and radius $\sqrt{4-k^2}$

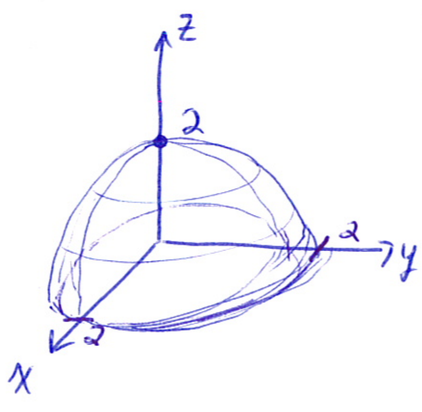
$k=0 \Rightarrow x^2+y^2=4$

$k=1 \Rightarrow x^2+y^2=3$

$k=2 \Rightarrow x^2+y^2=0$



(d) Sketch the graph of the function.

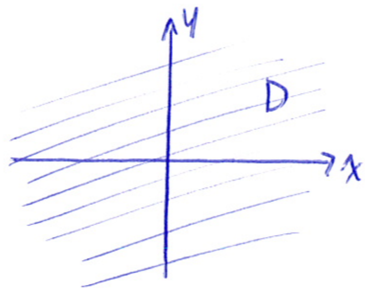


top half of a sphere w/ centre $(0,0,0)$ and radius 2

4. Let $g(x, y) = 8 + x^2 + y^2$.

(a) Find and sketch the domain.

domain : $\mathbb{R}^2$



(b) Determine the range.

$$z = \underbrace{8 + \underbrace{x^2 + y^2}_{\geq 0}}_{\geq 8}$$

$\therefore \; z \geq 8$

(c) Create a contour map for the function.

$8 + x^2 + y^2 = k$ where $k \geq 8$

$x^2 + y^2 = k - 8$

level curves are circles centred at $(0,0)$ w/ radius $\sqrt{k-8}$

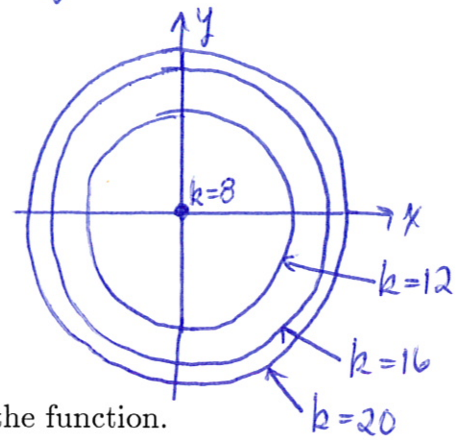
$k = 8 \Rightarrow x^2 + y^2 = 0 \quad (r=0)$

$k = 12 \Rightarrow x^2 + y^2 = 4 \quad (r=2)$

$k = 16 \Rightarrow x^2 + y^2 = 8 \quad (r \approx 2.8)$

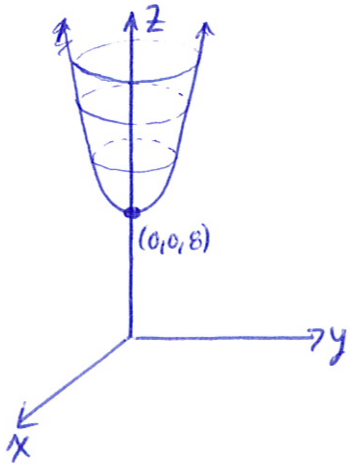
$k = 20 \Rightarrow x^2 + y^2 = 12 \quad (r \approx 3.5)$

contour map:



(d) Sketch the graph of the function.

paraboloid



THE END