Name: _____ Student Number:_____

MATH 1AA3, 1AA3E, 1ZB3 & 1ZB3E

Winter Session, 2017

Seating #1, Test #2, Version 1

C. McLean, A. Nicas,
P. Poudel, C.Wang

TERM TEST
DAY & EVENING CLASSES
DURATION OF TEST: 90 **min (**1.5 **hrs.**)
MCMASTER UNIVERSITY TERM TEST

Monday, March 13, 2017

THIS TEST INCLUDES 7 PAGES AND 18 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING YOUR COPY OF THE TEST IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

Instructions:

1. **NO** calculator is allowed to be used on this test.

2. Make sure your name and student number at the ***top of each page.***

3. For all questions, mark the answer in pencil on the OMR answer sheets according to the OMR instructions on page 2. Only solutions on the scan card will be graded.

4. Each correct answer is worth one mark.

5. A blank answer is an automatic zero for any question, even if the correct solution is circled on the question itself.

6. Incorrect or multiple answers are also worth zero marks. No negative marks or part marks will be assigned.

7. Scrap paper for rough work has been provided. *All* rough work, and this question sheet must be handed in with the test, but any solutions written either here or on the rough paper will ***NOT*** be graded.

8. *Good Luck!*

PLEASE READ THE OMR INSTRUCTIONS ON PAGE #2

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Name: _____ Student Number:_____

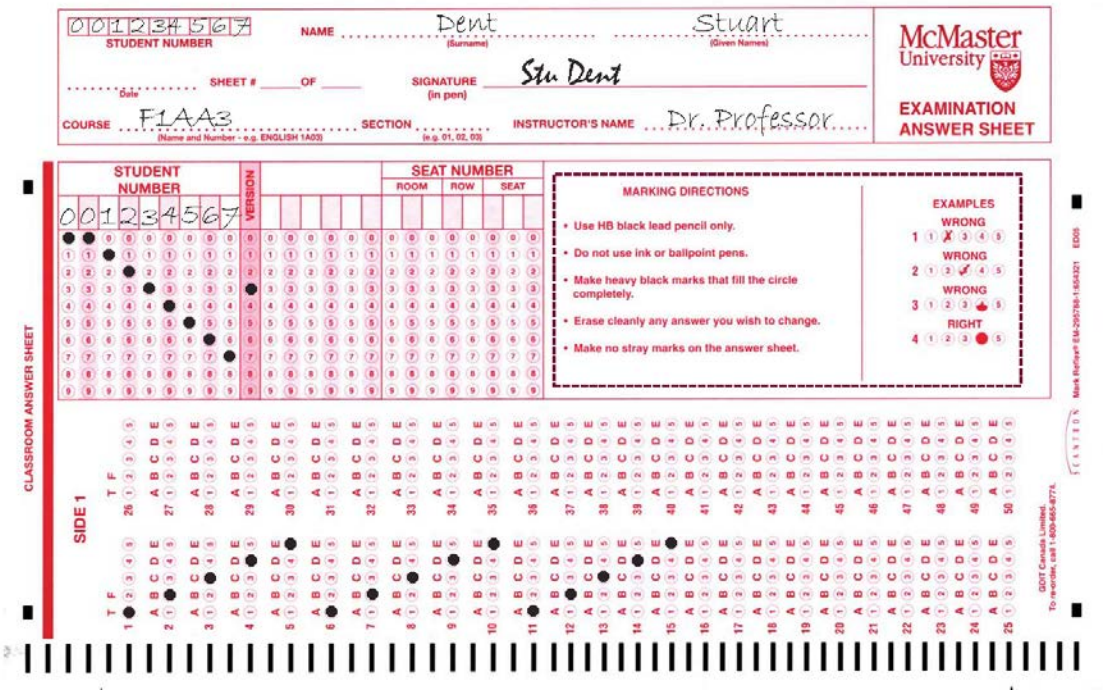
OMR EXAMINATION INSTRUCTIONS

NOTE: IT IS YOUR RESPONSIBILITY TO ENSURE THAT THE ANSWER SHEET IS PROPERLY COMPLETED: YOUR EXAMINATION RESULT DEPENDS ON PROPER ATTENTION TO THESE INSTRUCTIONS.

The scanner which reads the sheets senses the bubble-shaded areas by their non-reflection of light. A heavy mark must be made, completely filling the circular bubble, with an **HB pencil**. Marks made with a pen or a felt–tip marker will **NOT** be sensed. Erasures must be thorough or the scanner may still sense a mark. Do **NOT** use correction fluid on the sheets. Do **NOT** put any unnecessary marks or writing on the sheet.

1. On side 1 (red side) of the form, in the top box, in pencil, print your student number **(NOTE: 9 digits)**, name, course name, section number, instructor name and date in the spaces provided. Then you **MUST** sign in the space marked SIGNATURE**.**
2. In the second box, with a pencil, mark your student number, exam version number and course section number in the space provided and fill in the corresponding bubble numbers underneath.
3. To indicate your answers, mark only **ONE** choice from the alternatives (1, 2, 3, 4, 5, or A, B, C, D, E) provided for each question. The question number is to the left of the bubbles. Make sure that the number of the question on the scan sheet is the same as the question number on the test paper.
4. Pay particular attention to the Marking Directions on the form.
5. Begin answering the questions using the first set of bubbles, marked "1".

SAMPLE OMR CARD ONLY: DO NOT USE



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1. Which of the following is the Taylor series about $a = 5$ of the function $y = \ln(x)$?

a) $\ln(5) + \sum_{n=1}^{\infty} \frac{(-1)^n}{n\,5^n}(x-5)^n$ **b)** $\ln(5) + \sum_{n=1}^{\infty} \frac{(-1)^n}{5^n}(x-5)^n$ **c)** $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{5^n}(n-1)!(x-5)^n$

d) $\ln(5) + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n\,5^n}(x-5)^n$ **e)** $\ln(5) + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{5^n\,n!}(x-5)^n$

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2. Given $f(0) = 3$ and at each point, (x, y), the slope of the tangent line is given by $y\cos(x)$, find $f(\pi/2)$,

a) $e^{\sqrt{2}} + 2$ **b)** $3e$ **c)** 3 **d)** 4 **e)** 12

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3. A sample of radioactive isotope decays from 50 grams to 12 grams in 10 hours. Which of the following is equivalent to the isotope's half-life?

a) $\frac{5\ln(6)}{\ln(50)}$ **b)** $\frac{11}{2}$ **c)** $\frac{10\ln(2)}{\ln(25\ /\ 6)}$ **d)** $10\ln\left(\frac{12}{25}\right)$ **e)** $\frac{2\ln(10)}{\ln(50\ /\ 12)}$

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4. Find the Taylor error estimate for approximating $\frac{1}{\sqrt{x}}$ on the interval [3, 7] by its Taylor polynomial, $T_2(x)$, centred on $a = 5$.

a) $\frac{1}{5^{7/2}}$ **b)** $\frac{3}{2\left(3^{5/2}\right)}$ **c)** $\frac{5}{2\left(3^{7/2}\right)}$ **d)** $\frac{5}{2\left(7^{7/2}\right)}$ **e)** $\frac{3}{2\left(7^{5/2}\right)}$

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5. A crowd pours into a comic convention. By noon, 4000 people are on the convention floor. Due to restrictions, only 100 people can enter a minute, and at any time, roughly 5% of the current population of the room leaves every minute.

Which of the following represents the number of people in the convention by 1:00pm?

a) $4000e^{-3}$ **b)** $2000 - e^{-3}$ **c)** $4000 - 20e^{3}$

d) $2000(1+e^{-3})$ **e)** $\frac{4000}{1+100e^{3}}$

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Name: _____ Student Number:_____

6. Find the sum of the series $\sum_{n=1}^{\infty} n\left(\frac{x}{2}\right)^{n-1}$ at $x = 1/2$

a) $\frac{4n}{3}$ **b)** $\frac{16}{9}$ **c)** $\frac{4}{25}$ **d)** $\frac{2}{9}$ **e)** Cannot be determined

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7. Which of the following is the coefficient of x^n for $n > 1$ in the MacLaurin series of the function: $f(x) = (1 + x)^{1/3}$

a) $(-1)^{n-1}\frac{(1 \cdot 4 \cdot 7 \cdot \ldots \cdot (3n+1))}{n!\,3^n}$ **b)** $(-1)^{n-1}\frac{(2 \cdot 5 \cdot 8 \cdot \ldots \cdot (3n-4))}{n!\,3^n}$ **c)** $(-1)^{n}\frac{(3n-4)!}{n!\,3^n}$

d) $\frac{(1 \cdot 4 \cdot 7 \cdot \ldots \cdot (3n+1))}{(3n)!}$ **e)** $(-1)^{n}\frac{(2 \cdot 5 \cdot 8 \cdot \ldots \cdot (3n+2))}{n!\,3^n}$

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8. Which of the following represents the family of orthogonal trajectories to the family: $y = \frac{k}{x^3}$?

a) $y^2 = -\frac{1}{x^2} + C$ **b)** $y^3 = \frac{1}{x} + C$ **c)** $y = \frac{1}{x^3} + C$

d) $3y^2 - x^2 = C$ **e)** $y^2 x^2 = C$

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9. When working with MAPLE, a student defines an equation with dependent variable *y*, and independent variable *x*. Which of the following commands tell MAPLE to solve the differential equation, eq1, with the initial condition $y(1) = 2$?

a) dsolve({eq1, y(1)=2}, y(x)); **b)** eqsolve({eq1, y(1)=2}, y);

c) dsolve(eq1, (x,y)=(1,2)); **d)** eqsolve(eq1, (x,y)=(1,2)); **e)** solve(y, eq1, (x,y)=(1,2));

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10. Given the differential equation: $y'' - 5y' + 6y = 0$ find the value(s) of *k* such that $f(x) = e^{kx}$ is a solution.

a) − 1 only **b)** 0 only **c)** − 1 and 6 **d)** 2 and 3 **e)** 2 only

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11. Without solving the differential equation: $\frac{dy}{dx} = \sin^2(y)(x-5)$, which of the following can be said about the solutions?

a) All solutions have tangent lines with positive slope whenever $y > 0$.
b) All solutions have tangent lines with negative slopes whenever $x > 0$.
c) All solutions have horizontal tangent lines if $y = 0, \pm\pi, \pm 2\pi, \pm 3\pi$, etc.
d) The solutions all go to zero whenever $x = 5$
e) We can't say anything about the differential equation without solving it.

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12. Which of the following integrals represents the surface area of revolution of the function $f(x) = \ln(x)$, $1 \le x \le e$ rotated about the y axis?

a) $\int_1^e 2\pi \ln(x)\,dx$ **b)** $\int_1^e \pi(\ln(x))^2 \sqrt{1+\frac{1}{x^2}}\,dx$ **c)** $\int_1^e 2\pi \ln(x)\sqrt{1+\frac{1}{x^2}}\,dx$

d) $\int_1^e \pi\sqrt{x^2+x^4}\,dx$ **e)** $\int_1^e 2\pi x\sqrt{1+\frac{1}{x^2}}\,dx$

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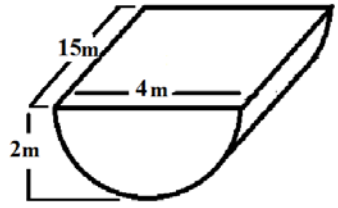
13. A tray in the form of a half-cylinder is filled with water. If the length of the tray is 15m, and the radius is 2m, what is the total hydrostatic force outward on one semicircular end of the tray?
(Remember, the density of water is 1000kg/m^3, For simplicity treat g = 10m/s^2 And don't forget: 1000N = 1kN)



a) $\frac{160}{3}$ kN **b)** $\frac{20}{3}$ kN **c)** 40π kN **d)** 20π kN **e)** 100 kN

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14. Given that $x(t) = \sqrt{t^2+2}$ and $y(t) = \cos(t)$, which of the following expressions describes the same graph?

a) $x^2 = \sec(y) + 2$ **b)** $y^2 = \cos\left(x^2 - 2\right)$ **c)** $y = \cos\left(\sqrt{x^2-2}\right)$

d) $y = \sqrt{\arccos(x-2)}$ **e)** $y = \frac{\sin(x)}{\sqrt{x^2-2}}$

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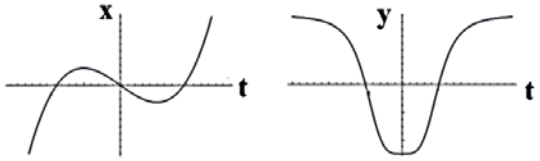
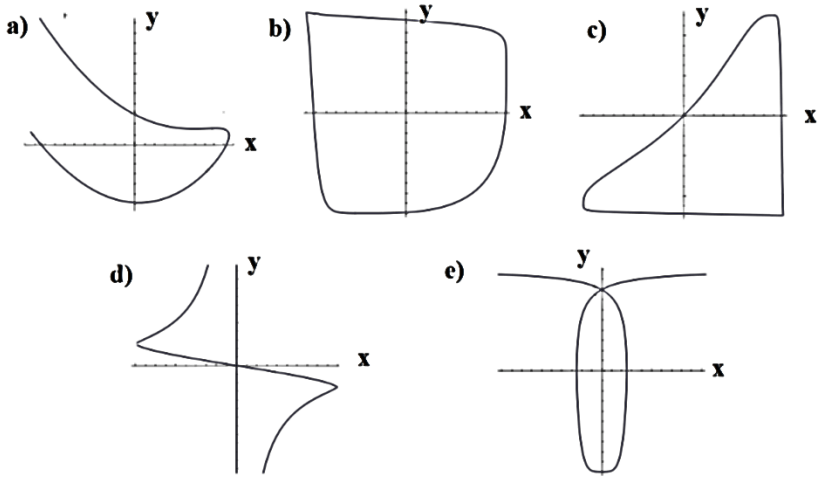
15. The graphs of $x(t)$ and $y(t)$ are given by:



Which of the following corresponds to the resulting parametric curve?



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16. Find the sum of the series: $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{2^{2n+1}(2n+1)!}$

a) 1 **b)** 0 **c)** $\sqrt{3}$ **d)** $\sqrt{2}$ **e)** Cannot be determined

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17. Find the second order Taylor polynomial, $T_2(x)$, centered at $a = 2$ for the function $f(x) = x \ln(x)$.

a) $\ln(4) + (1 + \ln 2)(x - 2) + \frac{(x-2)^2}{4}$ **b)** $\ln(4) + (1 + \ln 2)(x - 2) + \frac{(x-2)^2}{2}$

c) $(1 + \ln 2) + \frac{(x-2)}{2} - \frac{1}{8}(x-2)^2$ **d)** $(1 + \ln 2) + \frac{(x-2)}{2} - \frac{1}{4}(x-2)^2$.

e) $1 + \ln(2)(x - 2) + \frac{1}{2}(x-2)^2$

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18. Find the general solution to the differential equation: $x^2y' + 4xy = \frac{3e^x}{x^2}$,

a) $y = \frac{3e^{x-2x^2}}{4x^3} + \frac{Ce^{-2x^2}}{4x^3}$ **b)** $y = \frac{3e^x}{x^4} + \frac{C}{x^4}$ **c)** $y = \frac{3e^{-2x}}{x^2} + C$

d) $y = 2x^2e^x + C$ **e)** $y = \frac{3(x^2 - 1)e^x}{4x} + C$

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THE END

1AA3/1ZB3 Test #2 Sample Answer Key

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
D	B	C	C	D	B	B	D	A	D

Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18
C	E	A	C	E	A	A	B