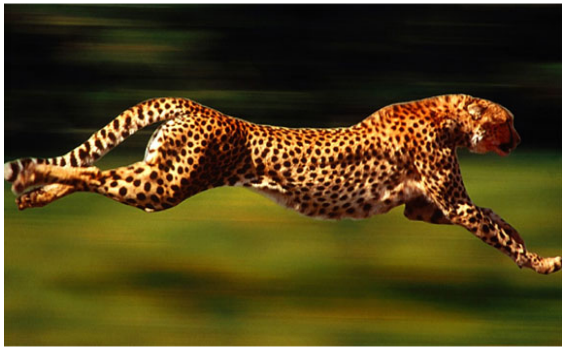
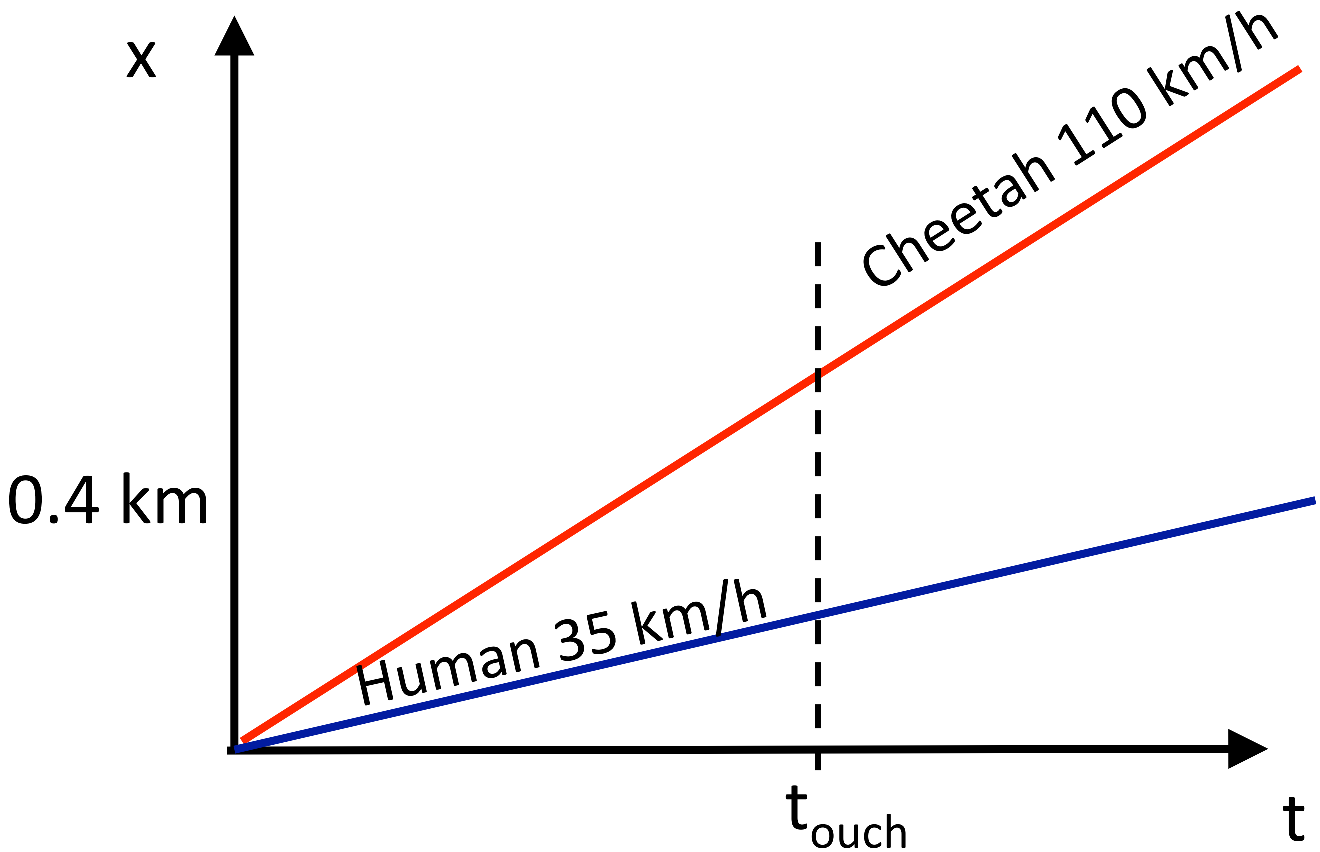
Theme 2
Mechanics

Module T2M1:
Kinematics

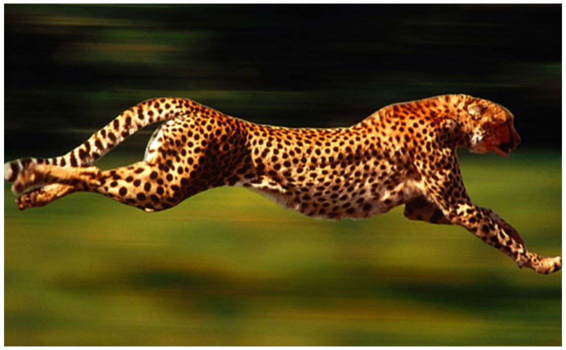
Cheetah Sprint



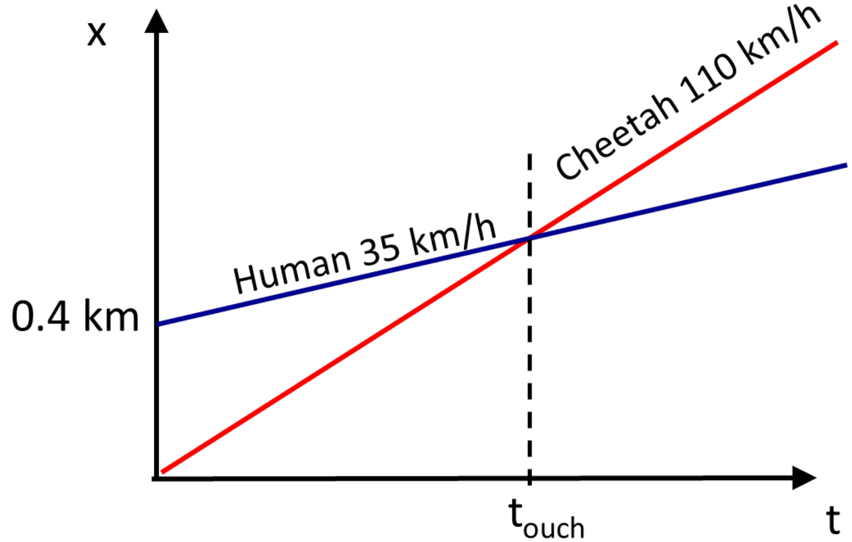
A cheetah can sprint at a speed of 110 km/h. The best a human is capable of is a speed of 35 km/h. A man and a cheetah are initially 0.400 km apart. Assuming that both man and cheetah are running at their top speed, how long does it take the cheetah to overtake the man?



Cheetah Sprint



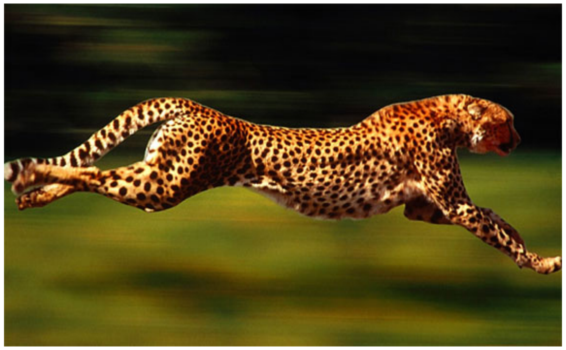
A cheetah can sp
human is capable
cheetah are initia
man and cheetah
long does it take



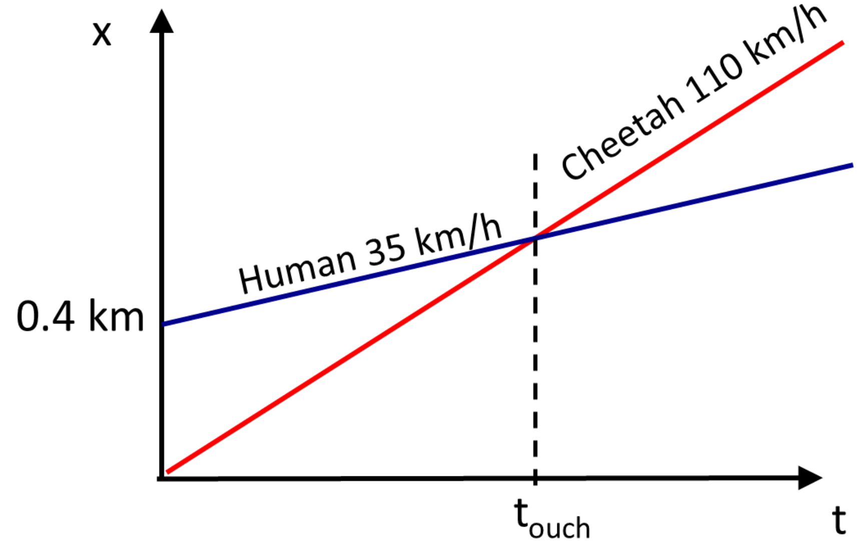
Lets write time in hours and distances in km

Cheetah	**Snack**
$x_i = 0$	$x_i = 0.4$
$x_f = ?$	$x_f = ?$
$v = 110$	$v = 35$
$a = 0$	$a = 0$
$t = ?$	$t = ?$

Cheetah Sprint



A cheetah can sp
human is capable
cheetah are initi
man and cheetah
long does it take



Lets write time in hours and distances in km

Cheetah	**Snack**
$x_i = 0$	$x_i = 0.4$
$x_f = ?$	$x_f = ?$
$v = 110$	$v = 35$
$a = 0$	$a = 0$
$t = ?$	$t = ?$

Use: $x_f = x_i + vt$

$$\text{Cheetah: } x_f = 0 + 110t$$
$$\text{Human: } x_f = 0.4 + 35t$$

$$110t = 0.4 + 35t$$
$$75t = 0.4$$
$$t = 5.33 \times 10^{-3}\ \text{h}$$
$$t = 19.2\ \text{s}$$

Kinematic equations of motion

constant *a*

$$v_f = v_i + a\Delta t$$

$$x_f = x_i + v_i\Delta t + \frac{1}{2}a\Delta t^2$$

$$v_f^2 = v_i^2 + 2a\cdot(x_f - x_i)$$

$$(x_f - x_i) = \frac{v_i + v_f}{2}\cdot\Delta t$$

$$\Delta x = x_f - x_i$$

a = 0

$$v_i = v_f$$

$$x_f = x_i + v\Delta t$$

What you need for every kinematics problem

$$a =$$

$$v_i =$$

$$v_f =$$

$$\triangle x =$$

$$\triangle t =$$

Example

The human body can survive trauma due to a sudden stop if the magnitude of acceleration is less than **245 m/s²**. Suppose you are in a car accident with an initial speed of **85 km/h** (**23.6 m/s**) and are stopped by an inflating air bag exactly at this limit (245 m/s²).

a) How long does it take to come to rest?

b) Over what distance must the air bag stop you if you are to survive the crash?

$$\Delta x = ?$$
$$v_1 = 23.6 \text{ m/s}$$
$$v_2 = 0$$
$$a =$$
$$t = ?$$

Is acceleration +ve or -ve???

Example

The human body can survive trauma due to a sudden stop if the magnitude of acceleration is less than **245 m/s²**. Suppose you are in a car accident with an initial speed of **85 km/h** (**23.6 m/s**) and are stopped by an inflating air bag exactly at this limit (245 m/s²).

a) How long does it take to come to rest?

b) Over what distance must the air bag stop you if you are to survive the crash?

$$\Delta x = ?$$
$$v_1 = 23.6 \text{ m/s}$$
$$v_2 = 0$$
$$a = -245 \text{ m/s}^2$$
$$t = ?$$

Part a)

Need t

$$v_f = v_i + a\Delta t$$

$$x_f = x_i + v_i\Delta t + \frac{1}{2}a\Delta t^2$$

$$v_f^2 = v_i^2 + 2a\cdot(x_f - x_i)$$

$$(x_f - x_i) = \frac{v_i + v_f}{2}\cdot\Delta t$$

Example

The human body can survive trauma due to a sudden stop if the magnitude of acceleration is less than **245 m/s²**. Suppose you are in a car accident with an initial speed of **85 km/h** (**23.6 m/s**) and are stopped by an inflating air bag exactly at this limit (245 m/s²).

a) How long does it take to come to rest?

b) Over what distance must the air bag stop you if you are to survive the crash?

$\Delta x = ?$

$v_1 = 23.6$ m/s

$v_2 = 0$

$a = -245$ m/s^2

$t = ?$

a) Use $v = v_o + a\, t$

$0 = 23.6 + (-245)\, t$ (add $-245\, t$ to each side)

$245\, t = 23.6$ (div by 245)

$t = 23.6/245 = 0.096$ s

Example

The human body can survive trauma due to a sudden stop if the magnitude of acceleration is less than **245 m/s²**. Suppose you are in a car accident with an initial speed of **85 km/h** (**23.6 m/s**) and are stopped by an inflating air bag exactly at this limit (245 m/s²).

a) How long does it take to come to rest?

b) Over what distance must the air bag stop you if you are to survive the crash?

$$\Delta x = ?$$
$$v_1 = 23.6 \text{ m/s}$$
$$v_2 = 0$$
$$a = -245 \text{ m/s}^2$$
$$t = ?$$

Part b)
Need Δx

$$v_f = v_i + a\Delta t$$

$$x_f = x_i + v_i\Delta t + \frac{1}{2}a\Delta t^2$$

$$v_f^2 = v_i^2 + 2a \cdot (x_f - x_i)$$

$$(x_f - x_i) = \frac{v_i + v_f}{2} \cdot \Delta t$$

Example

The human body can survive trauma due to a sudden stop if the magnitude of acceleration is less than **245 m/s²**. Suppose you are in a car accident with an initial speed of **85 km/h** (**23.6 m/s**) and are stopped by an inflating air bag exactly at this limit (245 m/s²).

a) How long does it take to come to rest?

b) Over what distance must the air bag stop you if you are to survive the crash?

$\Delta x = ?$

$v_1 = 23.6 \text{ m/s}$

$v_2 = 0$

$a = -245 \text{ m/s}^2$

$t = ?$

a) Use $v = v_o + a\,t$

$0 = 23.6 + (-245)\,t$ (add -245 t to each side)

$245\,t = 23.6$ (div by 245)

$t = 23.6/245 = 0.096 \text{ s}$

b) Use $v^2 = v_o{}^2 + 2\,a\,\Delta x$

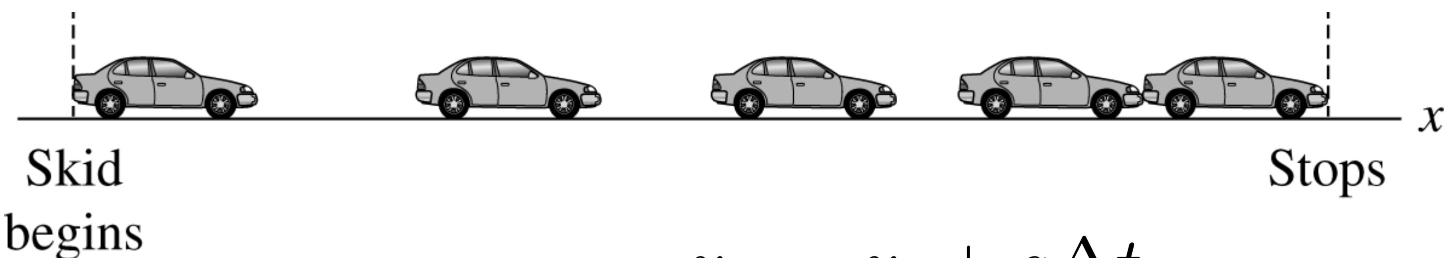
$0^2 = 23.6^2 + 2\,(-245)\,\Delta x$

$0 = 557 - 490\,\Delta x$

$\Delta x = 557/490 = 1.14 \text{ m}$

Try this on your own!

- A car brakes to avoid hitting a can of tuna. The brakes apply an acceleration of **4.5 m/s²**, and the car comes to rest over a distance of **27 m**. What was the speed of the car at the instant the brakes were applied?



$$v_f = v_i + a\Delta t$$

$$x_f = x_i + v_i\Delta t + \frac{1}{2}a\Delta t^2$$

$$v_f^2 = v_i^2 + 2a\cdot(x_f - x_i)$$

$$(x_f - x_i) = \frac{v_i + v_f}{2}\cdot\Delta t$$

$\Delta x =$

$v_1 =$

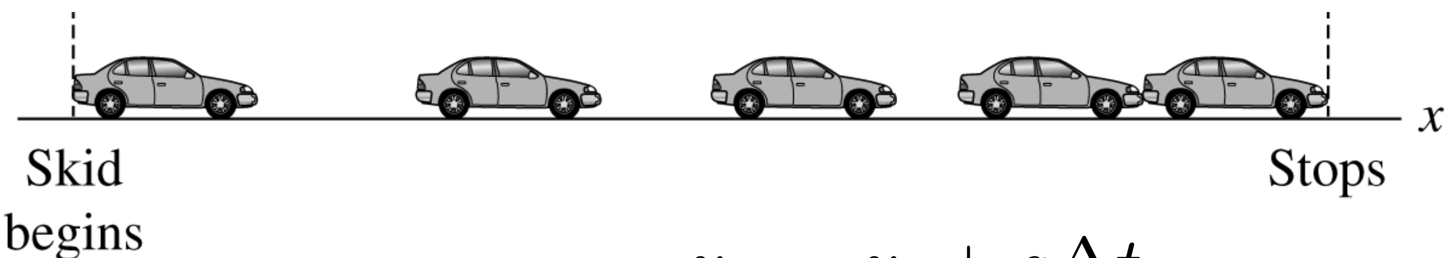
$v_2 =$

$a =$

$t =$

Try this on your own!

- A car brakes to avoid hitting a can of tuna. The brakes apply an acceleration of **4.5 m/s²**, and the car comes to rest over a distance of **27 m**. What was the speed of the car at the instant the brakes were applied?



$\Delta x = 27$ m

$v_1 = ?$

$v_2 = 0$

$a = -4.5$ m/s^2

$t = ?$

$$v_f = v_i + a\Delta t$$

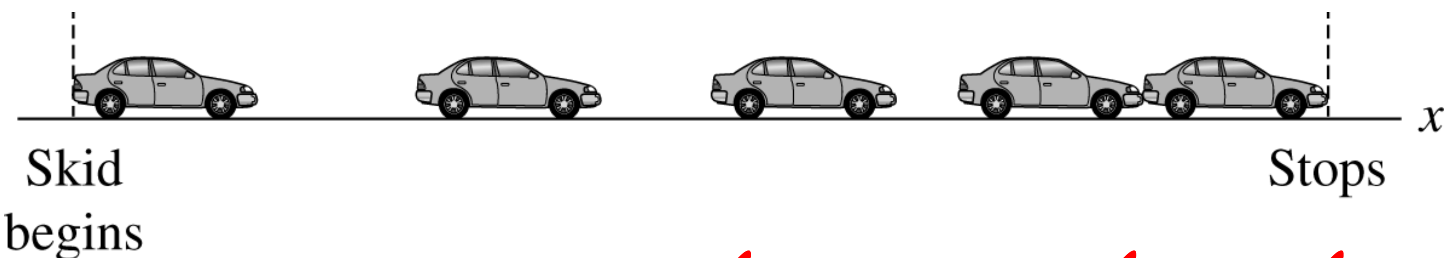
$$x_f = x_i + v_i\Delta t + \frac{1}{2}a\Delta t^2$$

$$v_f^2 = v_i^2 + 2a \cdot (x_f - x_i)$$

$$(x_f - x_i) = \frac{v_i + v_f}{2} \cdot \Delta t$$

Try this on your own!

- A car brakes to avoid hitting a can of tuna. The brakes apply an acceleration of **4.5 m/s²**, and the car comes to rest over a distance of **27 m**. What was the speed of the car at the instant the brakes were applied?



$$v_f^2 = v_i^2 + 2a \cdot (x_f - x_i)$$

$\Delta x = 27$ m

$v_1 = ?$

$v_2 = 0$

$a = -4.5$ m/s^2

$t = ?$

$0 = v_o^2 + 2(-4.5)(+27)$

$v_o^2 = 243$

$v_o = +15.6$ m/s

The quadratic equation – it happens to the best of us

- You may encounter a kinematics problem in which you are asked to solve for the time, using the equation:

$$x_f = x_i + v_i \Delta t + \frac{1}{2} a \Delta t^2 \longrightarrow \Delta y = v_{y0} t + \frac{1}{2} a_y t^2$$

where Δy, v_{y0} and a_y are known.

- You need to use the quadratic equation to solve for t:

$$0 = At^2 + Bt + C \qquad t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

All objects fall with the same acceleration

Apollo 15, 1971

Free fall

- All objects fall with the same acceleration:

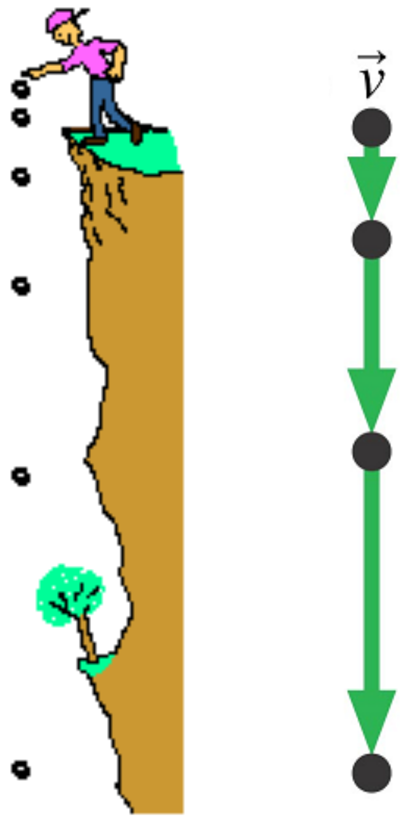
$$\vec{a} = \vec{g} = 9.8\ \mathrm{m/s^2}\ \ [down]$$

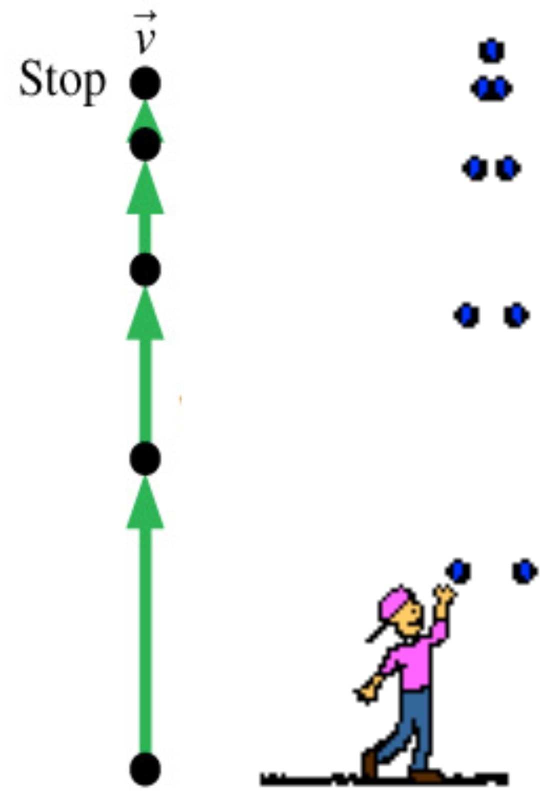
  - Valid near Earth's surface
  - Regardless of object's mass
  - Neglecting air resistance

- "Free fall" (i.e. motion with g) does not **ONLY** mean downward motion
  - Throw something up in the air – the moment it leaves your hand, it is in free fall (while moving upward as well as downward)

Free falling upward?

- “free fall just refers to acceleration = g





Example

Your frisbee is stuck in a branch that is 5.0 m above ground. You throw your shoe straight up to try to knock it down, but your shoe just reaches the frisbee before falling back down. What initial velocity did you give the shoe if it started at 1.5 m above ground?

Define up as the positive direction

What is Δy?

$\Delta y =$

$v_i =$

$v_f =$

$a =$

$\Delta t =$

$$v_f = v_i + a\Delta t$$

$$x_f = x_i + v_i\Delta t + \frac{1}{2}a\Delta t^2$$

$$v_f^2 = v_i^2 + 2a\cdot(x_f - x_i)$$

$$(x_f - x_i) = \frac{v_i + v_f}{2}\cdot\Delta t$$

Example

Your frisbee is stuck in a branch that is 5.0 m above ground. You throw your shoe straight up to try to knock it down, but your shoe just reaches the frisbee before falling back down. What initial velocity did you give the shoe if it started at 1.5 m above ground?

Define up as the positive direction

What is Δy?

$\Delta y = 3.5 \text{ m}$

$v_i = ?$ (must be +ve)

$v_f = 0$ (at the top)

$a = -9.8 \text{ m/s}^2$

$\Delta t = ?$

$$v_f = v_i + a\Delta t$$

$$x_f = x_i + v_i\Delta t + \frac{1}{2}a\Delta t^2$$

$$v_f^2 = v_i^2 + 2a \cdot (x_f - x_i)$$

$$(x_f - x_i) = \frac{v_i + v_f}{2} \cdot \Delta t$$

Example

Your frisbee is stuck in a branch that is 5.0 m above ground. You throw your shoe straight up to try to knock it down, but your shoe just reaches the frisbee before falling back down. What initial velocity did you give the shoe if it started at 1.5 m above ground?

Define up as the positive direction

What is Δy?

$\Delta y = 3.5 \text{ m}$

$v_i = ? \text{ (must be +ve)}$

$v_f = 0 \text{ (at the top)}$

$a = -9.8 \text{ m/s}^2$

$\Delta t = ?$

$$\checkmark \qquad \checkmark \quad \checkmark$$

$$v_f^2 = v_i^2 + 2a\Delta y$$

$$0 = v_i^2 + 2(-9.8)(3.5)$$

$$v_i = \sqrt{2(9.8)(3.5)} = 8.3 \text{ m/s}$$

Keep track of the units!!!

Would it matter if we had chosen down as the positive direction?

Clicker quiz

You are throwing a ball straight up in the air. At the highest point, the ball’s

a) velocity and acceleration are zero.

b) velocity is nonzero but its acceleration is zero.

c) acceleration is nonzero, but its velocity is zero.

d) velocity and acceleration are both nonzero.

Clicker quiz

You are throwing a ball straight up in the air. At the highest point, the ball's

a) velocity and acceleration are zero.

b) velocity is nonzero but its acceleration is zero.

c) acceleration is nonzero, but its velocity is zero.

d) velocity and acceleration are both nonzero.

Try this on your own!

- You are standing on 20 m tall stilts, and you throw a hotdog straight up into the air. It takes 5.0 seconds for the hotdog to hit the ground below you.

What was the initial velocity of the hotdog?

Define up as the positive direction (all units are in m and s)

$$\Delta y = -20$$
$$v_i = ?$$
$$v_f = ?$$
$$a = -9.8$$
$$\Delta t = 5.0$$

$$\Delta y = v_i t + \frac{1}{2} a t^2$$
$$-20 = v_i 5.0 + \frac{1}{2}(-9.8)5^2$$
$$v_i = 20.5 \text{ m/s}$$

Note: we did not have to break this problem into two parts (up and down), the math and proper signs takes care of it!!!

A note on the kinematic equations

- Our equations are valid for motion in which the acceleration is **constant**
- Of course, real motion can have changing acceleration
  - Consider this case of driving: Traffic light turns green, you start from rest, accelerate to 50 km/h, then continue on at that speed (acceleration becomes zero)
- When dealing with this motion, we can break it up into two parts (each with it's own constant acceleration

a. Accelerating up to 50 km/h

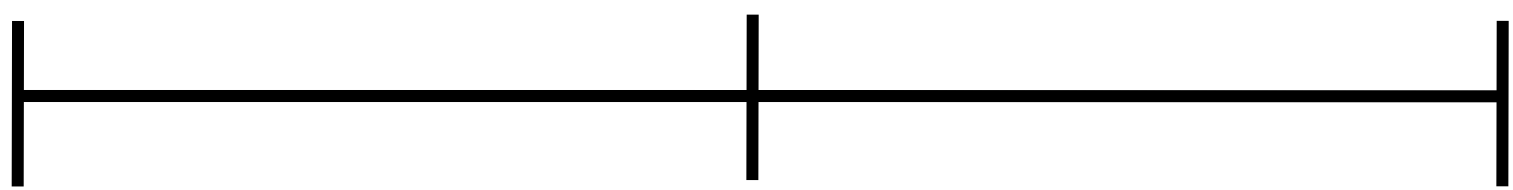
b. Continuing on at 50 km/h (with $a = 0$)

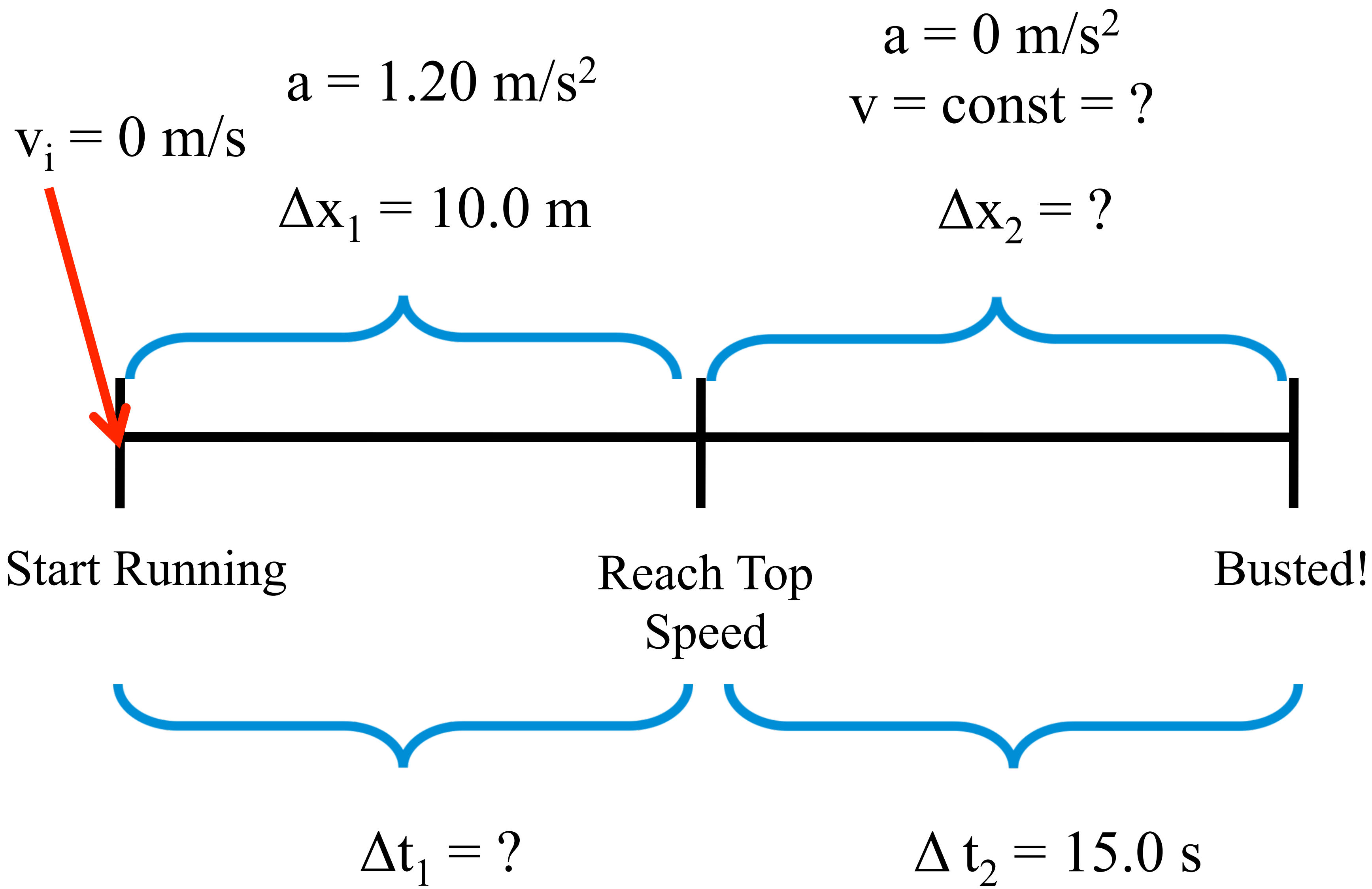
Example: 1D motion with two parts

Sitting beside an old lady in the park, you grab her purse and start running. Over the first 10.0 m, you accelerate at 1.20 m/s^2 up to your top running speed, and then continue to sprint at this speed for 15.0 s more before being tackled from behind by the old lady.

(a) How long did it take the old lady to catch you?

(b) How far from the bench did you get before being caught?




$a = 0\ m/s^2$
$v = const = ?$
$a = 1.20\ m/s^2$
$v_i = 0\ m/s$
$\Delta x_1 = 10.0\ m$
$\Delta x_2 = ?$
Start Running
Reach Top Speed
Busted!
$\Delta t_1 = ?$
$\Delta t_2 = 15.0\ s$

What do we know?

- **First Part**

$\Delta x_1 = 10.0$ m
$v_{1i} = 0$ m/s
$v_{1f} = ?$
$a_1 = 1.2$ m/s^2
$\Delta t_1 = ?$

- **Second Part**

$\Delta x_2 = ?$
$v_{2i} = ?$
$v_{2f} = ?$
$a_2 = 0$ m/s^2
$\Delta t_2 = 15.0$ s

same
(v constant)

Start Solving

- **First Part**

$$\Delta x_1 = v_{1i}\Delta t_1 + \frac{1}{2}a_1\Delta t_1^2$$

$$\Rightarrow \quad 10.0 = (0)\,\Delta t_1 + \tfrac{1}{2}(1.20)\,\Delta t_1^2$$

$$\Rightarrow \quad 10 = 0.6\,\Delta t_1^2$$

$$\Rightarrow \Delta t_1 = \sqrt{\frac{10}{0.6}} = 4.10\ s$$

$$v_{1f} = v_{1i} + a_1\Delta t_1$$

$$\Rightarrow v_{1f} = 0 + 1.2 \cdot 4.1 = 4.92\ m/s$$

- **Second Part**

$$\Delta x_2 = v_{2i}\Delta t_2 + \frac{1}{2}a_2\Delta t_2^2$$

$$\Rightarrow x_2 = (4.92)(15) + \tfrac{1}{2}(0)t^2$$

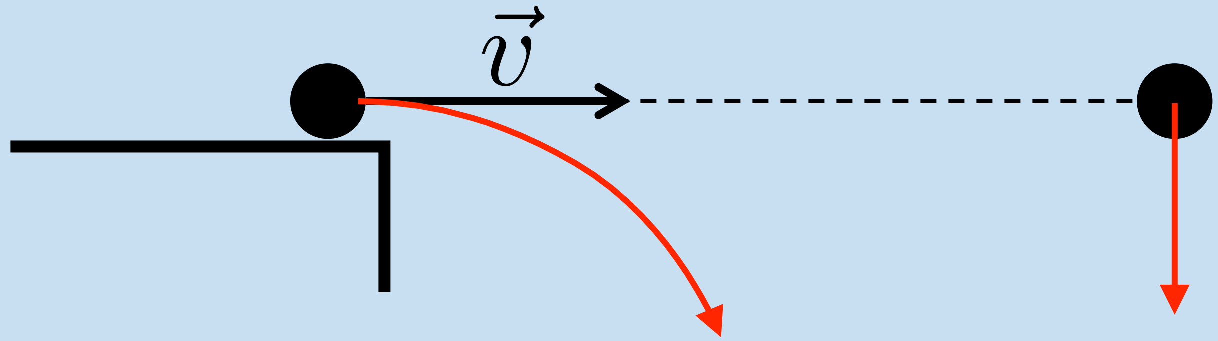
$$\Rightarrow x_2 = 73.8\ m$$

$$\Delta x_{TOT} = \Delta x_1 + \Delta x_2$$

$$\boxed{\Rightarrow x_{TOT} = 10.0 + 73.8 = 83.8\ m}$$

Concept Question

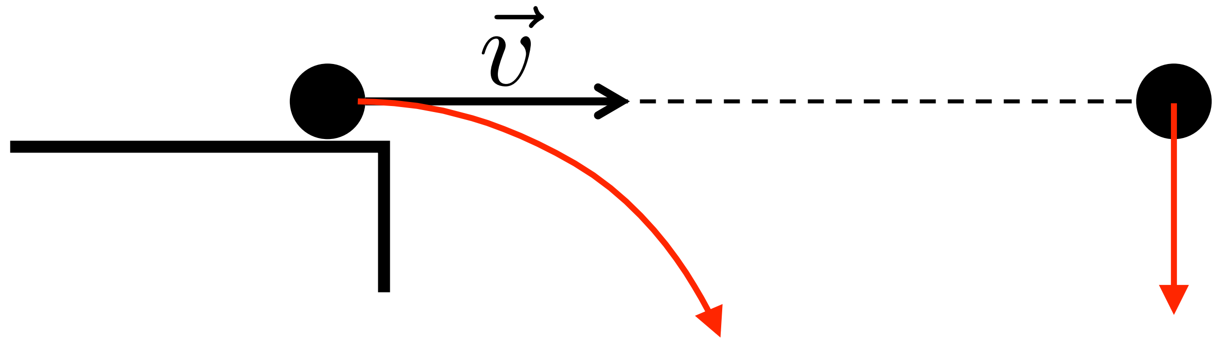
A ball is pushed off a table with some initial horizontal velocity, an other ball is released from rest from the same height at the same time. Which one hits the floor first?



A. The dropped ball
B. The pushed ball
C. They land at the same time

Concept Question

A ball is pushed off a table with some initial horizontal velocity, an other ball is released from rest from the same height at the same time. Which one hits the floor first?



A. The dropped ball
B. The pushed ball
C. They land at the same time

- What does this mean????
  - x and y motion are independent of each other!!

Components make life easy!

- Since all vector terms have *x* and *y* components, they separate:

<u>x motion</u>

$$v_{x,f} = v_{x,i} + a_x t$$

$$x_f = x_i + v_{x,i}t + \frac{1}{2}a_x t^2$$

$$x_f - x_i = \frac{1}{2}(v_{x,i} + v_{x,f})t$$

$$v_{x,f}^2 = v_{x,i}^2 + 2a_x \cdot (x_f - x_i)$$

<u>y motion</u>

$$v_{y,f} = v_{y,i} + a_y t$$

$$y_f = y_i + v_{y,i}t + \frac{1}{2}a_y t^2$$

$$y_f - y_i = \frac{1}{2}(v_{y,i} + v_{y,f})t$$

$$v_{y,f}^2 = v_{y,i}^2 + 2a_y \cdot (y_f - y_i)$$

Components make life easy!

- Since all vector terms have *x* and *y* components, they separate:

x motion

$$\boxed{x_f - x_i = v_x t}$$

y motion

$$v_{x,f} = v_{x,i} + \cancel{a_x} t \qquad v_{y,f} = v_{y,i} + a_y t$$

$$x_f = x_i + v_{x,i}t + \cancel{\frac{1}{2}a_x t^2} \qquad y_f = y_i + v_{y,i}t + \frac{1}{2}a_y t^2$$

$$x_f - x_i = \boxed{\frac{1}{2}(v_{x,i} + v_{x,f})t} \qquad y_f - y_i = \frac{1}{2}(v_{y,i} + v_{y,f})t$$

$$v_{x,f}^2 = v_{x,i}^2 + \cancel{2a_x} \cdot (x_f - x_i) \qquad v_{y,f}^2 = v_{y,i}^2 + 2a_y \cdot (y_f - y_i)$$

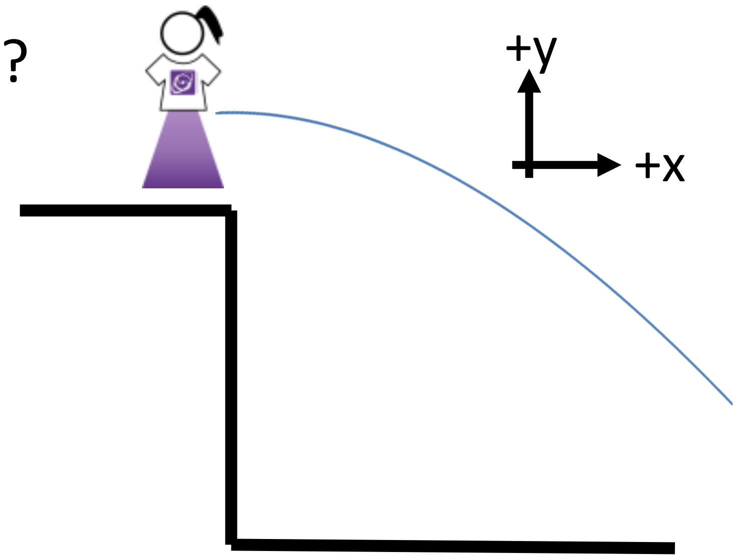
With projectile motion $a_x = 0$, $a_y = g$, life gets even easier!

Example 1

Physics Girl jumps from a balcony onto a trampoline with a horizontal velocity of 12 m/s, and lands 22 m below.

a) How long is she in the air?

b) How far from the balcony does she land?



x information

$\Delta x = ?$

$v_x = 12$

$\Delta t = ?$

y information

$\Delta y = -22$?

$v_{y,i} = 0$

$v_{y,f} = ?$

$a = -9.8$

$\Delta t = ?$

Part a)

$$\Delta y = v_{y,i}t + \frac{1}{2}at^2$$

$$-22 = 0 + \frac{1}{2}(-9.8)t^2$$

$$\frac{2(-22)}{-9.8} = t^2$$

$$t = 2.1 \text{ s}$$

Part b)

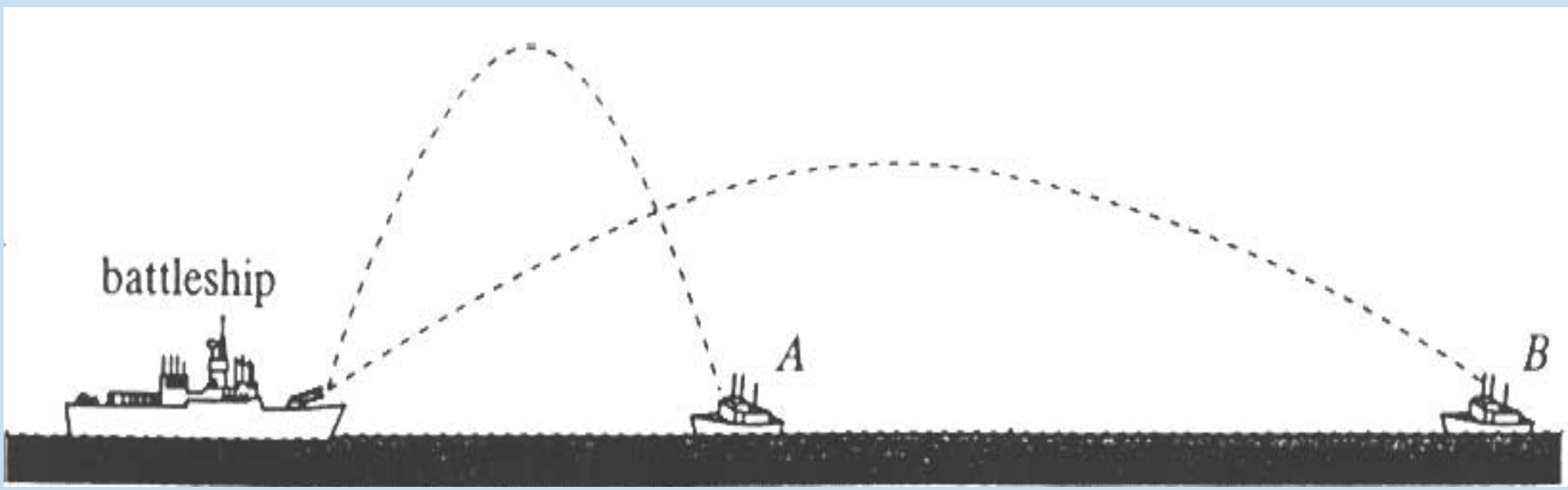
$$\Delta x = v_x t$$

$$\Delta x = 12(2.1)$$

$$\Delta x = 25 \text{ m}$$

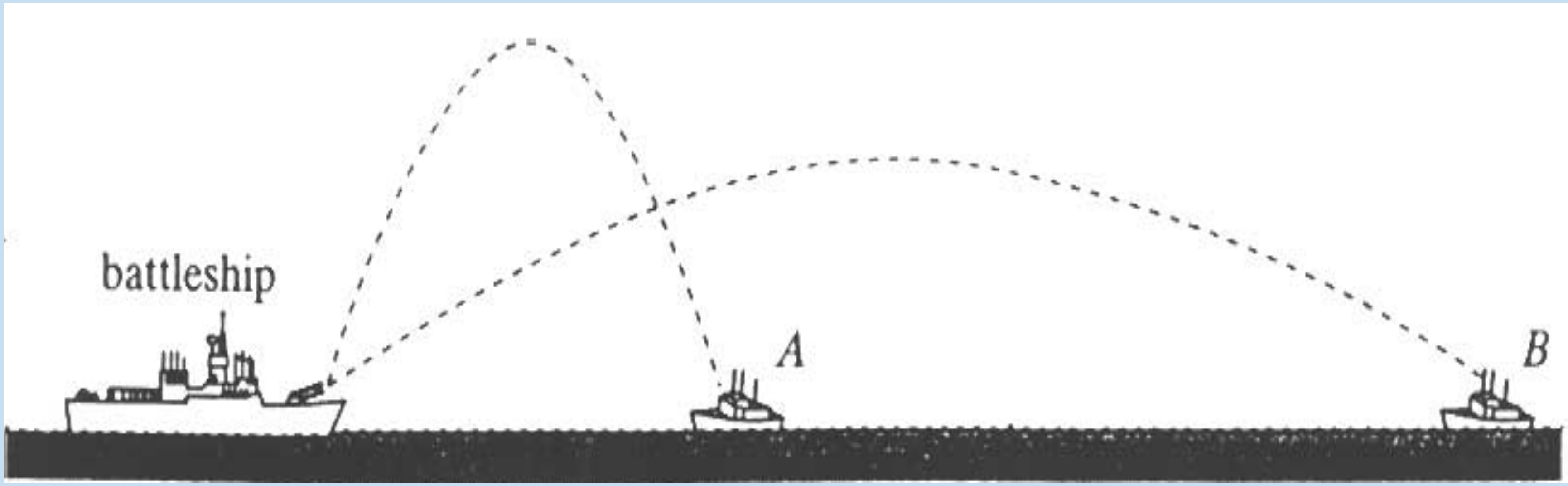
Concept Question

- A battleship simultaneously fires two shells at enemy ships. If the shells follow the parabolic trajectories shown, which ship gets hit first?



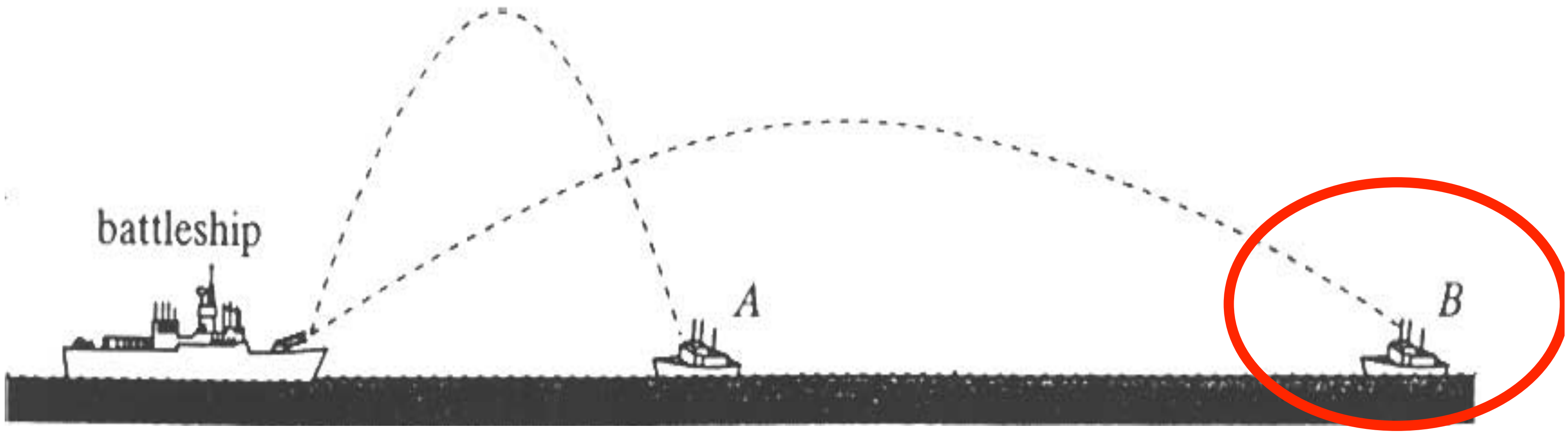
Concept Question

- A battleship simultaneously fires two containers of hot dogs and bananas at neighbouring ships. If the shells follow the parabolic trajectories shown, which ship gets hit first?



Concept Question

- A battleship simultaneously fires two containers of hot dogs and bananas at neighbouring ships. If the shells follow the parabolic trajectories shown, which ship gets hit first?



Remember, we can break up the motion into the x and y directions

Mid Term on Friday

- **You must write in your designated room:**

  - Mac ID: a___ to f____ write in BSB 147
  - Mac ID: g___ to kh___ write in ITB 137
  - Mac ID: ki___ to n___ write in JHE 376
  - Mac ID: o___ to sh___ write in MDCL 1102
  - Mac ID: si___ to z___ write in MDCL 1105

- **Check avenue carefully**