1.2.3

Last Day

Augmented Matrix Form

eg $\begin{cases} x + 2y - z = 3 \\ 2x - y + 5z = 2 \end{cases}$ $\rightsquigarrow$ $\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 2 & -1 & 5 & 2 \end{array}\right]$

x y z optional

eqn. 1

eqn. 2

Coeff. | constants

Elementary Row Ops.

1) Swap two rows

eg $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 4 & 5 & 6 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 4 & 5 & 6 \\ 1 & 2 & 3 \end{array}\right]$

2) Mult. a row by $k \neq 0$

eg $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 4 & 5 & 6 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 2 & 4 & 6 \\ 4 & 5 & 6 \end{array}\right]$

Row 1 · 2

3) Add a multiple of one row to another (in that location)

eg. $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 4 & 5 & 6 \end{array}\right]$ $\longrightarrow$ Row 2 − 4·Row 1

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & -3 & -6 \end{array}\right]$$

Row 2 → Row 2 − 4 Row 1

Goal: RREF (Reduced Row Echelon Form)

1) All 0 rows at bottom
2) First non-zero entry in any row is a (leading) 1
3) Each leading 1 is right of all leading 1's in rows above ("down & to the right!")

4) All entries below or above a leading 1, in same column must be 0

If entries below leading 1 are 0 but above are not => "Row Echelon Form" ie REF only!

Gaussian Elimination

Use elementary row ops. on aug. matrix

working starting at upper left corner &
working right to turn system into RREF

e.g. $\begin{cases} 3x + 3y - 6z = 3 \\ 2x + y + 3z = 1 \\ 5x + 3y + 5z = 0 \end{cases}$

Solve using Gauss. Elimination!

Solution

$$\left[\begin{array}{ccc|c} 3 & 3 & -6 & 3 \\ 2 & 1 & 3 & 1 \\ 5 & 3 & 5 & 0 \end{array}\right] \xrightarrow{\frac{1}{3} \cdot R_1} \left[\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 2 & 1 & 3 & 1 \\ 5 & 3 & 5 & 0 \end{array}\right]$$

die!

Row 2 → $R_2 - 2R_1$, then Row 3 → $R_3 - 5R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & -1 & 7 & -1 \\ 0 & -2 & 15 & -5 \end{array}\right] \xrightarrow[\text{Row 2} \cdot (-1)]{} \left[\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 1 & -7 & 1 \\ 0 & -2 & 15 & -5 \end{array}\right]$$

✓

Row 1 - Row 2 then Row 3 + 2Row 2

(4)

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 0 \\ 0 & 1 & -7 & 1 \\ 0 & 0 & 1 & -3 \end{array}\right]$$

✓ ✓

Row 2 + 7R3 then $R_1 - 5R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 15 \\ 0 & 1 & 0 & -20 \\ 0 & 0 & 1 & -3 \end{array}\right]$$

RREF ✓

$x = 15$

$y = -20$

$z = -3$

TAH DAH!!

Or · Slight shortcut!

(*) ⇒ $\left[\begin{array}{ccc|c} 1 & 0 & 5 & 0 \\ 0 & 1 & -7 & 1 \\ 0 & 0 & 1 & -3 \end{array}\right]$ } If we stop at any REF form

If in REF form can "back solve!"

$z = -3$

$y - 7z = 1 \Rightarrow y = 7z + 1 = 20$

$x + 5z = 0 \Rightarrow x = -5z = 15$

Note for a given system: RREF Unique to that system!
REF not.

Eg. of RREF

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{l} x = 3 \\ y = 4 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \rightarrow x = 2 \\ \rightarrow y = 5 \\ \rightarrow \cancel{0 = 0} \quad z = t \end{array}$$

a parameter!

z free variable

solution: $(x, y, z) = (2, 5, t)$

$$\left[\begin{array}{ccc|c} 1 & 8 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow \quad x + 8y = 2 \text{ so } x = 2 - 8y$$

$$\rightarrow \quad z = 3$$

$y = t$

y free $\Rightarrow$ t a parameter!

$x = 2 - 8t$

$y = t$

$z = 3$

$$\left[\begin{array}{ccc|c} 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \leftarrow z = 3$$

$x = t$
$y = s$
} two parameters!

$$(x, y, z) = (t, s, 3)$$

$$\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 1 \end{array}\right] \begin{array}{l} \rightarrow x = 0 \\ \rightarrow 0 = 1 \end{array}$$

Inconsistent! No solution!