McMaster University
Math 1LT3E

ASSIGNMENT 7

Sections 1, 2, 3, 4, and 5 in the Grey Module

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1. A population of lions p_t, where $t = 0, 1, 2...$, is modelled by $p_{t+1} = p_t + I_t$. The immigration term is equal to $I_t = 25$ with an 80% chance and $I_t = -30$ with a 20% chance. Assume that $p_0 = 100$.

(a) What is the deterministic part of this model? What is the stochastic part?

$$p_{t+1} = \underbrace{p_t}_{\text{deterministic term}} + \underbrace{I_t}_{\text{stochastic term}} \quad \text{where } I_t = \begin{cases} 25 & 80\% \text{ chance} \\ -30 & 20\% \text{ chance} \end{cases}$$

(b) What is your prediction for the population of lions in 10 years? Will the population increase, decrease, or remain about the same? Explain.

In 8 out of 10 years, we expect an increase of 25 lions/year.
In 2 out of 10 years, we expect a decrease of 30 lions/year.

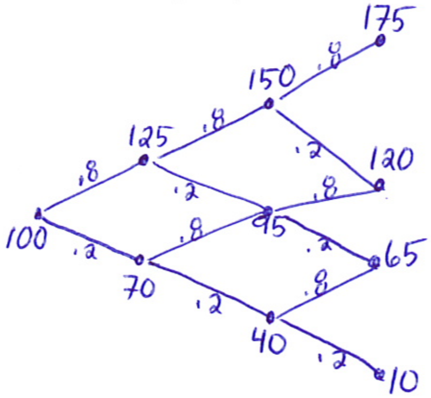
Expected net lions in 10 years:

$$8 \times 25 + 2 \times (-30) = 140$$

$\therefore$ After 10 years, a net increase of 140 lions is expected.

1. continued...

(c) Write the sample space for the population of lions after 3 years. (Hint: Draw a tree diagram).



$S = \{10, 65, 120, 175\}$

(d) Assuming that immigration from year to year is independent, determine probabilities for each outcome in the sample space in part (c).

Let X be the pop^N size after 3 years.

x	$p(x) = P(X = x)$
10	$(.2)^3 = 0.008$
65	$3(.2)^2(.8) = 0.096$
120	$3(.2)(.8)^2 = 0.384$
175	$(.8)^3 = 0.512$

(e) What is the probability that the population of lions will increase after 3 years?

$$P(X > 100) = P(X = 120) + P(X = 175)$$
$$= 0.384 + 0.512$$
$$= 0.896$$

$\therefore$ There is an 89.6 % chance that the pop^N will increase in 3 years.

2. A family has 5 children. Assume that female and male children are equally likely to be born.

(a) What is the probability that at least one child is a girl?

$G = \geq 1$ child is a girl

$G^c = <1$ child is a girl $= \{BBBBB\}$

$P(G^c) = \frac{|G^c|}{|S|} = \frac{1}{2^5} = 0.03125$

∵ equally likely

$\therefore P(G) = 1 - P(G^c) = 1 - 0.03125 = 0.96875$

∴ There is about a 97% chance that at least one child is a girl.

(b) What is the probability that exactly one child is a girl?

$G =$ exactly 1 child is a girl $= \{GBBBB, BGBBB, BBGBB, BBBGB, BBBBG\}$.

$P(G) = \frac{|G|}{|S|} = \frac{5}{2^5} = 0.15625$

∴ There is about a 16% chance that exactly one child is a girl.

(c) If it is known that at least one child is a boy, what is the probability that at least one child is a girl?

$B =$ at least one is a boy

$B^c = \{GGGGG\}$

$P(B) = 1 - P(B^c) = 1 - \frac{|B^c|}{|S|} = 1 - \frac{1}{2^5} = 0.96875 = \frac{31}{32}$

$G = \geq 1$ is a girl

$P(G|B) = \frac{P(G \cap B)}{P(B)}$ atleast one boy AND at least one girl

$(G \cap B)^c = \{BBBBB, GGGGG\}$

$= \frac{\frac{30}{32}}{\frac{31}{32}} = \frac{30}{31} \approx 0.9677$

∴ There is about a 97% chance at least one is a girl.

3. For the purposes of a study, university students were divided into two categories: those who work (at a paid job) throughout the school year and those who only work during the summer. Is this a partition of the sample space? If not, suggest a way in which the sample space could be partitioned based on the work habits of students.

This is NOT a partition!

① The two sets could intersect/overlap: students who work during the school year AND the summer + students who only work in the summer.

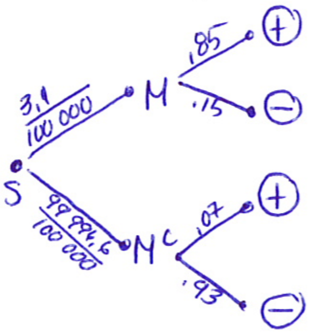
② The union of these 2 sets may not include the whole pop^N: what about students who don't work at all?

One possible partition:

E_1 = students who work at some point during a 12 month period

E_2 = students who do not work at all " " " " " .

4. The incidence of bacterial meningitis within a certain population was estimated to be about 3.4 cases per 100, 000 people during 2012. A test for meningitis shows a positive result in 85% of people who have it and in 7% of people who do not have it (false-positive). If you belong to this population and test positive for bacterial meningitis, what is the probability that you actually have it?



M = have meningitis

$$P(M\mid \oplus) = \frac{P(\oplus\mid M)\cdot P(M)}{P(\oplus\mid M)\cdot P(M) + P(\oplus\mid M^c)\cdot P(M^c)}$$

$$= \frac{(0{,}85)\left(\frac{3{,}4}{100\,000}\right)}{(.85)\left(\frac{3{,}4}{100\,000}\right) + (.07)\left(\frac{99\,996{,}6}{100\,000}\right)}$$

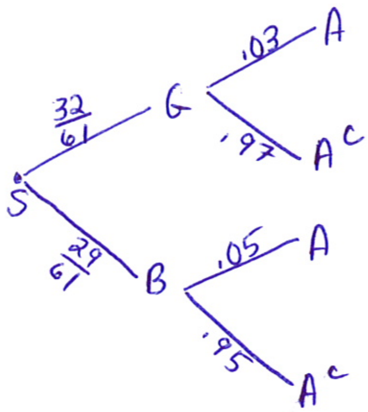
$$\approx 0.0004127$$

∴ There is only about a 0.04 % you actually have meningitis given that you tested positive for it.

5. It is estimated that ADHD affects 3 to 5 percent of school aged children globally, with males being diagnosed more frequently than females. Consider a population of school aged children comprised of 160 girls and 145 boys. Suppose that 3% of girls and 5% of boys within this population are estimated to be affected by ADHD.

(a) What is the probability that a randomly chosen child will be affected by ADHD?

A = affected by ADHD.



$$P(A) = P(A|G)P(G) + P(A|B)P(B)$$
$$= (.03)\left(\frac{32}{61}\right) + (.05)\left(\frac{29}{61}\right)$$
$$\approx 0.0395$$

$\therefore$ There is about a 3.95% chance that a randomly selected child from this pop^n will have ADHD.

(b) What is the probability that a child with ADHD is a girl?

$$P(G|A) = \frac{P(A|G)\cdot P(G)}{P(A)}$$
$$\approx \frac{(.03)\left(\frac{32}{61}\right)}{0.0395}$$
$$\approx 0.3983$$

$\therefore$ There is about a 39.83% chance that a child with ADHD is a girl.

6. In roulette, a wheel with numbered slots is spun and a ball is rolled in the opposite direction around the wheel. Players can bet on a single number or range of numbers based on where they expect the ball will stop. In American roulette, the wheel is numbered from 0 to 37 and the ball has an equally likely chance of stopping on any one of these numbers. If you always bet on 13, what is the probability of the ball stopping on 13 at least once in 10 rolls? What is the probability of the ball stopping on 13 for all 10 rolls?

$S = \{0, 1, 2, \ldots, 37\}$ $|S| = 38$ prob. of each outcome in $S = \frac{1}{38}$

Let X_i be the event that the ball stops on 13 on the i^{th} roll

$P(X_i) = \frac{1}{38}$ for $i = 1, 2, \ldots$

Let X = stops on 13 at least once in 10 rolls ∵ rolls are independent

$X^c = X_1^c \cap X_2^c \cap \cdots \cap X_{10}^c \quad (= \bigcap_{i=1}^{10} X_i)$

$P(X) = 1 - P(X^c) = 1 - [P(X_1^c) \cdot P(X_2^c) \cdot \ldots \cdot P(X_{10}^c)] \quad (= 1 - \prod_{i=1}^{10} P(X_i^c))$

$= 1 - (\frac{37}{38})^{10} = 0.2341$

Let Y = stops on 13 for all 10 rolls $= X_1 \cap X_2 \cap \cdots \cap X_{10}$

$P(Y) = P(X_1) \cdot P(X_2) \cdot \ldots \cdot P(X_{10}) = (\frac{1}{38})^{10} \approx 1.59 \times 10^{-16}$

∴ There is about a 23% chance the ball will stop on 13 at least once in 10 rolls but an almost 0% chance it will stop on 13 on all 10 rolls.

7. An online dating site claims that 1 out of 4 blind dates end in disappointment. To avoid disappointment, you decide to limit yourself to 3 blind dates in a year. What is wrong with this reasoning?

Each blind date is independent of the other blind dates. Each date, first, second, third, forth, etc. has an equally likely chance of ending badly (25% chance).