ln/log properties

0) $\log_a x = \frac{\ln x}{\ln a} = \left(\frac{1}{\ln a}\right) \ln x$

$= \# \cdot \ln x$

1) $\ln(ab) = \ln a + \ln b$

2) $\ln(a/b) = \ln a - \ln b$

3) $\ln(a^b) = b \ln a.$

big 3

Very Important.

these apply to $\log_a x$ as well

12A3

Expand $\ln\left(\dfrac{2^x \cdot (x^2-1)}{x(x^2+1)}\right)$

1) $\ln(ab) = \ln a + \ln b$

2) $\ln(a/b) = \ln a - \ln b$

3) $\ln(a^b) = b \ln a$.

Solution

$= \ln(2^x(x^2-1)) - \ln(x(x^2+1))$ by ②

$= \ln 2^x + \ln(x^2-1) - (\ln x + \ln(x^2+1))$ by ①

$\ln((x-1)(x+1))$

$= \ln 2^x + \ln(x^2-1) - \ln x - \ln(x^2+1)$

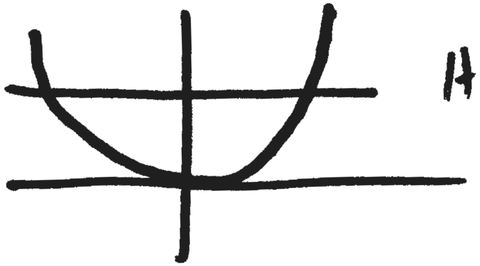
$= (\ln 2)\cdot x + \ln(x-1) + \ln(x+1) - \ln x - \ln(x^2+1)$

Stuck!

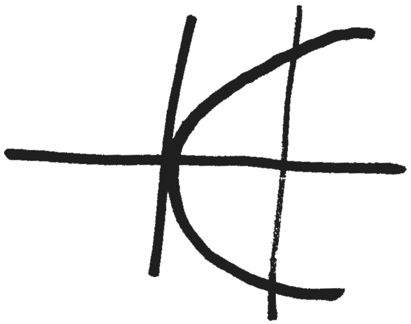
Back to $f^{-1}(x)$ Cosida: $f(x) = x^2$

$f^{-1}(x) = $ FAIL

$y = x^2$



HLT FAILS



not a func.

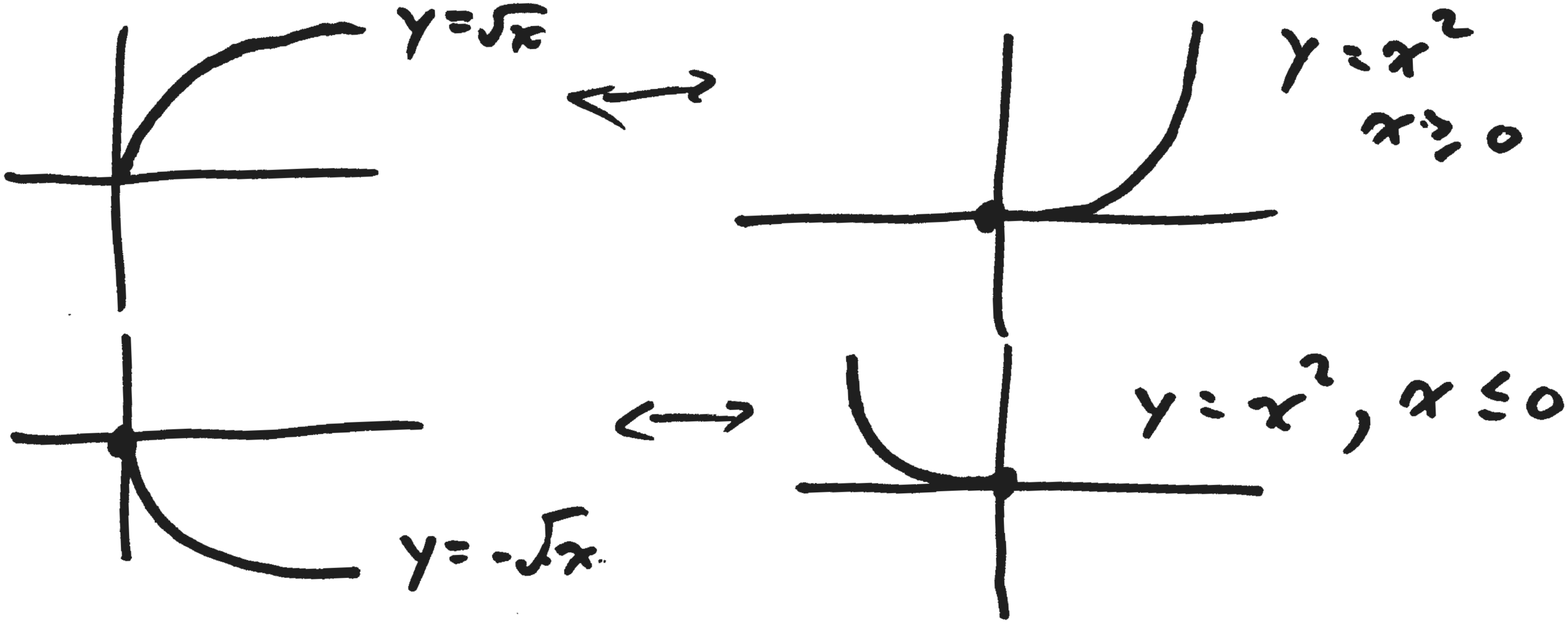
Now if $g(x) = \sqrt{x}$, what's $g^{-1}(x)$?

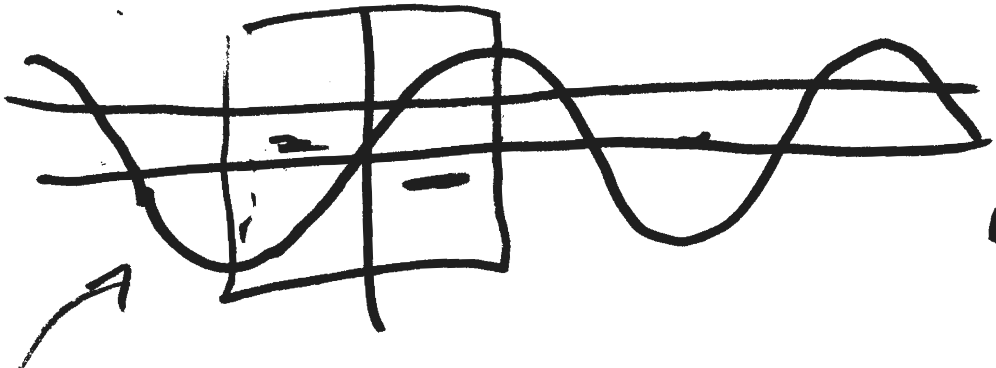
Solution $y = g(x) = \sqrt{x} \rightarrow$ now $x = \sqrt{y}$

$y = x^2$, $x \geq 0$ only

$g(x)$ inverse is $g^{-1}(x) = x^2$, $x \geq 0$ only

Inverse only on restricted domain!





$y = \sin x$

in → angle

out → ratio!

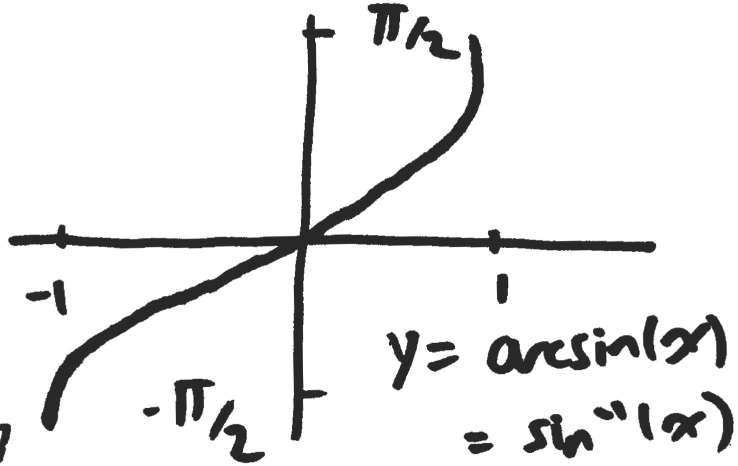
Fails HLT (badly)

but want angles! Badly! $\Rightarrow$ cheat Restrict Domain!

want to invert largest possible 1-1 interval

covers all range & preferably +ve acute angles!

$\Rightarrow$ invert $\sin(x)$ by default on $[-\frac{\pi}{2}, \frac{\pi}{2}]$



$f(x) = \sin x, \quad x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$f^{-1}(x) = \sin^{-1}(x)$

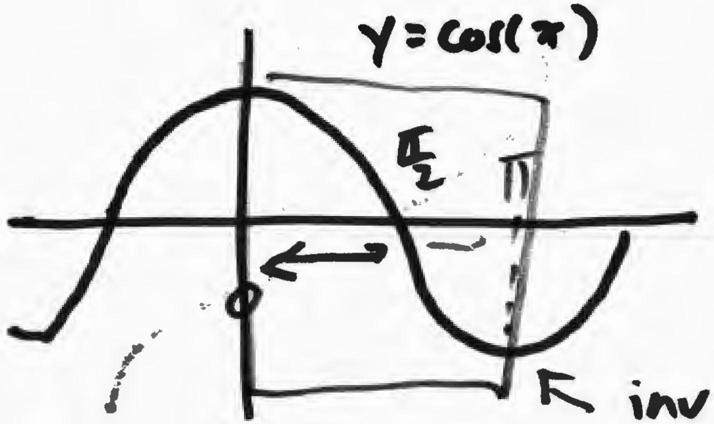
Note: $\sin^2 x = (\sin x)^2 = (\sin x)(\sin x)$

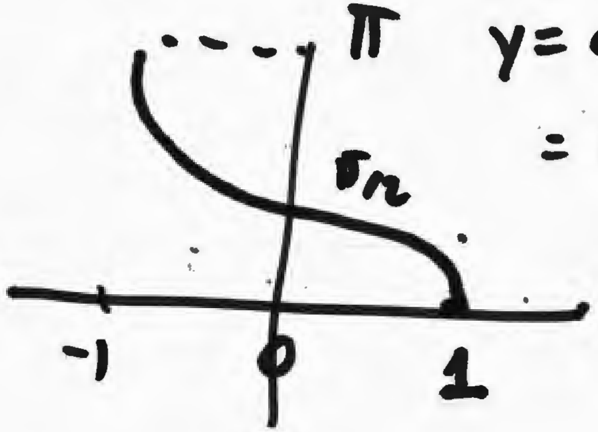
$\cdot\ \sin^5 x = (\sin x)^5 = \underbrace{(\sin x)(\sin(x)) \ldots (\sin(x))}_{5 \text{ copies!}}$

$\sin^{-1}(x) = \arcsin(x)$ but $(\sin(x))^{-1} = \frac{1}{\sin x} = \csc(x)$

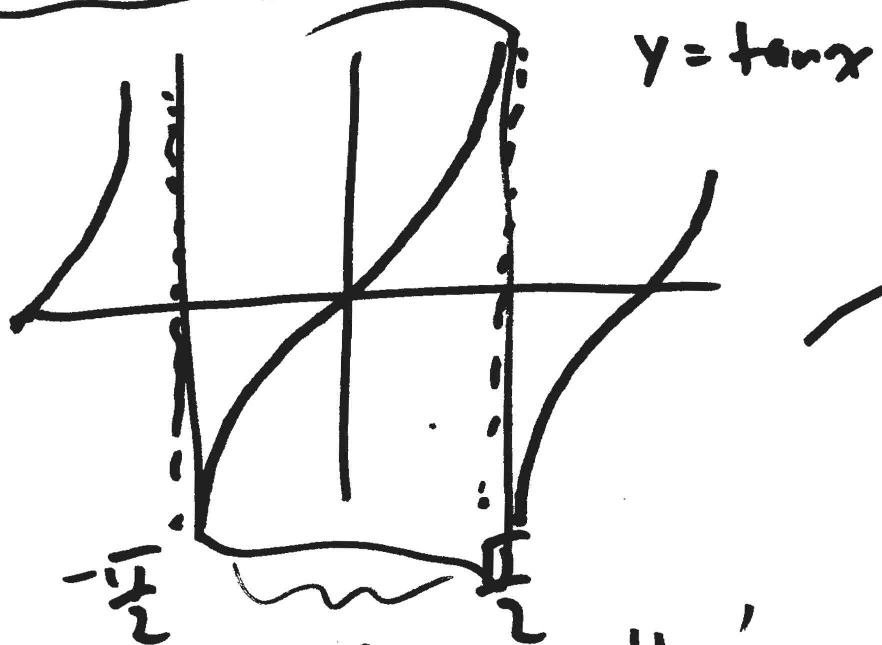
"inverse" of $\sin(x)$

Different!

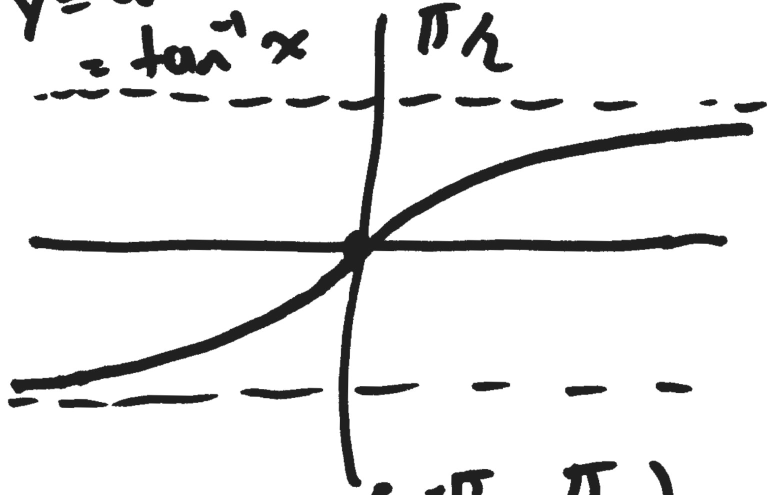




$y = \arccos(x) = \cos^{-1}(x)$



↑ invert this



range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Domain: $(-\infty, \infty)$

So notice:

$\tan^{-1}(1) = \pi/4$

$\sin^{-1}\left(\frac{1}{2}\right) = \pi/6 \qquad \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \pi/6$

$$\sin\Big(\underbrace{\sin^{-1}\left(\tfrac{\sqrt{3}}{2}\right)}_{\pi/3}\Big) = \sin(\pi/3) = \sqrt{3}/2$$

$$\sin^{-1}(\sin(3\pi/4)) = \pi/4$$

$1/\sqrt{2}$

not in $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$\sin^{-1}(x) \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$\sin(\pi/4) = \frac{1}{\sqrt{2}}$

$\pi/4 \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

Simplify $\tan(\arcsin(x))$

Solution let $\theta = \sin^{-1}(x) \rightsquigarrow \sin\theta = x$

$\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$$\tan(\arcsin(x)) = \tan\theta = ?$$

Draw:

$\sin\theta = \frac{x}{1} = \frac{\text{opp}}{\text{hyp}}$

← by pythag!

$\Rightarrow$ $\tan\theta = \text{opp}/\text{adj} = \frac{x}{\sqrt{1-x^2}} = \tan(\arcsin(x))$

Tah Dah!