New Sketch Example

Sketch $y = e^{1/x}$

Step #1 Symmetry: $f(-x) = e^{\frac{1}{(-x)}} = e^{-\frac{1}{x}} = \frac{1}{(e^{1/x})}$

$\neq f(x)$ (not even)

$\neq -f(x)$ (not odd)

} neither!

Is it periodic? ($f(x+k) = f(x)$?)

No, no try $\Rightarrow$ not usual sense of periodic.

can check rigorously, but no clearly not periodic!

Step #2 Domain (Range if easy)

Domain $f(x) = e^{1/x}$ $\frac{1}{x} \Rightarrow x \neq 0$ } $\Rightarrow e^{\frac{1}{x}}$ issue only is at $x=0$

e^x defined on all x

=> domain is all ~~x~~ $x \neq 0$

Range? $\frac{1}{x}$ has $y \neq 0$ all oth y allowed!

$e^{1/x}$ all +ve values except $e^0 = 1$

So range: all +ve y values except $y = 1$

$y \in (0,1) \cup (1, \infty)$

Step #3 **Intercepts**

x-int: $y=0$ $\quad 0 = e^{\frac{1}{x}}$ } never!

$e^{\text{stuff}} > 0$

y-int: $x=0 \Rightarrow$ not in domain! $\quad e^{\frac{1}{0}} = ??$

} No Intercepts.

Step #4: Asymptotes

HA

$$\lim_{x\to\infty} e^{\frac{1}{x}} = e^{\frac{1}{\infty}} = e^{0} = 1$$

$$\lim_{x\to-\infty} e^{\frac{1}{x}} = e^{\frac{1}{-\infty}} = e^{0} = 1$$

} $y=1$ HA at $\pm\infty$

VA Check our problem point at $x = 0$

$$\lim_{x \to 0} e^{1/x} = e^{1/0} = e^{?} = ?$$

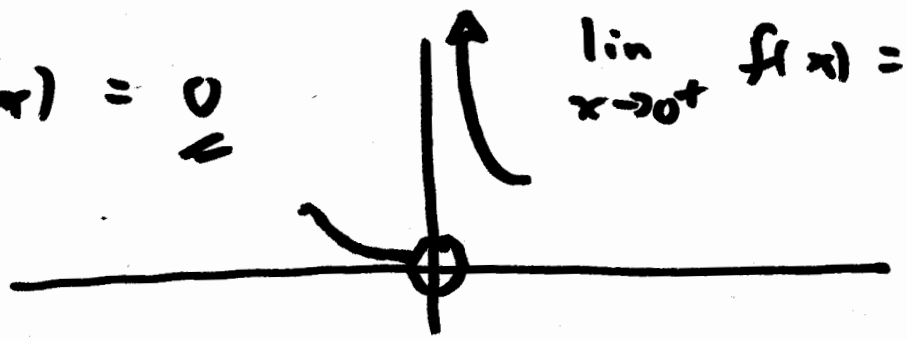
break it up!

$$\lim_{x \to 0^+} e^{1/x} = e^{\frac{1}{0^+}} = e^{+\infty} = \infty$$ } VA

$$\lim_{x \to 0^-} e^{\frac{1}{x}} = e^{\frac{1}{0^-}} = e^{-\infty} = 0$$ } not VA

$\lim_{x \to 0^-} f(x) = 0$

$\lim_{x \to 0^+} f(x) = \infty$ } VA at 0 from right only



Stop #5 $f'(x)$ & intervals of inc/dec

$f(x) = e^{1/x} \Rightarrow f'(x) = e^{1/x}\left(-\frac{1}{x^2}\right) = -\frac{e^{\frac{1}{x}}}{x^2}$

$f'(x) = 0 = -e^{1/x}/x^2 \Rightarrow e^{\frac{1}{x}} = 0$

never

$f'(x)$ DNE $\Rightarrow -e^{\frac{1}{x}}/x^2$ DNE $\Rightarrow$ $x=0$ only

not in domain! $\Rightarrow$ no C.N.

but since $x=0$ not in domain, still use $x=0$ in chart

	$(-\infty, 0)$	$(0, \infty)$
$-e^{1/x}$	$-$	$-$
x^2	$+$	$+$
$f'(x)$	$-$	$-$
$f(x)$	dec	dec

Step 6 — $f''(x)$ & Concavity

$$f'(x) = -e^{1/x}/x^2$$

$$f''(x) = \frac{(+e^{1/x}/x^2)\cdot x^2 - (2x)[-e^{1/x}]}{x^4}$$

$$= \left| \frac{e^{1/x}}{x^4} \right| [1 + 2x]$$

$\left| \frac{e^{1/x}}{x^4} \right| \geq 0$

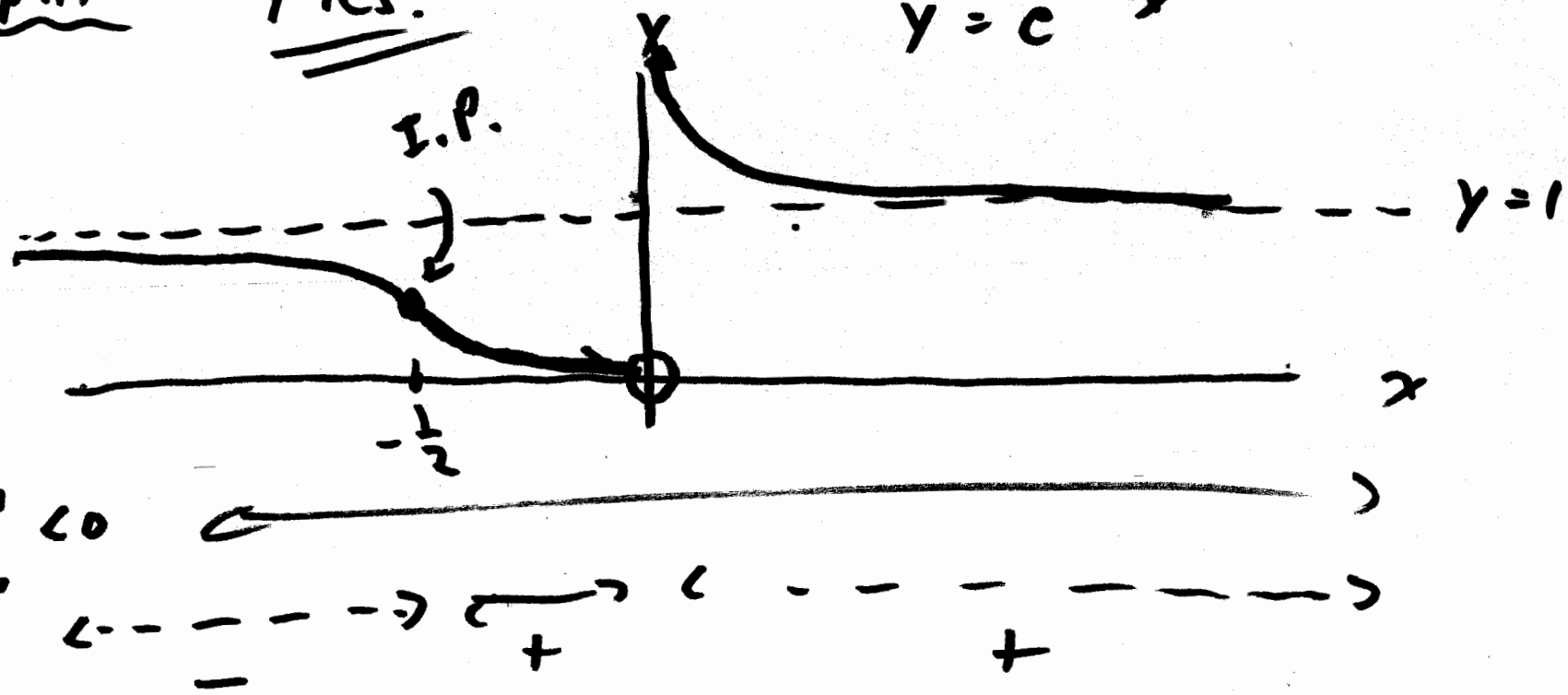
only potential I.P.

$f''(x) = 0 \Rightarrow 1 + 2x = 0 \Rightarrow \boxed{x = -\frac{1}{2}}$

$f''(x)$ DNE $\Rightarrow x = 0$ (not in domain)

	$(-\infty, -\frac{1}{2})$	$(-\frac{1}{2}, 0)$	$(0, \infty)$
$e^{1/x}/x^4$	+	+	+
$1+2x$	−	+	+
$f''(x)$	−	+	+
$f(x)$	CD $\cap$	CU $\cup$	CU $\cup$

Step #7 Pics!



The Killer App: Optimization

Step #3 Write known relations

$A = xy$ Fence Length $= 100$ m $= x + 2y$

Step #4 Reduce to 1 variable

$A = xy$ ← $x = 100 - 2y$

so $A = (100 - 2y)y = 100y - 2y^2$

Step #5 Derive

$dA/dy = 100 - 4y$

Step #6 Find abs. max

eg. A farmer has 100 m of fence. He wishes to enclose a rectangular llama pen. He wishes to use a nearby river as one side of the pen. What are the dimensions of the region if we maximize enclosed area

Solution

Step 1 P.i

Area = A

x, y dimension } Step A2 Name variables

$A' = 100 - 4y = 0 \Rightarrow$ **$y = 25$**

A' DNE never!

↑ only C.N.

but $y \in [0, 50]$

$= 100y - 2y^2 = 1250$.

$A(0) = 0$ $\quad A(25) = \# > 0$

$A(50) = 0$

clearly $y = 25 \Rightarrow$ abs. max. A

Step #7 Re-read the question to make sure you work what was asked

So **$x = 50, \ y = 25$**