MATHEMATICS 1LS3 TEST 2

Day Class
Duration of Examination: 60 minutes
McMaster University, 15 October 2012

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FIRST NAME (please print): SOLUTIONS

FAMILY NAME (please print): _____

Student No.: _____

THIS TEST HAS 8 PAGES AND 7 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE.

Total number of points is 40. Marks are indicated next to the problem number. Any non-graphing calculator is allowed.

USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

You need to show work to receive full credit.

Problem	Points	Mark
1	6	
2	6	
3	5	
4	6	
5	6	
6	5	
7	6	
TOTAL	40	

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1. **Multiple choice questions: circle ONE answer. No justification is needed.**

(a)[3] Which of the functions approach(es) ∞ faster than $x^{2.3}$ as $x \to \infty$?

(I) $f(x) = x^{2.2}$ (II) $f(x) = x^{2.4}$ (III) $f(x) = \ln x$

(A) none (B) I only (C) II only (D) III only
(E) I and II (F) I and III (G) II and III (H) all three

(b)[3] Assume that $f(x)$ is continuous at $x = 3$ and $f(3) = -2$. Which of the following statements is/are true for every function which satisfies these assumptions?

(I) $y = \sqrt{f(x)}$ is continuous at $x = 3$ → $y(3)$ is not defined
(II) $\lim_{x \to 3} f(x) = -2$ → this is definition of continuity
(III) $y = \dfrac{1}{f(x)}$ is continuous at $x = 3$ → quotient with denominator $\neq 0$

(A) none (B) I only (C) II only (D) III only
(E) I and II (F) I and III (G) II and III (H) all three

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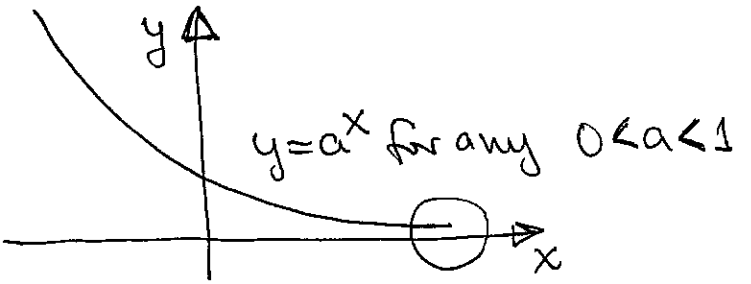
2. Identify each statement as true or false (circle your choice). No justification is needed.

(a)[2] $\lim_{x\to\infty} 0.78^x = 0.$

(TRUE) FALSE



(b)[2] The average rate of change of $f(x) = 3x - 17$ on the interval $[1.5, 1.5064]$ is 3.

(TRUE) FALSE

line of slope 3

(c)[2] If m^* is an equilibrium point of the discrete-time dynamical system $m_{t+1} = f(m_t)$, then $-m^*$ is the equilibrium point of the corresponding backwards discrete-time dynamical system.

TRUE (FALSE)

m^* is an equilibrium point of both the dyn. system and the backwards system

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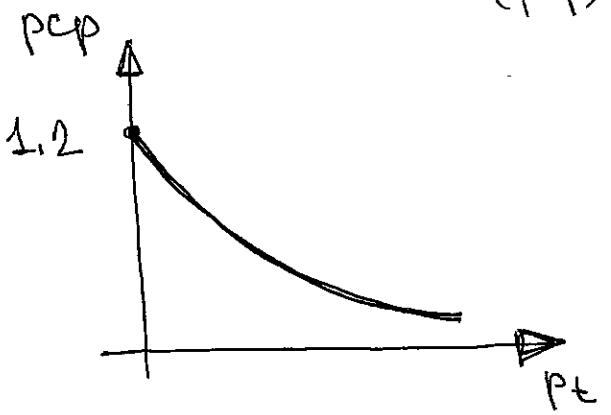
3. The dynamical system $p_{t+1} = \dfrac{12p_t}{10 + 0.002p_t}$ models the population of caribou in southern regions of Nunavut (p_t is the number of caribou and t is time in years).

(a)[2] Identify the per capita production rate and make a rough sketch of it.

per capita production (pcp) $= \dfrac{12}{10+0.002p_t}$



(b)[1] Does the dependence of the per capita production on the population size make sense? Explain why or why not.

YES. As the number of caribou increases, the pcp (ie, the number of new caribou per individual caribou) decreases → due to limited resources

(c)[2] Find all equilibrium points of the given system.

$$p^* = \frac{12p^*}{10 + 0.002p^*}$$

$$p^*\left(1 - \frac{12}{10+0.002p^*}\right) = 0$$

$p^* = 0$

$$\frac{12}{10+0.002p^*} = 1$$

$$0.002p^* = 2 \rightarrow p^* = 1000$$

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4. Consider the dynamical system $p_{t+1} = 0.5p_t(4 - p_t)$ where p_t is a population of bacteria in thousands and t is time in days.

(a)[1] For which values of p_t does the per capita production function make sense?

$0.5(4-p_t) \geq 0 \rightarrow p_t \leq 4$

population cannot be $\ominus$ $\rightarrow 0 \leq p_t \leq 4$

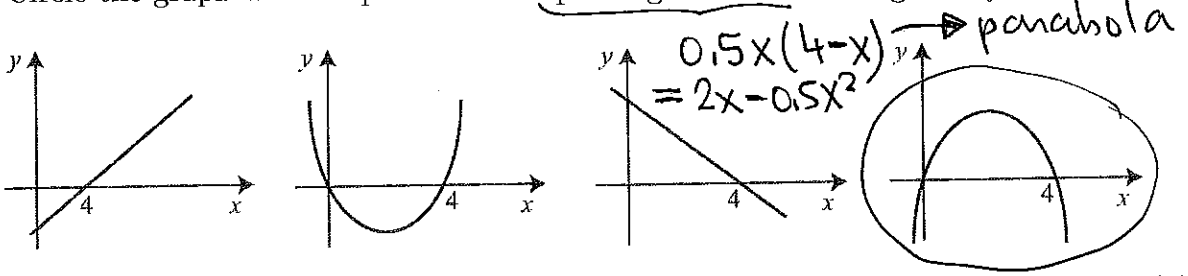
(b)[2] Find all equilibrium points of the given system.

$p^* = 0.5p^*(4-p^*)$

$p^*(1 - 0.5(4-p^*)) = 0$ $\rightarrow$ $p^* = 0$

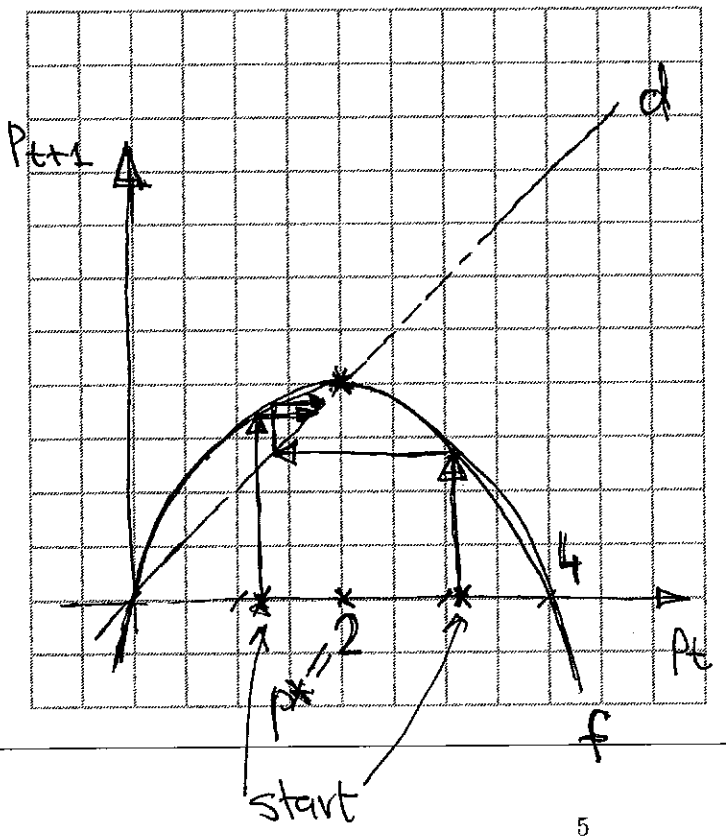
$0.5(4-p^*) = 1 \rightarrow p^* = 2$

(c)[1] Circle the graph which represents the updating function of the given system.



$0.5x(4-x) = 2x - 0.5x^2 \rightarrow$ parabola

(d)[2] Using cobwebbing, determine whether the largest equilibrium point you found in (b) is stable or unstable.



start on either side of $p^* = 2$

cobwebbing brings the values p_t closer to 2

$\rightarrow$ stable

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5. Find the following limits

(a)[2] Find $\lim_{x\to -2} \frac{x+2}{|x|-2}$ $= \lim_{x\to -2} \frac{x+2}{-x-2} = \lim_{x\to -2} \frac{x+2}{-(x+2)} = -1$

$|x| = -x$ because $x<0$

(b)[2] $\lim_{x\to 2} \frac{\frac{1}{4}-\frac{1}{x^2}}{x-2} = \frac{0}{0} = \lim_{x\to 2} \frac{\frac{x^2-4}{4x^2}}{x-2} = \lim_{x\to 2} \frac{(x-2)(x+2)}{4x^2(x-2)} = \frac{1}{4}$

(c)[2] $\lim_{x\to\infty} e^{x^2-x-4} = e^{\infty} = \infty$

because $\lim_{x\to\infty} (x^2-x-4) = \lim_{x\to\infty} x^2 = \infty$

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6. Consider the function

$$f(x) = \begin{cases} \dfrac{x-1}{x^3-x} & \text{if} \quad x \neq 1 \\ 1/2 & \text{if} \quad x = 1 \end{cases}$$

(a)[2] Is $f(x)$ continuous at $x = 0$? Explain why or why not.

near $x=0$, $f(x) = \dfrac{x-1}{x^3-x}$

$f(0)$ is not defined $\Rightarrow$ $f(x)$ is not continuous at $x=0$

(b)[3] Is $f(x)$ continuous at $x = 1$? Explain why or why not.

check to see if $\lim_{x \to 1} f(x) = f(1)$

$1/2$

$$= \lim_{x \to 1} \frac{x-1}{x^3-x} = \frac{0}{0} = \lim_{x \to 1} \frac{x-1}{x(x-1)(x+1)} = \frac{1}{2}$$

$f(x)$ is cont. at $x=1$

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7. Consider the alcohol consumption dynamical system $a_{t+1} = a_t - \frac{10.5a_t}{4.5 + a_t} + d$, where a_t is the amount of alcohol (in grams) at time t (measured in hours).

(a)[1] What is the meaning of the constant d?

amount of alcohol added (consumed) every hour

(b)[2] What is the meaning of the term $\frac{10.5a_t}{4.5 + a_t}$ in the formula for a_{t+1}? What are its units?

amount of alcohol absorbed by the body in an hour

units are grams

(c)[3] For which values of d does the given system have a meaningful equilibrium?

$a^* \geq 0$

$$a^* = a^* - \frac{10.5a^*}{4.5+a^*} + d$$

$$\frac{10.5a^*}{4.5+a^*} = d$$

$$10.5a^* = 4.5d + a^*d$$

$$a^*(10.5 - d) = 4.5d$$

$$a^* = \frac{4.5d}{10.5 - d}$$

meaningful: $d \geq 0$ and $10.5 - d > 0$ so $d < 10.5$

(a^* is positive ~~or zero~~)

THE END