12A3

Last Day **Sigma Notation**

eg $$\sum_{i=1}^{29} a_i = a_1 + a_2 + a_3 + \dots + a_{29}$$

Lazy forms

$$\sum_{i=1}^{n} a_i = \sum_{1}^{n} a_i = \underbrace{\sum}_{} a_i$$

if you know i is indexing

if "we" all know start & stop values!

often used to indicate sum over all possibilities

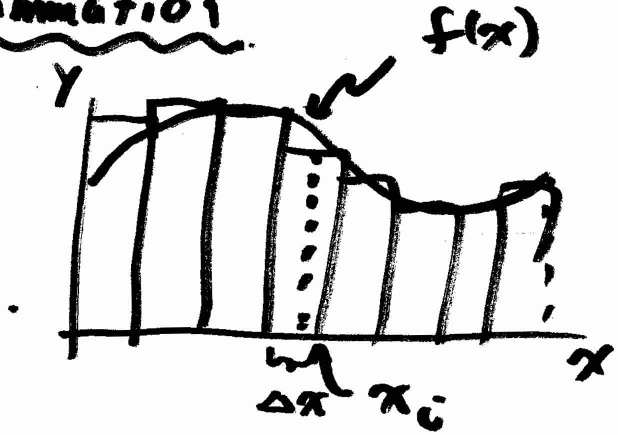
Don't forget special formulas:

$$\sum_{i=1}^{n} 1 = n, \quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\& \quad \sum_{i=1}^{n} i^3 = n^2(n+1)^2/4 = \left(\frac{n(n+1)}{2}\right)^2.$$

Know!

Areas & Summation



Area unda curve $\simeq$ Sum of blocks!

Area $\simeq \sum$ blocks $= \sum$ height $\cdot$ width

$$= \boxed{\sum_{i=1}^{n} f(x_i)\,\Delta x}$$

n blocks

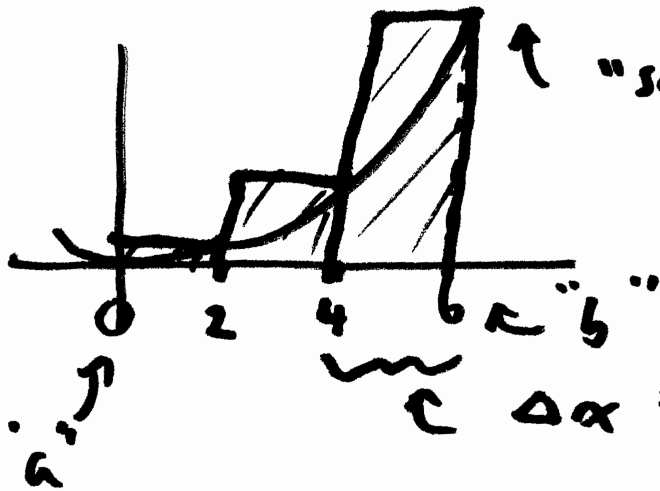
point where block height = curve height

= ith sample point

eg. $y = x^2$, $x \in [0,6]$ $\leftarrow [a,b]$, approx. area under the curve using 3 "sub-intervals" and right endpoint sample points.

(right end of each sub-int.)

<u>Solution</u>



"sample point" = right endpoint.

$$\Delta x = \frac{\text{width}}{n} = \frac{"b-a"}{n} = \frac{6-0}{3} = \underline{\underline{2}}$$

$$A \simeq \sum_{i=1}^{n} f(x_i)\,\Delta x = \sum_{i=1}^{3} (x_i)^2 \cdot 2$$

$$= 2\left(2^2 + 4^2 + 6^2\right)$$

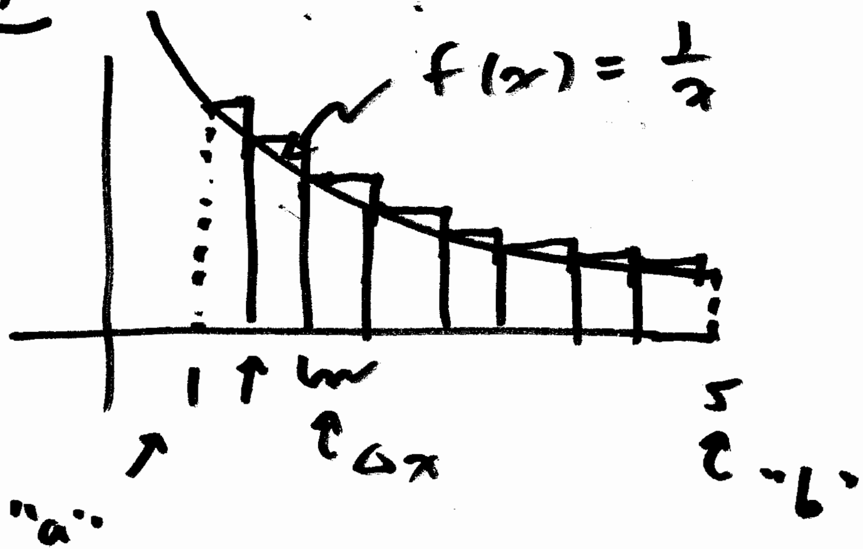
$$= 112$$

$\left.\begin{array}{l} x_1 = 2 \\ x_2 = 4 \\ x_3 = 6 \end{array}\right\}$ right ends!

In general! bigger $n \Rightarrow$ more blocks $\Rightarrow$ better approx!

eg. Approx. area under $y = \frac{1}{x}$ on $[1, 5]$ using 8 sub-intervals & <u>left</u> <u>endpoint</u> <u>sample</u> <u>points</u>

<u>Solution</u>



$$\Delta x = \frac{b-a}{n} = \frac{5-1}{8} = \frac{4}{8} = \frac{1}{2}$$

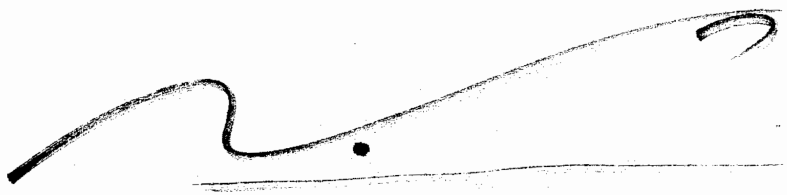
$a = x_1 = 1$

$x_2 = a + \Delta x = 1 + \frac{1}{2} = 3/2$

$x_3 = a + 2\Delta x = 1 + \frac{2}{2} = 2$

$$\text{Area} \simeq \sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{8} \underbrace{f(x_i)}_{x_i = ?} \frac{1}{2}$$

$$x_4 = a + 3\Delta x = 2.5 = 5/2$$



In general: left endpoints are $a + (i-1)\Delta x = x_i$

right endpoints are $a + i\Delta x = x_i$ ✓ Simplest!

midpoints are $a + (i - \frac{1}{2})\Delta x = x_i$

$$A = \sum_{i=1}^{8} f(x_i)\Delta x = \sum_{i=1}^{8} \frac{1}{x_i}\cdot\frac{1}{2}$$

$$x_i = a + (i-1)\Delta x = 1 + (i-1)\frac{1}{2} = \frac{1}{2} + \frac{i}{2}$$

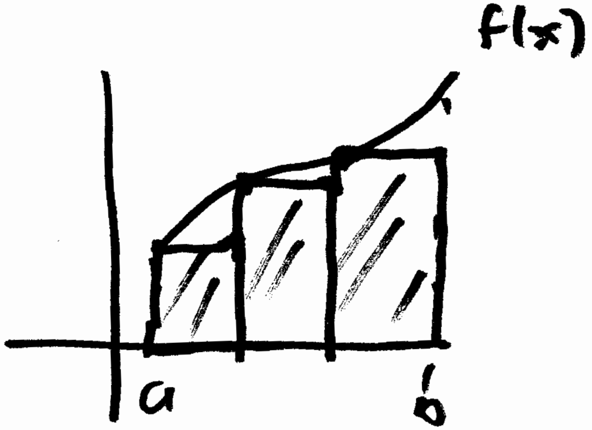
$$= \sum_{i=1}^{8} \frac{1}{\left(\frac{1}{2} + \frac{i}{2}\right)}\cdot\frac{1}{2}$$

$$= \boxed{\sum_{i=1}^{8} \frac{1}{1+i}}$$

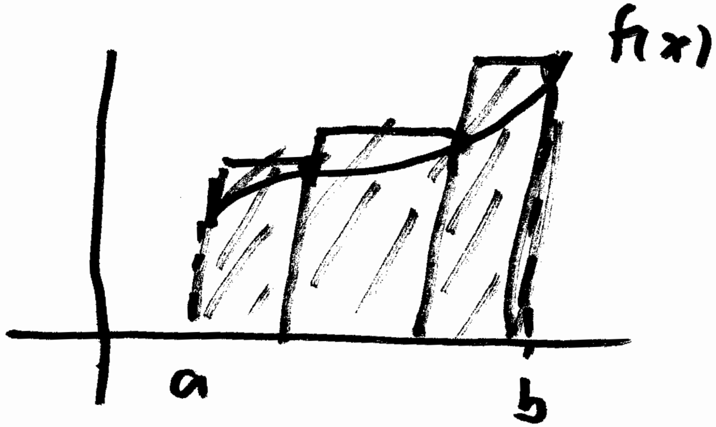
$$= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} = \#$$

Endpoint Meths : Over & Underestimates

Say $f'(x)>0$

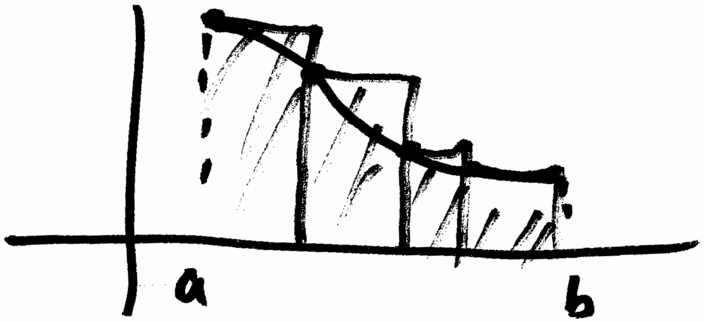


Left end. underestimate

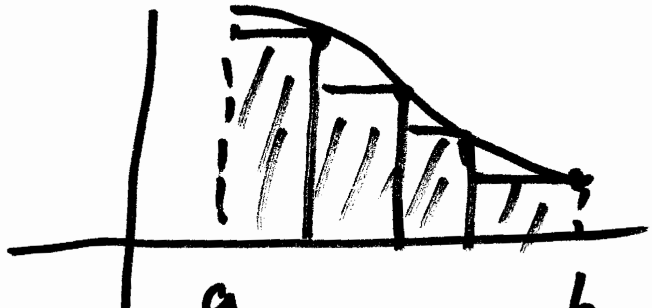


right end. overestimate

Say $f'(x)<0$



Left end. overestimate



right end. underestimate

Let's get exact area under $y = x^2$ on $[0,6]$

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \quad \Big\} \quad \Delta x = \frac{b-a}{n} = \frac{6-0}{n} = \frac{6}{n}$$

$* \; x_i = a + i\Delta x$

"easy right"

$= 0 + i(6/n) = \frac{6i}{n}$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{6i}{n}\right)\frac{6}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{6i}{n}\right)^2 \cdot \frac{6}{n}$$

$$= \lim_{n\to\infty} \frac{6^3}{n^3}\left(\sum_{i=1}^{n} i^2\right) = \lim_{n\to\infty} \frac{6^3}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

order < 3

$$= \lim_{n\to\infty} \frac{6^3}{n^3} \cdot \left(\frac{2n^3 + \text{stuff}}{6}\right)$$

$$= \lim_{n \to \infty} \frac{6^3}{6} \left(2 + \cancelto{0}{\left(\frac{\text{stuff order} \leq 2}{n^3} \right)} \right)$$

$$= \frac{1}{3} 6^3 = \underline{\underline{72}}$$

$$\downarrow$$

$$\int x^2 \, dx = \frac{1}{3} x^3 + c$$

Huh... <u>Curious</u>.