Math 1A03/1ZA3 (Version 2)
December, 2017

Name:_____
(Last Name) (First Name)

Student Number:_____

Day Class
Duration: 2.5 Hours **Instructors:** Childs, Hofscheier, Lin, McLean, Valeriote
Maximum Mark: 32

McMaster University Final Examination
This examination paper consists of 8 pages (including this one). This exam consists of 32 multiple choice questions worth 1 mark each (no part marks). The questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Use of Casio FX-991 MS or MS Plus calculator only is allowed.

Computer Card Instructions:

NOTE: IT IS YOUR RESPONSIBILITY TO ENSURE THAT THE ANSWER SHEET IS PROPERLY COMPLETED: YOUR EXAMINATION RESULT DEPENDS UPON PROPER ATTENTION TO THESE INSTRUCTIONS

The scanner, which reads the sheets, senses the shaded areas by their non-reflection of light. A heavy mark must be made, completely filling the circular bubble, with an HB pencil. Marks made with a pen or felt-tip marker will **NOT** be sensed. Erasures must be thorough or the scanner may still sense a mark. Do **NOT** use correction fluid on the sheets. Do **NOT** put any unnecessary marks or writing on the sheet.

1. Print your name, student number, course name, and the date in the space provided at the top of Side 1 (red side) of the form. Then the sheet **MUST** be signed in the space marked SIGNATURE.

2. Mark your student number in the space provided on the sheet on Side 1 and fill in the corresponding bubbles underneath.
3. Mark only **ONE** choice from the alternatives (A,B,C,D,E) provided for each question. If there is a True/False question, enter response of 1 (or A) as True, and 2 (or B) as False. The question number is to the left of the bubbles. Make sure that the number of the question on the scan sheet is the same as the question number on the test paper.
4. Pay particular attention to the Marking Directions on the form.
5. Begin answering questions using the first set of bubbles, marked "1".

1. Evaluate the following integral,

$$\int \frac{1}{x^2\sqrt{4-x^2}}\,dx$$

(a) $-\frac{x}{\sqrt{4-x^2}} + \frac{1}{4}\sin^{-1}\left(\frac{x}{2}\right) + C$ **(b)** $-\frac{\sqrt{4-x^2}}{4x} + C$ **(c)** $-\sqrt{4-x^2} + \sin^{-1}\left(\frac{x}{2}\right) + C$
(d) $-\frac{\sqrt{4-x^2}}{x} + \frac{1}{2}\sin^{-1}\left(\frac{x}{2}\right) + C$ **(e)** $-\frac{x}{\sqrt{4-x^2}} + C$

2. Find the value of B if

$$\frac{2x^2 - x + 4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 4}$$

(a) $-\frac{1}{2}$ **(b)** $\frac{1}{2}$ **(c)** 1 **(d)** -1 **(e)** 2

3. Evaluate the following integral,

$$\int \frac{1 - \tan^2 x}{\sec^2 x}\,dx$$

(a) $-\frac{1}{3\sin x}\cos^3 x - \frac{1}{3\cos x}\sin^3 x + C$ **(b)** $\frac{1}{3}\cos^3 x - \cos x + C$
(c) $\frac{1}{3}\cos^3 x - \frac{1}{3}\sin^3 x + C$ **(d)** $\frac{1}{3}\sin^3 x - \sin x + C$
(e) $\frac{1}{2}\sin 2x + C$

4. Perform the following division,

$$\frac{x^3 - 3x^2 + 1}{x - 2}.$$

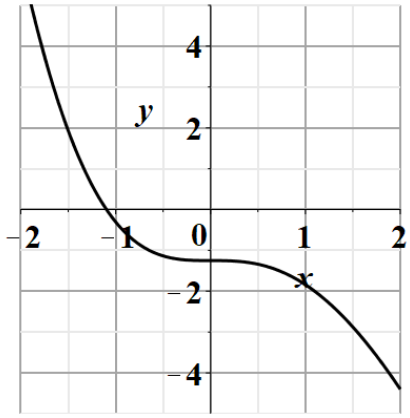
(a) $x^2 - 2x + 1 - \frac{1}{x-2}$ **(b)** $x - 2 - \frac{1}{x^2-2x+1}$ **(c)** $x^2 - 3x + \frac{1}{x-2}$
(d) $x - 2 - \frac{3}{x^2-x-2}$ **(e)** $x^2 - x - 2 - \frac{3}{x-2}$

5. Find the length of the following curve.

$$y = \sqrt{x - x^2} + \sin^{-1}\sqrt{x}, \;\; 0 \le x \le 1$$

(a) 2 **(b)** 1 **(c)** 3 **(d)** $\frac{1}{2}$ **(e)** $\frac{5}{2}$

6. Let f be the function whose graph is shown to the right. Find $f^{-1}(2)$.



(a) 2 **(b)** $-\frac{3}{2}$ **(c)** $-\frac{17}{4}$ **(d)** $-\frac{5}{4}$ **(e)** $\frac{5}{4}$

7. Evaluate the following integral,

$$\int \frac{2x-1}{x^2+9}\,dx$$

(a) $\frac{1}{2}\ln(x^2+9) - \frac{1}{9}\tan^{-1}\left(\frac{x}{9}\right) + C$ **(b)** $\ln(x^2+9) - \frac{1}{2x}\ln(x^2+9) + C$
(c) $-\frac{1}{(x^2+9)^2} - \frac{1}{9}\tan^{-1}\left(\frac{x}{3}\right) + C$ **(d)** $\ln(x^2+9) - \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$
(e) $-\frac{1}{(x^2+9)^2} - \frac{1}{3}\tan^{-1}\left(\frac{x}{9}\right) + C$

8. Evaluate the following integral,

$$\int \tan^3 x \sec^3 x\,dx$$

(a) $\frac{1}{4}\sec^4 x - \frac{1}{2}\sec^2 x + C$ **(b)** $\frac{1}{4}\tan^4 x \sec^3 x + \frac{1}{4}\tan^3 x \sec^4 x + C$
(c) $\frac{1}{4}\tan^4 x \sec^3 x - \frac{1}{4}\tan^3 x \sec^4 x + C$ **(d)** $\frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C$
(e) $\frac{1}{5}\tan^5 x + \frac{1}{3}\tan^3 x + C$

9. The following limit represents the derivative of some function f at some number a. Find such an f and a.

$$\lim_{x\to 2} \frac{\ln(x-1)}{x-2}$$

(a) $f(x) = \ln(x-3)$, $a = 4$ **(b)** $f(x) = \ln(x-2)$, $a = 1$
(c) $f(x) = \ln(x-1)$, $a = 2$ **(d)** $f(x) = \ln x$, $a = 2$
(e) none of the above

10. Let $f(x) = x + e^{x-3}$. Find $(f^{-1})'(4)$.
(a) $\frac{1}{2}$ **(b)** $\frac{1}{3}$ **(c)** $\frac{1}{4}$ **(d)** 1 **(e)** -1

11. Let $f(x) = \frac{x}{1+\ln(x-1)}$. Find the largest interval(s) on which f is continuous.

(a) $(1, e+1), (e+1, \infty)$ **(b)** $(1, e^{-1}+1), (e^{-1}+1, \infty)$ **(c)** $(1, \infty)$ **(d)** $(1, 2), (2, \infty)$
(e) $(1, 2), (2, e^{-1}+1), (e^{-1}+1, \infty)$

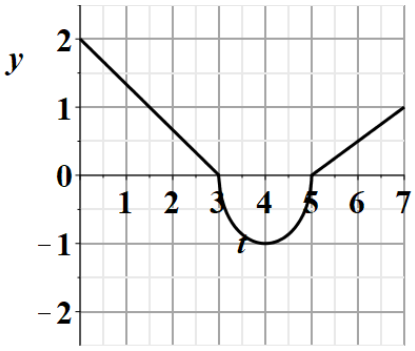
12. If $g(x) = xf(x)$, where $f(3) = 4$ and $f'(3) = -2$, find an equation of the tangent line to the graph of g at the point where $x = 3$.

(a) $y = 6x - 6$ **(b)** $y = -2x + 6$ **(c)** $y = -2x + 12$ **(d)** $y = -2x + 18$
(e) $y = -6x + 12$

13. Find all critical numbers of the function $f(x) = \frac{\sqrt{x}}{1+x^2}$.

(a) $0, \frac{1}{\sqrt{2}}$ **(b)** 0 **(c)** $0, 1$ **(d)** $0, \frac{1}{\sqrt{3}}$ **(e)** $0, \sqrt{2}$

14. Let f be the function whose graph is shown to the right and let $g(x) = \int_0^x f(t)dt$. Find the largest interval(s) on which g is concave up.



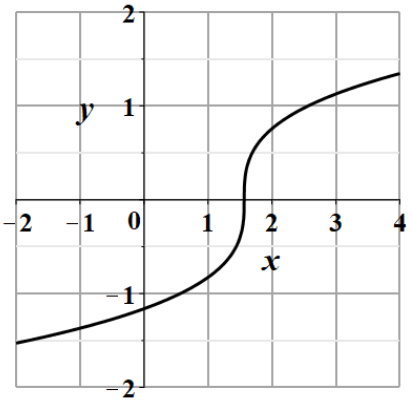
(a) $(3, 5)$ **(b)** $(4, 5), (5, 7)$ **(c)** $(2, 7)$
(d) $(0, 3), (5, 7)$ **(e)** $(0, 3), (3, 4)$

15. Find an expression for the area under the graph of f as a limit.

$$f(x) = \ln x, \quad 1 \le x \le 3$$

(a) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n} \ln\left(1 + \frac{3i}{n}\right)$ **(b)** $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n} \ln\left(\frac{2i}{n}\right)$ **(c)** $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n} \ln\left(1 + \frac{2i}{n}\right)$
(d) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n} \ln\left(\frac{3i}{n}\right)$ **(e)** $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n} \ln\left(1 + \frac{2i}{n}\right)$

16. Let f be the function whose graph is shown to the right. Suppose that Newton's method were used to find a root of the equation $f(x) = 0$ with the following initial guesses.



(i) $x_1 = 0$ (ii) $x_1 = 1$ (iii) $x_1 = 2$

For which if these initial guesses would Newton's method *fail* to converge?

(a) (i) only **(b)** (ii) only **(c)** (iii) only
(d) (i) and (iii) only **(e)** all of them

17. Evaluate the following limit.

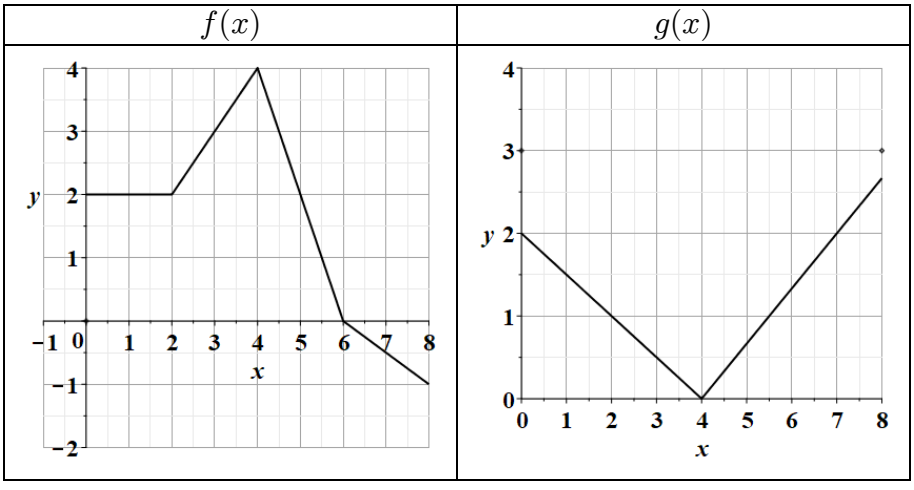
$$\lim_{x \to 0} \frac{\tan 4x}{x + \sin 2x}$$

(a) 1 **(b)** $\frac{1}{3}$ **(c)** $\frac{4}{3}$ **(d)** 0 **(e)** $\frac{1}{2}$

18. Let $f(x) = x^2 + 3x - 4$. Find the value of c that satisfies the conclusion of the mean value theorem on the interval $[1, 3]$.

(a) 2 **(b)** $\frac{3}{2}$ **(c)** $\frac{5}{2}$ **(d)** $\frac{3}{4}$ **(e)** $\frac{4}{3}$

19. Let f and g be the functions whose graphs are shown below, and let $h(x) = g(f(x))$. Find $h'(3)$.



(a) -2 **(b)** -1 **(c)** $\frac{1}{2}$ **(d)** 1 **(e)** $-\frac{1}{2}$

20. Find the value of x if $x > 0$ and $\cosh x = 2$.
(a) $\ln(3-\sqrt{2})$ **(b)** $\ln(3+\sqrt{2})$ **(c)** $\ln(2+\sqrt{3})$ **(d)** $e^{3+\sqrt{2}}$ **(e)** $e^{3-\sqrt{2}}$

21. Suppose that the *second derivative* of a function f is given by

$$f''(x) = x^2(7-x)^2(x-8).$$

How many inflection points does f have?
(a) 2 **(b)** 1 **(c)** 3 **(d)** 4 **(e)** 5

22. Suppose that $\int_1^3 f(x)dx = 4$ and $\int_1^6 f(x)dx = 2$. Evaluate $\int_3^6 [3f(x)+4]\,dx$

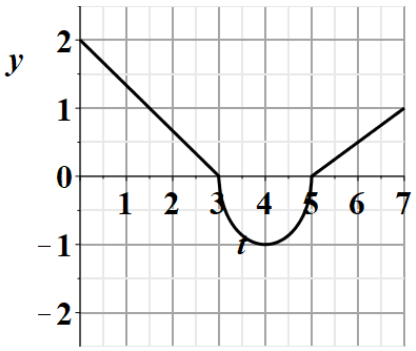
(a) 4 **(b)** 3 **(c)** 1 **(d)** 5 **(e)** 6

23. A farmer with r meters of fencing wants to enclose a rectangular region whose area is as large as possible. Find the dimensions of the rectangle.

(a) $x = \frac{r}{5}$, $y = \frac{3r}{10}$ **(b)** $x = \frac{r}{3}$, $y = \frac{r}{3}$ **(c)** $x = \frac{r}{2}$, $y = \frac{r}{2}$
(d) $x = \frac{r}{4}$, $y = \frac{r}{4}$ **(e)** $x = \frac{r}{6}$, $y = \frac{r}{3}$

24. Let f be the function whose graph is shown to the right and let $g(x) = \int_0^{2x} f(t)dt$. Find $g'(2)$.

(a) -2 **(b)** -1 **(c)** 4 **(d)** $\frac{1}{2}$ **(e)** 1



25. In Maple, what command could be used to evaluate the following integral?

$$\int_0^1 (2x+3)\,dx$$

(a) `>integral(2*x+3,x=0..1)`
(b) `>integrate(2*x+3,x=0..1)`
(c) `>antiderivative(2*x+3,x=0..1)`
(d) `>int(2*x+3,x=0..1)`
(e) `>antiderivative(2x+3,x=0..1)`

26. Find the volume of the solid obtained by rotating the region bounded by $x = y^2$, $x = 0$, $y = 2$, about the y-axis.

(a) 8π **(b)** $\frac{32\pi}{5}$ **(c)** $\frac{34\pi}{5}$ **(d)** $\frac{36\pi}{5}$ **(e)** $\frac{38\pi}{5}$

27. Evaluate the following integral,

$$\int x^2 \ln x \, dx$$

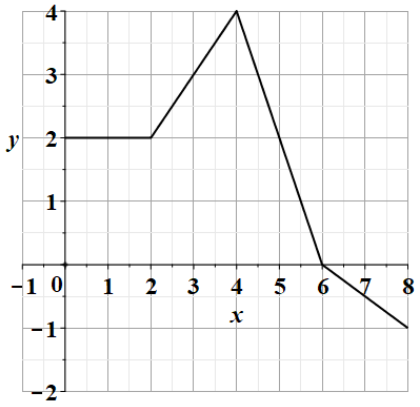
(a) $\frac{1}{3}x^3(9\ln x - 1) + C$ **(b)** $\frac{1}{9}x^3(3\ln x - 1) + C$ **(c)** $\frac{1}{3}x^3(2\ln x - 1) + C$
(d) $\frac{1}{3}x^3(2\ln x + 1) + C$ **(e)** $\frac{1}{9}x^3(2\ln x - 1) + C$

28. A chain lying on the ground is 10 ft long and weighs 120 lbs. How much work is required to raise one end of the chain to a height of 8ft?

(a) 382 ft-lb **(b)** 386 ft-lb **(c)** 388 ft-lb **(d)** 384 ft-lb **(e)** 380 ft-lb

29. Let f be the function whose graph is shown to the right. Find the average value of f on the interval $[4, 8]$.

(a) $\frac{3}{4}$ **(b)** $\frac{5}{4}$ **(c)** $\frac{7}{4}$ **(d)** 2 **(e)** 1



30. Which of the following integrals represent the volume of the solid obtained by rotating the region bounded by $y = x^3$, $x = 1$, $x = 2$, and the x-axis, about the line $y = -3$?

(a) $\int_1^2 \pi[(x^3 - 3)^2 - 9]\, dx$ **(b)** $\int_1^2 \pi[9 - (3 - x^3)^2]\, dx$ **(c)** $\int_1^2 \pi[(3 + x^3)^2 - 9]\, dx$
(d) $\int_1^2 \pi[x^6 - (3 - x^3)^2]\, dx$ **(e)** $\int_1^2 \pi[x^6 - (3 + x^3)^2]\, dx$

31. Which of the following is equal to

$$\int_0^1 x^n e^x \, dx \, ?$$

(a) $x^n e^x - n\int_0^1 x^{n-1} e^x \, dx$ **(b)** $\frac{e}{n+1} - \int_0^1 x^{n+1} e^x \, dx$ **(c)** $e - n\int_0^1 x^{n-1} e^x \, dx$
(d) $\frac{1}{n+1} x^{n+1} e^x - \frac{1}{n+1}\int_0^1 x^{n+1} e^x \, dx$ **(e)** $e(n-1) - n\int_0^1 x^{n-1} e^x \, dx$

32. By making an appropriate substitution, which of the following is equal to

$$\int \frac{\sqrt{x^2-1}}{x} \, dx \, ?$$

(a) $\int \tan^2\theta \, d\theta$ **(b)** $\int \frac{\sin\theta}{\cos^2\theta} \, d\theta$ **(c)** $\int \frac{\sin\theta}{\cos^4\theta} \, d\theta$ **(d)** $\int \frac{\sin^2\theta}{\cos^3\theta} \, d\theta$ **(e)** $\int \frac{\sin^3\theta}{\cos^2\theta} \, d\theta$

Don't forget to fill in the bubbles corresponding to both your student number <u>and</u> the version number on the scantron! You are writing VERSION 2.

(THE END)

Answers (Version 2)**:**

1. b **2.** c **3.** e **4.** e **5.** a **6.** b **7.** d **8.** d **9.** c **10.** a
11. b **12.** d **13.** d **14.** b **15.** c **16.** e **17.** c **18.** a **19.** e **20.** c
21. b **22.** e **23.** d **24.** a **25.** d **26.** b **27.** b **28.** d **29.** a **30.** c
31. c **32.** a

(THE END)