

**Limit Rules:** Assume that  $\{a_n\}, \{b_n\}$  are convergent.

1.  $\lim_{n \rightarrow \infty} ca_n =$

2.  $\lim_{n \rightarrow \infty} a_n + b_n =$

3.  $\lim_{n \rightarrow \infty} a_n - b_n =$

4.  $\lim_{n \rightarrow \infty} a_n b_n =$

5.  $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} =$

6. If  $f$  is continuous, then  $\lim_{n \rightarrow \infty} f(a_n) =$

**Examples:** Calculate

•  $\lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n}}{3}$

•  $\lim_{n \rightarrow \infty} \frac{n + 2n^2}{n^2 + 4}$

•  $\lim_{n \rightarrow \infty} f\left(\frac{1}{n}\right)$ , where  $f(x) = x^3$ .

•  $\lim_{n \rightarrow \infty} e^{\frac{1}{n}}$

**Squeeze Theorem:** If  $a_n \leq b_n \leq c_n$  for  $n \geq N$  and  $\lim_{n \rightarrow \infty} a_n = L$  and

$\lim_{n \rightarrow \infty} c_n = L$ , then \_\_\_\_\_.

Graphically:

**Example:** Find  $\lim_{n \rightarrow \infty} \frac{1}{n} \cos(n)$ .

**Example:** Find  $\lim_{x \rightarrow 0} x^2 \sin\left(\frac{\pi}{x}\right)$ .

We say that  $\{a_n\}$  is **monotone increasing** if \_\_\_\_\_.

We say that  $\{a_n\}$  is **monotone decreasing** if \_\_\_\_\_.

We say that  $\{a_n\}$  is **bounded from above** if \_\_\_\_\_.

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**Monotone convergence result:**

If  $\{a_n\}$  is **monotone increasing** and **bounded from above**,

then  $\{a_n\}$  is \_\_\_\_\_.

If  $\{a_n\}$  is **monotone decreasing** and **bounded from below**,

then  $\{a_n\}$  is \_\_\_\_\_.

**Example:** Decide if  $\left\{\frac{n}{n+1}\right\}$  converges.

**Example:** Let  $a_1 = 2$  and define  $a_n = \frac{a_{n-1}+6}{2}$  for all  $n > 1$ .

Decide if  $\{a_n\}$  converges.