Math 1B03 Term 2/1ZC3

1st Sample Test #2

Name:____

(Last Name) (First Name)

Student Number:______________________ **Tutorial Number:**_____________________

This test consists of 20 multiple choice questions worth 1 mark each (no part marks), and 1 question worth 1 mark (no part marks) on proper computer card filling. All questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Calculators are NOT allowed.

1. Solve the following system,

$$\begin{aligned} y_1' &= 8y_1 + 10y_2 \\ y_2' &= -5y_1 - 7y_2 \end{aligned}$$

(a) $y_1 = c_1e^{-2x} - 2c_2e^{3x}$, $y_2 = -c_1e^{-2x} + c_2e^{3x}$
(b) $y_1 = c_1e^{-3x} - 2c_2e^{2x}$, $y_2 = -c_1e^{-3x} + c_2e^{2x}$
(c) $y_1 = c_1e^{-x} - 2c_2e^{4x}$, $y_2 = -c_1e^{-x} + c_2e^{4x}$
(d) $y_1 = c_1e^{-4x} - 2c_2e^{x}$, $y_2 = -c_1e^{-4x} + c_2e^{x}$
(e) $y_1 = c_1e^{2x} - 2c_2e^{-x}$, $y_2 = -c_1e^{2x} + c_2e^{-x}$

2. Write the following differential equation as an equivalent system of linear differential equations.

$$y''' - 6y'' + 4y' - 3y = 0$$

(a) $y_1' = 2y_2 - y_3$, $y_2' = y_1 + 4y_3$, $y_3' = 3y_1 - 4y_2$
(b) $y_1' = 4y_1 + 6y_2 + y_3$, $y_2' = y_3$, $y_3' = y_1$
(c) $y_1' = y_2$, $y_2' = y_3$, $y_3' = 3y_1 - 4y_2 + 6y_3$
(d) $y_1' = y_2$, $y_2' = -3y_1 + 6y_2 - 4y_3$, $y_3' = y_1$
(e) $y_1' = y_2$, $y_2' = y_3$, $y_3' = 6y_1 - 4y_2 + 3y_3$

3. Find the reduced row echelon form of the following matrix.

$$\begin{bmatrix} -1 & -i & 1 \\ -i & 1 & i \\ 1 & i & -1 \end{bmatrix}$$

(a) $\begin{bmatrix} 1 & i & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & i \\ 0 & 0 & 0 \end{bmatrix}$ **(c)** $\begin{bmatrix} 1 & i & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ **(d)** $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
(e) $\begin{bmatrix} 1 & 0 & -i \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

4. Find $(\frac{1}{2} + \frac{\sqrt{3}}{2}i)^{60}$.
(a) 1 **(b)** -1 **(c)** $\frac{\sqrt{3}}{2} + \frac{1}{2}i$ **(d)** $\frac{\sqrt{3}}{2} - \frac{1}{2}i$ **(e)** $-\frac{\sqrt{3}}{2} + \frac{1}{2}i$

5. Recall that $\text{Re}(z)$ and $\text{Im}(z)$ denote, respectively, the real and imaginary parts of the complex number z. Consider the following statements.
(i) $\text{Re}(iz) = \text{Im}(z)$
(ii) $z - \overline{z} = 2i\,\text{Re}(z)$
Which of the above statements is always true?
(a) (i) only **(b)** (ii) only **(c)** (i) and (ii) **(d)** neither

6. Find the equation of the plane passing through $A(2,1,3)$, $B(3,-1,5)$, and $C(1,2,-3)$.
(a) $4x - 2z - 2 = 0$ **(b)** $10x + 4y - z - 21 = 0$
(c) $6x - 3z - 3 = 0$ **(d)** $8x + y - 3z - 8 = 0$
(e) $6x - 2y - 5z + 5 = 0$

7. Find the parametric equations of the line passing through the points $P(3,-1,4)$ and $Q(3,-1,5)$.

(a) $x = 3t$	**(b)** $x = 3$	**(c)** $x = 3 + t$	**(d)** $x = t$	**(e)** $x = 0$
$y = -t$	$y = -1$	$y = -1 + t$	$y = t$	$y = 0$
$z = 1{+}4t$	$z = 4 + t$	$z = 4 + t$	$z = 1 + 4t$	$z = t$

8. Find the volume of the parallelepiped determined by **w**, **u**, and **v** when:
$\mathbf{w} = (2,1,1)$, $\mathbf{v} = (1,0,2)$, and $\mathbf{u} = (2,1,-1)$.
(a) 1 **(b)** 2 **(c)** 3 **(d)** 4 **(e)** 5

9. Find the shortest distance between the following pairs of parallel lines.
$(x, y, z) = (2, -1, 3) + t(1, -1, 4)$
$(x, y, z) = (1, 0, 1) + t(1, -1, 4)$
(a) $\frac{5}{9}$ **(b)** $\frac{2}{9}$ **(c)** 1 **(d)** $\frac{2}{3}$ **(e)** $\frac{1}{\sqrt{2}}$

10. A parallelogram has sides $AB, BC, CD,$ and DA. Given $A(1, -1, 2), C(2, 1, 0)$, and the midpoint $M(2, 0, -3)$ of AB, find $\overrightarrow{BD}$.
(a) $(3, 1, -8)$ **(b)** $(-1, 0, 8)$ **(c)** $(2, 2, -10)$ **(d)** $(-3, -2, 18)$ **(e)** $(1, -1, 2)$

11. Consider the following statements regarding vectors in R^3.
(i) If $\|\mathbf{u}\| = \|\mathbf{v}\|$ then $\mathbf{u} + \mathbf{v}$ is orthogonal to $\mathbf{u} - \mathbf{v}$
(ii) If $\mathbf{v}$ is orthogonal to $\mathbf{w}_1$ and $\mathbf{w}_2$, then $\mathbf{v}$ is orthogonal to $\mathbf{u} = k_1\mathbf{w}_1 + k_2\mathbf{w}_2$ for all scalars k_1 and k_2.
(iii) $\|\mathbf{u} + \mathbf{v}\| = \|\mathbf{u} - \mathbf{v}\|$

Which of the above statements are always true?
(a) (i) only
(b) (ii) only
(c) (i) and (ii) only
(d) (iii) only
(e) (ii) and (iii) only

12. Assume $\mathbf{u}$ and $\mathbf{v}$ are nonzero vectors that are not parallel. Let $\mathbf{w} = \|\mathbf{u}\|\mathbf{v} + \|\mathbf{v}\|\mathbf{u}$. Find a simplified expression for the cosine of the angle between $\mathbf{u}$ and $\mathbf{w}$.
(a) $\frac{\|\mathbf{u}\|[(\mathbf{u}\cdot\mathbf{v})+\|\mathbf{v}\|]}{\|\mathbf{w}\|}$ **(b)** $\frac{(\mathbf{u}\cdot\mathbf{v})+\|\mathbf{v}\|\|\mathbf{u}\|^2}{\|\mathbf{w}\|}$ **(c)** $\frac{2(\mathbf{u}\cdot\mathbf{v})}{\|\mathbf{w}\|}$ **(d)** $\frac{(\mathbf{u}\cdot\mathbf{v})+\|\mathbf{v}\|\|\mathbf{u}\|}{\|\mathbf{w}\|}$ **(e)** $\frac{\|\mathbf{v}\|\|\mathbf{u}\|}{\|\mathbf{w}\|}$

13. Let V be the set of all ordered pairs of real numbers $\mathbf{u} = (u_1, u_2)$ with $u_1 > 0$, with the usual scalar multiplication, and consider the following operation on $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_1, v_2)$.

$$\mathbf{u} + \mathbf{v} = (u_1 v_1, u_2 + v_2)$$

If this set satisfies Axiom 4 of a vector space (the existence of a zero vector), what would be the zero vector?
(a) $(1, 0)$ **(b)** $(0, 0)$ **(c)** $(0, 1)$ **(d)** $(1, 1)$ **(e)** $(0, -1)$

14. Let V be the set of all ordered pairs of real numbers $\mathbf{u} = (u_1, u_2)$ with $u_1 > 0$, with the usual scalar multiplication, and consider the following operation on $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_1, v_2)$.

$$\mathbf{u} + \mathbf{v} = (u_1 v_1, u_2 + v_2)$$

If this set satisfies Axiom 5 of a vector space (the existence of the negative of a vector), what would be the value of $-\mathbf{u}$?
(a) $(\frac{1}{u_2}, -u_1)$ **(b)** $(-u_1, -u_2)$ **(c)** $(u_1, -u_2)$ **(d)** $(-u_1, u_2)$ **(e)** $(\frac{1}{u_1}, -u_2)$

For Questions 15-17, determine which of the following answers is correct for the given subset W of $\mathbb{R}^3$.
(a) W is a subspace
(b) W is closed under addition, but not closed under scalar multiplication
(c) W is closed under scalar multiplication, but not closed under addition
(d) W is not closed under scalar multiplication, and not closed under addition

15. $W =$ all vectors of the form $(a, b, 1)$
(a) **(b)** **(c)** **(d)**

16. $W =$ all vectors of the form (a, b, c) where $a - 2c = 0$
(a) **(b)** **(c)** **(d)**

17. $W =$ all vectors of the form (a, b, c) where $c \geq 0$
(a) **(b)** **(c)** **(d)**

18. Let $\mathbf{v}_1 = (-1, 1, 0)$, and $\mathbf{v}_2 = (0, 1, 1)$. Which of the following vectors are in span$\{\mathbf{v}_1, \mathbf{v}_2\}$?

(i) $(3, -1, 2)$
(ii) $(3, 1, -2)$
(iii) $(5, 1, 6)$

(a) (i) and (iii) only **(b)** all of them **(c)** (i) only **(d)** (iii) only **(e)** (ii) and (iii) only

19. Which of the following sets of vectors span R^3?

(i) $\{(1, 1, 0), (0, 1, 1)\}$
(ii) $\{(1, 1, 1), (1, 0, 2), (-1, -4, 2)\}$
(iii) $\{(1, 0, 1), (0, 1, 2), (-1, -4, 2)\}$

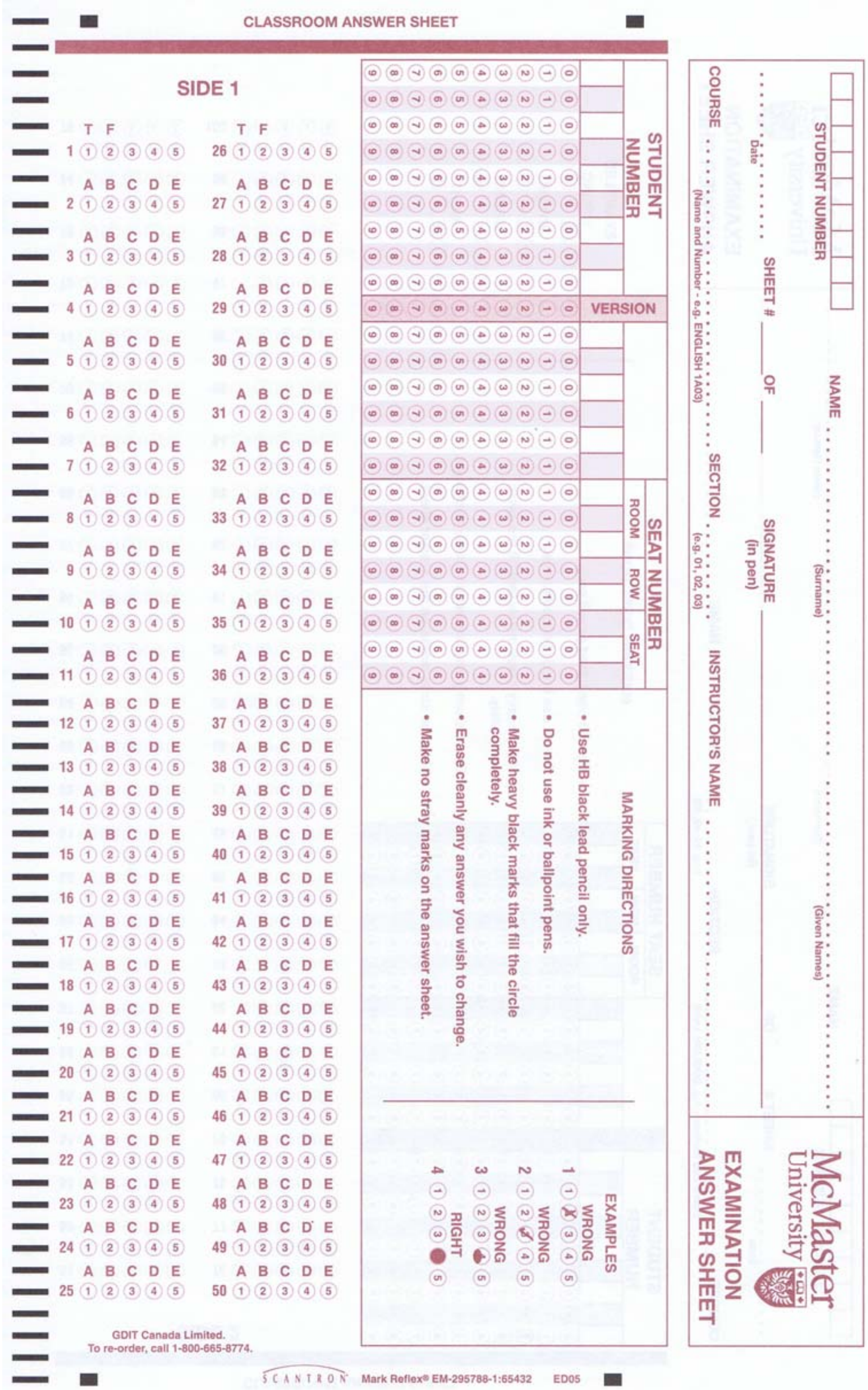
(a) (ii) and (iii) only **(b)** all of them **(c)** (ii) only **(d)** (iii) only **(e)** none of them

20. In Matlab, what command could be used to produce the following matrix?

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

(a) `>>diag(1,1,3,2)` **(b)** `>>block(eye(2),3,2)` **(c)** `>>ones(3,2)`
(d) `>>repmat(eye(2),3,2)` **(e)** `>>eye(3,2)`

21. Correctly fill out the bubbles corresponding to all 9 digits of your student number, as well as the version number of your test in the correct places on the computer card. (Use the below computer card for this sample test.)



Math 1B03 Term 2/1ZC3

2nd Sample Test #2

Name:____

(Last Name) (First Name)

Student Number:_____________________ **Tutorial Number:**____________________

This test consists of 20 multiple choice questions worth 1 mark each (no part marks), and 1 question worth 1 mark (no part marks) on proper computer card filling. All questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Calculators are NOT allowed.

1. Let $y(x)$ be a solution to the initial value problem $y'' + y' - 2y = 0$, $y(0) = 1$, $y'(0) = 2$. Find $y(\ln 2)$.

(a) $\frac{27}{12}$ **(b)** $\frac{29}{12}$ **(c)** $\frac{31}{12}$ **(d)** $\frac{25}{12}$ **(e)** $\frac{23}{12}$

2. Solve the following system

$$\begin{aligned} y_1' &= -y_1 - \frac{3}{2}y_3 \\ y_2' &= y_2 + \frac{1}{2}y_3 \\ y_3' &= 2y_3 \end{aligned}$$

(a) $\begin{aligned} y_1 &= c_1e^{-x} + 2c_3e^{2x} \\ y_2 &= c_2e^{x} + c_3e^{2x} \\ y_3 &= 2c_3e^{2x} \end{aligned}$ **(b)** $\begin{aligned} y_1 &= c_1e^{-x} - c_3e^{2x} \\ y_2 &= c_2e^{x} + c_3e^{2x} \\ y_3 &= 2c_3e^{2x} \end{aligned}$ **(c)** $\begin{aligned} y_1 &= c_1e^{-x} - c_3e^{2x} \\ y_2 &= c_2e^{x} - c_3e^{2x} \\ y_3 &= 2c_3e^{2x} \end{aligned}$

(d) $\begin{aligned} y_1 &= c_1e^{-2x} - c_3e^{x} \\ y_2 &= c_2e^{-2x} + c_3e^{x} \\ y_3 &= 2c_3e^{-2x} \end{aligned}$ **(e)** $\begin{aligned} y_1 &= c_1e^{-2x} + 2c_3e^{x} \\ y_2 &= c_2e^{-2x} + c_3e^{x} \\ y_3 &= 2c_3e^{-2x} \end{aligned}$

3. Solve the following equation for the complex number z. $z(1+i) = \overline{z} + (3+2i)$

(a) $-3+5i$ **(b)** $8-3i$ **(c)** $3+5i$ **(d)** $5-3i$ **(e)** $3+8i$

4. Express the following complex number in polar form. $z = -\sqrt{3} + i$.
(a) $2(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6})$ **(b)** $2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$ **(c)** $2(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3})$
(d) $2(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})$ **(e)** $2(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$

5. Find all complex numbers z such that

$$z^3 = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$$

(a) $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i,\ e^{i(5\pi/12)}, e^{i(7\pi/12)}$
(b) $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i,\ e^{i(11\pi/12)}, e^{i(19\pi/12)}$
(c) $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i,\ -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i,\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$
(d) $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i,\ e^{i(4\pi/5)}, e^{i(7\pi/5)}$
(e) $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i,\ e^{i(4\pi/5)}, e^{i(7\pi/5)}$

6. Let $\mathbf{u} = (1,1,2), \mathbf{v} = (0,1,2), \mathbf{w} = (1,0,-1)$, and $\mathbf{x} = (2,-1,6)$. Find the number c such that $\mathbf{x} = a\mathbf{u} + b\mathbf{v} + c\mathbf{w}$
(a) -8 **(b)** 8 **(c)** -9 **(d)** 9 **(e)** 10

7. Find the equation of the plane containing the lines $(x,y,z) = (1,-1,2) + t(1,1,1)$ and $(x,y,z) = (0,0,2) + t(1,-1,0)$.
(a) $x + y + z = 2$ **(b)** $x + y - 3z = -6$ **(c)** $x + y + 4z = 8$
(d) $x + y - 2z = -4$ **(e)** $x + y - z = -2$

8. Find the parametric equations of the line passing through $P(1,0,-3)$ and parallel to the line with parametric equations $x = -1 + 2t$, $y = 2 - t$, and $z = 3 + 3t$.

(a) $x = 1 - t$	**(b)** $x = 1 + 2t$	**(c)** $x = 1 - t$	**(d)** $x = 1$
$y = t$	$y = -t$	$y = 2t$	$y = 2t$
$z = -3 + t$	$z = -3 + 3t$	$z = -3 + 3t$	$z = -3$

(e) $x = 1 - 2t$
$y = 2t$
$z = -3 + 6t$

9. Compute the projection of $\mathbf{u}$ onto $\mathbf{v}$. $\mathbf{u} = (5,7,1), \mathbf{v} = (1,-1,3)$
(a) $(\frac{54}{11}, 8, \frac{7}{11})$ **(b)** $\frac{1}{11}(1,-1,3)$ **(c)** $\frac{1}{75}(5,7,1)$ **(d)** $\frac{1}{\sqrt{75}\sqrt{11}}$ **(e)** $(\frac{71}{75}, -\frac{82}{75}, \frac{224}{75})$

10. Let $P_1 = P_1(2, 1, -2)$ and $P_2 = P_2(1, -2, 0)$. Find the coordinates of the point P which is $\frac{1}{5}$ of the way from P_1 to P_2.
(a) $(\frac{3}{5}, -\frac{1}{5}, -\frac{2}{5})$ **(b)** $(-\frac{1}{5}, -\frac{3}{5}, \frac{2}{5})$ **(c)** $(\frac{6}{5}, -\frac{7}{5}, -\frac{2}{5})$
(d) $(-\frac{4}{5}, -\frac{12}{5}, \frac{8}{5})$ **(e)** $(\frac{9}{5}, \frac{2}{5}, -\frac{8}{5})$

11. Find the area of the triangle with vertices $A(1, 1, -1), B(2, 0, 1)$, and $C(1, -1, 3)$.
(a) $2\sqrt{5}$ **(b)** $\sqrt{5}$ **(c)** $\sqrt{3}$ **(d)** $2\sqrt{3}$ **(e)** $\sqrt{7}$

12. Find the point on the plane $3x - y + 4z = 1$ closest to the point $P(2, 1, -3)$.
(a) $(\frac{8}{5}, \frac{11}{13}, \frac{7}{8})$ **(b)** $(\frac{7}{3}, -\frac{2}{3}, \frac{5}{3})$ **(c)** $(\frac{25}{11}, -\frac{23}{11}, \frac{9}{11})$
(d) $(-\frac{38}{13}, \frac{11}{13}, \frac{23}{13})$ **(e)** $(\frac{38}{13}, \frac{9}{13}, -\frac{23}{13})$

13. Let $\mathbf{u}$ and $\mathbf{v}$ be vectors in $\mathbb{R}^3$, and consider the following statements.
(i) $\mathbf{u} \cdot \mathbf{v} = \frac{1}{2}\|\mathbf{u} + \mathbf{v}\|^2 - \frac{1}{2}\|\mathbf{u} - \mathbf{v}\|^2$
(ii) $\mathbf{v} - \mathbf{w}$ and $(\mathbf{u} \times \mathbf{v}) + (\mathbf{v} \times \mathbf{w}) + (\mathbf{u} \times \mathbf{w})$ are orthogonal.
(iii) If $\mathbf{u}$ is any vector then the projection of $\mathbf{u}$ on $\mathbf{v}$ equals the projection of $\mathbf{u} - \mathbf{v}$ on $\mathbf{v}$.

Which of the above statements are always true?
(a) (i) and (ii) only
(b) (i) and (iii) only
(c) (ii) and (iii) only
(d) (ii) only
(e) none of them

14. Let V be the set of all ordered pairs of real numbers, and consider the following addition and scalar multiplication operations on $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_1, v_2)$.

$$\mathbf{u} + \mathbf{v} = (u_1 + v_1, u_2 + v_2)$$

and

$$k\mathbf{u} = (ku_1 + 1, ku_2).$$

Recall axioms 7-9 of a vector space:
7. $k(\mathbf{u} + \mathbf{v}) = k\mathbf{u} + k\mathbf{v}$
8. $(k + m)\mathbf{u} = k\mathbf{u} + m\mathbf{u}$
9. $k(m\mathbf{u}) = (km)\mathbf{u}$

Which of these axioms are true?
(a) all of them **(b)** **7.** only **(c)** none of them **(d)** **7.** and **8.** only **(e)** **9.** only

For Questions 15-17, determine which of the following answers is correct for the given subset W of $\mathbb{R}^3$.
(a) W is a subspace
(b) W is closed under addition, but not closed under scalar multiplication
(c) W is closed under scalar multiplication, but not closed under addition
(d) W is not closed under scalar multiplication, and not closed under addition

15. $W =$ all vectors of the form (a, b, c) where $a - 2c - 1 = 0$
(a) **(b)** **(c)** **(d)**

16. $W =$ all vectors of the form (a, b, c) where the product $ab \geq 0$.
(a) **(b)** **(c)** **(d)**

17. Let **y** be a given vector. Let $W =$ all vectors **x** such that $\mathbf{y} \cdot \mathbf{x} = 0$.
(a) **(b)** **(c)** **(d)**

18. Which of the following statements are true?

(i) The polynomial $\mathbf{p} = 1 + 2x - x^2$ is in span$\{1 + x^2, 1 - 4x\}$
(ii) The set $S = \{1 + 2x, x + 3x^2, 2 + x^2\}$ spans P_2
(iii) $P_2 =$ span$\{1, x, x^2\}$

(a) all of them **(b)** (i) and (iii) only **(c)** (ii) only **(d)** (iii) only **(e)** (ii) and (iii) only

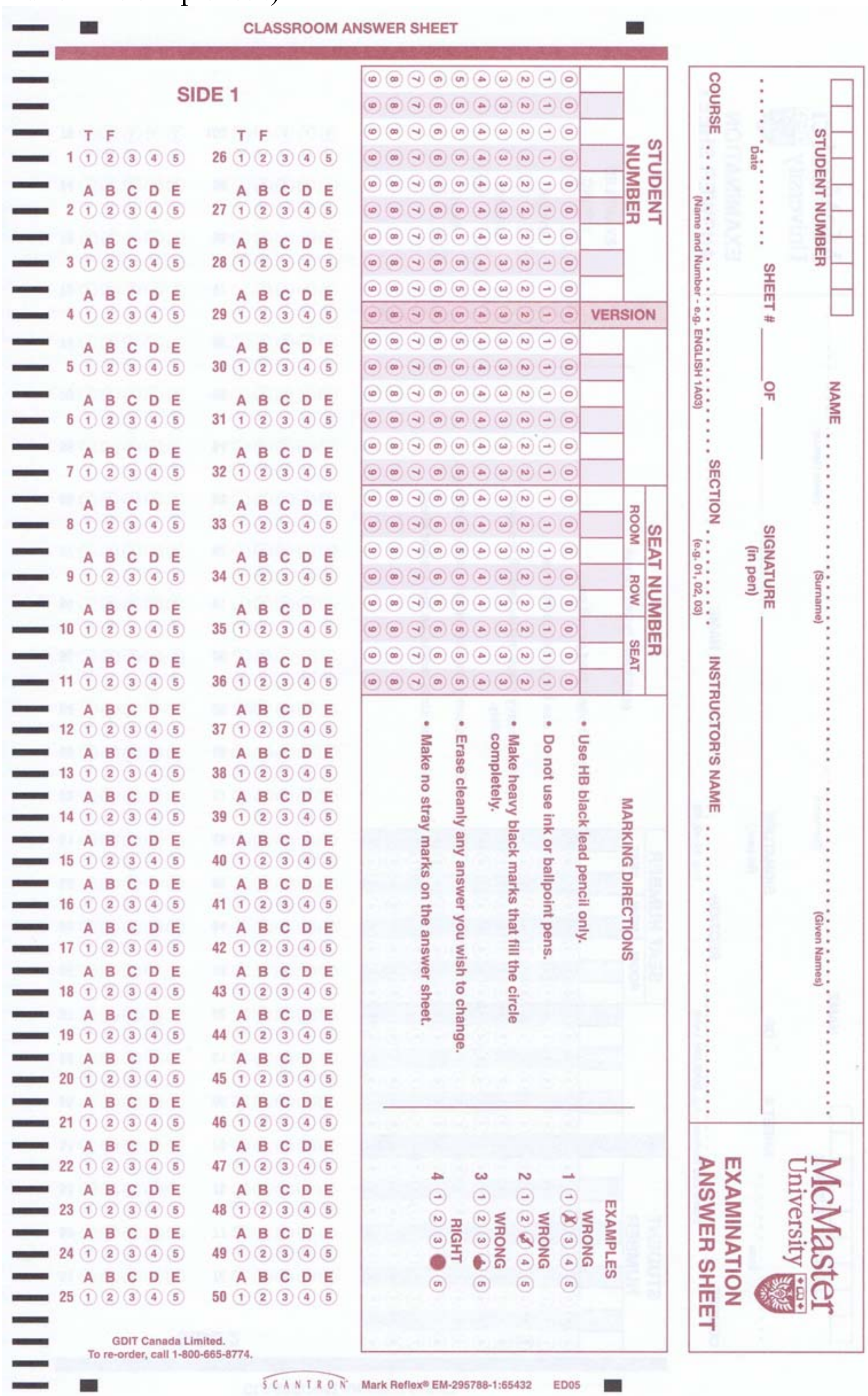
19. Which of the following statements are always true?

(i) If a set of vectors S spans a vector space V then every vector in V can be written as a linear combination of vectors in S.
(ii) If a set of vectors S spans a vector space V then no vector in S can be written as a linear combination of the remaining vectors in S.
(iii) If a set of vectors S in R^3 does not span R^3 then the set S must have less than 3 vectors in it.

(a) (i) and (iii) only **(b)** (i) only **(c)** (ii) only **(d)** (i) and (ii) only **(e)** none of them

20. In Matlab, what command could be used to find the modulus of the complex number z?
(a) `>>mod(z)` **(b)** `>>modulus(z)` **(c)** `>>abs(z)`
(d) `>>length(z)` **(e)** `>>z*z'`

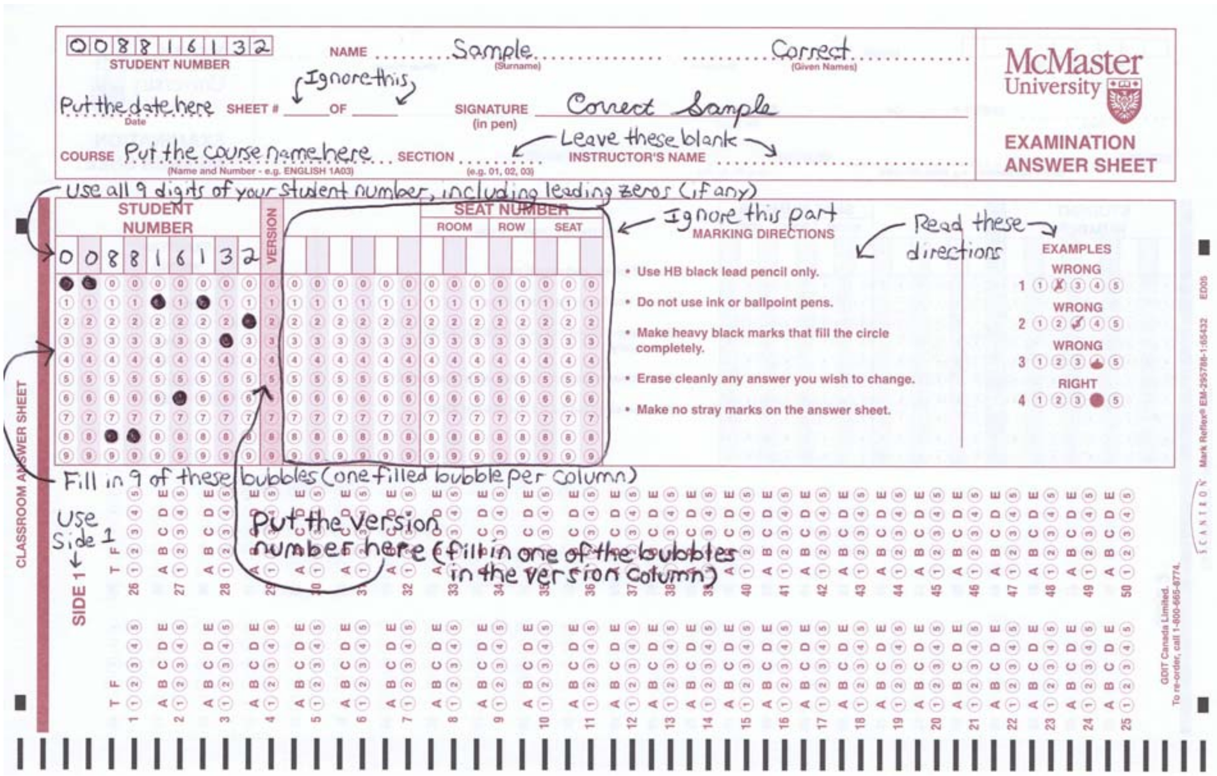
21. Correctly fill out the bubbles corresponding to all 9 digits of your student number, as well as the version number of your test in the correct places on the computer card. (Use the below computer card for this sample test.)



Answers for 1st Sample Test #2

1. a **2.** c **3.** a **4.** a **5.** d **6.** b **7.** b **8.** b **9.** d **10.** d
11. c **12.** d **13.** a **14.** e **15.** d **16.** a **17.** b **18.** a **19.** d **20.** d

21.



NOTE: On the sample tests, a version number is not given. On the actual tests, it will say "Version X" at the top, where X is the version number that you will have to fill in on the computer card. The sample above assumes that your student number is 008816132. On the actual test, you will have to fill in the bubbles corresponding to YOUR student number (not 008816132).

Answers for 2nd Sample Test #2

1. c **2.** b **3.** b **4.** a **5.** b **6.** a **7.** d **8.** b **9.** b **10.** e
11. b **12.** e **13.** e **14.** c **15.** d **16.** c **17.** a **18.** e **19.** b **20.** c
21. see the answer to #21 on the first sample test above.