COMPSCI 3MI3 : Assignment 5

Fall 2021

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1. (8 points) **Fibonacci Sequence**
   The Fibonacci numbers can be defined as follows:

$$Fib(n) = \begin{cases} 0 & n = 0 \\ 1 & n = 1 \\ Fib(n-1) + Fib(n-2) & n >= 2 \end{cases} \quad (1)$$

   Create a λ expression in the enriched λ-Calculus which, when input to the Fixed Point Combinator, takes in a position in the Fibonacci sequence, and outputs the corresponding Fibonacci number.

   Demonstrate your function works by evaluating the 4^{th} number in the sequence.

2. (6 points) **Determinacy of λ-Calculus**
   In UAE, we discussed the property of determinacy at some length. Is the call-by-value evaluation strategy of λ-Calculus determinate? If yes, provide a (traditional) proof of determinacy. If no, provide a (traditional) proof of non-determinacy.

   The operational semantics of the call-by-value strategy are given below.

   → *(untyped)*

   Syntax

t ::=		*terms:*
	x	*variable*
	λx.t	*abstraction*
	t t	*application*
v ::=		*values:*
	λx.t	*abstraction value*

   Evaluation $\quad t \longrightarrow t'$

$$\frac{t_1 \longrightarrow t_1'}{t_1\ t_2 \longrightarrow t_1'\ t_2} \quad \text{(E-App1)}$$

$$\frac{t_2 \longrightarrow t_2'}{v_1\ t_2 \longrightarrow v_1\ t_2'} \quad \text{(E-App2)}$$

$$(\lambda x.t_{12})\ v_2 \longrightarrow [x \mapsto v_2]t_{12} \quad \text{(E-AppAbs)}$$

3. (6 points) **Termination of λ-Calculus**
   A language is said to terminate if a finite set of terms will always result in a finite evaluation chain. This property does not hold in λ-Calculus, but no rigorous proof of this fact was provided in lecture.

   Create a rigorous (traditional) proof that the Termination property does not hold for λ-Calculus.

   HINT: Examining the proof of termination for UAE, as well as the section from slide set 6 on computability may help you structure this proof.