① (a) $\frac{6n^3}{(\log n+1)} = O(n^3)$

↳ $\frac{6n^3}{\log n+1} \leq cn^3 \Rightarrow 6n^3 \leq cn^3 \cdot (\log n+1)$

$\Rightarrow n^3(c\log n + c - 6) \geq 0$

pick $c=6$, $n_0=0$ $\Rightarrow$ the equation always meets.

---

① (b) $10n^3 + 9 = O(n)$

↳ $10n^3 + 9 \leq cn \rightarrow \boxed{10n^2 + \frac{9}{n} \leq c}$

The above equation cannot be satisfied since c must be a constant.

Q_2 - a Function that removes duplicates of a given list

```
public Queue<Item> ERase_Duplicates(Queue<Item> list)
{
    Queue<Item> new_List = new Queue<Item>();

    while(! list.empty())
    {
        ~~Node<R>~~
        Item t = list.dequeue()
        bool exist = false;
        Iterable iter = new_List.Iterable();

        while(iter.hasNext())
            if(iter.next().item == t)
                exist = true; break;

        if(! exist)
            new_List.enqueue(t);
    }
    return new_List;
{
```

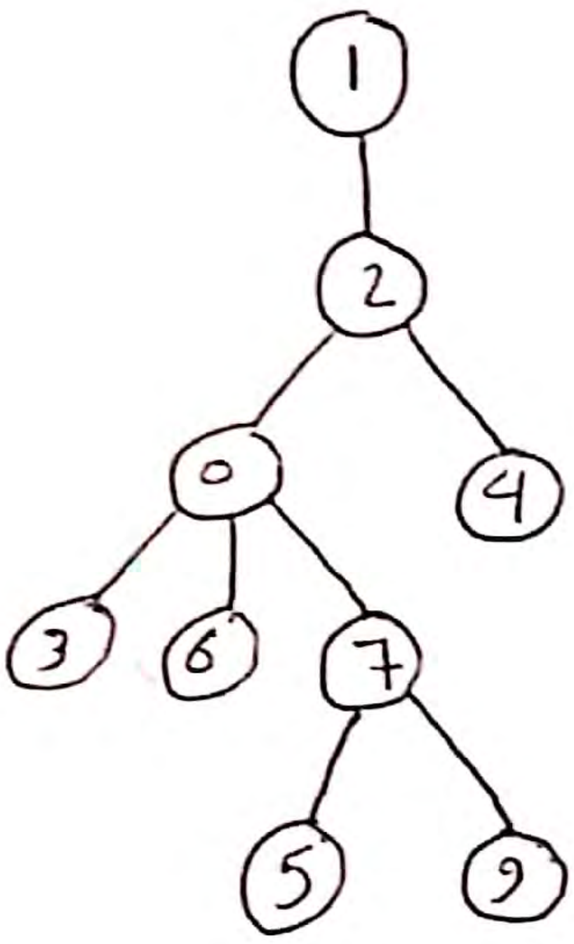
Q2.

```
public void ERase-Duplicants()        // Inside the class
                                      Node
{                                     Queue

    Node n1 = first;

    while (n1 != null)
    {
        Node n2 = n1.neat;
        Node temp = n1;

while (n2 != null)
{
    if (n2.item == n1.item)  // duplicate
    {
        temp.neat = n2.neat;
        N--;
    {
    else
    {
        temp = n2;
    {
        n2 = n2.next;
    {

    last = n1;
    n1 = n1.next;
{
{
```

(3)

0	1	2	3	4	5	6	7	8	9
2	1	1	0	2	7	0	0	8	7

a



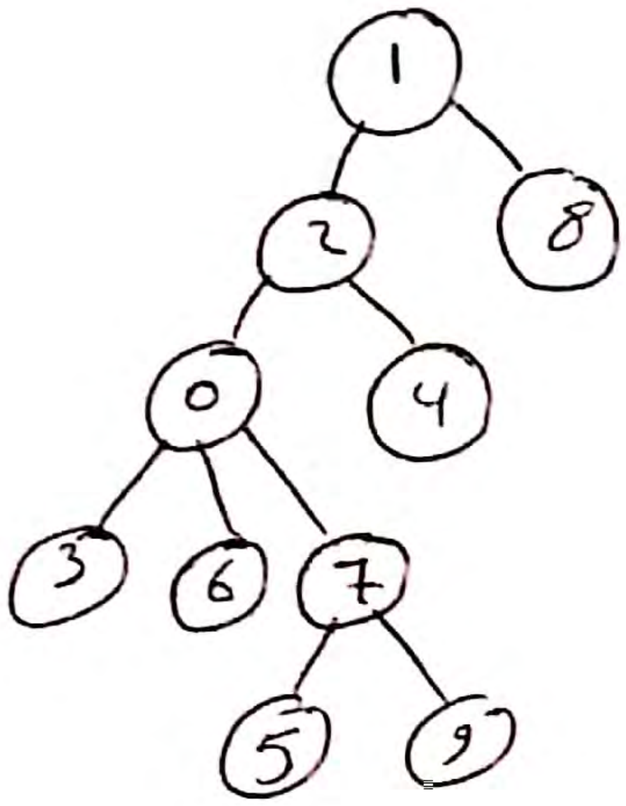
(8)

(b) Union (1, 8)

i = find (1) = 1 Size(1) = 8 9

j = find (8) = 8 Size(8/2 1

id[8] = i = 1 Size(1) = 9 + 1



Q3

(c) Find(5)

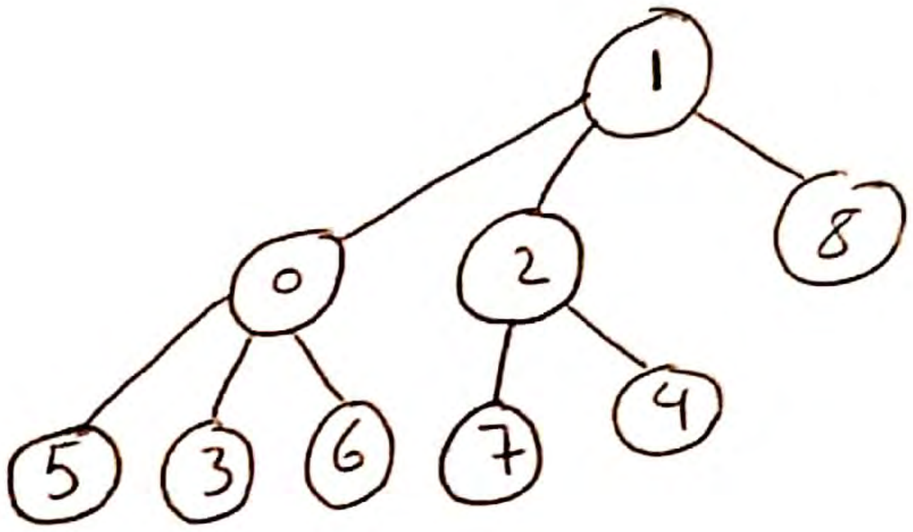
id[5] = id[id[5]] = id[7] = 0 , i = 7

id[7] = id[id[7]] = id[0] = 2 , i = 0

id[0] = id[id[2]] = id[1] = 1 , i = 2

id[2] = id[id[2]] = id[1] = 1 , i = 1

return i = 1

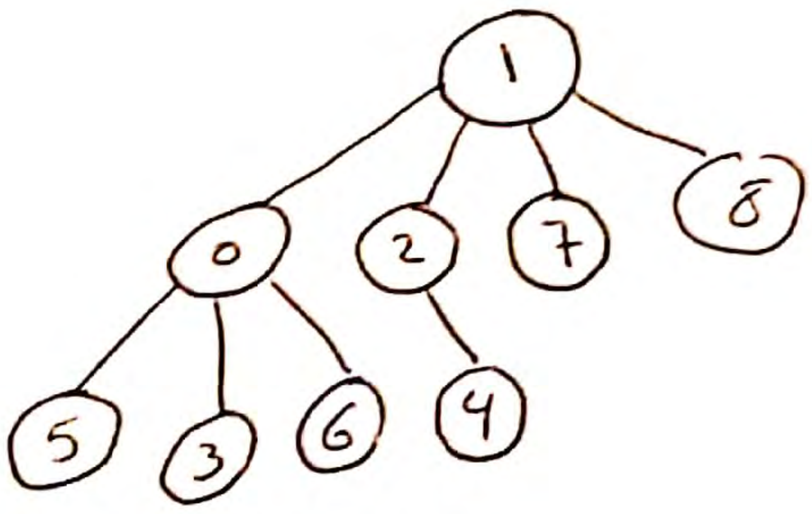


(d) Find(7)

id[7] = id[id[7]] = id[2] = 1 i = 2

id[2] = id[id[2]] = id[1] = 1 i = 1

return i = 1



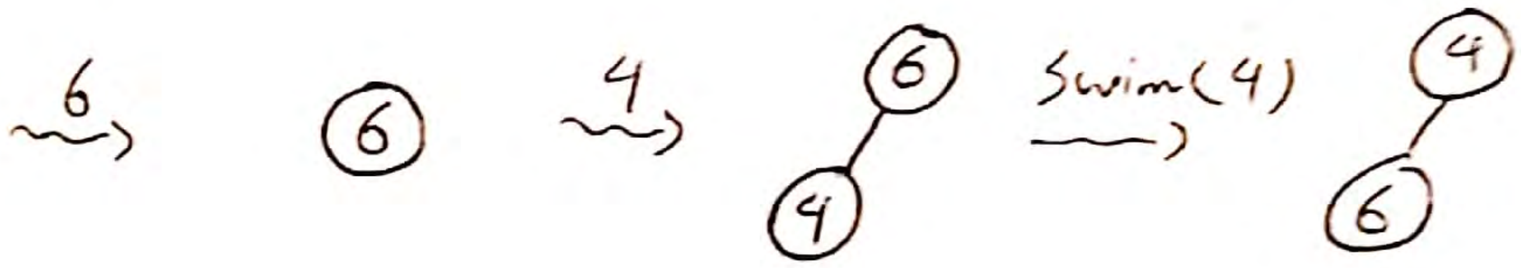
④ proof by induction:

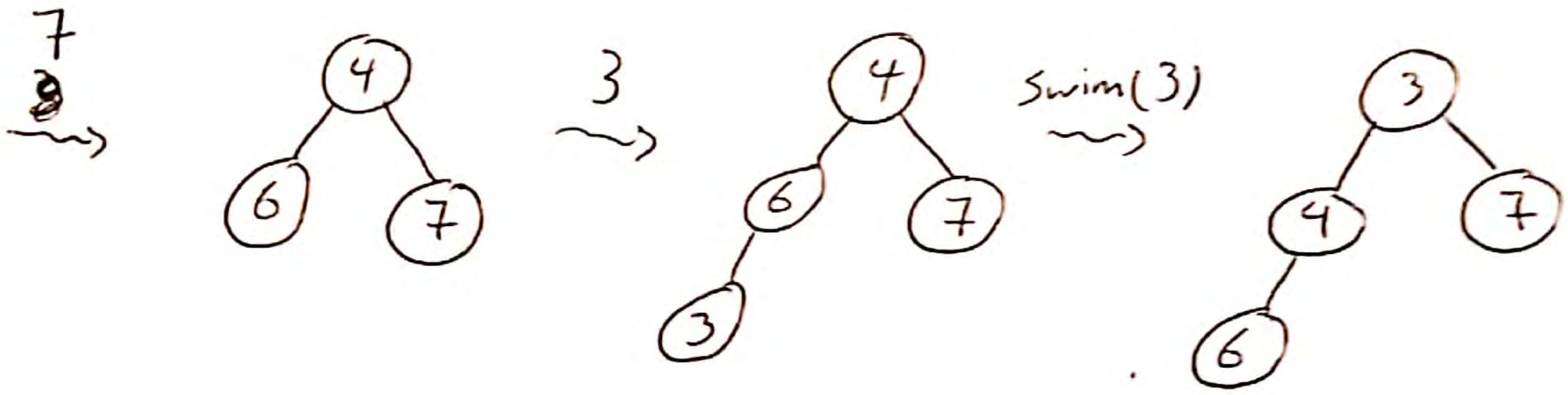
① Base Case: The nonempty tree with 0 internal node and one leaf. A full binary tree with one internal node has two leafnodes. $n \geq 0$ ✓ $n \geq 1$ ✓

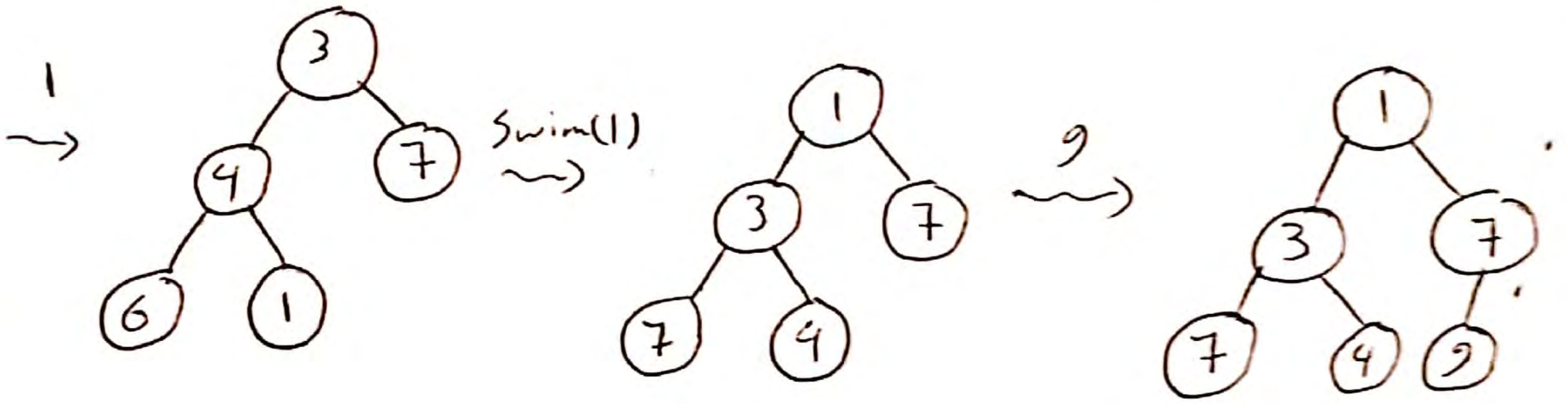
② induction Hypothesis: any full binary tree T containing n-1 internal nodes has n leaves.

③ Given tree T with n internal nodes. select an internal node that has Two children, remove it (along with it's children) now, T has n-1 internal node and from ②, it has n leaves.
now, restore T by adding the removed node and add Two children, now T has n internal node with n+2 children
However, the removed node counted as one of the leaves from the previous part, thus T has n+1 leaves.

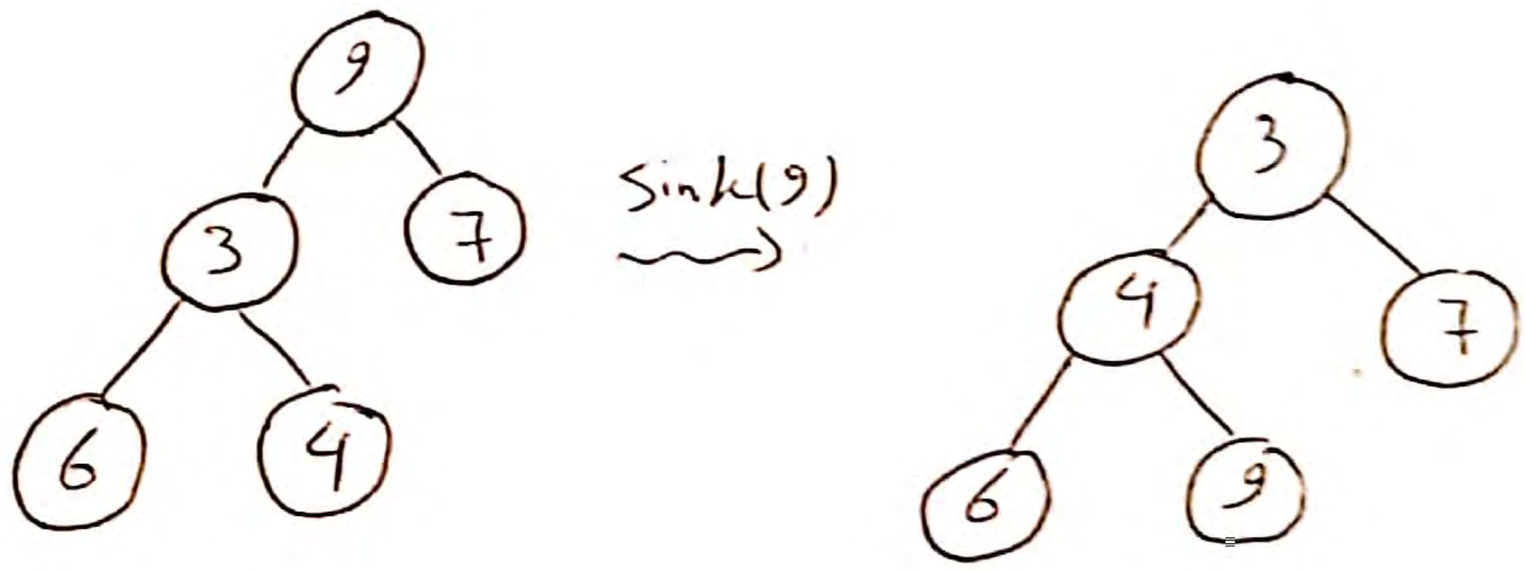
Q5 -


6
6
4
6
4
swim(4)
4
6

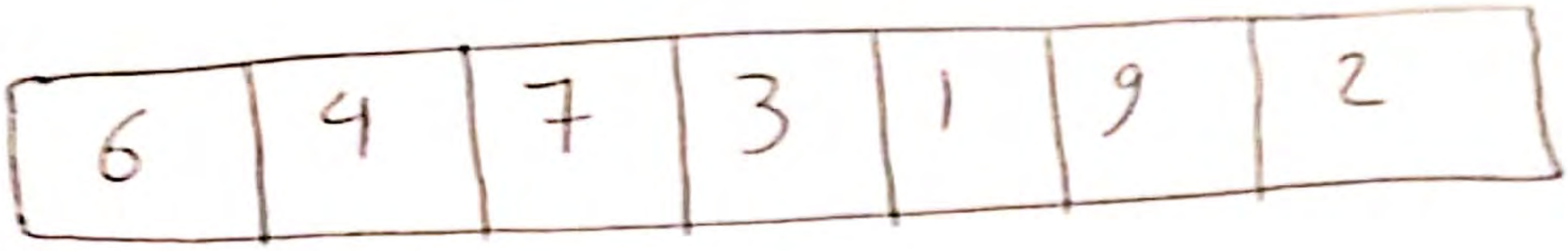

7
4
6
7
3
4
6
7
3
swim(3)
3
4
7
6


1
3
4
7
6
1
swim(1)
1
3
7
7
4
9
1
3
7
7
4
9

delMin

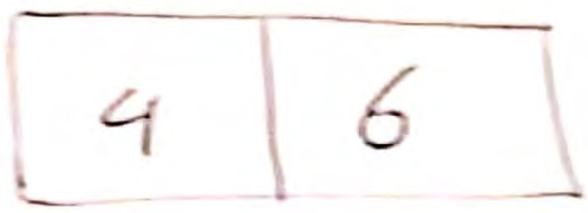

9
3
7
6
4
sink(9)
3
4
7
6
9

Q_6

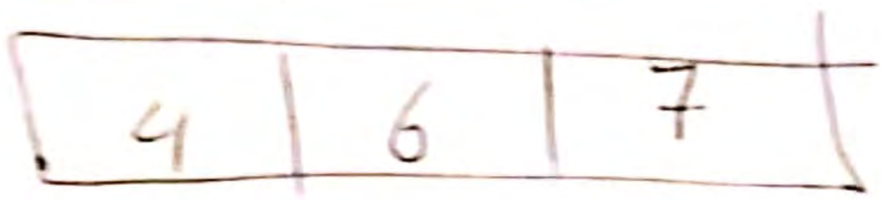


insertion sort

4 →

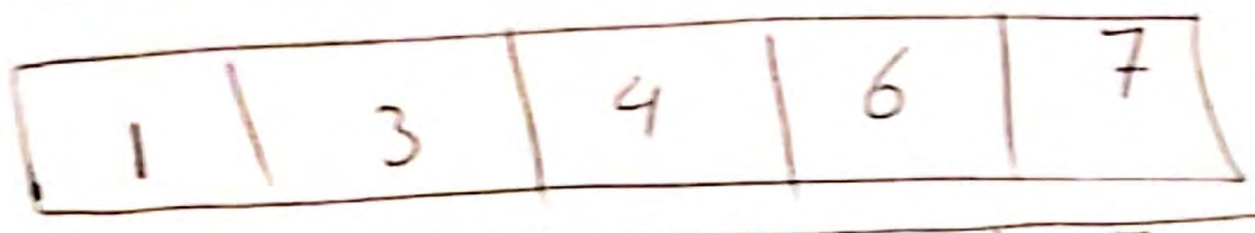


7 →

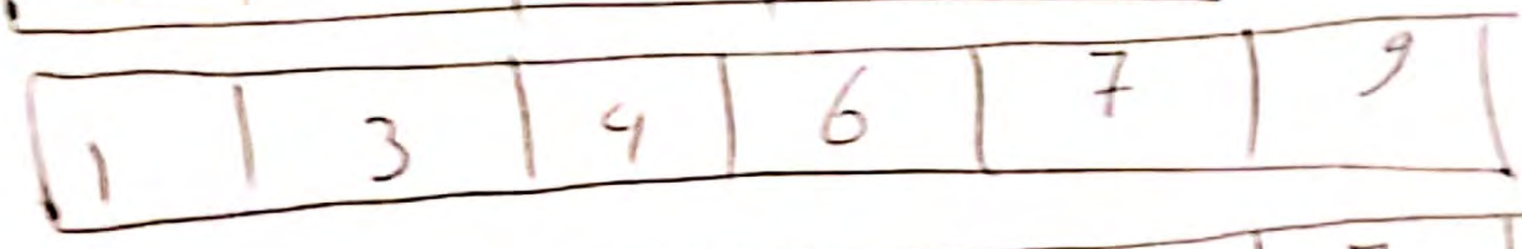


3 →

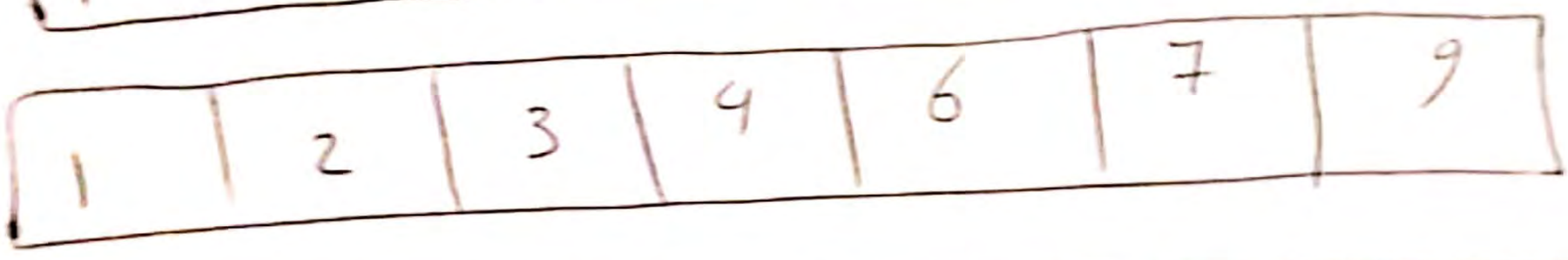
1 →



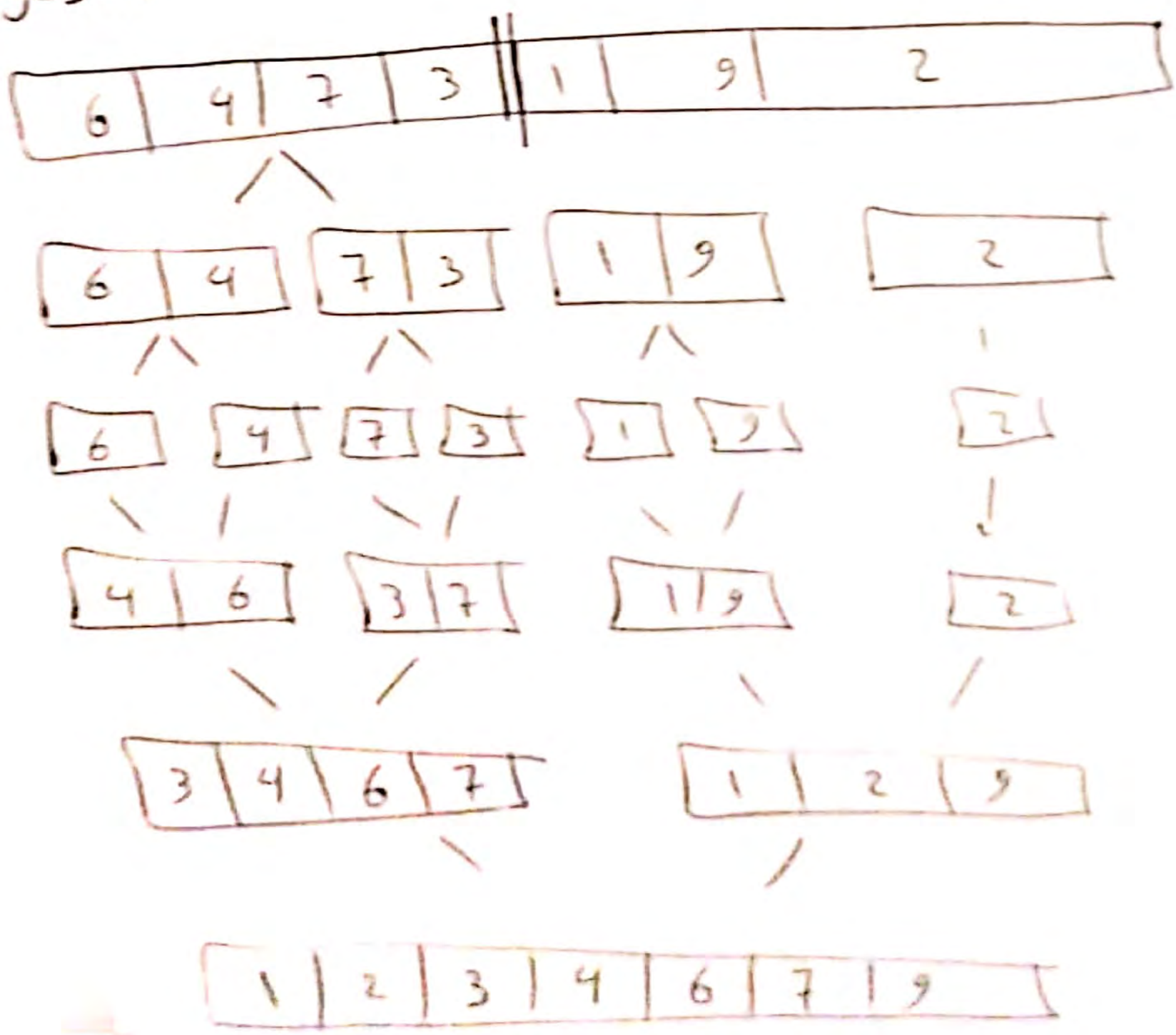
9 →



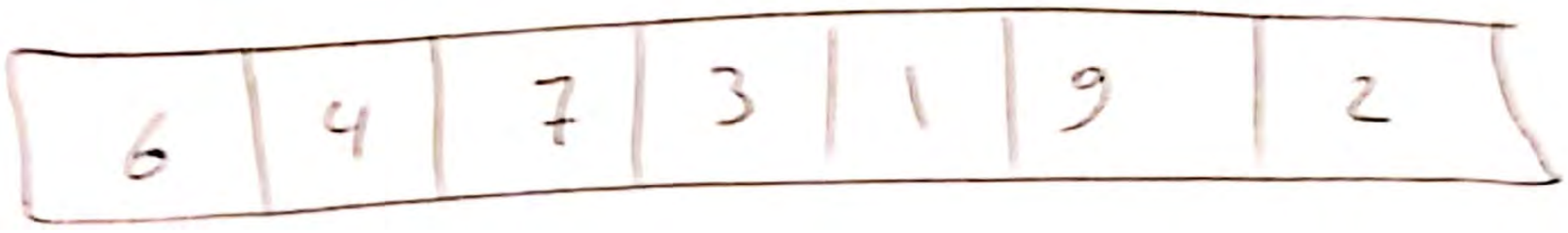
2 →



top-down mergeSort



Q6.

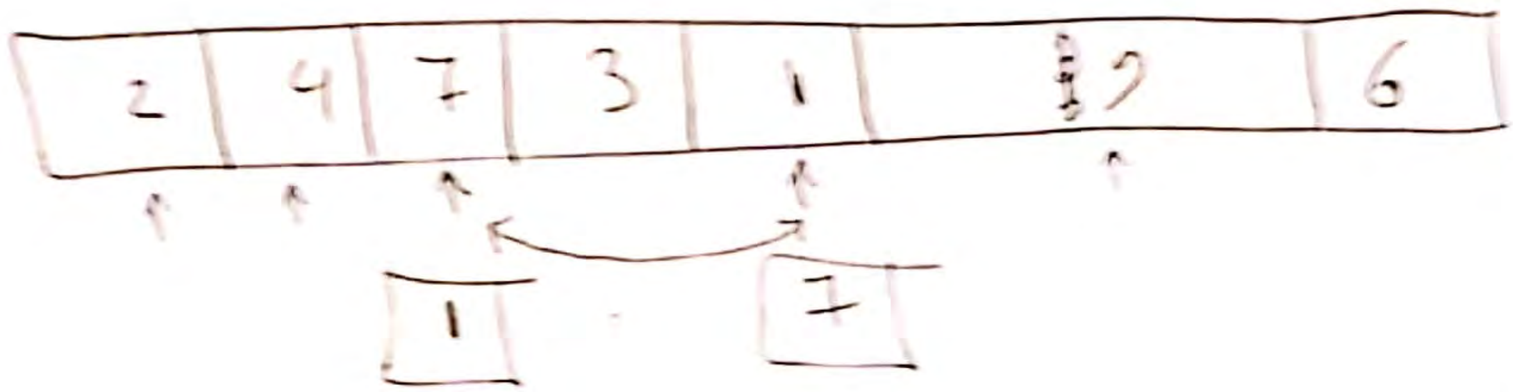


Quick Sort 1

pivot

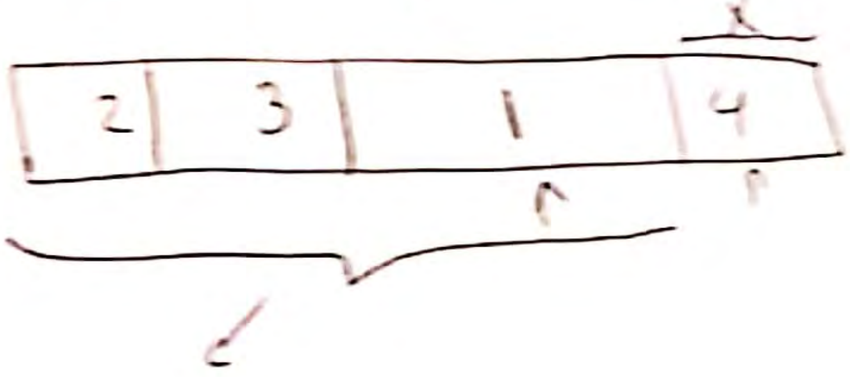
Largest(6,4) = 6

Swap(6,2)



Swap(7,6)

pivot = 4

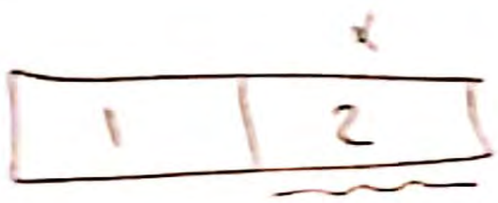


pivot = 9

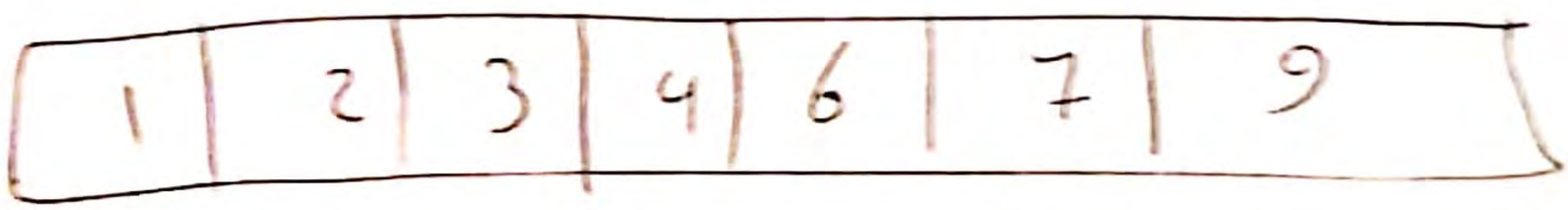
Done!

pivot = 3

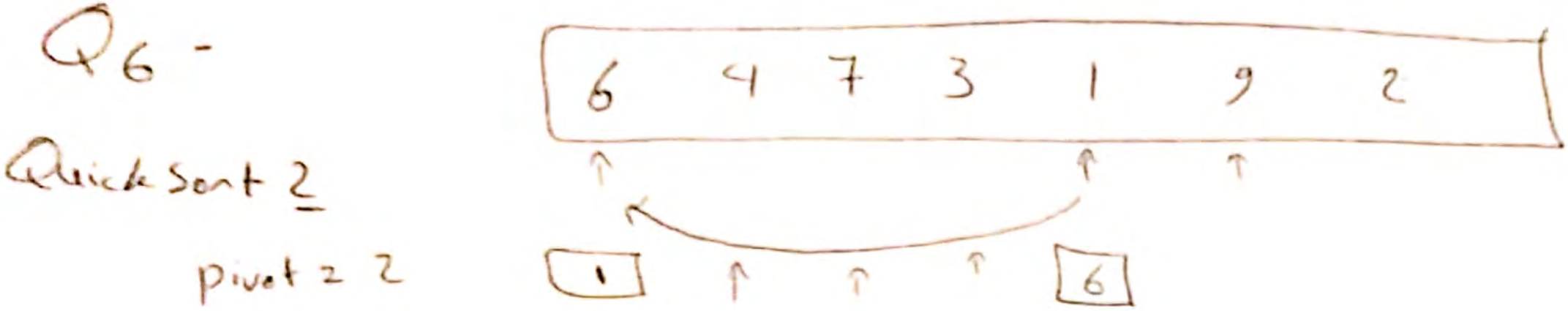
pivot = 2

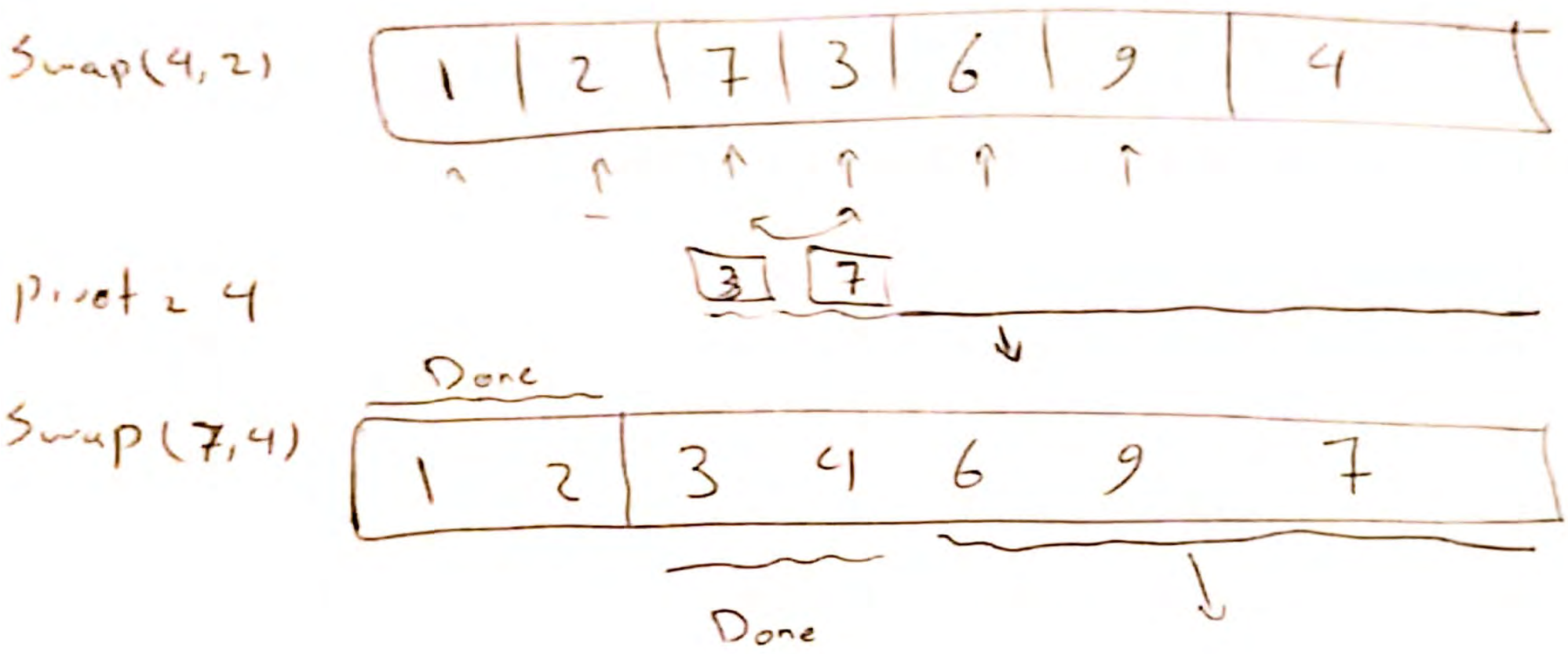


Done!



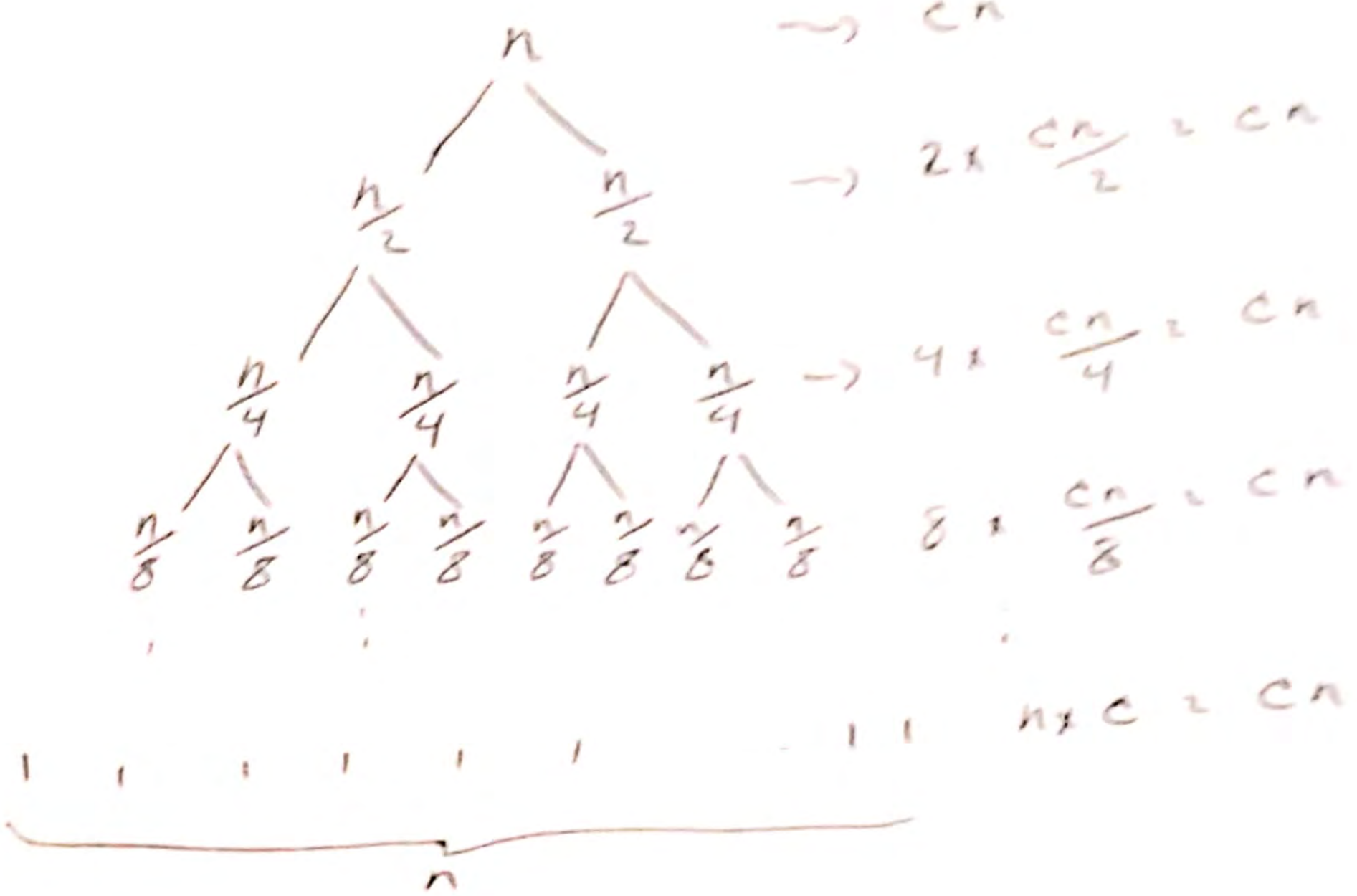
Q6 -


6 4 7 3 1 9 2
Quicksort 2
pivot = 2
1
6


Swap(4,2)
1 2 7 3 6 9 4
pivot = 4
3 7
Done
Swap(7,4)
1 2 3 4 6 9 7
Done
pivot = 7
6 9 7
Done!

Q7 - mergeSort Complexity

merging of 2 sorted arrays
↓
(for a constant c)



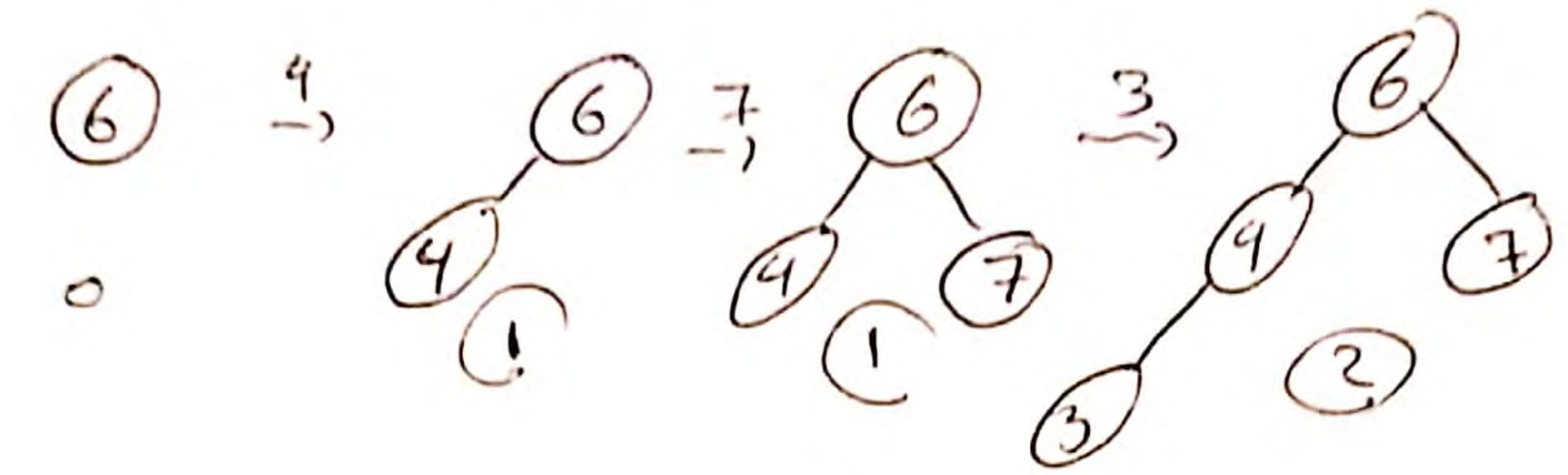
in total, the height of the tree is $\log n$ and the Complexity of each subproblem is cn where we have $cn \log n$ in total which means $O(n \log n)$

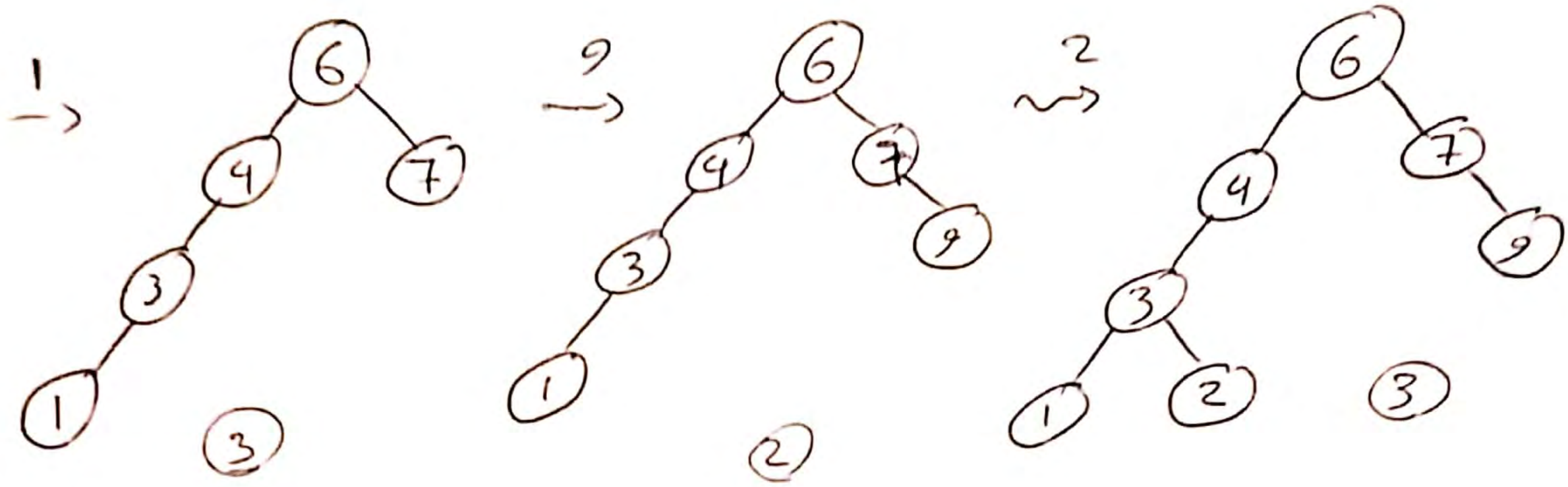
Q8.

Insertion sort as it is almost linear in this case. But be flexible and accept any reasonable arguments.
Since log 100000 ~ 17, some may argue that mergesort might work. Accept this if stated properly.

Definitely not selection sort as it is always ~ n^2 and not Quicksort as it is also ~ n^2 in this case.

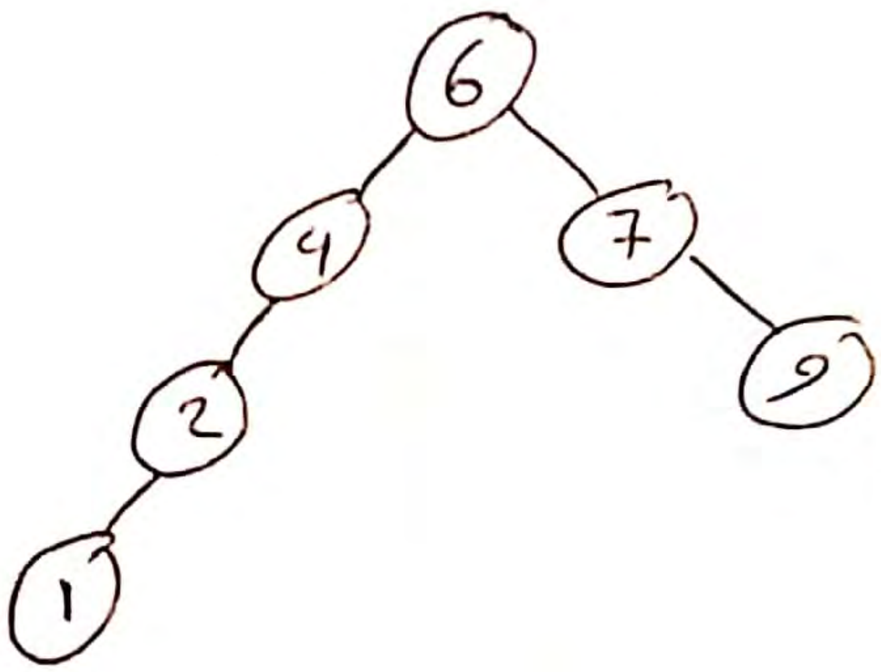
(9) (a)



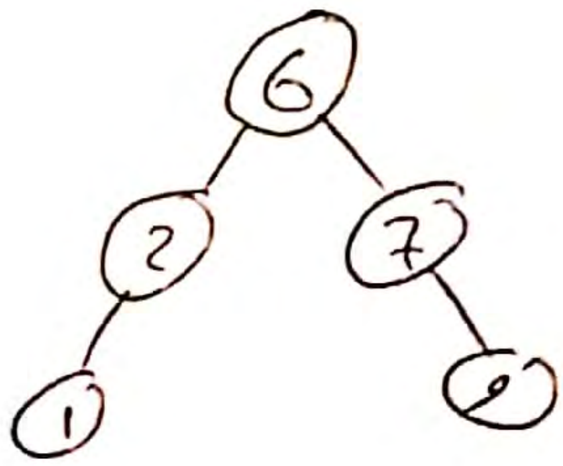


Comp. $0+1+1+2+3+2+3 = \boxed{12}$

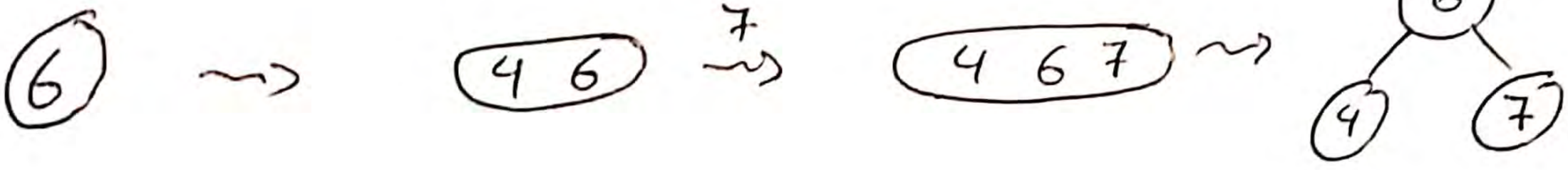
$dcl(3) \rightarrow \quad suc(3) = 2$

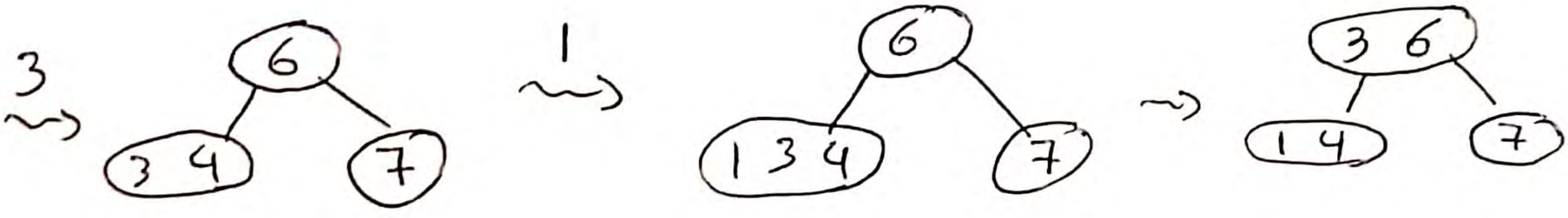


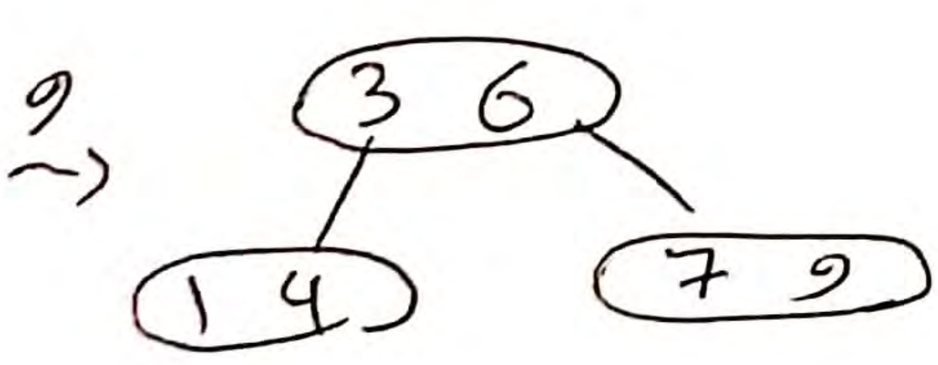
$dcl(4) \rightarrow \quad suc(4) = 6$

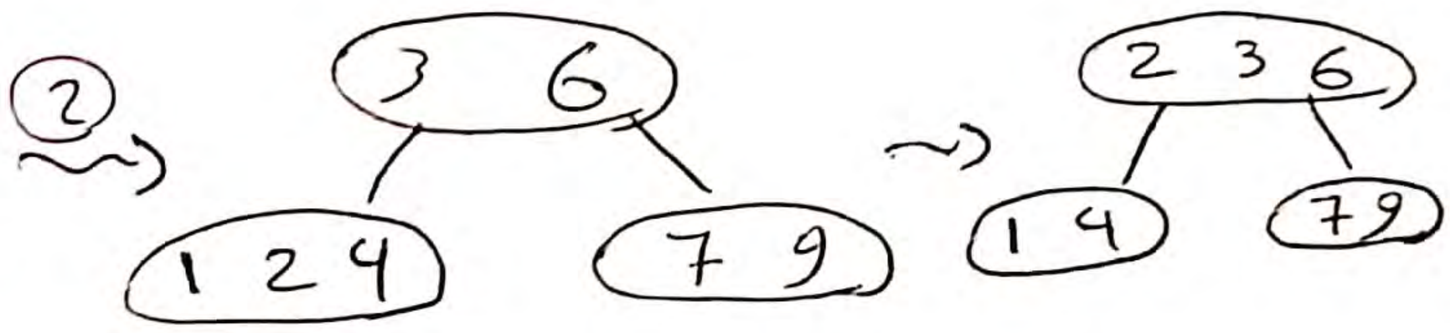


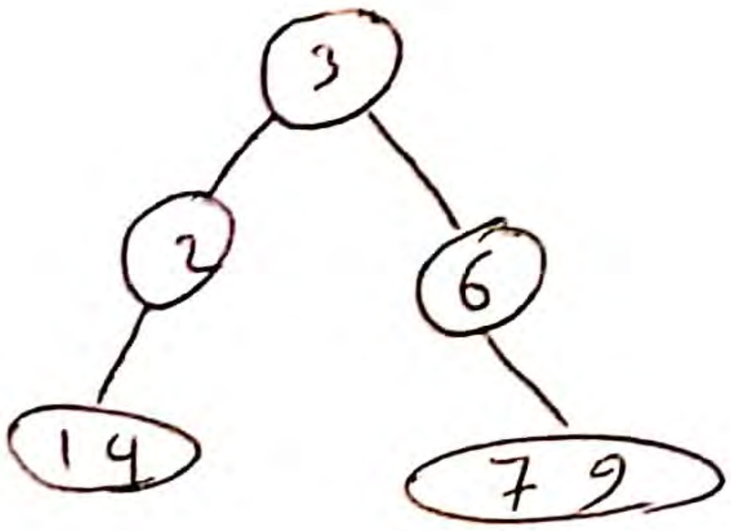
(10) a 6 4 7 3 1 9 2


6
4 6
7
4 6 7
6
4
7


3
6
3 4
7
1
6
1 3 4
7
3 6
1 4
7

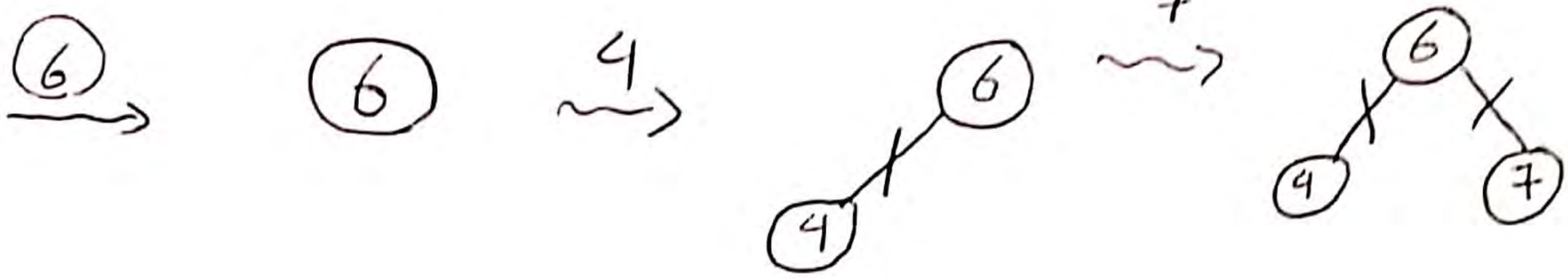

9
3 6
1 4
7 9

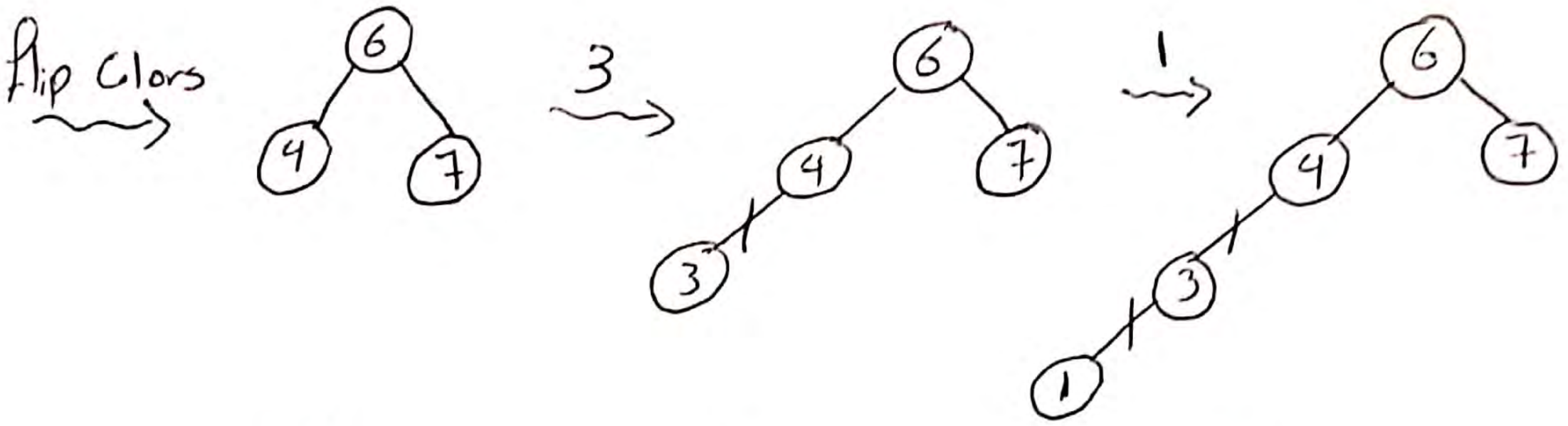

2
3 6
1 2 4
7 9
2 3 6
1 4
7 9

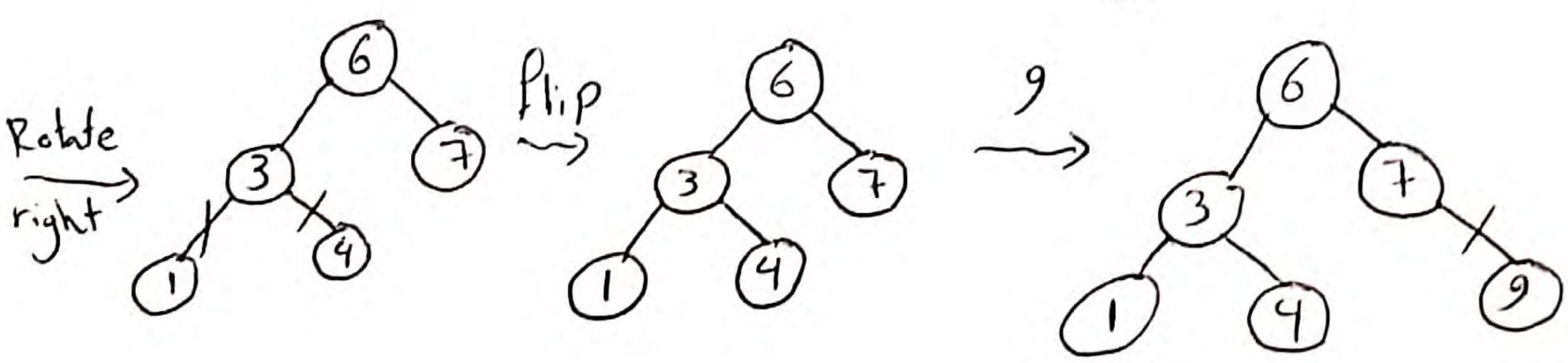

3
2
6
1 4
7 9

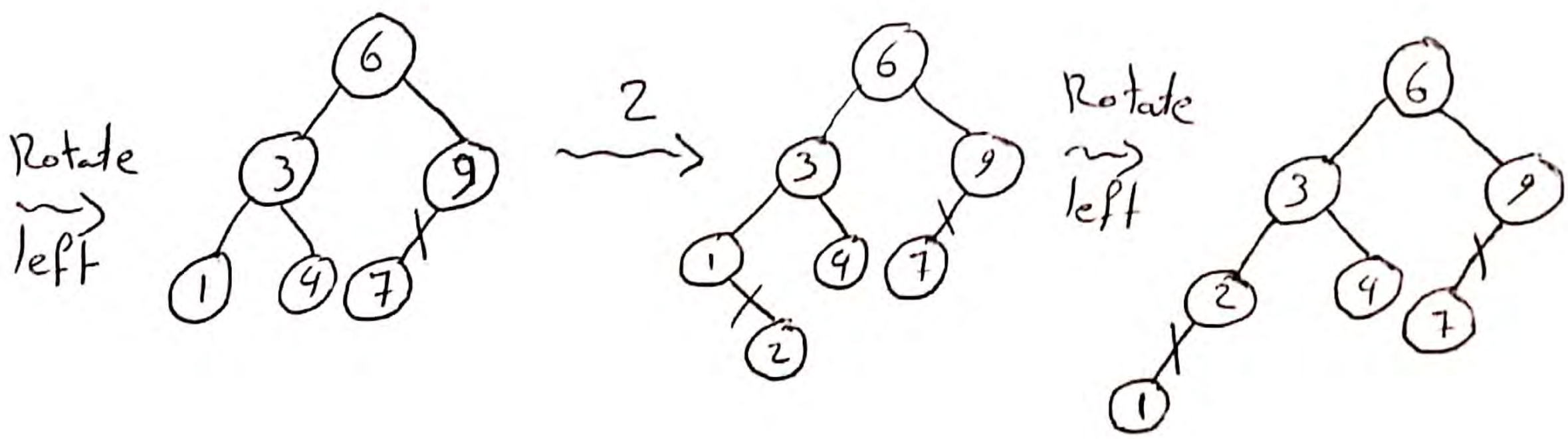
(1a)-b Red-black BST / showing red edge with a cross

6 4 7 3 1 9 2



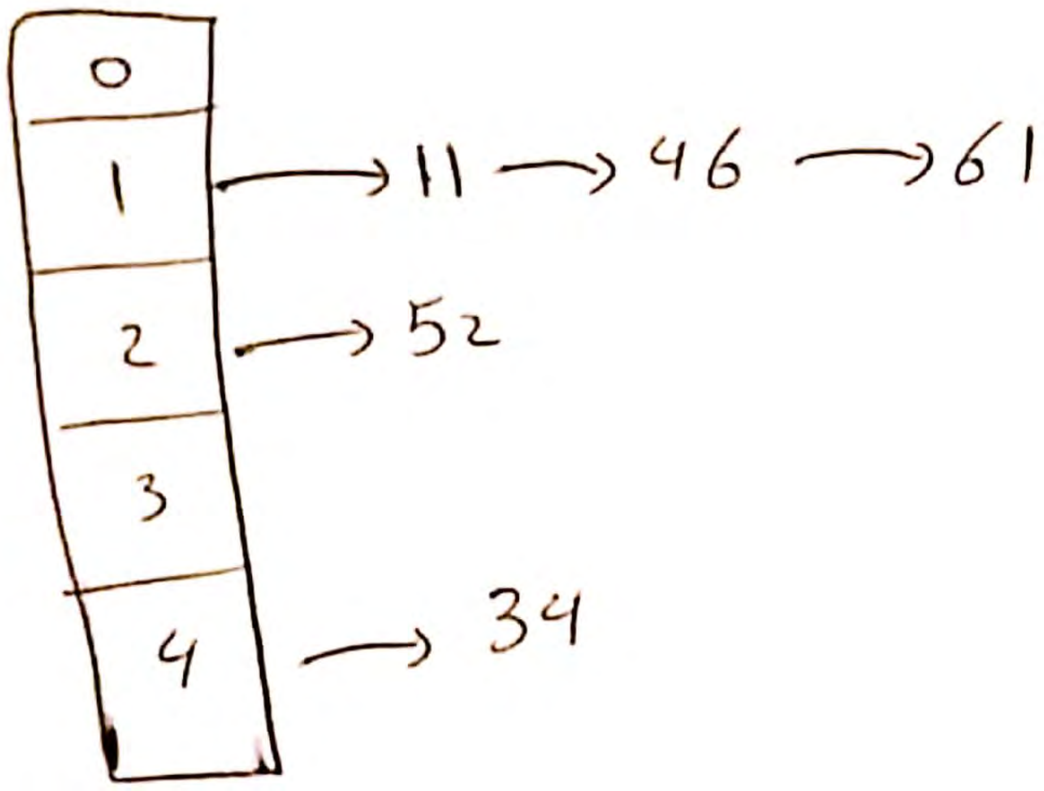






Q_{11} - $h(k) = k \bmod 5$

a



$h(11) = 1$

$h(46) = 1$

$h(52) = 2$

$h(61) = 1$

$h(34) = 4$

---

b