

Theme 1

Introductory Material

Module T1M1:
The Predictable Universe

T1M1 – Learning Objectives

- Identify the approach taken by physicists to understanding complex phenomena.
- Recognize that measurements are really comparisons with a standard **unit** of measure, and that different standard units can be related to each other.
- Distinguish between the specific units of a measured quantity, and the more general statement of the **dimensions** of the quantity.
- Recognize that the dimensions of a quantity are helpful at predicting the relationships that govern a system.
- Understand the idea of **proportionality** to describe the specific way in which quantities are related.

Module Clicker Quiz!

Now that you have had a chance to
review the entire first module, T1M1,
here is your first

module quiz!

Module Clicker Quiz!

Unit conversion (120 seconds)

- The walking speed limit in campus is 10. km/h. What is this speed in m/s?
- A. 3.6 m/s
 - B. 2.8 m/s
 - C. 1.7 m/s
 - D. 6.0 m/s
 - E. I don't know

Module Clicker Quiz!

Dimensional analysis (120 seconds)

- In the following formula, what are the dimensions of the variable ***E***?

$$U = \frac{EX^2}{T}$$

U – dimensions [M/T]

X – dimensions [L]

T – dimensions [M]

A. $\left[\frac{M^2}{L T} \right]$

C. $\left[\frac{1}{L^2 T} \right]$

E. I don't know

B. $\left[\frac{M}{L^2 T} \right]$

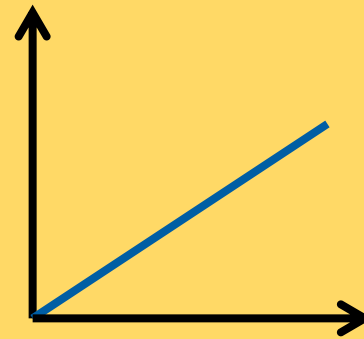
D. $\left[\frac{M^2}{L^2 T} \right]$

Module Clicker Quiz!

Proportionality (120 seconds)

- For the formula given, which relationships, when plotted on a graph, would yield a straight line, as shown:

$$U = \frac{EX^2}{T}$$



- A. T vs. X^2 (U, E held constant)
- B. U vs. T (E, X held constant)
- C. U vs. X (E, T held constant)
- D. E vs. X^2 (U, T held constant)
- E. I don't know

Proportionality

- The natural world rarely provides us with access to all the details to put together a complete picture!
- The trends and relationships that we observe allow us to infer general rules that we incorporate into a model

Proportionality: $x \propto y$ implies that

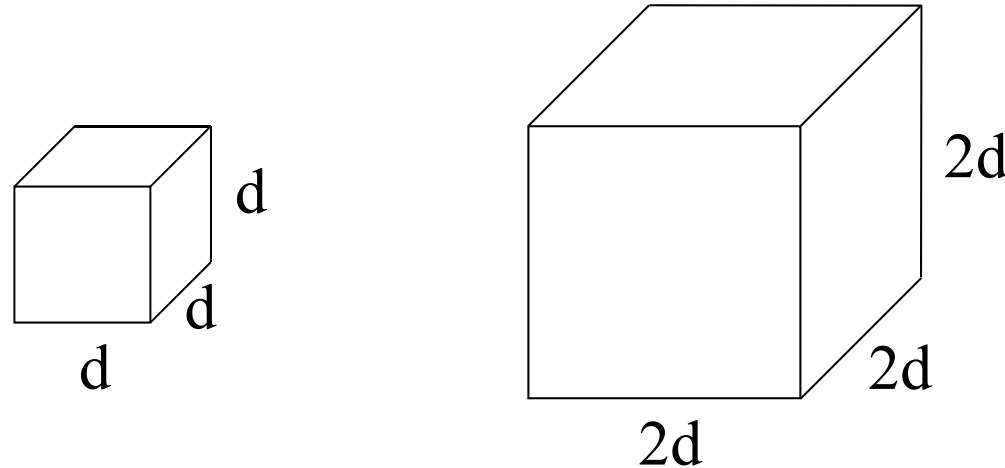
- If we double x , then y also doubles
 - If y is reduced to $1/7^{\text{th}}$ of its value, so is x .
- We would incorporate into our model

$$x = ay$$

where a is a constant of proportionality, independent of the actual values of x and y

Geometric proportionalities

- Isometric = same geometry, different size



- For a simple cube, various properties can be related to the side length:
 - If you double every dimension, how does area change?
Volume?
 - $area \propto d^2$
 - $volume \propto d^3$
- Does this change for a sphere (radius r)?

Using ratios to solve problems

- From previous slide: How many times more volume of large cube compared to small cube?

$$\frac{V_{big}}{V_{small}} = \frac{(2d)^3}{d^3} = 2^3 = 8$$

- What about two spheres of radii r and $2r$?

$$\frac{V_{big}}{V_{small}} = \frac{\frac{4}{3}\pi(2r)^3}{\frac{4}{3}\pi r^3} = \frac{(2r)^3}{r^3} = 2^3 = 8$$

- In both cases, when length scale increases 2x, the volume increases by factor $(2)^3 = 8$!
- *****The details of the shape don't matter (i.e. the $4/3\pi$ pre-factors cancel when you take a ratio)**

How do other quantities scale?

- Let's define a general length scale, L .
 - Could be side length, radius, diameter – any linear measurement of object's size
- We know that:
 - Area $\propto L^2$
 - Volume $\propto L^3$
- **What about Mass?** $\propto \text{Volume} \propto L^3$
 - Assume that two objects have the same density

Practical application: Clothing your clone

Mini me weighs exactly $1/8^{\text{th}}$ of Dr. Evil's mass. How much more material is needed for Dr. Evil's suit than for mini me's?

$$\frac{M_{DE}}{M_{mm}} = 8 = \left(\frac{L_{DE}}{L_{mm}} \right)^3$$

$$\frac{L_{DE}}{L_{mm}} = 8^{\frac{1}{3}} = 2$$

$$\frac{A_{DE}}{A_{mm}} = \left(\frac{L_{DE}}{L_{mm}} \right)^2 = 2^2 = 4$$



So, four times more material is needed for Dr. Evil's suit than for mini me's.

How do other quantities vary with size?

- Start by defining a length scale, L .
- We know that:
 - Area $\propto L^2$
 - Volume $\propto L^3$
- What about Mass? $\propto \text{Volume} \propto L^3$
- **What about:**
 - **flow into/out of an object – for example, heat flow?**
 - **Production of heat/energy/waste by an object?**

Clicker Quiz

How would you expect the amount of **body heat generated (G)** to scale with the linear dimension, L , of an organism?

- A. $G \propto L^{2/3}$
- B. $G \propto L^2$
- C. $G \propto L^{3/2}$
- D. $G \propto L^3$
- E. I have no idea

Clicker Quiz

How would you expect the amount of ***body heat lost (H)*** to scale with the linear dimension, ***L***, of an organism?

- A. $H \propto L^{2/3}$
- B. $H \propto L^2$
- C. $H \propto L^{3/2}$
- D. $H \propto L^3$
- E. I have no idea

Example: Rate of heat loss

- A grown adult weighs 70 kg while a newborn baby weighs 5 kg.

Compare the

(a) rate of heat loss, and

(b) heat loss rate *per kg* of each.

ratio of masses

$$\frac{M_a}{M_b} = 14 \Rightarrow \frac{L_a}{L_b} = 14^{\frac{1}{3}} = 2.41$$

$$\text{heat loss:} = \left(\frac{L_a}{L_b}\right)^2 = 2.41^2 = 5.81$$

$$\text{heat generated} = \left(\frac{L_a}{L_b}\right)^3 = 14$$



a) Rate of heat loss (H):

- The 3-step process to solving scaling problems:

1. Start with a known ratio:

$$\frac{M_{Big}}{M_{Small}} = \frac{70}{5} = 14$$

2. Get ratio of length scales:

$$\frac{M_B}{M_S} = \left(\frac{L_B}{L_S} \right)^3 \rightarrow \frac{L_B}{L_S} = \left(\frac{M_B}{M_S} \right)^{\frac{1}{3}} = 14^{\frac{1}{3}} = 2.41$$

3. Use this to find ratio of new quantity (H):

$$\frac{H_B}{H_S} = \left(\frac{L_B}{L_S} \right)^2 = 2.41^2 = 5.81$$

So, the big guy loses heat 5.81 times faster

b) Heat Generated:

1. This depends on mass

$$\frac{G_B}{G_S} = \frac{M_B}{M_S} = 14$$

c) Heat Loss per kg (or heat loss per heat generated):

$$H \propto L^2, \quad M \propto L^3 \Rightarrow \frac{H}{M} = \frac{L^2}{L^3} = \frac{1}{L} \rightarrow \frac{(H/M)_B}{(H/M)_S} = \frac{1/L_B}{1/L_S} = \frac{L_S}{L_B} = \frac{1}{2.41} = 0.415$$

- The adult loses more heat, the rate of heat loss per kilogram is greater for the baby. Remember the heat generation is proportional to mass (L^3), so this is exactly what we talked about in class.

Are we blowing this out of proportion?

- Where do we see this in the ‘real world’?

Cell size (from module) – surface area to volume

- there is a practical limit to the size of a cell

Allometry: The study of the relationship of body size to other characteristics of organisms (shape, anatomy, behaviour...)

- Metabolic rate $\propto (\text{Mass})^{3/4}$
- Breathing, heart rate $\propto (\text{Mass})^{1/4}$
- Optimal cruising speed (flight) $\propto (\text{Mass})^{1/6}$

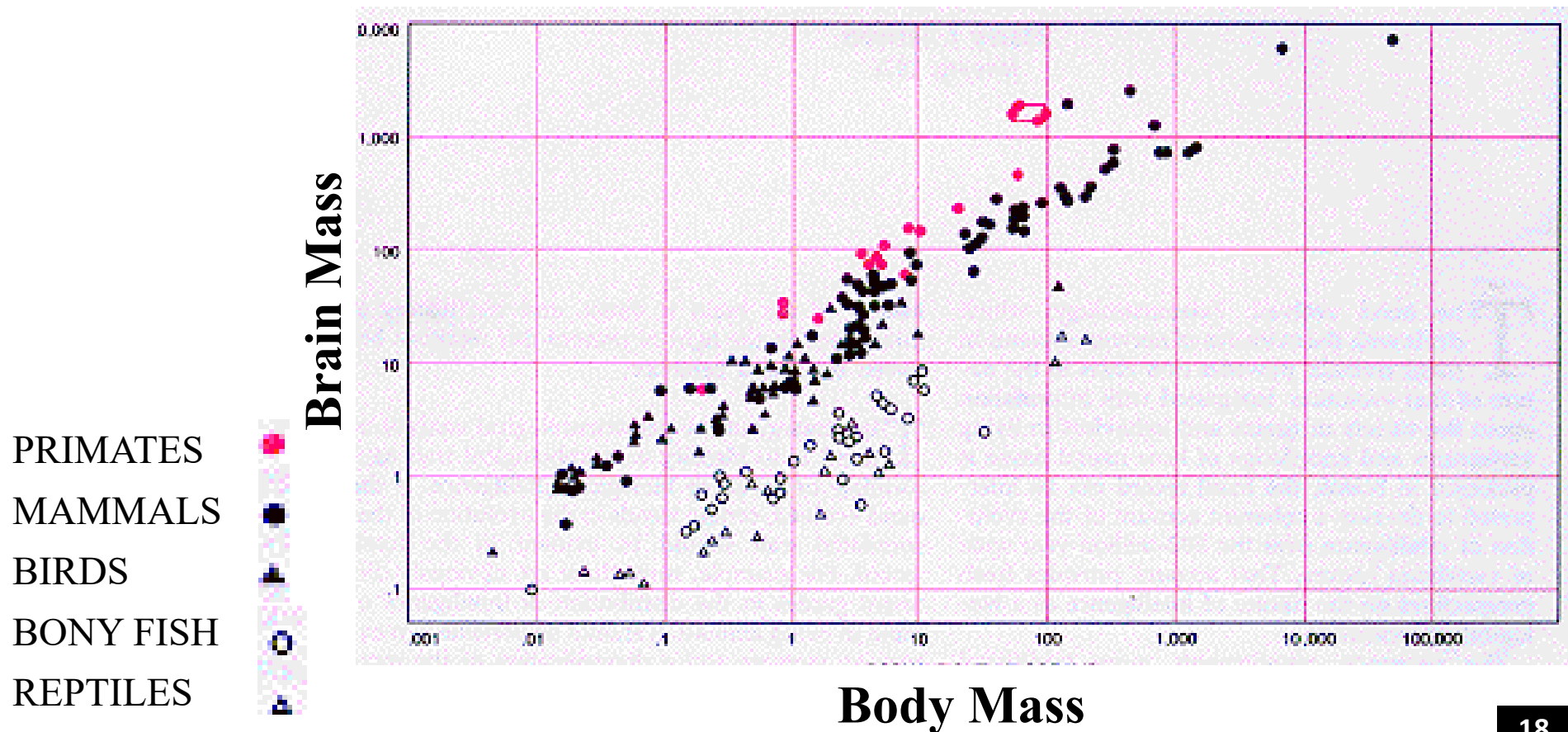
Societal implications

- Density of gas stations vs. city size
- Types of employment, wages vs. city size

What can we infer from these relationships?

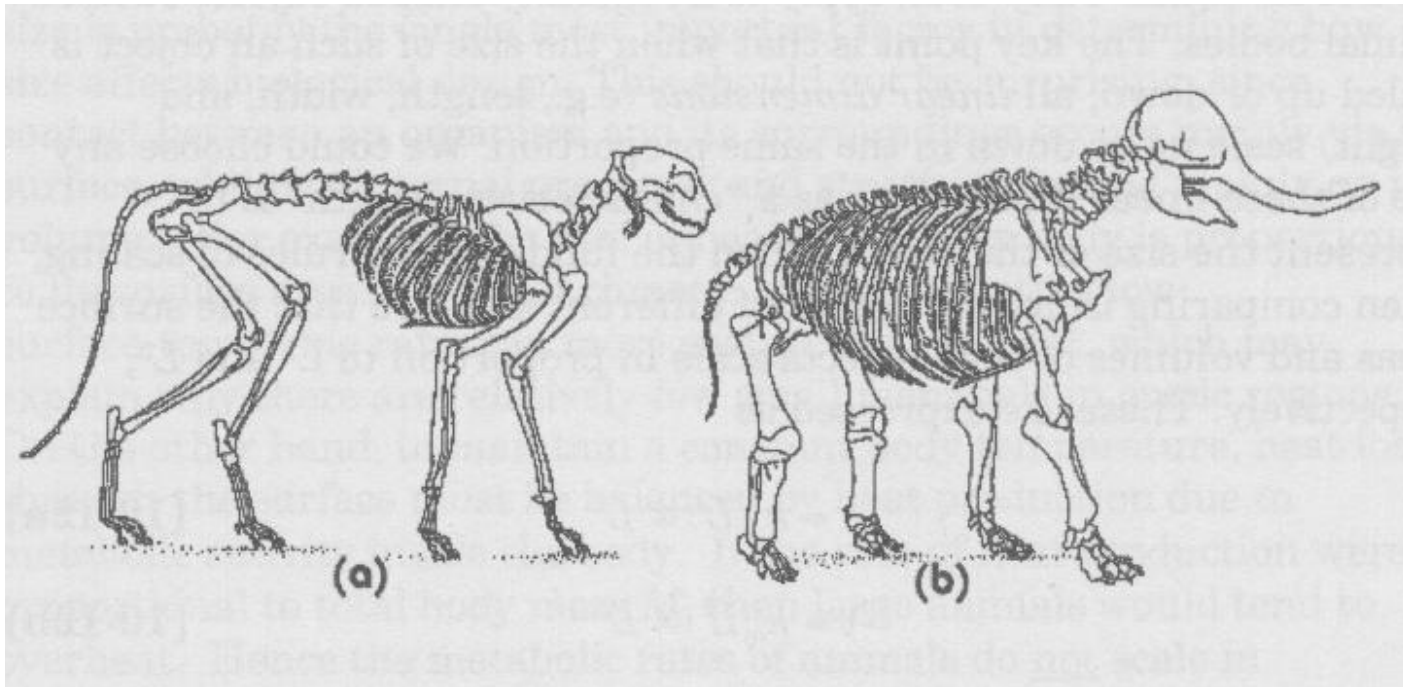
- Understanding how two quantities are connected helps us to understand the nature of the relationship!

What can this tell us about brain mass and intelligence?



Limitations on animal size, and mobility

- Skeleton of a house cat and an elephant



- What does this illustration tell you?