MATHEMATICS 1LS3 TEST 4

Day Class
Duration of Examination: 60 minutes
McMaster University, 25 November 2015

E. Clements, G. Dragomir, M. Lovrić

First name (PLEASE PRINT): SOLUTIONS

Family name (PLEASE PRINT): ______

Student No.: ______

THIS TEST HAS 8 PAGES AND 7 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE.

Total number of points is 40. Marks are indicated next to the problem number. Any non-graphing calculator is allowed.

USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

You must show work to receive full credit.

Problem	Points	Mark
1	4	
2	6	
3	9	
4	3	
5	6	
6	6	
7	6	
TOTAL	40	

Continued on next page

Name:________________
Student No.: ________________

1. **Multiple choice questions: circle ONE answer. No justification is needed.**

(a)[2] It is known that $\left(\frac{3x-1}{2x+1}\right)' = \frac{5}{(2x+1)^2}$. What is the value of $\int_0^{1/3} \frac{1}{(2x+1)^2}\,dx$?

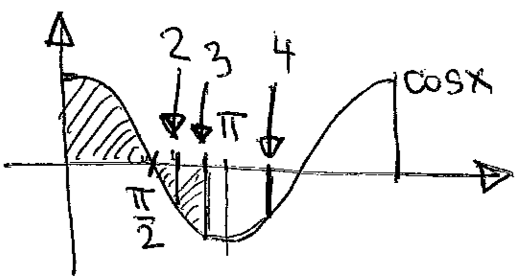
(A) 0	(B) 1/5	(C) 1/3	(D) 1/2
(E) 1	(F) 2	(G) 3	(H) 5

$$\int_0^{1/3} \frac{1}{(2x+1)^2} = \frac{1}{5}\cdot\frac{3x-1}{2x+1}\Big|_0^{1/3} = \frac{1}{5}\cdot 0 - \frac{1}{5}\cdot(-1)$$
$$= \frac{1}{5}$$

(b)[2] Which of the following definite integral(s) is/are positive?

(I) $\int_0^2 \cos x\,dx$ (II) $\int_0^3 \cos x\,dx$ (III) $\int_0^4 \cos x\,dx$

(A) none	(B) I only	(C) II only	(D) III only
(E) I and II	(F) I and III	(G) II and III	(H) all three



$\int_0^2, \int_0^3, \int_0^4$ = signed area

Continued on next page

Name:________________

Student No.: ________________

2. **Identify each statement as true or false (circle your choice). You do not need to justify your answer.**

(a)[2] The following calculation is correct:

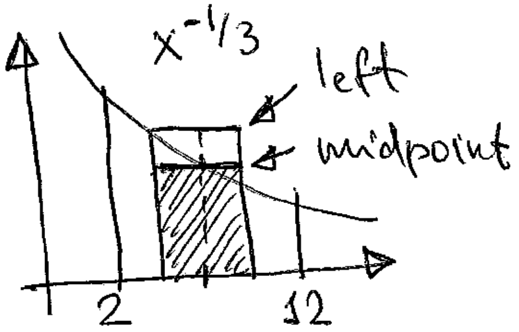
$$\int_{-2}^{1} \frac{1}{x}\,dx = \ln|x|\Big|_{-2}^{1} = \ln|1| - \ln|-2| = -\ln 2$$

not continuous on [-2,1]

TRUE FALSE

(b)[2] The left and the midpoint Riemann sums of $f(x) = x^{-1/3}$ on $[2, 12]$ satisfy $M_{15} < L_{15}$.

TRUE FALSE

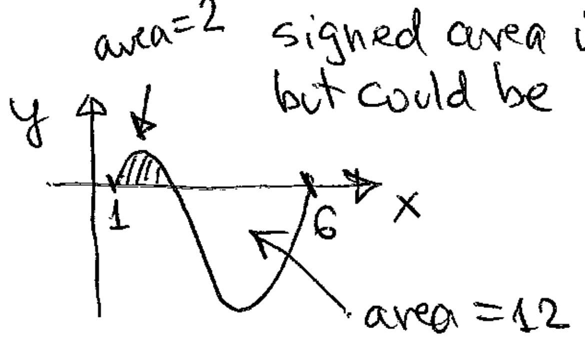


because f(x) is decreasing

(c)[2] It is known that $\int_1^6 f(x)\,dx = -10$. Thus, $f(x) < 0$ for all x in $[1, 6]$.

TRUE FALSE

signed area is -10
but could be



Continued on next page

Name:__________
Student No.: __________

Questions 3-7: You must show work to receive full credit.

3. (a)[2] Find the Taylor polynomial $T_3(x)$ for $f(x) = \sin x$ at $x = 0$.

$f, f', \dots$	at $x=0$
$f = \sin x$	0
$f' = \cos x$	1
$f'' = -\sin x$	0
$f''' = -\cos x$	-1

$$T_3(x) = f(0) + \frac{f'(0)}{1!}(x-0) + \frac{f''(0)}{2!}(x-0)^2 + \frac{f'''(0)}{3!}(x-0)^3$$

$$\rightarrow T_3(x) = x - \frac{x^3}{6}$$

(b)[1] Use your answer in (a) to find a polynomial approximation of $\sin(x^2)$.

$$\sin(x^2) \approx T_3(x^2) = x^2 - \frac{x^6}{6}$$

(c)[3] Use the polynomial from (b) to find an approximation of $\int_0^1 \sin(x^2)\, dx$.

$$\int_0^1 \sin(x^2)dx \approx \int_0^1 \left(x^2 - \frac{x^6}{6}\right)dx$$

$$= \frac{x^3}{3} - \frac{x^7}{6 \cdot 7}\Big|_0^1 = \frac{1}{3} - \frac{1}{42} = \frac{13}{42} \approx 0.31$$

Continued on next page

(d)[3] Use L_4 (left sum with four rectangles) to approximate $\int_0^1 \sin(x^2)\,dx$.

0 0,25 0,5 0,75 1

$$L_4 = \frac{1}{4}\sin(0^2) + \frac{1}{4}\sin(0{,}25^2) + \frac{1}{4}\sin(0{,}5^2) + \frac{1}{4}\sin(0{,}75)^2 \approx 0{,}21$$

4. [3] Find the average value of the function $f(x) = \dfrac{\sqrt{\ln x}}{x}$ on $[1, e]$.

$$\bar{f} = \frac{1}{e-1}\int_1^e \frac{\sqrt{\ln x}}{x}\,dx$$

$$\int_1^e \frac{\sqrt{\ln x}}{x}\,dx = \left\{ \begin{array}{l} u = \ln x \\ du/dx = 1/x\,dx \end{array} \right\} = \int_0^1 \sqrt{u}\,du$$

$$= \frac{2}{3}u^{3/2}\Big|_0^1 = \frac{2}{3}$$

$$\text{so } \bar{f} = \frac{1}{e-1}\cdot\frac{2}{3} = \frac{2}{3(e-1)} \approx 0{,}388$$

Continued on next page

5. The *rate of change* of the number of new individuals infected by a strain H2T1 of influenza A virus is given by the function $p(t) = 120te^{-0.1t}$. The variable t is time in days; the time $t = 0$ represents 1 February 2015.

(a)[4] Find $\int_0^4 120te^{-0.1t}\,dt$

$$= \left\{ \begin{array}{l} u = t \longrightarrow u' = 1 \\ v' = e^{-0.1t} \longrightarrow v = \frac{1}{-0.1} e^{-0.1t} \\ \qquad = -10e^{-0.1t} \end{array} \right\}$$

$$= 120\left(-10t\,e^{-0.1t} + \int 10e^{-0.1t}\,dt\right)$$

$$= 120\left(-10t\,e^{-0.1t} - 100\,e^{-0.1t}\right)\Big|_0^4$$

$$= 120\left(-10(4)\,e^{-0.4} - 100e^{-0.4}\right)$$

$$-120(0 - 100) \approx 738.62$$

(b)[2] What does the answer you obtained in (a) represent?

total number of new individuals infected between $t=0$ and $t=4$ is 738 or 739

$t=0$: 1 Feb 2015 $\quad$ $t=4$: 5 Feb 2015

Continued on next page

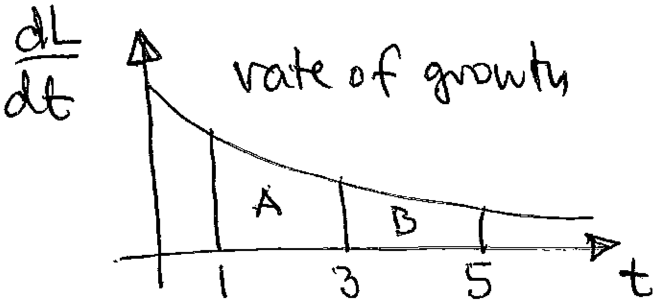
Name:________________
Student No.: ________________

6. The rate of change of the length of wild Pacific salmon is given by

$$\frac{dL}{dt} = 13.2e^{-0.1t+0.5}$$

where t is time in years and L is the length in centimetres.

(a)[2] Without evaluating integrals, explain why a wild Pacific salmon grows more (i.e., gains more in length) from year 1 to year 3, than from year 3 to year 5.



A = total growth from year 1 to year 3

B = total growth from year 3 to year 5

because rate of growth is decreasing, A > B

(b)[4] How much does a wild Pacific salmon grow in length (in cm) from year 3 to year 5?

$$\int_3^5 \frac{dL}{dt}\,dt = 13.2 \int_3^5 e^{-0.1t+0.5}\,dt$$

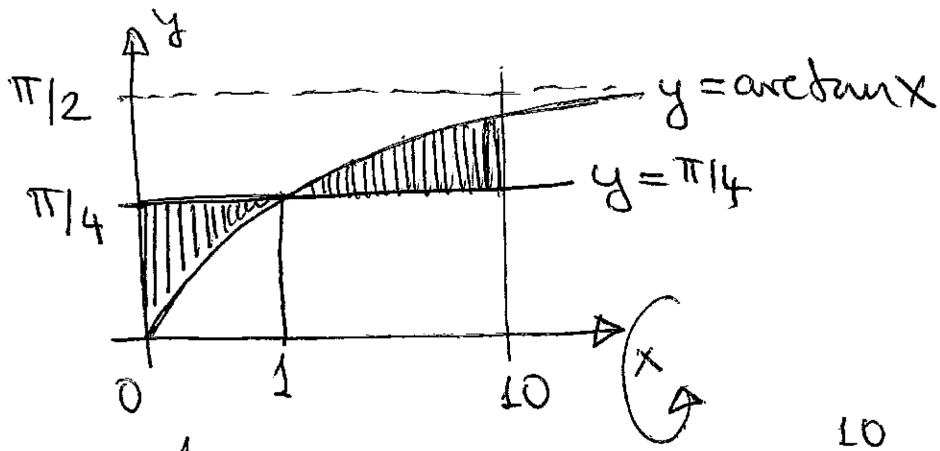
$$= 13.2 \frac{1}{-0.1} e^{-0.1t+0.5}$$

$$= -132 e^{-0.1t+0.5} \Big|_3^5$$

$$= -132e^{0} + 132e^{0.2} \approx 29.2 \text{ cm}$$

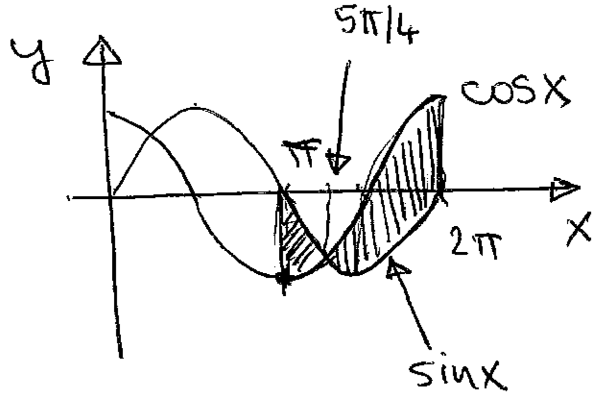
Continued on next page

7. (a)[3] Consider the region R bounded by the graphs of $y = \arctan x$, $x = 0$, $x = 10$, and $y = \pi/4$. Write a formula for the volume of the solid obtained by rotating the region R about the x-axis. You do NOT need to compute the volume.



$$V = \pi \int_0^1 \left(\left(\frac{\pi}{4}\right)^2 - \arctan^2 x\right) dx + \pi \int_1^{10} \left(\arctan^2 x - \left(\frac{\pi}{4}\right)^2\right) dx$$

(b)[3] Sketch (shade) the region bounded by the graphs of $y = \sin x$ and $y = \cos x$ on $[\pi, 2\pi]$. Write a formula for the area of this region. You do NOT need to compute the integral(s).



$\sin x = \cos x$

$\rightarrow \tan x = 1$

$x = \frac{\pi}{4} + \pi k$

$\rightarrow x = \frac{5\pi}{4}$

$$A = \int_{\pi}^{5\pi/4} (\sin x - \cos x)\, dx + \int_{5\pi/4}^{2\pi} (\cos x - \sin x)\, dx$$

THE END