**McMaster University**
**Math 1LT3E**

# ASSIGNMENT 2

## 6.1, 7.1, 7.2, 7.3

1. Write a differential equation and initial condition to describe each situation below. Identify the differential equation as pure-time, autonomous, or nonautonomous.

(a) The volume of a cell decreases at a rate inversely proportional to the time since the beginning of the experiment, at which time the cell had a volume of 100 $\mu$m$^3$.

(b) Volcanic mud, initially at $125^o C$, cools in contact with the surrounding air, whose temperature is $10^o C$. The rate of change of the temperature of the mud is proportional to the difference between the temperature of the mud and the temperature of the surrounding air.

2. Find an algebraic solution for the following initial value problems.

(a) $y' = 4x^3 - \sqrt{x} + \dfrac{1}{x}$ where $y(1) = 5$

(b) $\dfrac{dP}{dt} = 10te^{0.5t}$ where $P(0) = 500$

3. Verify that $y = \dfrac{1+e^x}{1-e^x}$ is a solution of the differential equation $y' = \dfrac{1}{2}y^2 - \dfrac{1}{2}$.

4. Consider the initial value problem $\dfrac{dP}{dt} = \dfrac{P}{1+t^2}$; $P(1) = 100$.

Use Euler's Method with a step size of $h = 0.5$ to find an approximation of $P(2)$.

5. Assuming that there is competition within a population of bacteria for resources, a model for limited bacterial growth is given by

$$\frac{db}{dt} = \lambda(b)b$$

where the per capita production rate, $\lambda$, is a decreasing function of the population size, $b$.

Suppose that the per capita production rate is a linear function of population size with a maximum of $\lambda(0) = 1$ and a slope of $-0.002$.

(a) Find $\lambda(b)$ and write the differential equation for $b$.

(b) For what values of $b$ will the population remain constant over time? For what values of $b$ will the population be increasing? Decreasing?

6. The modified logistic differental equation describing the growth of a population of sharks is given by

$$\frac{dP}{dt} = 0.8P\left(1 - \frac{P}{400}\right)\left(1 - \frac{30}{P}\right).$$

(a) Determine the equilibria for this population. What do these numbers represent?

(b) [2] Draw a phase-line diagram for this differential equation.

(c) Sketch the equilibrium solutions and typical solution curves corresponding to the initial conditions $P(0) = 20$, $P(0) = 300$, and $P(0) = 500$.

(d) Describe the dynamics of this population in words.

7. Consider the autonomous differential equation $\dfrac{dy}{dt} = ye^{-\beta y} - ay$, where $a$ and $\beta$ are parameters.

(a) Find the equilibria.

(b) Use the stability theorem to evaluate the stability of the equilibria. (Note: The stability depends on the parameter $a$ so you will need to consider different cases.)

(c) Suppose that $a = 0.5$ and $b = 1$. Draw the phase-line diagram for $\dfrac{dy}{dt} = ye^{-\beta y} - ay$.

(d) Suppose that $a = e$ and $b = 1$. Draw the phase-line diagram for $\dfrac{dy}{dt} = ye^{-\beta y} - ay$.

THE END