# MATHEMATICS 1LS3 TEST 1

Day Class
Duration of Examination: 60 minutes
McMaster University, 7 October 2015

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First name (PLEASE PRINT): _____

Family name (PLEASE PRINT): _____

Student No.: _____

THIS TEST HAS 8 PAGES AND 7 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE.

Total number of points is 40. Marks are indicated next to the problem number. Any non-graphing calculator is allowed.

USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

**You must show work to receive full credit.**

| Problem | Points | Mark |
|---|---|---|
| 1 | 4 | |
| 2 | 6 | |
| 3 | 8 | |
| 4 | 4 | |
| 5 | 6 | |
| 6 | 5 | |
| 7 | 7 | |
| TOTAL | 40 | |

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1. **Multiple choice questions: circle ONE answer. No justification is needed.**

(a)[2] Identify all correct statements about the function $f(x) = \dfrac{x^2-1}{x^2+1}$.

should be y=1

(I) $f(x)$ is continuous at $x = 0$.

(II) $x = -1$ is a vertical asymptote of the graph of $f(x)$.

(III) $x = 1$ is a horizontal asymptote of the graph of $f(x)$

(A) none (B) I only (C) II Only (D) III only
(E) I and II (F) I and III (G) II and III (H) ALL

(b)[2] The average half-life of acetaminophen (active ingredient in tylenol) is 2.5 hours. Assume that a patient is given a dose of 1000 mg of acetaminophen. Identify all correct statements.

(I) After 5 hours, 250 mg of acetaminophen is left unabsorbed in patient's body.

(II) After 2 hours, 450 mg of acetaminophen is left unabsorbed in patient's body.

(III) After 10 hours, less than 100 mg of acetaminophen is left unabsorbed in patient's body.

(A) none (B) I only (C) II Only (D) III only
(E) I and II (F) I and III (G) II and III (H) ALL

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2. **Identify each statement as true or false, or yes or no (circle your choice). You do not need to justify your answer.**

(a)[2] The fact that $\lim_{x\to 2} f(x) = 3$ implies that $f(x)$ is continuous at $x = 2$.

TRUE OR FALSE

(b)[2] The formula $H = Mx + 4$, where $M$ is a constant, represents a proportional relationship between $H$ and $x$.

TRUE OR FALSE

(c)[2] The average density of a human jaw bone is 1.07 oz/in$^3$, which is equivalent to approximately 1.85 g/cm$^3$. [Conversion factors: 1 oz = 28.35 g, 1 in = 2.54 cm]

TRUE OR FALSE

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**Questions 3-7: You must show work to receive full credit.**

3. Consider the function

$$f(x) = \begin{cases} \dfrac{x^3 - x}{x - 1} & \text{if} \quad x < 1 \\ 2x & \text{if} \quad x \geq 1 \end{cases}$$

(a)[4] Find $\lim_{x \to 1} f(x)$.

(b)[2] Is $f(x)$ continuous at $x = 1$? Explain why or why not.

(c)[2] Is $f(x)$ continuous at $x = -1$? Explain why or why not.

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4. The following excerpt is taken from *The laminar cortex model: a new continuum cortex model incorporating laminar architecture.* J. Du, V. Vegh, and D.C. Reutens. PLoS Computational Biology. 8.10 (Oct. 2012).

> the average of membrane potentials of neurons in the element, that is
>
> $$V = \frac{N_e V_e + N_i V_i}{N_e + N_i}$$
>
> where $N_e$, $N_i$ are the numbers of excitatory and inhibitory neurons and $V_e$ and $V_i$ are the (average) membrane potentials of excitatory and inhibitory neuron populations respectively.

View $V$ as a function of $N_i$.

By reading the text, we learn that $V$ is the average of membrane potentials and $N_i$ is the number of inhibitory neurons.

(a)[1] State (in one sentence) what question is answered by finding the inverse function of $V$.

(b)[3] Find a formula for the inverse function of $V$.

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5. (a)[3] Consider the formula for human population growth

$$P(t) = 4.43\left(\frac{\pi}{2} + \arctan\frac{t-2007}{42}\right)$$

where $t$ is a calendar year and $P(t)$ is in billions. Find the range of $P(t)$. Based on it, state the maximum world population predicted by this model.

(b)[3] A population of river sharks (freshwater sharks) in New Zealand changes periodically with a period of 12 months. In January, it reaches a maximum of 12,600, and in July it reaches a minimum of 5,800. By selecting an appropriate trigonometric function, find a formula which describes how the population of river sharks changes with time.

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6. The survival rate (i.e., percent) $S(D)$ of clonogenic cells (cancer cells) is modelled by

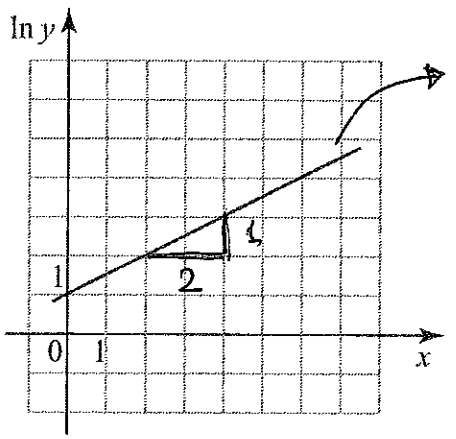
$$S(D) = e^{-0.6D}$$

where $D \geq 0$ represents the applied radiation dose (measured in grays, Gy).

(a)[1] Assume that the dose $D = 5$ Gy is applied to a cancer. What percent of cancer cells is going to survive this treatment? Nearest integer,

(b)[2] Sketch the semilog graph (use ln) of the survival rate. Label axes,

(c)[2] The linear graph below is a semilog graph of a function. Find an explicit formula for that function (i.e., write it in the form $y = \ldots$).

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7. (a)[3] Identify the domain of the function $f(x) = \ln(e^x - 1)$.

(b)[2] Find $\lim_{x \to \pi/2^+} \tan x$

(c)[2] Find $\lim_{x \to \infty} e^{-x^2}$

THE END