Name: _____ Student Number:_____

MATH 1AA3, 1AA3E, 1ZB3 & 1ZB3E

Winter Session, 2017

Test #1, Seating #1,Version #1

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TERM TEST
DAY & EVENING CLASSES
DURATION OF TEST: 90 **min (**1.5 **hrs.**)
MCMASTER UNIVERSITY TERM TEST

Monday, February 6, 2017

THIS TEST INCLUDES 6 PAGES AND 20 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING YOUR COPY OF THE TEST IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

Instructions:

1. **NO** calculator is allowed to be used on this test.

2. Make sure your name and student number at the ***top of each page.***

3. For all questions, mark the answer in pencil on the OMR answer sheets according to the OMR instructions on page 2. Only solutions on the scan card will be graded.

4. Each correct answer is worth one mark.

5. A blank answer is an automatic zero for any question, even if the correct solution is circled on the question itself.

6. Incorrect or multiple answers are also worth zero marks. No negative marks or part marks will be assigned.

7. Scrap paper for rough work has been provided. *All* rough work, and this question sheet must be handed in with the test, but any solutions written either here or on the rough paper will ***NOT*** be graded.

8. *Good Luck!*

PLEASE READ THE OMR INSTRUCTIONS ON PAGE #2

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Name: _____ Student Number:_____

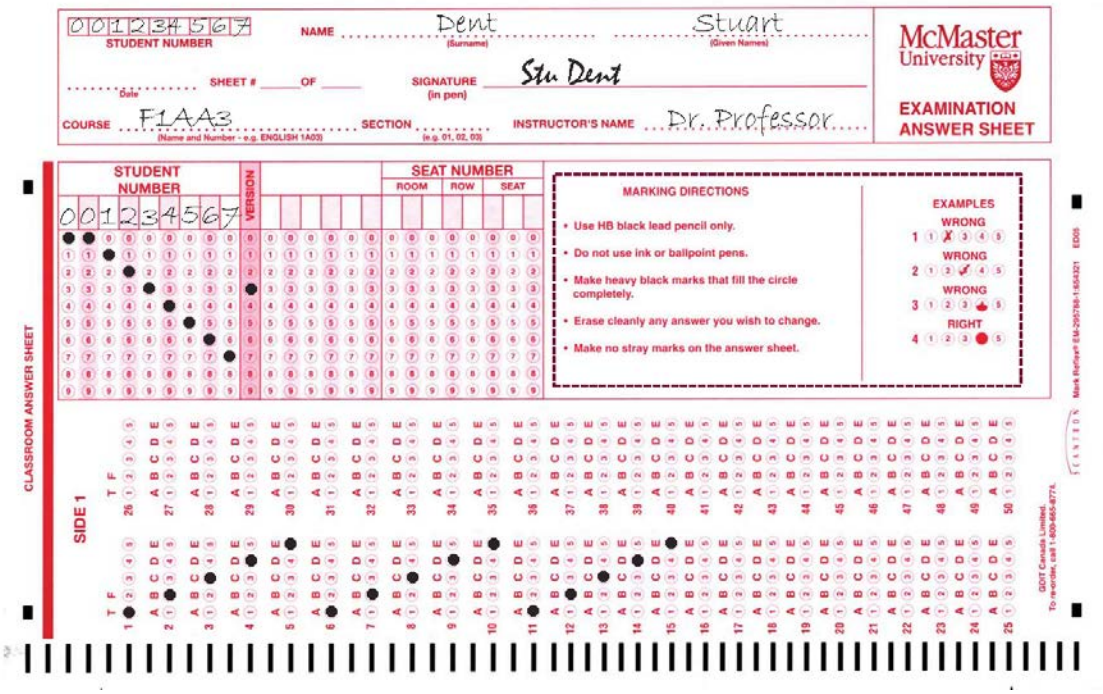
OMR EXAMINATION INSTRUCTIONS

NOTE: IT IS YOUR RESPONSIBILITY TO ENSURE THAT THE ANSWER SHEET IS PROPERLY COMPLETED: YOUR EXAMINATION RESULT DEPENDS ON PROPER ATTENTION TO THESE INSTRUCTIONS.

The scanner which reads the sheets senses the bubble-shaded areas by their non-reflection of light. A heavy mark must be made, completely filling the circular bubble, with an **HB pencil**. Marks made with a pen or a felt–tip marker will **NOT** be sensed. Erasures must be thorough or the scanner may still sense a mark. Do **NOT** use correction fluid on the sheets. Do **NOT** put any unnecessary marks or writing on the sheet.

1. On side 1 (red side) of the form, in the top box, in pencil, print your student number **(NOTE: 9 digits)**, name, course name, section number, instructor name and date in the spaces provided. Then you **MUST** sign in the space marked SIGNATURE**.**
2. In the second box, with a pencil, mark your student number, exam version number and course section number in the space provided and fill in the corresponding bubble numbers underneath.
3. To indicate your answers, mark only **ONE** choice from the alternatives (1, 2, 3, 4, 5, or A, B, C, D, E) provided for each question. The question number is to the left of the bubbles. Make sure that the number of the question on the scan sheet is the same as the question number on the test paper.
4. Pay particular attention to the Marking Directions on the form.
5. Begin answering the questions using the first set of bubbles, marked “1”.

SAMPLE OMR CARD ONLY: DO NOT USE



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Name: _____ Student Number:_____

1. Given $\sum_{n=1}^{m} a_n = \frac{2m-1}{m+2}$, find the value of a_2.

a) 1/4 **b)** 3/4 **c)** 5/12 **d)** 1 **e)** 1/3

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2. Which of the following series converge: $I)\ \sum_{n=2}^{\infty} \frac{n^3+2n+1}{5n^2+2n^5+6}$ $II)\ \sum_{n=1}^{\infty} \frac{\arctan(n)}{n}$

a) I only **b)** II only **c)** Both I and II **d)** Neither **e)** Cannot be determined

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3. If the series $S = \sum_{n=1}^{\infty} \frac{1}{n^3}$ is approximated by the partial sum $S_5 = \sum_{n=1}^{5} \frac{1}{n^3}$, use the integral error estimate to find an upper bound on the error $S - S_5$.

a) $\frac{1}{25}$ **b)** $\frac{1}{216}$ **c)** $\frac{1}{125}$ **d)** $\frac{1}{72}$ **e)** $\frac{1}{50}$

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4. Given $a_{n+1} = \frac{4a_n - 3}{a_n}$, and $a_2 = 2$, find $\lim_{n\to\infty} a_n$ if it exists, or state it diverges.

a) 0 **b)** 1 **c)** 2 **d)** 3 **e)** Diverges

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5. Given that $b_n > b_{n+1}$, and $0 < b_n < 1/n$ which of the following statements must **always** be true for the series: $\sum_{n=1}^{\infty} (-1)^n b_n$?

a) The series must diverge.
b) The series only converges conditionally.
c) The series must converge absolutely.
d) The series either diverges or it converges conditionally.
e) The series converges either conditionally or absolutely

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Name: _____ Student Number:_____

6. Which of the sequences converge: *I)* $a_n = (-1)^n\left(1 - \frac{1}{n^2}\right)$ *II)* $b_n = \cos\big((2n+1)\pi\big)$

a) I only **b)** II only **c)** Both I and II **d)** Neither **e)** Cannot be determined

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7. Evaluate the improper integral: $\int_0^e x\ln(x)\,dx$, or state it is divergent.

a) 0 **b)** $-1/4$ **c)** $e^2/4$ **d)** -1 **e)** Divergent

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8. Given the sequence: $a_{n+1} = \frac{1}{3}(4 + a_n),\ a_1 = 5$, we wish to show that it is monotonic, using mathematical induction. Which of the following statements corresponds to our induction step?

a) Assume $a_k \le a_{k+1}$, and show $\frac{1}{3}(4 + a_k) \le \frac{1}{3}(4 + a_{k+1})$

b) Assume $a_k \le a_{k+1}$, and show $\frac{1}{3}(4 + a_{k+1}) \le \frac{1}{3}(4 + a_{k+2})$

c) Assume $a_k \ge a_{k+1}$, and show $\frac{1}{3}(4 + a_{k+1}) \ge \frac{1}{3}(4 + a_{k+2})$

d) Assume $a_k \ge a_{k+1}$, and show $\frac{1}{3}(4 + a_k) \ge \frac{1}{3}(4 + a_{k+1})$

e) Assume $a_k \le a_{k+1}$, and show $\frac{1}{3}(4 + a_{k+1}) \le \frac{1}{3}(4 + a_k)$

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9. Which of the series converge: *I)* $\sum_{n=1}^{\infty} \frac{\sqrt{n^4 + n}}{n^3}$ *II)* $\sum_{n=1}^{\infty}\left(\frac{\sin(5n)}{3n}\right)^2$

a) I only **b)** II only **c)** Both I and II **d)** Neither **e)** Cannot be determined

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10. Evaluate the improper integral: $\int_0^{\infty} \frac{\arctan(x)}{1 + x^2}dx$, or state it is divergent.

a) 0 **b)** π **c)** $\frac{\pi^2}{8}$ **d)** $-\frac{\pi}{2}$ **e)** Diverges

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Name: _____ Student Number:_____

11. According to the alternating series error estimate, what is the minimum number of terms required to estimate the value of $\sum_{n=1}^{\infty} \frac{(-1)^n}{n2^n}$ within an error < 0.01?

a) 2 **b)** 3 **c)** 4 **d)** 5 **e)** 6

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12. Which of the following integrals converge: $I) \int_0^1 \frac{1+x^{3/2}}{x^2}dx$ $II) \int_2^{\infty} \frac{1+\sin x}{x^2}dx$

a) I only **b)** II only **c)** Both I and II **d)** Neither **e)** Cannot be determined

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13. Find the sum of the series: $\sum_{n=1}^{\infty} \frac{3^{n+1}}{4^{n-1}}$

a) 36 **b)** $\frac{27}{2}$ **c)** $\frac{1}{4}$ **d)** 3 **e)** Diverges

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14. Which of the series converge: $I) \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ $II) \sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln(n)}}$

a) I only **b)** II only **c)** Both I and II **d)** Neither **e)** Cannot be determined

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15. Which of the series converge: $I) \sum_{n=1}^{\infty} \left(\frac{4+n^2}{3n^2+1}\right)^n$ $II) \sum_{n=1}^{\infty} \frac{(2+n)^2}{n!}$

a) I only **b)** II only **c)** Both I and II **d)** Neither **e)** Cannot be determined

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16. Find the radius of convergence of the series: $\sum_{n=1}^{\infty} \frac{(2x-3)^n}{2n+4}$

a) 1/2 **b)** 3/2 **c)** 1/4 **d)** 1 **e)** 1/3

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Name: _____ Student Number:_____

17. In MAPLE, which of the following will calculate the fifth partial sum of a series with terms $a_n = n/(n+1)!$?

a) partial(n/(n+1)!,5); **b)** add(n/(n+1)!,5); **c)** sum(n/(n+1)!,n=1..5);

d) sum(n/(n+1)!,5); **e)** add(n/(n+1)!,,n=1..5);

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18. Find the value of the sum of the following series: $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$ or state it diverges.

a) 0 **b)** 1 **c)** e **d)** 3 **e)** Diverges

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19. A series of the form: $\sum_{n=1}^{\infty} c_n$ is known to converge. If each c_n may have values that are positive or negative, which one of the following statements must also **always** be true:

I) $\sum_{n=1}^{\infty} (-1)^n c_n$ must converge II) $\sum_{n=1}^{\infty} \frac{c_n}{2^n}$ must converge. III) $\sum_{n=1}^{\infty} 2^n c_n$ must diverge.

a) I only **b)** II only **c)** III only **d)** Both I and II **e)** All of I, II and III

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20. Which of the series converges only conditionally: $I)\ \sum_{n=1}^{\infty} (-1)^n \frac{n}{2^n}$ $II)\ \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$

a) I only **b)** II only **c)** Both I and II **d)** Neither **e)** Cannot be determined

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THE END

1AA3/1ZB3 Test #1 Answer Key

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
C	A	E	D	E	B	C	D	B	C

Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20
C	B	A	A	C	A	C	E	B	B