Example: Find

1. $\sum_{n=1}^{\infty} \frac{1}{5^n}$

2. $\sum_{n=0}^{\infty} 2^{n+1} 3^{-n}$

A **telescoping series** is a series, where the terms can be written as $a_n = c_n - c_{n+1}$ for some c_n.

Example: $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

Result: Assume $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n =$, because

$\Rightarrow$ **Test for Divergence**

Example: $\sum_{n=1}^{\infty} \frac{n}{n+1}$

Example: $\sum_{n=1}^{\infty} (-1)^n$

If $\lim_{n\to\infty} a_n = 0$, can we conclude that $\sum_{n=1}^{\infty} a_n$ converges?

Example: $\sum_{n=1}^{\infty} \frac{1}{n}$

Conclusion/Rule:

Limit Rules for Series: Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be convergent. Then,

a) $\sum_{n=1}^{\infty} (a_n + b_n) =$

b) $\sum_{n=1}^{\infty} (a_n - b_n) =$

c) $\sum_{n=1}^{\infty} c a_n =$

Example: $\sum_{n=1}^{\infty} \frac{2}{3^n} - \frac{1}{2^{n+1}}$

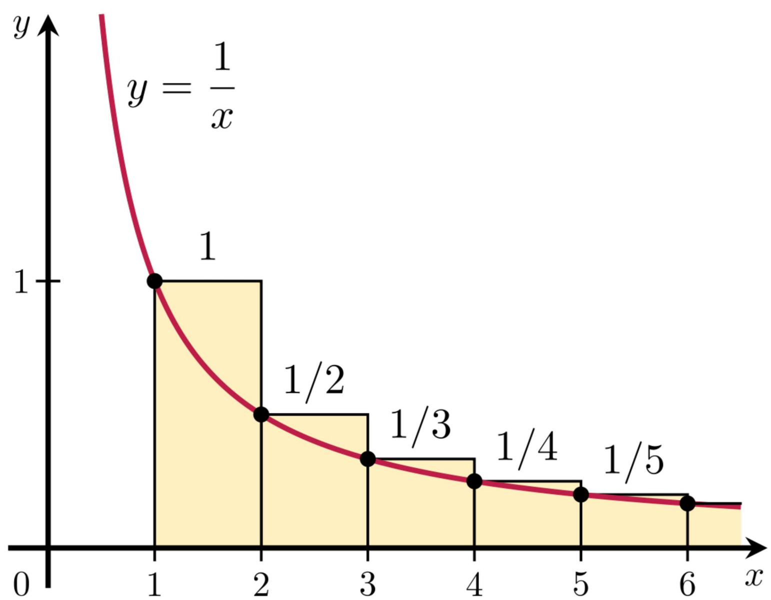
4.2 The Integral Test and Estimates of Sum (Chapter 11.3)

The Integral Test: Let $f(x)$ be a continuous, positive, decreasing function defined for $x \geq 1$. Let $a_n = f(n)$ for $n = 1, 2, 3, \ldots$.

Then, the **series** $\sum_{n=1}^{\infty} a_n$ is __________

if and only if

____



Example: Revisit $\sum_{n=1}^{\infty} \frac{1}{n}$

Example: $\sum_{n=1}^{\infty} \frac{1}{n^2}$

Does the integral test inform us about the value of the convergent series?

Example: $\sum_{n=3}^{\infty} \frac{1}{n\ln(n)}$