MATHEMATICS 1LS3 TEST 4

Day Class
Duration of Examination: 60 minutes
McMaster University, 19 November 2014

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FIRST NAME (please print): _____

FAMILY NAME (please print): _____

Student No.: _____

THIS TEST HAS 8 PAGES AND 7 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE.

Total number of points is 40. Marks are indicated next to the problem number. Any non-graphing calculator is allowed.

USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

You must show work to receive full credit.

Problem	Points	Mark
1	6	
2	6	
3	5	
4	5	
5	6	
6	7	
7	5	
TOTAL	40	

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1. **Multiple choice questions: circle ONE answer. No justification is needed.**

(a)[3] Which of the following limits is/are indeterminate form(s)?

(I) $\lim_{x\to 0} \dfrac{\cos x - x}{x}$ (II) $\lim_{x\to 0} \dfrac{e^x - 1 - x}{x^2}$ (III) $\lim_{x\to 0} \dfrac{\sec x - 1}{\tan x}$

(A) none (B) I only (C) II only (D) III only
(E) I and II (F) I and III (G) II and III (H) all three

(b)[3] Which of the following numbers is/are positive?

(I) $\int_0^1 (e^x - 1)\, dx$ (II) $\int_{0.1}^1 (\ln x - 1)\, dx$ (III) $\int_0^1 (\sec x - 1)\, dx$

(A) none (B) I only (C) II only (D) III only
(E) I and II (F) I and III (G) II and III (H) all three

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2. **Identify each statement as true or false, or yes or no (circle your choice). You do not need to justify your answer.**

(a)[2] It is known that $\int_a^b f(x)dx < 0$. This implies that the function $f(x)$ is negative, i.e., $f(x) < 0$ for all x in $[a, b]$.

TRUE FALSE

(b)[2] Consider $f(x) = e^{-0.8x}$ on $[0, 1]$. The left sum L_{10} is larger than the right sum R_{50}.

TRUE FALSE

(c)[2] The function $f(x) = \dfrac{1}{4-x}$ is one possible antiderivative of $g(x) = \dfrac{1}{(4-x)^2}$.

TRUE FALSE

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Questions 3-7: You must show work to receive full credit.

3. The rate at which new influenza cases occured in 2013 in Greater Vancouver Area follows the formula $125.4e^{0.3t} + 14.6e^{-0.1t}$ people/day. By t we represent the time in days measured from 1 December 2013 (so $t = 0$ represents 1 December 2013). On 1 December 2013 there were 56 cases of influenza.

(a)[2] Write a differential equation and the initial condition for the number $N(t)$ of influenza cases.

(b)[3] Solve the initial value problem in (a) to find the formula for $N(t)$.

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4. Consider the initial value problem $y' = 1 - t + t^2$, $y(0) = 2$.

(a)[2] Find an approximation of $y(0.4)$ using two steps of Euler's method with the step size $\Delta t = 0.2$. Do not round off the numbers, keep all decimals.

(b)[2] Using antidifferentiation, find the exact solution of the given initial value problem.

(c)[1] Using (b), find the true value of $y(0.4)$ and round off to three decimal places (thus checking your approximation in (a)).

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5. Consider the differential equation

$$P'(t) = 1.1P(t)\left(1 - \frac{P(t)}{1400}\right)^{1/3}$$

where $P(t)$ represents the number of elk in Douglas Provincial Park in Saskatchewan. The variable t represents time in years, with $t = 0$ representing 2006.

(a)[1] Classify the above differential equation as pure-time, autonomous, or neither pure-time nor autonomous.

(b)[2] For which values of $P(t)$ is the population increasing? Justify your answer.

(c)[2] For which values of $P(t)$ is the population decreasing? Justify your answer.

(d)[1] What is the biological meaning of the constant 1400?

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6. (a)[2] Find $\int_0^1 \left(\frac{5}{1+x^2} + \frac{1+x^2}{5}\right)\, dx$

(b)[2] Find $\int (\sec x \tan x + \pi)\, dx$

(c)[3] Compute $\int_{-2}^{4} (4 - 2|x|)\, dx$ by interpreting the definite integral in terms of area(s).

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7. Find the following limits.

(a)[3] Find $\lim_{x\to 0} \dfrac{e^{x^3} - 1 - x^3}{x^6}$

(b)[2] Find $\lim_{x\to 0^+} x^4 \ln x$