Summary of Theorems about Series (as of 02/07)

Definition $\sum a_n = \lim_{N\to\infty} \underbrace{\sum^{N} a_n}_{=S_N}$.

$\Longrightarrow$ $\sum a_n$ converges if $\{S_N\}$ converges.

Special Series

Geometric Series $\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$ $(|r| < 1)$

Harmonic Series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges

p-Test

$$\sum \frac{1}{n^p} \quad \begin{cases} \text{converges} & p > 1 \\ \text{diverges} & p \leq 1 \end{cases}$$

Test of Divergence

If $\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ diverges.

Integral Test

If $a_n = f(n)$, where f is positive, continuous, and decreasing.

Then $\sum_{n=1}^{\infty} a_n$ converges if and only if $\int_1^{\infty} f(x)\,dx$ converges.

Comparison Test

Let $0 \leq a_n \leq b_n$.

If $\sum b_n$ converges, then $\sum a_n$ converges.

If $\sum a_n$ diverges, then $\sum b_n$ diverges.

Limit Comparison Test

Let $0 < a_n, b_n$.

If $\lim_{n\to\infty} \frac{a_n}{b_n} = c \in (0, \infty)$, then $\sum a_n$ and $\sum b_n$ either both converge or they both diverge.

Alternating Series Test

Let $b_n > 0$.

If $b_{n+1} \leq b_n$ and $\lim_{n\to\infty} b_n = 0$, then $\sum (-1)^{n-1} b_n$ converges.