THEOREM

The Intermediate Value Theorem (IVT)

Suppose that f is **continuous** on the **closed** interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c in the **open** interval (a, b) such that $f(c) = N$.

FORMULA

Trig Identity Between $\cos^2(\theta)$ *and* $\sin^2(\theta)$

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

FORMULA

Trig Identity Between $\tan^2(\theta)$ *and* $\sec^2(\theta)$

$$1 + \tan^2(\theta) = \sec^2(\theta)$$

FORMULA

Double Angle Formula for $\sin(x)$

$$\sin(2x) = 2\sin(x)\cos(x)$$

Definition

Hyperbolic Sine

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

Definition

Hyperbolic Cosine

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

Formula

Identity Between $\cosh^2(x)$ *and* $\sinh^2(x)$

$$\cosh^2(x) - \sinh^2(x) = 1$$

Formula

Derivative of Inverse Function

$$\frac{d}{dx} f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}$$

Formula

Newton's Method Formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Definition

Derivative as a Limit

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

or

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

Question

What is x_{n+1} in Newton's Method?

x_{n+1} in the equation

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

is the x-intercept of the tangent line drawn to the function f at the point $(x_n, f(x_n))$.

Rule

Constant Multiple Rule

If c is a constant, then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

Rule

Power Rule

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

Rule

Sum Rule and Difference Rule

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

Rule

Product Rule

$$\frac{d}{dx}[f(x) \cdot g(x)] = f(x) \cdot \frac{d}{dx}[g(x)] + g(x) \cdot \frac{d}{dx}[f(x)]$$

Rule

Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) \cdot \frac{d}{dx}[f(x)] - f(x) \cdot \frac{d}{dx}[g(x)]}{[g(x)]^2}$$

Rule

Chain Rule

$$[f(g(x))]' = f'(g(x)) \cdot g'(x)$$

Algorithm

Implicit Differentiation

Step 1: Take the derivative of both sides of the equation

Step 2: Solve for the derivative

Algorithm

Logarithmic Differentiation

Step 1: Apply $\ln(x)$ to both sides of the equation

Step 2: Take the derivative of both sides of the equation

Step 3: Solve for the derivative

Note:

$$\frac{d}{dx}\ln(f(x)) = \frac{f'(x)}{f(x)}$$

Algorithm

How to find the equation of a tangent line?

To find the equation of the tangent line to the function $f(x)$ at the point (x_0, y_0)...

Step 1: Calculate $f'(x_0)$

Step 2: Use the formula

$$y = f'(x_0) \cdot (x - x_0) + y_0$$

Note: This is a rewritten version of the equation

$$y - y_0 = m(x - x_0)$$