12A3

Intervals of Inc. / Dec.

Method

Look for c.p. & discont ⇒ where $f'(x) = 0$
or
$f'(x)$ DNE

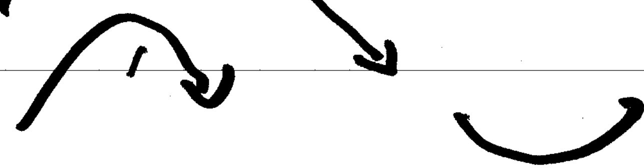
⇒ break into intervals of inc/dec
(ie $f'(x) > 0$ or $f'(x) < 0$ same sign on
entire interval.

⇒ check sign! $f' > 0 \Rightarrow$ inc
$f' < 0 \Rightarrow$ dec.

eg <u>Last day</u> $y = x^3 - 3x + 6$

$y' = 0 = 3x^2 - 3$ at $\underline{x = \pm 1}$

	$(-\infty, -1)$	$(-1, \cancel{0}\,1)$	$(1, \infty)$
f'	+	−	+
f	~~dec~~ inc	~~inc~~ dec	~~dec~~. inc.
	↗	↘	↗

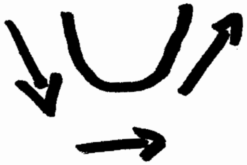


max | min

The First Derivative Test

If $x = c$ is a c.n. & $f'(x) < 0$ for $x < c$

$f'(x) > 0$ for $x > c$



for all x in some neighbourhood of c

(ie small open interval)

Then we have a local (relative) minimum point!

Similarly if $f'(x) > 0,\ x < c$ & $f'(x) < 0,\ x > c$

for all x in a neighbourhood of c, then

Then we have a local maximum point!

eg. Given $f(x) = x^{2/3}$ $\left(= (x^2)^{1/3} \right)$

find all c.n. & classify as local max, local min or other

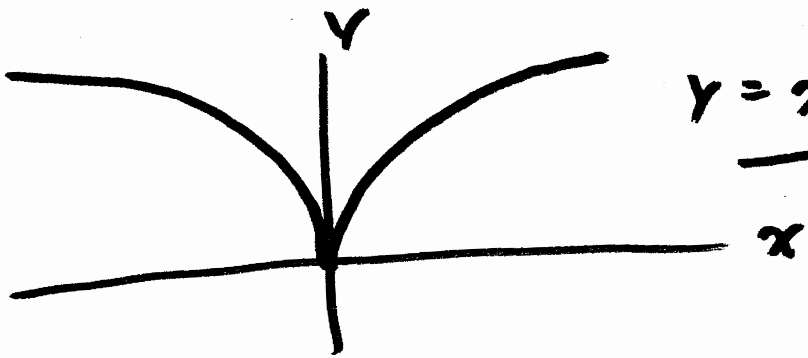
<u>Solution</u> $f'(x) = \frac{2}{3} x^{-\frac{1}{3}} = \frac{2}{3x^{1/3}}$

$f'(x) \neq 0$, $f'(x)$ DNE at $\underline{\underline{x = 0}}$ $\leftarrow$ $f\underline{\underline{(0) = 0}}$

$\Rightarrow$ in domain!

	$(-\infty, 0)$	$(0, \infty)$
$f' = \frac{2}{3x^{1/3}}$	$-$	$+$
$f(x)$	dec $\searrow$	inc $\nearrow$

$\Rightarrow$ at $x = 0$ **local min**



$y = x^{2/3}$

2nd Derivative & Concavity

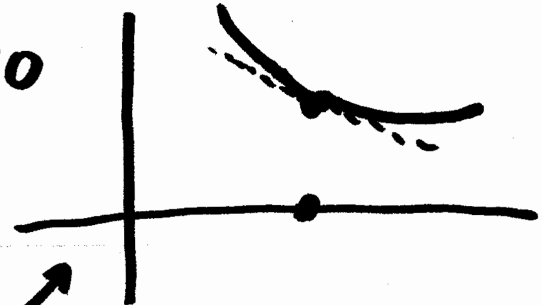
Remember $f''(x) = \frac{d}{dx} f'(x)$ } is rate of change of tangent slopes.

$f' > 0$

$f'' > 0$

$f' < 0$

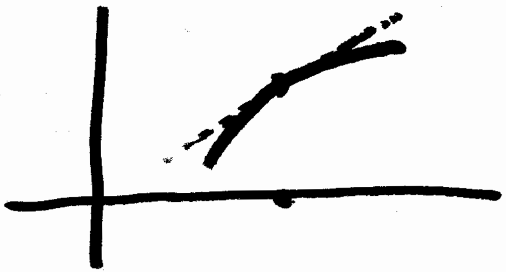
$f'' > 0$



"concave up"

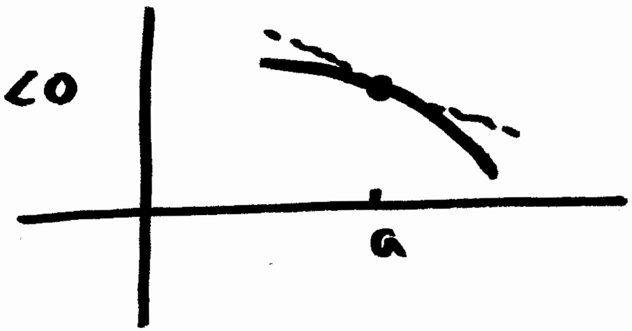
$f' > 0$

$f'' < 0$



$f' < 0$

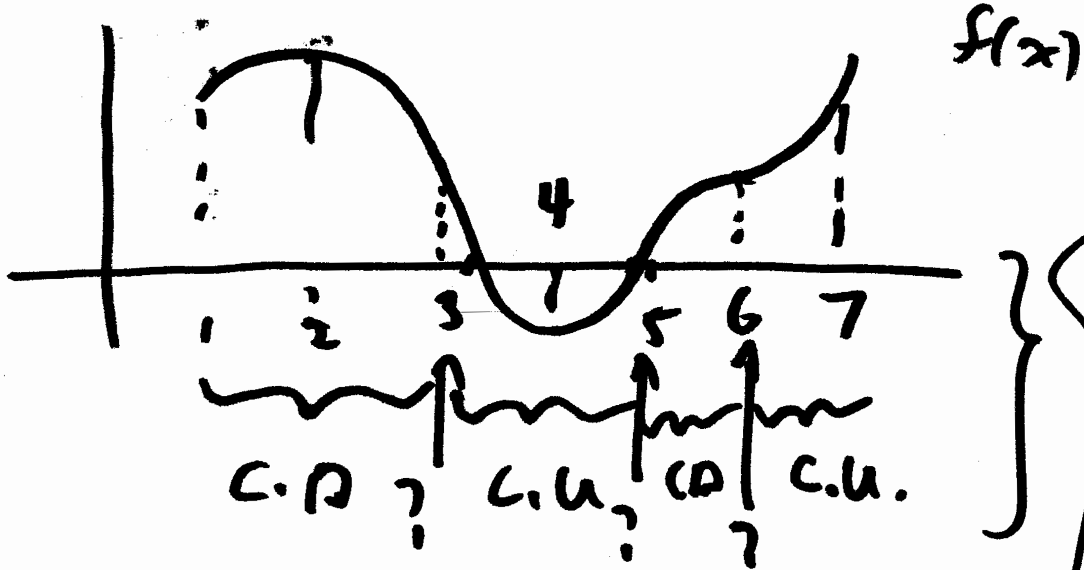
$f'' < 0$



"concave down"

$$\begin{cases} f'' > 0 \Rightarrow \text{"concave up"}, \ C.U. \ \smile \\ f'' < 0 \Rightarrow \text{"concave down"}, \ C.D. \ \frown \end{cases}$$

eg. Find Intervals of Concavity of:



at $x = 3, 5, 6$ concavity changes sign!

$\Rightarrow$ Inflection Points!

I.P.

$f'' < 0$, C.D. on $(1,3)$ & $(5,6)$

$f'' > 0$, C.U. on $(3,5)$ & $(6,7)$

eg. Given $y = x^3 - 3x + 6$, find all inflection points & intervals of concavity

Solution Let's find where $f'' = 0$ or f'' DNE

i.e. only points where f'' ~~change~~ can change sign!

Here $f'(x) = 3x^2 - 3$, $f''(x) = 6x$

$f''(x) = 6x = 0$ if $x = 0$ & $f''(x)$ always defined

$\Rightarrow$ $x = 0$ only possible point!

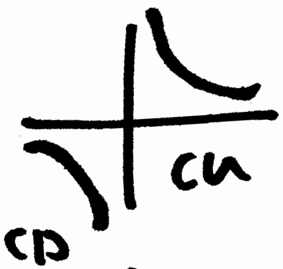
Chart!

	$(-\infty, 0)$	$(0, \infty)$
$f''(x) = 6x$	$-$	$+$
$f(x)$	C.D.	C.U.

concavity changes at $x=0$ $\Rightarrow$ I.P. at $x=0$

Watch out 1) no point $\Rightarrow$ no I.P.

eg. $y = \frac{1}{x}$



$\Rightarrow y' = \frac{-1}{x^2}$, $y'' = \frac{2}{x^3}$

$x=0$ not in Domain

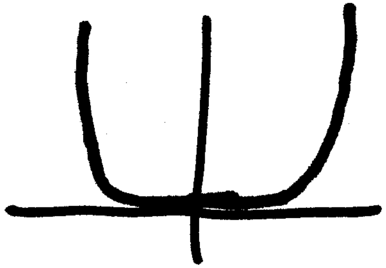
	$(-\infty, 0)$	$(0, \infty)$
y''	$-$	$+$
y	C.D	C.U.

Conc. changes at $x=0$ but pointless!

no I.P.

2) $f''(x)=0$ does not mean you have I.P.

eg. $y = x^4$



c.u.

$y' = 4x^3$, $y'' = 12x^2$ $\Rightarrow$ $y''=0$ at $x=0$

	$(-\infty,0)$	$(0,\infty)$
y''	+	+
y	c.u	c.u

no conc. change!

So no I.P.

3) Inflection points (IP) are <u>max</u> / <u>min</u> of rate of change!

<u>Why?</u>

I.P. at $x = c$

$\Rightarrow$

$x < c$	$x > c$
CU	CD
$f'' > 0$	$f'' < 0$
$\frac{d}{dx} f' > 0$	$\frac{d}{dx} f' < 0$
f' inc	f' dec.

↑

local max of $f'(x)$ at $x = c$

$x < c$	$x > c$
CD	CU
$f'' < 0$	$f'' > 0$
$\frac{d}{dx} f' < 0$	$\frac{d}{dx} f' > 0$
f' dec	f' inc

↑

local <u>min</u> of $f'(x)$ at $x = c$

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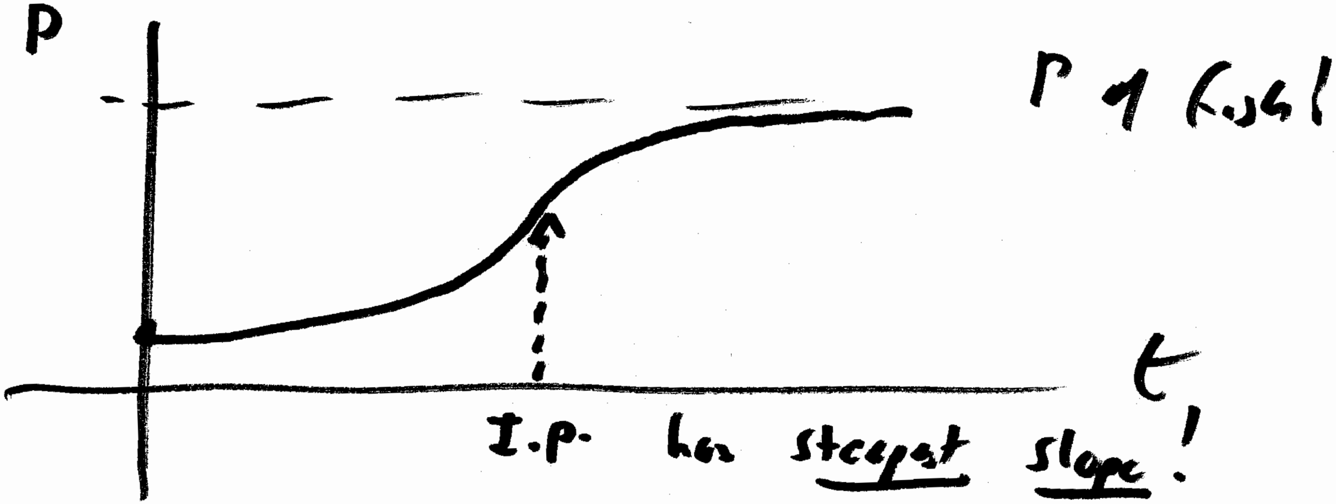
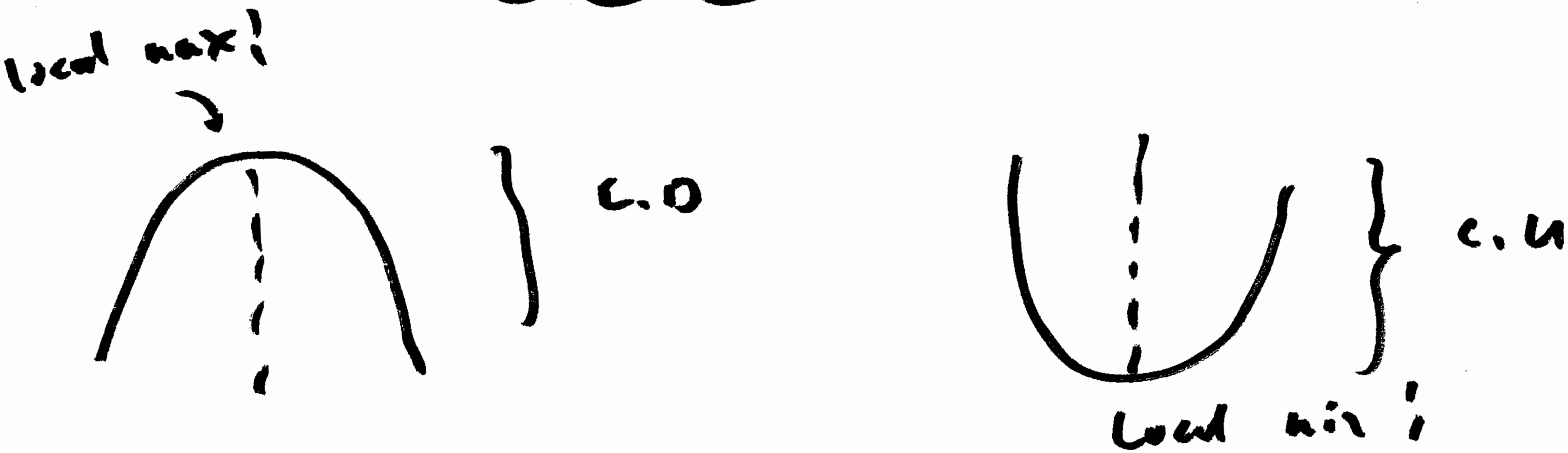


I.p. has <u>steepest</u> <u>slope</u>!

<u>Last thing</u> 2nd Derivative Test



2nd Derivative Test

If $x=c$ is a c.n. with $f'(x)=0$

& $f''(x)$ exists

then $f''(x) < 0 \Rightarrow$ local max at $x=c$

$f''(x) > 0 \Rightarrow$ local min at $x=c$

$f''(x) = 0 \Rightarrow$??? no idea!

use 1st deriv. test!