

## ASSIGNMENT 4

### Sections 1, 2, and 3 in the Red Module

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1. Consider the function  $f(x, y) = \frac{e^x}{y}$ .

(a) Find and sketch the domain of  $f$ .

(b) Determine the range of  $f$ .

(c) Sketch a contour map of  $f$ . Include at least 5 level curves.

(d) Treat  $y$  as a parameter and sketch a graph in two-dimensions to illustrate how  $f$  depends on  $x$ . (Consider the case when  $y < 0$  and then when  $y > 0$ .)

(e) Treat  $x$  as a parameter and sketch a graph in two-dimensions to illustrate how  $f$  depends on  $y$ .

2. Find and sketch the domain of the following functions.

(a)  $f(x, y) = \ln(1 + x - y)$

(b)  $g(x, y) = \frac{3x + 1}{xy^2 - x}$

3. Let  $f(x, y) = \sqrt{4 - x^2 - y^2}$ .

(a) Find and sketch the domain.

(b) Determine the range.

(c) Create a contour map for the function.

(d) Sketch the graph of the function.

4. Let  $g(x, y) = 8 + x^2 + y^2$ .

(a) Find and sketch the domain.

(b) Determine the range.

(c) Create a contour map for the function.

(d) Sketch the graph of the function.

5. Question 4 on page 28 in the Functions of Several Variables module (section 2)

6. Question 32 on page 30 in the Functions of Several Variables module (section 2)



7. (a) In your own words, explain what is meant by  $\lim_{(x,y) \rightarrow (a,b)} f(x,y) = L$ .

(b) Explain how you would show that  $\lim_{(x,y) \rightarrow (a,b)} f(x,y)$  does not exist.

8. Show that the following limits do not exist. Sketch the domains and paths involved.

(a)  $\lim_{(x,y) \rightarrow (0,0)} \frac{(x-y)^2}{x^2 + y^2}$

(b)  $\lim_{(x,y) \rightarrow (0,0)} \frac{2xy^2}{x^2 + y^4}$

9. (a) Explain what you would have to show in order to prove that a function  $f(x, y)$  is continuous at  $(a, b)$ .

(b) Find a function  $g$  such that  $\lim_{(x,y) \rightarrow (5,4)} g(x, y)$  exists but  $g$  is not continuous at  $(5, 4)$ .

(c) Find and sketch the largest domain on which  $z = \ln(y - x) + \sqrt{y + x}$  is continuous.

10. Use the definition of continuity to show that

$$h(x, y) = \begin{cases} 4 - e^{-x-y+2} & \text{if } (x, y) \neq (1, 1) \\ 3 & \text{if } (x, y) = (1, 1) \end{cases}$$

is continuous at  $(1, 1)$ .

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THE END