MATHEMATICS 1LS3 TEST 4

Day Class
Duration of Examination: 60 minutes
McMaster University, 28 November 2016

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First name (PLEASE PRINT): _____

Family name (PLEASE PRINT): _____

Student No.: _____

THIS TEST HAS 8 PAGES AND 6 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE.

Total number of points is 40. Marks are indicated next to the problem number. Any non-graphing calculator is allowed.

EXCEPT ON QUESTIONS 1 AND 2, YOU MUST SHOW CORRECT WORK TO EARN CREDIT.

USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

You must show work to receive full credit.

Problem	Points	Mark
1	4	
2	6	
3	8	
4	6	
5	6	
6	10	
TOTAL	40	

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1. **Multiple choice questions: circle ONE answer. No justification is needed.**

(a)[2] Which of the following improper integrals are convergent?

(I) $\int_1^\infty x^{-1.5}\,dx$ (II) $\int_1^\infty x^{-1}\,dx$ (III) $\int_1^\infty x^{-0.5}\,dx$

(A) none (B) I only (C) II only (D) III only
(E) I and II (F) I and III (G) II and III (H) all three

(b)[2] Identify all correct statements about the dynamical system $m_{t+1} = 1.4m_t$, $m_0 = 1$.

(I) The updating function is $f(m_t) = 1.4$

(II) The corresponding backward dynamical system is $m_t = \dfrac{1.4}{m_{t+1}}$

(III) $m_t = 1.4^t$ for all $t \geq 1$.

(A) none (B) I only (C) II only (D) III only
(E) I and II (F) I and III (G) II and III (H) all three

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2. **Identify each statement as true or false (circle your choice). You do not need to justify your answer.**

(a)[2] A population of bacteria triples every hour. Every hour, before reproduction, 700 bacteria are removed. The population starts with 1,000 bacteria. Let p_t denote the population size (i.e., number of bacteria) at time t. The dynamical system which describes this population is given by $p_{t+1} = 3(p_t - 700)$, $p_0 = 1000$.

TRUE FALSE

(b)[2] The solid of revolution whose volume is given by $\pi \int_0^2 x^2\,dx$ is a cone of base radius 4 and height 2.

TRUE FALSE

(c)[2] The dynamical system $h_{t+1} = 1.5h_t + 0.45$ describes the height of a tree in metres, where t is time in years. Converted so that the height is in centimetres, this dynamical system reads $H_{t+1} = 150H_t + 45$.

TRUE FALSE

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Questions 3-6: You must show CORRECT work to receive full credit.

3. (a)[4] Sketch (shade) the region bounded by the graphs of $y = 3e^x$ and $y = e^{2x}$ on $[0, 4]$. Write a formula for its area. Your answer should not contain absolute value. **Do not evaluate the integral(s) involved.**

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(b)[4] Consider the region bounded by the graphs of $y = \sin x$, $y = 1/2$, $x = 0$ and $x = \pi/2$. Write a formula for the volume of the solid obtained by revolving this region about the x-axis. **Do not evaluate the integral(s) involved.**

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4. (a)[1] Write the Taylor polynomial $T_2(x)$ for the function $f(x) = e^x$ at $x = 0$.

(b)[2] Use (a) to show that the function $f(x) = xe^{-x^2}$ can be approximated by the polynomial $T(x) = x - x^3 + \dfrac{x^5}{2}$ near $x = 0$.

(b)[3] Use your answer to (b) to find an estimate for $\displaystyle\int_0^1 xe^{-x^2}\,dx$.

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5. (a)[3] Determine whether the improper integral $\int_5^6 \frac{1}{\sqrt{x-5}}\,dx$ is convergent or divergent. If convergent, find its value.

(b)[3] Determine whether the improper integral $\int_1^\infty \frac{3}{(1+x)^{4/3}}\,dx$ is convergent or divergent. If convergent, find its value.

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6. (a)[3] Find $\int x^2 \ln x \, dx$.

(b)[3] Find $\int \frac{(1+\sqrt{x})^3}{\sqrt{x}} \, dx$.

(c)[4] Find the most general antiderivative of the function $f(x) = \frac{3-2x}{1+x^2}$.

THE END