

## assignment 22

①  $\int x \ln x \, dx$

$u = \ln x$	$dv = x \, dx$
$du = \frac{1}{x} \, dx$	$v = \frac{x^2}{2}$

$$= (\ln x) \left( \frac{x^2}{2} \right) - \int \left( \frac{x^2}{2} \right) \left( \frac{1}{x} \right) dx$$

$$= \frac{x^2 \ln x}{2} - \frac{1}{2} \int x \, dx = \frac{x^2 \ln x}{2} - \frac{1}{2} \left( \frac{x^2}{2} \right) + C$$

$$= \frac{x^2 \ln x}{2} - \frac{x^2}{4} + C$$

②  $\int x \sin x \, dx$

$u = x$	$dv = \sin x \, dx$
$du = dx$	$v = -\cos x$

$$= (x)(-\cos x) - \int (-\cos x) \, dx$$

$$= -x \cos x + \int \cos x \, dx = -x \cos x + \sin x + C$$

③  $\int_0^2 x e^{-3x} \, dx$

$u = x$	$dv = e^{-3x} \, dx$
$du = dx$	$v = -\frac{e^{-3x}}{3}$

step 1.

$$= (x) \left( -\frac{e^{-3x}}{3} \right) - \int \left( -\frac{e^{-3x}}{3} \right) dx$$

$$= -\frac{x e^{-3x}}{3} + \frac{1}{3} \int e^{-3x} \, dx = -\frac{x e^{-3x}}{3} - \frac{e^{-3x}}{9} + C$$

step 2.

$$\int_0^2 x e^{-3x} \, dx = \left. -\frac{x e^{-3x}}{3} - \frac{e^{-3x}}{9} \right|_0^2$$

cont'd...

$$= \left( -\frac{(2)e^{-3(2)}}{3} - \frac{e^{-3(2)}}{9} \right) - \left( 0 - \frac{1}{9} \right) \quad (2)$$

$$= -\frac{7e^{-6}}{9} + \frac{1}{9}$$

common denominator to subtract fractions

④  $\int x \cos(\pi x) dx$

$u = x \quad dv = \cos(\pi x) dx$   
 $du = dx \quad v = \frac{\sin(\pi x)}{\pi}$

$$= (x) \left( \frac{\sin(\pi x)}{\pi} \right) - \int \frac{\sin(\pi x)}{\pi} dx$$

$$= \frac{x \sin(\pi x)}{\pi} + \frac{\cos(\pi x)}{\pi^2} + C$$

⑤  $\int \arcsin(2x) dx$

$u = \arcsin(2x) \quad dv = dx$   
 $du = \frac{1}{\sqrt{1-(2x)^2}} 2 dx \quad v = x$   
 $= \frac{2 dx}{\sqrt{1-4x^2}}$

$$= \arcsin(2x)(x) - \int (x) \left( \frac{2 dx}{\sqrt{1-4x^2}} \right)$$

$$= x \arcsin(2x) - 2 \int \frac{x}{\sqrt{1-4x^2}} dx$$

substitution:  
 $w = 1-4x^2$   
 $dw = -8x dx$   
 $-\frac{dw}{8} = x dx$

$$= x \arcsin(2x) - 2 \int \frac{1}{w^{1/2}} \left( -\frac{dw}{8} \right)$$

$\frac{1}{w^{1/2}} = w^{-1/2}$

$$= x \arcsin(2x) + \frac{1}{4} \int w^{-1/2} dw$$

$$= x \arcsin(2x) + \frac{1}{4} \left( \frac{w^{1/2}}{1/2} \right) + C$$

$$= x \arcsin(2x) + \frac{(1-4x^2)^{1/2}}{2} + C$$



$$\textcircled{8} \int_1^2 (2x+1)e^x dx \quad \boxed{\begin{array}{ll} u=2x+1 & dv=e^x dx \\ du=2dx & v=e^x \end{array}} \quad \textcircled{4}$$

step 1.

$$\begin{aligned} &= (2x+1)(e^x) - \int e^x (2dx) = (2x+1)e^x - 2 \int e^x dx \\ &= (2x+1)e^x - 2e^x + c \end{aligned}$$

step 2.

$$\begin{aligned} \int_1^2 (2x+1)e^x dx &= (2x+1)e^x - 2e^x \Big|_1^2 \\ &= [(2(2)+1)e^2 - 2e^2] - [(2(1)+1)e^1 - 2e^1] \\ &= 3e^2 - e \end{aligned}$$

$$\textcircled{9} \int x^2 e^{-x} dx \quad \boxed{\begin{array}{ll} u=x^2 & dv=e^{-x} dx \\ du=2x dx & v=-e^{-x} \end{array}}$$

$$\begin{aligned} &= (x^2)(-e^{-x}) - \int (-e^{-x})(2x dx) \\ &= -x^2 e^{-x} + 2 \int x e^{-x} dx \end{aligned}$$

By parts

$$\boxed{\begin{array}{ll} u=x & dv=e^{-x} dx \\ du=dx & v=-e^{-x} \end{array}}$$

$$\begin{aligned} &= -x^2 e^{-x} + 2 \left[ (x)(-e^{-x}) + \int e^{-x} dx \right] \\ &= -x^2 e^{-x} + 2 \left[ -x e^{-x} - e^{-x} \right] + c \\ &= -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + c \end{aligned}$$

$$\textcircled{10.} \int x^2 \cos x dx \quad \boxed{\begin{array}{ll} u = x^2 & dv = \cos x dx \\ du = 2x dx & v = \sin x \end{array}} \quad \textcircled{5}$$

$$= (x^2)(\sin x) - \int (\sin x)(2x dx)$$

$$= x^2 \sin x - 2 \int \underline{x \sin x} dx$$

by parts  
twice

$$\boxed{\begin{array}{ll} U = x & dV = \sin x dx \\ dU = dx & V = -\cos x \end{array}}$$

$$= x^2 \sin x - 2 \left[ (x)(-\cos x) - \int (-\cos x) dx \right]$$

$$= x^2 \sin x - 2 \left[ -x \cos x + \int \cos x dx \right]$$

$$= x^2 \sin x - 2 \left[ -x \cos x + \sin x \right] + C$$

$$= x^2 \sin x + 2x \cos x - 2 \sin x + C \quad \square$$