

Math 1AA3/1ZB3: Exam Review

April 3, 2019

Please note that this problem set does *not* cover the last 2 weeks of course material. This is intended to help you remember material from earlier in the course that you may have forgotten. If there's recent material that you feel you don't understand, make sure you practice it! Sample Test 3 is a good place to start.

1. Determine whether the following integrals converge or diverge.

(a)

$$\int_0^1 \frac{x+1}{\sqrt{x}} dx$$

(b)

$$\int_0^\infty \frac{x^2}{1+e^x} dx$$

2. Consider the recursive sequence given by $a_n = 2(a_{n-1})^2 - 5$ for $n \geq 2$ and $a_1 = 2$.
 - (a) Is this sequence strictly increasing, strictly decreasing, or neither?
 - (b) Find the limit of this sequence as $n \rightarrow \infty$.
3. Determine whether the series converges conditionally, converges absolutely, or diverges:

$$\sum_{n=1}^{\infty} \frac{(-2)^n \cdot n^3}{(2n-1)!}$$

4. Let $f(x) = \ln(x^2)$.
 - (a) Find the Taylor series for f centered at $x = 2$ and compute its radius of convergence.
 - (b) Use the Taylor remainder estimate to find an error bound for the quadratic Taylor polynomial centered at $x = 2$ on the interval $[1, 3]$.
5. A triangular prism lies on a riverbed; each end of the prism is equilateral with side length $2m$. If the river is $5m$ deep, compute the hydrostatic force exerted on one end of the prism.
6. A tank containing $10L$ of pure water is mixed with a brine solution containing 10% salt at a rate of $1L$ per hour, and the tank is simultaneously drained at the rate of 10% of its contents per hour. Additionally, 2% of the tank's water evaporates each hour. After 5 hours, how much salt is in the tank?
7. Consider the parametric curve $x = \sin t$, $y = \sin t \cos t$.

- (a) Find an expression for the arclength of the curve. Do *not* solve the resulting integral.
- (b) Given that $0 \leq t \leq \frac{\pi}{2}$, find the area under the corresponding function $y(x)$.
8. Consider the polar curve $r = \sin \theta \cos \theta$ where $0 \leq \theta \leq 2\pi$.
- (a) At how many values of θ does the curve pass through the origin?
- (b) Is the resulting curve traced multiple times on the given domain for θ ?
9. Compute the following limit, or show that it does not exist:

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^3}{x^4 + x^2y^2}$$

10. Sketch level curves to the function $z = \ln x + 2 \ln y$.