ASSIGNMENT 6

1.(a) looking for $f(x)$ such that $P_{t+1} = f(P_t)$

thus $f(x) = 4.6 - 2x$

(b) $f(x) = 2x^2 - 1.44$ (c) $f(x) = \frac{x^2}{x+2}$

(d) $f(x) = \frac{3.2}{x^3}$

2.(a) solve for P_t : $P_t = \frac{P_{t+1} - 4.6}{-2} = -\frac{1}{2} P_{t+1} + 2.3$

(b) $2k_t^2 = k_{t+1} + 1.44$, $k_t^2 = \frac{1}{2} k_{t+1} + 0.72$

$$k_t = \pm\sqrt{\frac{1}{2} k_{t+1} + 0.72}$$

(c) $N_{t+1}(N_t + 2) = N_t \rightarrow N_t N_{t+1} + 2N_{t+1} = N_t$

so $N_t(1 - N_{t+1}) = 2N_{t+1}$

$$N_t = \frac{2N_{t+1}}{1 - N_{t+1}}$$

(d) $P_{t+1} P_t^3 = 3.2 \rightarrow P_t^3 = \frac{3.2}{P_{t+1}} \rightarrow P_t = \sqrt[3]{\frac{3.2}{P_{t+1}}}$

3.(a) $P_{t+1} = 0.4 P_t$

$P_0 = 1000$, $P_1 = 0.4 \cdot P_0 = 400$

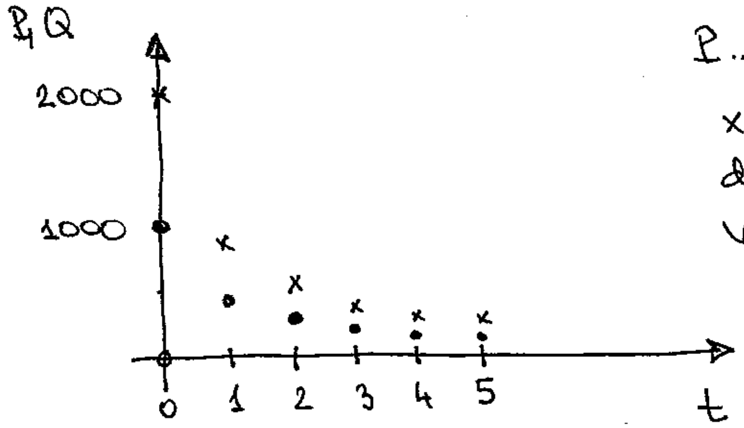
$P_2 = 0.4 \cdot P_1 = 0.4 \cdot 400 = 160$

$P_3 = 0.4 \cdot 160 = 64$, $P_4 = 25.6$, $P_5 = 10.24$

$Q_{t+1} = 0.4\, Q_t$, $Q_0 = 2000$

$Q_1 = 0.4 \cdot 2000 = 800$

$Q_2 = 320$, $Q_3 = 128$, $Q_4 = 51.2$, $Q_5 = 20.48$



P.. •, Q... x

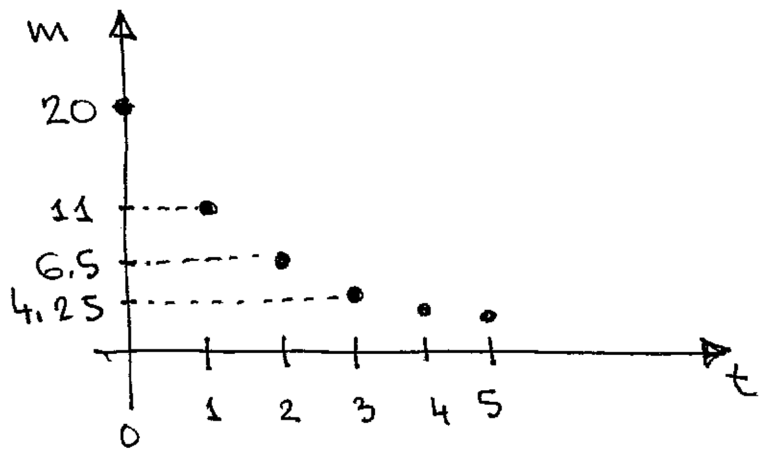
x values are double of • values

(b) $m_{t+1} = 0.5 m_t + 1$ $\quad m_0 = 20$

$m_1 = 0.5 m_0 + 1 = 0.5 \cdot 20 + 1 = 11$

$m_2 = 0.5 \cdot 11 + 1 = 6.5$

$m_3 = 4.25$, $m_4 = 3.125$, $m_5 = 2.5625$



4. (a) $Q_0 = 2000$

$Q_1 = 0.4\, Q_0 = 0.4 \cdot 2000$

$Q_2 = 0.4\, Q_1 = 0.4\,(0.4 \cdot 2000) = 0.4^2 \cdot 2000$

$Q_3 = 0.4 \cdot Q_2 = 0.4\,(0.4^2 \cdot 2000) = 0.4^3 \cdot 2000$

$\vdots$

so $Q_t = 0.4^t \cdot 2000$

(b) $Q_5 = 0.4^5 \cdot 2000 = 0.01024 \cdot 2000 = 20.48$

(c) $Q_{14} = 0.4^{14} \cdot 2000 \approx 0.00537$

5. (a) f_{t+1} = 10% increase over f_t

$= f_t + 0.1 f_t = 1.1 f_t$

(b) $f_0 \to f_1 = 1.1 f_0 \to f_2 = 1.1 f_1 = 1.1(1.1 f_0) = 1.1^2 f_0$

$\cdots \quad f_t = 1.1^t \cdot f_0 = f_0 \cdot 1.1^t$

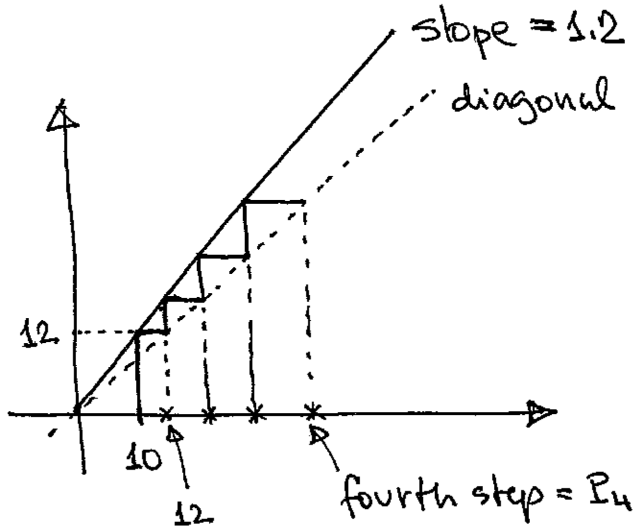
if $f_0 = 0.0001$ then $f_t = 0.0001 \cdot 1.1^t$

(c) f_t is exponential function, so as $t \to \infty$ it approaches ∞ as well $\to$ will reach 1.0

NOTE: but there might not be <u>integer</u> t so that $f_t = 1.0$

no values > 1.0 make sense for f_t since f_t is a fraction of population (and cannot be > 1, ie $> 100\%$)

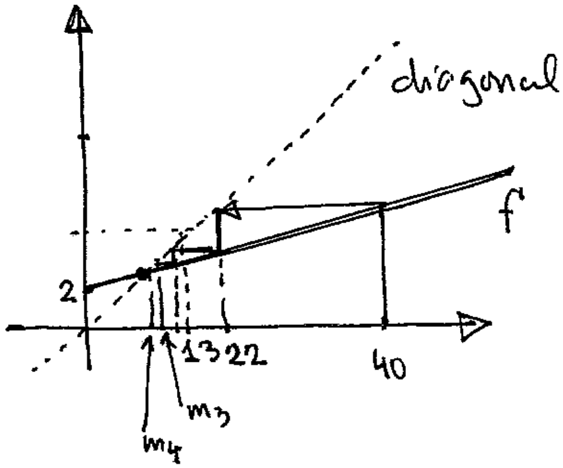
6. (a)



$P_{t+1} = 1.2\,P_t \longrightarrow P_t = P_0 \cdot 1.2^t = 10 \cdot 1.2^t$

so $P_1 = 12$, $P_2 = 14.4$, $P_3 = 17.28$, $P_4 = 20.736$

(b)



updating function:
$f(x) = 0.5x + 2$

$f(x) = x \to 0.5x + 2 = x$

$0.5\,\cancel{1.5}x = 2$

$x = \frac{2}{0.5} = \cancel{4}\,4$

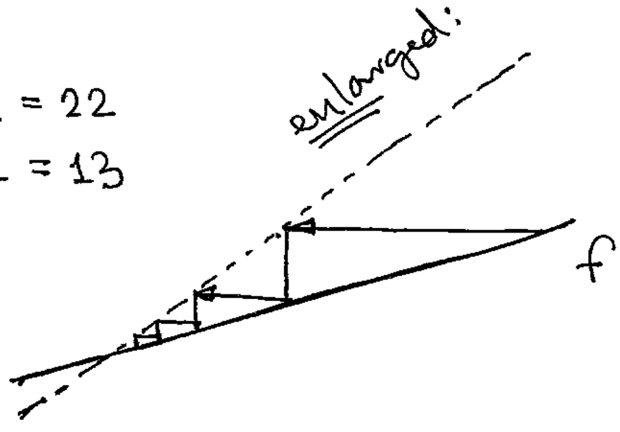
$m_{t+1} = 0.5 m_t + 2$

$m_0 = 40 \ldots \quad m_1 = 0.5 \cdot 40 + 2 = 22$

$m_2 = 0.5 \cdot 22 + 2 = 13$

$m_3 = 8.5$

$m_4 = 6.25$



[difficult to do without graph paper]

7. (a) $f(x) = 0.4x + 3$

equilibrium $0.4x + 3 = x$

$\to 0.6x = 3, \; x = \underline{\underline{5}}$

start at $N_0 = 2$ → see next page for cobwebbing (on graph paper)

can calculate, to check cobwebbing

$N_{t+1} = 0.4 N_t + 3$

$N_0 = 2 \to N_1 = 0.4(2) + 3 = 3.8$

$N_2 = 0.4(3.8) + 3 = 4.52$

(b) $m_{t+1} = -0.5 m_t + 2 \rightsquigarrow f(x) = -0.5x + 2$

equilibrium $-0.5x + 2 = x$

$1.5x = 2, \; x = 2/1.5 \approx 1.3\dot{3}$

see page 6

again, if needed, we check by algebraic calculations

(c) $m_{t+1} = -m_t^2 + 2 \to f(x) = -x^2 + 2$

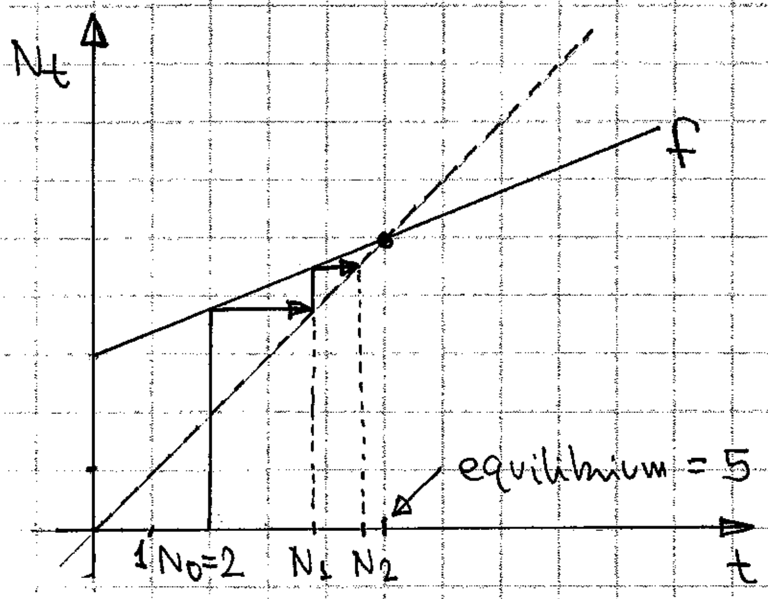
equilibrium: $-x^2 + 2 = x \to x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

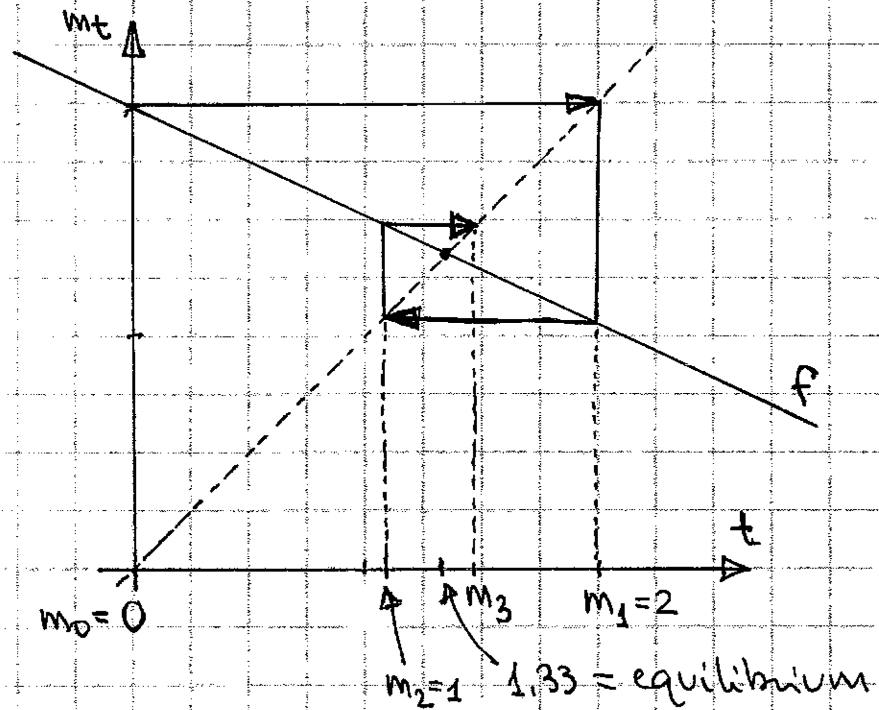
$x = -2, 1$

see page 7

7. (a)



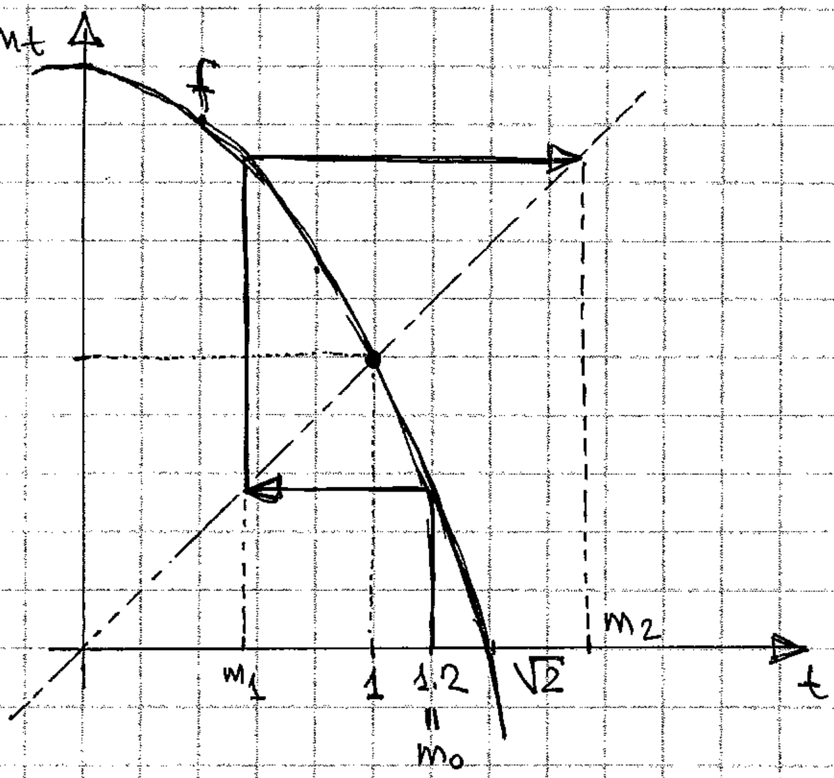
(b)



$$m_{t+1} = -0.5\, m_t + 2$$

$$m_0 = 0 \longrightarrow m_1 = 2 \longrightarrow m_2 = 1 \longrightarrow m_3 = 1.5$$

(c)



$$m_{t+1} = -m_t^2 + 2$$

$$m_0 = 1.2 \longrightarrow m_1 = -1.2^2 + 2 = 0.56$$

$$m_2 = -0.56^2 + 2 \approx 1.69$$

8.(a) $f(x) = 3x-1 = x \rightarrow 2x=1,\ x=\underline{\underline{1/2}}$

(b) $f(x) = -0.5x+2 = x \rightarrow 1.5x = 2$

$$x = \frac{2}{1.5} = \frac{4}{3}$$

(c) $f(x) = \dfrac{0.5x}{x+1} = x \rightarrow x - \dfrac{0.5x}{x+1} = 0$

$$x\Big(\underbrace{1 - \frac{0.5}{x+1}}_{=0}\Big) = 0$$

$\underline{\underline{x=0}}$

$$\frac{0.5}{x+1} = 1$$

$$x+1 = 0.5$$

$$\underline{\underline{x = -0.5}}$$