12A3

Note: No office hours today only
(I have to oversee a mid-term)

Last Day

$$\int f(x)\,dx = F(x) + C$$

C is arbitrary constant
$F(x)$ is <u>any</u> antiderivative of $f(x)$ $\left(\frac{d}{dx}F(x) = f(x)\right)$

$\int f(x)\,dx$ is <u>general antiderivative</u>
"indefinite integral"

Appendix E: Sigma notation:

terminal value

eg $$\sum_{i=1}^{10} i^2 = \underbrace{1^2 + 2^2 + 3^2 + \dots + 10^2}_{\text{"i" increment by 1 only}}$$

GREEK

indexing variable

initial value

Greek capital letter Σ, "sigma" ie. greek capital "S"

eg. $$\sum_{i=10}^{1000} \frac{1}{i} = \frac{1}{10} + \frac{1}{11} + \dots + \frac{1}{10000}$$

$$\sum_{i=6}^{12} \sqrt{2i} = \sqrt{2(6)} + \sqrt{2(7)} + \dots \sqrt{2(12)}$$

$$= \sqrt{12} + \sqrt{14} + \sqrt{16} + \dots \sqrt{24}.$$

e.g. Say I had a <u>table of data</u>

e.g.

i	1	2	3	4	5
a_i	0.4	2.1	6.8	0	-0.2

$$\sum_{i=1}^{5} a_i = a_1 + a_2 + a_3 + a_4 + a_5$$

$$= 0.4 + 2.1 + 6.8 + 0 - 0.2 = \#$$

e.g. Let $b_i = \sin(i\pi/6)$

$$\sum_{i=2}^{4} b_i = b_2 + b_3 + b_4$$

$$= \sin(2\pi/6) + \sin(3\pi/6) + \sin(4\pi/6)$$

$$= \ldots\ldots$$

Sigma Notation Rules

① $\displaystyle\sum_{i=1}^{n}(a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$

② $\displaystyle\sum_{i=1}^{n} k a_i = k \sum_{i=1}^{n} a_i$

k is const. in i

eg $\displaystyle\sum_{i=1}^{5} \frac{6}{i} + 2i^2 = 6\sum_{i=1}^{5}\frac{1}{i} + 2\sum_{i=1}^{5} i^2$

$\displaystyle = 6\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right)$

$\displaystyle + 2\left(1^2+2^2+3^2+4^2+5^2\right).$

etc.

3) $\sum_{i=1}^{n} a_i = \sum_{i=1}^{m} a_i + \sum_{i=m+1}^{n} a_i$

eg $\sum_{i=1}^{6} i^4 = \underbrace{1^4 + 2^4 + 3^4 + 4^4}_{} + \underbrace{5^4 + 6^4}_{}$

$= \sum_{i=1}^{4} i^4 + \sum_{i=5}^{6} i^4$

4) Index shifting

in subscript!

$\sum_{i=1}^{n} a_i = \sum_{i=2}^{n+1} a_{i-1}$

$= a_1 + a_2 + a_3 + \dots + a_n$

$= a_{2-1} + a_{3-1} + \dots + a_{n+1-1}$

$= a_1 + a_2 \dots + a_n$

eg $\sum_{i=1}^{6} e^{i} = e^{1} + e^{2} + e^{3} + e^{4} + e^{5} + e^{6}$ ✓

$= \sum_{i=2}^{7} e^{i-1} = e^{2-1} + e^{3-1} + \dots + e^{7-1}$

$= \sum_{i=8}^{13} e^{i-7} = e^{8-7} + e^{9-7} + \dots + e^{13-7}$

$= +7$

$= e^{1} + e^{2} + \dots + e^{6}$ ✓

$= \sum_{i=-2}^{3} e^{i+3} = e^{-2+3} + e^{-1+3} + e^{0+3} + \dots + e^{3+3}$

-3

$= e^{1} + e^{2} + \dots e^{6}$ ✓

Special Sum Formulas

$$\sum_{i=1}^{n} 1 = \underbrace{1+1+\cdots+1}_{n \text{ tim}} = n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = n(n+1)(2n+1)/6$$

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^2(n+1)^2}{4}$$

$$\begin{array}{r}
1+2+\cdots\cdots+100 \\
100+99+\cdots\cdots+1 \\
\hline
\underbrace{101+101+\cdots\cdots+101}_{100 \text{ terms!}}
\end{array}$$

$$= 100(101)$$

$$\Rightarrow 1+2+\cdots+100 = \frac{100(101)}{2}$$

g. $$\sum_{i=1}^{20} i^2 - 2i + 1 = \sum_{i=1}^{20} i^2 - 2\sum_{i=1}^{20} i + \sum_{i=1}^{20} 1$$

$$= \frac{(20)(21)(41)}{6} - \cancel{2}\cdot\frac{20(21)}{\cancel{2}} + 20$$

$$= \#$$

$$\sum_{i=1}^{20} (i-1)^2 = \sum_{i=0}^{19} i^2 = \left(\sum_{i=1}^{19} i^2\right) + \underbrace{\cancel{0^2}}_{i=0 \text{ value!}}$$

$+1$

$$= 19(20)(39)/6 = \#$$ same as before!

(prettier!)

eg. $\sum_{i=10}^{100} i^3 = \sum_{i=1}^{100} i^3 - \sum_{i=1}^{9} i^3$

$$= \left(\frac{100(101)}{2}\right)^2 - \left(\frac{9(10)}{2}\right)^2 = \#\#$$

eg $\sum_{i=1}^{25} \left(\frac{1}{i} - \frac{1}{i+1}\right)$

$$= \left(1 - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right)$$

$$\left(\cancel{\frac{1}{23}} - \cancel{\frac{1}{24}}\right) + \left(\cancel{\frac{1}{24}} - \cancel{\frac{1}{25}}\right) + \left(\cancel{\frac{1}{25}} - \frac{1}{26}\right)$$

$$= 1 - \frac{1}{26}$$