Math 1AA3/1ZB3 Sample Exam

Name:____
(Last Name) (First Name)

Student Number:________________________

This test consists of 40 multiple choice questions worth 1 mark each (no part marks). Questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Calculators are NOT allowed.

1. What is the minimum number of terms of the following series that we have to add in order to estimate the following sum accurate to within .1?

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$$

(a) 4 **(b)** 5 **(c)** 1 **(d)** 2 **(e)** 3

2. Find an equation of the ellipse with foci $(-1,2)$, $(-1,10)$, and with vertex $(-1,11)$.

(a) $\dfrac{(x+1)^2}{9} + \dfrac{(y-6)^2}{25} = 1$ **(b)** $\dfrac{(x+1)^2}{4} + \dfrac{(y-6)^2}{25} = 1$
(c) $\dfrac{(x-1)^2}{9} + \dfrac{(y-6)^2}{4} = 1$ **(d)** $\dfrac{(x-1)^2}{9} + \dfrac{(y+6)^2}{25} = 1$ **(e)** $\dfrac{(x+1)^2}{4} + \dfrac{(y+6)^2}{25} = 1$

3. Evaluate the following sum.

$$\sum_{n=2}^{\infty} \frac{1}{n(n-1)}$$

(a) $\frac{1}{2}$ **(b)** $-\frac{1}{2}$ **(c)** 1 **(d)** $\frac{1}{3}$ **(e)** $\frac{1}{6}$

4. Determine whether the following sequences converge or diverge. If they converge, find the limit.

(i) $a_n = \dfrac{3n^3 + 4n^2 - 2}{5n^3 - n + 1}$ **(ii)** $a_n = \cos(2/n)$

(a) $\frac{3}{5}$, diverge **(b)** 0, diverge **(c)** diverge, diverge **(d)** $\frac{3}{5}, 1$ **(e)** $1, 1$

5. A sequence is defined by $a_1 = 1$, $a_{n+1} = \frac{1}{2}(a_n + 3)$. Which of the following statements is true?
(a) $\{a_n\}$ is increasing and bounded above by $\frac{5}{2}$.
(b) $\{a_n\}$ is decreasing and bounded below by 1.
(c) $\{a_n\}$ is decreasing and bounded below by 2.
(d) $\{a_n\}$ is decreasing and bounded above by 2.
(e) $\{a_n\}$ is increasing and bounded above by 3.

6. Evaluate the following sum.

$$\sum_{n=1}^{\infty} 4^{n+1} 3^{-2n}$$

(a) $\frac{16}{5}$ **(b)** $\frac{13}{5}$ **(c)** $\frac{4}{7}$ **(d)** $\frac{12}{7}$ **(e)** divergent

7. Determine which of the following series converge.
(i) $\displaystyle\sum_{n=1}^{\infty} \frac{2n-1}{3n+1}$ **(ii)** $\displaystyle\sum_{n=1}^{\infty} \frac{4^{n-1}}{3^n}$
(a) (i) only **(b)** neither **(c)** (ii) only **(d)** (i) and (ii) **(e)** 36

8. Evaluate the following integral.

$$\int_1^e (\ln x)^2 \, dx$$

(a) $3e - 2$ **(b)** $e - 2$ **(c)** $2e - 1$ **(d)** $2e + 3$ **(e)** $2e + 1$

9. Determine which of the following series converge.
(i) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ **(ii)** $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{3n^2-2}$
(a) (ii) only **(b)** neither **(c)** (i) and (ii) **(d)** (i) only **(e)** 81

10. Estimate the volume of the solid that lies below the surface $z = xy$ and above the rectangle $R = \{(x, y) \,|\, 0 \le x \le 2,\ 0 \le y \le 4\}$ using a Riemann sum with $m = 2$, $n = 2$, and taking the sample point to be the lower right corner of each square.
(a) 24 **(b)** 16 **(c)** 4 **(d)** 12 **(e)** 32

11. A bacterial culture contains 200 cells initially and grows at a rate proportional to its size. After half an hour there were 400 bacteria. When will the population reach 1000?

(a) $\frac{\ln 5}{2\ln 2}$ **(b)** $\frac{\ln 2}{5\ln 5}$ **(c)** $\frac{\ln(2/5)}{2\ln 5}$ **(d)** $\frac{\ln(2/5)}{5\ln 2}$ **(e)** $\frac{\ln 2}{2\ln 5}$

12. Use the Comparison Theorem to determine which of the following integrals converge.

(i) $\displaystyle\int_{-\infty}^{0} \frac{1}{1+e^{-x}}\,dx$ **(ii)** $\displaystyle\int_{1}^{\infty} \frac{\arctan x}{1+x^3}\,dx$

(a) (ii) only **(b)** (i) only **(c)** (i) and (ii) **(d)** neither **(e)** 452

13. Determine which of the following series converge.

(i) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^3-1}$ **(ii)** $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{n^2-3}}{n^2}$

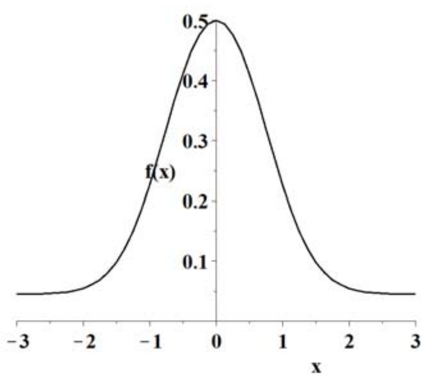
(a) (ii) only **(b)** neither **(c)** (i) only **(d)** (i) and (ii) **(e)** 45

14. Find the radius of convergence of the following power series.

$$\sum_{n=1}^{\infty} (-1)^n \frac{nx^n}{2^n}$$

(a) 1 **(b)** 3 **(c)** 2 **(d)** 4 **(e)** $\frac{1}{4}$

15. The function with the below graph is a solution to which one of the following differential equations?



(a) $\dfrac{dy}{dx} = xe^{-x^2-y^2}$ **(b)** $\dfrac{dy}{dx} = -xe^{x^2+y^2}$ **(c)** $\dfrac{dy}{dx} = x^2\,e^{-x^2-y^2}$ **(d)** $\dfrac{dy}{dx} = -xe^{-x^2-y^2}$

(e) $\dfrac{dy}{dx} = xe^{x^2+y^2}$

16. By reversing the order of integration, which of the below integrals is equal to

$$\int_0^1 \int_{4x}^4 f(x,y)\,dy\,dx\ ?$$

(a) $\int_{4x}^4 \int_0^1 f(x,y)\,dx\,dy$ **(b)** $\int_1^4 \int_{y/4}^1 f(x,y)\,dx\,dy$ **(c)** $\int_0^1 \int_{y/4}^1 f(x,y)\,dx\,dy$
(d) $\int_1^4 \int_y^1 f(x,y)\,dx\,dy$ **(e)** $\int_0^4 \int_0^{y/4} f(x,y)\,dx\,dy$

17. Let $f(x,y) = y^2/x$. Find the maximum rate of change of f at the point $(2,4)$.
(a) $\sqrt{32}$ **(b)** $\sqrt{8}$ **(c)** 4 **(d)** $\sqrt{24}$ **(e)** $\sqrt{48}$

18. Express the following function as a power series.

$$f(x) = \ln(1-x)$$

(a) $-\sum_{n=0}^{\infty} \frac{1}{n+1} x^{n+1}$ **(b)** $-\sum_{n=1}^{\infty} \frac{1}{n} x^{n+1}$ **(c)** $\sum_{n=0}^{\infty} n x^{n-1}$ **(d)** $-\sum_{n=1}^{\infty} \frac{1}{n} x^{n-1}$ **(e)** $\sum_{n=0}^{\infty} \frac{1}{n+1} x^n$

19. Which of the following series are conditionally convergent?
(i) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{\sqrt{n^2+1}}$ **(ii)** $\sum_{n=1}^{\infty} \frac{(-1)^n}{6^{n-1}}$
(a) neither **(b)** (i) only **(c)** (i) and (ii) **(d)** (ii) only **(e)** 47

20. Let $f(x,y) = e^{-xy}\cos y$. Find the linearization $L(x,y)$ at the point $(\pi, 0)$.
(a) $1 - x + \pi y$ **(b)** $1 - \pi y$ **(c)** $2 + \pi y - 2x$ **(d)** $\pi x + 3y - 1$ **(e)** $2\pi x - y + 2$

21. What is the minimum number of terms that need to be added in order to estimate the following sum to within .001?

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

(a) 3 **(b)** 1001 **(c)** 4 **(d)** 250 **(e)** 500

22. Determine which of the following series converge.
(i) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{3/2}}$ **(ii)** $\sum_{n=1}^{\infty} \frac{\sqrt{1+e^{1/n}}\, e^{1/n}}{n^2}$
(a) (i) only **(b)** (ii) only **(c)** (i) and (ii) **(d)** neither **(e)** 28

23. Suppose that the concentration $y(t)$ of glucose in the bloodstream satisfies the following differential equation,

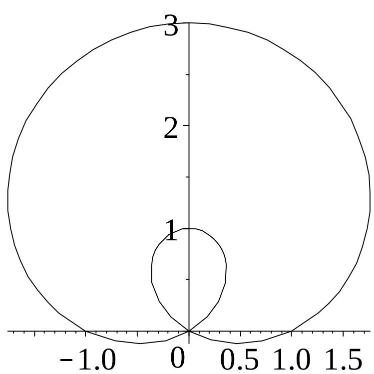
$$\frac{dy}{dt} = r - ky,$$

where r and k are positive constants. Determine the concentration at any time t by solving the above differential equation.

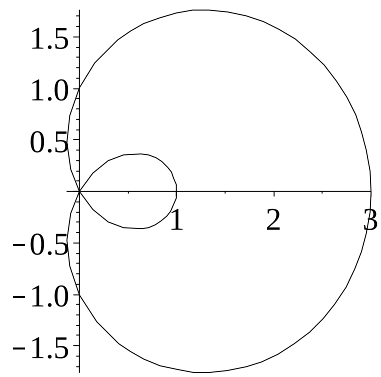
(a) $y = 1 + Ce^{-(r/k)t}$ **(b)** $y = t + Ce^{-(r/k)t}$ **(c)** $y = \frac{t}{k} + Ce^{-(r/k)t}$ **(d)** $y = \frac{r}{k} + Ce^{-kt}$
(e) $y = C + \frac{r}{k}e^{-kt}$

24. Sketch the polar curve $r = -1 + 2\cos\theta$.

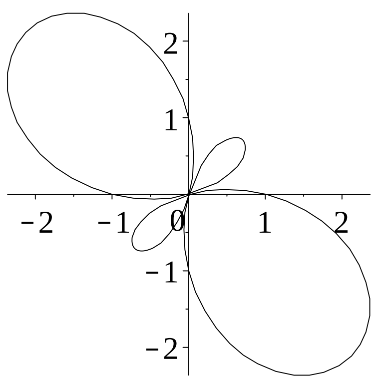
(a)



(b)



(c)



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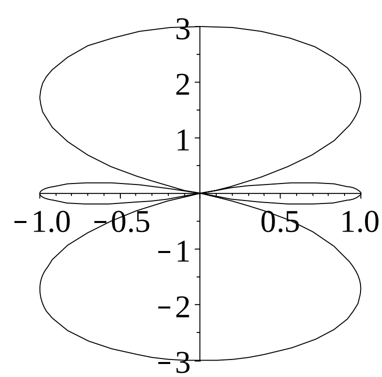
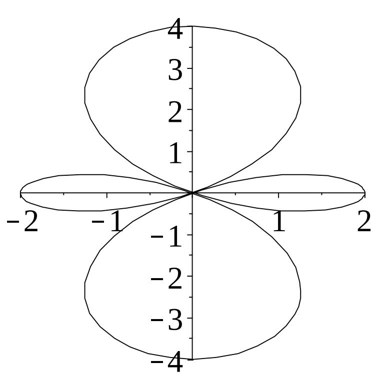


(e)



25. Evaluate

$$\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+y^2}$$

(a) d.n.e. **(b)** 0 **(c)** $\frac{1}{4}$ **(d)** 1 **(e)** $\frac{1}{3}$

26. Find the orthogonal trajectories of the family of curves $y^2 = kx^3$.

(a) $\dfrac{y^2}{3} + \dfrac{x^2}{2} = C$ **(b)** $\dfrac{y^3}{3} + \dfrac{x^2}{2} = C$ **(c)** $\dfrac{y^2}{2} + \dfrac{x^2}{3} = C$ **(d)** $\dfrac{y^2}{3} + \dfrac{x^3}{2} = C$
(e) $\dfrac{y^3}{2} + \dfrac{x^3}{3} = C$

27. Evaluate the following integral.

$$\int_{-1}^{2} \frac{1}{x^2} dx$$

(a) $-\frac{3}{2}$ **(b)** $\frac{3}{2}$ **(c)** $\frac{1}{2}$ **(d)** divergent **(e)** $-\frac{1}{2}$

28. Find the values of θ on the polar curve $r = 2\sin\theta,\ 0 \le \theta \le \pi,$ where the tangent line is horizontal or vertical.
(a) $0, \dfrac{\pi}{2}, \pi, \dfrac{\pi}{3}, \dfrac{2\pi}{3}$ **(b)** $\dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{\pi}{6}, \dfrac{5\pi}{6}$ **(c)** $\dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{\pi}{2}, \dfrac{\pi}{6}$ **(d)** $0, \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{\pi}{4}, \dfrac{3\pi}{4}$
(e) $0, \dfrac{\pi}{2}, \pi, \dfrac{\pi}{4}, \dfrac{3\pi}{4}$

29. Find the second degree polynomial $T_2(x)$ for the function $f(x) = \sqrt[3]{x}$ at $a = 8$.
(a) $2 + \dfrac{1}{6}(x-8) - \dfrac{1}{256}(x-8)^2$ **(b)** $2 + \dfrac{1}{12}(x-8) - \dfrac{1}{288}(x-8)^2$
(c) $2 + \dfrac{1}{12}(x-8) + \dfrac{1}{256}(x-8)^2$ **(d)** $4 + \dfrac{1}{6}(x-8) - \dfrac{1}{256}(x-8)^2$
(e) $2 + \dfrac{1}{12}(x-8) - \dfrac{1}{144}(x-8)^2$

30. The parametric curve $x = \cos t,\ y = \sin t \cos t$ has two tangent lines at $(0,0)$. Find their slopes.
(a) $\frac{1}{2}, -1$ **(b)** $-\frac{1}{2}, 1$ **(c)** $-1, 1$ **(d)** $0, 1$ **(e)** $0, -1$

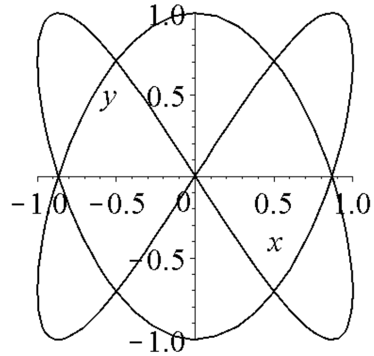
31. Let $x = t^2 + t,\ y = e^t$. Find $\frac{d^2y}{dx^2}$.

(a) $\dfrac{e^t(2t-1)}{(2t+1)^3}$ **(b)** $\dfrac{e^t(t+2)}{(2t+1)^3}$ **(c)** $\dfrac{e^t(2t-1)^2}{(2t+1)^2}$ **(d)** $\dfrac{e^t(2te^t-1)}{(2t+1)^2}$ **(e)** $\dfrac{e^{2t}(2t+1)}{(2t+1)^3}$

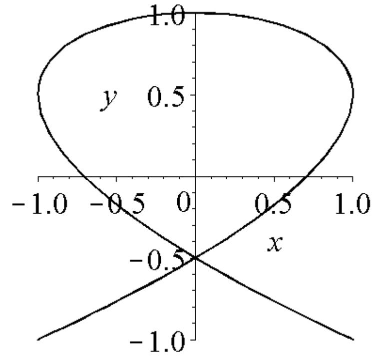
32. Plot the following parametric curve.

$$x = \sin 3t, \quad y = \cos 2t$$

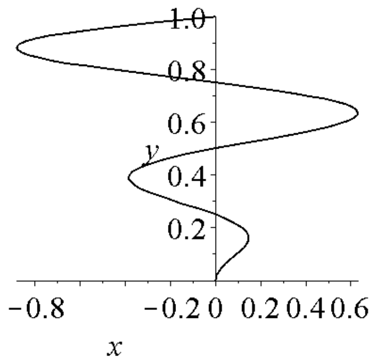
(a)



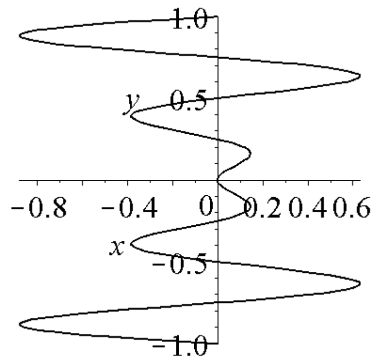
(b)



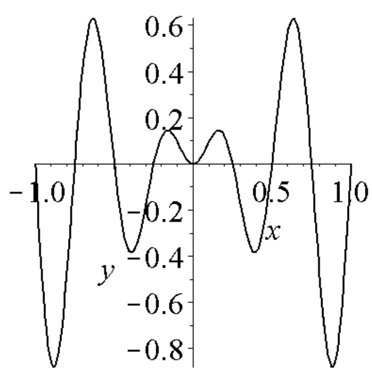
(c)



(d)



(e)



33. Find an integral which represents the area of the surface obtained by rotating the following curve about the y-axis.

$$y = \int_1^x \sqrt{\sqrt{t} - 1}\, dt, \quad 1 \le x \le 16$$

(a) $\displaystyle\int_1^{16} 2\pi x^2\, dx$ **(b)** $\displaystyle\int_1^{16} 2\pi x^{7/4}\, dx$ **(c)** $\displaystyle\int_1^{16} 2\pi x^{9/4}\, dx$ **(d)** $\displaystyle\int_1^{16} 2\pi x^{5/4}\, dx$
(e) $\displaystyle\int_1^{16} 2\pi x\sqrt{x-1}\, dx$

34. Solve the following differential equation.

$$y' + \frac{1}{x\ln x} y = \frac{1}{\ln x}\, e^{-x}, \;\; x > 0.$$

(a) $y = \frac{e^{-x}}{\ln x} + C$ **(b)** $y = e^{-x} + C\ln x$ **(c)** $y = \frac{\ln x}{e^x} + C$ **(d)** $y = \frac{C + \ln x}{e^x}$ **(e)** $y = \frac{C - e^{-x}}{\ln x}$

35. Let $u = \sqrt{x^2 + 2y^2}$, $x = s + r\cos t$, $y = r + s\sin t$. Find $\frac{\partial u}{\partial t}$ when $s = 1, r = 2, t = 0$.
(a) $\frac{3}{\sqrt{15}}$ **(b)** $\frac{1}{\sqrt{13}}$ **(c)** $\frac{4}{\sqrt{17}}$ **(d)** $\frac{3}{\sqrt{17}}$ **(e)** $\frac{2}{\sqrt{15}}$

36. Find the length of the following parametric curve

$$x = e^t + e^{-t}, \quad y = 5 - 2t \qquad 0 \le t \le 1$$

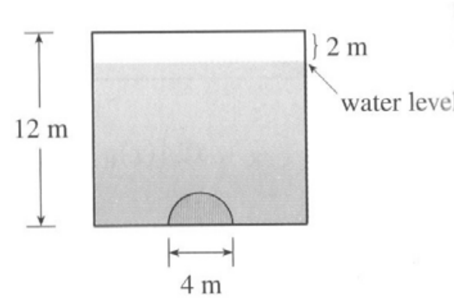
(a) $\frac{1}{2}e(e-1)$ **(b)** $e(e-1)$ **(c)** $e^2 - 1$ **(d)** $e - \frac{1}{e}$ **(e)** $\frac{1}{2}(e^2 - e^{-2} + 4)$

37. Evaluate the following integral,

$$\int_0^4 \int_0^{\sqrt{y}} xy^2 dx\, dy$$

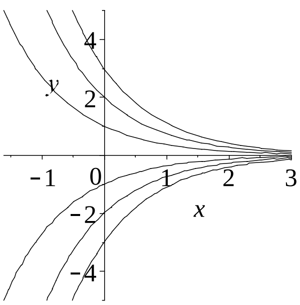
(a) 64 **(b)** 16 **(c)** 48 **(d)** 24 **(e)** 32

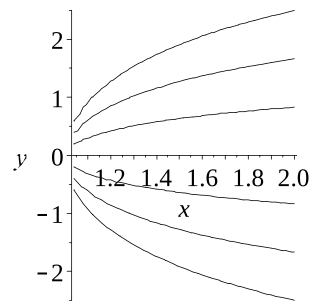
38. A vertical dam has a semicircular gate as shown in the figure to the right. Find the hydrostatic force against the gate.

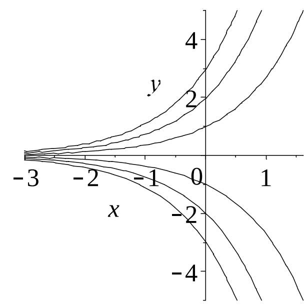


(a) $\displaystyle\int_0^4 2\rho g(12-y)\sqrt{4-y^2}\,dy$ **(b)** $\displaystyle\int_0^{10} \rho g(10-y)\sqrt{4-y^2}\,dy$
(c) $\displaystyle\int_0^2 2\rho g(2-y)\sqrt{2-y^2}\,dy$ **(d)** $\displaystyle\int_0^4 2\rho g(10-y)\sqrt{2-y^2}\,dy$
(e) $\displaystyle\int_0^2 2\rho g(10-y)\sqrt{4-y^2}\,dy$

39. Draw a contour map of the function $f(x,y)=\frac{y}{e^x}$.

(a)


(b)


(c)


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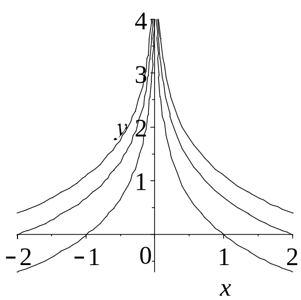
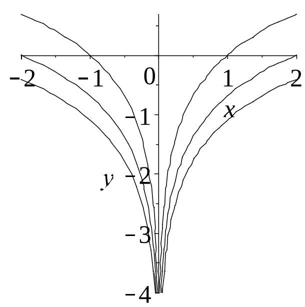


(e)


40. Supose that over a certain region of space the electrical potential V is given by $V(x,y,z)=2x^2-3xy+xyz$. Find the rate of change of the potential at the point $P(0,1,2)$ in the direction of the vector $\mathbf{v}=\mathbf{i}+\mathbf{j}-\mathbf{k}$.
(a) 0 **(b)** $\frac{2}{\sqrt{3}}$ **(c)** 1 **(d)** -1 **(e)** $-\frac{1}{\sqrt{3}}$

Answers:

1. e **2.** a **3.** c **4.** d **5.** e **6.** a **7.** b **8.** b **9.** d **10.** d
11. a **12.** c **13.** c **14.** c **15.** d **16.** e **17.** a **18.** a **19.** a **20.** b
21. b **22.** c **23.** d **24.** b **25.** a **26.** c **27.** d **28.** e **29.** b **30.** c
31. a **32.** b **33.** d **34.** e **35.** c **36.** d **37.** e **38.** e **39.** c **40.** e