Math 1A03/1ZA3 (Version 1)
December, 2016

Name:_____
(Last Name) (First Name)

Student Number:_____

Day Class
Duration: 2.5 Hours **Instructors:** Childs, McLean, Moore, Poudel, Tran
Maximum Mark: 32

McMaster University Final Examination

This examination paper consists of 8 pages (including this one). This exam consists of 32 multiple choice questions worth 1 mark each (no part marks). The questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Only the McMaster standard calculator Casio fx-991 is allowed.

Computer Card Instructions:

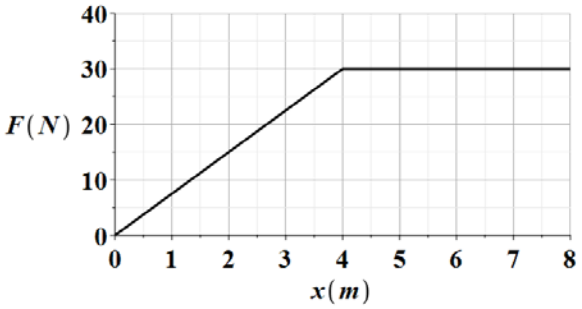
NOTE: IT IS YOUR RESPONSIBILITY TO ENSURE THAT THE ANSWER SHEET IS PROPERLY COMPLETED: YOUR EXAMINATION RESULT DEPENDS UPON PROPER ATTENTION TO THESE INSTRUCTIONS

The scanner, which reads the sheets, senses the shaded areas by their non-reflection of light. A heavy mark must be made, completely filling the circular bubble, with an HB pencil. Marks made with a pen or felt-tip marker will **NOT** be sensed. Erasures must be thorough or the scanner may still sense a mark. Do **NOT** use correction fluid on the sheets. Do **NOT** put any unnecessary marks or writing on the sheet.

1. Print your name, student number, course name, and the date in the space provided at the top of Side 1 (red side) of the form. Then the sheet **MUST** be signed in the space marked SIGNATURE.

2. Mark your student number in the space provided on the sheet on Side 1 and fill in the corresponding bubbles underneath.
3. Mark only **ONE** choice from the alternatives (A,B,C,D,E) provided for each question. If there is a True/False question, enter response of 1 (or A) as True, and 2 (or B) as False. The question number is to the left of the bubbles. Make sure that the number of the question on the scan sheet is the same as the question number on the test paper.
4. Pay particular attention to the Marking Directions on the form.
5. Begin answering questions using the first set of bubbles, marked "1".

1. The graph to the right is a continuous force function (in Newtons) that increases to its maximum value and then remains constant. How much work is done by the force in moving an object a distance of 8 m?

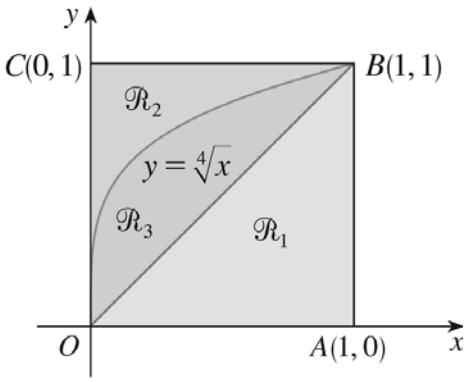


(a) 220 J **(b)** 160 J **(c)** 200 J
(d) 180 J **(e)** 240 J

2. Find the average value of the function $f(x) = \frac{e^x}{1+e^x}$ on the interval $[0, \ln 2]$.

(a) 1 **(b)** $\frac{\ln(\frac{3}{2})}{\ln 2}$ **(c)** $\frac{\ln 3}{\ln 2}$ **(d)** $\frac{\ln 2}{\ln 3}$ **(e)** $\frac{\ln(\frac{2}{3})}{\ln 3}$

3. Find the volume of the solid obtained by rotating the region $\mathcal{R}_3$ (in the figure to the right) about the line OC.



(a) $\frac{\pi}{9}$ **(b)** $\frac{2\pi}{9}$ **(c)** $\frac{2\pi}{3}$ **(d)** $\frac{\pi}{3}$ **(e)** $\frac{3\pi}{2}$

4. Which of the following is equal to

$$\int (\ln x)^5 \, dx \, ?$$

(a) $5x(\ln x - 1) + C$ **(b)** $5(\ln x)^4 - \int 5(\ln x)^4 \, dx$ **(c)** $x(\ln x)^5 - \int 5(\ln x)^4 \, dx$
(d) $x(\ln x)^5 - \int \frac{1}{6}(\ln x)^6 \, dx$ **(e)** $x(\ln x)^5 - \int \frac{1}{6}\frac{(\ln x)^6}{x} \, dx$

5. A chain lying on the ground is 18 ft long and weighs 90 lbs. How much work is required to raise one end of the chain to a height of 6 ft?
(a) 90 ft-lb **(b)** 80 ft-lb **(c)** 70 ft-lb **(d)** 60 ft-lb **(e)** 50 ft-lb

6. Use integration by parts to evaluate the following integral,

$$\int x \sec^2 x \, dx$$

(a) $x^2 \sec x - x + \frac{1}{3}x^3 \sec^3 x + C$ **(b)** $x \tan x + \frac{1}{2}\tan^2 x + C$ **(c)** $x \tan x + \ln|\sec x| + C$
(d) $x \tan x + \ln|\cos x| + C$ **(e)** $x^2 \sec x - \frac{1}{3}x^3 + x^2 \sec^3 x + C$

7. Evaluate the following integral,

$$\int \frac{1}{\sqrt{4+x^2}}\,dx$$

(a) $\ln|\frac{1}{\sqrt{4+x^2}}+x|+C$ **(b)** $\ln|\frac{1}{2}\sqrt{4+x^2}+\frac{x}{2}|+C$ **(c)** $\ln|\sqrt{4+x^2}-\frac{x}{2}|+C$
(d) $\frac{\sqrt{4+x^2}}{2x}+x+C$ **(e)** $\frac{2x}{\sqrt{4+x^2}}+\sqrt{4+x^2}+C$

8. Evaluate the following integral,

$$\int \frac{\sqrt{x^2-1}}{x^4}\,dx$$

(a) $\frac{x}{\sqrt{x^2-1}}+\ln|x+\sqrt{x^2-1}|+C$ **(b)** $\frac{\sqrt{x^2-1}}{x}+\frac{1}{2}\ln|x+\sqrt{x^2-1}|+C$ **(c)** $\frac{(x^2-1)^{3/2}}{3x^3}+C$
(d) $\frac{x}{2\sqrt{x^2-1}}+\ln|\frac{x}{2}+\sqrt{x^2-1}|+C$ **(e)** $\frac{x}{\sqrt{x^2-1}}-\sec^{-1}x+C$

9. Write the form of the partial fraction decomposition of

$$\frac{x^2+4x}{(x^2-9)(x+1)^2}$$

(a) $\frac{Ax+B}{x^2-9}+\frac{Cx+D}{(x+1)^2}$ **(b)** $\frac{Ax+B}{x^2-9}+\frac{C}{(x+1)}+\frac{D}{(x+1)^2}$ **(c)** $\frac{A}{x-3}+\frac{B}{x+3}+\frac{Cx+D}{(x+1)^2}$
(d) $\frac{A}{x-3}+\frac{B}{x+3}+\frac{C}{x+1}+\frac{D}{(x+1)^2}$ **(e)** $\frac{A}{x-3}+\frac{B}{x+3}+\frac{C}{x+1}+\frac{Dx+E}{(x+1)^2}$

10. Evaluate the following integral,

$$\int \frac{x^3+4}{x^2-1}\,dx$$

(a) $\frac{1}{3}\ln|x-1|-\frac{4}{3}\ln|x+1|+C$ **(b)** $\frac{1}{2}x^2+4x+\frac{1}{3}\ln|x-1|-\frac{4}{3}\ln|x+1|+C$
(c) $\frac{1}{2}x^2+4x+\frac{5}{2}\ln|x-1|-\frac{3}{2}\ln|x+1|+C$ **(d)** $\frac{1}{2}x^2+\frac{1}{3}\ln|x-1|-\frac{4}{3}\ln|x+1|+C$
(e) $\frac{1}{2}x^2+\frac{5}{2}\ln|x-1|-\frac{3}{2}\ln|x+1|+C$

11. Evaluate the following integral,

$$\int \frac{1}{x^2+2x+5}$$

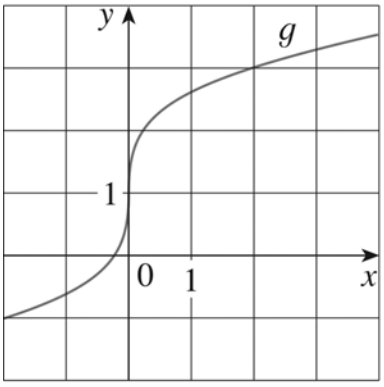
(a) $\frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right)+C$ **(b)** $\frac{1}{4}\tan^{-1}\left(\frac{x+1}{4}\right)+C$ **(c)** $2x+1+\ln|x^2+2x+5|+C$
(d) $x+1+\frac{1}{2}\ln|x^2+2x+5|+C$ **(e)** $2x+1+\tan^{-1}\left(\frac{x+1}{2}\right)+C$

12. Evaluate the following integral,

$$\int \cos^2 x \sin^3 x \, dx$$

(a) $\sin x - \frac{1}{3}\sin^3 x + \frac{1}{4}\cos^4 x + C$ **(b)** $-\frac{1}{4}\cos^4 x + \frac{1}{6}\cos^6 x + C$
(c) $\cos x - \frac{1}{3}\cos^3 x + \frac{1}{4}\sin^4 x + C$ **(d)** $-\frac{1}{4}\sin^4 x + \frac{1}{6}\sin^6 x + C$
(e) $-\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C$

13. Let g be the function whose graph is shown to the right. Find $g^{-1}(3)$.



(a) 0 **(b)** 1 **(c)** 3.3 **(d)** 2 **(e)** 3.5

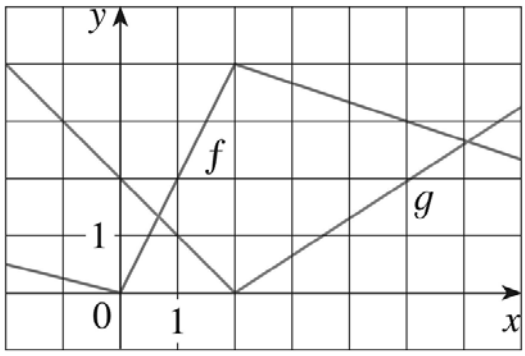
14. Consider the following equations. In each case suppose that we apply the Intermediate Value Theorem using the interval $[0, 1]$. (i.e., we take $a = 0$, $b = 1$ in the Intermediate Value Theorem.)

(i) $x^2 + x - 4 = 0$
(ii) $2e^x = x - 1$
(iii) $\ln(x + 1) = 1 - 2x$

For which equations does the Intermediate Value Theorem conclude that there must be a root of the equation in the interval $(0, 1)$?

(a) (i) only and (ii) only **(b)** (ii) only **(c)** (ii) and (iii) only
(d) none of them **(e)** (iii) only

15. Let f and g be the functions whose graphs are shown to the right. Let $h(x) = f(g(x))$. Find $h'(-1)$.



(a) $\frac{1}{3}$ **(b)** $\frac{1}{2}$ **(c)** $-\frac{1}{3}$ **(d)** $-\frac{1}{2}$ **(e)** 0

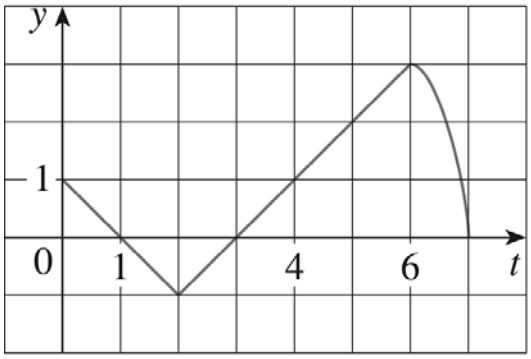
16. Evaluate the following integral,

$$\int \tan^4 x \, dx$$

(a) $\frac{1}{3}\tan^3 x - \tan x + x + C$ **(b)** $\frac{1}{5}\tan^5 x - \frac{1}{3}\tan^3 x + C$ **(c)** $\frac{1}{3}\sec^3 x + \frac{1}{5}\sec^5 x + C$
(d) $\sec x - \frac{1}{3}\sec^3 x - \frac{1}{5}\sec^5 x + C$ **(e)** $\frac{1}{5}\tan^5 x \sec^2 x + \frac{1}{3}\tan^3 x \sec x + C$

17. Find an expression for the arc length of the upper half of a circle of radius 9 on the interval $[0, t]$ (where $0 < t < 9$).
(a) $\frac{1}{3}\tan^1\left(\frac{t}{3}\right)$ **(b)** $9\sin^{-1}\left(\frac{t}{9}\right)$ **(c)** $\frac{1}{3}\sin^{-1}\left(\frac{t}{3}\right)$ **(d)** $\frac{1}{9}\tan^{-1}\left(\frac{t}{9}\right)$ **(e)** $3\tan^{-1}\left(\frac{t}{3}\right)$

18. Let f be the function whose graph is shown to the right, and let $g(x) = \int_0^x f(t)\, dt$ Find an equation of the tangent line to the graph of g at the point where $x = 5$.



(a) $y = \frac{3}{2}x - 2$ **(b)** $y = 2x - \frac{3}{2}$ **(c)** $y = 2x - \frac{17}{2}$ **(d)** $y = \frac{3}{2}x - 1$ **(e)** $y = \frac{3}{2}x - 3$

19. Find y' if $x^y = y^x$.

(a) $\frac{\ln x - y}{\ln y - x}$ **(b)** $\frac{\ln y - \frac{y}{x}}{\ln x - \frac{x}{y}}$ **(c)** $\frac{y^{x-2}}{x^{y-2}}$ **(d)** $\frac{y^{x-2}}{x^{y-2}}\frac{\ln y}{\ln x}$ **(e)** $\frac{y^{x-2}}{x^{y-2}}\frac{\ln x}{\ln y}$

20. If $f(1) = 8$ and $f'(x) \geq 5$ for $1 \leq x \leq 5$ then, according to the Mean Value Theorem, what is the smallest possible value for $f(5)$?

(a) 22 **(b)** 24 **(c)** 26 **(d)** 30 **(e)** 28

21. Let $f(x) = e^{1/x}$. Find the largest interval(s) on which f is concave up.

(a) $(-\frac{1}{2}, 0), (0, \infty)$ **(b)** $(-\frac{1}{2}, 0)$ **(c)** $(-\infty, -\frac{1}{2}), (0, \infty)$ **(d)** $(-\infty, 0), (0, \frac{1}{2})$ **(e)** $(0, \frac{1}{2})$.

22. Which of the following is equal to $\cosh^2 x + \sinh^2 x$?

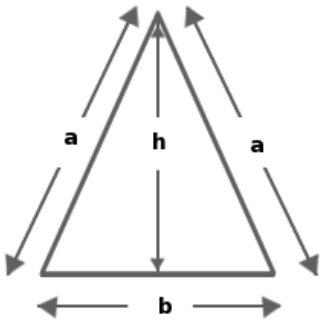
(a) -1 **(b)** 1 **(c)** $\cosh 2x$ **(d)** $\sinh 2x$ **(e)** $\frac{1}{2}e^{2x}$

23. Evaluate the following limit,

$$\lim_{x \to \infty} x^3 e^{-x^2}$$

(a) 1 **(b)** ∞ **(c)** 0 **(d)** e **(e)** e^{-1}

24. If the two equal sides of an isosceles triangle have length a, find the length of the third side that maximizes the area of the triangle.
(i.e., find b in terms of a in the figure to the right)



(a) $2a$ **(b)** $\frac{1}{2}a$ **(c)** a **(d)** $\sqrt{2}\,a$ **(e)** $\frac{1}{\sqrt{2}}a$

25. Find the absolute maximum and absolute minimum of the following function

$$f(x) = \frac{\sqrt{x}}{1 + x^2}$$

on the interval $[0, 2]$.

(a) $\frac{1}{4}3^{3/2}, 0$ **(b)** $\frac{1}{4}3^{3/2}, -\frac{1}{5}\sqrt{2}$ **(c)** $\frac{1}{4}3^{3/4}, -\frac{1}{5}\sqrt{2}$ **(d)** $\frac{1}{4}3^{3/4}, 0$ **(e)** $\frac{1}{5}\sqrt{2}, 0$

26. Suppose that the *derivative* of a function f is given by

$$f'(x) = \frac{4-x}{x^{1/3}(6-x)^{2/3}}$$

Find the largest interval(s) on which f is increasing.

(a) $(0,6)$ **(b)** $(0,4)$ **(c)** $(0,4),(6,\infty)$ **(d)** $(-\infty,0),(4,6)$ **(e)** $(-\infty,0),(4,\infty)$

27. If $f'(x) = \frac{\sin\sqrt{x}}{\sqrt{x}}$ and $f(0) = 1$, find $f(\pi^2/9)$.

(a) -1 **(b)** 0 **(c)** 3 **(d)** 1 **(e)** 2

28. If $g(x) = \int_x^{2x} \ln t\, dt$. Find $g'(2)$.

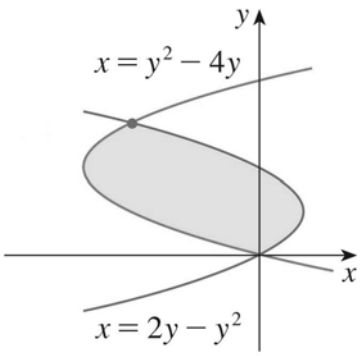
(a) $2\ln 2$ **(b)** $3\ln 2$ **(c)** $\ln 2$ **(d)** $4\ln 2$ **(e)** $5\ln 2$

29. Evaluate the following limit by first expressing it as a definite integral,

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n} e^{1+2i/n}$$

(a) $e(e^2-1)$ **(b)** e^2-1 **(c)** $e-1$ **(d)** $e(e-1)$ **(e)** e^2

30. Find the area of the region enclosed by the two curves in the figure to the right.



(a) 7 **(b)** 8 **(c)** 9 **(d)** 6 **(e)** 5

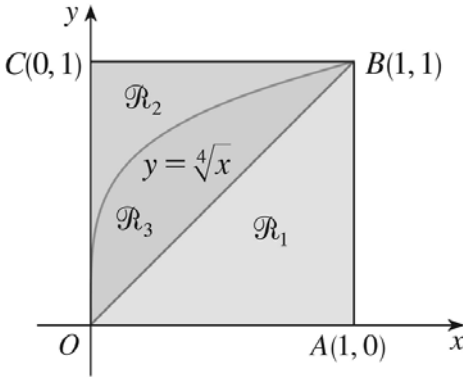
31. Suppose that we were to use Maple to evaluate

$$\int x^2 e^x \, dx$$

using integration by parts with $u = x^2$ and $dv = e^x \, dx$, and that the above integral was given the name `B`. Which command would be used?

(a) `>IntegrationByParts(B,u=x^2,dv=exp(x))`
(b) `>Integrate(B,u=x^2,dv=exp(x))`
(c) `>Int(B,u=x^2,dv=exp(x))`
(d) `>Parts(B,x^2)`
(e) `>IBP(B,u=x^2,dv=exp(x))`

32. Find the volume of the solid obtained by rotating the region $\mathcal{R}_1$ (in the figure to the right) about the line OA.



(a) π **(b)** $\frac{\pi}{2}$ **(c)** $\frac{\pi}{3}$ **(d)** $\frac{2\pi}{3}$ **(e)** $\frac{3\pi}{2}$

(THE END)

Answers:

1. d **2.** b **3.** b **4.** c **5.** a **6.** d **7.** b **8.** c **9.** d **10.** e
11. a **12.** e **13.** d **14.** e **15.** a **16.** a **17.** b **18.** c **19.** b **20.** e
21. a **22.** c **23.** c **24.** d **25.** d **26.** b **27.** e **28.** b **29.** a **30.** c
31. d **32.** c

(THE END)