COMPSCI/SFWRENG 2FA3

Discrete Mathematics with Applications II

Winter 2020

Assignment 11

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Assignment 11 consists of two problems. You must write your solutions to the problems using LaTeX.

Please submit Assignment 11 as two files, `Assignment_11_`*`YourMacID`*`.tex` and `Assignment_11_`*`YourMacID`*`.pdf`, to the Assignment 11 folder on Avenue under Assessments/Assignments. *`YourMacID`* must be your personal MacID (written without capitalization). The `Assignment_11_`*`YourMacID`*`.tex` file is a copy of the LaTeX source file for this assignment (`Assignment_11.tex` found on Avenue under Contents/Assignments) with your solution entered after each problem. The `Assignment_11_`*`YourMacID`*`.pdf` is the PDF output produced by executing

```
pdflatex Assignment_11_YourMacID
```

This assignment is due **Sunday, April 12, 2020 before midnight.** You are allow to submit the assignment multiple times, but only the last submission will be marked. **Late submissions and files that are not named exactly as specified above will not be accepted!** It is suggested that you submit your preliminary `Assignment_11_`*`YourMacID`*`.tex` and `Assignment_11_`*`YourMacID`*`.pdf` files well before the deadline so that your mark is not zero if, e.g., your computer fails at 11:50 PM on April 12.

Although you are allowed to receive help from the instructional staff and other students, your submission must be your own work. Copying will be treated as academic dishonesty! If any of the ideas used in your submission were obtained from other students or sources outside of the lectures and tutorials, you must acknowledge where or from whom these ideas were obtained.

Problems

1. **[10 points]** Let $\Sigma = \{a, b\}$ and

$$L = \{x \in \Sigma^* \mid \#a(x) \text{ is even and } \#b(x) \text{ is even}\}.$$

Construct a total Turing machine that accepts L.

Jatin Chowdhary — Chowdhaj — April 12^{th}

Put the description of your Turing machine here.

First of all, since zero is an even number (*By Parity of Zero*), the Turing Machine accepts the empty string as valid (does not reject it).

Let $M = (Q, \Sigma, \Gamma, \vdash, \llcorner\!\lrcorner, \delta, s, t, r)$ be the Turing Machine, where:

$Q = \{s,\ O_o,\ O_e,\ E_o,\ E_e,\ t,\ r\}$

$\Sigma = \{a,\ b\}$

$\Gamma = \Sigma \cup \{\vdash,\ \llcorner\!\lrcorner\} = \{a,\ b,\ \vdash,\ \llcorner\!\lrcorner\}$

s = Start State

t = Accepted State

r = Rejected State

δ is defined by the following table:

	$\vdash$	a	b	$\llcorner\!\lrcorner$
s	(s, $\vdash$, R)	(O_e, a, R)	(E_o, b, R)	(t, $\llcorner\!\lrcorner$, R)
O_o	(O_o, $\vdash$, R)	(E_o, a, R)	(O_e, b, R)	(r, $\llcorner\!\lrcorner$, R)
O_e	(O_e, $\vdash$, R)	(E_e, a, R)	(O_o, b, R)	(r, $\llcorner\!\lrcorner$, R)
E_o	(E_o, $\vdash$, R)	(O_o, a, R)	(E_e, b, R)	(r, $\llcorner\!\lrcorner$, R)
E_e	(E_e, $\vdash$, R)	(O_e, a, R)	(E_o, b, R)	(t, $\llcorner\!\lrcorner$, R)
t	(t, $\vdash$, R)	(t, a, R)	(t, b, R)	(t, $\llcorner\!\lrcorner$, R)
r	(r, $\vdash$, R)	(r, a, R)	(r, b, R)	(r, $\llcorner\!\lrcorner$, R)

Note: The 'O'/'o' means Odd and the 'E'/'e' means Even. The first letter corresponds to 'a' and the second letter corresponds to 'b'. So the state E_o means that there are an even number of the letter 'a' and odd number of the letter 'b'.

2. **[10 points]** Let $\Sigma = \{a, b\}$ and

$$L = \{xn \in \Sigma^* \mid \#a(x) = \#b(x)\}.$$

Construct a total Turing machine that accepts L.

Hint: Model your Turing machine on the Turing machine described in Example 2 of the 6 Turing Machines and Computability slides that accepts the language $\{a^n b^n c^n \mid n \geq 0\}$.

Jatin Chowdhary — Chowdhaj — April 12^{th}

Put the description of your Turing machine here.

$M = (Q, \Sigma, \Gamma, \vdash, \sqcup, \Delta, s, t, r)$ where:

$Q = \{s, q_1, q_2, q_3, q_3, q_4, q_5, q_6, q_7, t, r\}$

$\Sigma = \{a, b\}$

$\Gamma = \Sigma \cup \{\vdash, \dashv, \sqcup\} = \{a, b, \vdash, \dashv, \sqcup\}$

δ is defined by the following table:

	$\vdash$	a	b	$\dashv$	$\sqcup$
s	$(s, \vdash, R)$	(q_1, a, R)	(q_1, b, R)	$(r, \dashv, R)$	$(t, \sqcup, R)$
q_1	$(r, \vdash, R)$	(q_1, a, R)	(q_1, b, R)	$(r, \dashv, R)$	$(q_2, \dashv, L)$
q_2	$(q_4, \vdash, R)$	$(q_3, \sqcup, L)$	(q_8, b, L)	$(q_2, \dashv, L)$	$(q_2, \sqcup, L)$
q_3	$(q_4, \vdash, R)$	(q_3, a, L)	$(q_2, \sqcup, L)$	$(q_3, \dashv, L)$	$(q_3, \sqcup, L)$
q_4	$(q_4, \vdash, R)$	$(q_5, \sqcup, R)$	(q_4, b, R)	$(q_6, \dashv, L)$	$(q_4, \sqcup, R)$
q_5	$(q_5, \vdash, R)$	(q_4, a, R)	$(q_4, \sqcup, R)$	$(q_6, \dashv, L)$	$(q_5, \sqcup, R)$
q_6	$(t, \vdash, R)$	$(q_3, \sqcup, L)$	$(q_2, \sqcup, L)$	$(q_6, \dashv, L)$	$(q_7, \sqcup, L)$
q_7	$(t, \vdash, R)$	$(q_3, \sqcup, L)$	$(q_2, \sqcup, L)$	$(q_6, \dashv, L)$	$(q_7, \sqcup, L)$
t	$(t, \vdash, R)$	(t, a, R)	(t, b, R)	$(t, \dashv, R)$	$(t, \sqcup, R)$
r	$(r, \vdash, R)$	(r, a, R)	(r, b, R)	$(r, \dashv, R)$	$(r, \sqcup, R)$