1ZC3

Remember extra $\mathbb{C}$ chapter 10.1, 10.2, 10.3

Online only

Linked with textbook practice problems!

Last Day z in betta polar form!

$$z = re^{i\theta}$$

$$e^{i\theta} = \cos\theta + i\sin\theta = \text{cis}\,\theta$$

r = modulus of $z = \sqrt{a^2+b^2} = |z|$

$$e^{i\pi} = \cos\pi + i\sin\pi$$

$\Rightarrow e^{i\pi} = -1$

$\theta = \arg(z) \Rightarrow \tan\theta = b/a$

Choose $\theta \in (-\pi, \pi]$ (or in some texts: $\in [0, 2\pi)$)

Roots in $\mathbb{C}$ Find all 4th roots of $16i$

Solution 4th root is z $\Rightarrow$ $z^4 = 16i$

In polar $z = re^{i\theta}$

$16i \Rightarrow |16i| = 16$

$\arg(16i) = \pi/2$

(by inspection!)

$$(re^{i\theta})^4 = 16e^{i\pi/2}$$

$$r^4 e^{i4\theta} = 16e^{i\pi/2}$$

$\Rightarrow$ modulus equal $\Rightarrow$ $r^4 = 16$ $\Rightarrow$ $r = 16^{1/4} = \underline{\underline{2}}$ $(r>0)$

$\Rightarrow$ equivalent arguments $\Rightarrow$ $4\theta = \pi/2 + 2n\pi$

$$\theta = \pi/8 + \cancel{\frac{2n\pi}{4}} \to n\pi/2$$

$\underline{\underline{n=0}}$ $\quad \theta = \pi/8$

$\underline{\underline{n=1}}$ $\quad \theta = \pi/8 + \pi/2 = 5\pi/8$

$\underline{\underline{n=2}}$ $\quad \theta = \pi/8 + 2\cancel{\pi/2} = 9\pi/8$

$+\pi$

$\underline{\underline{n=3}}$ $\quad \theta = \pi/8 + 3\pi/2 = 13\pi/8$

$\underline{\underline{n=4}}$ $\quad \theta = \pi/8 + \cancel{4\pi/2} \to 2\pi$

$\underline{\underline{\text{Solution}}}$ of $\quad z^4 = 16i \Rightarrow z = 2 \cdot e^{i\pi/8}$ (1)

or $2 \cdot e^{i5\pi/8}$ (2)

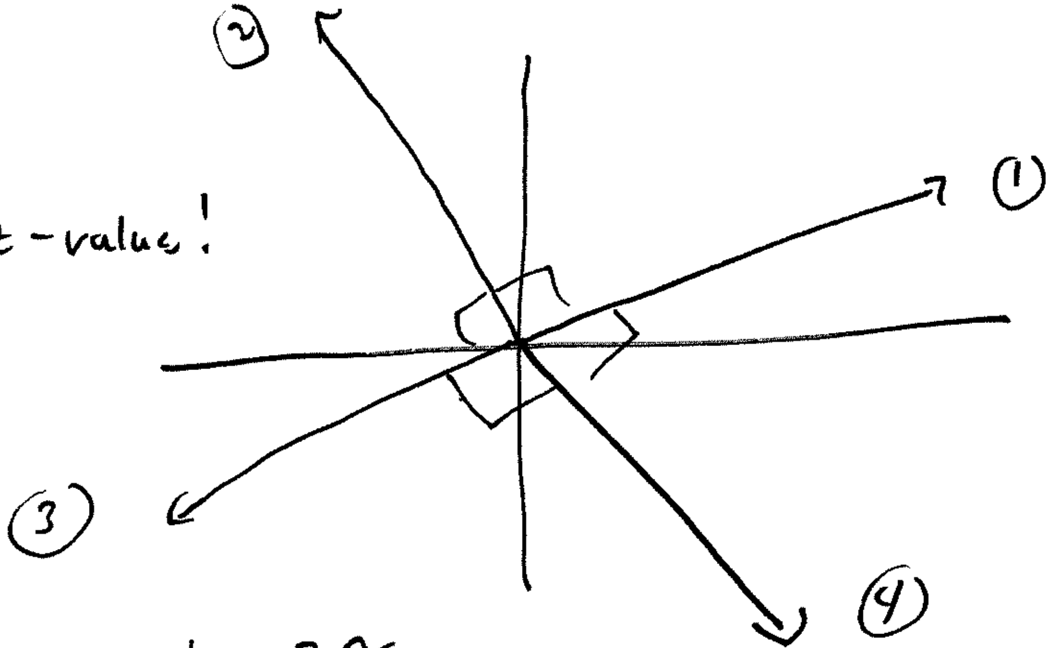
or $2 \cdot e^{i9\pi/8}$ (3)

or $2 \cdot e^{i13\pi/8}$ (4)

If $z^n = w$

n possible z-values!

$|z| = |w|^{1/n}$



4th root

$\Rightarrow$ 4 roots!

arguments differ by $\frac{2\pi}{n}$.

Now we can have complex matrices!

eg $A = \begin{bmatrix} 2i & 6-i \\ 7i & +2-3i \end{bmatrix}$

Complex λ

Complex $\det(A)$!

Turns out i was there all along!

eg $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$$C_B(\lambda) = |B - \lambda I|$$

$$= \begin{vmatrix} -\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = \underline{\underline{\lambda^2 + 1}}$$

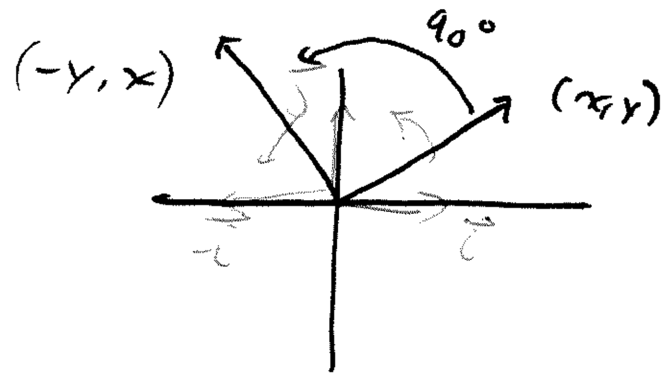
$$\lambda^2 + 1 = 0 \Rightarrow \lambda = \pm i$$

<u>notice</u>

$$B \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$B \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$B \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ x \end{bmatrix}$$



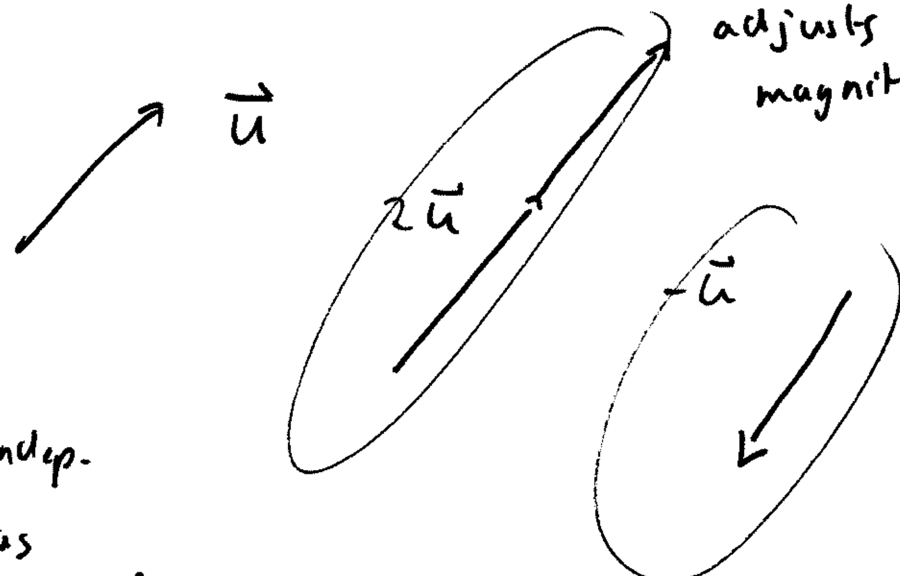
Vectors

old definition

"a vector is a quantity with direction & magnitude"

eg <u>velocity</u>, <u>force</u>, <u>displacement</u>, $\vec{E}$ <u>etc.</u>

eg



adjusts magnitude!

reverse direction

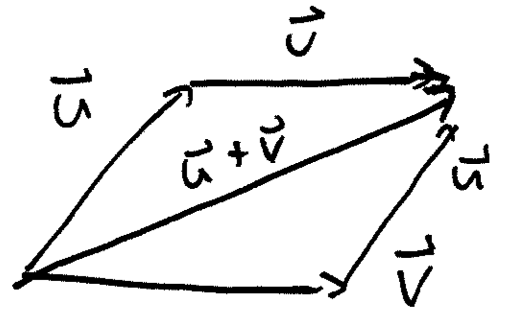
same magnitude

location indep.

"=" as long as same direction & <u>magnitude</u>!

<u>optional</u> double bars for <u>vector magnitude</u>!

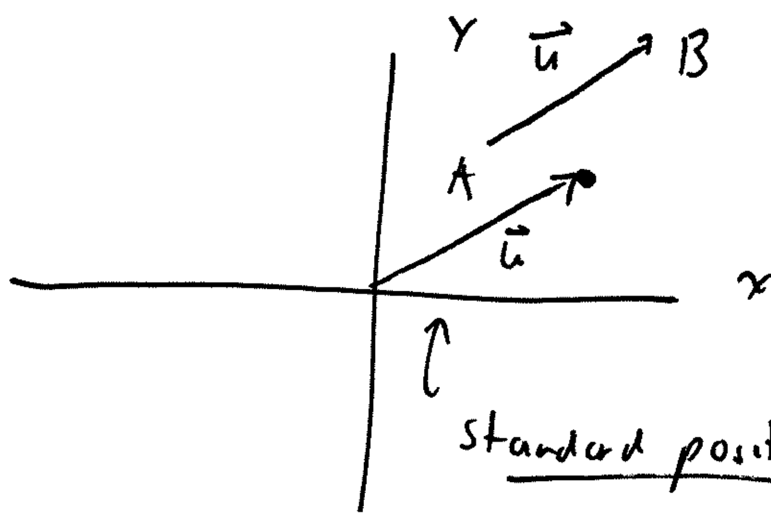
$$\|k\vec{u}\| = |k|\,\|\vec{u}\|$$



$\vec{u} + \vec{v} = \vec{v} + \vec{u}$

<u>parallelogram law</u> or "<u>$\parallel$grm</u>" law

<u>Descarte</u>!



$A = (a_1, a_2)$

$B = (b_1, b_2)$

$\vec{u} = \overrightarrow{AB}$

<u>standard position</u>! (<u>"start" at $(0,0)$</u>)

$\vec{u}$ related to <u>position</u> of <u>tip</u> $= (b_1 - a_1,\ b_2 - a_2)$

$= (u_1, u_2)$

$\|\vec{u}\| = \sqrt{u_1^2 + u_2^2}$ } by <u>pythagoras</u>!

"<u>Euclidean norm</u>"

$k\vec{u} = (ku_1, ku_2) \Rightarrow \|k\vec{u}\| = |k| \, \|\vec{u}\|.$

$-\vec{u} = (-u_1, -u_2) \Rightarrow$ opposite direction!

$\vec{u} + \vec{v} = (u_1, u_2) + (v_1, v_2) = (u_1 + v_1, u_2 + v_2)$

$= \vec{v} + \vec{u},$ just like llgrm law

e.g. $(2, 7) + (1, 1) = (3, 8)$

$3(-6, 1) = (-18, 3)$

notice adding follows: $\vec{u} + \vec{v} = \vec{v} + \vec{u}$

$k(\vec{u} + \vec{v}) = k\vec{u} + k\vec{v})$

$\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w} = \vec{u} + \vec{v} + \vec{w}$