COMPSCI/SFWRENG 2FA3

Discrete Mathematics with Applications II

Winter 2020

Week 08 Exercises

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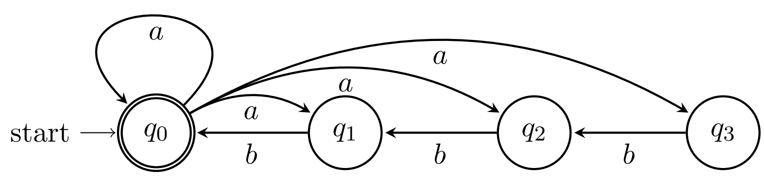
Revised: February 24, 2020

Exercises

1. Construct a NFA $N = (Q, \Sigma, \Delta, S, F)$ such that $\Sigma = \{0, 1\}$ and $L(N)$ is the set of strings in Σ^* that contain two consecutive 0s or two consecutive 1s.

2. Consider the following NFA N over $\{a, b\}$ defined by the following transition diagram:



   a. What is $L(N)$?

   b. Construct a DFA M equivalent to N that has no inaccessible states.

3. Construct an NFA N for the alphabet $\Sigma = \{a, b\}$ such that $L(N)$ is the set of all strings $x \in \Sigma^*$ in which at least one of the last three symbols of x is an a if $|x| \geq 3$ and at least one of the symbols of x is an a if $|x| < 3$. Present N as a transition diagram.

4. Consider the NFA $N = (Q, \Sigma, \Delta, S, F)$ defined by the following transition table:

	Q \ Σ	0	1
start →	p	$\{p, q\}$	$\{p\}$
	q	$\{r\}$	$\{r\}$
	r	$\{s\}$	$\{\}$
final →	s	$\{s\}$	$\{s\}$

Construct a DFA M equivalent to N that has no inaccessible states.

5. Let the *reverse* of a string x, written, $\mathsf{rev}(x)$, be the string x written backwards. Also, for $A \subseteq \Sigma^*$, let $\mathsf{rev}(A) = \{\mathsf{rev}(x) \mid x \in A\}$. Prove that, if $A \subseteq \Sigma^*$ is regular, then so is $\mathsf{rev}(A)$.

6. Let $N = (Q_N, \Sigma, \Delta_N, S_N, F_N)$ be an NFA and $M = (Q_M, \Sigma, \delta_M, s_M, F_M)$ be obtained from N by the subset construction. Prove by induction on $|x|$, that

   $$\hat{\delta}_M(A, x) = \hat{\Delta}_N(A, x)$$

   for all $A \subseteq Q_N$ and $x \in \Sigma^*$.