MATHEMATICS 1LS3 TEST 1

Day Class
Duration of Examination: 60 minutes
McMaster University, 7 October 2015

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First name (PLEASE PRINT): SOLUTIONS

Family name (PLEASE PRINT): ____________

Student No.: ____________

THIS TEST HAS 8 PAGES AND 7 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE.

Total number of points is 40. Marks are indicated next to the problem number. Any non-graphing calculator is allowed.

USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

You must show work to receive full credit.

Problem	Points	Mark
1	4	
2	6	
3	8	
4	4	
5	6	
6	5	
7	7	
TOTAL	40	

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1. **Multiple choice questions: circle ONE answer. No justification is needed.**

should be y=1

(a)[2] Identify all correct statements about the function $f(x) = \dfrac{x^2-1}{x^2+1}$.

(I) $f(x)$ is continuous at $x = 0$.
(II) $x = -1$ is a vertical asymptote of the graph of $f(x)$.
(III) $\cancel{x} = 1$ is a horizontal asymptote of the graph of $f(x)$.

(A) none (B) I only (C) II only (D) III only
(E) I and II (F) I and III (G) II and III (H) all three

(b)[2] The average half-life of acetaminophen (active ingredient in tylenol) is 2.5 hours. Assume that a patient is given a dose of 1000 mg of acetaminophen. Identify all correct statements.

(I) After 5 hours, 250 mg of acetaminophen is left unabsorbed in patient's body.
(II) After 2 hours, 450 mg of acetaminophen is left unabsorbed in patient's body.
(III) After 10 hours, less than 100 mg of acetaminophen is left unabsorbed in patient's body.

(A) none (B) I only (C) II only (D) III only
(E) I and II (F) I and III (G) II and III (H) all three

$1000 \xrightarrow{2{,}5\text{hrs}} 500 \xrightarrow{2{,}5\text{hrs}} 250 \xrightarrow{2{,}5\text{hrs}} 125 \xrightarrow{2.5\text{hrs}} 62{,}5$

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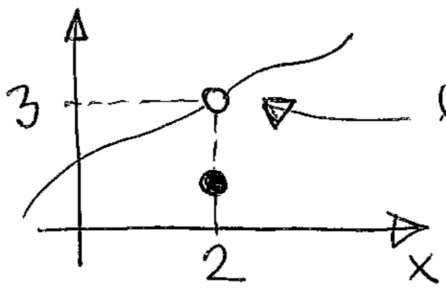
2. **Identify each statement as true or false, or yes or no (circle your choice). You do not need to justify your answer.**

(a)[2] The fact that $\lim_{x\to 2} f(x) = 3$ implies that $f(x)$ is continuous at $x = 2$.

TRUE FALSE



limit is 3, but
$f(2) \neq 3$
so not continuous
at $x=2$

(b)[2] The formula $H = Mx + 4$, where M is a constant, represents a proportional relationship between H and x.

TRUE FALSE

nonzero - so not proportional

(note: $H = Mx$ is a proportional relationship between H and x)

(c)[2] The average density of a human jaw bone is 1.07 oz/in^3, which is equivalent to approximately 1.85 g/cm^3. [Conversion factors: 1 oz = 28.35 g, 1 in = 2.54 cm]

TRUE FALSE

$$1.07 \frac{oz}{in^3} = 1.07 \cdot \frac{28.35\ g}{2.54^3\ cm^3}$$

$$= 1.85112$$

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Questions 3-7: You must show work to receive full credit.

3. Consider the function

$$f(x) = \begin{cases} \dfrac{x^3 - x}{x-1} & \text{if} \quad x < 1 \\ 2x & \text{if} \quad x \geq 1 \end{cases}$$

(a)[4] Find $\lim\limits_{x\to 1} f(x)$.

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(2x) = 2(1) = 2$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \frac{x^3-x}{x-1} = \lim_{x\to 1^-} \frac{x\cancel{(x-1)}(x+1)}{\cancel{x-1}}$$

$$= \lim_{x\to 1^-} x(x+1) = 2$$

thus, $\lim\limits_{x\to 1} f(x) = 2$

(b)[2] Is $f(x)$ continuous at $x = 1$? Explain why or why not.

yes, because $\lim\limits_{x\to 1} f(x) = 2$ and $f(1) = 2(1) = 2$

(c)[2] Is $f(x)$ continuous at $x = -1$? Explain why or why not.

near $x = -1$, $f(x) = \dfrac{x^3 - x}{x-1}$

this is a rational function with non-zero denominator $\rightarrow$ so continuous at $x = -1$

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4. The following excerpt is taken from *The laminar cortex model: a new continuum cortex model incorporating laminar architecture.* J. Du, V. Vegh, and D.C. Reutens. PLoS Computational Biology. 8.10 (Oct. 2012).

> the average of membrane potentials of neurons in the element, that is
>
> $$V = \frac{N_e V_e + N_i V_i}{N_e + N_i}$$
>
> where N_e, N_i are the numbers of excitatory and inhibitory neurons and V_e and V_i are the (average) membrane potentials of excitatory and inhibitory neuron populations respectively.

View V as a function of N_i.

By reading the text, we learn that V is the average of membrane potentials and N_i is the number of inhibitory neurons.

(a)[1] State (in one sentence) what question is answered by finding the inverse function of V.

How does N_i depend on V?
How does the number of inhibitory neurons depend on average of membrane potentials

(b)[3] Find a formula for the inverse function of V.

solve $V = \dfrac{N_e V_e + N_i V_i}{N_e + N_i}$ for N_i

$$V(N_e + N_i) = N_e V_e + N_i V_i$$
$$V N_e + V N_i = N_e V_e + N_i V_i$$
$$V N_i - V_i N_i = N_e V_e - V N_e$$
$$N_i (V - V_i) = N_e V_e - V N_e$$
$$N_i = \frac{N_e V_e - V N_e}{V - V_i} = \frac{N_e (V_e - V)}{V - V_i}$$

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5. (a)[3] Consider the formula for human population growth

$$P(t) = 4.43\left(\frac{\pi}{2} + \arctan\frac{t-2007}{42}\right)$$

where t is a calendar year and $P(t)$ is in billions. Find the range of $P(t)$. Based on it, state the maximum world population predicted by this model.

range of $\arctan\left(\frac{t-2007}{42}\right)$ .... $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

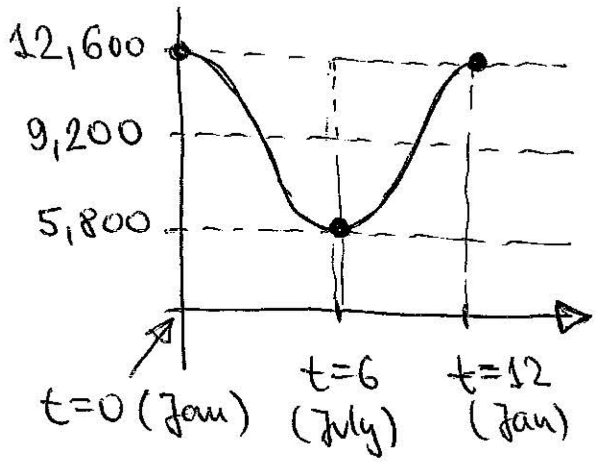
$\frac{\pi}{2} + \arctan$ .... $(0, \pi)$

$4.43\left(\frac{\pi}{2} + \arctan\right)$ ... $(0, 4.43\pi)$

≈ 13.92

max world population (in billions)

(b)[3] A population of river sharks (freshwater sharks) in New Zealand changes periodically with a period of 12 months. In January, it reaches a maximum of 12,600, and in July it reaches a minimum of 5,800. By selecting an appropriate trigonometric function, find a formula which describes how the population of river sharks changes with time.



use $\cos t$ ... period $= \frac{2\pi}{a} = 12$

so $a = \frac{2\pi}{12} = \frac{\pi}{6}$

so $\boxed{\cos\left(\frac{\pi}{6}t\right)}$

average $= \frac{12{,}600 + 5{,}800}{2} = 9{,}200$

amplitude $= 3{,}400$

$$P(t) = 9{,}200 + 3{,}400 \cdot \cos\left(\frac{\pi}{6}t\right)$$

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6. The survival rate (i.e., percent) $S(D)$ of clonogenic cells (cancer cells) is modelled by

$$S(D) = e^{-0.6D}$$

where $D \geq 0$ represents the applied radiation dose (measured in grays, Gy).

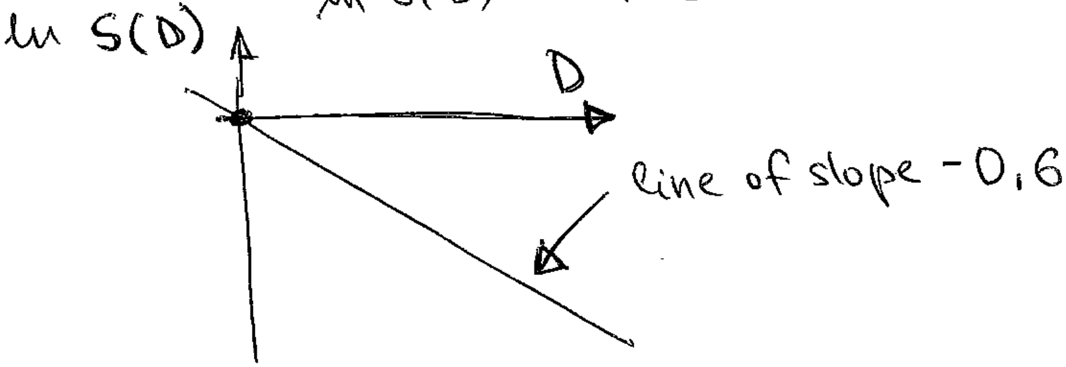
(a)[1] Assume that the dose $D = 5$ Gy is applied to a cancer. What percent of cancer cells is going to survive this treatment? Nearest integer;

$$S(5) = e^{-0.6(5)} = e^{-3} \approx 0.049787$$

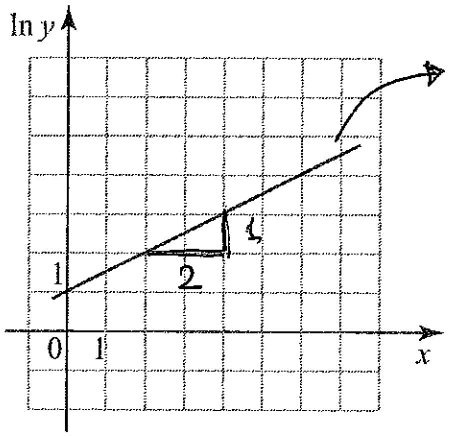
ie, 4.97 or approximately 5%

(b)[2] Sketch the semilog graph (use ln) of the survival rate. Label axes,

$$\ln S(D) = -0.6D$$



(c)[2] The linear graph below is a semilog graph of a function. Find an explicit formula for that function (i.e., write it in the form $y = \ldots$).



line of slope = 1/2, intercept 1

$$\cancel{y} = \frac{1}{2}x + 1$$

semilog, so this is ln y; i.e.

$$\ln y = \frac{1}{2}x + 1$$

$$\rightarrow y = e^{\frac{1}{2}x + 1}$$

(can simplify: $y = e^{1/2 x} \cdot e^1 = e \cdot e^{x/2}$)

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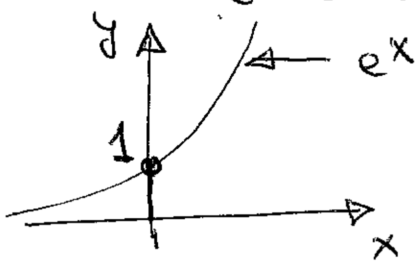
7. (a)[3] Identify the domain of the function $f(x) = \ln(e^x - 1)$.

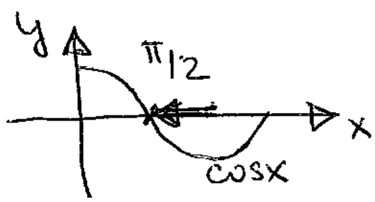
$e^x - 1 > 0 \therefore \underline{e^x > 1}$



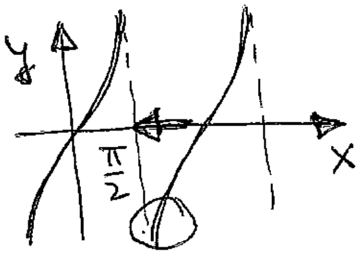
from the graph:
$e^x > 1$ when $x > 0$

$\Rightarrow$ domain: $x > 0$ or $(0, \infty)$

(b)[2] Find $\lim_{x\to\pi/2^+} \tan x = \lim_{x \to \frac{\pi}{2}+} \frac{\sin x}{\cos x} = \frac{1}{0} = \frac{\oplus}{\ominus} = -\infty$

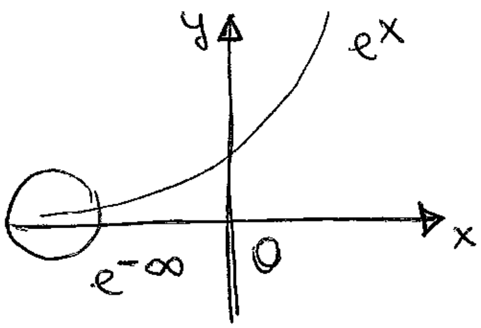


or: sketch the graph:



(c)[2] Find $\lim_{x\to\infty} e^{-x^2} = e^{-\infty} = 0$

(note: $-x^2 = -(\infty)^2 = -\infty$)



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