

MATHEMATICS 1LS3 TEST 3

Day Class

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Duration of Examination: 60 minutes

McMaster University, 5 November 2014

FIRST NAME (please print): _____

FAMILY NAME (please print): _____

Student No.: _____

THIS TEST HAS 8 PAGES AND 7 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE.

Total number of points is 40. Marks are indicated next to the problem number. Any non-graphing calculator is allowed.

USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

You must show work to receive full credit.

Problem	Points	Mark
1	6	
2	6	
3	7	
4	7	
5	5	
6	6	
7	3	
TOTAL	40	

1. Multiple choice questions: circle ONE answer. No justification is needed.

(a)[3] The slope of the tangent to the curve given implicitly by $xy^4 + \cos(\pi x) = 2$ at the point $(1, 1)$ is

- | | | | |
|-------------|-------------|-----------|-----------|
| (A) $\pi/4$ | (B) $\pi/2$ | (C) 1 | (D) -1 |
| (E) $-1/4$ | (F) $-1/2$ | (G) $1/2$ | (H) $1/4$ |

(b)[3] It is known that $f(4) = 0$ and $f'(4) = 0$. Which statements is/are true for all functions $f(x)$ which satisfy these two conditions?

- (I) $f(4) = 0$ is a local (relative) minimum of $f(x)$
- (II) the tangent line to the graph of $f(x)$ at $x = 4$ is $y = 0$
- (III) $f(4) = 0$ is a global (absolute) minimum of $f(x)$ on the interval $[-2, 0]$

- | | | | |
|--------------|---------------|----------------|---------------|
| (A) none | (B) I only | (C) II only | (D) III only |
| (E) I and II | (F) I and III | (G) II and III | (H) all three |

2. Identify each statement as true or false, or yes or no (circle your choice). You do not need to justify your answer.

(a)[2] $x = 0$ is a critical point (critical number) of the function $f(x) = \sqrt[3]{x}$.

TRUE

FALSE

(b)[2] If $f(x) = \arcsin(e^x - x - 1)$, then $f'(0) = 1$.

TRUE

FALSE

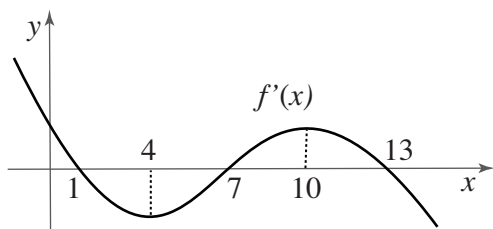
(c)[2] The formula $\lim_{h \rightarrow 0} \frac{e^{2+h} - e^2}{h} = e^2$ is correct.

YES

NO

Questions 3-7: You must show work to receive full credit.

3. The graph of the **derivative** f' of a function f is given below.



Answer the following questions. In order to receive credit, you need to justify your answers.

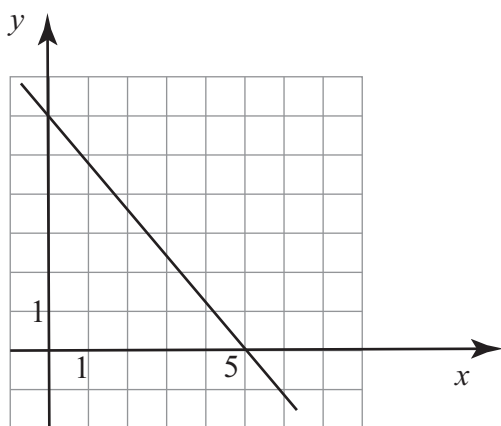
(a)[2] On which interval(s) is f decreasing?

(b)[2] On which interval(s) is f concave up?

(c)[3] At which value(s) of x does f have a maximum?

4. (a)[2] Find $f'(1)$, if $f(x) = 2^{\ln x} + (\ln 5)^2$.

(b)[3] Let $h(x) = x \sin(f(x))$. The graph of $f(x)$ is a line shown below. Find $h'(5)$.



(c)[2] Find $f'(0)$ if $f(x) = \ln \frac{ae^x + b}{ce^{-x} + d}$.

5. (a)[2] In the article *Migration behaviour of grizzly bears in Northern British Columbia: contribution to a modelling approach*. G. Brown et al, Bear Science 4 (June 2012), we find the formula

$$P(t) = \arctan(1.67t) + 4.71$$

where t represents time.

Next, we read “initially, $P(t) \approx 1.67t + 4.71$, which gives a linear relationship.” Explain why this statement is correct. [Hint: Think in terms of the linear approximation at $t = 0$.]

(b)[3] The linear model for the ratio S of cancer cells surviving radiation treatment states that

$$S(x) = e^{-ax+b}$$

where a and b are constants and x is a radiation dose. This formula is sometimes simplified using a quadratic approximation near $x = 0$. Find that approximation.

6. (a)[2] The function $f(x) = x^2e^{4x}$ has two critical points. Find them.

(b)[2] Give a statement of the Extreme Value Theorem. Make sure to clearly identify assumption(s) and conclusion(s).

(c)[2] Find the absolute maximum and the absolute minimum of the function $f(x) = x^2e^{4x}$ on the interval $[0, 1]$.

7. [3] The interstitial fluid pressure p at a location r mm from the centre of a tumour is given by

$$p(r) = 0.267p_i + \frac{2 \sinh(0.4r)}{r}$$

where p_i is the atmospheric pressure (assumed constant). Searching Wikipedia, you found that $\sinh x = \frac{1}{2} (e^x - e^{-x})$.

Researchers claim that p is an increasing function of r when $r > 2.5$. Justify their claim.