12A3

Last Day : Limits & Continuity

Recap

$$\lim_{x \to a} f(x) = L$$ } Limit of $f(x)$ as x approaches a is L

$$\underbrace{\lim_{x \to a^+}}\; f(x)$$

"right-handed limit"

approach from $x > a$ only

$$\underbrace{\lim_{x \to a^-}}\; f(x)$$

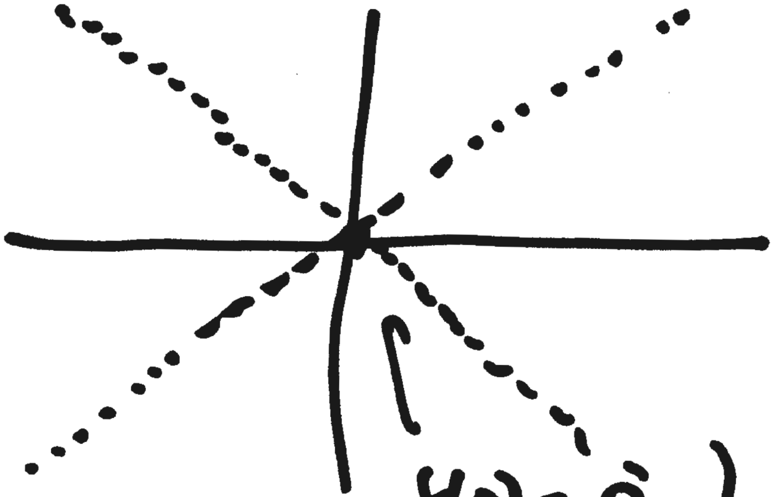
left-handed limit.

approach $x < a$ only

Also

$f(x)$ cont. at $x = a$ if $\lim_{x \to a} f(x) = f(a)$

eg



$f(x) = \begin{cases} x, & x \text{ rational} \\ -x, & x \text{ irrational} \end{cases}$

$\left.\begin{array}{l} f(0) = 0 \\ \& \displaystyle\lim_{x \to 0} f(x) = 0 \end{array}\right\} \Rightarrow$ cont at $x = 0$.

(nowhere else!)

$f(x)$ is left-cont. at $x=a$ if $\lim_{x\to a^-} f(x) = L$

$f(x)$ is right-cont at $x=a$ if $\lim_{x\to a^+} f(x) = L$

$f(x)$ cont. at a <u>iff</u> <u>both</u> left & right cont. at $x=a$

<u>limit laws</u>

$$\lim_{x\to a} f(x) \pm \lim_{x\to a} g(x) = \lim_{x\to a} f(x) \pm g(x)$$

$$\lim_{x\to a} f(x)\,g(x) = \left(\lim_{x\to a} f(x)\right)\left(\lim_{x\to a} g(x)\right)$$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x\to a} f(x) \Big/ \lim_{x \to a} g(x)$$

if not 0.

All applies to left & right handed limits as well!

Continuity Law If $f(x)$ & $g(x)$ are cont. at $x = a$.

Then $f(x) \pm g(x)$, $f(x)/g(x)$ $(g(x) \neq 0)$, $f(x)g(x)$

are all cont. at $x = a$.

Notice Say $\lim_{x \to a} f(x) = L$ & $g(x)$ is cont. for all x near L

Then $\lim_{x \to a} g \circ f(x) = \lim_{x \to a} g(f(x))$

$= g\left(\lim_{x \to a} f(x)\right) = g(L)$

eg. $\lim_{x \to 2} e^{\left(\frac{\frac{1}{x} - \frac{1}{2}}{x-2}\right)} = e^{\lim_{x \to 2}\left(\frac{\frac{1}{x} - \frac{1}{2}}{x-2}\right)}$

$= e^{\left(\lim_{x \to 2} \frac{\left[\frac{2-x}{2x}\right]}{[x-2]}\right)} = e^{\left(\lim_{x \to 2} \frac{-1}{2x}\right)}$

$= e^{-1/4}$

note if $f(x)$ cont. at a, g cont. at $f(a)$

then $g(f(x)) = g \circ f(x)$ is **cont** at a.

Define $f(x)$ cont. on (a, b)

means $f(x)$ cont. for all $x \in (a, b)$.

$f(x)$ cont. on $[a, b]$

means $f(x)$ cont. on (a, b) & left-cont. at b

& right-cont at a

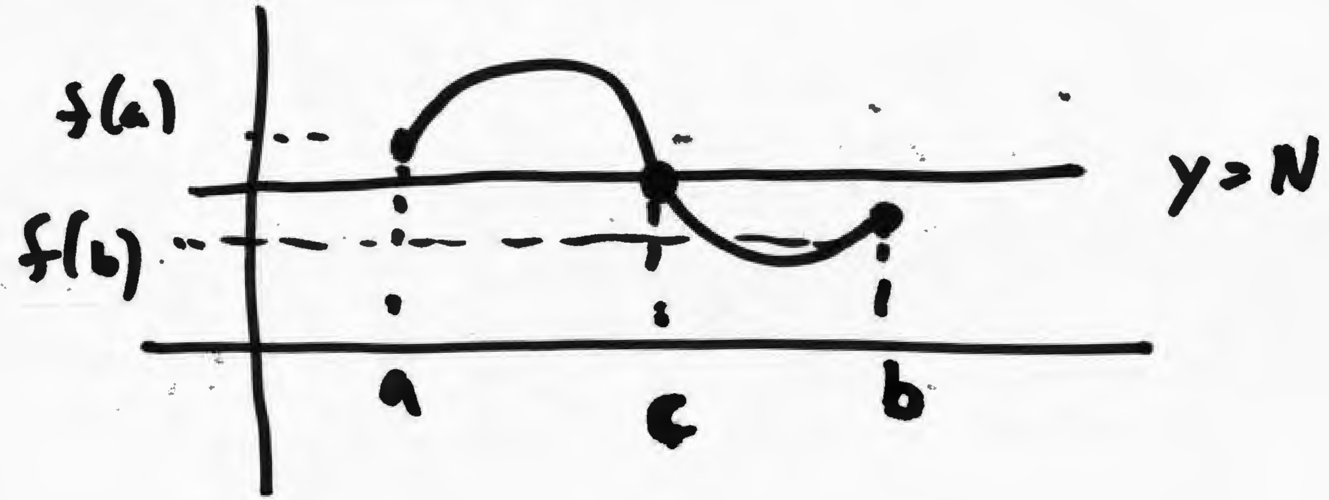
Intermediate Value Theorem (IVT)

If $f(x)$ is cont. on $[a,b]$

and $f(a) < N < f(b)$ (or $f(b) < N < f(a)$)

then there exists <u>at least</u> one $c \in (a,b)$

such that $f(c) = N$



$f(0) = 10^0 - 0 - 2 = -1$

$f(100) = \text{big} + \# > 0$

$f(x)$ cont. on $[0, 100]$

$f(0) < 0 < f(100)$

by IVT $f(c) = 0$ for some $c \in (0, 100)$

eg. Show $e^x = -x^3$ has a root
(ie cross! $e^x = -x^3$ for some x)

Solution $e^x = -x^3$ if $e^x + x^3 = 0$

$f(x)$

proceed as above!

$$f(0) = 1 + 0 = 1 > 0$$

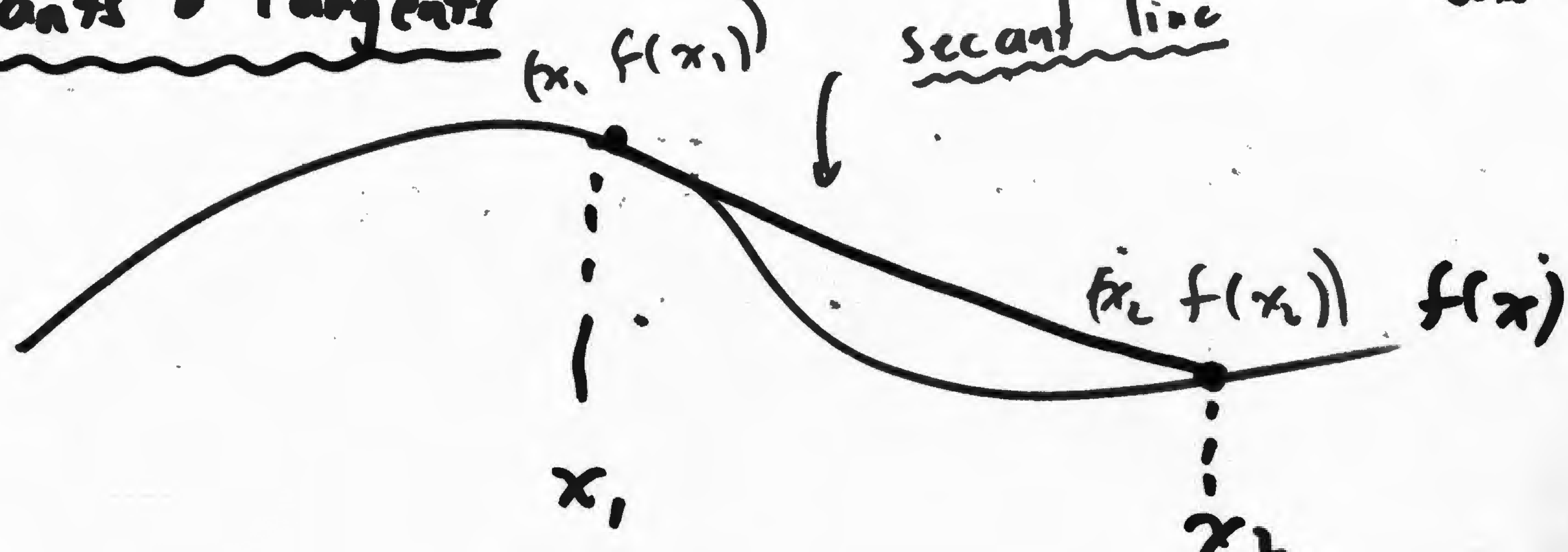
$$f(-100) \simeq -1000000 < 0$$

$$\Rightarrow \quad f(c) = 0, \qquad c \in (-100, 0)$$

$e^c = c^3$ i.e. such a c exists!

What is c? What value? Who knows? Who cares?

Secants & Tangents

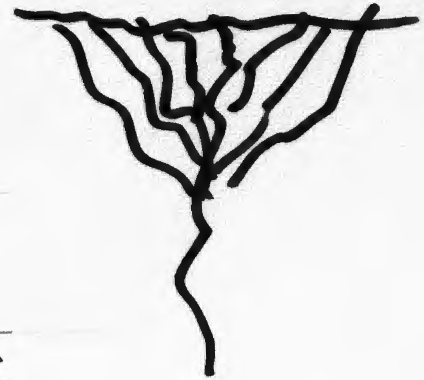


$m_{secant} = \frac{\text{rise}}{\text{run}} = \Delta y / \Delta x$

$= \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

= ave rate of change of f on $[x_1, x_2]$

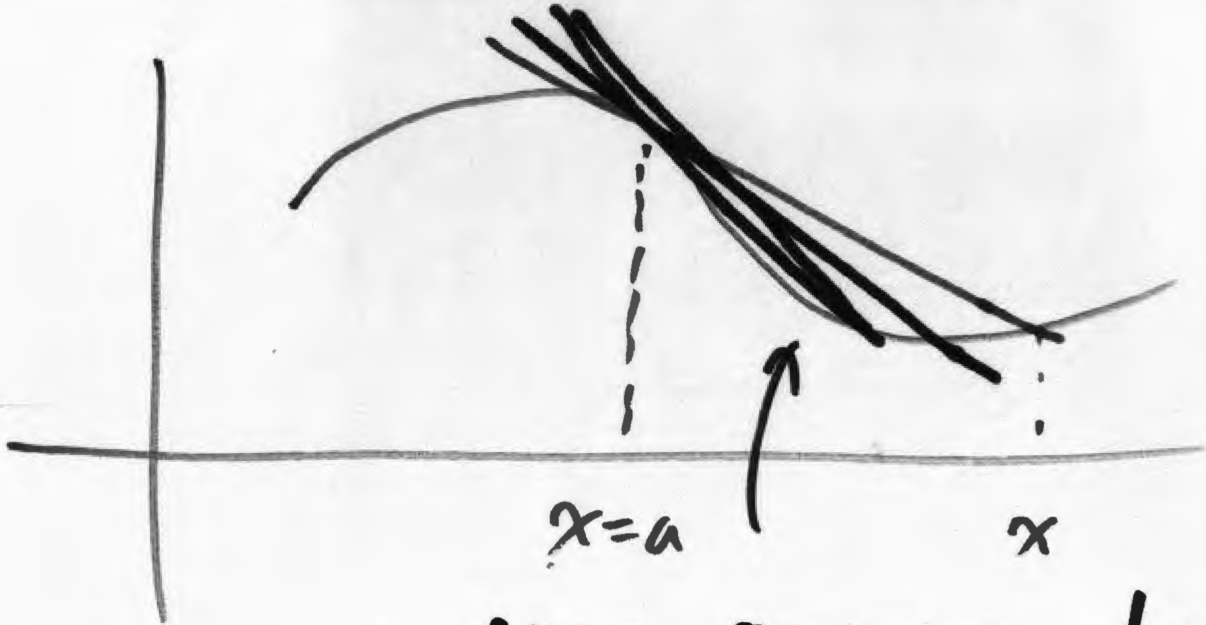
River Delta



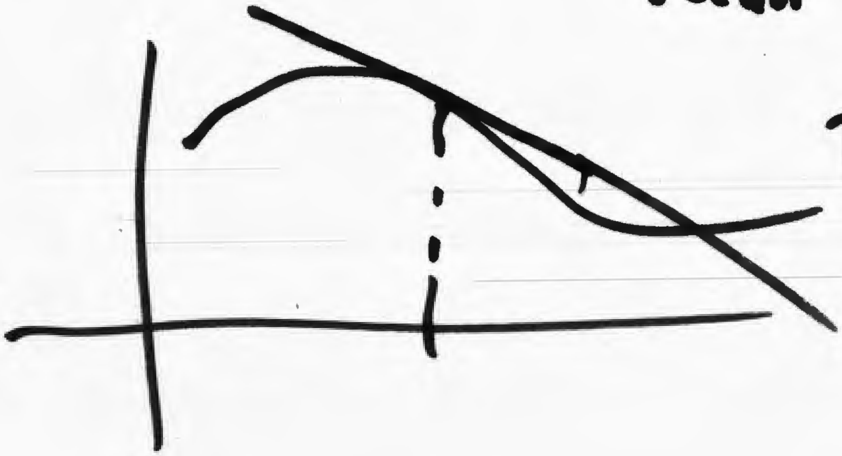
To get Instantaneous rate of change

take the limit!

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$



secant approaches <u>tangent</u> <u>line</u>



tangent lines "<u>kiss</u>" f(x)