

Conditional probability (Section 4)

---

**1.** Let  $S = \{1, 2, 3, 4, 5\}$  be a sample space, and assume that  $P(1) = 0.15$ ,  $P(2) = 0.25$ ,  $P(3) = 0.2$ ,  $P(4) = 0.3$ , and  $P(5) = 0.1$ .

(a) Find two sets  $A$  and  $B$  so that  $P(A|B) \neq P(B|A)$ .

(b) Find two sets  $A$  and  $B$  so that  $P(A|B) = P(B|A)$ .

**2.** The sample space is  $S = \{1, 2, 3, 4, 5\}$ , with  $P(1) = 0.2$ ,  $P(2) = 0.1$ ,  $P(3) = 0.15$ ,  $P(4) = 0.45$ ,  $P(5) = 0.1$ ; and  $A = \{1, 2\}$ ,  $B = \{3, 4, 5\}$ . Find  $P(A \cap B)$ ,  $P(A|B)$ , and  $P(B|A)$ .

---

Continued on next page

**3.** Using *conditional probability*, answer the following questions. Then, check your answers by using other means.

(a) A coin is tossed three times. Find the probability that exactly two heads occurred given that at least one toss resulted in heads.

(b) Two dice are rolled. Find the probability that one die is a 4 given that the sum is 6.

---

Continued on next page

4. Within a population of tigers, the trait of the dominant allele P is a spotted tail, and the trait of the recessive allele T is a striped tail. A baby tiger, born to genotype PT parents, has a spotted tail. What is the probability that it is of genotype PT?

5. Within a population of tigers, the trait of the dominant allele P is a spotted tail, and the trait of the recessive allele T is a striped tail. A baby tiger, born to genotype PT and TT parents, is known to have one T allele. What is the probability that it has a striped tail?

---

Continued on next page

(a) Find three mutually exclusive subsets of  $S$  that do not form a partition of  $S$ .

(c) Find a partition of  $S$  that contains three sets.

7. Within a population with a 1:1 sex ratio, 15% of females have blond hair and 5% of males have blond hair.

(a) What is the probability that a randomly chosen person has blond hair?

(b) What is the probability that a randomly selected blond person is female?

**8.** A certain medical condition (could be high blood pressure) comes in three forms,  $X$ ,  $Y$ , and  $Z$ , with prevalences of 45%, 35%, and 20%, respectively. The probability that a person will need emergency medical attention is 10% if he has the  $X$  form, 5% if he has the  $Y$  form, and 45% if he has the  $Z$  form. What is the probability that a person who has the condition will require emergency medical attention?

**9.** There is an open-air concert tomorrow, but the weather forecast does not look good: a 60% chance of rain. You need transportation to go to the concert and know the following: if it rains, there is a 30% chance that you will have a car available, but if it does not rain, the chance is 90%. How likely is it that you will go to the concert tomorrow?

---

Continued on next page

**10.** The average incidence of autism spectrum disorder is 45 cases per 10,000. [Source: Rutter, M. (2005). Incidence of autism spectrum disorders: changes over time and their meaning. *Acta Paediatrica*, 94 (1), 2-15.] A test for the disorder shows a positive result in 96% of people who have the disorder, and in 1% of people who do not have it.

(a) What is the probability that a randomly selected person tests positive for the disorder?

(b) If a person tests positive for the disorder, what is the probability that they have it?