1A03/17A3

Last Day Inverse Functions

eg $f(3) = 2 \Rightarrow f^{-1}(2) = 3$

f does $\Rightarrow$ f^{-1} undoes

$\Rightarrow$ $f(f^{-1}(x)) = x$

$f^{-1}(f(x)) = x$

eg. Given $f(x) = x^3$, find $f^{-1}(x)$

Solution $y = x^3$ $\xrightarrow{\text{new!}}$ $x = y^3$

$y = x^{1/3} = f^{-1}(x)$

eg. $f(x) = x + e^x$

a) Find $f^{-1}(f(12))$

b) Find $f^{-1}(1)$

Solution a) $f(12) = ??$

$f^{-1}(??) = 12$

b) What people try $f(x) = x + e^x = y$

$\hookrightarrow y + e^y = x$ solve for y

IVoooooo! FAIL

Only wanted $f^{-1}(1)$ not $f^{-1}(x)$

$f^{-1}(1) = a$ if $f(a) = 1$

$a + e^a = 1$

Let's try a pretty # $a = 0$? $0 + e^0 = 1$ ✓

$a = 0$

$\boxed{f^{-1}(1) = 0}$ Done!

Graphs of Inverse Functions

$f^{-1}(x)$ domain = range of $f(x)$

$f^{-1}(x)$ range = domain of $f(x)$

since $x \leftrightarrow y$ swapped

$\Rightarrow$ (a,b) on $f(x)$ if (b,a) on $f^{-1}(x)$

HLT horizontal line test

If our graph of $f(x)$ has any horizontal line crosses at most once $\Rightarrow$ $f^{-1}(x)$ exists (is a funct.)

$\Rightarrow$ $f(x)$ invertible

$\Rightarrow$ each x has a unique y & y has a unique x

$f(x)$ is one-to-one " 1-1 "

Classic Example of $f^{-1}(x)$ $\quad y = f(x) = a^x \quad y = f^{-1}(x) = \log_a x$

eg $2^3 = 8 \quad \log_2 8 = 3$

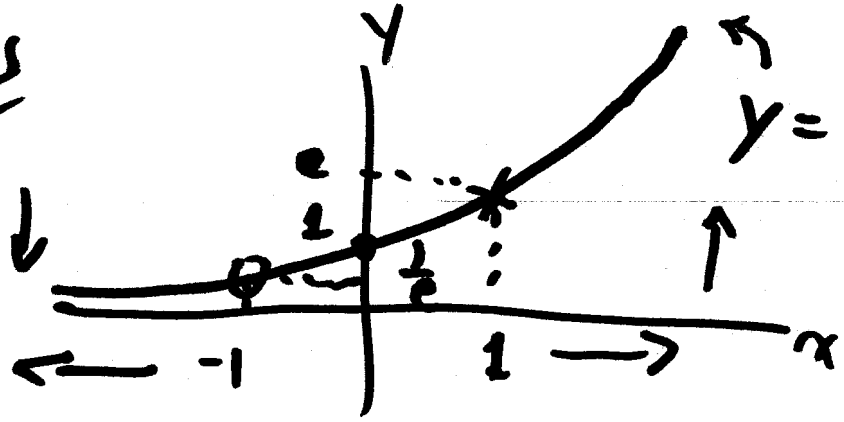
$f(x) = 2^x$

exponent to #

$f^{-1}(x) = \log_2 x$

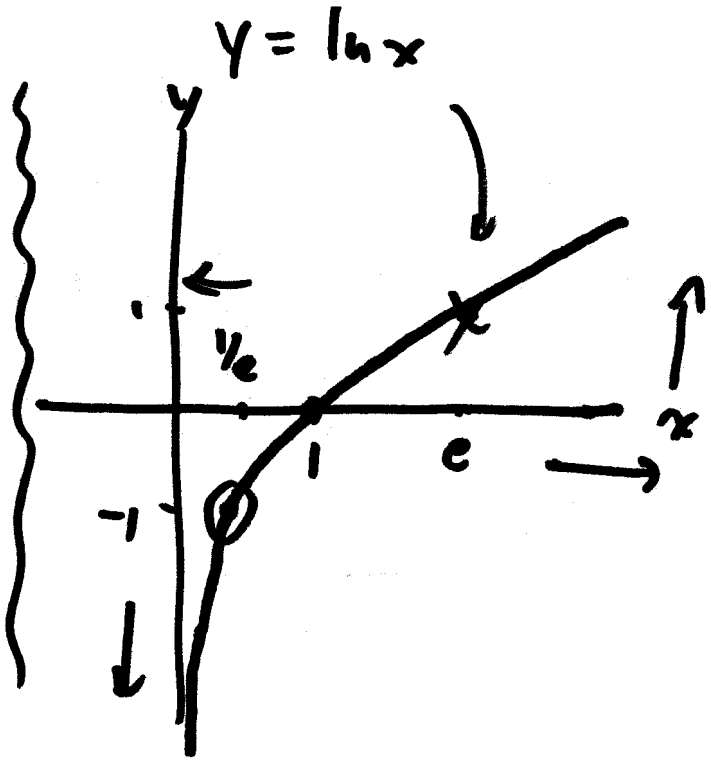
to exponent

Pics



$y = e^x$

$\frac{1}{e} = e^{-1}$



$e^{\infty} = \infty$, $e^{-\infty} = 0$

$e^x > 0$ } range

$\ln(\infty) = \infty$

$\ln(0^+) = -\infty$

Domain { $\ln x$ defined for $x > 0$

e.g. $5^2 = 25$ $\quad \log_5 25 = 2$.

$3^4 = 81$ $\quad \log_3 81 = 4$ $\quad \log_9 81 = 2$

$9^2 = 81$

Note $\quad a^{\log_a x} = x \qquad \log_a(a^x) = x$

$y = e^x \longrightarrow \log_e x = \ln x$

e "Euler's Number"

Watch out: $\log_e x = \ln x = \log x$ ~~(in math)~~

but $\log_{10} x = \log x$ ~~(in eng.)~~

we avoid by not using this!

eg $7e^x = 5e^x + 4$ find x

Solution $7e^x - 5e^x = 4 \Rightarrow 2e^x = 4$

$e^x = 2$

$\ln(e^x) = \ln 2$

x

$x = \ln 2$

$e^{\ln x} = \ln e^x = x$