McMaster University
Math 1LT3E

ASSIGNMENT 1

Brief Review of Integration (6.2 - 6.5), 6.6, 6.7

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1. Evaluate the following.

(a) $\int_0^1 \left(e^{0.2x} - 5x^4 + \frac{10}{(1+x)^2} \right) dx$

(b) $\int \frac{3x^2 + 5x + \sqrt{x}}{x}\, dx$

(c) $\int_1^2 x \ln x \, dx$

(d) $\int x e^{5x^2} \, dx$

2. (a) Graph $f(x) = \arctan x$ and shade the area corresponding to $\int_0^1 \arctan x \, dx$.

(b) Determine a Taylor polynomial of degree 3, $T_3(x)$, for $f(x) = \arctan x$ when x is near 0.

(c) Use your answer in part (b) to approximate $\int_0^1 \arctan x \, dx$. How could you improve on this approximation?

(d) Describe another technique that could be used to approximate $\int_0^1 \arctan x \, dx$.

(e) Evaluate $\int_0^1 \arctan x \, dx$ using the Fundamental Theorem of Calculus.

(f) Explain why $\int_{-1}^1 \arctan x \, dx = 0$.

3. (a) Sketch and shade the area bounded by the curves $f(x) = e^{2x}$, $g(x) = e^{-x}$, $x = -1$, and $x = 2$.

(b) Approximate the area of this region using a left, right, or midpoint Riemann sums and 3 rectangles. Draw the rectangles on the graph in part (a).

(c) Calculate the area of the bounded region by evaluating $\int_{-1}^{2} |e^{2x} - e^{-x}|\, dx$.

4. Calculate the area bounded by the given curves. Sketch the curves and shade the bounded region.

(a) $f(x) = \dfrac{1}{x}$, $g(x) = \dfrac{1}{x^2}$, $x = 1$, and $x = 3$

(b) $f(x) = e^x$, $g(x) = x + 1$, $x = -2$, and $x = 1$

5. Evaluate the following improper integrals.

(a) $\int_0^\infty \frac{1}{(1+2x)^{\frac{3}{2}}}\,dx$

(b) $\int_{10}^\infty \frac{1}{x^2}\,dx$

(c) $\int_1^\infty e^{-0.5x}\,dx$

(d) $\int_1^\infty \frac{\ln x}{x^4}\,dx$

6. The rate of change of the amount of pollutant (such as heavy metals) leaking into a lake is given by $A'(t) = 1.2te^{-0.2t}$, where t is time in hours and $A(0) = 0$. $A(t)$ is measured in grams.

(a) How much pollutant will leak into the lake during the first 24 hours?

(b) How much pollutant in total will eventually leak into the lake?

THE END