McMaster University
Math 1LT3

2nd Edition, Geese:
Review of Chapter 7 + Section 8.1

ASSIGNMENT 1

Review of Chapter 6 + Section 7.1

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1. Verify that $y(x) = e^{1-\sqrt{1+x^2}} - 1$ is a solution of the differential equation $\dfrac{dy}{dx} = -\dfrac{x(1+y)}{\sqrt{1+x^2}}$.

2. Sketch the graph of $f'(x) = 10e^{-2x} - 40$. Use this to sketch the graph of $f(x)$ given that $f(0) = 0$.

3. Find the general solution of the following pure-time differential equations.

(a) $\dfrac{dy}{dx} = \dfrac{x}{\sqrt{1-x^2}}$

(b) $y' = xe^{2x}$

4. Consider the pure-time differential equation $\dfrac{dw}{dt} = \dfrac{2}{1+t}$ with initial condition $w(0) = 3$.

(a) Solve the IVP. What is the value of $w(1)$?

(b) Apply Euler's method using a step size of $h = 0.25$ and starting from the initial condition $w(0) = 3$ to estimate $w(1)$.

5. In the textbook, read example 7.1.4 **Newtons Law of Cooling** (pages 520-521) and answer the following on p. 527:

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Read example 8.1.4 (pages 594-595) and answer the following on page 601

(a) Question 50.

(b) Question 52.

6. Psychologists interested in learning theory study **learning curves**. A learning curve is the graph of a function $P(t)$, the performance of someone learning a skill as a function of the training time t. The derivative dP/dt represents the rate at which performance improves.

(a) When do you think P increases most rapidly? What happens to dP/dT as t increases? Explain.

(b) If M is the maximum level of performance of which the learner is capable, explain why the differential equation

$$\frac{dP}{dt} = k(M - P) \qquad k \text{ a positive constant}$$

is a reasonable model for learning.

(c) Make a rough sketch of a possible solution of this differential equation.

7. Consider the modified logistic differential equation $P' = 2P(1 - \frac{120}{P})$.

(a) For which values of P is the population increasing? For which values of P is the population decreasing?

(b) Check that the constant function $P(t) = 120$ is a solution of the equation. What is special about it?

7. continued....
(c) Apply Euler's method using a step size of $h = 5$ and starting from the initial condition $P(0) = 200$ to estimate $P(15)$.

(d) Make a rough sketch of the solution.

THE END