1ZC3

Don't Forget   Tutorials Start this week

Help Centre opens today at 2:30pm

Assignment #1 Due this week!

Today   Matrices & Matrix Arithmetic

eg.

$$A = \underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}}_{3 \text{ columns}} \Big\} \text{ 2 rows}$$

name matrices using upper case letters

so called 2x3 matrix

In general: m rows, n col. $\Rightarrow$ $m \times n$ matrix

"matrix dimensions"

values in matrix A are entries / elements of matrix A

indicated by lower case & subscripts

eg $a_{11} = \left(\begin{array}{l}\text{element} \\ \text{in row 1, col 1.}\end{array}\right) = 1$, $a_{23} = 6$

$a_{12} = 2$ a_{34} = undefined } no element outside size of matrix

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Special Cases "Square matrices": "$n \times n$ matrices"

eg $\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$ eg $\begin{bmatrix} 1 & 1 & 7 \\ -1 & 2 & 5 \\ 0 & 0 & 1 \end{bmatrix}$ } number rows = # columns

Row Matrices ("Row vector")

eg $\underbrace{\begin{bmatrix} 1 & 2 \end{bmatrix}}_{1\times 2}$, $\underbrace{\begin{bmatrix} 2 & 5 & 7 \end{bmatrix}}_{1\times 3}$ } 1 row, n col. ie $1 \times n$

Column Matrices ("column vector")

eg $\begin{bmatrix} 2 \\ 7 \end{bmatrix} = \vec{a}$ eg $\begin{bmatrix} 1 \\ 5 \\ 4 \\ 2 \end{bmatrix} = \vec{b}$ In general:

2×1 4×1 $m \times 1$ matrices!

...

Matrix Math

eg $7 \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} = \underset{\uparrow \text{ scalar (ie real \#)}}{7} \cdot \underline{A}$

$$= \begin{bmatrix} 14 & 7 \\ -7 & 0 \end{bmatrix}$$

in general kA
mult. all entries by scalar k.

Matrix Addn. eg.

$$\begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -1 & 7 \\ 2 & 5 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 & 12 \\ 2 & 6 & 4 \end{bmatrix}$$

Add corresponding entries

eg. $\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 5 & 8 \end{bmatrix} = \underline{\underline{\text{FAIL}}}$

Only add matrices if <u>dimensions match</u>

$\Rightarrow$ If dimensions of matrices are equal, add, subtract & scalar multiply element to element.

<u>Matrix / Matrix Multiplication</u>

eg. $A = \begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 & 3 \\ -1 & -2 & 5 \end{bmatrix}$

$$AB = \overset{2\times 2}{\begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}} \overset{2\times 3}{\begin{bmatrix} 1 & 1 & 3 \\ -1 & -2 & 5 \end{bmatrix}} = \overset{2\times 3}{\begin{bmatrix} -1(1)+2(-1) & -1-4 & -3+1 \\ 0(1)+3(-1) & 0+3(-2) & 0+15 \end{bmatrix}}$$

$$= \begin{bmatrix} -3 & -5 & 7 \\ -3 & -6 & 15 \end{bmatrix}$$

In General If A is $m \times p$ & B is $p \times n$

then AB is $m \times n$ dimension

must equal

$$\underset{m \times p}{A} \; \underset{p \times n}{B} = \underset{m \times n}{C}$$

If # col. of $A \neq$ # rows of $B \Rightarrow AB$ undefined

eg

$$BA = \underset{2 \times 3}{\begin{bmatrix} 1 & 1 & 3 \\ -1 & -2 & 5 \end{bmatrix}} \underset{2 \times 2}{\begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}} = \text{FAIL}$$

DNE

no match!

In general Even if dimensions match $AB \neq BA$

"A & B do not commute"

Except in very rare cases!

eg. $$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 0+3 & 0+4 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}$$ not equal!

Mult. properties

$$A(B+C) = AB + AC$$
$$(B+C)A = BA + CA$$
not equal to each other!

$$(kA)B = A(kB) = k(AB)$$

watch out!

If $A = B$

$CA = CB \neq BC$ In general!

Transpose of a Matrix

e.g. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ then transpose of A

$$= A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

not an exponent

rows go to col., col. go to rows!

In general If $B = A^T$ then $b_{ij} = a_{ji}$

note: a_{11}, a_{22} etc. <u>don't move</u> under transpose.

"<u>Principal</u> ~~Diag~~ <u>Diagonal</u>"

<u>So</u> transpose reflects entries across "Principal diagonal"

eg $\begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 5 \\ 1 & 1 & 2 \end{bmatrix}^T = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 1 \\ 3 & 5 & 2 \end{bmatrix}$

principal diag.

& <u>Note:</u> If A is $\underline{m \times n}$ A^T is $n \times m$

<u>Properties of Transpose</u>

$$(A+B)^T = A^T + B^T$$

$$(kA)^T = kA^T$$

$$(AB)^T = B^T A^T$$

note $\left(\underset{m\times p}{A}\ \underset{p\times n}{B}\right)^T = (\underset{m\times n}{AB})^T = \underbrace{(AB)^T}_{n\times m} = \underset{n\times p}{B^T}\ \underset{p\times m}{A^T}$

Side Note If A is • square ie $n\times n$

trace of A = $tr(A)$ = sum on principal diagonal

eg $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ $tr(A) = 1+5+9 = 15$

eg $B = \begin{bmatrix} -1 & 0 \\ 2 & 5 \end{bmatrix}$ $\quad tr(B) = -1 + 5 = \underline{\underline{4}}$

<u>Note</u>

$$tr(kA) = k\, tr(A)$$

$$tr(A+B) = tr(A) + tr(B)$$

$$tr(\underrightarrow{ABC}) = tr(\underrightarrow{BCA}) = tr(\underrightarrow{CAB})$$