1A03/12A3

Interval Notation

$1 \leq x \leq 2 \rightsquigarrow x \in [1, 2]$ ← closed set

$\in$ "an element of"

$1 < x < 2 \rightsquigarrow x \in (1, 2)$ ← open set.

$\cong \quad x \in]1, 2[$

$x > 2 \rightsquigarrow x \in (2, \infty)$

$3 \leq x < 5 \rightsquigarrow x \in [3, 5)$

$x \neq 0 \rightsquigarrow x \in (-\infty, 0) \cup (0, \infty)$

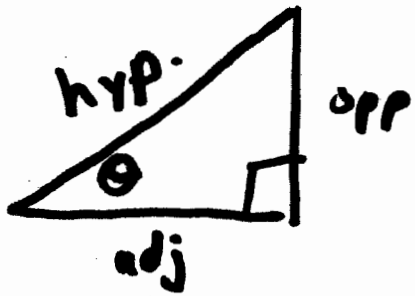
"union"

Algebra Trig!

radians! 2π rad = 360°

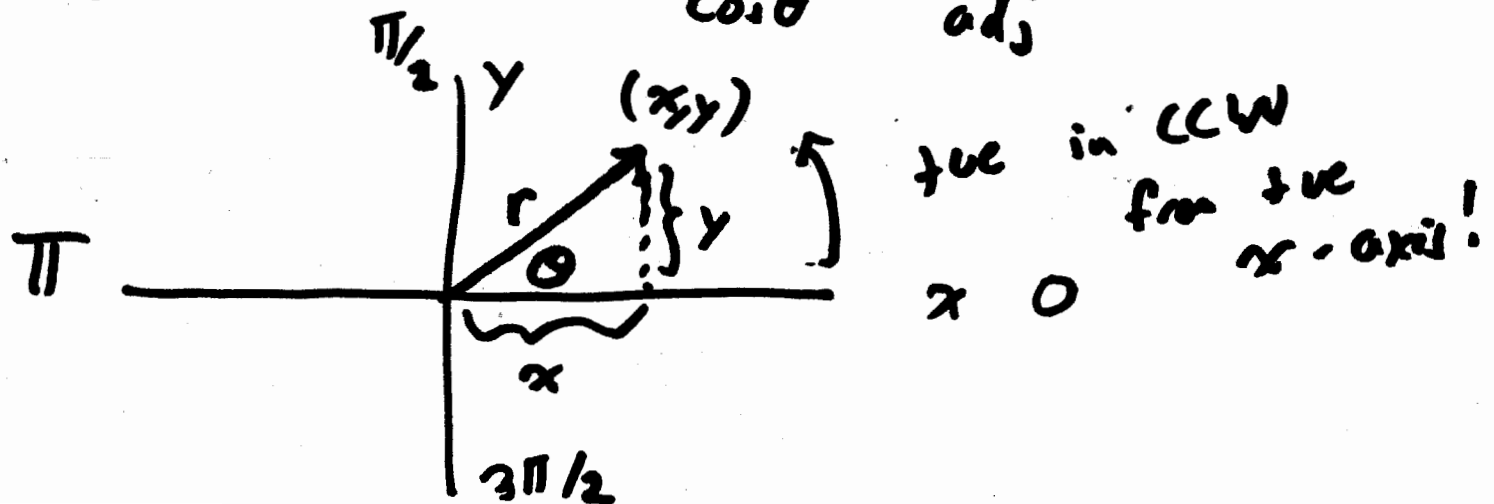
$180° = \pi$, $90° = \pi/2$, $45° = \pi/4$

$60° = \pi/3$ $30° = \pi/6$ ∴



$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp}$$

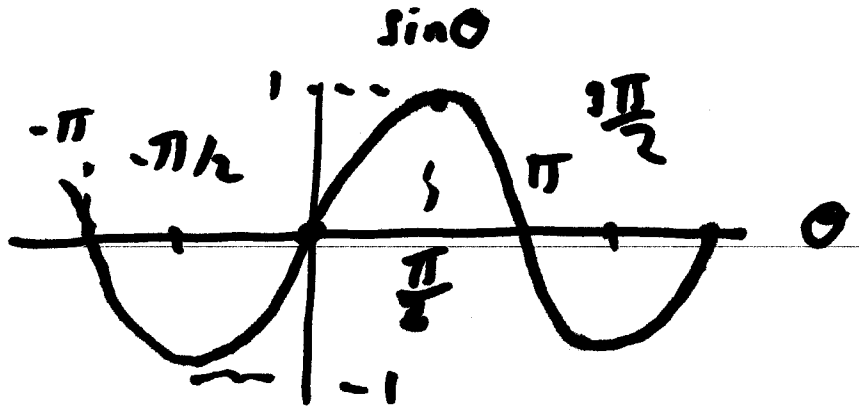
$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{opp}{adj}$$



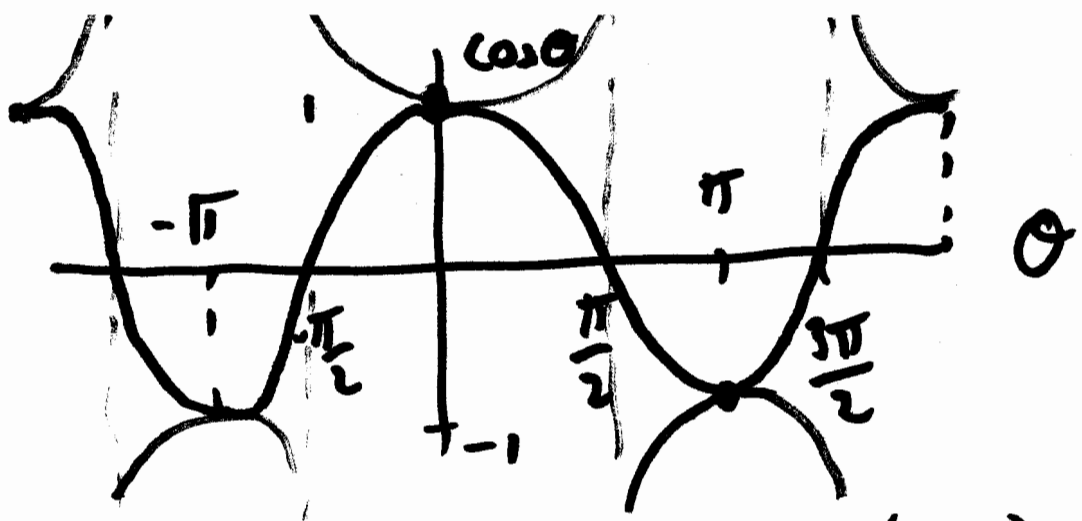
$\sin(\theta) = y/r$ $\quad \cos\theta = x/r$ $\quad \tan\theta = y/x$

$\csc(\theta) = r/y = 1/\sin\theta$ $\quad \sec(\theta) = r/x = 1/\cos\theta$ $\quad \cot\theta = x/y = 1/\tan\theta$.

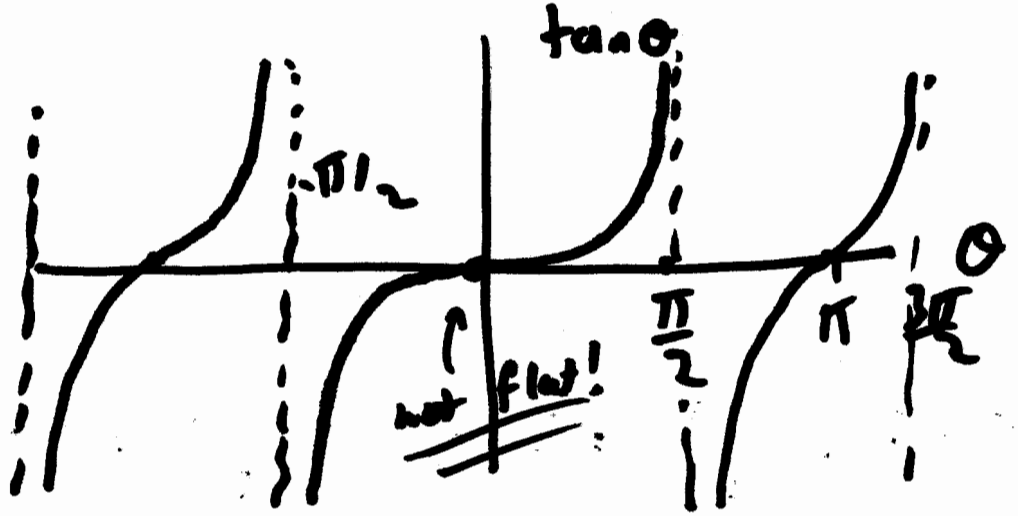
Draw graphs!



2π periodic $\quad$ "odd" $\quad \sin(-\theta) = -\sin(\theta)$



2π periodic "even" $\cos(-\theta) = \cos(\theta)$



π-periodic

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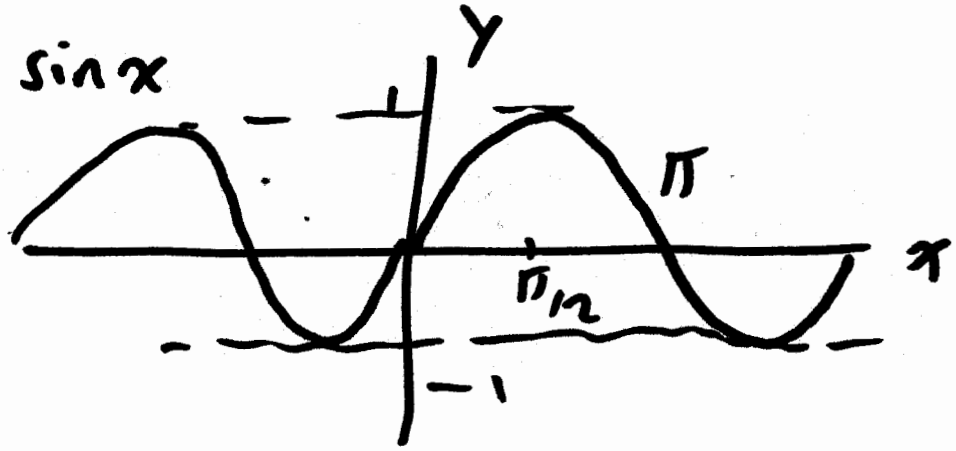
note $\tan\theta$, $\sin\theta$ etc are functions

often call "in" x "out" y } not x-y of course!

eg $f(x) = \sin x$



Identities You "Know"

$$\cos^2 x + \sin^2 x = 1$$ Important

$$1 + \tan^2 x = \sec^2 x$$

$$\cot^2 x + 1 = \csc^2 x$$ Forgettable!

$$\sin(2x) = 2\sin x \cos x$$

$$\cos(2x) = 1 - 2\sin^2 x$$

$$= 2\cos^2 x - 1$$

$= \cos^2 x - \sin^2 x$

$\sin^2 x = \frac{1}{2}(1 - \cos(2x))$

$\cos^2 x = \frac{1}{2}(1 + \cos(2x))$

$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$

$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$

Functions

$y = f(x)$

f = function

x = "inputs"

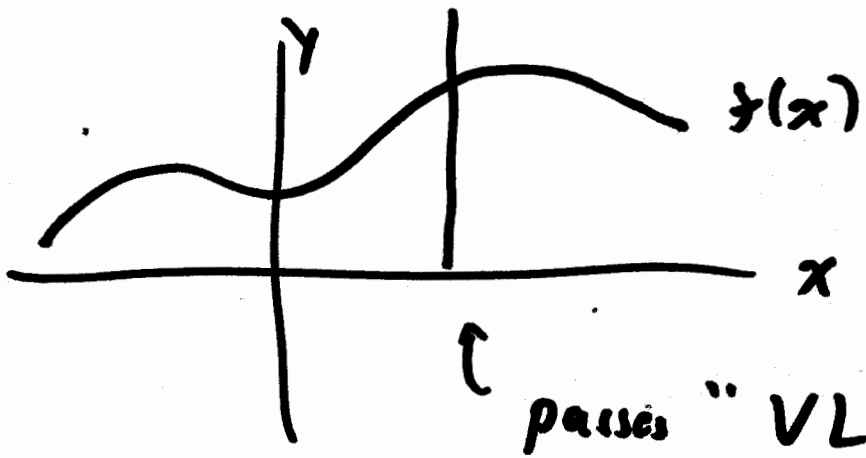
"independent variable"

y = "dependant" variable

= outputs

usable x-values = domain

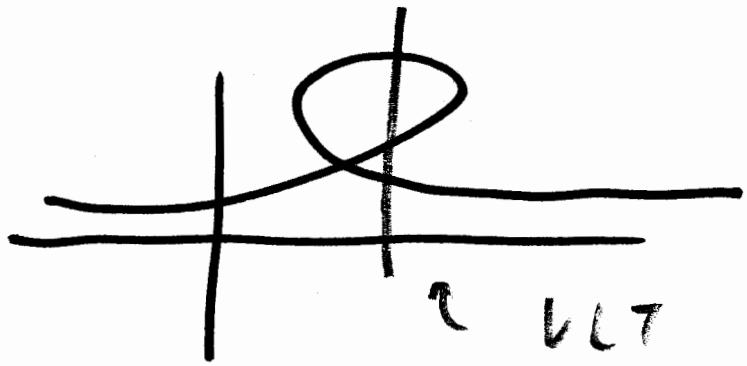
↳ range = all possible outputs!



f(x) has exactly one output for each input!

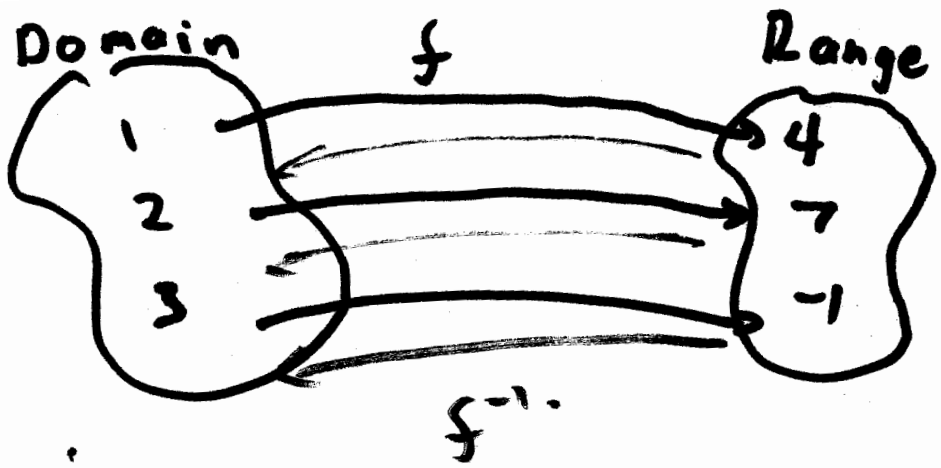
passes "VLT" vertical line test

any vertical line crosses f(x) at most once!



} not a function

Inverse Functions: Reversed Functions!



∴ $f(x)$ is our function ⇒ $f^{-1}(x)$ inverse funct.

eg $f(1) = 4$ $\quad f^{-1}(4) = 1$

$f(2) = 7$ $\quad f^{-1}(7) = 2$

so $f^{-1}(f(x)) = x$

$f(f^{-1}(x)) = x$

} Note

$f^{-1}(x)$ inverse is $f(x)$

Eg. Given $f(x) = \frac{1}{1+x}$, find $f^{-1}(x)$

Solution

Step 1: reverse x & y names

$y = \frac{1}{1+x}$ ⇝ now $x = \frac{1}{1+y}$

Step 2: Solve for y

$x = \frac{1}{1+y}$ ⇒ $x(1+y) = 1$

$x + xy = 1$

$xy = 1 - x$

$\boxed{y = \frac{1-x}{x}} = f^{-1}(x)$