In a desperate attempt to come up with an equation to use during an examination, a student tries the following equations. Use dimensional analysis to determine which of these equations could be correct. Here x, v, and a, are the position, velocity and acceleration.

1. $v^2=ax$
2. $a=v^2/x$
3. $x=v^2/a$
4. $a=xv^2$
5. $x=av^2$
6. $v=ax$
7. $v=a/x$
8. $x=av$
9. $a=xv$
10. $v=a/t$
11. $v=at$
12. $v=t/a$
13. $a=v/t$
14. $t=av$
15. $t=v/a$
16. $t=a/v$
17. $a=t/v$

$$x \Rightarrow [L]$$

$$v \Rightarrow \left[\frac{L}{T}\right]$$

$$a \Rightarrow \left(\frac{m}{s^2}\right) \Rightarrow \left[\frac{L}{T^2}\right]$$

$$t \Rightarrow [T]$$

1) $v^2 \Rightarrow \frac{L^2}{T^2}$ $\quad ax \Rightarrow \frac{L}{T^2}\cdot L = \frac{L^2}{T^2}$ True

2) $a \Rightarrow \frac{L}{T^2}$ $\quad \frac{v^2}{x} \Rightarrow \frac{L^2}{T^2}\cdot\frac{1}{L} = \frac{L}{T^2}$ True

3) $x \Rightarrow L$ $\quad \frac{v^2}{a} \Rightarrow \frac{L^2}{T^2}\cdot\frac{T^2}{L} = L$ True

4) $a \Rightarrow \frac{L}{T^2}$ $\quad xv^2 \Rightarrow L\cdot\frac{L^2}{T^2} = \frac{L^3}{T^2}$ False

5) $x \Rightarrow L$ $\quad av^2 \Rightarrow \frac{L}{T^2}\cdot\frac{L^2}{T^2} = \frac{L^3}{T^4}$ False

⋮

etc. you get the idea!!!

Bicyclists in the Tour de France reach speeds of 28.7 miles per hour (mi/h) on flat sections of the road. What is the speed in kilometers per hour (km/h)? **46.2 km/h**

$$28.7 \frac{\cancel{\text{miles}}}{h} \cdot \left(\frac{1.609\,km}{1\,\cancel{\text{mile}}}\right) = 46.2\ km/h$$

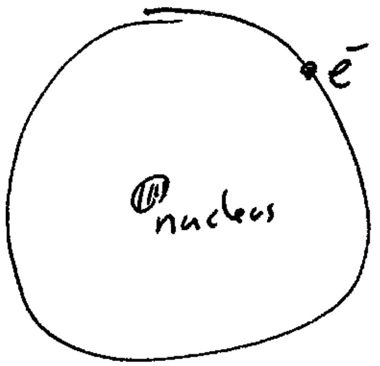
<u>Speed in m/s:</u>

$$46.2 \frac{km}{h} \cdot \left(\frac{1000\ m}{1\ km}\right) \cdot \underbrace{\left(\frac{1\ h}{60\ min}\right)\left(\frac{1\ min}{60\ s}\right)}$$

$$= 12.8\ m/s$$

or could just do 3600 s = 1 h in one multiplication

A hydrogen atom has a diameter of approximately 1.05E-10m, as defined by the diameter of the spherical electron cloud around the nucleus. The hydrogen nucleus has a diameter of approximately 2.30E-15m. For a scale model, represent the diameter of the hydrogen atom by the length of an American football field (100 yards = 300ft), and determine the diameter of the model nucleus in millimeters.



H diameter $\Longrightarrow$ 1.05×10^{-10} m

nucleus diameter $\Longrightarrow$ 2.30×10^{-15} m

a)

$$\frac{d_{atom}}{d_{nuc}} = \frac{100 \text{ yards}}{x \text{ yards}}$$

$$x \text{ yards} = 100 \text{ yards} \cdot \frac{d_{nuc}}{d_{atom}} = 100 \cdot \frac{2.3 \times 10^{-15}}{1.05 \times 10^{-10}}$$

$$= 2.19 \times 10^{-3} \text{ yards}$$

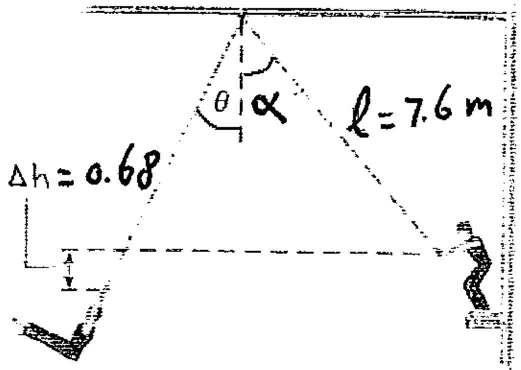
$$= 2.19 \times 10^{-3} \text{ yards} \left(\frac{0.9144 \text{ m}}{1 \text{ yard}} \right) = 0.02 \text{ m}$$

$$= \underline{\underline{2.00}} \text{ mm}$$

b) How much bigger?

$$\text{Volume} \propto L^3 \Longrightarrow \left(\frac{d_{atom}}{d_{nuc}} \right)^3 = \underline{\underline{9.51 \times 10^{13}}}$$

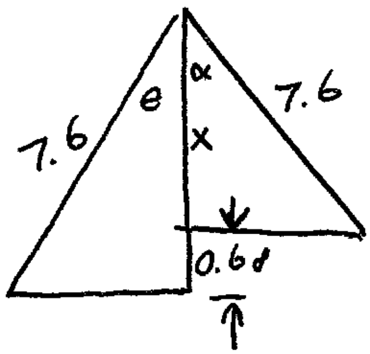
A aerialist on a high platform holds on to a trapeze attached to a support by a 7.6-m cord. Just before he jumps off the platform, the cord makes an angle α of 40.9deg with the vertical. He jumps, swings down, then back up, releasing the trapeze at the instant it is 0.68 m below its initial height. Calculate the angle θ that the trapeze cord makes with the vertical at this instant. **32.3 deg**



$\alpha = 40.9^\circ$

$h = 0.68\ m$

$\theta = ?$



Let's call x the vertical section as shown. (small Δ)

SOH CAH TOA

$\cos\alpha = \frac{x}{7.6} \Rightarrow x = 7.6 \cdot \cos(40.9)$

$= 5.744$

Large Δ :

$\cos\theta = \frac{(x + 0.68)}{7.6} = \frac{5.74 + 0.68}{7.6} = 0.845$

$\theta = 32.4^\circ$

An auditorium measures 44.0m × 20.0m × 11.0m. The density of air is 1.20 kg/m^3. What is the volume of the room in cubic feet? (Do not input units.) What is the weight of air in the room in pounds? (Do not input units.)

$$V = (44.0)(20.0)(11.0)\ m^3 = \cancel{880\ m^3} = 9680\ m^3$$

$$9680\ \cancel{m^3} \left(\frac{3.28\ ft}{1\ \cancel{m}}\right)^{3}$$

$$= 341583.5\ ft^3$$

$$3.42 \times 10^5\ ft^3$$

too many sig figs!

<u>Weight:</u>

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \Rightarrow M = \text{Density} \times \text{Vol}$$

$$M = (44.0)(20.0)(11.0)\,m^3 \left(1.2\ \frac{kg}{m^3}\right) = 11616\ kg$$

$$11616\ \cancel{kg} \left(\frac{2.205\ lb}{1\ \cancel{kg}}\right) = 2.56 \times 10^4$$

The amount of water in reservoirs is often measured in acre-feet. One acre-foot is a volume that covers an area of 1 acre to a depth of 1ft. An acre is an area of 43560ft^2. Calculate the volume in SI units of a reservoir containing 25.9 acre-ft of water.

$$25.9\,(acre \cdot ft)$$

$\Downarrow$

note: this makes sense in terms of dimensions $\Rightarrow$ acre is an area $[L^2]$

ft is a length $[L]$

$\therefore$ acre $\cdot$ ft $\Rightarrow [L^3]$ which is a volume.

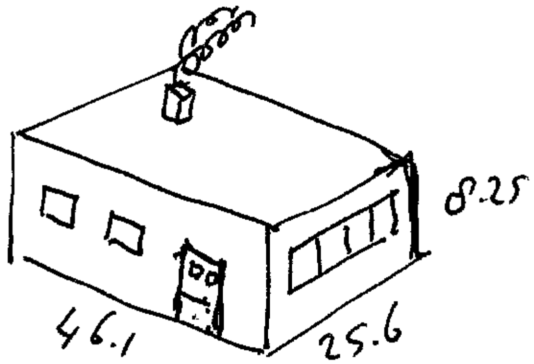
back to it:

$$25.9\,(\cancel{acre} \cdot ft)\left(\frac{43560\ ft^2}{1\ \cancel{acre}}\right) = \cancel{1.13}$$

$$= 1.13 \times 10^6\ ft^3$$

$$\cancel{1.13/A} \quad 1.13 \times 10^6\ ft^3 \left(\frac{1\ m}{3.28\ ft}\right)^3 = \underline{\underline{3.20 \times 10^4\ m^3}}$$

A house is 46.1ft long and 25.6ft wide, and has 8.25ft high ceilings.
What is the volume of the interior of the house in cubic meters?
What is the volume of the interior of the house in cubic centimeters?

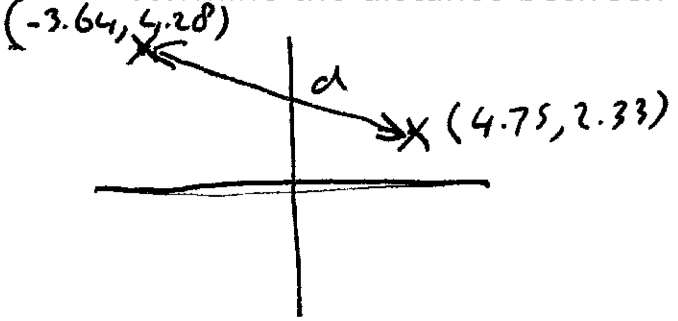


$$V = 46.1 \cdot 25.6 \cdot 8.25 \text{ ft}^3 \left(\frac{1 \text{ m}}{3.28 \text{ ft}}\right)^3$$

$$= 276. \text{ m}^3 = \underline{\underline{2.76 \times 10^2 \text{ m}^3}}$$

$$= 2.76 \times 10^2 \cancel{\text{m}^3} \left(\frac{100 \text{ cm}}{1 \cancel{\text{m}}}\right)^3 = 2.76 \times 10^8 \text{ cm}^3$$

Two points in a rectangular coordinate system have the coordinates (4.75,2.33) and (-3.64,4.28), where the units are centimeters. Determine the distance between these points.



$$d = \sqrt{\Delta x^2 + \Delta y^2}$$
$$= \sqrt{(4.75 + 3.64)^2 + (2.33 - 4.28)^2}$$
$$= \sqrt{74.19} = \underline{\underline{8.61}} \text{ cm}$$

The volume of liquid flowing per second is called volume flow rate Q and has the dimensions of [L]³/[T]. The flow rate of liquid through a hypodermic needle during an injection can be estimated with the following equation: Q=(πRⁿ(P2-P1))/(8η L) . The length and radius of the needle are L and R , respectively both of which have the dimension [L]. The pressures at opposite ends of the needle P_2 and P_1, both which have dimensions of [M]/[L][T]². η represents the viscosity of the liquid and has dimensions of [M]/[L][T]. π stands for pi and, like the number 8 and the exponent n, has no dimensions. Using dimension analysis, determine the value of n in the expression for Q.

$$Q = \frac{\pi R^n (P_2 - P_1)}{8 \eta L} \qquad Q \Rightarrow \frac{L^3}{T}$$

$$R.H.S \Rightarrow \frac{L^n \cdot \cancel{\frac{M}{LT^2}}}{\cancel{\frac{M}{LT}} \cdot \cancel{L}} = \frac{L^{n-1}}{T}$$

if $\underline{\underline{n = 4}}$, then $\frac{L^3}{T}$

Note: $P_2 - P_1$, since they have the same dimensions, subtraction of $P_2 - P_1$ does not change the dimensions

Also, π, 8 don't have dimensions.

A partly full paint can has 0.797 U.S. gallons of paint left in it. What is the volume of the paint in cubic meters? If all the remaining paint is used to coat a wall evenly (wall area = 13.8 m²), how thick is the layer of wet paint? Give your answer in meters.

$$0.797 \text{ gal} \cdot \left(\frac{3.79\, \ell}{\text{gal}}\right)\left(\frac{1\ m^3}{1000\, \ell}\right) = \underline{\underline{3.02 \times 10^{-3}}}\ m^3$$

$$V = (\text{area})(\text{thickness}).$$

$$\therefore\ \text{thickness} = \frac{V}{\text{area}} = \frac{3.02 \times 10^{-3}\ m^3}{13.8\ m^2}$$

$$= \underline{\underline{2.19 \times 10^{-4}}}\ m$$

Incidentally, and just for fun(!): human hair is about 100 μm in diameter (or 10^{-4} m) So, the paint layer is about 2 hairs thick,

A house is 51.7ft long and 21.8ft wide, and has 7.83ft high ceilings. What is the volume of the interior of the house in cubic meters? Do not enter units. What is the volume of the interior of the house in cubic centimeters?

Same as earlier question

Two points in a rectangular coordinate system have the coordinates (5.07,2.69) and (-3.71,3.12), where the units are centimeters. Determine the distance between these points.

Same as earlier question.