Theme 1
Introductory Material

Module T1M1:

The Predictable Universe

If you have questions or want to learn more about physics programs, summer research, or physics-related careers please contact:

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McMaster University SCIENCE

McMaster University Department of Physics & Astronomy

Lab sections:

- Better when the TA/student ratio is higher:
- We are encouraging students to migrate:
  - ***only* from sections L01-L05, L08, L10**
  - **to sections L09, L13-L18**.

- First come first serve basis.
- Email our Senior Lab Supervisor, Dr. Vorobyov voroby@mcmaster.ca

- Email with: name, student ID number, your current section, and the one you would like to switch to
- Dr. Vorobyov will then respond to you to let you know if the switch was possible.

T1M1 – Learning Objectives

- Identify the approach taken by physicists to understanding complex phenomena.
- Recognize that measurements are really comparisons with a standard ***unit*** of measure, and that different standard units can be related to each other.
- Distinguish between the specific units of a measured quantity, and the more general statement of the ***dimensions*** of the quantity.
- Recognize that the dimensions of a quantity are helpful at predicting the relationships that govern a system.
- Understand the idea of ***proportionality*** to describe the specific way in which quantities are related.

Module Clicker Quiz!

Now that you have had a chance to review the entire first module, T1M1, here is your first

module quiz!

Module Clicker Quiz!

Dimensional analysis (120 seconds)

- In the following formula, what are the dimensions of the variable ***E***?

$$U = \frac{AE}{X^2}$$

U – dimensions [M/T]

X – dimensions [L]

A – dimensions [M]

A. $\left[\frac{L^2}{M^2\ T}\right]$

B. $\left[\frac{M}{L^2 T}\right]$

C. $\left[\frac{1}{L^2 T}\right]$

D. $\left[\frac{L^2}{T}\right]$

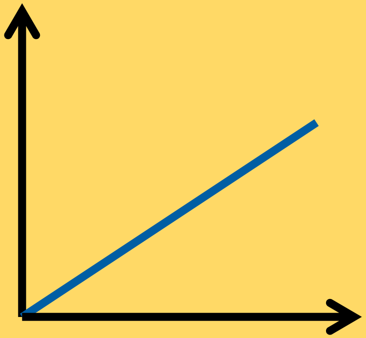
E. I don't know

Module Clicker Quiz!

Proportionality (120 seconds)

- For the formula given, which relationships, when plotted on a graph, would yield a straight line, as shown:

$$U = \frac{AE}{X^2}$$



A. ***E*** vs. ***A*** (***U***, ***X*** held constant)

B. ***A*** vs. $\boldsymbol{X^2}$ (***U***, ***E*** held constant)

C. ***U*** vs. ***X*** (***E***, ***A*** held constant)

D. ***U*** vs. $\boldsymbol{X^2}$ (***E***, ***A*** held constant)

E. I don't know

Module Clicker Quiz!

Unit conversion (120 seconds)

- As you learn physics, your brain gets more massive (unverified). If your brain grows 1 kg in 12 weeks, what is the growth rate in grams per hour?

A. 0.50 g/hr

B. 3.5 g/hr

C. 12 g/hr

D. 24 g/hr

E. I don’t know

Answers to Module Clicker Quiz

• Dimensional analysis	D
• Proportionality	B
• Unit conversion	A

Proportionality

- The natural world rarely provides us with access to all the details to put together a complete picture!
- The trends and relationships that we observe allow us to infer general rules that we incorporate into a model

Proportionality: $x \propto y$ implies that

- If we double x, then y also doubles
- If y is reduced to 1/7th of it's value, so is x.

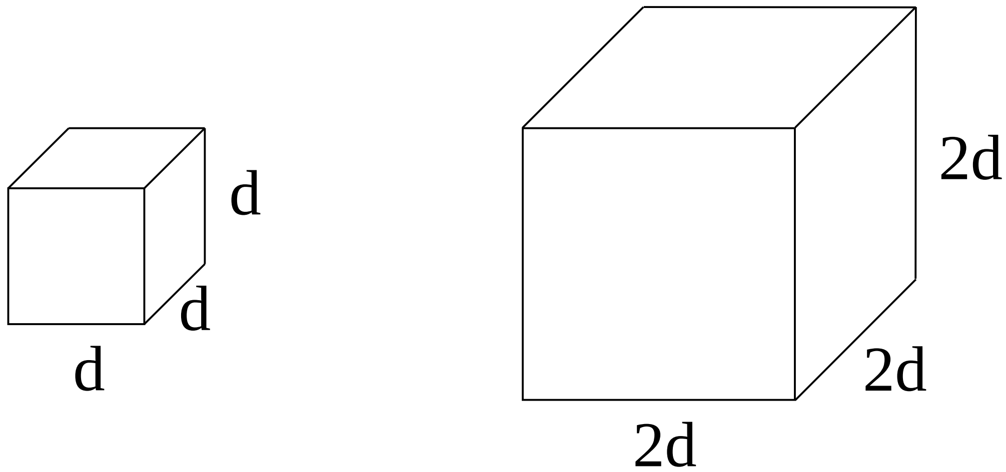
- We would incorporate into our model

$$\boldsymbol{x = ay}$$

where *a* is a constant of proportionality, independent of the actual values of *x* and *y*

Geometric proportionalities

- Isometric = same geometry, different size



- For a simple cube, various properties can be related to the side length:
  - If you double every dimension, how does area change? Volume?
  - $area \propto d^2$
  - $volume \propto d^3$
- Does this change for a sphere (radius *r*)?

Using ratios to solve problems

- From previous slide: How many times more volume of large cube compared to small cube?

$$\frac{V_{big}}{V_{small}} = \frac{(2d)^3}{d^3} = 2^3 = 8$$

- What about two spheres of radii *r* and 2*r*?

$$\frac{V_{big}}{V_{small}} = \frac{\frac{4}{3}\pi(2r)^3}{\frac{4}{3}\pi r^3} = \frac{(2r)^3}{r^3} = 2^3 = 8$$

- In both cases, when length scale increases 2x, the volume increases by factor $(2)^3 = 8$!
- *****The details of the shape don't matter (i.e. the 4/3π pre-factors cancel when you take a ratio)**

How do other quantities scale?

- Let's define a general length scale, **L**.
  - Could be side length, radius, diameter – any linear measurement of object's size

- We know that:
  - Area $\propto$ **L**^{2}
  - Volume $\propto$ **L**^{3}
- **What about Mass? $\propto$ Volume $\propto$ L^3**
  - Assume that two objects have the same density

Practical application: Clothing your clone

Mini me weighs exactly 1/8th of Dr. Evil's mass. How much more material is needed for Dr. Evil's suit than for mini me's?

$$\frac{M_{DE}}{M_{mm}} = 8 = \left(\frac{L_{DE}}{L_{mm}}\right)^3$$

$$\frac{L_{DE}}{L_{mm}} = 8^{\frac{1}{3}} = 2$$

$$\frac{A_{DE}}{A_{mm}} = \left(\frac{L_{DE}}{L_{mm}}\right)^2 = 2^2 = 4$$

So, four times more material is needed for Dr. Evil's suit than for mini me's.

How do other quantities vary with size?

- Start by defining a length scale, **L**.
- We know that:
  - Area $\propto L^2$
  - Volume $\propto L^3$
- What about Mass? $\propto$ Volume $\propto L^3$
- **What about:**
  - **flow into/out of an object – for example, heat flow?**
  - **Production of heat/energy/waste by an object?**

Clicker Quiz

How would you expect the amount of ***body heat generated (G)*** to scale with the linear dimension, ***L***, of an organism?

A. $\mathrm{G} \propto \mathrm{L}^{2/3}$

B. $\mathrm{G} \propto \mathrm{L}^{2}$

C. $\mathrm{G} \propto \mathrm{L}^{3/2}$

D. $\mathrm{G} \propto \mathrm{L}^{3}$

E. I have no idea

Clicker Quiz

How would you expect the amount of ***body heat lost (H)*** to scale with the linear dimension, ***L***, of an organism?

A. $\mathrm{H} \propto \mathrm{L}^{2/3}$
B. $\mathrm{H} \propto \mathrm{L}^{2}$
C. $\mathrm{H} \propto \mathrm{L}^{3/2}$
D. $\mathrm{H} \propto \mathrm{L}^{3}$
E. I have no idea

Example: Rate of heat loss

- A grown adult weighs 70 kg while a newborn baby weighs 5 kg.

  Compare the

  (a) rate of heat loss, and

  (b) heat loss rate *per kg* of each.

Example: Rate of heat loss

- A grown adult weighs 70 kg while a newborn baby weighs 5 kg.

$$\frac{M_{Big}}{M_{Small}} = \frac{70}{5} = 14$$

- The adult is 14 times as massive. Let's relate mass to a characteristic length (height, for example)

$$\frac{M_B}{M_S} = \left(\frac{L_B}{L_S}\right)^3 \longrightarrow \frac{L_B}{L_S} = \left(\frac{M_B}{M_S}\right)^{\frac{1}{3}} = 14^{\frac{1}{3}} = 2.41$$

- This tells us that the adult is roughly 2.5 times taller

Example: Rate of heat loss

- A grown adult weighs 70 kg while a newborn weighs 5 kg.

Since we now have a ratio of lengths, we can use to learn about:

(a) rate of heat loss

$$\frac{H_B}{H_S} = \left(\frac{L_B}{L_S}\right)^2 = 2.41^2 = 5.81$$

Heat loss is 6x greater for the adult.

(b) heat loss rate *per kg* of each.

$$\frac{G_B}{G_S} = \left(\frac{L_B}{L_S}\right)^3 = 2.41^3 = 14$$

BUT, heat generated by adult is 14x greater!

a) Rate of heat loss (H):

- The 3-step process to solving scaling problems:
  1. Start with a known ratio: $$\frac{M_{Big}}{M_{Small}} = \frac{70}{5} = 14$$
  2. Get ratio of length scales: $$\frac{M_B}{M_S} = \left(\frac{L_B}{L_S}\right)^3 \rightarrow \frac{L_B}{L_S} = \left(\frac{M_B}{M_S}\right)^{\frac{1}{3}} = 14^{\frac{1}{3}} = 2.41$$
  3. Use this to find ratio of new quantity (H): $$\frac{H_B}{H_S} = \left(\frac{L_B}{L_S}\right)^2 = 2.41^2 = 5.81$$

So, the big guy loses heat 5.81 times faster

b) Heat Generated:

1. This depends on mass $$\frac{G_B}{G_S} = \cdot \frac{M_B}{M_S} = 14$$

c) Heat Loss per kg (or heat loss per heat generated):

$$H \propto L^2, \quad M \propto L^3 \quad \Rightarrow \quad \frac{H}{M} = \frac{L^2}{L^3} = \frac{1}{L} \rightarrow \frac{(H/M)_B}{(H/M)_S} = \frac{1/L_B}{1/L_S} = \frac{L_S}{L_B} = \frac{1}{2.41} = 0.415$$

- The adult loses more heat, the rate of heat loss per kilogram is greater for the baby. Remember the heat generation is proportional to mass (L^3), so this is exactly what we talked about in class.

Are we blowing this out of proportion?

- Where do we see this in the ‘real world’?

Cell size (from module) – surface area to volume

- there is a practical limit to the size of a cell

<u>Allometry:</u> The study of the relationship of body size to other characteristics of organisms (shape, anatomy, behaviour…)

- Metabolic rate $\propto$ $(\text{Mass})^{3/4}$
- Breathing, heart rate $\propto$ $(\text{Mass})^{-1/4}$
- Optimal cruising speed (flight) $\propto$ $(\text{Mass})^{1/6}$

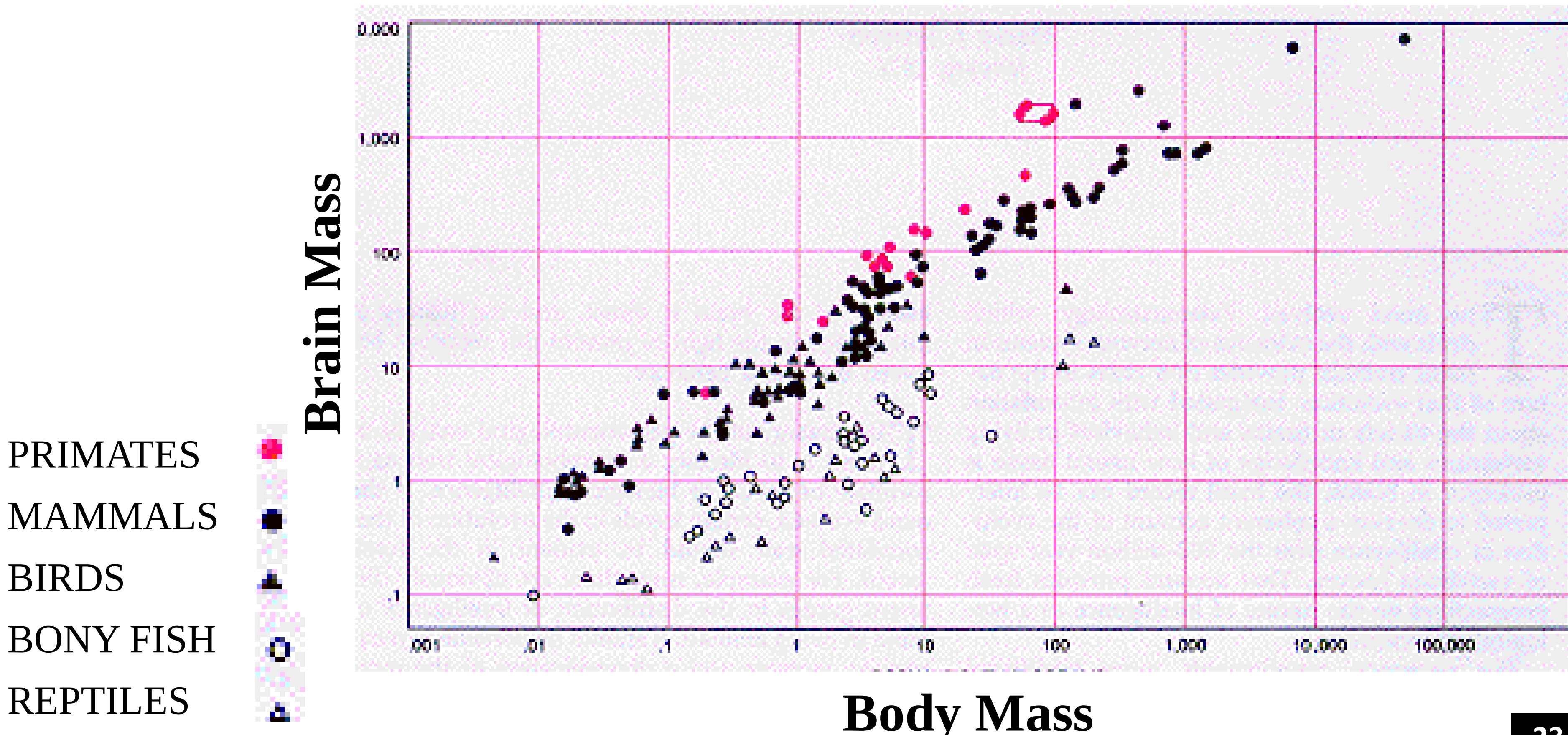
Societal implications

- Density of gas stations vs. city size
- Types of employment, wages vs. city size

What can we infer from these relationships?

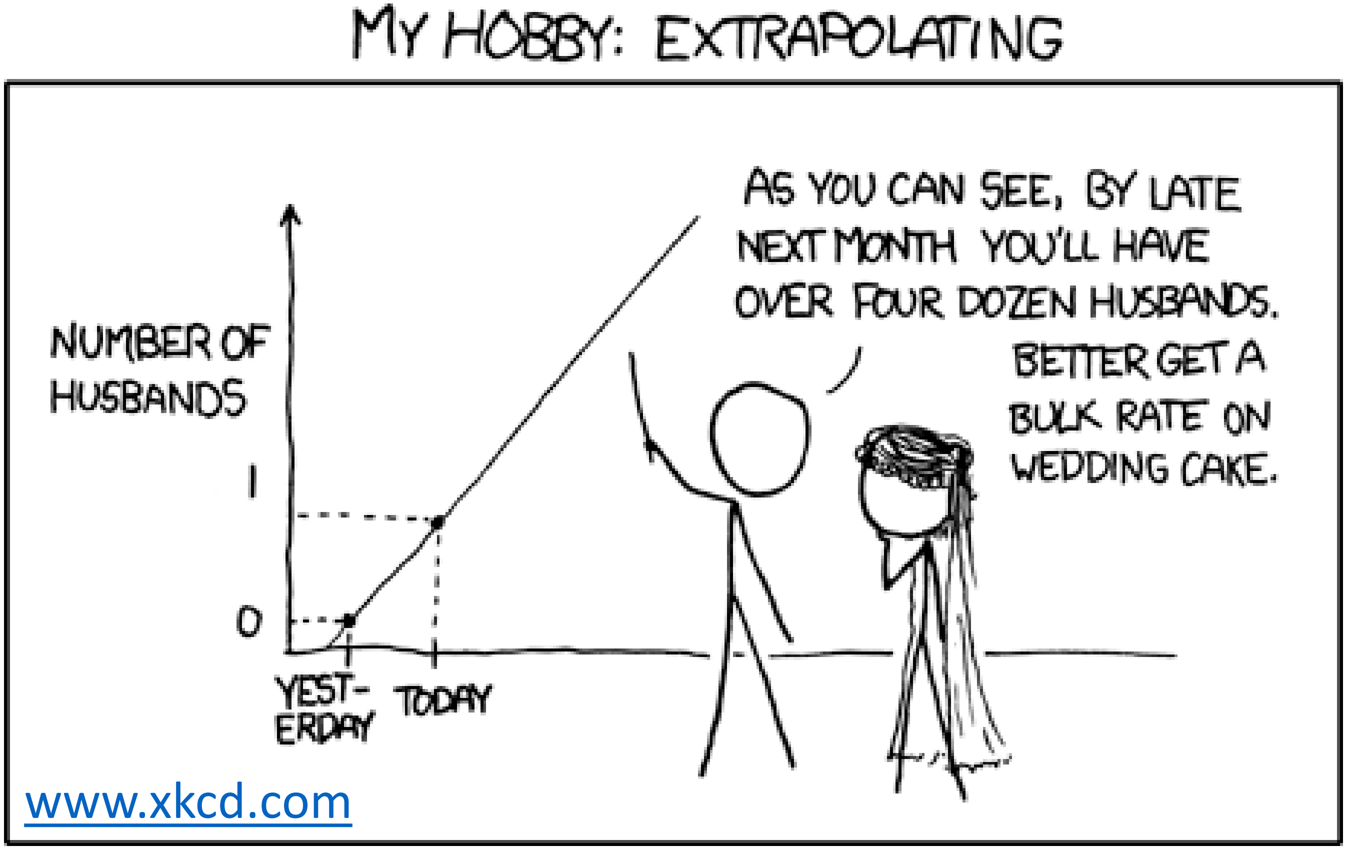
- Understanding how two quantities are connected helps us to understand the nature of the relationship!

What can this tell us about brain mass and intelligence?



Inferences using proportionality

- Of course, it's not enough to just have a proportionality; there's more to modeling than that!!



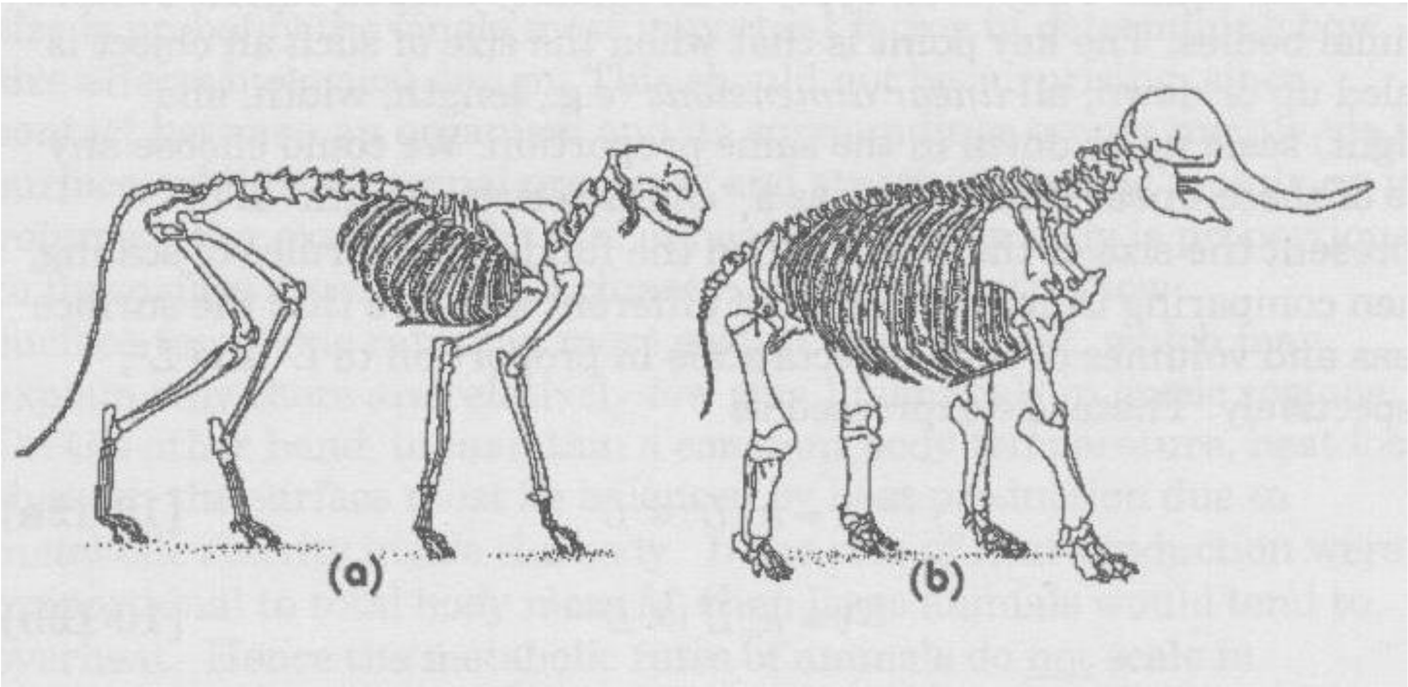
How do other quantities vary with size?

- Start by defining a length scale, **L**.
- We know that:
    - Area $\propto L^2$
    - Volume $\propto L^3$
    - Mass? $\propto$ Volume $\propto L^3$
- Flow (heat, chemical, electrical) $\propto L^2$
- Heat production $\propto L^3$

- **What about strength?**

Limitations on animal size, and mobility

- Skeleton of a house cat and an elephant



- What does this illustration tell you?

Something to think about

Could King Kong exist as shown in the movies?

Gorilla: 180 kg, 1.7 m tall, (eats ~25 kg food/day)

King Kong – 7x scaled up version of a gorilla

Vectors

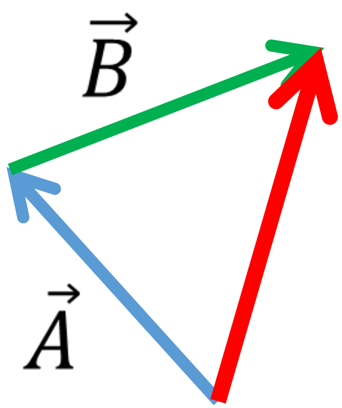
- Scalars
  - Answers a question like:
    - How hot?
    - How heavy?
    - How far can you throw the person sitting next to you?
- Vectors
  - Used when a number doesn't give enough info
    - Velocity $\vec{v} = 20\ m/s$ [north]

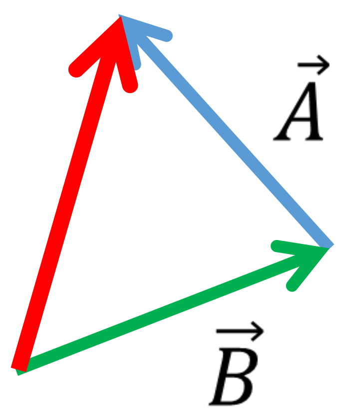
You will want to become comfortable with vector addition & subtraction, and in particular, working with vector components

Vector Addition

- When adding vectors, line them up ‘tip-to-tail’

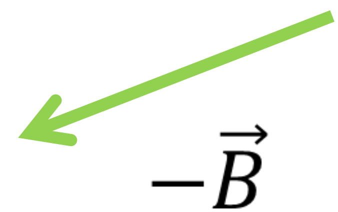
$$\vec{A}+\vec{B} = \vec{B}+\vec{A}$$





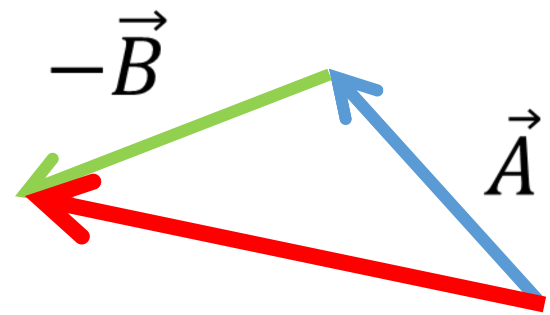
Vector Subtraction

- What about $\vec{A} - \vec{B}$?
  - Let's create a vector $-\vec{B}$



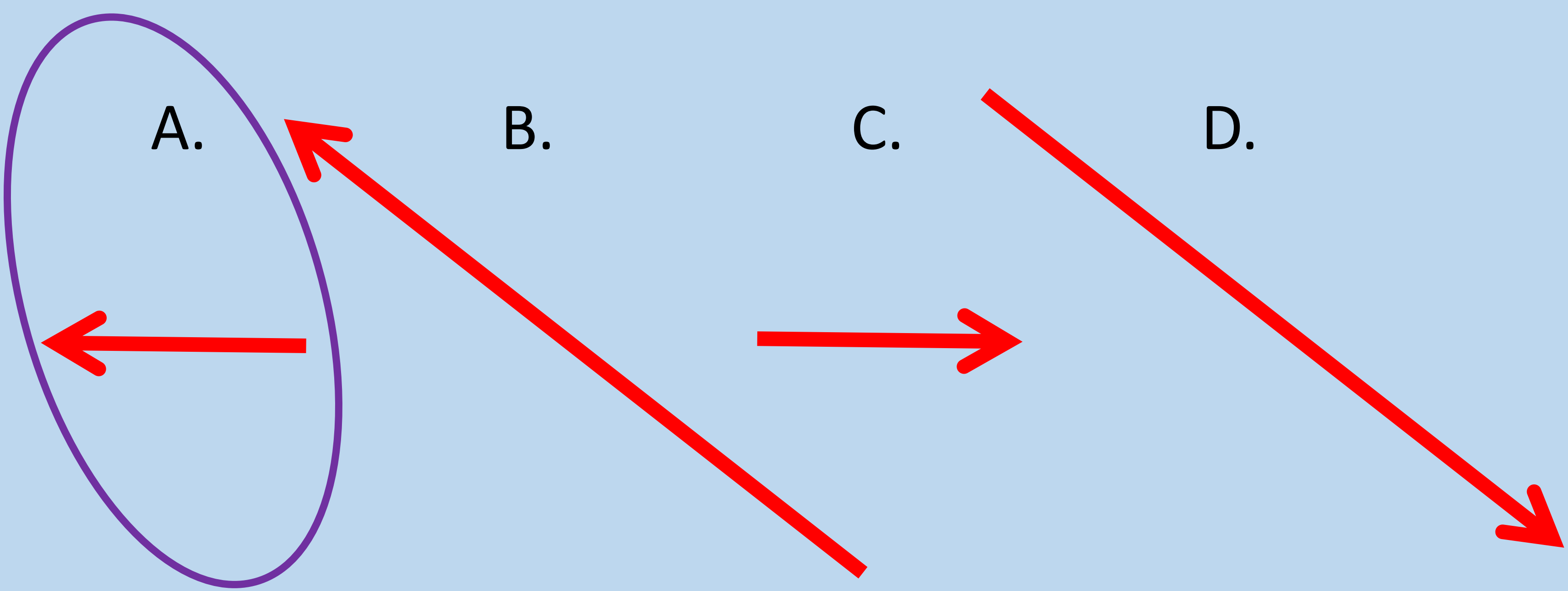
- Now we can simply add:

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$



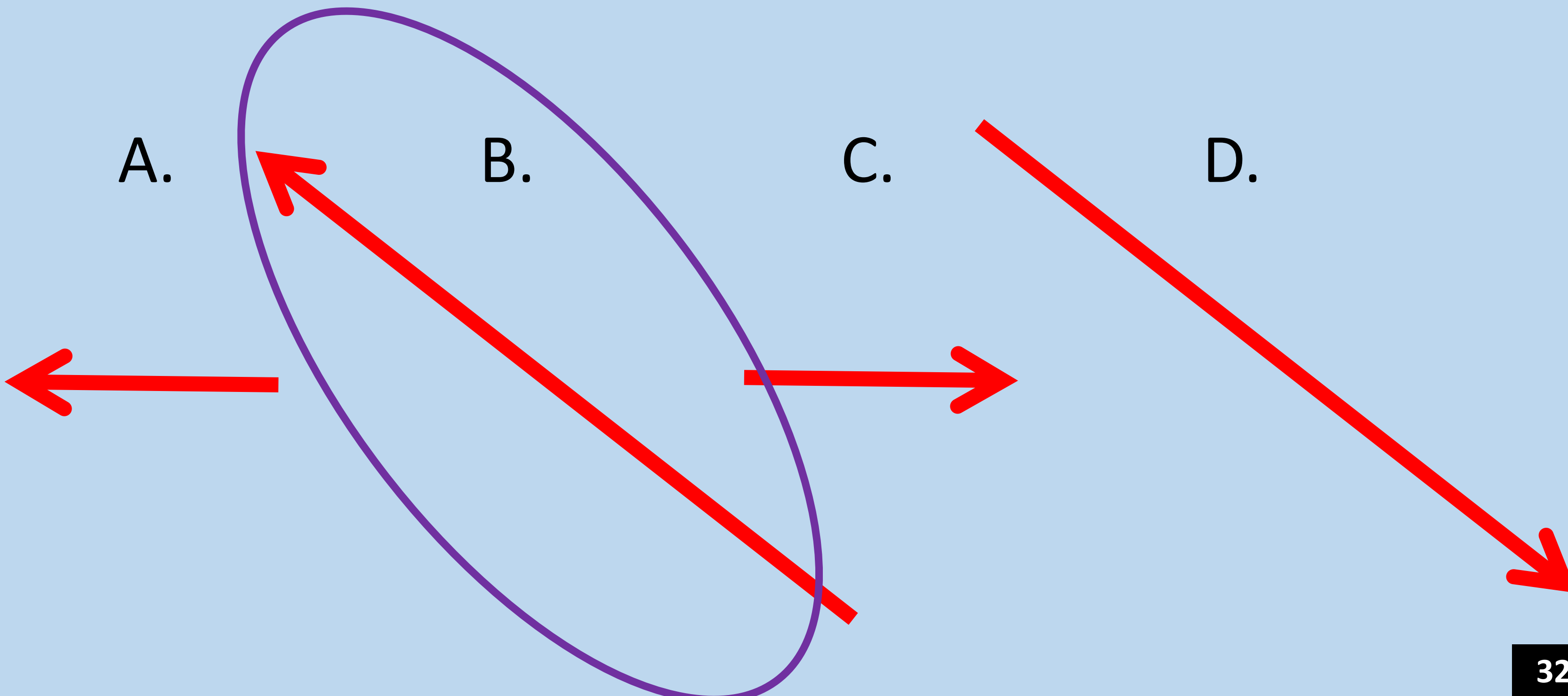
Clicker Quiz

- For the two vectors shown, what is $\vec{A} + \vec{B}$

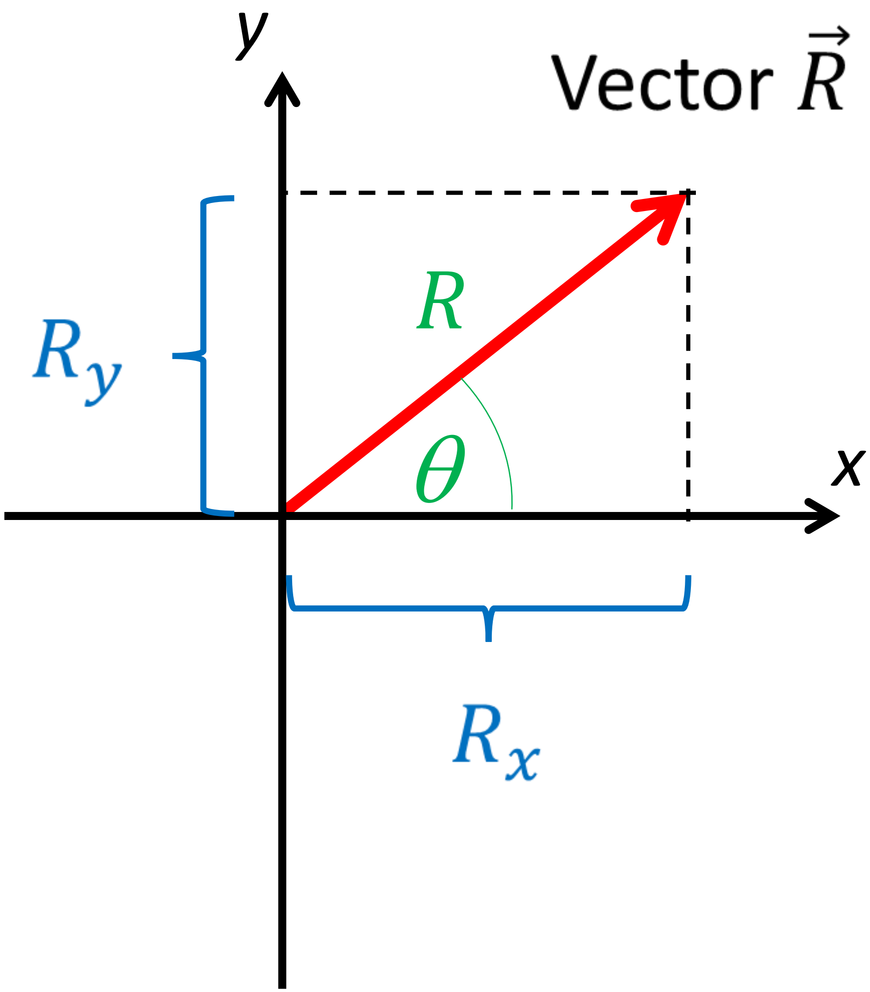


Clicker Quiz

- For the two vectors shown, what is $\vec{A} - \vec{B}$



Vector Notation



1) $\vec{R} = (R, \theta)$

- $R = |\vec{R}|$ is the 'magnitude'
- θ is the direction, relative to the +x axis

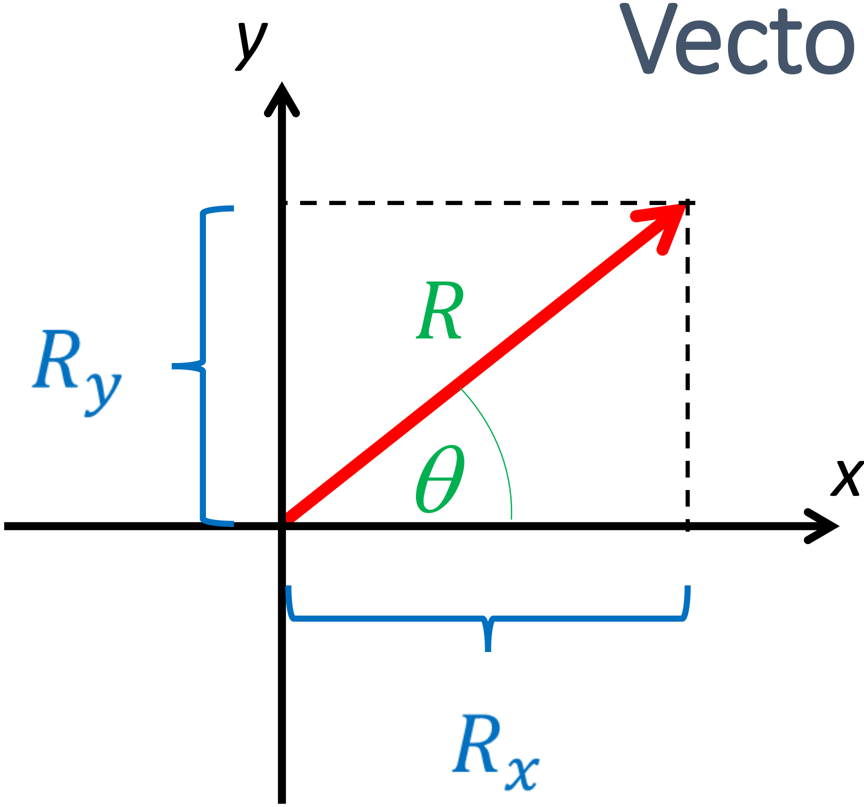
2. $\vec{R} = (R_x, R_y)$

- ***"vector components"***
- Sometimes also write

$$\vec{R} = R_x\hat{\imath} + R_y\hat{\jmath}$$

where $\hat{\imath}$ and $\hat{\jmath}$ indicate the +x and +y directions

Vector Notation



1. $(R, \theta) \rightarrow (R_x, R_y)$

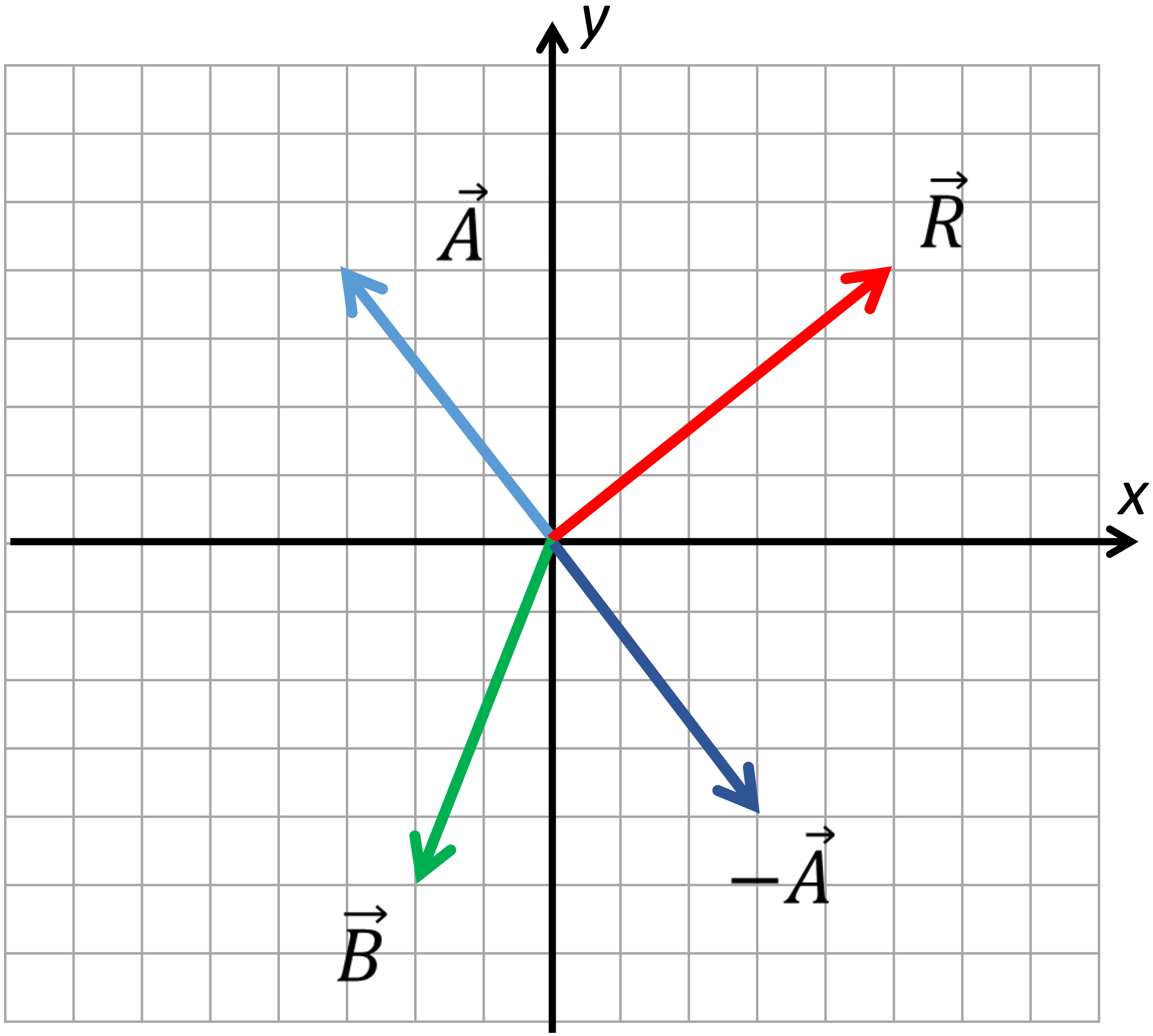
- $R_x = R\cos(\theta)$
- $R_y = R\sin(\theta)$

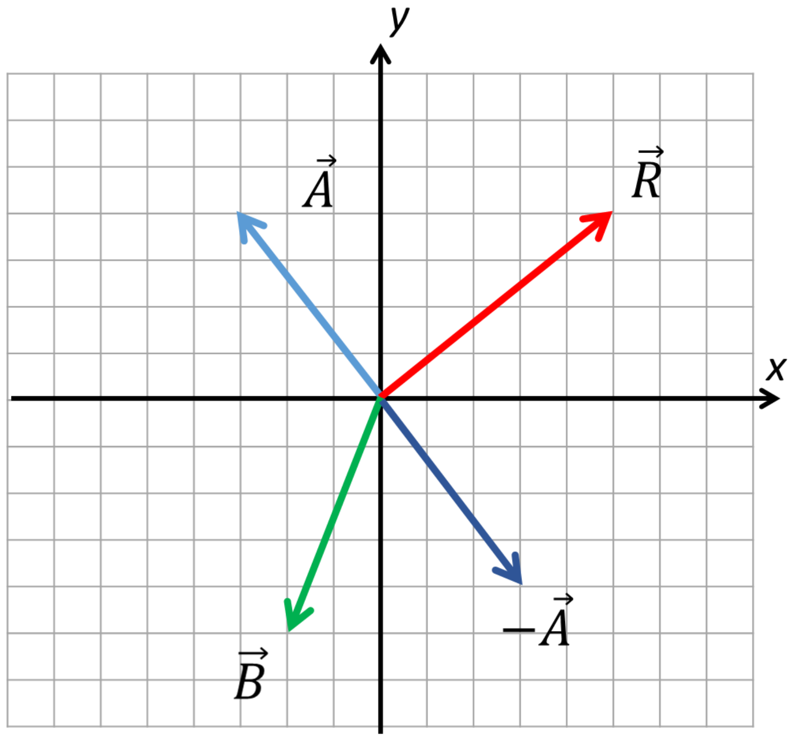
2. $(R_x, R_y) \rightarrow (R, \theta)$

- $R = \sqrt{(R_x)^2 + \left(R_y\right)^2}$

- $\tan(\theta) = \frac{R_\mathrm{y}}{R_\mathrm{x}}$

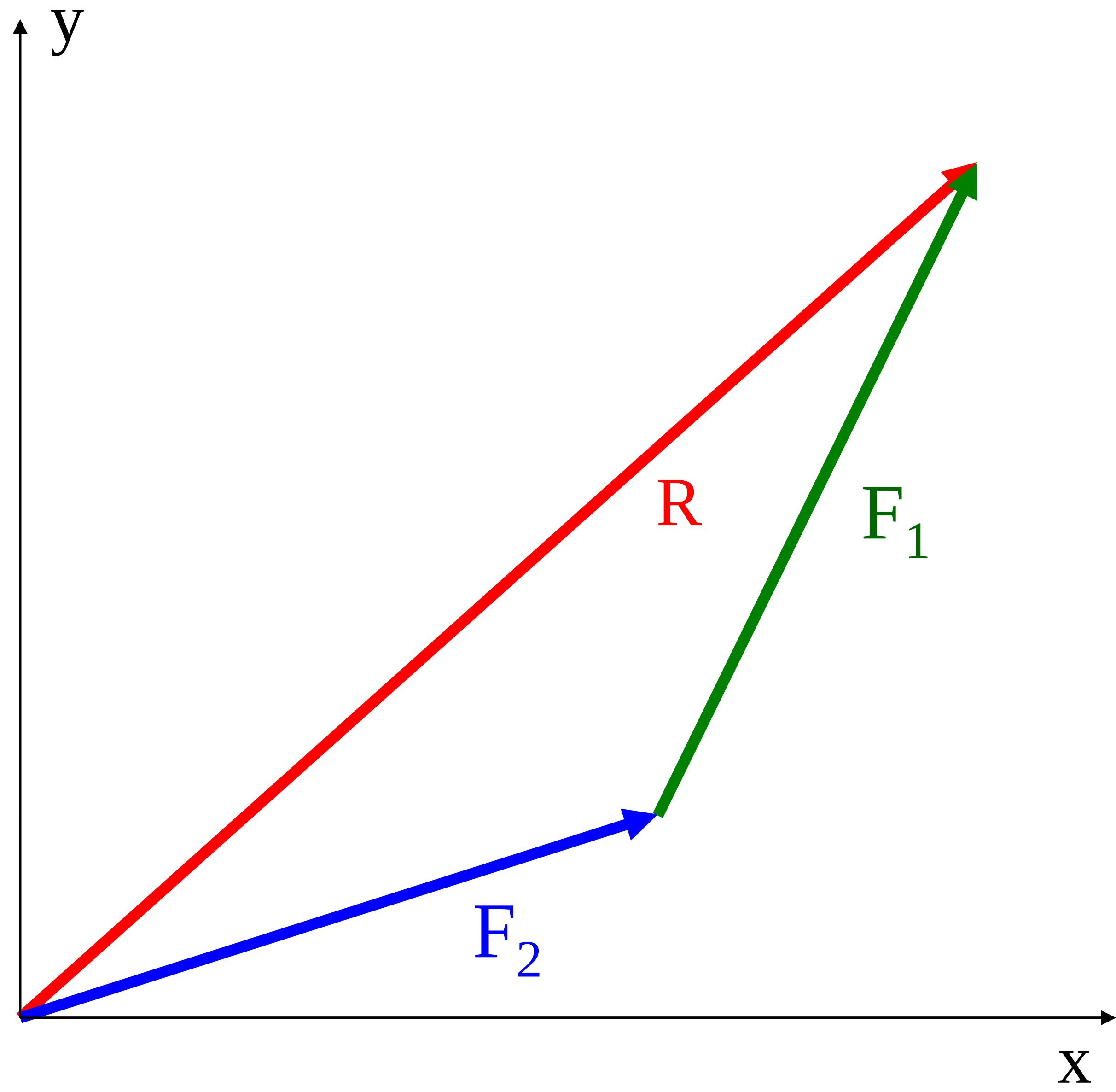
Vector Notation – Can you express these vectors in terms of (R, θ) and (R_x, R_y)?





	(R_x,R_y)	(R,θ)
• R:	(5,4),	(6.4, 38.7^o [0.675 rad])
• A:	(-3,4),	(5, 126.9^o [2.21 rad])
• B:	(-2,-5),	(5.39, -111.8^o [-1.95 rad])
• -A:	(3,-4),	(5, -53.1^o [-0.93 rad])

Adding two vectors using components



Adding two vectors using components

$$F_{2x} = F_2 \cos\alpha$$

$$F_{2y} = F_2 \sin\alpha$$

$$F_{1x} = F_1 \cos\beta$$

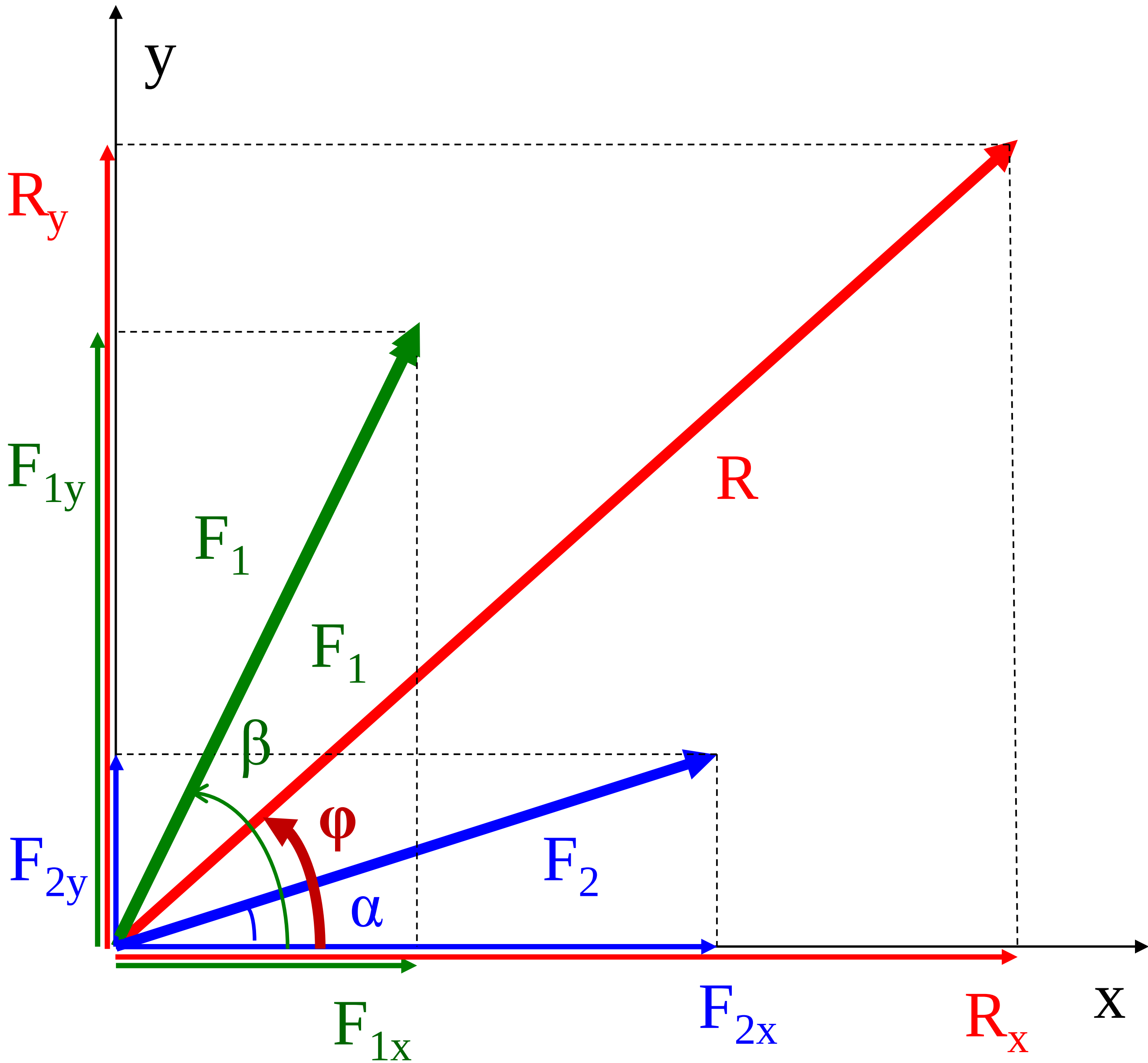
$$F_{1y} = F_1 \sin\beta$$

$$R_x = F_{1x} + F_{2x}$$

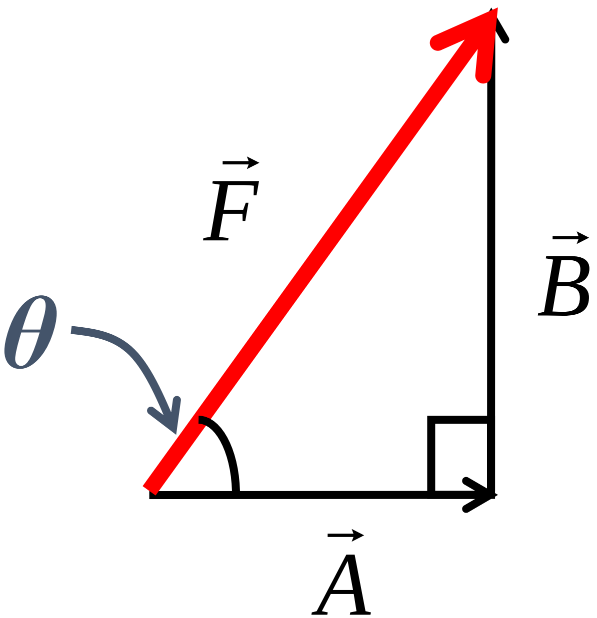
$$R_y = F_{1y} + F_{2y}$$

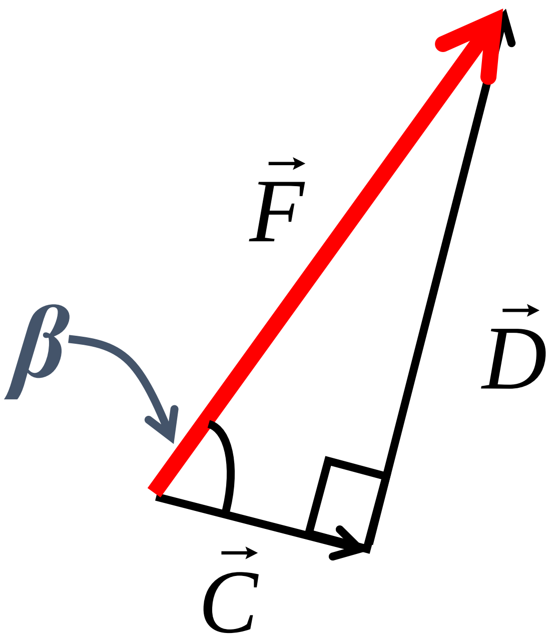
$$R = \sqrt{R_x^2 + R_y^2}$$

$$\tan\varphi = R_y / R_x$$



Vector Components





$$\vec{F} = \vec{A} + \vec{B}$$

$$\left|\vec{A}\right| = F\cos\theta$$

$$\left|\vec{B}\right| = F\sin\theta$$

$$\vec{F} = \vec{C} + \vec{D}$$

$$\left|\vec{C}\right| = F\cos\beta$$

$$\left|\vec{D}\right| = F\sin\beta$$

Vector Components

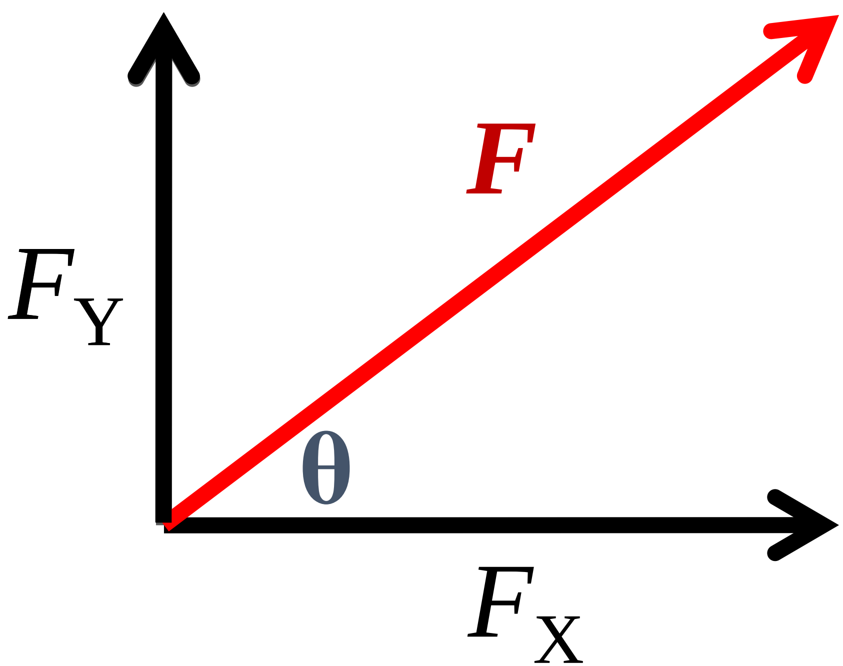
Referring to previous slide:

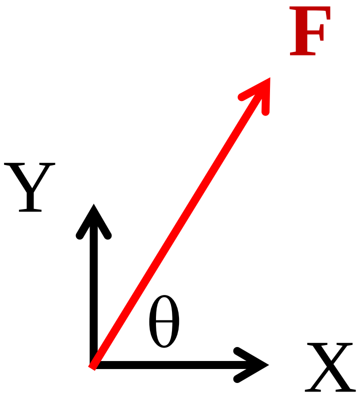
- Vector components are two perpendicular vectors, which add to give the total vector ("*F*")
- There are an infinite number of pairs that will do this
    - i.e. components are **NOT** just a horizontal/vertical pair!
- However, for specified values of (F, θ) there is one corresponding set of components (F_x, F_y), and vice versa

- It will be very helpful later in the course to be comfortable with finding the values of components!

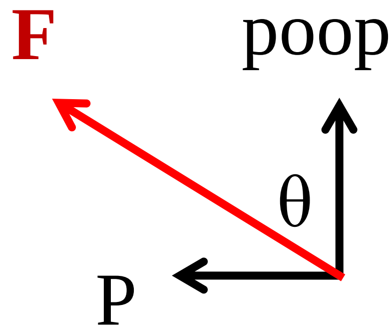
sin(θ) or cos(θ)?

SOH CAH TOA	PEN SWIPE
• **S**in = **O**pposite/**H**ypoteneuse • **C**os = **A**djacent/**H**ypoteneuse • **T**an = **O**pposite/**A**djacent	• Swipe over θ gives ‘cos’ • Swipe away from θ gives ‘sin’

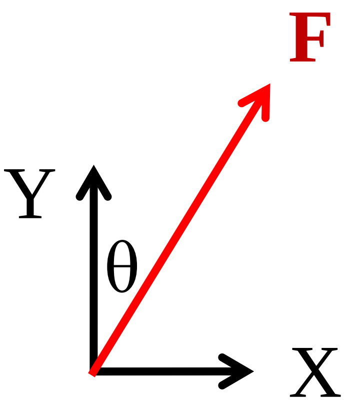




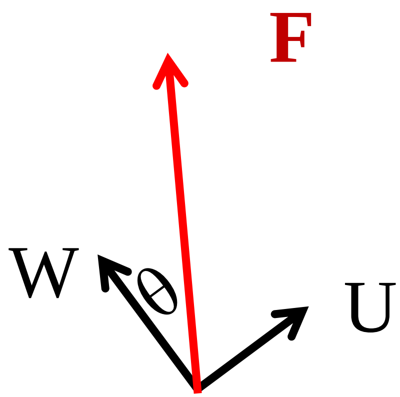
$$F_X = F\cos(\theta)$$



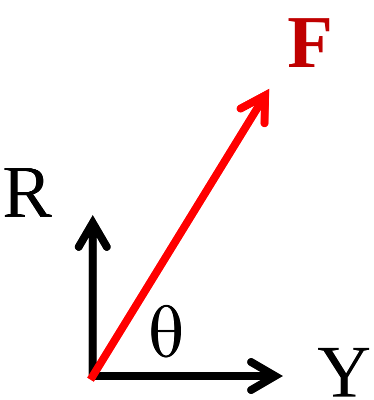
$$F_{poop} = F\cos(\theta)$$



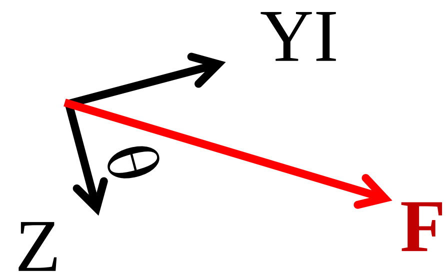
$$F_Y = F\cos(\theta)$$



$$F_U = F\sin(\theta)$$



$$F_R = F\sin(\theta)$$



$$F_{YI} = F\sin(\theta)$$

Theme 1
Introductory Material

Module T1M2:

Precision and Estimation

Learning Objectives

- Recognize that the presentation of a numerical quantity, using **significant figures** and **scientific notation**, reflects the accuracy of a measurement.
- Carry the appropriate significant figures through simple arithmetic calculations.
- Appreciate the importance of **estimating unknown quantities** as a means of understanding a system and predicting outcomes.
- Develop the skill of making an estimate and performing '**order of magnitude' approximations**.

Significant Figures

- When expressing a quantity, we want to communicate how precisely we know its value

- Example: My height is 16.705423 TP-sheets

- Is this a trustworthy statement?

- We usually count as ***"significant"*** all digits up to the first uncertain one
  - Based on how the measurement was made
  - Could be statistically determined (result of variations observed over successive measurements)

Significant Figures

- For a properly written quantity

  - 43.84
  - 27

  Non-zero digits are typically assumed to be significant

  - 5.0502
  - 2001

  Zeroes on the interior of a number are significant

  - 14.50
  - 270.

  Zeros on the right side of the number are typically significant

  - 0.013
  - 0.04710
  - 270

  The exception: sometimes zeros are used as placeholders, and are not significant.

How many sig figs?

Assuming these quantities have been properly expressed, how many significant figures does each have?

- 26.38 (*4 sig figs*),
- 27 (*2 sig figs*),
- 0.005<u>**00**</u> (*3 sig figs*),
- 0.03<u>**0**</u>4<u>**0**</u> (*4 sig figs*),
- 3.<u>**0**</u>88<u>**0**</u> (*5 sig figs*),
- 0.00418 (*3 sig figs*),
- 3.2088×10^6 (*5 sig figs*).

Sig Figs & Arithmetic

- When **adding/subtracting** quantities which have a specific number of significant figures

  the number of decimal places in our answer must match that of the least reliable measurement

- Examples:

a) 2.54 cm + 1.2 cm = ? = 3.7 cm (Not 3.74)

b) 7.432 cm + 2 cm = ? = 9 cm (Not 9.432)

- Don't forget to round!

c) 7.632 cm + 2 cm = ? = 10 cm (Not 9.632)

Sig Figs & Arithmetic

- When **multiplying/dividing** quantities which have a specific number of significant figures

  the number of significant figures in our answer must match that of the least reliable measurement

- Example:

a) 56.78 cm × 2.45 cm = ?

= 139 cm^2 (Not 139.111 cm^2)

b) 813.2 m ÷ 35 s = ?

= 23 m/s (Not 23.234 m/s)

Sig Figs & Scientific Notation

- What about this one?

a) $8132 \text{ m} \div 35 \text{ s} = ?$

$= 232 \text{ m/s}$?

$= 230 \text{ m/s}$?

Use Scientific Notation: $8132 \text{ m} \div 35 \text{ s} = 2.3 \times 10^2 \text{ m/s}$

Clicker Quiz

- How many significant figures should be written in the sum of:

14.65 g +9.023 g + 850.0078 g + 26540.4390 + 0.80 g?

A. 3
B. 4
C. 5
D. 6
E. 7

Clicker Quiz

- A parking lot is 134.3 m long and 37.66 m wide. The parking lot area is

  A. 5.05774×10^3 m^2
  B. 5.0577×10^3 m^2
  C. 5.058×10^3 m^2
  D. 5.06×10^3 m^2
  E. 5.1×10^3 m^2

Answers to Clickers

- Addition of terms: E
- Calculation of area: C