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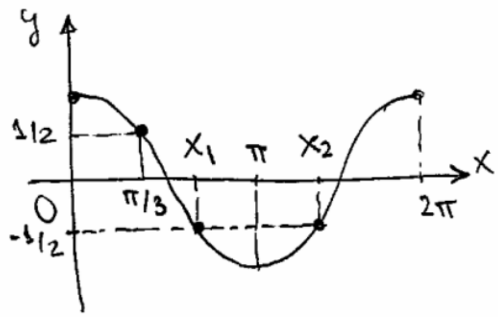
Student No.:

1. (a)[2] Find all solutions of the equation $\cos x = -1/2$.

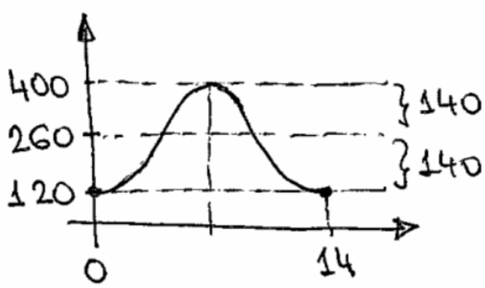


Know: $\cos\frac{\pi}{3} = \frac{1}{2}$

$$x_1 = \pi - \frac{\pi}{3} = \frac{2\pi}{3} + 2\pi k$$

$$x_2 = \pi + \frac{\pi}{3} = \frac{4\pi}{3} + 2\pi k$$

(b)[3] A population $P(t)$ of brown wolves in a small region of southern Nunavut has been known to fluctuate between the low of 120 volwes and high of 400 wolves, with a period of 14 years (i.e., increases from 120 to 400 and decreases back to 120 in 14 years). Use a trigonometric function to find a formula for $P(t)$ as function of time t, measured in years.



Use cosine

period $\frac{2\pi}{a} = 14 \rightarrow a = \frac{\pi}{7}$

$\cos\left(\frac{\pi}{7}x\right) \rightarrow$ adjust for min/max

$-140\cos\left(\frac{\pi}{7}x\right) + 260$

so $P(t) = 260 - 140\cos\left(\frac{\pi}{7}t\right)$

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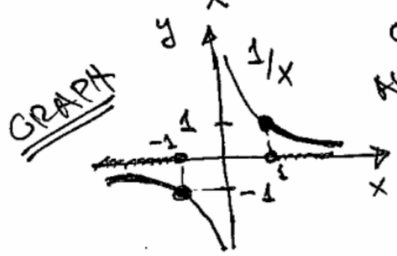
2. (a)[2] Is the formula $\arcsin(\sin x) = x$ true for all real numbers x? Justify your answer.

no, only true for x in $[-\frac{\pi}{2}, \frac{\pi}{2}]$

for example, $\arcsin(\sin\pi) = \arcsin(0) = 0 \neq \pi$

(b)[2] Find the domain of the function $f(x) = \arcsin(1/x)$. Explain your answer.

$\frac{1}{x}$ .... $x \neq 0$, arcsin ... $-1 \leq \frac{1}{x} \leq 1$



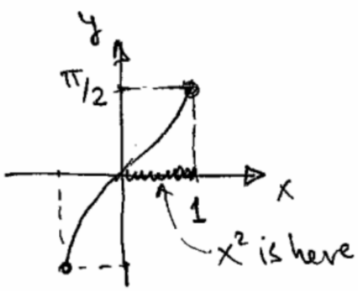
OR ALGEBRA

$\frac{1}{x} \leq 1 \rightarrow x \geq 1$ ... $[1, \infty)$ and

$\frac{1}{x} \geq -1$ ... $x \leq -1$ .... $(-\infty, -1]$

$(-\infty, -1]$ and $[1, \infty)$

(c)[2] Find the range of the function $y = \arcsin(x^2)$. Explain your answer.



$x^2 \geq 0$

so $\arcsin(x^2) \geq 0$

range $= [0, \frac{\pi}{2}]$

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3. Find the following limits

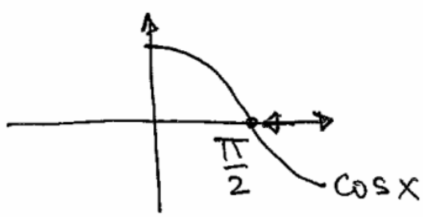
(a)[3] $\lim_{x\to 4} \dfrac{\frac{1}{\sqrt{x}} - \frac{1}{2}}{x-4}$. $= \dfrac{0}{0}$

$$= \lim_{x\to 4} \frac{\frac{2-\sqrt{x}}{2\sqrt{x}}}{x-4} = \lim_{x\to 4} \frac{2-\sqrt{x}}{2\sqrt{x}} \cdot \frac{1}{x-4}$$

$$= \lim_{x\to 4} \frac{2-\sqrt{x}}{2\sqrt{x}(\sqrt{x}-2)(\sqrt{x}+2)} =$$

$$= \lim_{x\to 4} \frac{-1}{2\sqrt{x}(\sqrt{x}+2)} = -\frac{1}{16}$$

(b)[2] Find $\lim_{x\to(\pi/2)^+} x^2 \sec x$. $= x^2 \cdot \dfrac{1}{\cos x} = \left(\dfrac{\pi}{2}\right)^2 \cdot \dfrac{1}{0 \ominus} = -\infty$



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4. Consider the function

$$f(x) = \begin{cases} \dfrac{x-1}{x^3-x} & \text{if} \quad x \neq 1 \\ 1/2 & \text{if} \quad x = 1 \end{cases}$$

(a)[2] Is $f(x)$ continuous at $x = -2$? Explain why or why not.

near $x=-2$ ... $f(x) = \dfrac{x-1}{x^3-x}$ is continuous as quotient of continuous functions with denominator $(-2)^3-(-2) = -6 \neq 0$

(b)[3] Is $f(x)$ continuous at $x = 1$? Explain why or why not.

$$\lim_{x\to1} f(x) = \lim_{x\to1} \frac{\cancel{x-1}}{x(\cancel{x-1})(x+1)} = \frac{1}{2} = f(1)$$

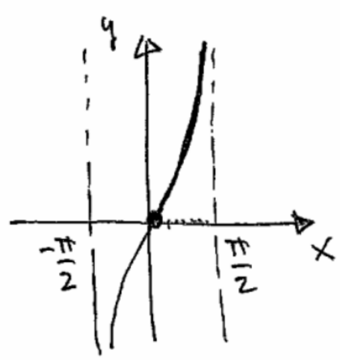
YES

(c)[2] Find all real numbers x where the function $f(x) = \sqrt{\tan x}$ is continuous.

$\Rightarrow \tan x \geq 0$

x is in $[0, \frac{\pi}{2})$

so $[0 + \pi k, \frac{\pi}{2} + \pi k)$



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5. Let $f(x) = \sqrt{x}$.

(a)[1] Find the average rate of change of $f(x)$ on $[4, 4.2]$.

$$\frac{f(4.2) - f(4)}{0.2} = 0.2469$$

(b)[1] Find the average rate of change of $f(x)$ on $[4, 4.1]$.

$$\frac{f(4.1) - f(4)}{0.1} = 0.2485$$

(c)[3] What number are the values in (a) and (b) supposed to approach? Explain why they approach that number and not some other number.

$$f'(x) = \frac{1}{2\sqrt{x}} \qquad f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

they are supposed to approach $\frac{1}{4} = 0.25$

since average rate of change approaches instantaneous rate of change as interval shrinks to a point

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6. (a)[2] Find $f'(x)$, if $f(x) = 3^{\ln x} + (\ln x)^3 + (\ln 3)^3$.

$$f'(x) = 3^{\ln x} \ln 3 \cdot \frac{1}{x} + 3(\ln x)^2 \cdot \frac{1}{x} + 0$$

(b) [2] Find $f'(0)$, if $f(x) = \arcsin x + (\arcsin x)^2$.

$$f'(x) = \frac{1}{\sqrt{1-x^2}} + 2\arcsin x \cdot \frac{1}{\sqrt{1-x^2}}$$

$$f'(0) = \frac{1}{\sqrt{1}} + 2\underbrace{\arcsin 0}_{0} \cdot \frac{1}{\sqrt{1}} = \underline{\underline{1}}$$

(c)[3] Find the equation of the line tangent to the graph of $y = \dfrac{e^x + 1}{x}$ at the point where $x = 1$.

$$x = 1 \rightarrow y = e+1$$

$$y' = \frac{e^x \cdot x - (e^x+1) \cdot 1}{x^2} \quad \therefore \; y'(1) = \frac{e - (e+1)}{1} = -1$$

so $y - (e+1) = -1(x-1)$

$$y = \underline{\underline{-x + e + 2}}$$

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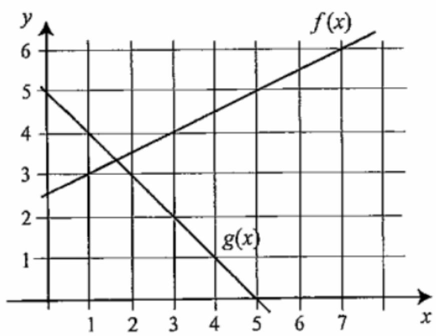
7. (a)[2] Let $g(x) = x^2\sqrt{f(x)}$, where f is a differentiable function such that $f(1) = 4$ and $f'(1) = 1$. Find $g'(1)$.

$$g'(x) = 2x\sqrt{f(x)} + x^2 \cdot \frac{1}{2}(f(x))^{-1/2} \cdot f'(x)$$

$$g'(1) = \underbrace{2 \cdot 1 \cdot \sqrt{4}}_{4} + 1 \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{4}} \cdot 1 = 4 + \frac{1}{4} = \frac{17}{4}$$

(b)[3] The graphs of the functions $f(x)$ and $g(x)$ are given below. Compute $(fg)'(3)$, i.e. compute the derivative of the product of $f(x)$ and $g(x)$ when $x = 3$.



$(fg)'(3)$

$= f'(3) \cdot g(3) + f(3) \cdot g'(3)$

$= \frac{1}{2} \cdot 2 + 4 \cdot (-1)$

$= -3$

THE END