Math 1B03/1ZC3 Test #1 (Version 1)
February 15th, 2017

Name:____
(Last Name) (First Name)

Student Number:____________________

This test consists of 20 multiple choice questions worth 1 mark each (no part marks), and 1 question worth 1 mark (no part marks) on proper computer card filling. Questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Calculators are NOT allowed.

1. Suppose that A and B are $n \times n$ invertible matrices, and consider the following statements.

(i) $(A+B)^T = A^T + B^T$
(ii) $(A+B)^{-1} = A^{-1} + B^{-1}$
(iii) $(AB)^{-1} = A^{-1}B^{-1}$

Which of the above statements are always true?

(a) all of them **(b)** (i) and (iii) only **(c)** (i) only **(d)** (i) and (ii) only **(e)** (ii) and (iii) only

2. Find all values of k, if any, that satisfy the following equation.

$$\begin{bmatrix} 2 & 2 & k \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 3 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ k \end{bmatrix} = 0$$

(a) $-2, -10$ **(b)** no solution **(c)** $1, 5$ **(d)** $-1, -5$ **(e)** $-2, 5$

3. Suppose that A is a 4×2 matrix, C is a 3×5 matrix, and that the product $(C^T B)A^T$ is defined. How many columns does the matrix B have?

(a) 5 **(b)** 2 **(c)** 3 **(d)** 4 **(e)** 6

4. Consider the following statements.

(i) A linear system whose equations are all homogeneous must be consistent.
(ii) If the number of equations in a linear system exceeds the number of unknowns then the system must be inconsistent
(iii) The linear system

$$\begin{aligned} x - y &= 3 \\ 2x - 2y &= k \end{aligned}$$

cannot have a unique solution, regardless of the value of k.

Which of the above statements are true?

(a) (ii) only **(b)** (i) only **(c)** (i) and (iii) only **(d)** all of them **(e)** (i) and (ii) only

5. Which of the following matrices are in reduced row-echelon form?

(i) $\begin{bmatrix} 1 & -7 & 5 & 5 \\ 0 & 1 & 3 & 2 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 0 & 0 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 3 & 0 & 7 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(a) (ii) and (iii) only **(b)** (iii) only **(c)** none of them **(d)** (ii) only **(e)** (i) only

6. Find the matrix A if

$$(I + 2A)^{-1} = \begin{bmatrix} 1 & 3 \\ -2 & -4 \end{bmatrix}^T$$

(a) $\begin{bmatrix} -\frac{1}{2} & -2 \\ \frac{3}{2} & 1 \end{bmatrix}$ **(b)** $\begin{bmatrix} 2 & \frac{3}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}$ **(c)** $\begin{bmatrix} -1 & -\frac{3}{2} \\ \frac{1}{4} & \frac{1}{2} \end{bmatrix}$ **(d)** $\begin{bmatrix} -\frac{3}{2} & \frac{1}{2} \\ -\frac{3}{4} & -\frac{1}{4} \end{bmatrix}$ **(e)** $\begin{bmatrix} -\frac{1}{2} & \frac{3}{4} \\ 1 & -\frac{1}{4} \end{bmatrix}$

7. Let $A = \begin{bmatrix} 1 & 3 & -2 \\ 4 & 1 & 6 \\ 2 & -5 & 10 \end{bmatrix}$ and let $B = \begin{bmatrix} 1 & 3 & -2 \\ 0 & -11 & 14 \\ 2 & -5 & 10 \end{bmatrix}$. Let E be an elementary matrix such that $EA = B$. Find $e_{21} + e_{22} + e_{23}$ (i.e., find the sum of the entries in the 2nd row of the matrix E).

(a) -3 **(b)** -4 **(c)** 2 **(d)** 3 **(e)** 4

8. For what value of k does the following matrix fail to be invertible?

$$A = \begin{bmatrix} 3 & 1 & 0 & 0 & 0 \\ k & 2 & 0 & 0 & 0 \\ -1 & 4 & -2 & 0 & 0 \\ 2 & 8 & -6 & -1 & 0 \\ 12 & 3 & 1 & 5 & 1 \end{bmatrix}$$

(a) $\frac{1}{3}$ **(b)** 2 **(c)** 1 **(d)** 3 **(e)** 6

9. Determine the condition on the b_i's, if any, in order to guarantee that the following linear system is consistent.

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= b_1 \\ 3x_1 + 6x_2 - x_3 &= b_2 \\ 5x_1 + 10x_2 - 7x_3 &= b_3 \end{aligned}$$

(a) consistent for all b_i's **(b)** $b_3 = b_2 + 2b_1$ **(c)** $b_1 = 2b_3 + 3b_2$ **(d)** $b_2 = b_3 - \frac{1}{2}b_1$
(e) $b_2 = \frac{1}{2}b_3 + 3b_1$

10. Consider the following matrix A and let $B = A^{-1}$. Find b_{31} (i.e., find the entry in row 3, column 1 of A^{-1}).

$$A = \begin{bmatrix} 1 & 1 & -2 \\ -2 & -1 & 0 \\ -2 & -1 & 2 \end{bmatrix}$$

(a) $\frac{1}{2}$ **(b)** $\frac{3}{2}$ **(c)** -1 **(d)** $-\frac{3}{2}$ **(e)** 0

11. Which of the following statements are true?

(i) If A is an invertible matrix then the linear system $A\mathbf{x} = \mathbf{0}$ has nontrivial solutions.
(ii) If A is a square matrix and the linear system $A\mathbf{x} = \mathbf{b}$ has a unique solution then the linear system $A\mathbf{x} = \mathbf{c}$ also must have a unique solution.
(iii) If A and B are square matrices satisfying $BA = I$ then $B = A^{-1}$.

(a) all of them **(b)** (iii) only **(c)** (ii) and (iii) only **(d)** (i) only **(e)** (ii) only

12. Find the determinant of the following matrix.

$$A = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 2 & -4 & 0 & -3 \\ 1 & 0 & -4 & 3 \\ 1 & 0 & 4 & 0 \end{bmatrix}$$

(a) 24 **(b)** 6 **(c)** 8 **(d)** 4 **(e)** -4

13. In Matlab, what command could be used to define the following matrix.

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 5 & 9 \end{bmatrix}$$

(a) `>>A=matrix(2,3,[1 3 4; 2 5 9])`
(b) `>>A=matrix([1 3 4; 2 5 9])`
(c) `>>A=mat(1 3 4; 2 5 9)`
(d) `>>A=[1 3 4; 2 5 9]`
(e) `>>A:=matrix([1 3 4; 2 5 9])`

14. Which of the following statements are true?

(i) If A and B are symmetric $n \times n$ matrices then the matrix AB is also symmetric.
(ii) If $A + B$ is a symmetric matrix then A and B must also be symmetric
(iii) If A and B are skew-symmetric $n \times n$ matrices then $A - B$ is also skew-symmetric (recall that a matrix A is skew-symmetric if $A^T = -A$)

(a) (i) and (ii) only **(b)** (i) and (iii) only **(c)** (ii) and (iii) only **(d)** all of them
(e) (iii) only

15. Suppose that

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 4$$

Evaluate the following determinant.

$$\begin{vmatrix} a & b & c \\ 2d & 2e & 2f \\ g+3a & h+3b & i+3c \end{vmatrix}$$

(a) 24 **(b)** 8 **(c)** 32 **(d)** 288 **(e)** 6

16. Consider the following matrix.

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

Given that $\lambda = 2$ is an eigenvalue of A, which of the following is a basis for the eigenspace corresponding to $\lambda = 2$?

(a) $\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ **(b)** $\left\{ \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$ **(c)** $\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ **(d)** $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$
(e) $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

17. Suppose that A is a 4×4 matrix with $\det(A) = 3$. Find $\det(2A^T A^{-1})$.

(a) 2 **(b)** 144 **(c)** 16 **(d)** 48 **(e)** 18

18. Consider the following matrix.

$$A = \begin{bmatrix} -6 & -1 & -8 \\ 2 & 4 & -3 \\ 3 & 4 & 1 \end{bmatrix}$$

Let $B = \text{adj}(A)$. Find b_{32} (i.e., find the entry in row 3, column 2 of the adjoint of A).

(a) -21 **(b)** -62 **(c)** 18 **(d)** 21 **(e)** 62

19. Find the eigenvalues of the following matrix.

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

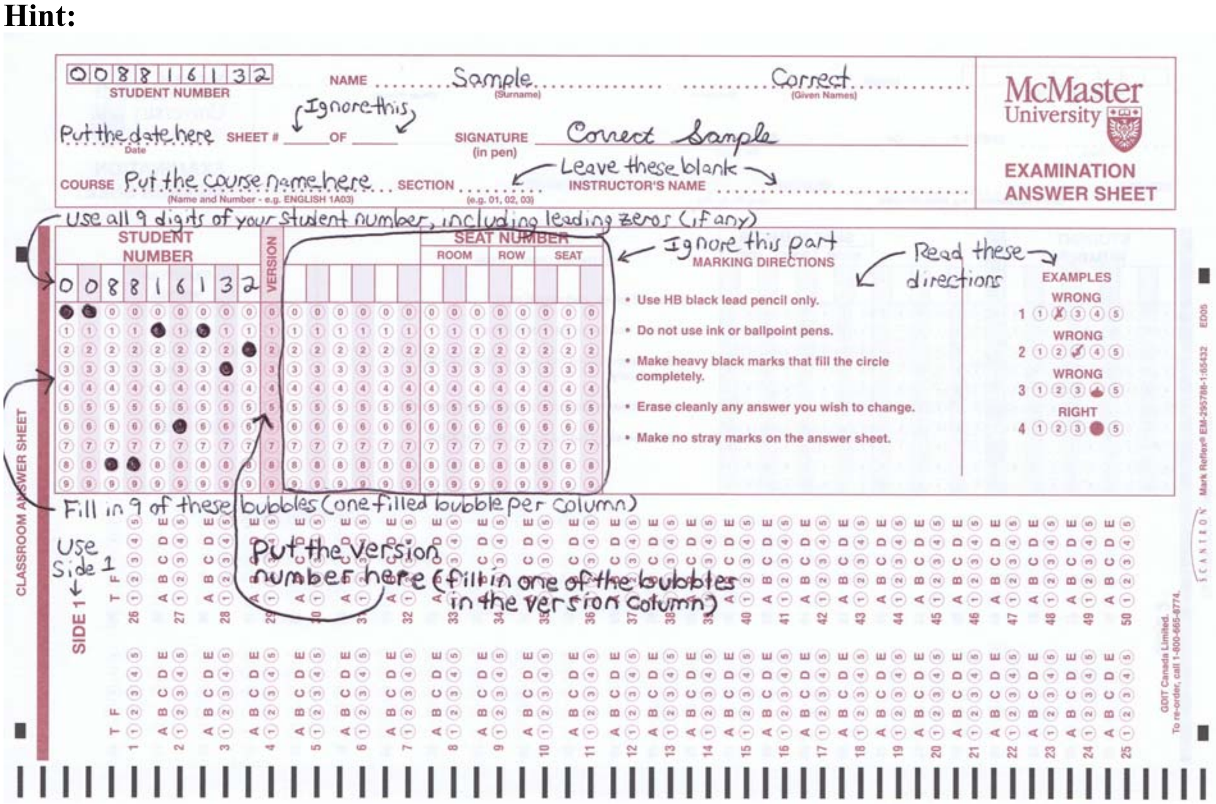
(a) $-2, 1$ **(b)** $3, -1$ **(c)** $2, -1$ **(d)** $1, 4$ **(e)** $-1, 4$

20. Use Gauss-Jordan elimination to solve the following system of equations.

$$\begin{aligned} 2x + 2y - 6z &= 2 \\ x + 6y - 3z &= 4 \\ 6x + 16y - 18z &= 12 \end{aligned}$$

(a) $x = \frac{1}{2} + \frac{25}{6}t$, $y = \frac{1}{2}$, $z = t$ **(b)** $x = \frac{2}{5} + \frac{21}{5}t$, $y = \frac{3}{5}$, $z = t$ **(c)** $x = \frac{8}{5} + 3t$, $y = \frac{3}{5}$, $z = t$ **(d)** $x = \frac{2}{5} + 3t$, $y = \frac{3}{5}$, $z = t$
(e) $x = \frac{1}{2} + 3t$, $y = \frac{1}{2}$, $z = t$

21. Correctly fill out the bubbles corresponding to all 9 digits of your student number, as well as the version number of your test in the correct places on the computer card.
Note: You are writing **Version 1**.
Hint:



Answers (Version 1)**:**

1. c **2.** a **3.** b **4.** c **5.** b **6.** d **7.** a **8.** e **9.** b **10.** e
11. c **12.** a **13.** d **14.** e **15.** b **16.** a **17.** c **18.** d **19.** b **20.** d