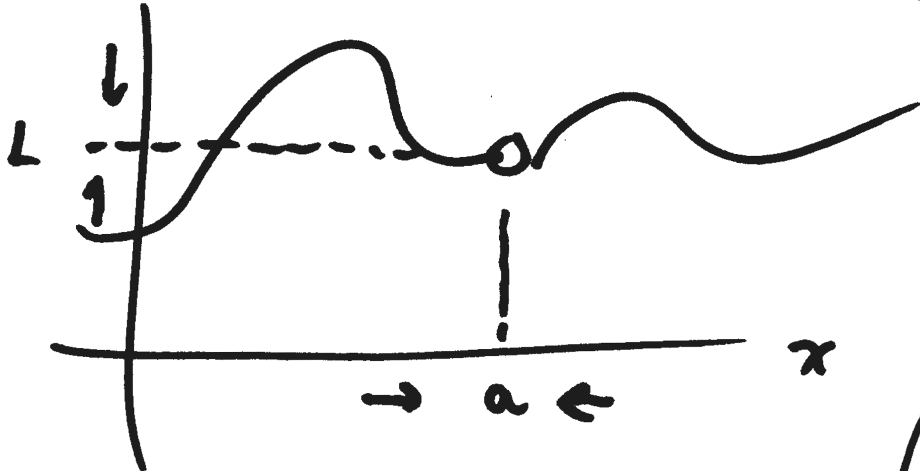
12A3

$$\boxed{\lim_{x \to a} f(x) = L}$$

$\boxed{\text{Formal } \varepsilon, \delta \text{ Definition} \quad \text{Please Ignore!}}$



as x approaches a

$y = f(x)$ approaches L

"limit of $f(x)$ as x approaches a is L"

eg $\lim_{x \to 2} x^2 - 5 = 2^2 - 5 = -1$

If $f(x)$ is continuous at $x = a$

then $\boxed{\lim_{x \to a} f(x) = f(a)}$

All usual functions: poly, rationals, trig etc.
all cont. on their domain!

But eg. $\displaystyle\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$ } 4 is not in domain of our func.

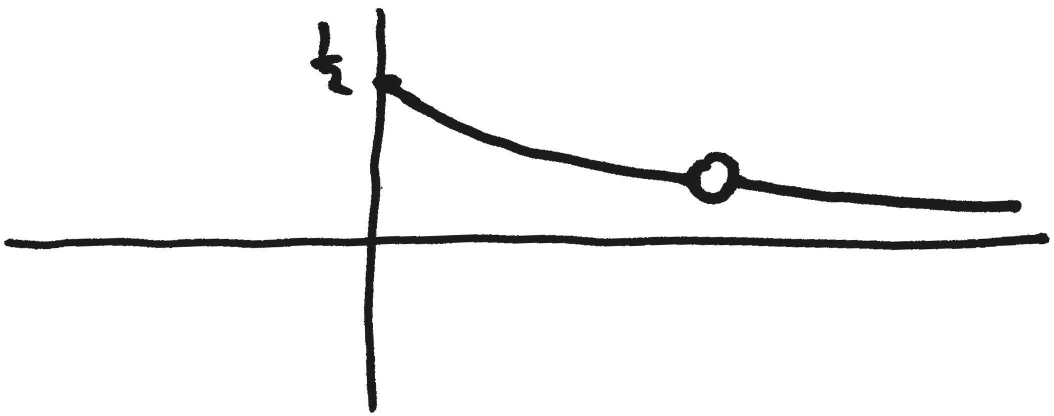
plug in? get "$\frac{0}{0}$" = **Bad**

$$\lim_{x \to 4} \frac{(\sqrt{x} - 2)}{x - 4} \cdot \frac{(\sqrt{x} + 2)}{(\sqrt{x} + 2)} = \lim_{x \to 4} \frac{\cancel{(x - 4)}}{\cancel{(x - 4)}(\sqrt{x} + 2)}$$

$$= \lim_{x \to 4} \frac{1}{\sqrt{x} + 2} = \frac{1}{2+2} = 1/4$$

<u>notice</u> $\frac{\sqrt{x} - 2}{x - 4}$ <u>not</u> cont. at $x = 4$

but $= \frac{1}{2 + \sqrt{x}}$, $x \neq 4$



"Point Discontinuity"

ie $\underbrace{\lim_{x \to a} f(x) = L}_{\text{Limit exists!}} \neq f(a)$ ← not cont.

Approaches, but does not arrive!

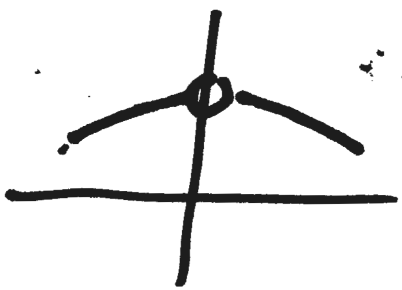
What else can go wrong?

Point hole!

eg $\frac{\sin x}{x}$ } → at $x = 0 \Rightarrow \frac{0}{0}$

but $\lim_{x \to 0} \frac{\sin x}{x} = 1$ } Ugly proof Do later!

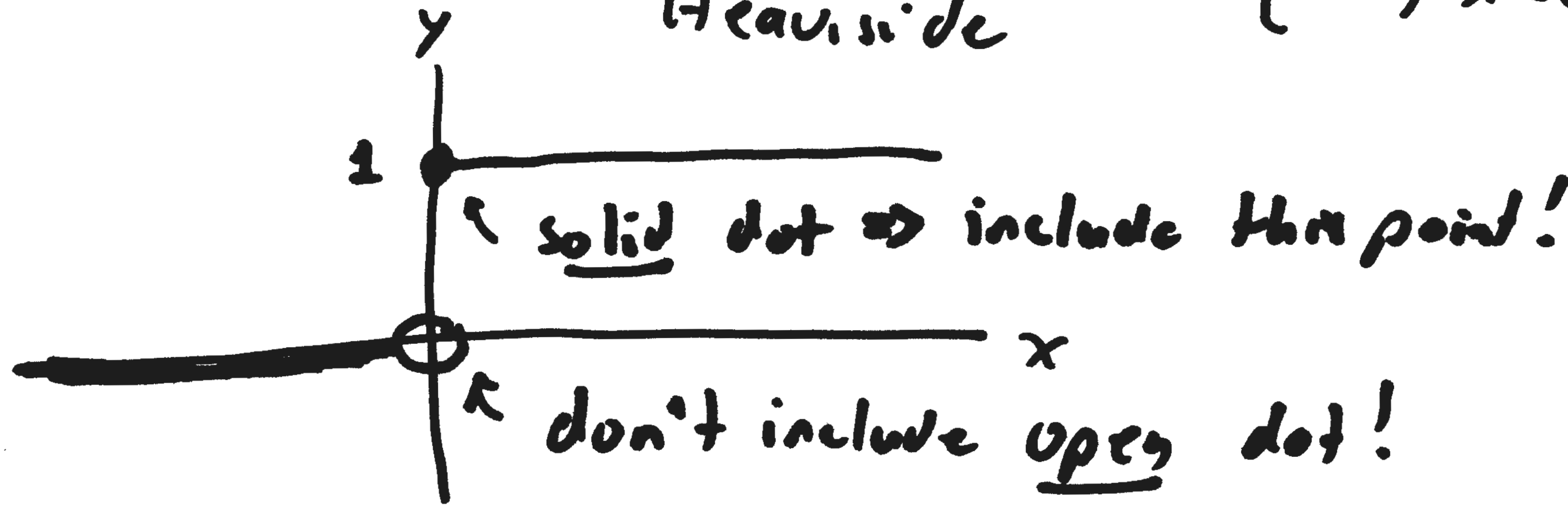
Pic



Jump Discontinuity

e.g. $H(x) = \mathcal{U}(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$

"Heaviside"



$\lim_{x \to 0} H(x) = ?$ DNE (does not exist!)

$\underbrace{\lim_{x \to 0^+} H(x) = 1}$, $\underbrace{\lim_{x \to 0^-} H(x) = 0}$

The limit from the right

The limit from the left

Notice $\lim_{x\to a} f(x)$ **exists**

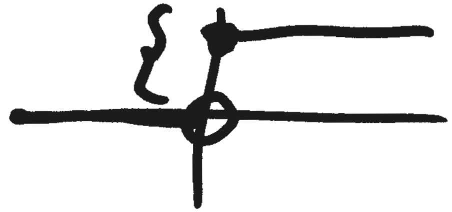
iff (if and only if) $\lim_{x\to a^+} f(x)$, $\lim_{x\to a^-} f(x)$ exist

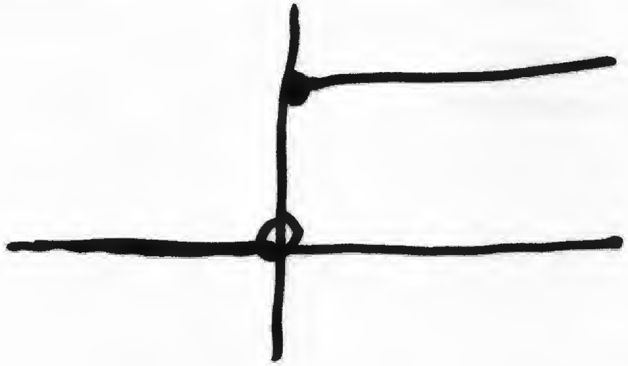
and $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = \lim_{x\to a} f(x)$

But if $\lim_{x\to a^-} f(x)$ **exists** & $\lim_{x\to a^+} f(x)$ **exists**

but $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$

$\Rightarrow$ Jump Discontinuity





$$\lim_{x \to 0^+} H(x) = 1 \qquad \lim_{x \to 0^-} H(x) = 0$$

$$H(0) = 1$$

Since $\lim_{x \to 0^+} H(x) = H(0) \Rightarrow$ $H(x)$ is right cont. at $x = 0$

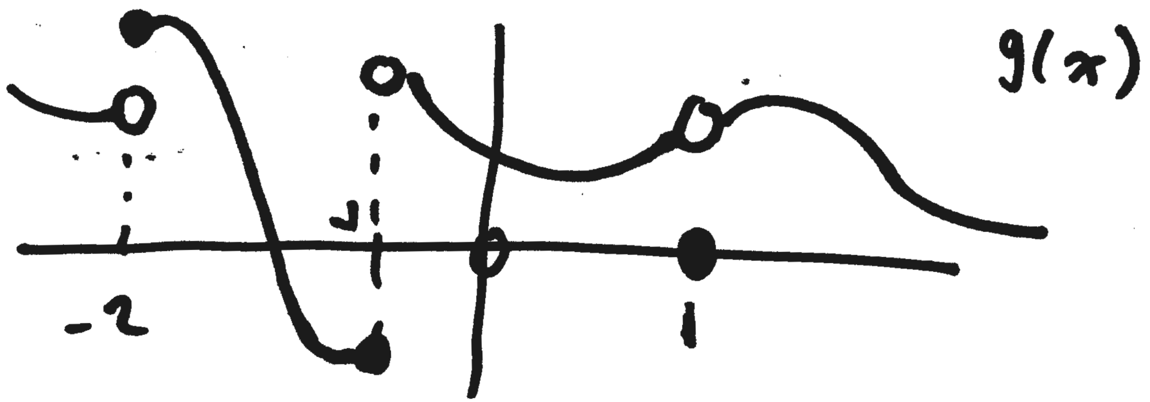
In general

$f(x)$ is left. cont. at a if $\lim_{x \to a^-} f(x) = f(a)$

" " right cont. at a if $\lim_{x \to a^+} f(x) = f(a)$

And cont. at a iff both left & right cont. at $x=a$.

eg



$g(x)$ disc. at $x=1, -1, -2$

$g(x)$ is only right cont at $x=-2$

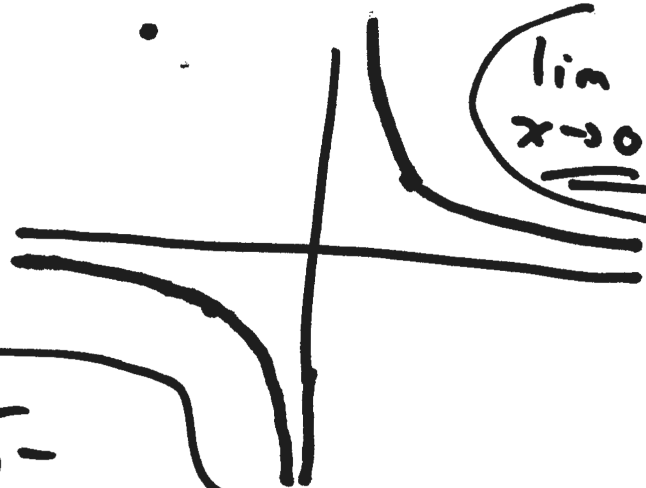
$g(x)$ is left cont. only at $x=-1$

note at $x=1$ limit $\lim_{x\to 1} g(x)$ exists but not

, not cont. , not right or left cont

∞ jumps, ∞ discont.

eg. $f(x) = \frac{1}{x}$



$$\lim_{x \to 0^+} \frac{1}{x} = \frac{1}{0^+} = \frac{1}{\text{small} +} = \text{big} + = +\infty$$

DNE

$$\lim_{x \to 0^-} f(x) = \frac{1}{0^-} = \frac{1}{\text{small} -} = \text{big} - = -\infty$$

DNE

& "and"

$$\lim_{x \to 0} \frac{1}{x} = \frac{1}{0}$$ DNE undefined

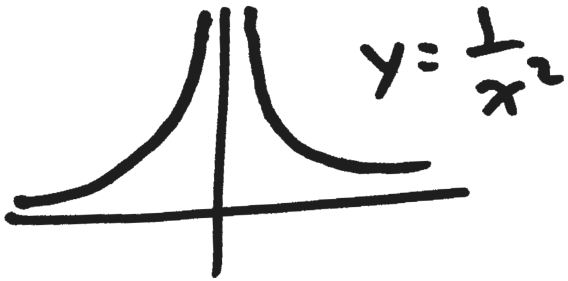
why !..

∞ not reals

Note ∞ not a real #

So if $\lim_{x \to a} f(x) = \pm\infty \Rightarrow$ DNE, but we know how it DNE!

It blows up!

eg 

$$\lim_{x \to 0} \frac{1}{x^2} = \frac{1}{0^2} = \frac{1}{0^+_{\text{e}}} \text{ small +ve}$$

$$= +\infty.$$

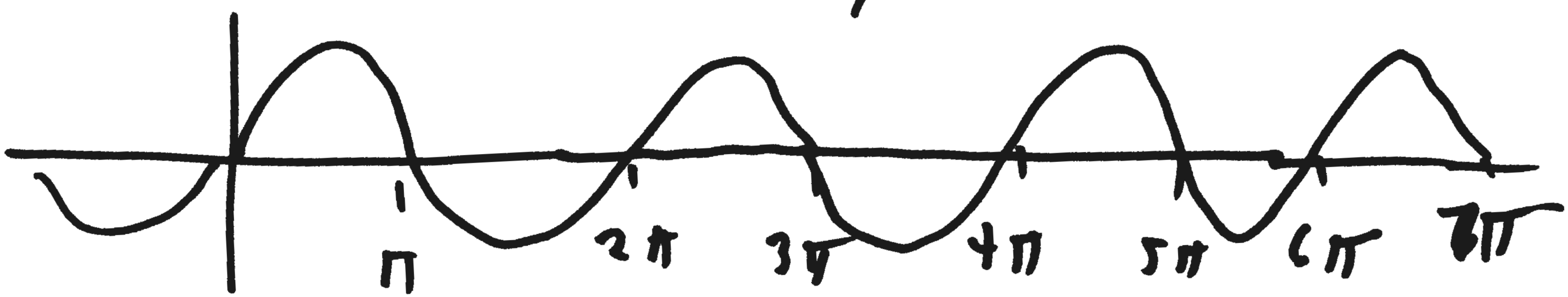
still DNE

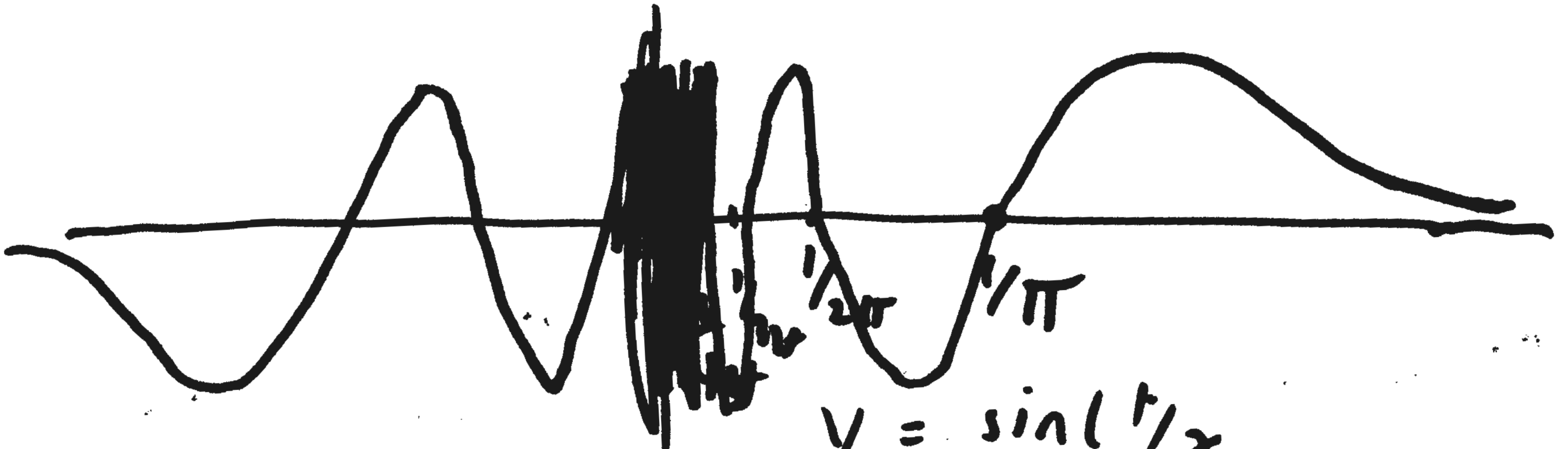
Try This

$$\lim_{x \to 0} \sin\left(\frac{1}{x}\right) = ?$$

Pic

$y = \sin x$





$y = \sin(1/x)$

∞ oscillations between ± 1