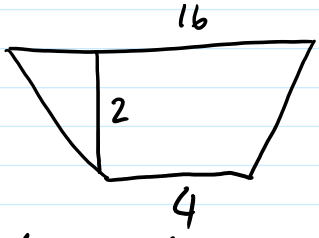
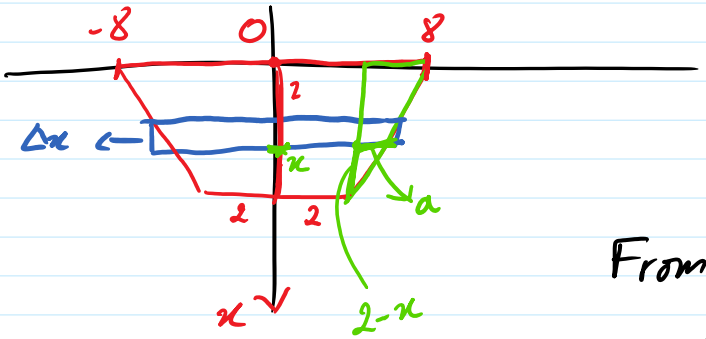
Hydrostatic force :

Suppose a gate of an irrigation canal is a trapezoid :



Find the hydrostatic force if the gate is fully submerged in water.

Textbook way



$F = P \cdot A$ (depth)

$= g \cdot \rho \cdot d \cdot A$ → area.

↑ gravitational acceleration; → density of liquid

From the green triangle, we have

$$\frac{a}{2-x} = \frac{6}{2}$$

$$a = 3(2-x).$$

$\Rightarrow$ the width of the blue strip

$$w = 2a + 4 = 2 \cdot 3(2-x) + 4$$

$$= 16 - 6x$$

$\Rightarrow$ Area of the blue strip $= (16-6x)\Delta x$

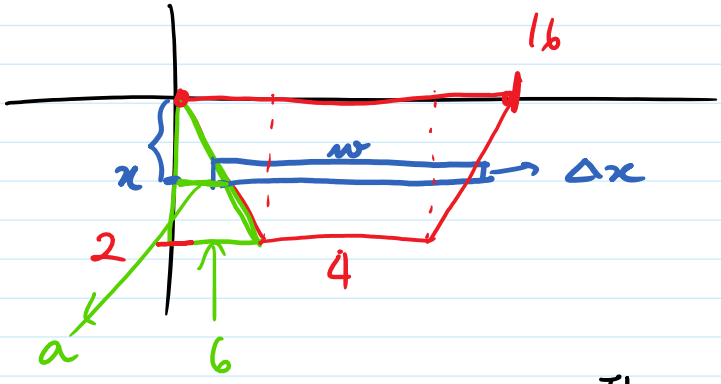
Hydrostatic force on the strip

$$= g \cdot \rho \cdot x \cdot (16-6x)\Delta x.$$

Hydrostatic on the whole domain

$$= g \cdot \rho \int_0^2 x(16 - 6x)\, dx \quad .$$

Small variation in the set up compared to the book



Similar triangle.

$$\frac{a}{x} = \frac{6}{2} \Rightarrow a = 3x .$$

The width of the blue strip

$$w = 16 - 2a = 16 - 6x .$$

Area of the blue strip

$$w \cdot \Delta x = (16 - 6x)\Delta x .$$

$\Rightarrow$ Hydrostatic for on the whole trapezoid

$$F = g \cdot \rho \int_0^2 x \cdot (16 - 6x)\, dx .$$