Extra practice with extreme values (Section 10)

1. Answer the following questions without calculating critical points and/or using the second derivatives test.

(a) Explain why $f(x, y) = 3x - y + 4$ has no local minimum values.

(b) Explain why $f(x, y) = 4x^2 + y^4$ has a local minimum at $(0, 0)$.

(c) Explain why $f(x, y) = 4x^2 + y^4$ has no local maximum values.

(d) Explain why every point on the line $x - y - \pi/2 = 0$ is a local maximum of the function $f(x, y) = \sin(x - y)$.

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(e) Find all relative minimum values of $g(x, y) = |x| + y^2$.

(f) Find all relative minimum values of $g(x, y) = |x + y^2|$.

(g) Explain why $f(x, y) = e^{-x^2-2y^2}$ has a local maximum at $(0, 0)$.

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2. Draw a contour diagram of a function that has a minimum at $(2,2)$ and a saddle point at $(-1,1)$.

3. Assume that a differentiable function f has a local maximum at (a,b). Define a function $g(y)=f(a,y)$ and use it to prove that $f_y(a,b)=0$.

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4. Find the local minimum and maximum values and saddle points (if any) of each function.

(a) $f(x, y) = x^2 + y^2 + 2xy^2$

(b) $f(x, y) = x^3 - 2y^2 + 3xy + 4$

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(c) $f(x,y) = xye^{-x-y}$

(d) $f(x,y) = x + \dfrac{x+y}{xy}$

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5. Consider the functions $f(x, y) = x^2$, $g(x, y) = 3 - x^2$, and $h(x, y) = x^3$.

(a) Find the critical points of each function.

(b) Show that, in each case, $D(x, y) = 0$.

(c) Sketch the graphs of the three functions, and use them to determine what happens at their critical points.

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