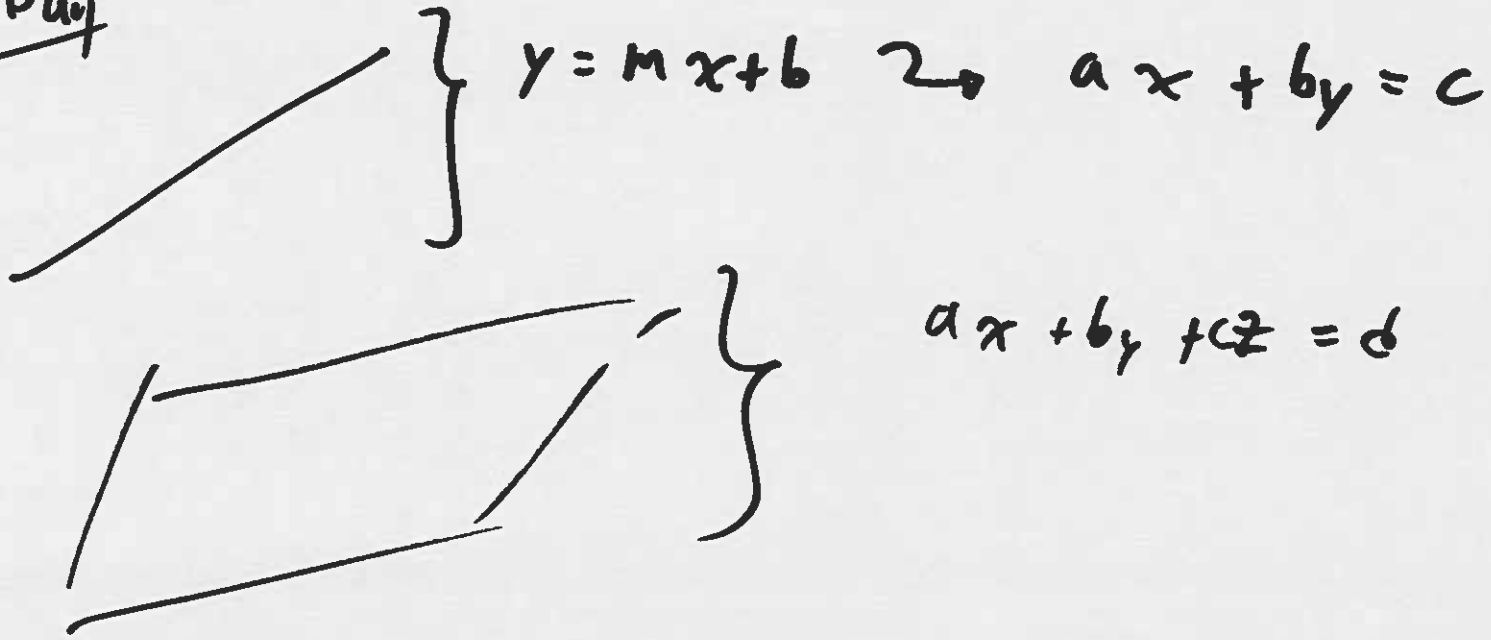
1263

Note Office hours start Thurs

Tutorials & Help Centre start Monday

Note: Two ~~assignments~~ 1 Lab & 1 Assgn. are already up!

Last Day



Linear Equation $a_1 x_1 + a_2 x_2 + a_3 x_3 + \cdots + a_n x_n = b$

a_i = coefficients (real const.), b = constant (real).

x_i = variables

Linear System: A collection of linear eqns. in the same variables

eg $\begin{cases} 2x + y = 1 \\ x - y = 2 \end{cases}$

eg $\begin{cases} x + y - 7z + 2w = 5 \\ 2x - y + 12z - w = 7 \end{cases}$

eg $\begin{cases} x + y = 1 \\ x - x = 3 \\ y = 12 \end{cases}$

A Solution to a linear system is a set of values of the variables that satisfies all of the equations (at the same time!)

eg $\begin{cases} x+y-z=0 \\ x-y+2z=0 \\ y+z=0 \end{cases}$ $(x,y,z)=(0,0,0)$ is a solution!

(not only!)

Solving Systems

eg. $\begin{cases} x+y=2 \\ x-y=5 \end{cases}$ $\rightarrow$ $x=2-y$

$(2-y)-y=5$

$2-2y=5$

Only solution

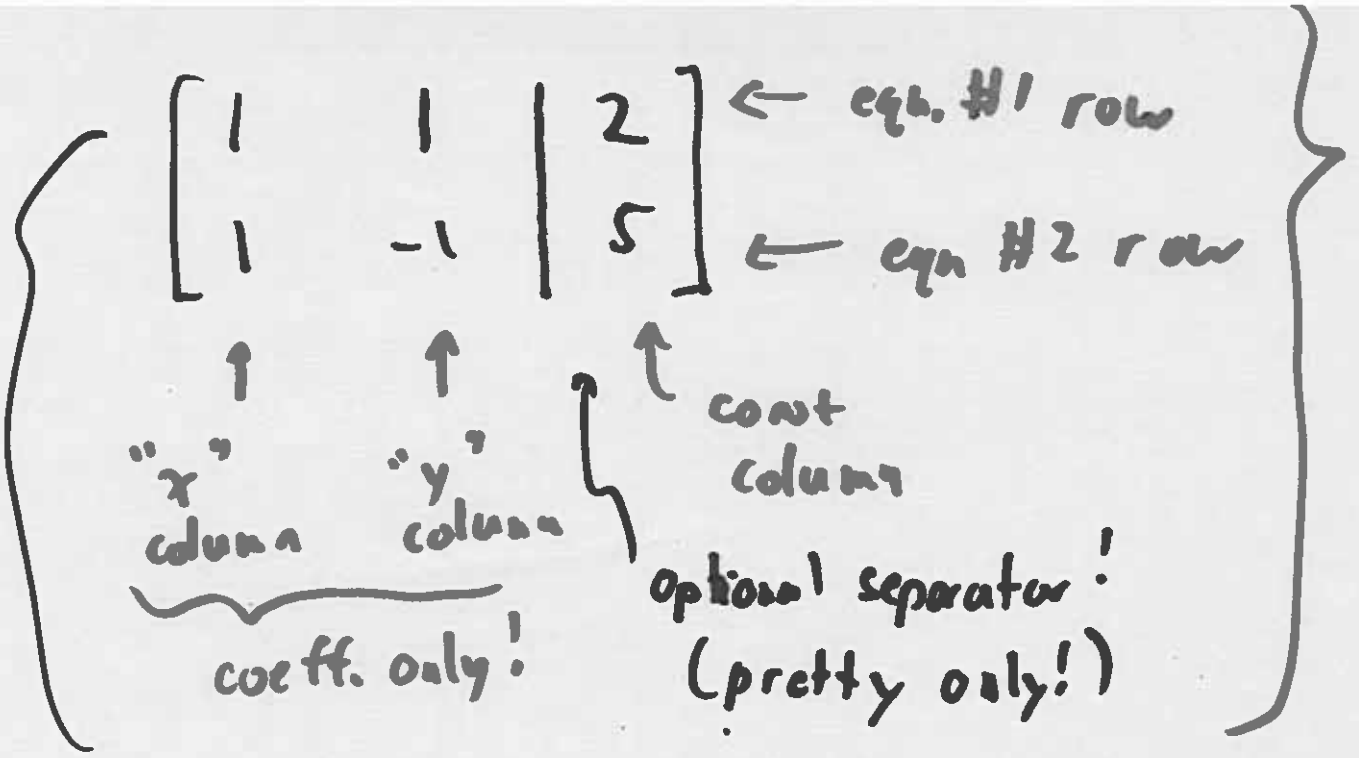
$$(x, y) = \left(\frac{7}{2}, -\frac{3}{2}\right) \Leftarrow \begin{cases} y = -3/2 \\ x = 2 - \frac{3}{2} = \frac{7}{2} \end{cases}$$

Or solve using equation arithmetic (equation operations)

eg. $\begin{cases} x + y = 2 \\ x - y = 5 \end{cases}$ now add!

$$2x + 0y = 7 \rightsquigarrow \Rightarrow \begin{cases} x = \frac{7}{2} \\ y = -\frac{3}{2} \end{cases}$$

Or Write into an "augmented matrix"



↳ Solve system once in "augmented matrix" form using "3 elementary row operations":

Op #1 Swap rows (change order of equations)

eg $\begin{cases} 2x - y = 5 \\ x - 2y = 12 \end{cases} \Rightarrow \left[\begin{array}{cc|c} 2 & -1 & 5 \\ 1 & -2 & 12 \end{array}\right]$

$\text{Row}_1 \leftrightarrow \text{Row}_2$

$\begin{cases} x - 2y = 12 \\ 2x - y = 5 \end{cases} \Leftarrow \left[\begin{array}{cc|c} 1 & -2 & 12 \\ 2 & -1 & 5 \end{array}\right]$

Op #2 Divide / Multiply a row by a non-zero constant

eg $\begin{cases} 2x + 4y = 6 \\ x - y = 12 \end{cases} \Rightarrow \left[\begin{array}{cc|c} 2 & 4 & 6 \\ 1 & -1 & 12 \end{array}\right]$

$(\text{Row } 1) \cdot \frac{1}{2}$

$\begin{cases} x + 2y = 3 \\ x - y = 12. \end{cases} \Leftarrow \left[\begin{array}{cc|c} 1 & 2 & 3 \\ 1 & -1 & 12 \end{array}\right]$

Op #3 Add a multiple of one row to another in its location

eg. $\left\{ \begin{array}{l} x + y = 2 \\ 3x - y = 5 \end{array} \right\} \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 3 & -1 & 5 \end{array} \right]$

Row 2 $\rightarrow$ Row 2 - 3Row 1.

$\left\{ \begin{array}{l} x + y = 2 \\ -4y = -1 \end{array} \right\} \Leftarrow \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & -4 & -1 \end{array} \right]$

$-1 - 3(1)$ $\quad$ $5 - 3(2)$

Ops. in action!

eg. $\begin{cases} 3x - 6y = 12 \\ 2x - y = 0 \end{cases}$ convert! $\Longrightarrow$ $\left[\begin{array}{cc|c} 3 & -6 & 12 \\ 2 & -1 & 0 \end{array}\right]$

Now solve!

$R_{ow}1 \to R_{ow}1 \cdot (\frac{1}{3})$ $\left[\begin{array}{cc|c} 1 & -2 & 4 \\ 2 & -1 & 0 \end{array}\right]$

$Row\,2 \to Row\,2 - 2 \cdot Row\,1$ $\left[\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 3 & -8 \end{array}\right]$

$Row\,2 \cdot (\frac{1}{3})$ $\left[\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 1 & -8/3 \end{array}\right]$

$Row\,1 \to Row\,1 + 2Row\,2$ $\left[\begin{array}{cc|c} 1 & 0 & -4/3 \\ 0 & 1 & -8/3 \end{array}\right]$

$x = -4/3$

$y = -8/3$

Answer!

Goal: To get as close to $x = \#$ as possible!
$y = \#$
$z = \#$

"Reduced Row Echelon Form" (RREF)

1) Any all-zero rows go at the bottom
2) The first non-zero entry in any row must be a $\underline{1}$ ("Leading 1")
3) Each leading 1 is right of any in rows above.

(ie "down & to the right)

4) Any entry below or above a leading 1 is $\underline{0}$.