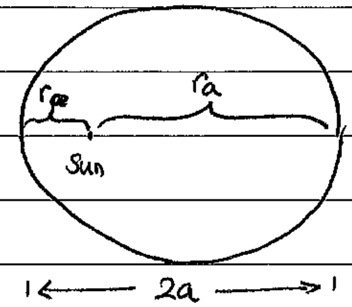
<u>1F03 : ASSIGNMENT 1 SOLUTIONS</u>

Q1. For the comet, perihelion distance $r_p = 1$ A.U.
aphelion distance $r_a = 71$ A.U.



Semi-major axis (a) :

$$2a = r_a + r_p$$

$$\Rightarrow a = \frac{r_a + r_p}{2} = \frac{1+71}{2} = \boxed{36 \text{ A.U.}}$$

$$r_a = a(1+e) \qquad \text{OR} \qquad r_p = a(1-e)$$

$$\Rightarrow e = \frac{r_a}{a} - 1 \qquad\qquad \Rightarrow e = 1 - \frac{r_p}{a}$$

Eccentricity $e = \frac{71}{36} - 1 = \boxed{\frac{35}{36} \cong 0.972}$

Kepler's 3rd Law : P^2 (yrs) $= a^3$ (A.U.).

$$\Rightarrow P = a^{3/2} = (36)^{3/2}$$

$$= 216 \text{ yrs.}$$

But, period is total time for comet to travel from perihelion to aphelion back to perihelion. $\therefore$ Time to travel from aphelion to perihelion $= P/2 = \boxed{108 \text{ yrs}}$

Q2. a) The physical ideas behind Kepler's Laws still hold for any 2 bodies gravitationally, so long as the central object's mass (e.g. Earth) greatly exceeds the mass of the object orbiting it.

Since the standard version of Kepler's Laws apply to objects orbiting the sun, they need to be modified to describe objects orbiting the Earth.

For the 1st & 2nd Laws, this simply requires us to change "Sun" to "Earth" & "planet" to "orbiting object).
For the 3rd Law, $P^2 = a^3$ is only true for objects orbiting the sun & when P is in yrs & a in A.U.

For any other 2-body system, the 3rd Law can be stated as $P^2 = Ca^3$ where the constant of proportionality c depends on the mass of the central object. as well as the units chosen.

b) Since $P^2 = Ca^3$

$$P_1^2 = Ca_1^3 \quad \text{and} \quad P_2^2 = Ca_2^3$$

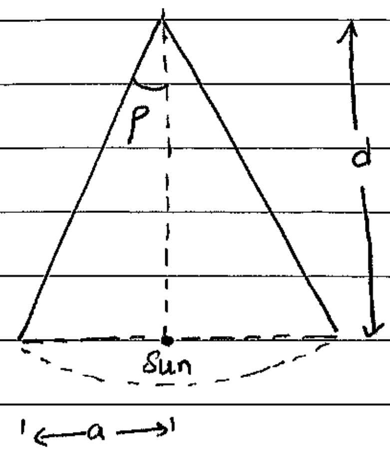
$$\Rightarrow \left(\frac{P_1}{P_2}\right)^2 = \frac{\cancel{C}a_1^3}{\cancel{C}a_2^3} = \left(\frac{a_1}{a_2}\right)^3$$

For the Moon, $P_1 = 27$ days, $a_1 = 60 R_{Earth}$

For the satellite, $a_2 = R_{Earth}$.

$$\therefore \left(\frac{27}{P_2}\right)^2 = \left(\frac{60\cancel{R_{Earth}}}{\cancel{R_{Earth}}}\right)^3 \Rightarrow P_2 = \frac{27}{60^{3/2}} = \boxed{0.058 \text{ days}}$$

Q3.



Distance to the star $d = 20\,pc$.

Since $d \gg a$, where a is the semi-major axis of any planet orbitting the Sun, we can approximate the parallax angle as:

$$p = \frac{a}{d}$$

If this were not true, we could use $\tan p = \frac{a}{d}$.

From Earth, the parallax angle is

$$p = \frac{1\text{ A.U.}}{20\text{ pc}} = \frac{1.496\times10^{11}\text{ m}}{20\times3.086\times10^{16}\text{ m}} = 2.424\times10^{-7}\text{ rad}$$

$$= (2.424\times10^{-7}\text{ rad})\times\left(\frac{360^\circ}{2\pi\text{ rad}}\right)\times\left(\frac{3600''}{1^\circ}\right) = \boxed{0.05''}$$

From Jupiter, the parallax angle is

$$p = \frac{5.203\text{ A.U.}}{20\text{ pc}} = 1.261\times10^{-6}\text{ rad} = \boxed{0.26''}$$

NOTE: 1 pc is defined as the distance at which 1 A.U. subtends an angle of 1".

$$\therefore\quad p\,(\text{arcsec}) = \frac{a\,(\text{A.U.})}{d\,(\text{pc})}$$

$$\therefore\quad p_{earth} = \frac{1}{20} = 0.05''$$

$$p_{Jupiter} = \frac{5.203}{20} = 0.26''$$

Question 4

$r_c = 0.073\, r_\oplus \quad M_c = 0.00015\, M_\oplus$

$$F_{ceres} = \frac{G M_c M_{astro}}{r_c^2} \qquad F_\oplus = \frac{G M_\oplus M_{astro}}{r_\oplus^2}$$

$$\frac{F_{ceres}}{F_\oplus} = \frac{\cancel{G} M_c \cancel{M_{astro}}}{r_c^2} \cdot \frac{r_\oplus^2}{\cancel{G} M_\oplus \cancel{M_{astro}}}$$

$$= \frac{0.00015\, \cancel{M_\oplus}}{(0.073\, \cancel{r_\oplus})^2} \cdot \frac{\cancel{r_\oplus^2}}{\cancel{M_\oplus}}$$

$$\longrightarrow \frac{F_{ceres}}{F_\oplus} = \frac{0.00015}{(0.073)^2} = \boxed{0.028}$$

Question 5

$\Theta = 2'$ $\quad \lambda = 21\ cm = 0.21\ m$

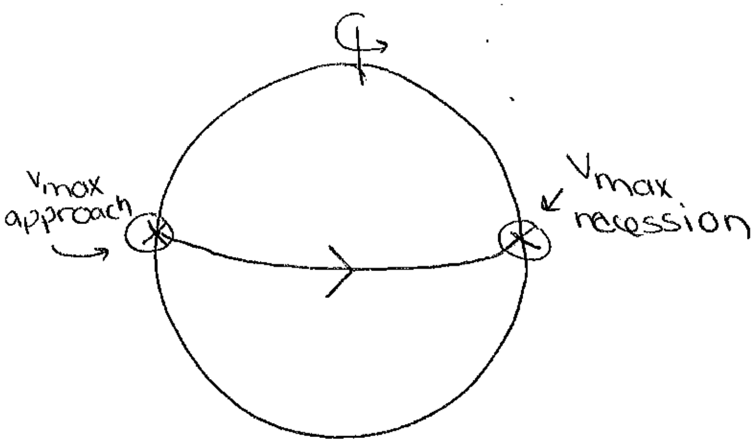
$$\Theta(rad) = \frac{1.22\,\lambda(m)}{D\ (m)}$$

→ convert Θ from arcminutes to radians

$$2' \cdot \frac{1\ deg}{60'} \cdot \frac{\pi\ rad}{180\ deg} = 5.81 \times 10^{-4}\ rad$$

$$\rightarrow D = \frac{1.22\lambda}{\Theta} = \frac{1.22 \cdot 0.21\,m}{5.81 \times 10^{-4}\ rad} = \boxed{441\ m}$$

Question 6



$P_{\odot} = 25$ days
$= 2.16 \times 10^6$ s

$R_{\odot} = 700\,000$ km
$= 7 \times 10^8$ m

$$v_{max} = \frac{d}{t} = \frac{2\pi R_{\odot}}{P_{\odot}} = \frac{2\pi \cdot 7\times10^8 \text{m}}{2.16\times10^6 \text{s}} = 2035 \text{ m/s}$$

$$\frac{\Delta\lambda}{\lambda_{emit}} = \frac{v_{max}}{c} = \frac{2035 \text{ m/s}}{3\times10^8 \text{ m/s}} = \boxed{6.78\times10^{-6}}$$