Extra Inequality References

There are several online references that are quite handy for manipulating inequalities.

One of the more straightforward examples can be found here:

http://www.mathcentre.ac.uk/search/?q=inequalities

Particularly handy is the following link for dealing with squares and absolute values:

http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-inequalities-2009-1.pdf

None of this helps cleanly explain our fraction-type problems, so let's review the role of inequalities with the function $1/x$.

Inequalities in Reciprocals

eg. #1

Say we are given the inequality:

$$\frac{1}{x} \geq \frac{3}{2}$$

If we attempt to solve for x, we get rid of $1/x$, by effectively multiplying both sides by x.

This produces different results, depending on the sign of x.

If $x > 0$ then:

$$2 \geq 3x \Rightarrow \frac{2}{3} \geq x \Leftrightarrow x \leq \frac{2}{3}$$

But, we've already said that for this case, $x > 0$, so we get:

$$0 < x \leq \frac{2}{3}$$

On the other hand, if $x < 0$,

$$\frac{1}{x} \geq \frac{3}{2} \Rightarrow \frac{2}{3x} \geq 1 \Rightarrow \frac{2}{3} \leq x$$

since we are multiplying by a negative, the inequality order has swapped.
But a number cannot both be negative and bigger than 2/3, so this case is impossible.

We're left with the first case as our only usable result:

$$0 < x \leq \frac{2}{3}$$

Using interval notation, we can also write:

$$x \in (0, 2/3)$$

eg. #2

What if our inequality was reversed?

$$\frac{1}{x} \leq \frac{3}{2}$$

Again, we'd have two cases:

If $x > 0$,

$$\frac{1}{x} \leq \frac{3}{2} \Rightarrow \frac{2}{3} \leq x \Rightarrow x \geq \frac{2}{3}$$

and since this means $x > 0$, and $x \geq 2/3$, we can just say:

$$x \geq \frac{2}{3}$$

Conversely, if $x < 0$, then

$$\frac{1}{x} \leq \frac{3}{2} \Rightarrow 2 \geq 3x \Rightarrow \frac{2}{3} \geq x \Rightarrow x \leq \frac{2}{3}$$

Notice, then this means that both $x < 0$, and $x \leq 2/3$, so we can just say:

$$x \leq 0$$

So our result is that:

$$\frac{1}{x} \le \frac{3}{2}$$

means that

$$x < 0, \quad or \quad x \ge \frac{2}{3}$$

And, as in the other example, we can write this using interval notation as:

$$x \in (-\infty, 0) \cup [2/3, \infty)$$