
Binomial distribution (Section 10)

1. (a) A shipment of 2,000 containers has arrived at the port of Vancouver. As part of the customs inspection, a container is selected at random and checked for contraband (say, illegal drugs). Then, of the remaining 1,999 containers, another container is selected at random and checked for illegal drugs. This routine is repeated 25 times. Assume that all containers have the same probability of carrying illegal drugs, and let X = “number of containers that contain illegal drugs.” Is X a binomial variable? Why or why not?

(b) If your answer to (a) is NO, suggest a modification that to the routine that will make X a binomial variable. Comment on whether or not it makes sense in reality.

2. Suppose that the probability of a success is 0.3. Make a complete list of ways in which we can obtain exactly two successes in five trials. Based on your list, find the probability of obtaining exactly two successes in five independent trials. Compare with the formula for the binomial distribution.

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3. Find each quantity and explain what it represents.

(a) $\binom{11}{3}$

(b) $C(12, 4)$

(c) $b(2, 5; 0.6)$

(d) $b(1, 7; 0.8)$

4. Express each quantity in the form of a binomial coefficient and find its value.

(a) The number of ways we can obtain six heads in seven tosses of a coin.

(b) The number of ways of selecting a team of 11 players from a group of 14 available players.

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5. A random variable X is distributed binomially with probability of success $p = 0.6$. Using the notation $b(k, n; p)$ for the probabilities associated with the binomial distribution, say what you would need to calculate to answer each question.

(a) Find the probability of at least two successes in five trials.

(b) Find the probability of at most three successes in eight trials.

(c) Find the probability that there are more than 2 successes in 50 trials.

(d) Find the probability that there are between 3 and 7 successes in 23 trials.

6. The random variable X is binomially distributed with parameters $n = 3$ (number of trials) and $p = 0.4$ (probability of success).

(a) Find the probability distribution of X .

(b) Draw a histogram of X .

(c) Find the mean and the variance of X using the probabilities you found in (a).

(d) Find the mean and the variance using formulas (10.4) and (10.5) and compare with your answers to (c).

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7. About 10% of Canadians have latent TB (tuberculosis) infection (i.e., they have been infected, are not infectious, but can develop tuberculosis at some point in their lives). [Source: Canadian Institute for Health Information, Canadian Lung Association, Health Canada Statistics Canada, *Respiratory Disease in Canada*, September 2001. Available at www.phac-aspc.gc.ca/publicat/rdc-mrc01/.]

(a) What is the expected number of people with latent TB infection in Winnipeg, Manitoba (population 633 thousand)? Define the binomial variable involved, say what constitutes a success, and state the probability of success.

(b) What is the probability that in a randomly chosen sample of ten people in Winnipeg nobody has a latent TB infection?

8. It has been determined that 15% of all tomato plants in a greenhouse have been infested with hornworms. You pick ten plants at random. What is the probability that none of them have been infested with hornworms?

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9. Various surveys have found that about 95% of claims that certain products are “green” (or “eco-friendly” or “organic”) are either misleading or not true at all.

(a) What is the expected number of truly “green” products out of 1,000 products that are claimed to be “green?”

(b) You buy five products that are claimed to be “green.” What is the probability that none of them are truly “green”? What is the probability that all of them are truly “green”?