McMaster University
Math 1LT3E

ASSIGNMENT 6

Sections 6, 7, 9, and 10 in the Red Module

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1. Suppose that $z = F(x, g(x, y), h(x, y))$. Sketch a tree diagram and find formulas for $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

2. Wheat production W in a given year depends on the average temperature T and the annual rainfall R. Scientists estimate that the average temperature is rising at a rate of 0.15^oC/year and rainfall is decreasing at a rate of 0.1 cm/year. They also estimate that, at current production levels, $\frac{\partial W}{\partial T} = -2$ and $\frac{\partial W}{\partial R} = 8$.

(a) What is the significance of the signs of these partial derivatives?

(b) Estimate the current rate of change of wheat production, $\frac{dW}{dt}$.

3. Use the chain rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

(a) $z = y^2 e^{-x}$, $x = 2s - 5t$, $y = -s - 4t$

(b) $z = \dfrac{ab - 1}{b^2 + 1}$, $a = 3s$, $b = st$

4. Find all second-order partial derivatives of $f(x, y) = \dfrac{xy}{x^2 + 1}$.

5. (a) Compute the quadratic approximation of the function $f(x, y) = x^2 \arctan(y)$ at (1,0).

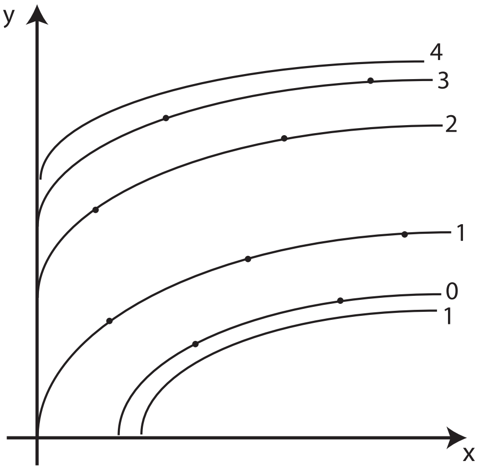
(b) Use your formula in part (a) to approximate the value of the function at (1.05, 0.05) and compare this to the actual value of $f(1.05, 0.05)$.

6. (a) Find the directional derivative of the function $f(x, y) = x \ln y^2 + \frac{x}{y}$ at the point (2,1) in the direction of the vector $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j}$.

(b) What does this number tell us about the function f at the point (2,1)?

(c) Is it possible that (in some direction other than that specified by the vector $\mathbf{v}$ in part (a)) the directional derivative of f at (2,1) is equal to 3? Explain.

7. On the contour diagram for $f(x, y)$ below, draw gradient vectors at the indicated points.



8. Find the maximum rate of change of the function $f(x, y) = 2ye^x + e^{-x}$ at the point (0, 0) and the direction in which it occurs.

9. Draw a contour diagram of a function that has a minimum at (-1,0) and a saddle point at (1,1).

10. Reason geometrically (i.e., without the second derivatives test) to show that the function $f(x, y) = y^3 - 4x^2y$ has a saddle point at $(0, 0)$.

11. Find the local minimum and maximum values and saddle points (if any) of each function.

(a) $f(x, y) = x^3 - 2y^2 + 3xy + 4$

(b) $f(x, y) = xye^{-x-y}$

THE END