Problem Set: Hypothesis Testing in Practice

These problems will reinforce sampling distributions, hypothesis testing, and p-values using normal and uniform distributions in `scipy.stats`.

1 Does a New Teaching Method Impact Exam Scores?

A professor claims that students' exam scores follow a normal distribution with a mean of 75 and a standard deviation of 10. However, an experimental teaching method is introduced, and a new class is tested. The observed sample mean is slightly lower than 75.

Task: Follow these steps to determine whether the new teaching method significantly lowers exam scores:

1. Use the provided dataset of 30 exam scores from students under the new teaching method:

```
68, 72, 75, 70, 71, 73, 74, 76, 69, 72,
74, 70, 73, 71, 75, 74, 72, 73, 70, 71,
73, 75, 72, 74, 76, 71, 73, 72, 70, 74
```

2. Compute the observed mean from the dataset.
3. Generate a large number of samples (e.g., 10,000) from a normal distribution with $\mu = 75, \sigma = 10$ to create the null distribution, sampling one instance at a time.
4. Store the mean of each sample to build the null distribution.
5. Convert the null distribution to a cumulative distribution function (CDF).
6. Compute the p-value as the probability of obtaining the observed mean or lower under the null distribution.
7. Compare the p-value to $\alpha = 0.05$ to determine whether the new method significantly lowers scores.

Hypotheses:

- H_0: The mean exam score is at least 75 ($\mu \geq 75$).
- H_A: The mean exam score is lower than 75 ($\mu < 75$).

Hint: Since this is a one-tailed test, the p-value is calculated as the proportion of the null distribution that is less than or equal to the observed mean.

Additional Challenge: Modify the sampling process to iteratively store sample means in a loop rather than vectorized operations, ensuring students understand the iterative nature of hypothesis testing simulations.

2 Does a New Training Program Improve Pass Rates?

A company claims that the pass rate for its employee certification test is 60%. A new training program is introduced, and a sample of employees undergoes the new training and takes the test. The observed pass rate is different from 60%.

Task: Follow these steps to determine whether the new training program significantly changes pass rates:

1. Use the provided dataset of 100 employees who took the test after the new training program. Each employee either passed (1) or failed (0):

```
1, 1, 0, 1, 0, 1, 1, 0, 1, 1, 1, 0, 1, 1, 1, 1, 0, 0, 1, 1,
0, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 0, 1, 0, 1, 1,
1, 1, 1, 0, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1,
1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 0, 1, 1,
1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0
```

2. Compute the observed pass rate from the dataset.
3. Generate a large number of samples (e.g., 10,000) from a binomial distribution with $p = 0.60$ (the claimed pass rate) to create the null distribution.
4. Store the proportion of successes in each sample to build the null distribution.
5. Convert the null distribution to a cumulative distribution function (CDF).
6. Compute the p-value as the probability of obtaining the observed pass rate or more extreme under the null distribution.
7. Compare the p-value to $\alpha = 0.05$ to determine whether the new training program significantly impacts pass rates.

Hypotheses:

- H_0: The pass rate is 60% ($p = 0.60$).
- H_A: The pass rate is different from 60% ($p \neq 0.60$).

Hint: Since this is a two-tailed test, the p-value is calculated as the proportion of the null distribution that is as extreme or more extreme than the observed pass rate.

Additional Challenge: Modify the sampling process to iteratively store pass rates in a loop rather than vectorized operations, ensuring students understand the iterative nature of hypothesis testing simulations.