CS3400 Machine Learning Problem Set 1: Linear Algebra Review Name: Salvin

1) For each of the following, indicate if it is a vector or matrix and its dimensions:

1. $u \in \mathbb{R}^5$ vector
2. $A \in \mathbb{R}^{3\times4}$ matrix
3. A^T where $A \in \mathbb{C}^{4\times8}$ matrix
   $A^T \in \mathbb{C}^{8\times4}$

$A \in \mathbb{R}^n \rightarrow$ vectors
$A \in \mathbb{R}^{m\times n} \rightarrow$ matrix
A^T is just transpose
$\rightarrow$ m is row
n is column

2) Given the values of the entries indicated below.

$$U = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 3 & 5 \\ 7 & 11 & 13 \\ 17 & 19 & 23 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}$$

$$v = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 8 \\ 10 \end{bmatrix}$$

$U_{x,y}$ $x \rightarrow$ row
$y \rightarrow$ column
V_x $x \rightarrow$ index position
"diagonal elements" top left to bottom right

1. $U_{1,1}$ $= 1$
2. $U_{2,3}$ $= 13$
3. v_4 $= 8$
4. The diagonal elements of U. $=$ 1, 11, 23

3) Linear algebra operations require that the shapes of the matrices and/or vectors match up. For each operation below, indicate if it is valid. If it is valid, give the dimensions of the resulting object. Note that $N \times 1$ and $1 \times N$ are used to indicate column and row vectors, respectively.

1. $u \cdot v$ where $u, v \in \mathbb{R}^{5\times1}$
2. uv $(5\times1)\cdot(5\times1)$ not allowed
3. $u^T v$ $(1\times5)\cdot(5\times1)$ allowed
4. uv^T $(5\times1)\cdot(1\times5)$ allowed
5. $u + v$ $(5\times1)+(5\times1)$ allowed
6. UV where $U \in \mathbb{R}^{5\times6}, V \in \mathbb{R}^{6\times7}$
7. $U^T V$ $(6\times5)\cdot(6\times7)$ not allowed
8. UV^T $(5\times6)\cdot(7\times6)$ not allowed

dot product rule: column of first vector = rows of second vector
$\rightarrow$ produces a scalar result

cross product rule: only works on 3D vectors
$\rightarrow$ can't take cross product of different dimensions

o to add two matrix, must be same shape

4) Perform the following linear algebra operations and write the result.

$$u \cdot v = \begin{bmatrix}1\\3\\5\end{bmatrix} \cdot \begin{bmatrix}7\\11\\13\end{bmatrix} = (1)(7)+(3)(11)+(5)(13) = 105$$

$$u = \begin{bmatrix}1\\3\\5\end{bmatrix} \qquad v = \begin{bmatrix}7\\11\\13\end{bmatrix}$$

$$u \times v^T = \begin{bmatrix}1\\3\\5\end{bmatrix} \times \begin{bmatrix}7 & 11 & 13\end{bmatrix} = \begin{bmatrix}7 & 11 & 13\\21 & 33 & 39\\35 & 55 & 65\end{bmatrix}$$

1. $u \cdot v$ = 105
2. uv^T $\begin{bmatrix}7 & 11 & 13\\21 & 33 & 39\\35 & 55 & 65\end{bmatrix}$
3. $u^T v$ = 105

$$u^T \times v = \begin{bmatrix}1 & 3 & 5\end{bmatrix} \times \begin{bmatrix}7\\11\\13\end{bmatrix} = 1(7)+3(11)+5(13) = 105$$

4. UV where $U = \begin{bmatrix}1 & 3 & 5\\7 & 11 & 13\\17 & 19 & 23\end{bmatrix}$ and $V = \begin{bmatrix}2 & 4 & 6\\8 & 10 & 12\\14 & 16 & 18\end{bmatrix}$ $UV = \begin{bmatrix}96 & 114 & 132\\284 & 346 & 408\\508 & 626 & 744\end{bmatrix}$

5) Give the vectors β and x that make the following equations equivalent.

$$\beta = \begin{bmatrix}\beta_0\\\beta_1\\\beta_2\\\beta_3\end{bmatrix} \qquad x = \begin{bmatrix}1\\x_1\\x_2\\x_3\end{bmatrix}$$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$$

$$y = x^T \beta$$

$$x^T\beta = \begin{bmatrix}1, x_1, x_2, x_3\end{bmatrix} \begin{bmatrix}\beta_0\\\beta_1\\\beta_2\\\beta_3\end{bmatrix} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$$

6) Norms and distance.

1. Write the squared norm $\|v\|^2$ of a vector v in terms of a dot product. $\|v\|^2 = v \cdot v = v^T \cdot v$
2. Convert the equation for the Euclidean distance between two vectors u and v into vector notation using vector arithmetic and norms.

$$d(u, v) = \sqrt{(u_1 - v_1)^2 + (u_2 - v_2)^2 + \cdots + (u_n - v_n)^2}$$

if $w = u - v$, then $w_i = u_i - v_i$

$$\sum_{i=1}^{n} (u_i - v_i)^2 = \|u - v\|^2 \qquad d(u,v) = \|u - v\|$$

$$d(u,v) = \sqrt{\|u-v\|^2}$$

4. $1(2)+3(8)+5(14) = 96$ $\quad 7(2)+11(8)+13(14) = 284$ $\quad 17(2)+19(8)+23(14) = 508$
$1(4)+3(10)+5(16) = 114$ $\quad 7(4)+11(10)+13(16) = 346$ $\quad 17(4)+19(10)+23(16) = 626$
$1(6)+3(12)+5(18) = 132$ $\quad 7(6)+11(12)+13(18) = 408$ $\quad 17(6)+19(12)+23(18) = 744$