(a) Consider the log link model with interaction term

$$\mathcal{M}_{12}: \quad \log\left(\frac{\mu_i}{t_i}\right) = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i1} x_{i2}.$$

Calculate the maximum likelihood estimate for the expected value μ_{i_*} when $x_{i_*1} = 4$, $x_{i_*2} = 0.75$, and $t_{i_*} = 64070$.

(b) Calculate the maximum likelihood prediction for the ratio

$$\frac{Y_f}{t_f}$$

when $x_{f1} = 4$, $x_{f2} = 0.75$. Also, create suitable prediction intervals for the ratio $\frac{Y_f}{t_f}$.

(c) Assume that $\mathrm{Var}(Y_i) = \phi\mu_i$. Test at 5% significance level, is the explanatory variable X_2=`doserate` statistically significant variable in the model

$$\mathcal{M}_{12}: \quad \log\left(\frac{\mu_i}{t_i}\right) = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i1} x_{i2},$$

Calculate the value of the test statistic.

(d) Consider the model

$$\mathcal{M}_{12}: \quad \log\left(\frac{\mu_i}{t_i}\right) = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i1} x_{i2}.$$

Under which distribution, the model $\mathcal{M}_{12}$ fits best on data in your opinion?

i. Y_i follows Poisson distribution with the variance $\mathrm{Var}(Y_i) = \mu_i$,

ii. Y_i follows quasi-Poisson distribution with the variance $\mathrm{Var}(Y_i) = \phi\mu_i$,

iii. Y_i follows negative binomial distribution $Y_i \sim NegBin(\mu_i, \theta)$.