1. In research, it was investigated how the tensile strength of a paper depends on the percentage of the hardwood portion in raw material mixture X_1=Hardwood and (mechanical) scrubbing pressure X_2=pressure during the manufacturing process of paper. Below is a part of the material available in the study. The entire material can be found in dataset paper.txt.

	strength	hardwood	pressure
1	196.6	2	400
2	197.7	2	500
3	199.8	2	650
4	198.4	2	400
.			
35	197.8	8	500
36	199.8	8	650

Denote explanatory variables as X_1=hardwood and X_2=pressure. Consider modelling the response variable Y=strength by following two different models:

$$\begin{aligned} \mathcal{M}_1 : &\quad Y_i = \beta_0 + \beta_1 x_{i1} + \varepsilon_i, \\ \mathcal{M}_{1|2} : &\quad Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \varepsilon_i, \end{aligned}$$

where in each model the random error term ε_i is assumed to follow normal distribution $\varepsilon_i \sim N(0, \sigma^2)$.

(a) Under the model $\mathcal{M}_{1|2}$, calculate the maximum likelihood estimate for the parameter β_1.

(b) Under the model $\mathcal{M}_{1|2}$, find the restricted maximum likelihood estimate, i.e., an unbiased estimate $\tilde{\sigma}^2$ for the the variance parameter σ^2.

(c) Under the model $\mathcal{M}_{1|2}$, calculate the fitted value $\hat{\mu}_{36}$ for the last observation $i = 36$ in the data set.

(d) Under the model $\mathcal{M}_{1|2}$, calculate maximum likelihood estimate for the expected value μ_{i_*}, when $\mathrm{x}_{i_*1} = 8$ and $x_{i_*2} = 550$.

(e) Under the model $\mathcal{M}_{1|2}$, calculate the 80% prediction interval for the new observation Y_{i_*}, when $x_{i_*1} = 8$ and $x_{i_*2} = 550$. Particularly, what is your estimate for lowerbound of the prediction interval?