Dose Herbicide DryMatter
1 1 Glyphosate 4.7

2 1 Glyphosate 4.6

3 1 Glyphosate 4.1

4 1 Glyphosate 4.4

Data are from an experiment, comparing the potency of the two herbicides glyphosate and bentazone in white mustard Sinapis alba.

Dose - a numeric vector containing the dose in g/ha.

Herbicide - a factor with levels Bentazone Glyphosate (the two herbicides applied).

DryMatter - a numeric vector containing the response (dry matter in g/pot).

Christensen, M. G. and Teicher, H. B., and Streibig, J. C. (2003)

Linking fluorescence induction curve and biomass in herbicide screening, Pest Management Science, 59, 1303?1310.

Denote the variables as Y = DryMatter, $X_1 = Dose$, and $X_2 = Herbicide$.

(a) Let us assume $Y_i \sim Gamma(\mu_i, \phi)$. Consider modeling the expected value μ_i of the response variable Y = DryMatter by the following model

$$\mathfrak{M}: \quad \frac{1}{\mu_i} = \beta_0 + \beta_1 x_{i1} + \alpha_j + \gamma_j x_{i1},$$

where index j is related to the categories of X_2 . Calculate the maximum likelihood estimate for the expected value μ_{i_*} when $X_1=50$ and $X_2=$ Glyphosate.

(b) Let us continue to assume $Y_i \sim Gamma(\mu_i, \phi)$, and let us continue to model the expected value μ_i by the model

$$\mathcal{M}: \quad \frac{1}{\mu_i} = \beta_0 + \beta_1 x_{i1} + \alpha_j + \gamma_j x_{i1},$$

Create 80 % prediction interval for new observation y_f , when $X_1 = 50$ and $X_2 = \text{Glyphosate}$. Particularly, what is your estimate for lowerbound of the prediction interval?

(c) Let us assume $Y_i \sim IG(\mu_i, \phi)$. Consider modeling the expected value μ_i of the response variable Y = DryMatter by the following model

$$\mathcal{M}: \quad \frac{1}{\mu_i^2} = \beta_0 + \beta_1 x_{i1} + \alpha_j + \gamma_j x_{i1},$$

where index j is related to the categories of X_2 . Calculate the 95% confidence interval estimate for the expected value μ_{i_*} when $X_1 = 50$ and $X_2 = \text{Glyphosate}$. Particularly, what is your estimate for lowerbound of the confidence interval?