Shiny Application Assignment: Bivariate Poisson PDF

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1. Overview

The bivariate Poisson models have emerged to address a wide range of applications in various fields where paired count data are correlated. Leiter and Hamadan (1973) suggested bivariate probability models applicable to traffic accidents and fatalities. The bivariate Poisson distribution has been developed following various assumptions. Among those, the most comprehensive one has been proposed by Kocherlakota and Kocherlakota (1992). Islam and Chowdhury (2015) proposed bivariate Poisson model for covariate dependence based on the extended generalized linear model.

In practice, we may not observe the full range of outcomes as the theoretical range of outcomes has no upper limits. This right truncation may occur for both the variables due to restrictions in the definition of the outcome variables. Therefore, the total probability may not sum to one.

A shiny application is developed to compute the probability of both events based on different ranges of two outcomes and corresponding parameter values. This shiny application also, produce plot of bivariate Poisson PDF.

2. The Bivariate Poisson-Poisson Model

Consider, Y_1 be the number of occurrences of the first event in a given interval follow Poisson distribution with parameter λ_1 and the probability of the second event, Y_2, for given Y_1, where $Y_2 = Y_{21} + ... + Y_{2y_1}$, be Poisson with parameter, $\lambda_1 y_1$. The joint pdf of Y_1 and Y_2 is shown as

$$g(y_1, y_2) = \frac{e^{-\lambda_1}\lambda_1^{y_1} e^{-\lambda_2 y_1}(\lambda_2 y_1)^{y_2}}{(y_1! y_2!)}, y_1 = 0, 1, ...; y_2 = 0, 1, ...,$$

where $\lambda_1 > 0$ and $\lambda_2 > 0$.

3. The Shiny Apps

There are four sliders in the left pane to select Y_1, Y_2, λ_1 and λ_2 values. Right pane have two tabs for **Joint Probability** and **Plot**. By default **Joint Probability** tab is shown which is a table with Y_1 values in rows and Y_2 values in columns. Last row and column are the marginals. Cells in the table show the joint probability of two outcomes. Last cell in the table is the total probability which shows how much right truncation is there. Lesser the truncation, this total probability is closer to one. Pressing **Plot** tab shows the the plot of bivariate pdf. Figure 1 shows the **Joint Probability** tab of the shiny apps. There are four sliders as explained above.
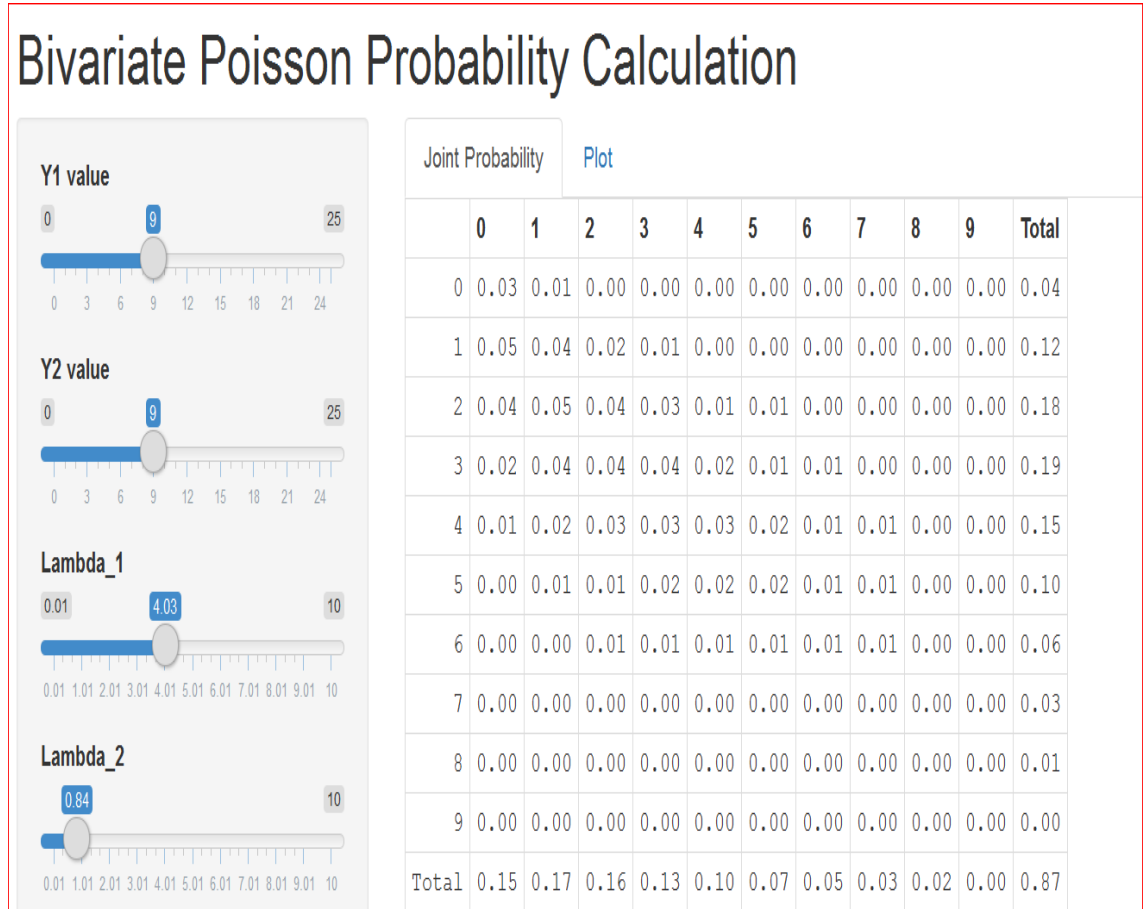

Figure 1. Joint Probability: Bivariate Poisson

Figure 2 displays the bivariate Poisson PDF. As values of Y_1, Y_2, λ_1 and λ_2 are changed the corresponding plot and table values will be chnaged.
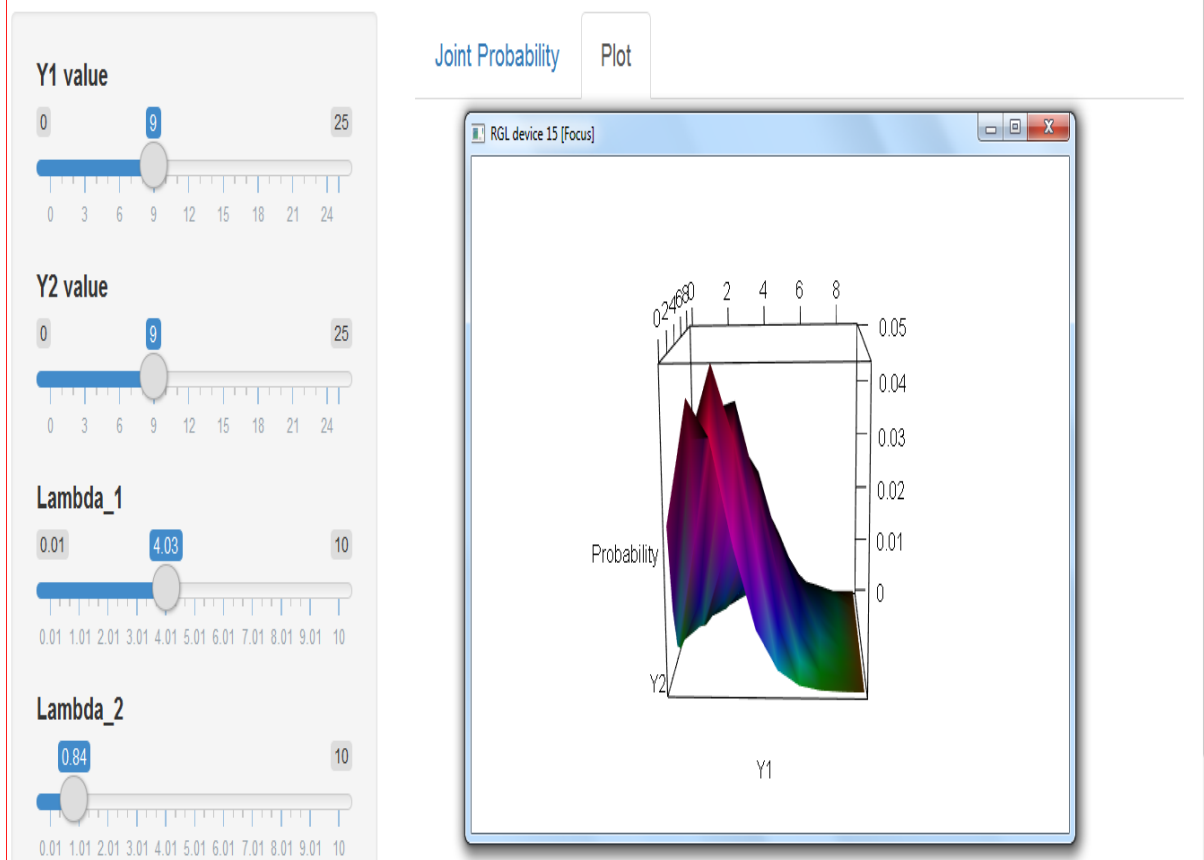
Figure 2. Joint Probability Plot: Bivariate Poisson

References

Islam, M.A. and Chowdhury, R.I. (2015). A Bivariate Poisson Model with Covariate Dependence. *Bulletin of the Calcutta Mathematical Society*, 107(1), 11-20.

Kocherlakota S, Kocherlakota K. (1992). *Bivariate Discrete Distributions.* Marcel Dekker, New York.

Leiter RE and Hamdan MA. Some Bivariate Probability Models Applicable to Traffic Accidents and Fatalities. (1973), *International Statistical Review,* 41(1): 87-100.