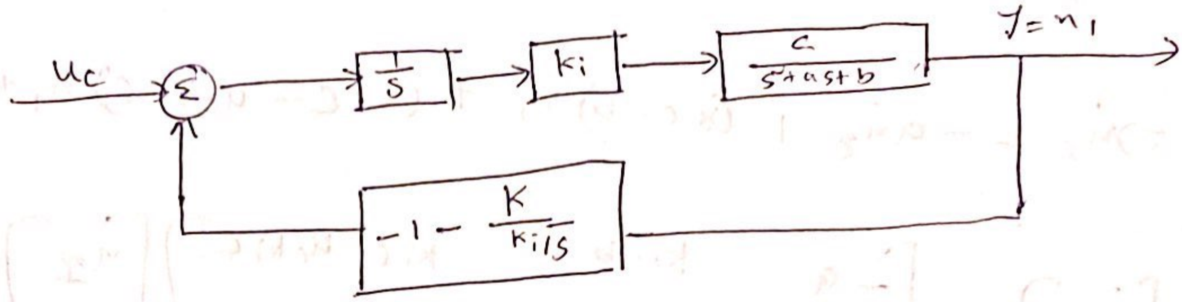
Model Reference adaptive controller for MLS system:



$$G_m = \left(\frac{k_i}{s}\right)\left(\frac{c}{s^2+as+b}\right)$$

$$G_c = \frac{\left(\frac{k_i}{s}\right)\left(\frac{c}{s^2+as+b}\right)}{1+\left(-1-\frac{k_s}{k_i}+u_c\right)\frac{k_i c}{s(s^2+as+b)}}$$

$$= \frac{k_i c}{s^3+as^2+bs-k_i c-k_s c+u_c k_i c}$$

$$\Rightarrow \frac{y}{u} = \frac{k_i c}{s^3+as^2+s(b-kc)-k_i c+u_c k_i c} \quad \text{--- (1)}$$

$x_1 = y$

$x_2 = \dot{x}_1 = \dot{y}$

$x_3 = \dot{x}_2 = \ddot{y}$

$x_4 = \dot{x}_3 = \dddot{y}$

from (1),

$$\dddot{y} + a\ddot{y} + (b-kc)\dot{y} - (k_i c - u_c k_i c)\,y = k_i c u$$

$$\Rightarrow \dot{x}_3 + a\dot{x}_2 + (b - kc)\dot{x}_1 - (k_i c - u_c k_i c)x_1 = k_i c u$$

$$\Rightarrow \dot{x}_3 = -a\dot{x}_2 + (kc - b)\dot{x}_1 + (k_i c - u_c k_i c)x_1 + k_i c u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -a & kc-b & k_i c - u_c k_i c \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} k_i c u \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{y} \\ \dot{y} \\ y \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u$$

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from system identification toolbox,

$$G_m(s) := \frac{0.009885}{s^3 + 0.5245\, s^2 + 0.1058\, s + 0.009876}$$

using matlab,

$$A_m = \begin{bmatrix} -0.5245 & -0.1058 & -0.099 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\frac{dx_3^m}{dt} = -0.5245\, x_3^m - 0.1058\, x_2^m - 0.0099\, x_1^m + u$$

$$\frac{dx_3}{dt} = -a\dot{x}_2 + (kc - b)\dot{x}_1 + (k_i c - u_c k_i c)x_1 + k_i c u$$

Comparing these eq^{n}s,

$$b - kc = 0.1058 \Rightarrow k = \frac{b - 0.1058}{c} = k^0$$

$$k_i(c - u_c c) = -0.0099$$

$$\Rightarrow k_1 = \frac{0.0099}{u_c c - c} = k_1^0$$

Now, $e = x - x_m$

$$\frac{de_1}{dt} = \frac{dx_1}{dt} - \frac{dx_m}{dt} = x_2 - x_2^m = e_2$$

$$\frac{de_2}{dt} = \frac{dx_2}{dt} - \frac{dx_2^m}{dt} = e_1$$

$$\frac{de_3}{dt} = \frac{dx_3}{dt} - \frac{dx_3^m}{dt}$$

$$= -ax_3 + (kc - b)x_2 + (k_i c - u_c k_i c)x_1 + k_i c u$$

$$\cancel{=}\ -0.5245\, x_3^m - 0.1058\, x_2^m - 0.0099\, x_1^m + u$$

$$A_m = \begin{bmatrix} 0.5245 & 0.1058 & 0.0099 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\theta = \begin{bmatrix} k \\ k_i \\ 0 \end{bmatrix}, \quad \theta^0 = \begin{bmatrix} k^0 \\ k_i^0 \\ 0 \end{bmatrix} \quad \varphi = \begin{bmatrix} -cx_1 & cx_2 & cx_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Now, Lyapunov function;

$$V = \gamma e^T P e + (\theta - \theta^0)^T (\theta - \theta^0)$$

where, $A_m^T P + P A_m = -Q$

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad P = \begin{bmatrix} P_1 & P_{12} & P_{13} \\ P_{12} & P_2 & P_{23} \\ P_{13} & P_{23} & P_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0.5245 & 1 & 0 \\ 0.1058 & 0 & 1 \\ 0.0099 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_1 & P_{12} & P_{13} \\ P_{12} & P_2 & P_{23} \\ P_{13} & P_{23} & P_3 \end{bmatrix} + \begin{bmatrix} P_1 & P_{12} & P_{13} \\ P_{12} & P_2 & P_{23} \\ P_{13} & P_{23} & P_3 \end{bmatrix} \begin{bmatrix} 0.525 & 0.105 & 0.009 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= -Q$$

Solving this by MATLAB,

$P_1 = -2.9238$ $\qquad P_{12} = 1.0336$

$P_2 = -0.6514$ $\qquad P_{23} = -0.6093$

$P_3 = -0.0147$ $\qquad P_{31} = 0.4187$

If $\dfrac{d\theta}{dt} = -\gamma \varphi^T P e$

Then, $\dfrac{dV}{dt} = \dfrac{-\gamma}{2} e^T Q e$

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$$\varphi^T P e = \begin{bmatrix} -bx_1 & 0 & 0 \\ bx_2 & 0 & 0 \\ bx_3 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_1 & P_{12} & P_{13} \\ P_{12} & P_2 & P_{23} \\ P_{13} & P_{23} & P_3 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

$$= \begin{bmatrix} -cx_1 (P_1 e_1 + P_{12} e_2 + P_{13} e_3) \\ cx_2 (P_{12} e_1 + P_2 e_2 + P_{23} e_3) \\ cx_3 (P_{13} e_1 + P_{23} e_2 + P_3 e_3) \end{bmatrix}$$

$$\therefore \frac{dK}{dt} = c\nu (P_1 e_1 + P_{12} e_2 + P_{13} e_3) x_1$$

$$\frac{dk_i}{dt} = -\nu c (P_{12} e_1 + P_2 e_2 + P_{23} e_3) x_2$$