Three Degree of Freedom Two-Bar Vibrational System

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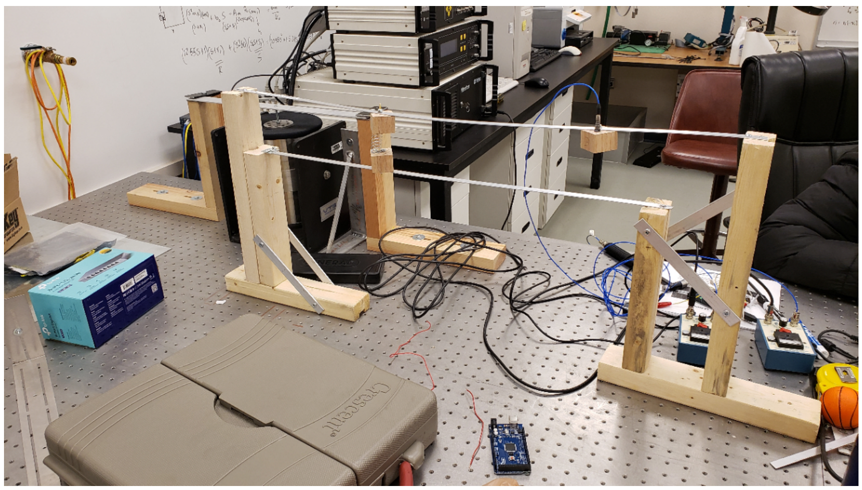

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1 System Description

1.1 Project Objectives

The objective of this project is to create a complex vibrational system to analyze using excitation with an impact hammer or a shaker table. Specific to this group, the system is to include at least one fixed-fixed beam.

The proposed system is to be mathematically modeled, and examined for expected behavior. The equations of motions are to be made and examined so that the proper count and variation of shape of the modes can be estimated.

Once the system has been built, it should be tested using equipment such as an impact hammer or shaker table for controlled input into the system, and sensors such as an accelerometer or laser vibrometer to measure the outputs of the system when excited.

If an impact hammer is used, each output should be separately monitored reacting to input at each measurement point. Due to the system being approximately linear, each output will follow a sum of sinusoids representing each mode of vibration in the system. The results can be processed by frequency to yield the natural frequencies and mode shapes of the system.

If a shaker table is used, the system must be oriented to have some component of the base excitation parallel with the direction of motion of the beams. The base excitation can then be slowly changed in frequency. The system will exhibit resonance as a natural frequency is passed, and the mode shapes can then be observed by the output at that frequency.

The mode shapes and natural frequencies can then be used to construct an approximate state-space model of the system using Fourier analysis and control theory.

1.2 Materials

To build the system described in this paper, certain tools and materials are required. These include the following:

Tools:

- Circular Saw (For cutting wood for support and masses)
- Metal saw or large clippers (For cutting beams to desired size)
- Power drill
- Drill bits, for drilling wood and metal
- Driver bits, large and fine Phillips head (heads may vary depending on screws used)
- A small wrench

Building materials:

- Wood beams, 2"x4", 8 ft or more. Scrap wood is fine.
- Small wood or metal pieces to reinforce the supports, to prevent propagation of motion out of the beams.
- Thin metal beams. Should be easily flexible by hand. No longer than 4 ft.
- Heavy wood (e.g. oak) or metal cubes, 1.5-2.5 inches wide, for masses
- Small, flexible coil springs, one for each pair of linked beams. They should be easily compressed and stretched by hand.
- *(Optional)* Small cabinet hinges. These can be fastened on the ends of the beams in lieu of simple cantilever joints for a lower frequency vibration.
- Small plastic or metal strips to hold springs to masses.

Fasteners:

- 10-32*1/2" Metal screws, one per hinge.
- 10-32*(2 or 3)" Metal screws, long enough to reach through mass cubes and beams.
- 10-32 lock nuts, one per screw.
- 10-32 washers, two per screw.
- 3" Wood screws, approx. 25
- 1/2" wood screws, four per spring, and two per hinge if not supplied with hinges

Devices:

- Impact hammer and/or shaker table
- Accelerometer and/or laser vibrometer
- Signal conditioners
- Arduino microcontroller or oscilloscope
- 18-gauge wire
- Signal wire with alligator clips on one end
- A computer to receive the data

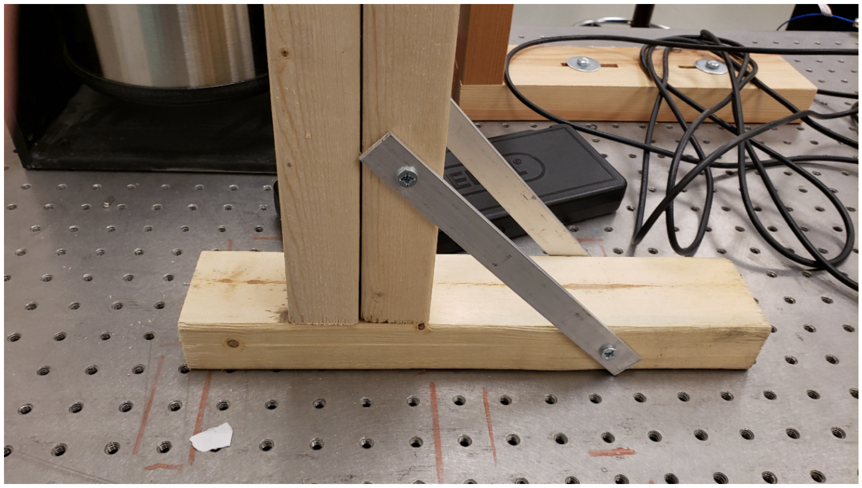

Fig. 1: Supports

1.3 Construction of system

Cut the wooden beams into two base pieces, and two support pieces for each beam being mounted. Make sure that the heights of the beam supports are sufficiently close to allow the springs to attach between the beams without being significantly stretched or compressed. The base must also be large enough and have enough space available to be held or fastened down while testing.

Use the drill and wood screws to build the supports into the desired shape to hold the beams. The beams supports should be arranged such that the beams will be placed in descending order of length from top to bottom, so that the shorter beams' supports will not interfere with longer beams. Attach small metal or wood pieces to hold the supports rigid. If any supports are butted together, screws should fasten them together.

Cut the metal beams to size, and drill holes in each end for fastening to hinges or directly to supports. If a beam end will be attached to a hinge, drill a hole in the center of the hinge plate, if necessary. Use a short metal screw, nut, and washers to fasten the hinge to the beam.

Make note of how the beams will sit relative to each other, and pick locations for the mass blocks. Drill holes in the beams at those locations, and through each mass block. Drill two 1/2" holes in each mass where a spring will be fastened, and similarly in each small fastening strip. Attach each strip with one of its two wood screws to their mass blocks, and attach the masses to their places on the beams. Attach the beam ends and hinges to the tops of their respective supports with appropriate wood screws.

Loosen the screws holding each fastening strip so that it has room to slip a spring under it. Place the end of a spring under the strip, then screw the other end of the strip in. Tighten the strip's screws to hold the spring without slipping. Repeat for all fastening points of springs.

Set the electronics up to measure and control the inputs and outputs. For example, if using an impact hammer, accelerometer, and an Arduino, plug the Arduino into the computer, and wire the hammer and accelerometer into analog inputs and a ground. Use the Arduino IDE or MATLAB/Simulink to record and save the data received. Test the system and check for unexpected behavior.

1.4 Details and Mathematical Model

The system built by this group uses two linked beams, with three masses on the beams. The large upper beam has two masses mounted roughly at third-intervals, with one mass connected by spring to the mass on the small lower beam. Each beam segment between masses, or between a mass and a mounting point, is modeled as a spring. The beams are asymmetrical, and the smaller beam has a hinge at one end. The beams are mathematically modeled as springs. This is detailed in the Theory section.

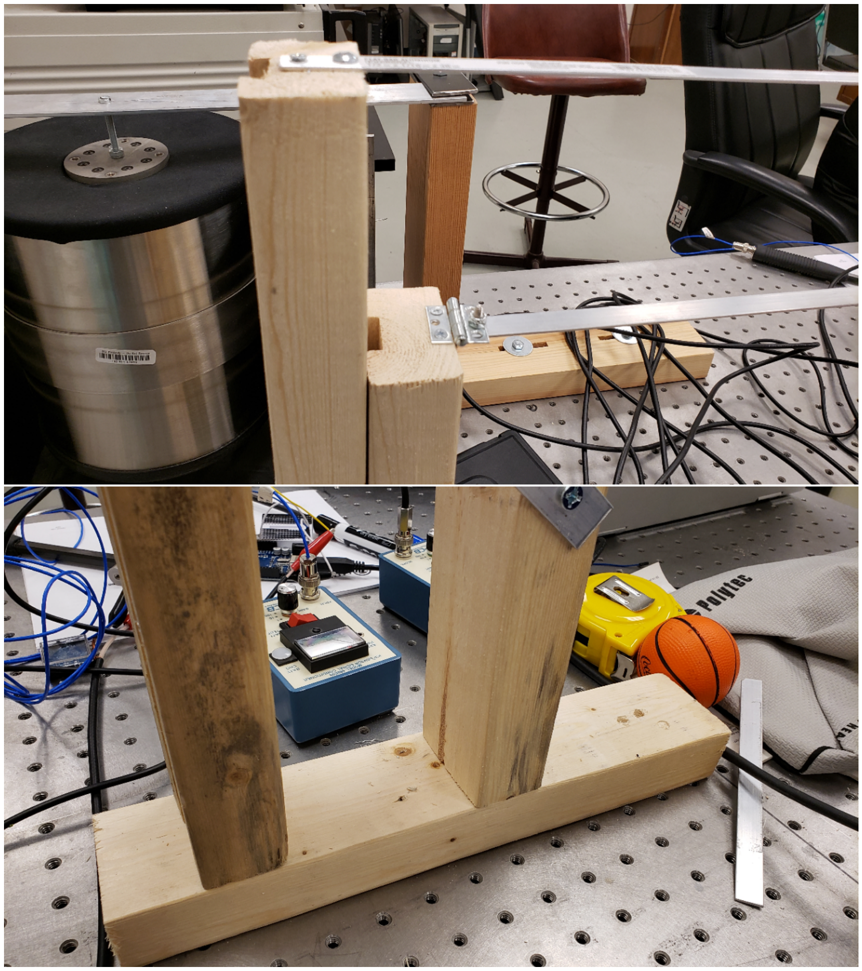

Fig. 2: Supports

Fig. 3: Mounting of beams

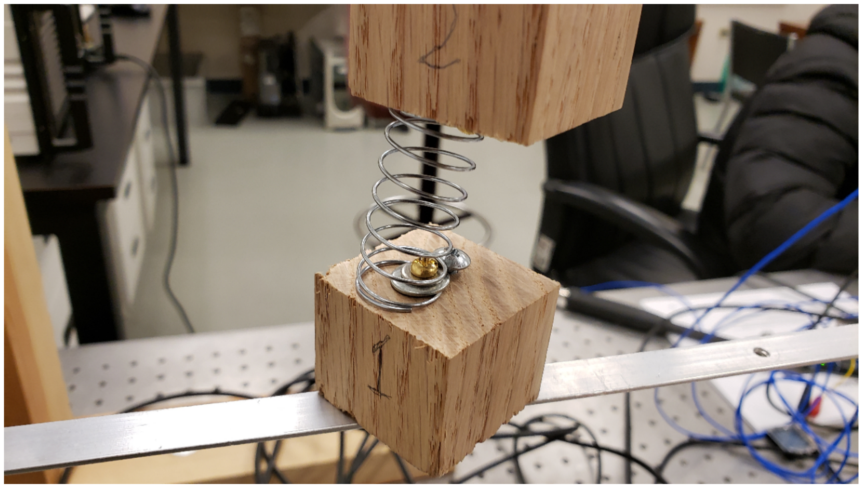

Fig. 4: Spring attachment

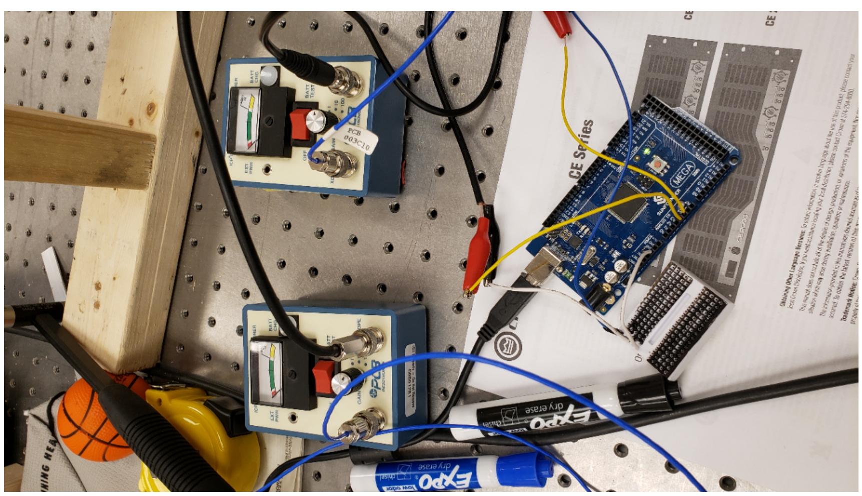

Fig. 5: Arduino Setup

2 Theory

2.1 Multi-DOF vibration systems

$$M\ddot{\boldsymbol{y}} + C\dot{\boldsymbol{y}} + K\boldsymbol{y} = \boldsymbol{f}(t) \tag{1}$$

This experiment relies on multi-degree-of-freedom vibrational analysis theory. It expands upon simple vibration theory of single isolated masses, introducing masses that are linked to each other with elastic members approximated with springs. This section covers the theory necessary to adequately understand the nature of this system and the experiment it is used for.

2.2 Beam deflection

$$\theta = \frac{TL}{JG} \tag{2}$$

When a beam bends, it exhibits an internal moment that opposes the bending of the beam. This bending moment at any cross-section of the beam is proportional to the beam's curvature, which is the beam's spacial second derivative. If treating the beam with its own mass distributed along its length, this creates a partial differential equation in space and time to solve the beam's vibrational properties.

Beams fixed at the ends must bend in the middle to reach nonzero values, as are cantilever beams. Since the bending moment also causes a shear force opposing the local displacement of the beam, which allows the beam to be modeled as a spring in a vibrating system.

2.3 Continuous vibration, Partial differential equations

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial l^2} \tag{3}$$

If a continuous vibration approach is used, the system can be modeled with a partial differential equation (PDE)known as the wave equation, an equation that involves both second-order spacial derivatives and second-order time derivatives.

Boundary conditions are given that correspond to the fixed endpoints of each beam. The initial condition is now a function across the beam's space, and its own Fourier analysis determines what modes the initial conditions hold. This initial condition must itself satisfy the boundary conditions.

Linear partial differential equations are solved using the Separation-of-Variables technique. For flat beams, the eigenfunctions found are an infinite number sines and cosines in space and time. Each eigenfunction has an eigenvalue representing have wave number and natural frequency of vibration. These eigenfunctions and their coefficients sum to the true solution of the vibration problem. Each eigenfunction also represents a mode of vibration in the beam.

2.4 Lumped mass simplification

$$\boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \tag{4}$$

However, there are a number of issues with PDEs that can complicate their use. If additional masses are added at discrete points, the solution of the equations becomes much more difficult. Numerical solutions to PDEs are much less thoroughly researched and require simplifications that may or may not be applicable. In addition, the higher-frequency modes may be undetectable with sensor equipment.

To simplify the mathematical model of this system, instead of treating the beam as a continuous mass, we lump their inertial effects in with the oak masses themselves. This reduces the number of degrees of freedom, and therefor the number of mode shapes, to the number of lumped masses in the system. This is a much easier way to treat the system, as it simplifies the measurement process considerably. It also makes numerical system identification techniques and application of control theory much more feasible. In this case, the beams and springs are assumed mass-less, and the masses perfectly rigid.

2.5 Base excitation with a shaker table

$$M\ddot{\boldsymbol{y}} + C\dot{\boldsymbol{y}} + K\boldsymbol{y} = K\boldsymbol{\phi} + C\dot{\boldsymbol{\phi}} \tag{5}$$

If the system is excited with a shaker table, the forcing function becomes the springs and damping linked to the table's movement function. The tables excitation should be at least partially in the direction designed to exhibit motion. When testing, steadily increase the frequency and observe the system's response to the excitation. As the excitation approaches a natural frequency, the system response will increase and exhibit resonance. Take note of the motion of the system, as at resonance the mode shape for this natural frequency should be evident.

2.6 Impulse with an impact hammer

$$M\ddot{\boldsymbol{y}} + C\dot{\boldsymbol{y}} + K\boldsymbol{y} = \boldsymbol{u}(t) \tag{6}$$

If the system is excited with an impact hammer, the forcing function closely resembles a simple impulse when the hammer strikes. With an accelerometer on the first mass, hit each mass with the hammer, and record the response from the accelerometer. Repeat the process with the accelerometer on each other mass. The accelerometer's responses should be sums of all of the system mode shapes, exhibiting free vibration in each of the system's natural frequencies. These waveforms allow the system to be properly identified using Fourier analysis.

2.7 System identification

$$\begin{aligned} \dot{\boldsymbol{x}} &= A\boldsymbol{x} + B\boldsymbol{u} \\ \boldsymbol{y} &= C\boldsymbol{x} + D\boldsymbol{u} \end{aligned} \tag{7}$$

The results taken from either method above identify the system using a numerical package or program such as MATLAB. A Fast Fourier Transform of the output signals received will show the natural frequencies and their magnitudes on each of the masses. Looking at the magnitudes of a specific natural frequency on all of the masses will show the mode shape for that frequency. The natural frequencies and mode shapes can then be used to build an eigenvalue-eigenvector state-matrix to construct a good estimation of the system.

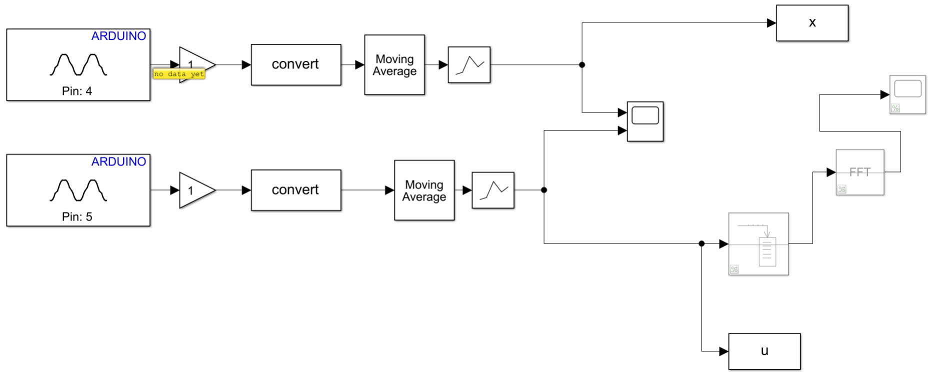

Fig. 6: Simulink diagram for taking data

```
t1=ScopeData.time;
x1=ScopeData.signals(1).values;
x1=x1(:);
u1=ScopeData.signals(2).values;
u1=u1(:);
```

Fig. 7: Raw data from the Arduino Simulink program

3 Experimental Procedure

For doing the experiment, we used accelerometer and impact hammer. We connected the sensor and impact hammer with the transducer and then connected to computer via Arduino mega board. After that, we used Simulink Arduino toolbox to get the analog input data from the accelerometer and impact hammer. We filter the signal using moving average and masking. Then we use scope to monitor the data and from scope we send the data to workspace as scope data. The Simulink diagram is given in Figure 6.

We used accelerometer in mass 3 and give input using impact hammer in three mass position. For each mass position, we give three times impact because we used system identification tool where some data need for validation. We count first impact for estimation and other two for the validation. We did this for three position. In each position, one impact is for estimation and other two is for validation. By Simulink, we send scope data to workspace. So now from scope data, we need to separate input and output. Here, accelerometer data is counted as output and signal by impact hammer is considered as input. We separate this input and output from scope data for each three cases. Extracting input and output from scope data is done by the code in Figure 7.

We did this for three different position and save the data as input U and output X. After that, we needed to combine three of the data from different impact position. But the data size is different. For that reason, we created a function which makes three data of same size. The code is given in Figure 8.

We created a matrix ‘X’ with three input X1, X2 and X3 and did same this for the output. When plot ‘X’, it shows as in Figure 9.

Then we save all of our data to a separate file so that we can use it from anywhere by loading in editor. Now the data collection portion is over. We can use this data anytime. Now, we used System identification toolbox for determining the natural frequency and mode shape. We loaded the data from saved place. We used the first impact data from each of the three place for the estimation and used other two for validation. Figure 10 is the estimation vs validation graph.

```
%iff x1>x2>x3
x2=[x2;zeros(length(x1)-length(x2),1)];
x3=[x3;zeros(length(x1)-length(x3),1)];
u2=[u2;zeros(length(u1)-length(u2),1)];
u3=[u3;zeros(length(u1)-length(u3),1)];
X=[x1,x2,x3];
U=[u1,u2,u3];
```

Fig. 8: Splitting the responses up by impacts

After that, we created frequency response function and find the frequency and mode shape with the help of system identification toolbox which will be further discussed in MATLAB publication section.

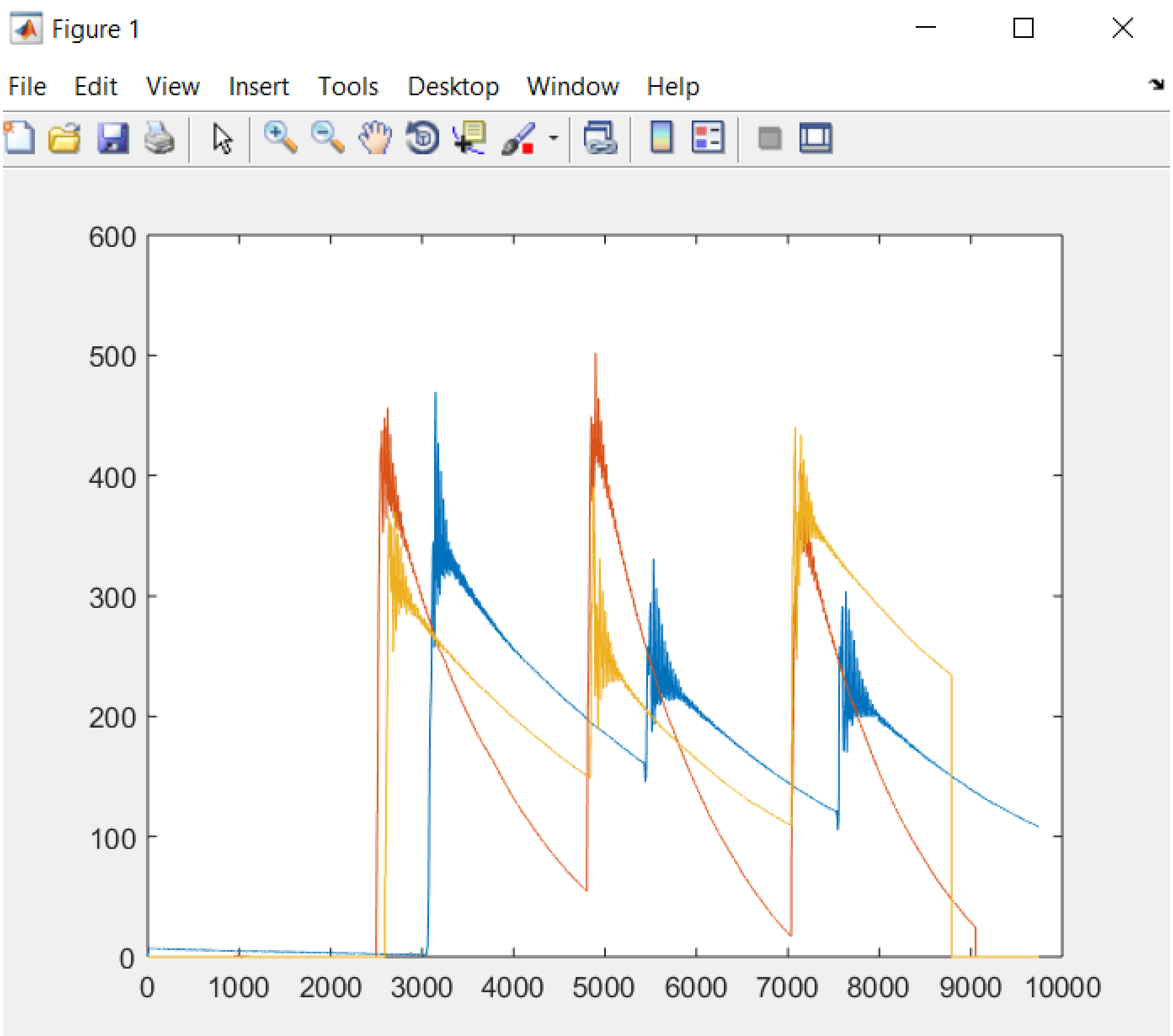

Fig. 9: All response data superimposed. Note that even with a stopwatch, the intervals are not identical and require adjustment.

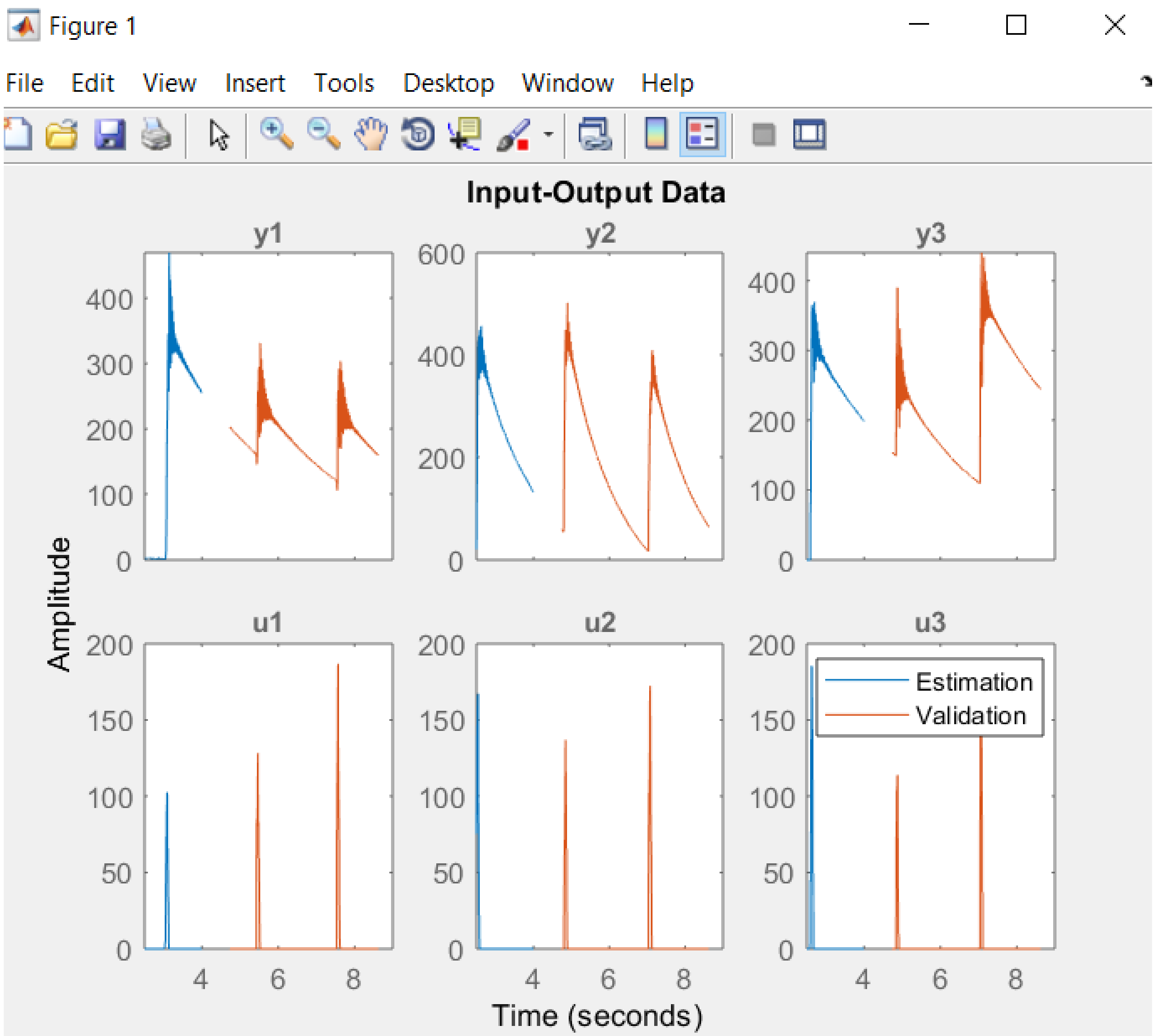

Fig. 10: Separated time data. Note the different responses from each hammer hit at different locations.

4 References

References

[1] Singiresu S. Rao. *Mechanical Vibrations*. Prentice Hall, fifth edition edition, 2011.

5 Appendix