FRA501 Class Project: 1-Dimension Jumping Robot

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December 16, 2019

We model and simulate the dynamics and control of a 1-dimension Jumping robot with a linear actuator.

The robot consists of two links; a linear actuator with mass m_a and a moving thrust rod m_r. The bottom of the rod is connected to a massless spring, with a stiffness of k and a drag coefficient b. Figure 1 shows the system diagram.

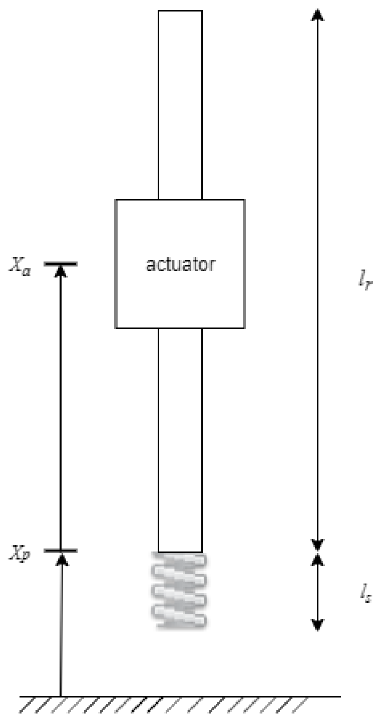

Figure 1: The jumping robot.

The joint space of the robot is defined by

$$\vec{q} = \begin{bmatrix} x_p \\ x_a \end{bmatrix} \tag{1}$$

We use Lagrangian dynamics to model the system's behaviour. Note that for convenience's sake, we will consider the spring force as an external force, as the links in motion must be rigid bodies

The Lagrangian $\mathcal{L}$ of a mechanical system is defined by a the difference between its kinetic and potential energy

$$\mathcal{L} = K - P \tag{2}$$

$$\mathcal{L} = \frac{1}{2}m_a(\dot{x_a} + \dot{x_p})^2 + \frac{1}{2}m_r\dot{x_p}^2 - m_a g(x_a + x_p) - m_r g(x_p + \frac{l_r}{2}) \tag{3}$$

The Euler-Lagrange equations are then

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\vec{q}}} - \frac{\partial L}{\partial \vec{q}} = Su \tag{4}$$

where, S is the selection matrix of the input u. For this robot, $S = \begin{bmatrix} 0 & 1 \end{bmatrix}$

The input force given by the actuator is $u = F_a$. From (4) and the Lagrangian, we have

$$\begin{bmatrix} m_a + m_r & 0 \\ 0 & m_a \end{bmatrix} \begin{bmatrix} \ddot{x_p} \\ \ddot{x_a} \end{bmatrix} + \begin{bmatrix} m_a + m_r \\ m_a \end{bmatrix} g = \begin{bmatrix} 0 \\ 1 \end{bmatrix} F_a \tag{5}$$

With no external force acting on the system, (5) represents the dynamics of the robot in mid-air. Once the spring touches the ground($x_p < l_s$)and starts to compress, we can add its external force $f_s = -(k(l_s - x_p) + b\dot{x_p})$ to the system via its transposed Jacobian $J_s^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ to get

$$\begin{bmatrix} m_a + m_r & 0 \\ 0 & m_a \end{bmatrix} \begin{bmatrix} \ddot{x_p} \\ \ddot{x_a} \end{bmatrix} + \begin{bmatrix} m_a + m_r \\ m_a \end{bmatrix} g = \begin{bmatrix} 0 \\ 1 \end{bmatrix} F_a - \begin{bmatrix} 1 \\ 0 \end{bmatrix} (k(l_s - x_p) + b\dot{x_p}) \tag{6}$$

(6) represents the dynamics of the system when the robot is on the ground. While in midair, the robot's actuation of x_a has a limit of $[0, l_r]$, at the limits, we can classify the sytem's behaviour into two constrained states. At this point, we introduce a discrete valued state variable $x_d \in \{0, 1, 3, 4, 5\} \subset \mathbb{Z}$ that encodes the contact situations as assigned in Figure 2.

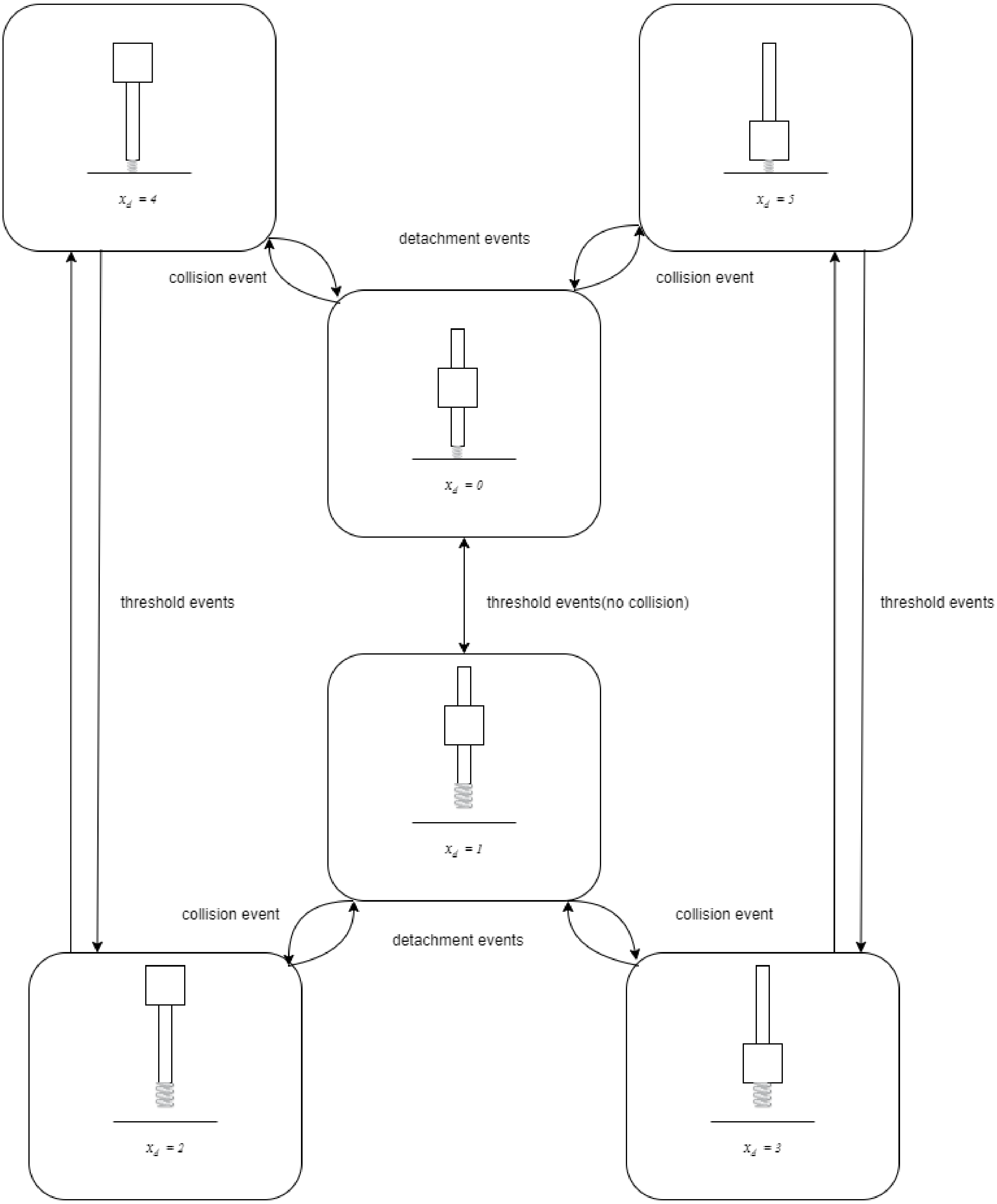

Figure 2: Transition graph of the robot.

The contact force for each contact situation is

$$sdf\,sdf \tag{7}$$

We implement the simulation and control in MATLAB, using according to the following control diagram.

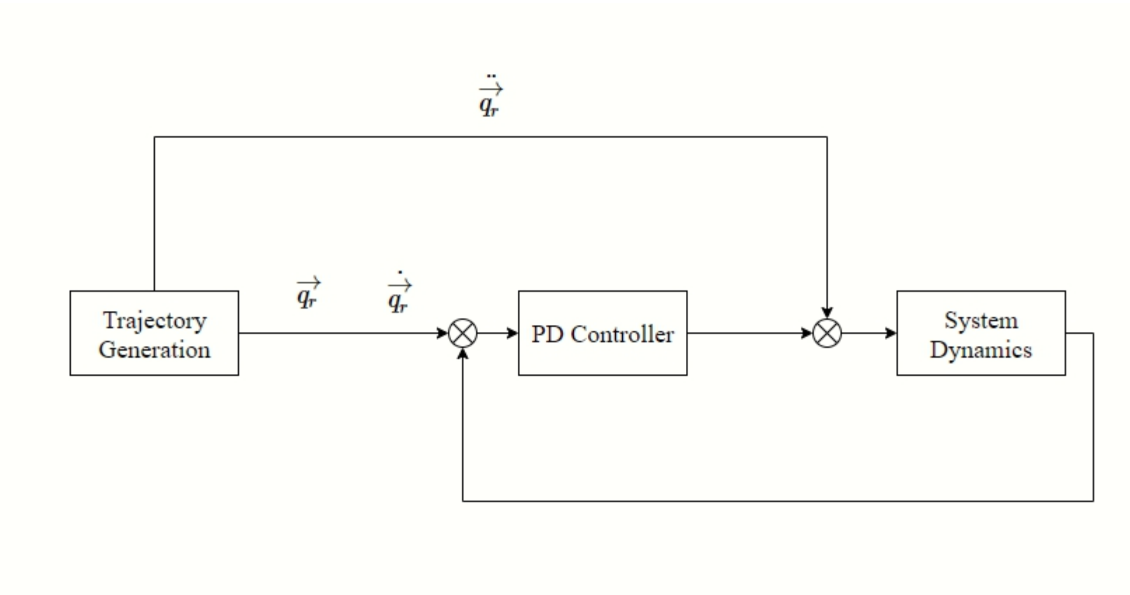

Figure 3: Control of the robot.