The spatial separation between sinusoidal trajectories and average speeds

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1 Analytic derivation

Let $\Delta\theta$ stand for the difference in trajectories between target and masker as a function of time. Let ω_i be the angular velocities and let ϕ_i be the initial phases for each source. The amplitude of the trajectories, A, is equal to 90°. Then, the long-term RMS separation between the trajectory is obtained by the following integral:

$$\Delta\theta_{\text{RMS}} = \sqrt{\lim_{T\to\infty} \frac{1}{T} \int_0^T \left[A\sin\left(\omega_1 t + \phi_1\right) - A\sin\left(\omega_2 t + \phi_2\right)\right]^2 dt} \tag{1}$$

Constant amplitude A can be factored out of the above equation. Let $\xi_1 = \omega_1 t + \phi_1$ and $\xi_2 = \omega_2 t + \phi_2$. Making the substition, above equation can be written as:

$$\Delta\theta_{\text{RMS}} = A\sqrt{\lim_{T\to\infty} \frac{1}{T} \int_0^T \left[\sin\xi_1 - \sin\xi_2\right]^2 dt} \tag{2}$$

1.1 Equal velocities, symmetric motion

When the sources have velocities with equal magnitude but opposite directions (i.e. *pi* radians out of phase), we have $\xi_1 = \xi_2 + \pi$. This means $\sin\xi_1 = -\sin\xi_2$ and Eq. (2) simplifies thusly:

$$\Delta\theta_{\text{RMS}} = 2A\sqrt{\lim_{T\to\infty} \frac{1}{T} \int_0^T \sin^2\xi dt} \tag{3}$$

In the limit as $T \to \infty$, the expression inside the square root converges to $\frac{1}{2}$. Therefore, $\Delta\theta_{\text{RMS}} = \sqrt{2}A \approx 127.3°$.

1.2 Differential velocities motion

In this case, we have $\omega_1 \neq \omega_2$ and $\phi_1 \neq \phi_2$. Evaluating the indefinite integral gives:

$$\begin{aligned}
&\int \left[\sin\xi_1 - \sin\xi_2\right]^2 dt \\
&= t - \frac{1}{2\xi_1}\sin\xi_1\cos\xi_1 - \frac{1}{2\xi_2}\sin\xi_2\cos\xi_2 \\
&+ \frac{1}{\omega_1+\omega_2}\sin\left(\xi_1+\xi_2\right) + \frac{1}{\omega_1-\omega_2}\sin\left(\xi_1-\xi_2\right) + C
\end{aligned} \tag{4}$$

In the limit as $T \to \infty$, all sinusoidal terms in Eq. (4) above vanish to 0. Therefore, the only term left inside the square root of the original Eq. (2) will be 1. This implies that the RMS spatial separation will be $A = 90°$.

2 Speeds being used

Following speeds will be considered:

Velocity conditions	f [Hz]	ω [rad/s]
Static (ST)	0	0
Very slow (VS)	0.5	π
Slow (SL)	1	2π
Fast (FS)	2	4π
Very fast (VF)	5	10π

For sense of scale, in the next table we convert the angular velocities to equivalent radial speed (s) at $r = 1$ m and show their corresponding RMS speeds ($\bar{s}$) (excluding the static condition):

Vel. cond.	s [m/s]	$\bar{s}$ [m/s]
VS	3.14	2.22
SL	6.28	4.44
FS	12.56	8.89
VF	31.52	22.21

The average human walking speed of 4 mph is approximately 1.79 m/s. Even the SL condition is about 2.5 times faster than someone walking by 3 feet away!