Assessing the Efficacy of Kaufman's Adaptive Moving Average on Trading the S&P 500 Index

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Content

Introduction

Moving Averages

Ordinary:

- Simple Moving Average (SMA)

Weighted:

- Sinus Weighted Moving Average (SWMA)
- Weighted Moving Average (WMA)
- Exponential Moving Average (EMA)
- Double Exponential Moving Average (DEMA)
- Zero-lag Moving Average (ZMA)

Adaptive:

- Kaufman’s Adaptive Moving Average (KAMA)

Simple Moving Average (SMA)

$$SMA_i = \sum_{j=i-n+1}^{i} w_j X_j \ , \ w_j = \frac{1}{n}$$

n : number of periods

w_j: weight factor for day j

X_j: price on day j

Sinusoidal Weighted Moving Average (SWMA)

$$SWMA_i = \sum\nolimits_{j=i-n+1}^{i} w_j X_j \,, \; w_j = \frac{\sin\left(\frac{180}{6} i\right)}{\sum\nolimits_{j=i-n+1}^{i} \sin\left(\frac{180}{6} i\right)}$$

Exponential Moving Average (EMA)

$$EMA_i = \sum_{j=i-n+1}^{i} w_j X_j \, , w_j = \frac{(1-\alpha)^{j-i}}{\sum_{k=i-n+1}^{i} (1-\alpha)^{k-i}}$$

Or, iteratively,

$$EMA_n(X)_i = \alpha X_i + (1-\alpha) EMA_n(X)_{i-1} \, , \; \alpha = \frac{2}{n+1}.$$

α: smoothing factor

n : number of periods

X_i: price on day i

Weighted Moving Average (WMA)

$$WMA_i = \sum_{j=i-n+1}^{i} w_j X_j \; , \; w_j = \frac{j-i}{n}$$

n : number of periods

X_i: price on day i

Double Exponential Moving Average (DEMA)

$$DEMA_n(X) = 2 \times EMA_n(X) - EMA_n(EMA_n(X)),$$

n : number of periods

X_i: price on day i

Zero-Lag Exponential Moving Average (ZLEMA)

ZLEMA = EMA of (close + (close-close[n]))

n : number of periods

X_i: price on day i

Kaufman’s Adaptive Moving Average (KAMA)

Efficiency ratio:

$$\epsilon = \frac{|X_i - X_{i-n}|}{n\sum_{i=1}^{n}|X_n - X_{n-1}|}$$

Smoothing constant:

$$c = \left[\epsilon\left(\frac{f}{f+1} - \frac{s}{s+1}\right) + \frac{s}{s+1}\right]^2$$

KAMA:

$$KAMA_i = KAMA_{i-1} + c(X_i - KAMA_{i-1})$$

X_i: price on day i

ε: efficiency ratio

c: smoothing constant

KAMA $_i$: KAMA value on day i

n: number of periods for the efficiency ratio 2

s: number of periods for the slowest EMA 30

f: number of periods for the fastest EMA 10

Kaufman’s Suggested Values

Research Questions

1. Can KAMA generate timely trading signals to generate profits in trading S&P 500 Index?
2. What is the optimal set of parameter values of KAMA for generating profitable trades?
3. How to design an objective function for optimizing the parameter values of KAMA that can maximize the risk-adjusted return and prevent overtrading and undertrading?

Methodology

Data

S&P 500 Index

Period: 2000-01-03 ~ 2021-11-09 (21 years)

Data: Log of close price

Frequency: Daily

Assumptions

1. All entry and exit prices are on the next trading day of the signal at the close.
2. Commission fees and taxes are assumed to be 5% of the total trading amount.
3. Limit and stop order options are not allowed at this stage.
4. Risk-free rate is assumed to be zero.

Trading Strategy

Signals:

```
if KAMA(today)/KAMA(n days ago)-1 > uptrend_threshold then
    Generate a buy-signal
else if KAMA(today)/KAMA(n days ago)-1 < downtrend_threshold then
    Generate a sell-signal
```

Trading Strategy

Enter/exit actions:

if *a buy-signal is generated n days ago* **then**

    Buy at today's close

else if *a sell-signal is generated n days ago* **then**

    Sell at today's close

Training and Test Set

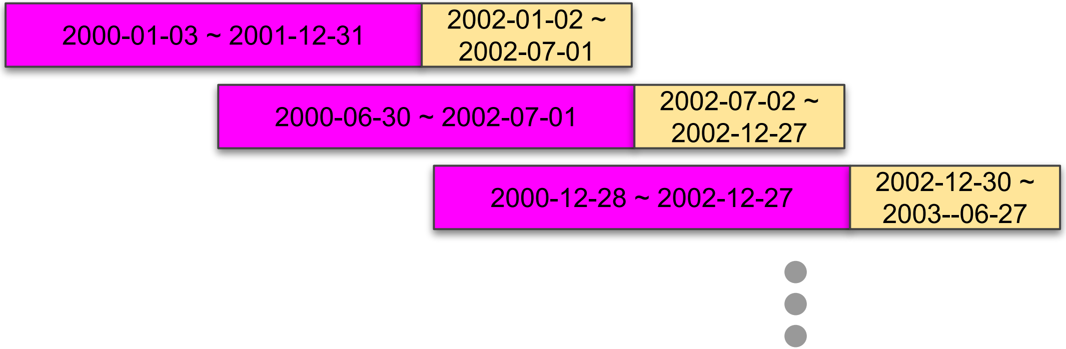
Train-Test Split = 8 : 2

Timeframe	Training (days)	Test (days)	Total (days)	Total (years)
A	200	50	250	~ 1
B	500	125	625	~ 2
C	1000	250	1250	~ 5

Training and Test Set

Sliding Window of Horizon

eg Timeframe B: 500 days (Training) + 125 days (Test)



Parameters to be Optimized

```
bounds = [(3, 20),
          (2, 5),
          (10, 100),
          (0, 0.1),
          (-0.1, 0),
          (2, 5)]
```

1. KAMA
   a. Number of periods for the efficiency ratio (*n_delta*)
   b. Number of periods for the fastest EMA (*n_fast*)
   c. Number of periods for the slowest EMA (*n_slow*)

2. Threshold values
   a. Uptrend (*delta_lim_up*)
   b. Downtrend (*delta_lim_dn*)
   c. Number of days for trend determination (*delta_lim_len*)

Objective Functions

Maximize:

1. r
2. $r \times w$
3. $r \times \left(\frac{w+\epsilon}{l+\epsilon}\right)^{sgn(r)}$, $\epsilon = 1$
4. $r \times \left(\frac{w+\epsilon}{l+\epsilon}\right)^{sgn(r)}$, $\epsilon = 10^{-6}$
5. $r \times (w - l) \times I$,

   where $I = -1$ if $r < 0$ and $w < l$, or otherwise, 1.

6. $r \times (w^2 - l^2) \times I$,

   where $I = -1$ if $r < 0$ and $w < l$, or otherwise, 1.

r : return

w : number of winning trades

l : number of losing trades

\# Note:

Directly maximizing Sharpe or Sortino ratios raises errors during training due to zero-trade issue.

Optimization Method

Dual Annealing

- `scipy.optimize.dual_annealing` in Python SciPy module
- Max. number of global search iterations (maxiter): 500
- Seed = 1
- Other input values: default

  (*initial_temp=5230.0, restart_temp_ratio=2e-05, visit=2.62, accept=-5.0, maxfun=10000000.0, no_local_search=False, callback=None, x0=None*)

Metrics for Trading Performance Evaluation

1. Sharpe ratio
2. Sortino ratio
3. Return

Results and Discussion

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Training and Test Set

Train-Test Split = 8 : 2

Timeframe	Training (days)	Test (days)	Total (days)	Total (years)
A	200	50	250	~ 1
B	500	125	625	~ 2
C	1000	250	1250	~ 5

Highlighted Results

Result 1:

Trading Performance

- Sharpe Ratio

Objective Functions

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1. r
2. $r \times w$
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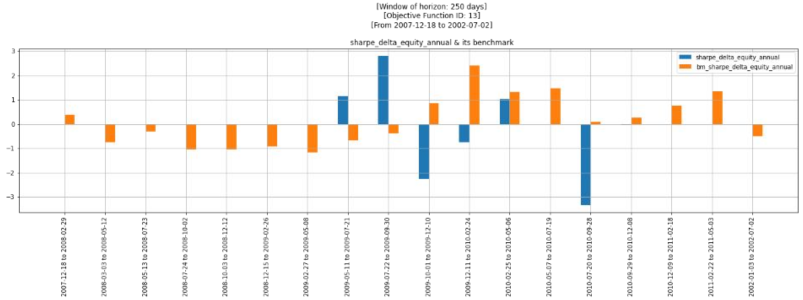
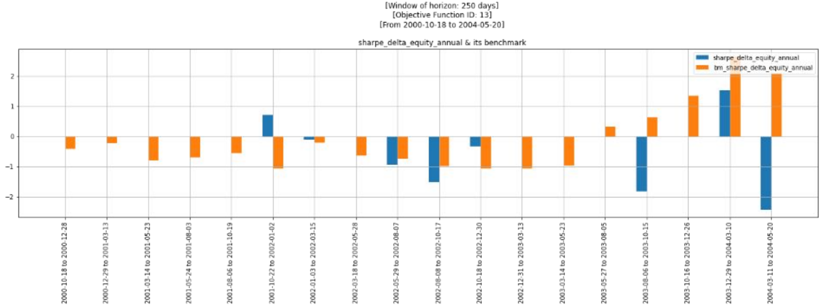
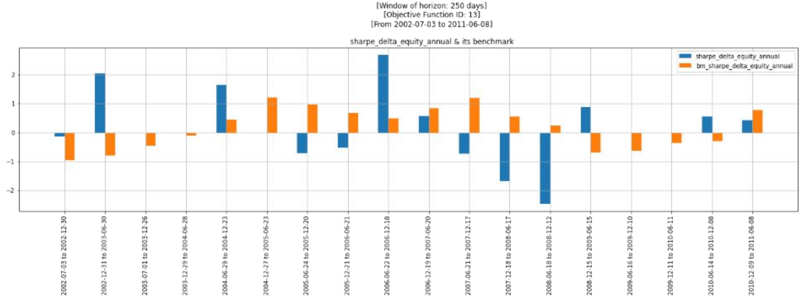
Note:

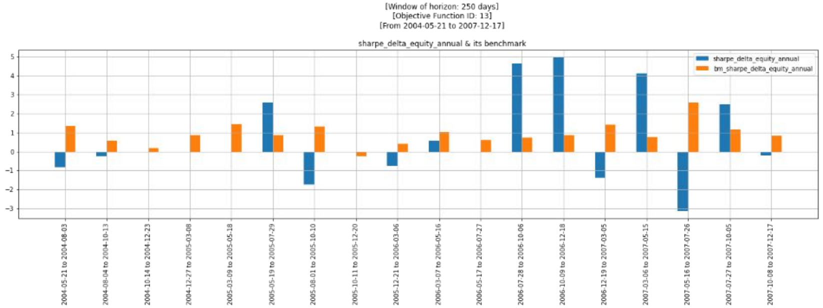
Directly maximizing Sharpe or Sortino ratios raises errors during training due to zero-trade issue.

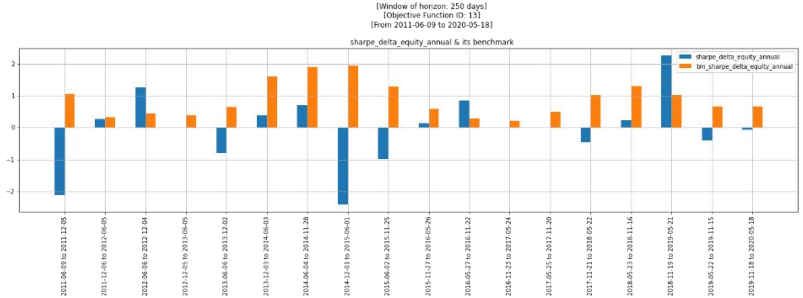
Sharpe (200+50 days)



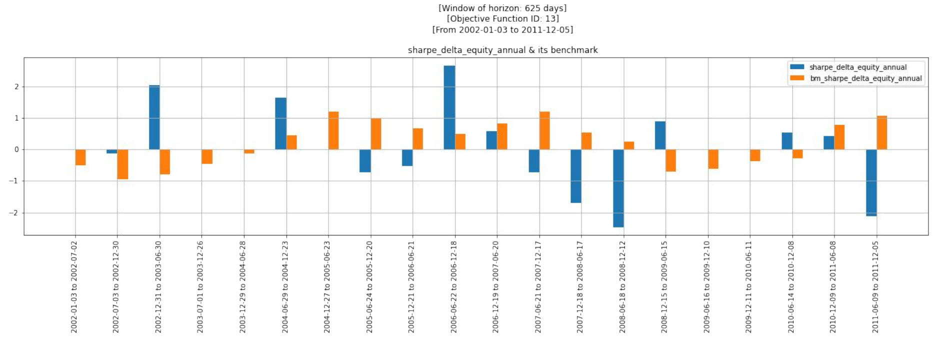


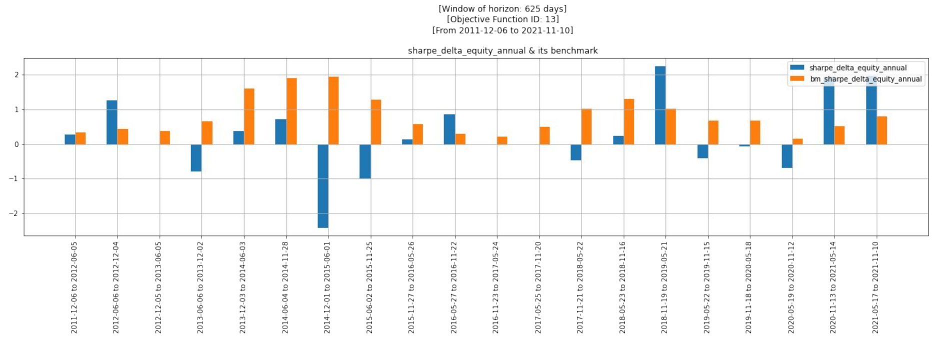




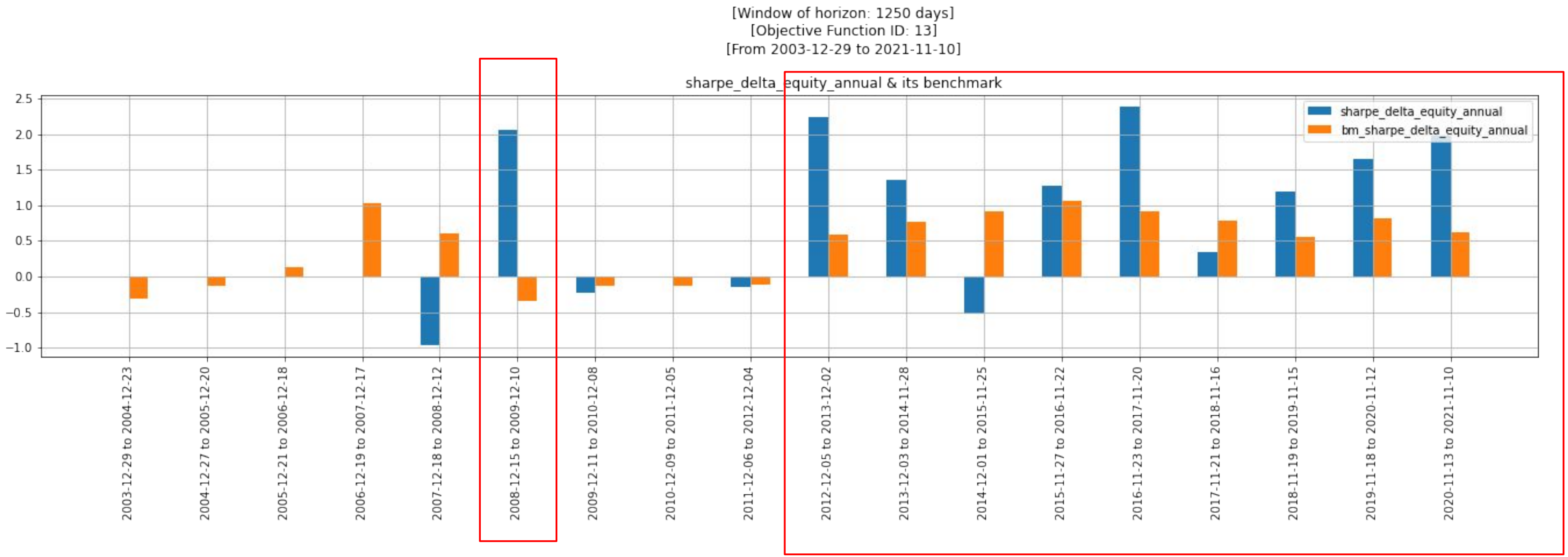


Sharpe (500+125 days)





Sharpe (1000+250 days)



Objective Functions

Maximize:

1. r
2. $r \times w$
3. $r \times \left(\dfrac{w+\epsilon}{l+\epsilon}\right)^{sgn(r)}, \epsilon = 1$
4. $r \times \left(\dfrac{w+\epsilon}{l+\epsilon}\right)^{sgn(r)}, \epsilon = 10^{-6}$

r : return

w : number of winning trades

l : number of losing trades

5. $r \times (w - l) \times I$,

   where $I = -1$ if $r < 0$ and $w < l$, or otherwise, 1 .

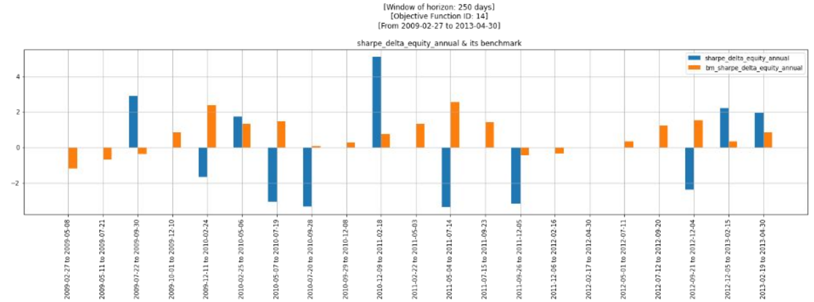
6. $r \times (w^2 - l^2) \times I$,

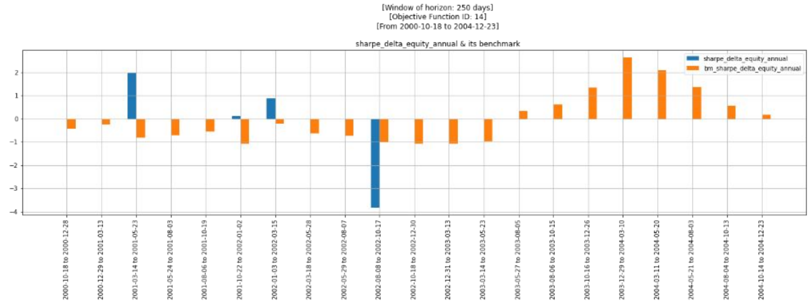
   where $I = -1$ if $r < 0$ and $w < l$, or otherwise, 1 .

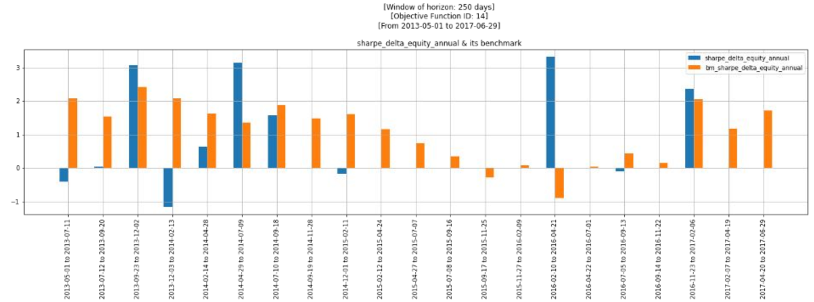
Note:

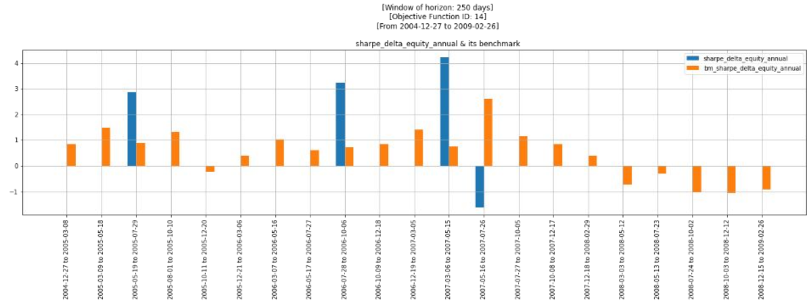
Directly maximizing Sharpe or Sortino ratios raises errors during training due to zero-trade issue.

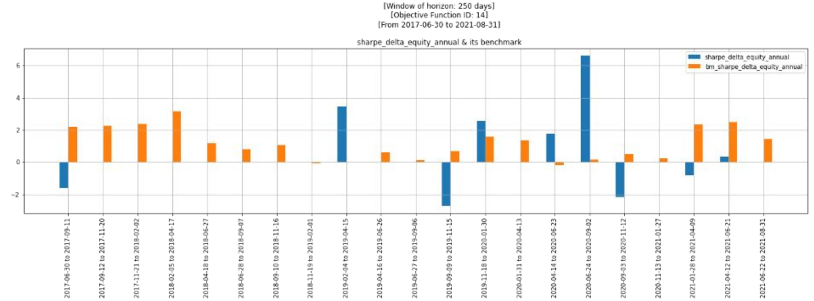
Sharpe (200+50 days)



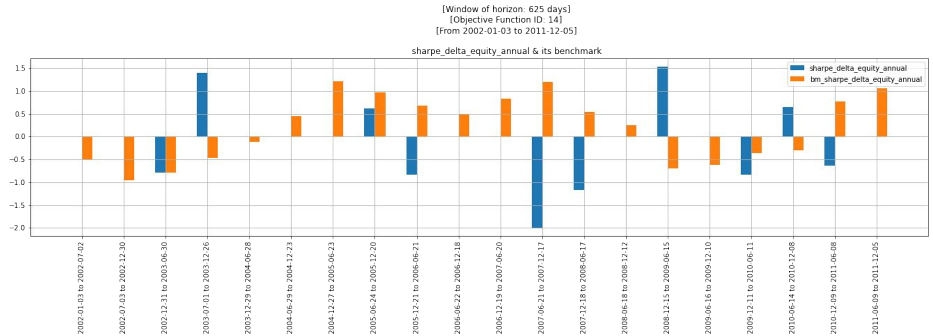


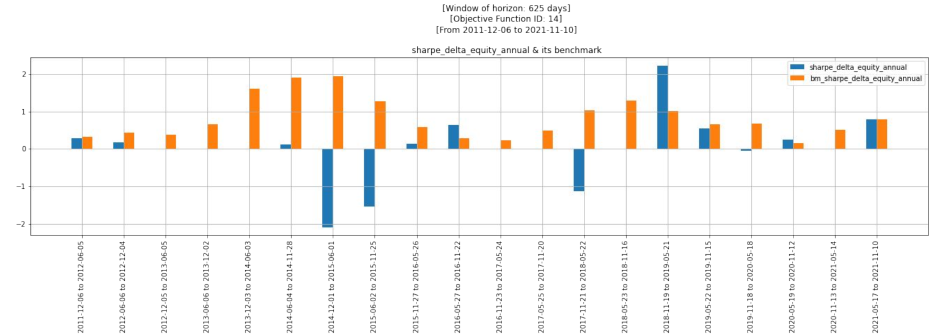




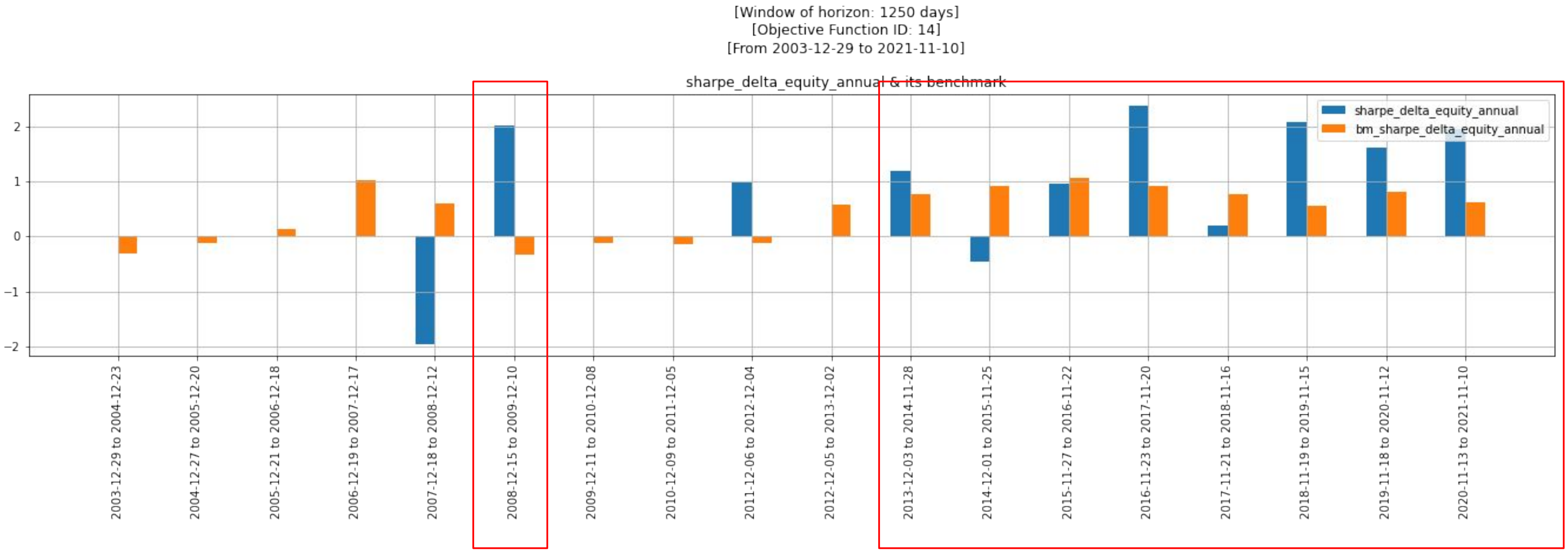


Sharpe (500+125 days)





Sharpe (1000+250 days)



Result 2:

Parameters of KAMA

Parameters

1. KAMA
    a. Number of periods for the efficiency ratio (*n_delta*)
    b. Number of periods for the fastest EMA (*n_fast*)
    c. Number of periods for the slowest EMA (*n_slow*)

2. Threshold values
    a. Uptrend (*delta_lim_up*)
    b. Downtrend (*delta_lim_dn*)
    c. Number of days for trend determination (delta_lim_len)

	n_delta	n_fast	n_slow	delta_lim_up	delta_lim_dn	delta_lim_len
count	1245.000000	1245.000000	1245.000000	1245.000000	1245.000000	1245.000000
mean	7.849032	3.005112	39.095305	0.008596	-0.029199	3.192675
std	3.850146	0.873958	29.246344	0.011540	0.035967	0.955119
min	3.001896	2.000721	10.010294	0.000002	-0.099981	2.003630
25%	4.401306	2.260350	11.088855	0.000776	-0.076729	2.277016
50%	7.522795	2.566505	29.374803	0.002696	-0.004983	2.924820
75%	10.089374	4.160973	63.722799	0.009125	-0.001579	4.103031
max	19.950902	4.991453	99.583507	0.030233	-0.000007	4.999012
Kaufman's	10	2	30			

Conclusion

Research Questions

1. Can KAMA generate timely trading signals to generate profits in trading S&P 500 Index?

   Longer time frame (eg 1000+250 days) can potentially achieve that.

Research Questions

2. What is the optimal set of parameter values of KAMA for generating profitable trades?

   The optimal set varies with market condition.

   It can be slightly or significantly different from the set suggested by Kaufman.

Research Questions

3. How to design an objective function for optimizing the parameter values of KAMA that can maximize the risk-adjusted return and prevent overtrading and undertrading?

   Sharpe ratio and Sortino ratio may be appropriate for this purpose.

   Further study on how to implement or modify the objective functions is needed due to zero-trade issue.