# Statistik IV Assignment 9

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### No. 16

First, read the data

```
train <- read.csv('data_train.csv')
validation <- read.csv('data_valid.csv')
```

## a)

Get an estimate of the covariance matrix and its inverse

```
cov_estimate <- cov(train[, 1:2])
cov_inv <- solve(cov_estimate)
```

Now, get the means for each class

```
mean_1 <- colMeans(train[train$Class==1, 1:2])
mean_2 <- colMeans(train[train$Class==2, 1:2])
mean_3 <- colMeans(train[train$Class==3, 1:2])
```

Build the discrimination functions

```
# Calculates the value of the discrimination function for a given class
discriminator <- function(x, mean, cov_inv) {
  t(mean) %*% cov_inv %*% x - 1/2 %*% t(mean) %*% cov_inv %*% mean
}

# Returns the predicted class
d <- function(x) {
  scores <- c(discriminator(x, mean_1, cov_inv),
              discriminator(x, mean_2, cov_inv),
              discriminator(x, mean_3, cov_inv))
  return(which.max(scores))
}
```

Get the predictions

```
predictions_lda_train <- apply(train[, 1:2], 1, d)
predictions_lda_validation <- apply(validation[, 1:2], 1, d)
```

Estimate the error on the training as well as on the validation set

```
(error_lda_train <- 1 - mean(predictions_lda_train == train$Class))
```

```
## [1] 0.3929619
```

```
(error_lda_validation <- 1 - mean(predictions_lda_validation == validation$Class))
```

```
## [1] 0.3919598
```

## b)

Do a quadratic discriminant analysis on the training set assuming equal a'priori probabilities

```
qda_model <- qda(Class ~ X1 + X2, data = train, prior = c(1/3, 1/3, 1/3))
```

Get the predictions

```
predictions_qda_train <- predict(qda_model, train)$class
predictions_qda_validation <- predict(qda_model, validation)$class
```

Estimate the error on the training as well as on the validation set

```
(error_qda_train <- 1 - mean(predictions_qda_train == train$Class))
```

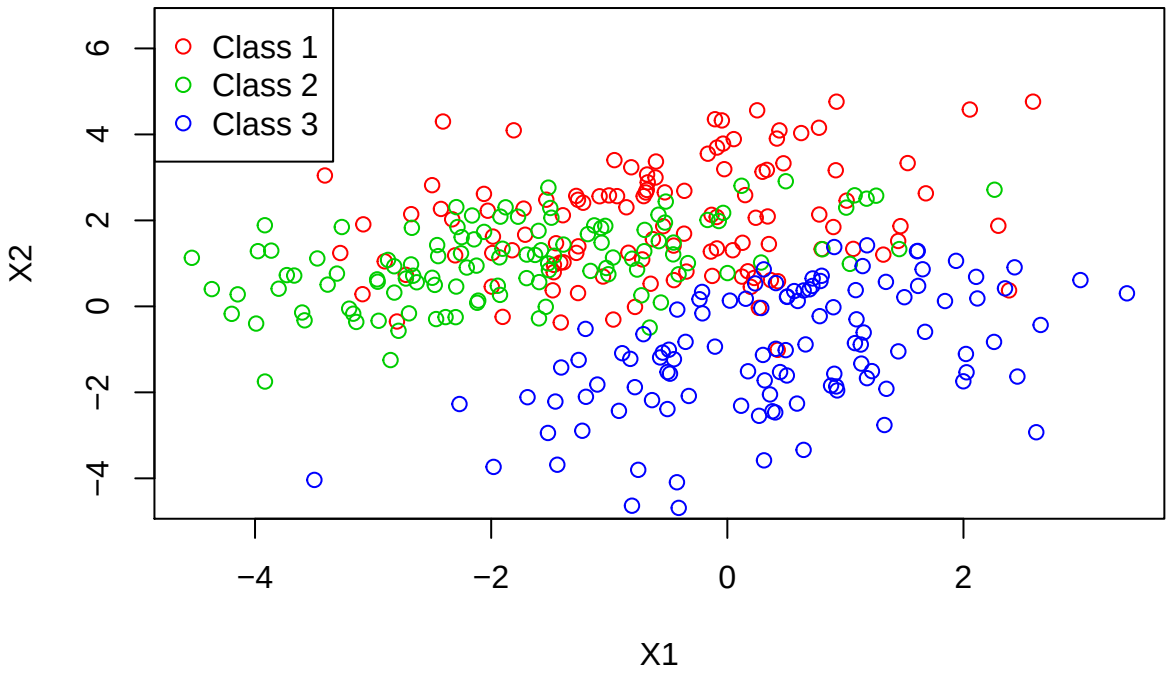
```
## [1] 0.2199413
```

```
(error_qda_validation <- 1 - mean(predictions_qda_validation == validation$Class))
```

```
## [1] 0.2361809
```

## c)

Visualize the data as well as the decision boundaries for the lda as well as for the qda




## d)

Implement the k-Nearest-Neighbor classifier based on the training set

```
knn_classifier <- function(x, k) {
  distances <- apply(train[, 1:2], 1, function(w) norm(x-w, type='2'))
```

```
  neighbor_indices <- order(distances)[1:k]
  neighbor_votes <- table(train$Class[neighbor_indices])
  vote_result <- as.integer(names(which.max(neighbor_votes)))
  return(vote_result)
}
```

Get the predictions on the validation set using k=4

```
predictions_knn_validation <- apply(validation[, 1:2], 1, function(x) knn_classifier(x, 4))
```

Estimate the validation error

```
(error_knn_validation <- 1 - mean(predictions_knn_validation == validation$Class))
```

```
## [1] 0.2763819
```

# e)

As we can see by looking at the error rates, the simple lda delivered the worst results, it's validation error being roughly 39.2%. The error on the validation set is roughly the same as the error on the training set for this method. The qda lead to better results, showing a validation error of about 23.6%. This is most likely due to the higher complexity of this model.

With a validation error of roughly 27.6%, the kNN classifier ranks second among the models. The interesting thing about this is that this error is only about 4 percentage points worse than that of the qda despite not making any assumptions on the distribution of the data.