Boosting

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What you will know

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$\rightarrow$ How to create a strong and efficient learning algorithm

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$\rightarrow$ What is AdaBoost and why is it so successful?

Let’s talk about training a model

How to train a machine learning model

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But there is one problem...

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...so what can we do?

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Let's call ERM on a simple class a **weak learner**. We will formally define it later...

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But first, let's get back to weak learning.

Weak Learnability

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$\boxed{\text{In weak learning, we only want the error to be less than 50\%.}}$

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- ...$L_{(\mathcal{D},f)}(h) < \frac{1}{2} - \gamma$ with confidence $1 - \delta$
- ...for every labeling function f and every distribution $\mathcal{D}$ (if RA holds)

AdaBoost

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Conclusion

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The End.

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