Boosting

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What you will know

$\rightarrow$ The idea behind boosting

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$\rightarrow$ How to create a strong and efficient learning algorithm

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$\rightarrow$ How to create a strong and efficient learning algorithm

$\rightarrow$ What is AdaBoost and why is it so successful?

Let’s talk about training a model

How to train a machine learning model

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But there is one problem...

The problem with ERM

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...so what can we do?

A first idea...

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Let's call ERM on a simple class a **weak learner**. We will formally define it later...

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But first, let's get back to weak learning.

Weak Learnability

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In weak learning, we only want the error to be less than 50%.

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...but how does this help us?

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Lets look at an example (Decision Stumps)

Spam detection with decision stumps

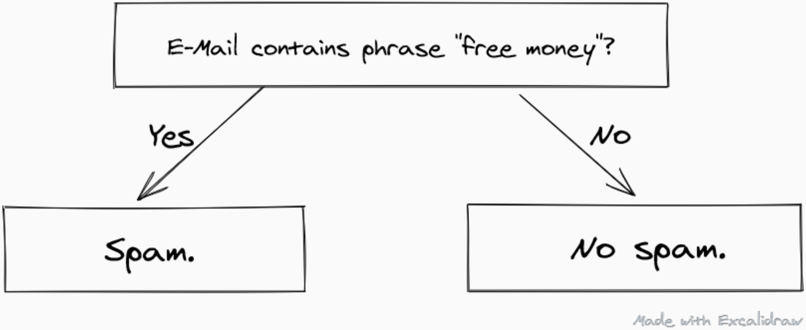

Figure 1: This is a Decision Stump.

AdaBoost

AdaBoost

Conclusion

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The End.

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