Boosting

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What you will know

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$\rightarrow$ How to create a strong and efficient learning algorithm

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$\rightarrow$ What is AdaBoost and why is it so successful?

Let’s talk about training a model

How to train a machine learning model

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But there is one problem...

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...so what can we do?

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Let's call ERM on a simple class a **weak learner**. We will formally define it later...

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But first, let's get back to weak learning.

Weak Learnability

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In weak learning, we only want the error to be less than 50%.

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...but how does this help us?

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Lets look at an example (Decision Stumps)

Spam detection with decision stumps

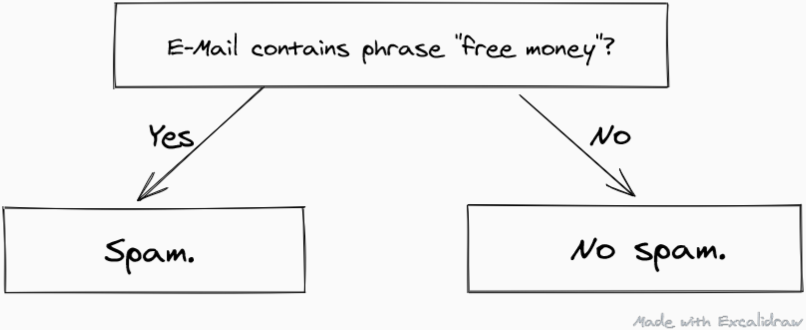

Figure 1: This is a Decision Stump.

ERM for decision stumps is efficient

- Decision Stumps partition the instance space $\mathcal{X}$ along a single dimension

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$$\min_{j \in [d]} \min_{\theta \in \mathbb{R}} \left(\sum_{i: y_i = 1}^{m} D_i \mathbb{1}_{[x_{i,j} > \theta]} + \sum_{i: y_i = -1}^{m} D_i \mathbb{1}_{[x_{i,j} \leq \theta]} \right)$$

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$\boxed{\text{This can be solved in } \mathcal{O}(dm)\text{!}}$

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  - $\rightarrow$ Use an efficient weak learner to create weak hypotheses. Combine them to get a strong hypothesis.

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  - → An algorithm that finds a hypothesis that performs better than random
- What are decision stumps?
  - → A simple hypothesis class where ERM is efficient. Decision stumps partition the feature space along a single dimension.
- What is the idea behind boosting?
  - → Use an efficient weak learner to create weak hypotheses. Combine them to get a strong hypothesis.

But how to do that? The AdaBoost algorithm will tell us...

AdaBoost

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Conclusion

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The End.

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