Dual Form of SVM

Soft-margin SVM

$$\min_{w,b} \frac{1}{2}\|w\| + c\sum_{i=1}^{n} \varepsilon_{i}$$

$$\text{s.t. } y_i(w^T x_i + b) \geq 1 - \varepsilon_i, \forall i$$

$$\varepsilon_i \geq 0$$

Primal Problem

The Dual Form of SVM is

$$\max_{\alpha_i} \sum_{i=1}^{n} \alpha_i - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i \alpha_j y_i y_j x_i^T x_j$$

$$\text{s.t. } \alpha_i \geq 0 \text{ and } \sum_{i=1}^{n} \alpha_i y_i = 0$$

In SVM instead of solving the Primal Problem, we solve Dual ~~Problem~~ Form

By comparing ~~the~~ both the form, we can say that:

① For every x_i we have corresponding α_i $\quad x_i \rightarrow \alpha_i$

② x_i and x_j, x_i's occur only in the form of $x_i^T x_j$

③ $f(x_q) = \sum_{i=1}^{n} \alpha_i y_i x_i^T x_q + b$

$$\because \underbrace{x_q}_{\text{query Point}} \rightarrow \underbrace{(w^T x_q + b)}_{y_q} = \underbrace{f(x_q)}_{+ve / -ve}$$

④ $\alpha_i > 0$, only for ~~s~~ support vectors.

and $\alpha_i = 0$, for non-support vectors.

$f(x) = \sum_{i=1}^{n} \alpha_i y_i x_i^T x_q + b$

for SVS $\alpha_i > 0$

for NSVS $\alpha_i = 0$

$\therefore$ to compute $f(x_q)$ the only point that matters is your support vectors.

Hence the behaviour of SVM, the only points that matters is Support Vectors.

This is the Reason the whole algorithm is named as $\boxed{\underline{\underline{\text{support}}}\ \underline{\underline{\text{vector}}}\ \underline{\underline{\text{machine}}}}$

Now, let's look over the formulation one more time.

$$\max_{\alpha_i} \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j y_i y_j x_i^T x_j$$

$$\text{s.t. } \alpha_i \geq 0 \longrightarrow \alpha_i > 0 \text{ for SVS}$$

$$\alpha_i = 0 \text{ for NSV}$$

$$\sum_{i=1}^{n} \alpha_i y_i = 0$$