A force-based model to reproduce stop-and-go waves in pedestrian dynamics

M. Chraibi, A. Tordeux and A. Schadschneider

Abstract Stop-and-go waves in single-file movement are a phenomenon that is observed empirically in pedestrian dynamics. It manifests itself by the co-existence of two phases: moving and stopping pedestrians. We show analytically based on a simplified one-dimensional scenario that under some conditions the system can have instable homogeneous solutions. Hence, oscillations in the trajectories and instabilities emerge during simulations. To our knowledge there exists no force-based model which is collision- and oscillation-free and meanwhile can reproduce phase separation. We develop a new force-based model for pedestrian dynamics able to reproduce qualitatively the phenomenon of phase separation. We investigate analytically the stability condition of the model and define regimes of parameter values where phase separation can be observed. We show by means of simulations that the predefined conditions lead in fact to the expected behavior and validate our model with respect to empirical findings.

1 Introduction

In vehicular traffic, the formation of jams and the dynamics of traffic waves have been studied intensively [3, 7]. Particular car-following models including spacing and speed difference variables have been shown to reproduce realistic stop-and-go phenomena [13, chap. 15]. In pedestrian dynamics this phenomenon has been observed empirically, especially when the density exceeds a critical value [11, 9]. Jams can be reproduced as a result of phase transitions from a stable homogeneous

Mohcine Chraibi and Antoine Tordeux
Forschungszentrum Jülich GmbH, Jülich Supercomputing Centre, Jülich 52428, Germany, e-mail: m.chraibi@fz-juelich.de, a.tordeux@fz-juelich.de

Andreas Schadschneider
Institute for Theoretical Physics, Universität zu Köln, 50937 Köln, Germany, e-mail: as@thp.uni-koeln.de

configuration to an unstable configuration. In the literature some space-continuous models [8, 12, 6, 5] reproduce partly this phenomenon. However, force-based models generally fail to describe pedestrian dynamics in jam situations correctly. Often uncontrollable oscillations in the direction of motion occur, which lead to unrealistic dynamics in form of collisions and overlappings [4].

In this work we present a force-based model that is able to reproduce stop-and-go waves for certain parameter values. By means of a linear stability analysis we derive conditions to define parameter regions, where the described system is unstable.

We study by numerical simulations if the system behaves realistically, i.e. jams emerge without any collisions in agreement with experimental results [9]. Furthermore, we validate the model by comparing the fundamental diagram with experiments. We conclude this paper with a discussion of the results and the limitations of the proposed model.

2 Model definition

The phenomenon of stop-and-go waves in pedestrian dynamics was investigated experimentally in one-dimensional scenarios [9]. Therefore, we limit our analysis to 1D systems. Consider N pedestrians distributed uniformly in a narrow corridor with closed boundary conditions and neglect the effects of walls on pedestrians. Furthermore, for interactions among N pedestrians, we assume that pedestrian n is only influenced by the pedestrian right in front.

For the state variables position x_n and velocity $\dot{x}_n = \frac{dx_n}{dt}$ of pedestrian n we define the distance of the centers Δx_n and speed difference $\Delta \dot{x}_n$ of two successive pedestrians as

$$\Delta x_n = x_{n+1} - x_n, \qquad \Delta \dot{x}_n = \dot{x}_{n+1} - \dot{x}_n\,. \tag{1}$$

In general, pedestrians are modeled as simple geometric objects of constant size, e.g. a circle or ellipse. In one-dimensional space the size of pedestrians is characterized by a_n (Fig. 1), i.e. their length is $2a_n$. However, it is well-known that the space

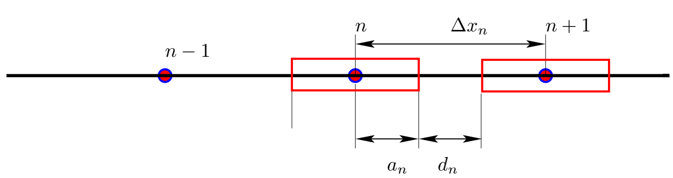

Fig. 1 Definition of the quantities characterizing the single-file motion of pedestrians (represented by rectangles).

requirement of a pedestrian depends on its velocity and can be characterized by a linear function of the velocity [14, 2, 10]

$$a_n = a_0 + a_v \dot{x}_n\,, \tag{2}$$

with a_0, characterizing the space requirement of a standing person and $a_v \geq 0$ a parameter for the speed dependence with the dimension of time. The effective distance (distance gap) d_n of two consecutive pedestrians is then

$$d_n = \Delta x_n - a_n - a_{n+1} = \Delta x_n - a_v\,(\dot{x}_n + \dot{x}_{n+1}) - 2a_0. \tag{3}$$

At each time the change of state variables of pedestrian n is given by superposition of driving and repulsive terms. Thus, in general the equation of motion for pedestrian n described by a force-based model is given by

$$\ddot{x}_n = f\Big(\dot{x}_n, \Delta\dot{x}_n, \Delta x_n\Big) + \frac{v_0 - \dot{x}_n}{\tau}. \tag{4}$$

Typical values for the parameters are $\tau = 0.5$ s for the relaxation time and $v_0 = 1.2$ m/s for the desired speed.

For f we propose the following expression

$$f(\Delta x_n, \dot{x}_n, \dot{x}_{n+1}) = -\frac{v_0}{\tau} \ln\Big(c \cdot R_n + 1\Big), \tag{5}$$

with

$$R_n = r_\varepsilon\Big(\frac{\Delta x_n}{a_n + a_{n+1}} - 1\Big), \qquad c = e - 1. \tag{6}$$

$r_\varepsilon(x)$ is an approximation of the non-differentiable ramp function

$$r_\varepsilon(x) = \varepsilon \ln(1 + e^{-x/\varepsilon}) \qquad (0 < \varepsilon \ll 1). \tag{7}$$

Pedestrians anticipate collisions when their distance to their predecessors is smaller than a critical distance $a = a_n + a_{n+1}$. Therefore, a_n does not only model the body of pedestrian n but represents also a “personal” safety distance. Assuming that $\dot{x}_n = 0$, for $\Delta x_n = 0$, i.e., $R_n = 1$, the repulsive force reaches the value $-v_0/\tau$ (at the limit $\varepsilon \to 0$) to nullify the effects of the driving term (Fig. 2).

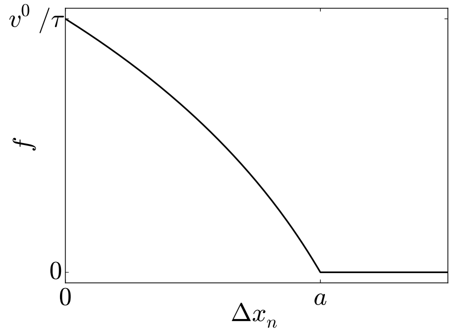

Fig. 2 The absolute value of the repulsive force according to Eq. (5) (at the limit $\varepsilon \to 0$).

3 Linear dynamics

In this section, we investigate the stability of the system (4). The position of pedestrian n in the homogeneous steady state is given by

$$x_n = \frac{n}{\rho} + vt\,, \tag{8}$$

so that $x_{n+1} - x_n = \frac{1}{\rho} = \Delta x$, $\dot{x}_n = v$, being speed for the equilibrium of uniform solution. $\ddot{x}_n = 0$ for all n, where derivatives are taken with respect to t. For $\Delta y = \Delta x_n / a_0$ we consider small (dimensionless) perturbations ε_n of the steady state positions of the form

$$\varepsilon_n(t) = \alpha_n e^{zt}\,, \tag{9}$$

with $\alpha_n, z \in \mathbb{C}$. Replacing in (4) and expanding to first order yields a second-order equation for z. To obtain stability, one needs to ensure $\Re(z) < 0$ for the real part of all solutions z with the exception of the solution $z = 0$.

For the system (4) with the repulsive force (5) we obtain the following stability condition

$$\Phi := \Big(\frac{1}{1+2\xi a'_v \Delta y}\Big)\Big(\frac{\xi}{1+2\xi a'_v \Delta y} + \xi a'_v \Delta y\Big) - 1/2 < 0, \tag{10}$$

with $\xi = \frac{c}{d_0}\frac{v'_0}{a'}, a' = \frac{a}{a_0}, v'_0 = v_0 \frac{\tau}{a_0}$ and $d_0 = 1 + c(1 - \Delta y / a')$.

Fig. 3 shows the stability behavior of the system with respect to the dimensionless parameters v'_0 and $\tilde{a}_v = a_v / \tau$. The system becomes increasingly unstable with increasing v'_0 (for a relatively small and constant $\tilde{a}_v$). Assuming that the free flow speed v_0 is constant, this means that increasing the reaction time τ or diminishing the safety space leads to unstable behavior of the system. This results is well-known in traffic theory (see for instance [1]).

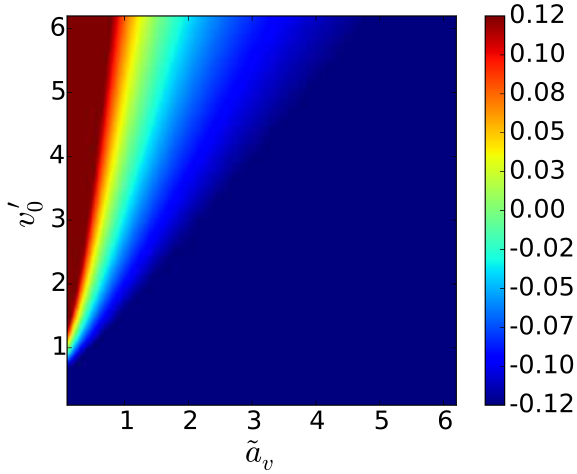

Fig. 3 Stability region in the $(\tilde{a}_v, v_0')$-space for $\Delta y = 1.5$. The colors are mapped to the values of Φ and $(\tilde{a}_v, v_0')$ are the dimensionless parameters in Eq. (10).

4 Simulations

We perform simulations with the introduced model to analyze the unstable dynamics. For $a_v = 0$, $v_0' = 1$ and $\Delta y_n = 1.5$ we calculate the solution for 3000 s. These parameters lay in the unstable regime of the model (Fig. 3). Thus, jam waves are expected to emerge. Fig. 4 shows the trajectories of 133 pedestrians. ε in Eq. (7) is set to 0.01.

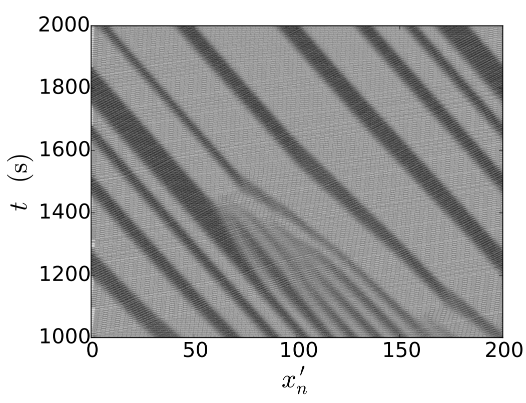

Fig. 4 Trajectories for $\Delta y_n = 1.5$ show stop-and-go waves.

We observe jam waves propagating in the system. Note that the observed jam waves last for a long period of time (here 3000 s), which is a indication that they are not dependent on the initial conditions of the simulation and are “stable” in time.

As shown in Fig. 5 the speed does not become negative, therefore backward movement is not observed. This condition favors the appearance of stable jams.

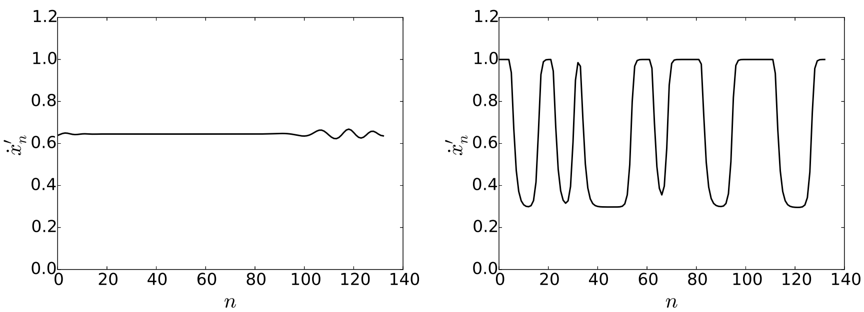

Fig. 5 Speed of pedestrians at different time steps. Left: $t = 300$ s, right: $t = 2000$ s.

Having reproduced stop-and-go waves, the model will be further tested by comparing qualitatively the produced density-velocity relation (fundamental diagram). The same setup as above is simulated several times. In order to scan a sufficiently large density interval, the number of pedestrians N is increased after each simulation. Fig. 6 shows a comparison of the simulation results with experimental data from [11]. The observed fundamental diagram is composed of two different regimes: free flow regime, where the speed of pedestrians does not depend on the density ($\rho < 0.5\,\mathrm{m}^{-1}$), and a regime where the speed decreases with increasing density. Here we observe that the correct shape of the fundamental diagram is reproduced quite well, although the velocity is slightly higher than the experimental velocities for $\rho > 2\,\mathrm{m}^{-1}$.

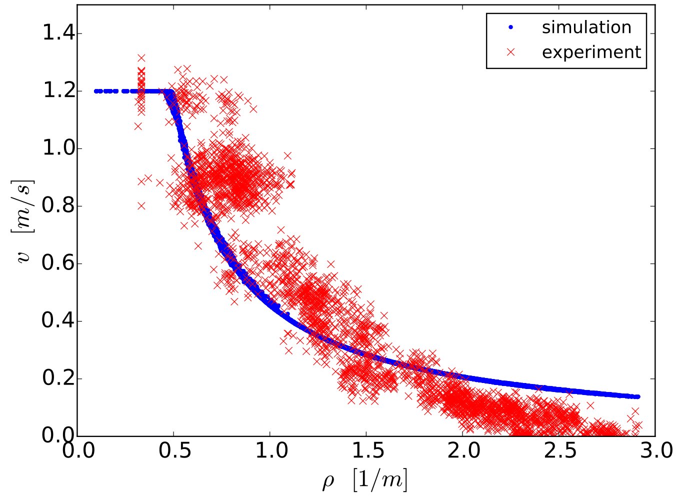

Fig. 6 Fundamental diagram: comparison with experiments from [11].

5 Discussion

We have introduced a simple force-based model for which uniform solutions can be unstable. By simulations we observe that the proposed model shows phase separation in its unstable regime, in agreement with empirical results [9].

The linear stability condition of the models shows that we can find *realistic* parameter values in the unstable regime. However, depending on the chosen values for the (rescaled) desired speed v_0', collisions *can* occur, as a result of backwards movement and negative speeds.

Further investigations remain to be carried out to determine the set of parameter values for which the model have unstable solutions with realistic (i.e. collision-free) stop-and-go phenomena and meanwhile a better *quantitative* agreement with the experimental data e.g. in form of the fundamental diagram.

Acknowledgements In memory of Matthias Craesmeyer.

M.C. thanks the Federal Ministry of Education and Research (BMBF) for partly supporting this work under the grant number 13N12045. A.S. thanks the Deutsche Forschungsgemeinschaft (DFG) for support under grant “Scha 636/9-1”.

References

1. Bando, M., Hasebe, K., Nakayama, A., Shibata, A., Sugiyama, Y.: Dynamical model of traffic congestion and numerical simulation. Phys. Rev. E **51**(2), 1035–1042 (1995)

2. Jelić, A., Appert-Rolland, C., Lemercier, S., Pettré, J.: Properties of pedestrians walking in line. ii. stepping behavior. Phys. Rev. E **86**, 046,111 (2012)
3. Chowdhury, D., Santen, L., Schadschneider, A.: Statistical physics of vehicular traffic and some related systems. Phys. Rep. **329**(4–6), 199–329 (2000)
4. Chraibi, M., Ezaki, T., Tordeux, A., Nishinari, K., Schadschneider, A., Seyfried, A.: Jamming transitions in force-based models for pedestrian dynamics. Phys. Rev. E **92**, 042,809 (2015)
5. Eilhardt, C., Schadschneider, A.: Stochastic headway dependent velocity model for 1d pedestrian dynamics at high densities. Transportation Research Procedia **2**(0), 400 – 405 (2014)
6. Lemercier, S., Jelic, A., Kulpa, R., Hua, J., Fehrenbach, J., Degond, P., Appert-Rolland, C., Donikian, S., Pettré, J.: Realistic following behaviors for crowd simulation. Computer Graphics Forum **31**, 489–498 (2012)
7. Orosz, G., Wilson, R.E., Stepan, G.: Traffic jams : dynamics and control. Proc. R. Soc. A **368**(1957), 4455–4479 (2010)
8. Portz, A., Seyfried, A.: Modeling stop-and-go waves in pedestrian dynamics. In: R. Wyrzykowski, J. Dongarra, K. Karczewski, J. Wasniewski (eds.) PPAM 2009, Part II, pp. 561–568. Springer, Berlin Heidelberg (2010)
9. Portz, A., Seyfried, A.: Analyzing stop-and-go waves by experiment and modeling. In: R. Peacock, E. Kuligowski, J. Averill (eds.) Pedestrian and Evacuation Dynamics 2010, pp. 577–586. Springer (2011)
10. Seitz, M.J., Köster, G.: Natural discretization of pedestrian movement in continuous space. Phys. Rev. E **86**, 046,108 (2012)
11. Seyfried, A., Boltes, M., Kähler, J., Klingsch, W., Portz, A., Rupprecht, T., Schadschneider, A., Steffen, B., Winkens, A.: Enhanced empirical data for the fundamental diagram and the flow through bottlenecks. In: W.W.F. Klingsch, C. Rogsch, A. Schadschneider, M. Schreckenberg (eds.) Pedestrian and Evacuation Dynamics 2008, pp. 145–156. Springer, Berlin Heidelberg (2010)
12. Seyfried, A., Portz, A., Schadschneider, A.: Phase coexistence in congested sates of pedestrian dynamics. Lect. Notes Comp. Sc. **6350**, 496–505 (2010)
13. Treiber, M., Kesting, A.: Traffic Flow Dynamics. Springer, Berlin (2013). ISBN: 978-3-642-32459-8
14. Weidmann, U.: Transporttechnik der Fussgänger. Tech. Rep. Schriftenreihe des IVT Nr. 90, Institut für Verkehrsplanung,Transporttechnik, Strassen- und Eisenbahnbau, ETH Zürich, ETH Zürich (1993). 2nd Edition