Lad os kigge på formlen fra notation and integration.pdf

$$U(F_I) = p_d(a, \psi_{0,F_I}) - p_d(a, \psi_0) \tag{1}$$

Begge led i summen kan skrives som produkter

$$\begin{aligned}
& p_d(a, \psi_{0,F_I}) - p_d(a, \psi_0) && (2)\\
&= P_d(a)\Bigg(\prod_{j=1}^{n_d} \frac{g_{j,d}(R^{1,j,d}(\psi_{0,F_I}^{1,j,d}), \ldots, R^{k_{j,d},j,d}(\psi_{0,F_I}^{k_{j,d},j,d}))}{\mathrm{norm}^{j,d}(l(a))} - && (3)\\
&\qquad \prod_{j=1}^{n_d} \frac{g_{j,d}(R^{1,j,d}(\psi_0^{1,j,d}), \ldots, R^{k_{j,d},j,d}(\psi_0^{k_{j,d},j,d}))}{\mathrm{norm}^{j,d}(l(a))}\Bigg)\\
&= P_d(a)\frac{1}{\prod_{j=1}^{n_d} \mathrm{norm}^{j,d}(l(a))}\Bigg(\prod_{j=1}^{n_d} g_{j,d}(R^{1,j,d}(\psi_{0,F_I}^{1,j,d}), \ldots, R^{k_{j,d},j,d}(\psi_{0,F_I}^{k_{j,d},j,d})) - \\
&\qquad \prod_{j=1}^{n_d} g_{j,d}(R^{1,j,d}(\psi_0^{1,j,d}), \ldots, R^{k_{j,d},j,d}(\psi_0^{k_{j,d},j,d}))\Bigg) && (4)\\
& && (5)
\end{aligned}$$

Hvis vi antager at g'erne er produkter, står der

$$\begin{aligned}
&= P_d(a)\frac{1}{\prod_{j=1}^{n_d} \mathrm{norm}^{j,d}(l(a))}\Bigg(\prod_{j=1}^{n_d} R^{1,j,d}(\psi_{0,F_I}^{1,j,d}) \cdots R^{k_{j,d},j,d}(\psi_{0,F_I}^{k_{j,d},j,d}) - && (6)\\
&\qquad \prod_{j=1}^{n_d} R^{1,j,d}(\psi_0^{1,j,d}) \cdots R^{k_{j,d},j,d}(\psi_0^{k_{j,d},j,d})\Bigg) && (7)
\end{aligned}$$

Inde i den store parentes står der et objekt af formen

$$r_1 r_2 \cdots r_n - r_{1,0} r_{2,0} \cdots r_{n,0} \tag{8}$$

Det kan skrives som

$$\begin{aligned}
&(r_{1,0} + (r_1 - r_{1,0})) \cdots (r_{n,0} + (r_n - r_{n,0})) - r_{1,0} r_{2,0} \cdots r_{n,0} && (9)\\
&\quad = r_{1,0} \cdots r_{n,0} + (r_1 - r_{1,0}) r_{2,0} \cdots r_{n,0} + \cdots + (r_n - r_{n,0}) r_{1,0} \cdots r_{n-1,0} + R - r_{1,0} \cdots r_{n,0} && (10)\\
&\quad = (r_1 - r_{1,0}) r_{2,0} \cdots r_{n,0} + \cdots + (r_n - r_{n,0}) r_{1,0} \cdots r_{n-1,0} + R && (11)
\end{aligned}$$

Vi vil nu dele R ud på risk factorne svarende til r_1, $r_2 \dots, r_n$. Formlen i notation and integration.pdf ser således ud

$$S^{(1)}(F_i) = \frac{S(F_i)}{\sum_{j=1}^n S(F_j)} \cdot U_* \tag{12}$$

hvilket siger at den ekstra vægt (dvs. $U_*/(\sum_{j=1}^n S(F_j))$) skal ~~deles ud proportionalt til r_i/norm^i~~ deles ud proportionalt til

$$r_{1,0} \cdots r_{i-1,0} \cdot (r_i - r_{i,0}) \cdot r_{i+1,0} \cdots r_n \tag{13}$$

hvilket er upåvirket af omskaleringer af hele risk ratiotabeller.

For klarifikation kan vi skrive

$$\begin{aligned} &P_d(a) \frac{1}{\prod_{j=1}^{n_d} \text{norm}^{j,d}(l(a))} \Bigg(\prod_{j=1}^{n_d} R^{1,j,d}(\psi_{0,F_I}^{1,j,d}) \cdots R^{k_{j,d},j,d}(\psi_{0,F_I}^{k_{j,d},j,d}) - \qquad (14) \\ &\qquad \prod_{j=1}^{n_d} R^{1,j,d}(\psi_0^{1,j,d}) \cdots R^{k_{j,d},j,d}(\psi_0^{k_{j,d},j,d}) \Bigg) \\ &= \sum_{j=1}^n S(F_i) + P_d(a) \frac{1}{\prod_{j=1}^{n_d} \text{norm}^{j,d}(l(a))} \Big(R \Big) = U_* \qquad (15) \end{aligned}$$

På branchen prototype brugte vi ~~i stedet for~~ denne udgave af formel (12)

$$S^{(1)}(F_j) = \frac{\frac{r_j - r_{j,0}}{r_{j,0}}}{\sum_{i=1}^n \frac{r_i - r_{i,0}}{r_{i,0}}} U_* \tag{16}$$

Generelt

Lad som sædvanlig $U(F_I) = p_d(a, \psi_{F_I}) - p_d(a, \psi_0)$ og $U_* = U(F_{\{1,\dots,N\}})$. Vi ønsker at lave dekomponeringen

$$U_* = \sum_i S(F_i) \tag{17}$$

og vi ønsker at $S(F_i)$ på en eller anden måde er tæt på $U(F_i)$. I multinomialfordelingen ville det fortolkes som at produktet

$$\prod_{i=1}^N \big(S(F_i)\big)^{U(F_i)} \tag{18}$$

er stort. Man kan vise at

$$\arg \max_{S(F_1),\ldots,S(F_N)\in\mathbb{R}_+} \prod_{i=1}^{N} \big(S(F_i)\big)^{U(F_i)}, \tag{19}$$

$$\text{med betingelsen } \sum_{i=1}^{N} S(F_i) = U_* \tag{20}$$

giver løsningen

$$\S(F_i) = \frac{U(F_i)}{\sum_{j=1}^{N} U(F_j)} U_* \tag{21}$$

hvilket er identisk med formel (12).