Covid-19 epidemics by SIR models

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The standard SIR model has state variables $S(t)$ the susceptibles (to infection) in a population at time $t > 0$, $I(t)$ the number of infected and $R(t)$ the number of persons either recovered and immune, or dead from an infection, e.g. Covid-19. In this exercise, we will assess different hypotheses in relation to epidemics and epidemics management, with Denmark as example.

(a) The infectious period Covid-19 is approximately 14 days; at the start of the epidemic ($t = 0$), where 15 persons are known to be infected, the reproductive number Q_0 is 1.8. Assuming Covid-19 infection gives complete subsequent immunity, setup the standard SIR model in MatLab, corresponding to this situation for Denmark ($S_0 = 6 \cdot 10^6$). Solve the equations and plot the solution.

(b) How long time does the epidemics last and when does it peak, and what fraction of the danish population avoid getting sick. Compare with r inceased by 10 % and comment.

(c) Make phase space plots of (S, I) for different numbers of initially infected persons $I(t = 0)$. At what $I(t = 0)$ does the number of infected not increase for $t > 0$?

(d) A certain fraction h of the infected needs intensive care. In Denmark the patient capacity is 50000. Find a reasonable value for h (in Denmark) and figure out how much authorities need to reduce r, to make sure all patients needing hospitalization at *any* time can be admitted, assuming 15 persons are infected at $t = 0$. Is this reasonable ? Is it reasonable to apply same h for all countries?

A characteristic feature of real epidemics is that several epidemic waves often emerge in first season. The basic SIR model does not predict this and we will explore one or several extensions that may explain multi-wave epidemics. Explore several of the hypotheses below as extensions of the basic SIR model that allows multi-wave epidemics

(e) Loss of immunity *(easy/moderate)*. Virus mutate quickly, leading to loss of immunity. Assume reasonable time scales of loss of immunity extend the SIR model to cover this.

(f) Seasonal dependence of r, e.g. contact reductions in the summer vacation period *(easy/moderate)*. Try a seasonally depending $r(t)$ (possibly combined with loss of immunity).

(g) r depends on the perceived risk of infection by each susceptible *(tricky)*. Express the risk of infection per susceptible per time x, and propose a reasonable form of $r(x)$, reflecting cautious behavior at increasing risk.

(h) Heterogeneity of contact patterns *(moderately complicated)*. Different population segments (e.g.young and old) have different contact patterns; Extend the SIR model so that $S \to (S_y, S_o)$ and $I \to (I_y, I_o)$ and assume reasonable values for $(r_{yy}, r_{oo}, r_{oy}, r_{yo})$ reflecting different contact patterns between population segments.

(i) Is recirculation a possible mechanism within the SIR model (e.g. a traveller brings back the same virus after first wave) (hint: consider the phase space analysis of the SIR model) *(theoretical question)*?

(j) Propose alternative hypotheses of multi-wave epidemics yourself, and test them as a modified SIR model, if possible.