Week 7: Trophic Control

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1 Model description: K-dependence without predators.

$$\frac{dN}{dt} = \left[r\left(1 - \frac{N}{K}\right) - b_c C \right] N \tag{1}$$

$$\frac{dC}{dt} = \left(\varepsilon_c b_c N - d_c\right) C \tag{2}$$

the equilibria for this system can be found for the three regions:

Region	N	C
Trivial	0	0
$K < K_{crit}$	K	0
$K > K_{crit}$	K_{crit}	$\frac{r}{b_c}\left(1 - \frac{K_{crit}}{K}\right)$

where $K_{crit} = \frac{d_c}{\varepsilon_c b_c}$. This value is found from eq. 2. The K-dependent equilibrium $K > K_{crit}$, is found from settinbg to zero 1.

2 Model description: K-dependence with predators.

$$\frac{dN}{dt} = \left[r\left(1 - \frac{N}{K}\right) - b_c C \right] N \tag{3}$$

$$\frac{dC}{dt} = \left(\varepsilon_c b_c N - d_c - b_c P\right) C \tag{4}$$

$$\frac{dP}{dt} = \left(\varepsilon_p b_p C - d_p\right) P \tag{5}$$

The equilibria for this system can be found as follows:

Region	N	C	P
Trivial	0	0	0
$K < K_{crit1}$	K	0	0
$K_{crit1} < K < K_{crit2}$	K_{crit}	$\frac{r}{b_c}\left(1 - \frac{K_{crit}}{K}\right)$	0
$K_{crit2} < K$	$K(1 - \frac{b_c}{r}\frac{d_p}{\varepsilon p b_p})$	$\frac{d_p}{\varepsilon p b_p}$	$K(1 - \frac{b_c}{r}\frac{d_p}{\varepsilon p b_p})\varepsilon_c b_c - d_c$

3 Model description: K-dependence with predators.

The equations are identical to eq. - . The equilibria for this system can be found as follows:

Region	N	C	P
Trivial	0	0	0
$r < r_{crit1}$	$\frac{d_c}{\varepsilon_c b_c}$	$\frac{r}{b_c}\left(1 - \frac{1}{K}\frac{d_c}{\varepsilon_c b_c}\right)$	0
$r_{crit1} < r$	$K\left(1 - \frac{b_c}{r}\frac{d_p}{\varepsilon_p b_c}\right)$	$\frac{d_p}{\varepsilon_p b_c}$	$\frac{\varepsilon_c b_c}{b_p} K\left(1 - \frac{b_c}{r}\frac{d_p}{\varepsilon_p b_c}\right) - \frac{d_c}{b_p}$