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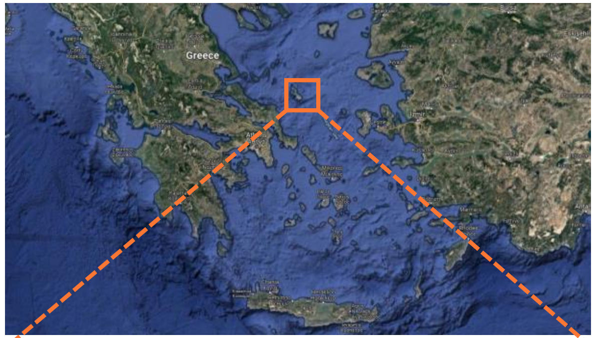
Predator-prey systems

The case: insular geckos on an Aegean islet

LOCATION:
Exo Diavates, $V = 20$ [sqm]



PREDATOR - Lizard

Species: *Mediodactylus kotschyi*

Life history:
Longevity: 4.5 [years]
Death rate: d=1/4.5 [1/years]
Clutch size: 2 [offspring/female]
Clutch frequency: 2 [clutches/year]
(Çiçek *et al.*, 2015; Schwarz *et al.*, 2020)

How many offspring?
φ= 2 [offspring/lizard/year]

Feeding:
Feeding frequency: $f_{fr} = 1/3$ [1/days]
Feeding quantity: $f_q = 1$ [beetle]
Feeding per year:
$f_{yr} = 365 \cdot f_{fr} \cdot f_q = 122$ [beetles/year]

Efficiency:
$\varepsilon = \varphi / f_{yr} = 1/61$ [offspring/beetles/year]

PREY – Beetle

Species: *Tenebrio molitor*

(Hypothetical) life history:
Longevity: 1 [year]
Death rate: d_b =1/1 [1/years]
Offspring: φ_b=50 [offspring/beetle/year]
Offspring survival: 10%

Reproductive rate:
$r = 0.1 \cdot \varphi_b - d_b$= 4 [offspring/beetle/year]

Clearance rate:
1 square meter in 3 days for 1 lizard:
$b = 1/(3 \cdot 1)$=0.33 [sqm/days/lizard]

Lotka-Volterra Predator-Prey equations

PREDATOR – Lizard – P

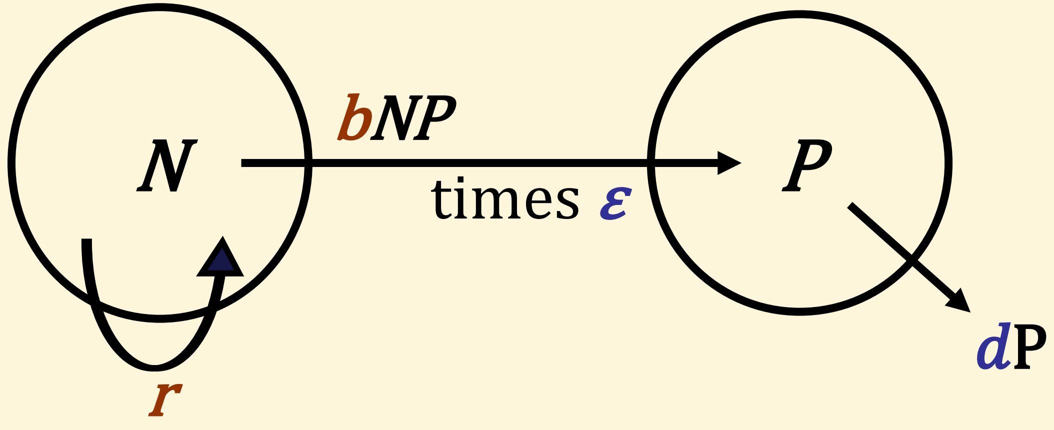
Death rate: d=1/4.5 [1/years]

Efficiency:
$\boldsymbol{\varepsilon} = \mathbf{1/61}$ [offspring/beetles/year]

PREY – Beetle – N

Reproductive rate:
$\boldsymbol{r} = \mathbf{4}$ [offspring/beetle/year]

Clearance rate: $\boldsymbol{b} = \mathbf{0.33}$ [sqm/days/lizard]



units

equations

$$\left[\frac{\#P}{V\cdot T}\right] \quad \frac{dP}{dt} = \varepsilon bNP - dP$$

$$\left[\frac{\#N}{V\cdot T}\right] \quad \frac{dN}{dt} = -bNP + rN$$

equilibria

$$\bar{P} = {}^{r}/_{b} = 12\ [\#/\text{sqm}]\text{=}240[\#]$$

$$\bar{N} = \frac{d}{\varepsilon\cdot b} = 40.5[\#/\text{sqm}]\text{=}811[\#]$$

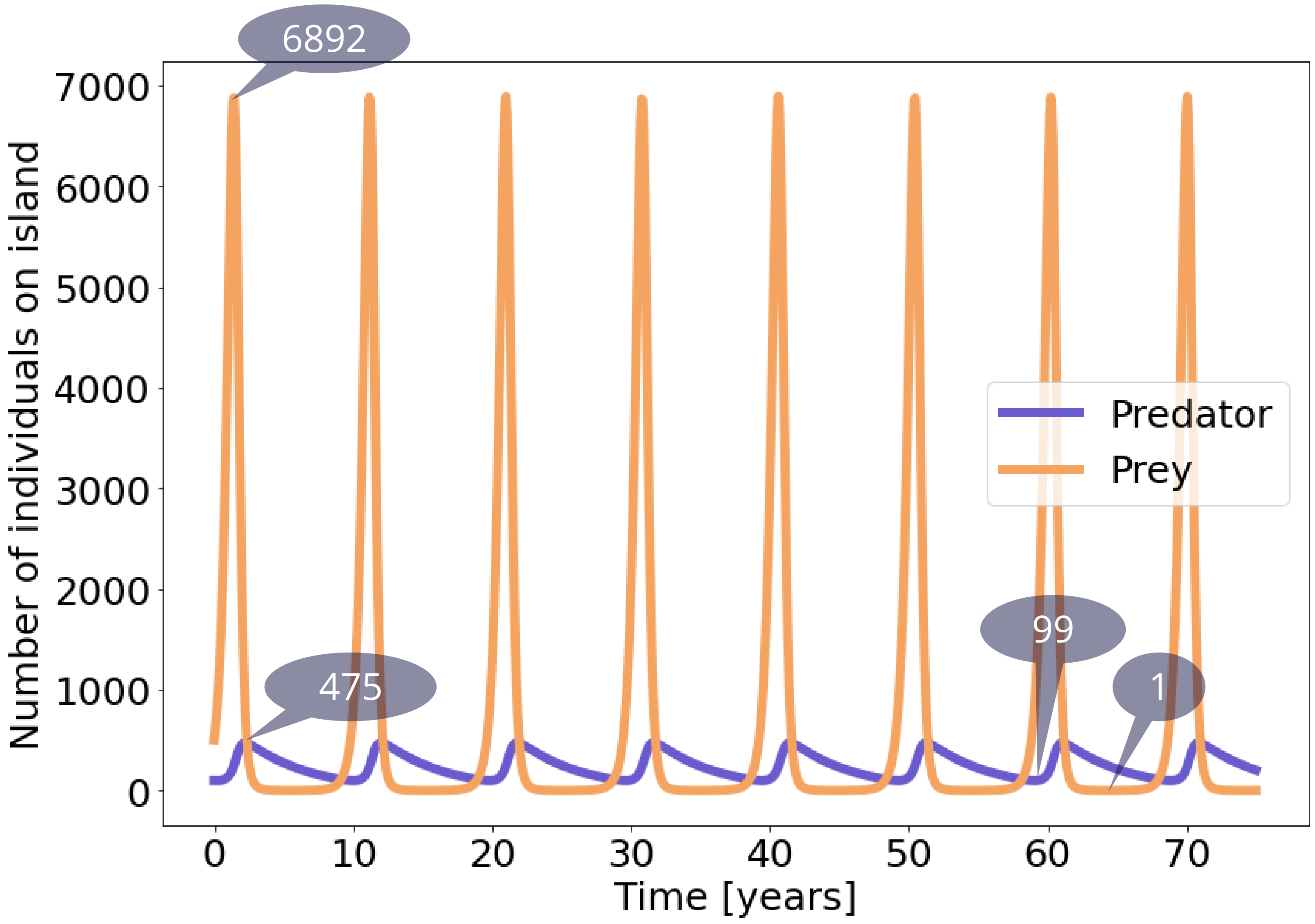
Lotka-Volterra Predator-Prey, plot in time (75 years)

Initial populations:

$$P_0 = 100/V = 5$$
$$N_0 = 500/V = 25$$
$$\text{With } V = 20 \text{ [sqm]}$$

$$\frac{dP}{dt} = \varepsilon bNP - dP,$$
$$\frac{dN}{dt} = -bNP + rN$$

Populations of beetles and lizards oscillate every, approximately, 10 years.



Lotka-Volterra Predator-Prey, phase plane plot

Initial populations:

$\boldsymbol{P_0 = 100/V = 5}$

$\boldsymbol{N_0 = 500/V = 25}$

With $\boldsymbol{V = 20}$ **[sqm]**

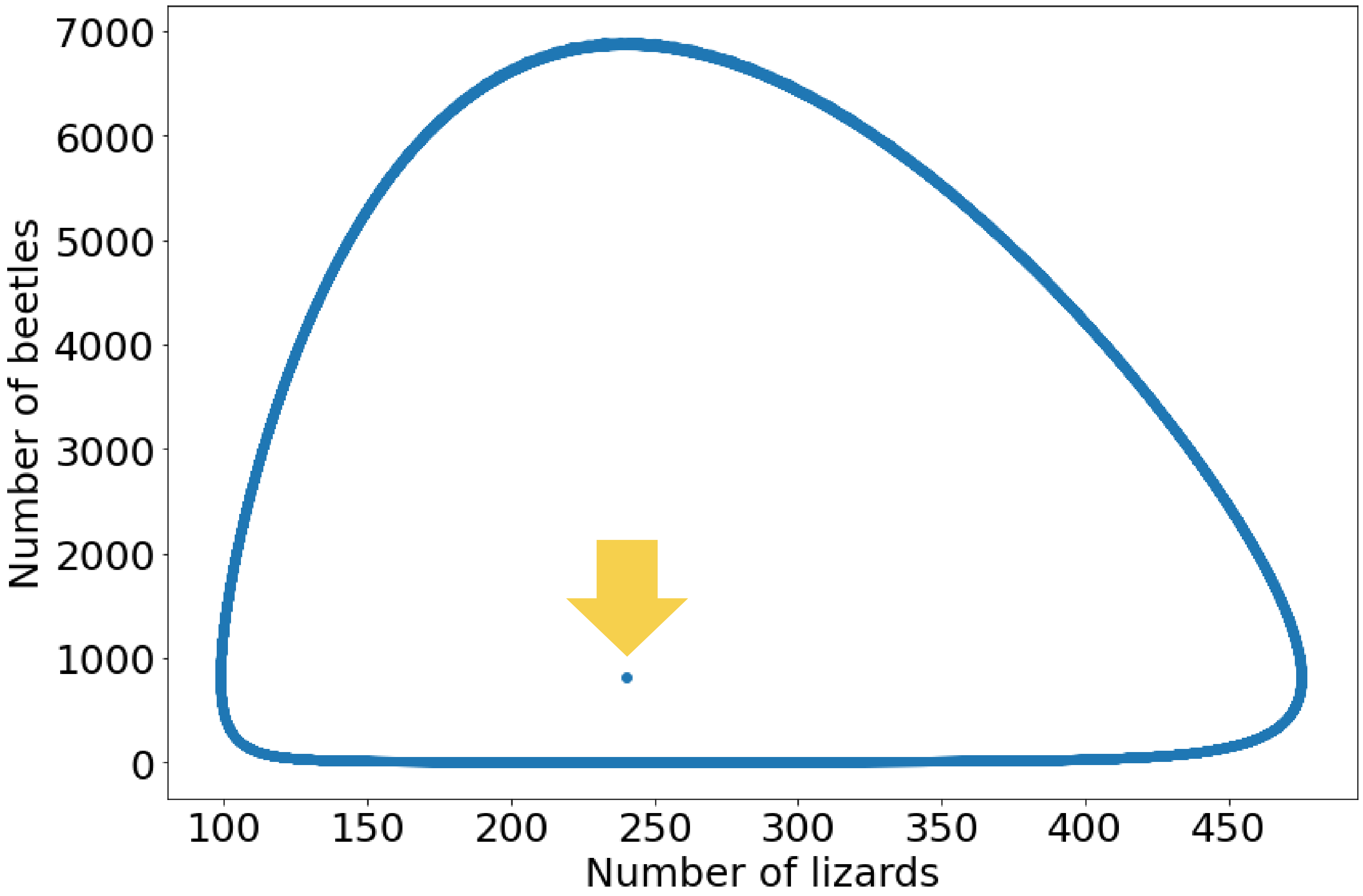
$$\frac{dP}{dt} = \varepsilon bNP - dP,$$

$$\frac{dN}{dt} = -bNP + rN$$

The populations never reach the equilibrium.

$$\bar{P} = {}^{r}/_{b} = 12\ [\#/\mathrm{sqm}]{=}240[\#]$$

$$\bar{N} = \frac{d}{\varepsilon \cdot b} = 40.5[\#/\mathrm{sqm}]{=}811[\#]$$



Lotka-Volterra Predator-Prey: Mortality due to hunger

Parameters:

Same P_0, N_0, V ε, r, d, b as before.

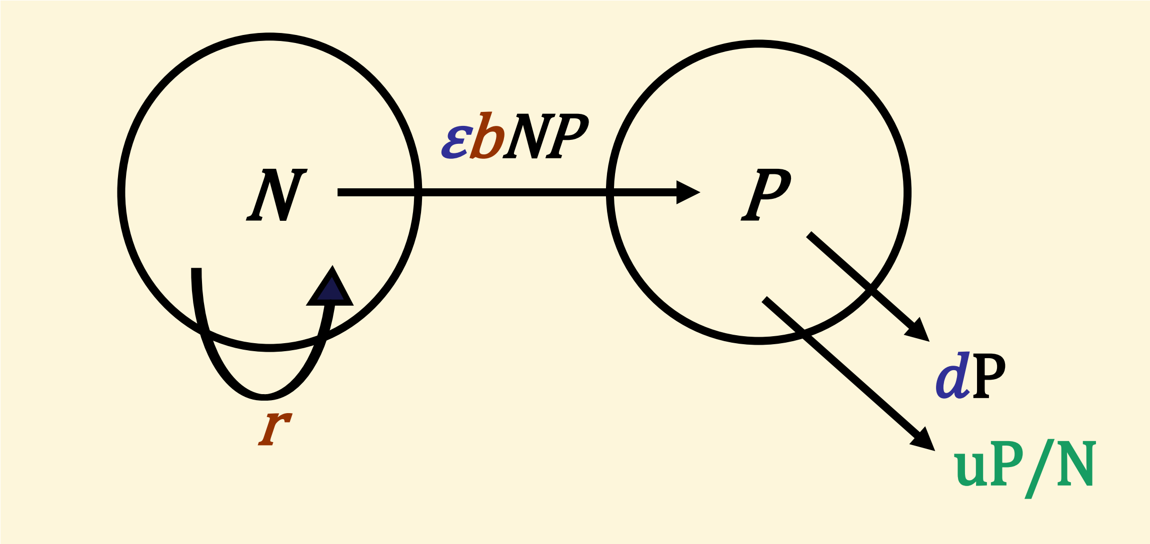
u = 2 (arbitrary)

units

equations

$$\left[\frac{\#P}{V\cdot T}\right] \quad \frac{dP}{dt} = \varepsilon bNP - dP - u\frac{P}{N}$$

$$\left[\frac{\#N}{V\cdot T}\right] \quad \frac{dN}{dt} = -bNP + rN$$



When N is very small
(i.e. not much prey available),
the $u\frac{P}{N}$ factor becomes very large,
and many predators die (due to hunger).
When N is very big, then the $u\frac{P}{N}$ factor becomes very small.

equilibria

$$\bar{P} = {}^{r}/_{b} = 12\ [\#/\text{sqm}]=240[\#]$$

$$\bar{N} = \frac{d+\sqrt{d^2+4\cdot b\cdot \varepsilon\cdot u}}{2\cdot\varepsilon\cdot b} = 48[\#/\text{sqm}]=962[\#]$$

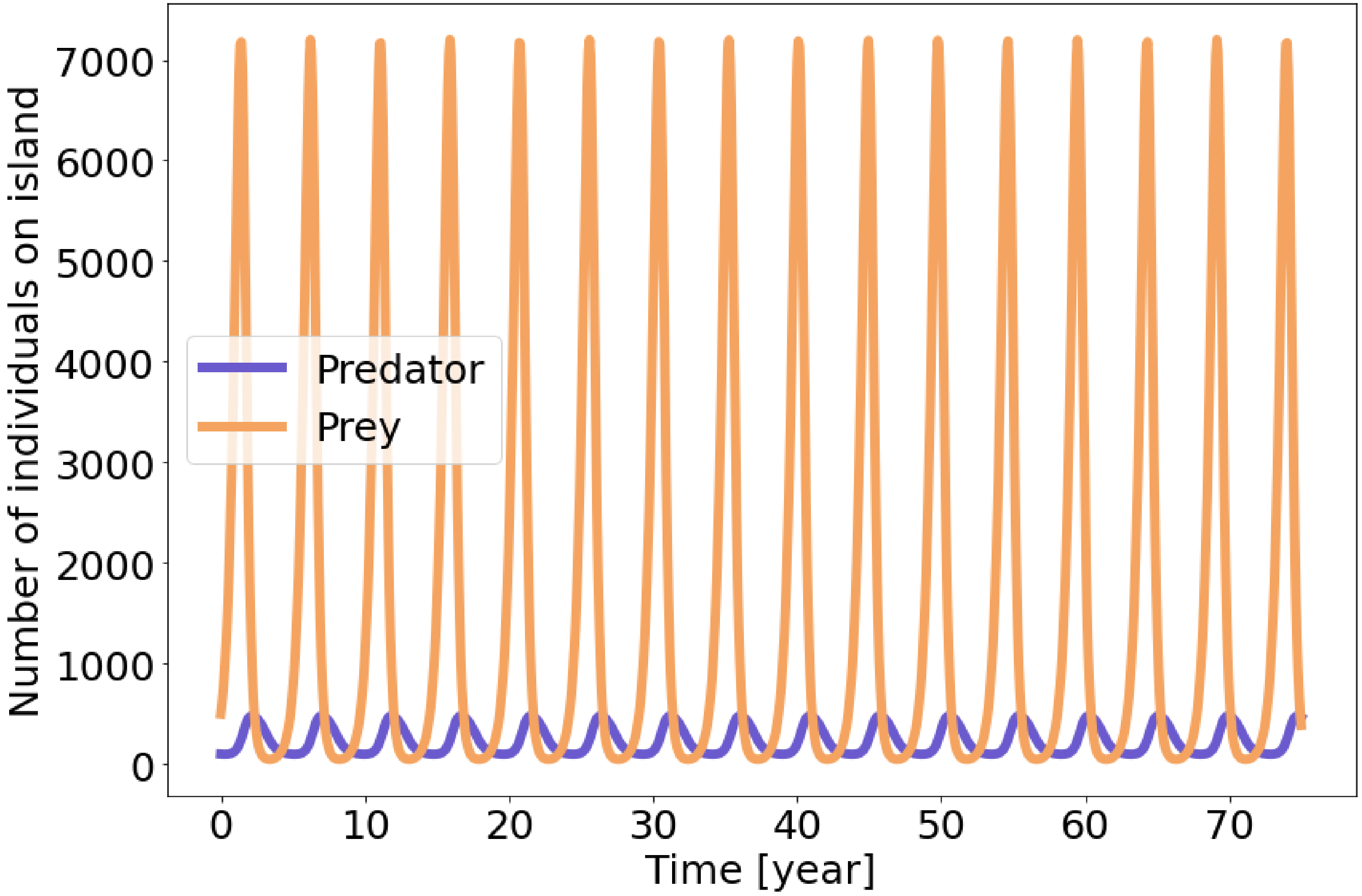
Lotka-Volterra Predator-Prey: Mortality due to hunger

$$\frac{dP}{dt} = \varepsilon bNP - dP - u\frac{P}{N},$$

$$\frac{dN}{dt} = -bNP + rN$$

Populations of beetles and lizards oscillate every, approximately, 5 years.

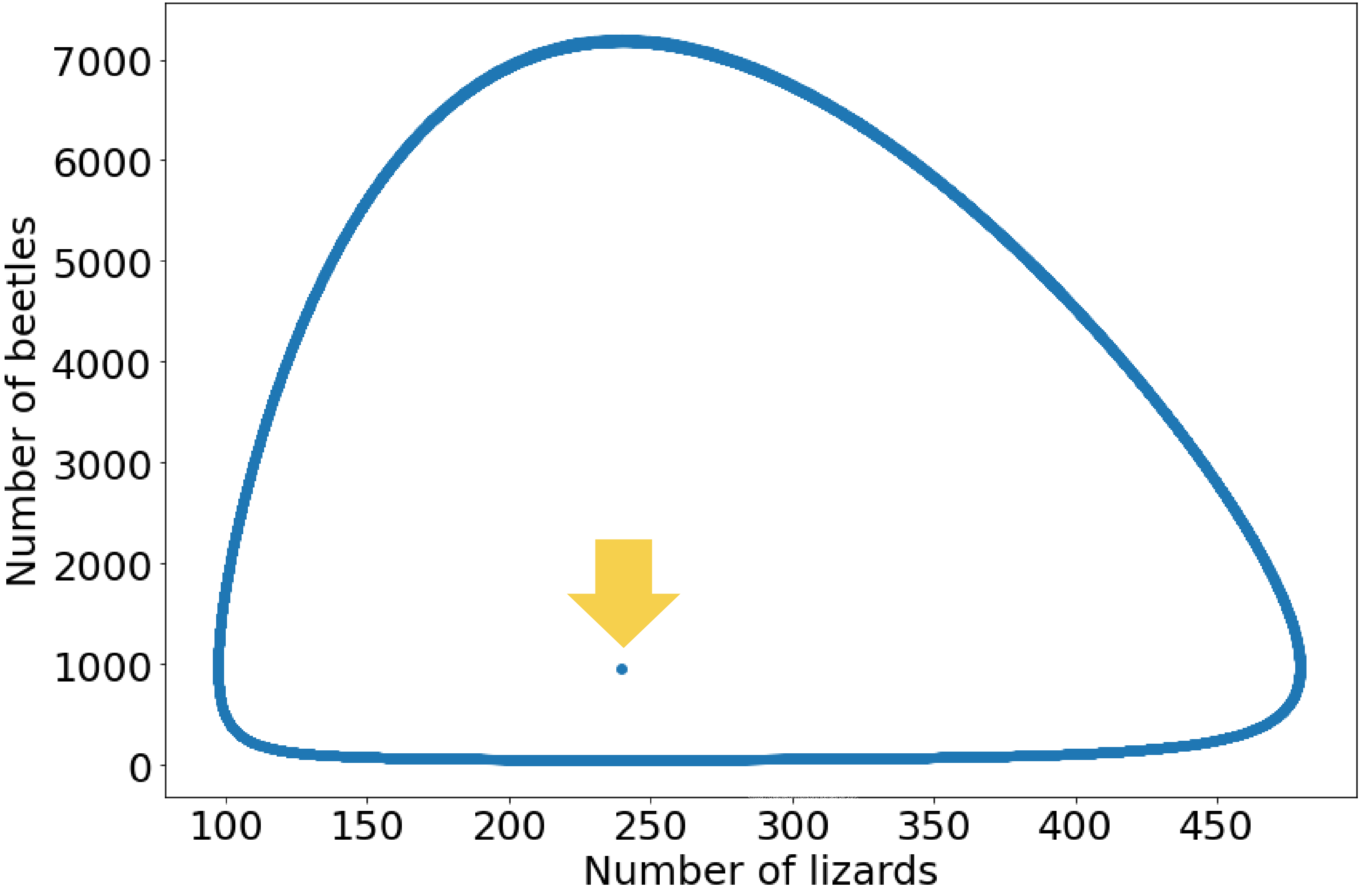
This is because predators die faster when there is little prey.
Then, prey gets the change to recover also faster (compared to the first case).
So, the system oscillates more often than without the hunger factor.



Lotka-Volterra Predator-Prey: Mortality due to hunger

$$\frac{dP}{dt} = \varepsilon bNP - dP - u\frac{P}{N},$$

$$\frac{dN}{dt} = -bNP + rN$$

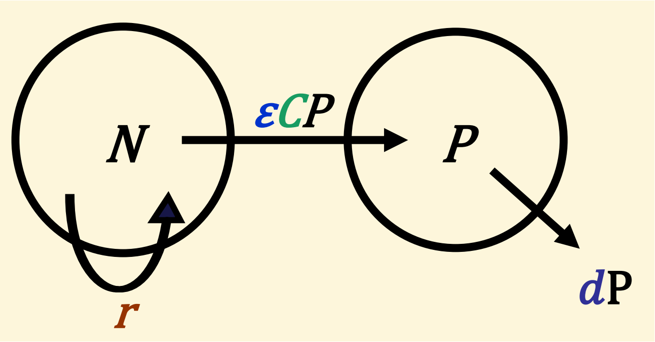


The populations never reach the equilibrium.

$$\bar{P} = {}^{r}/_{b} = 12\ [\#/\mathrm{sqm}]{=}240[\#]$$

$$\bar{N} = \frac{d+\sqrt{d^2+4\cdot b\cdot \varepsilon\cdot u}}{2\cdot \varepsilon\cdot b} = 48[\#/\mathrm{sqm}]{=}962[\#]$$

Lotka-Volterra Predator-Prey: Type II functional response



Parameters:

Same P_0, N_0, V ε, r, d, b as before.

$$C = \frac{b \cdot N \cdot c_{max}}{b \cdot n + c_{max}},$$

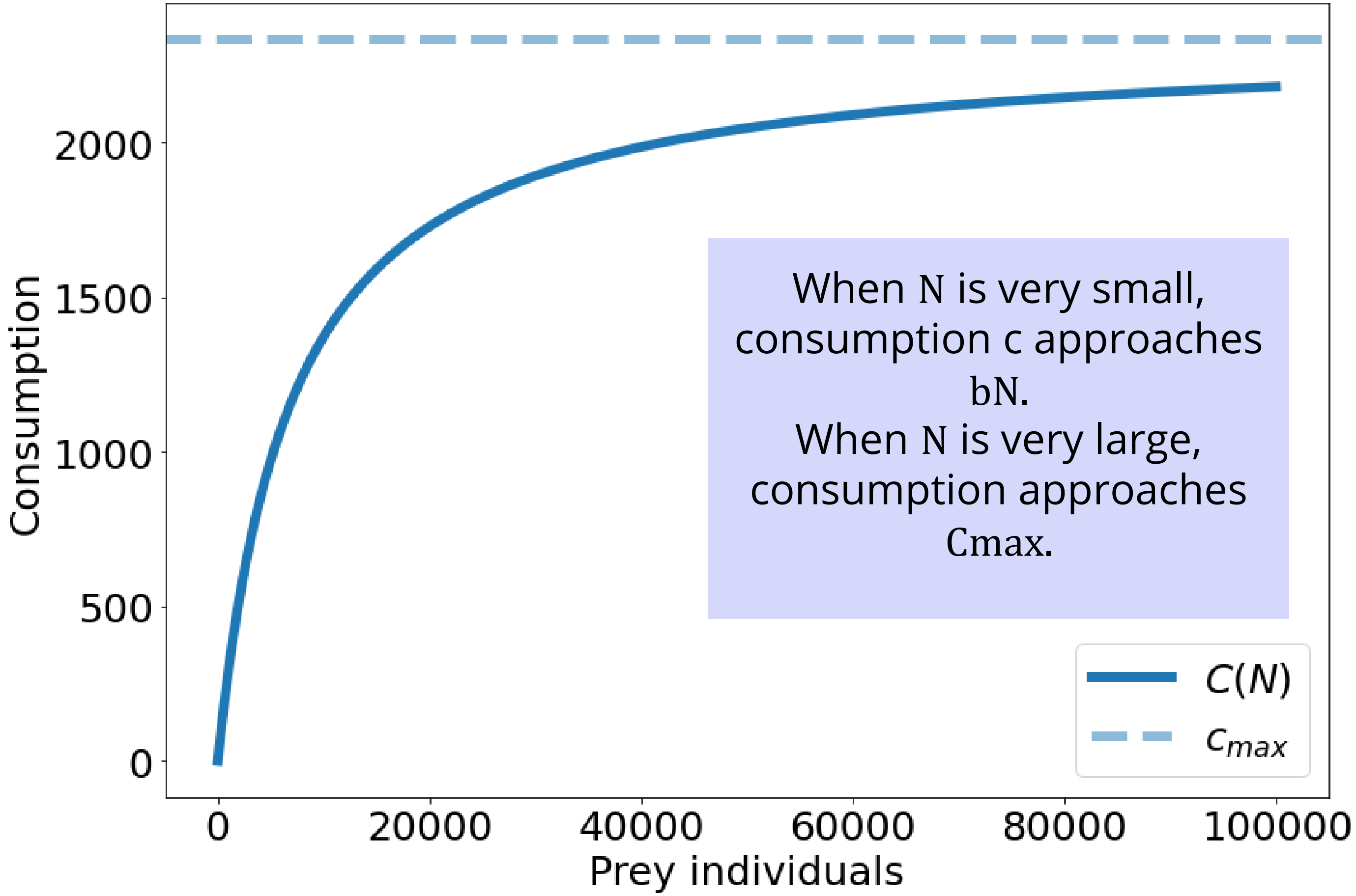
equations

$$\frac{dP}{dt} = \varepsilon CP - dP, \qquad \frac{dN}{dt} = -CP + rN$$

equilibria

$$\bar{P} = {}^{c_{max} \cdot \varepsilon \cdot r}/_{b(c_{max} \cdot \varepsilon - d)} = 12\ [\#/\text{sqm}]{=}241[\#]$$

$$\bar{N} = {}^{c_{max} \cdot d}/_{b(c_{max} \cdot \varepsilon - d)} = 41[\#/\text{sqm}]{=}815[\#]$$



When N is very small, consumption c approaches bN.
When N is very large, consumption approaches Cmax.

Lotka-Volterra Predator-Prey: Type II functional response

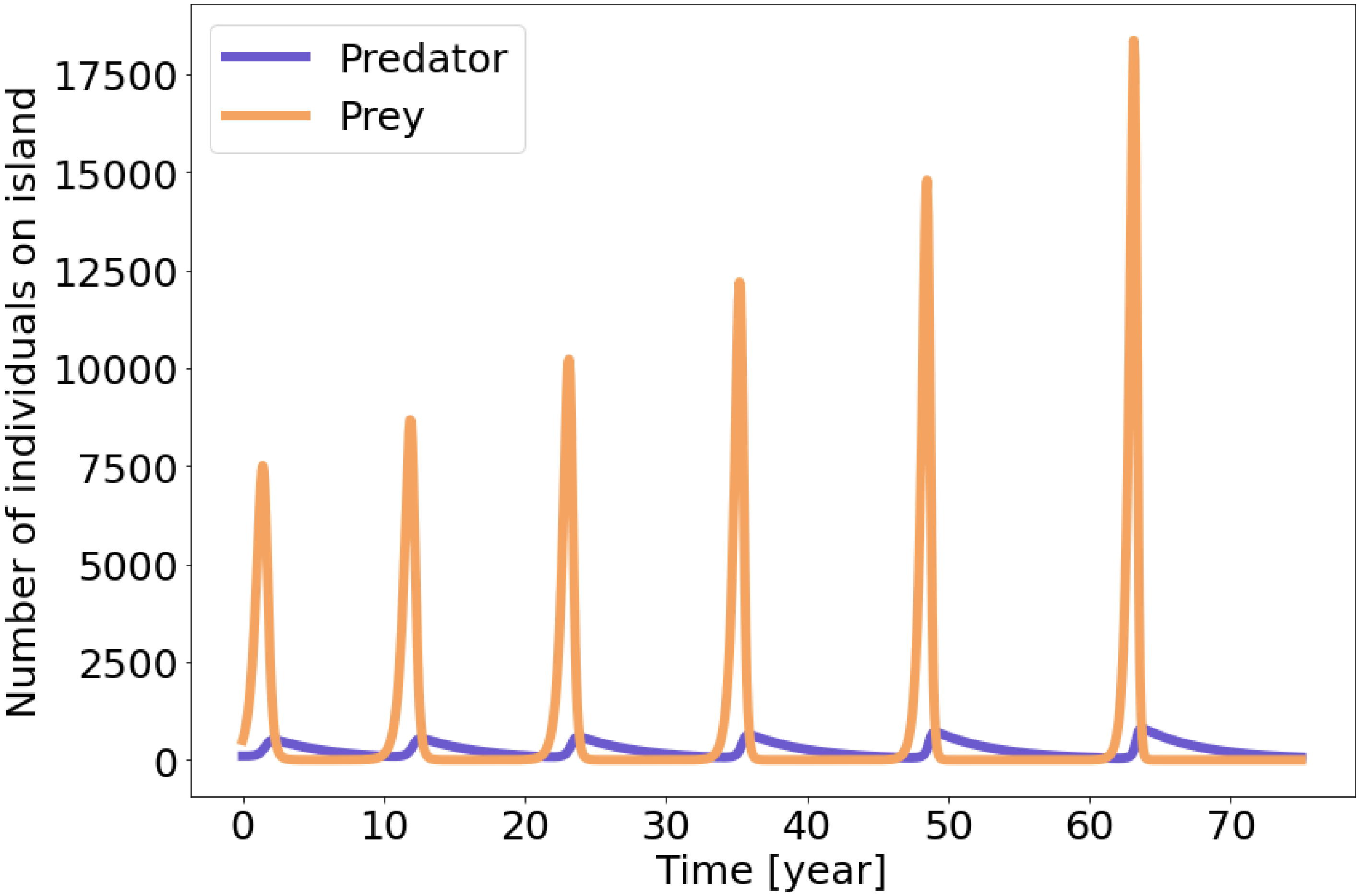
$$C = \frac{b \cdot N \cdot c_{max}}{b \cdot n + c_{max}},$$

$$\frac{dP}{dt} = \varepsilon CP - dP, \qquad \frac{dN}{dt} = -CP + rN$$

Populations of beetles and lizards oscillate every, approximately,10 years.

Because the predators are satiated at Cmax, the prey has a chance to recover to larger numbers.

Then, at the next cycle, the higher prey numbers can support more predators.
Yet again, because they are satiated, the prey can recover to even higher numbers



Lotka-Volterra Predator-Prey: Type II functional response

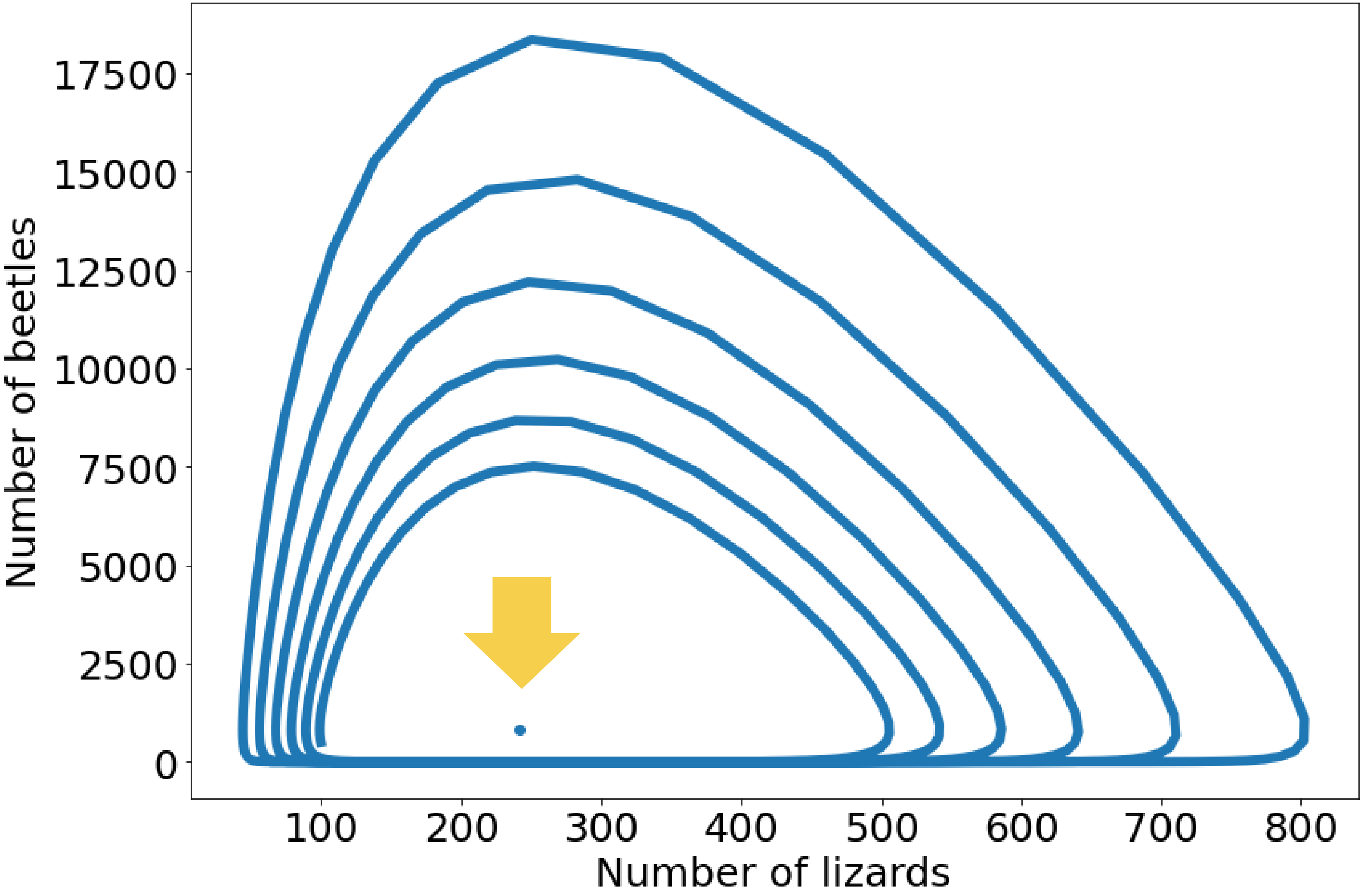
$$C = \frac{b \cdot N \cdot c_{max}}{b \cdot n + c_{max}},$$

$$\frac{dP}{dt} = \varepsilon C P - dP, \qquad \frac{dN}{dt} = -CP + rN$$

The populations never reach the equilibrium, but the curve of the phase plane plot keeps expanding with time (to higher respective numbers of predators and prey) .

$$\bar{P} = {}^{c_{max} \cdot \varepsilon \cdot r}/_{b(c_{max} \cdot \varepsilon - d)} = 12\ [\#/\text{sqm}] = 241[\#]$$

$$\bar{N} = {}^{c_{max} \cdot d}/_{b(c_{max} \cdot \varepsilon - d)} = 41[\#/\text{sqm}] = 815[\#]$$

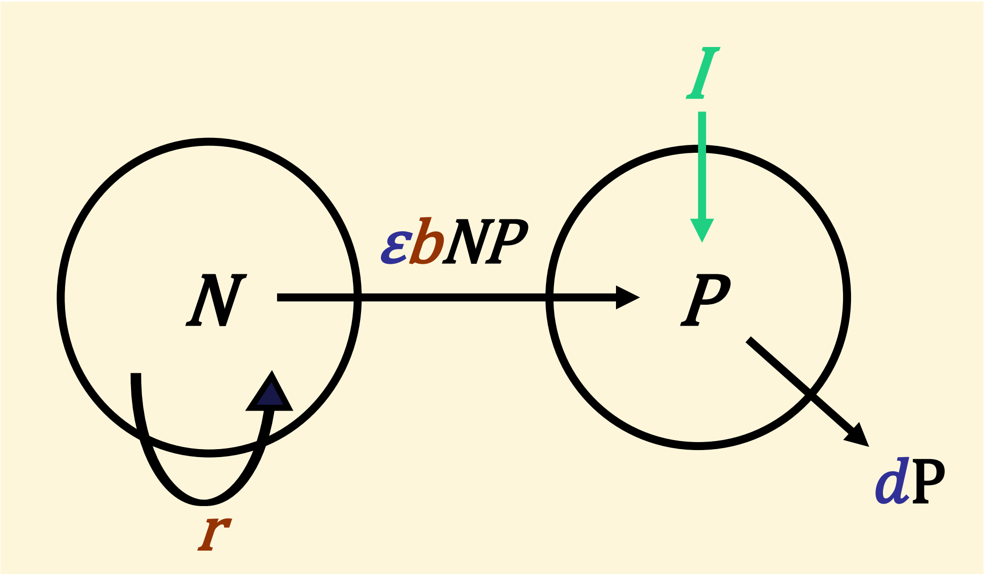


Lotka-Volterra Predator-Prey: Immigration of predators

Parameters:

Same P_0, N_0, V ε, r, d, b as before.

I = 1 [#/V] (arbitrary, & means immigration of 20 lizards per year for the entire island)



units

equations

$$\left[\frac{\#P}{V \cdot T}\right] \quad \frac{dP}{dt} = I + \varepsilon bNP - dP,$$

$$\left[\frac{\#P}{V \cdot T}\right] \quad \frac{dN}{dt} = -bNP + rN$$

equilibria

$$\bar{P} = {}^{r}/_{b} = 12\ [\#/\mathrm{sqm}]=240[\#]$$

$$\bar{N} = \frac{-I + d \cdot r/d}{\varepsilon \cdot r} = 25[\#/\mathrm{sqm}]=506[\#]$$

Lotka-Volterra Predator-Prey: Immigration of predators

$$\frac{dP}{dt} = I + \varepsilon bNP - dP, \frac{dN}{dt} = -bNP + rN$$

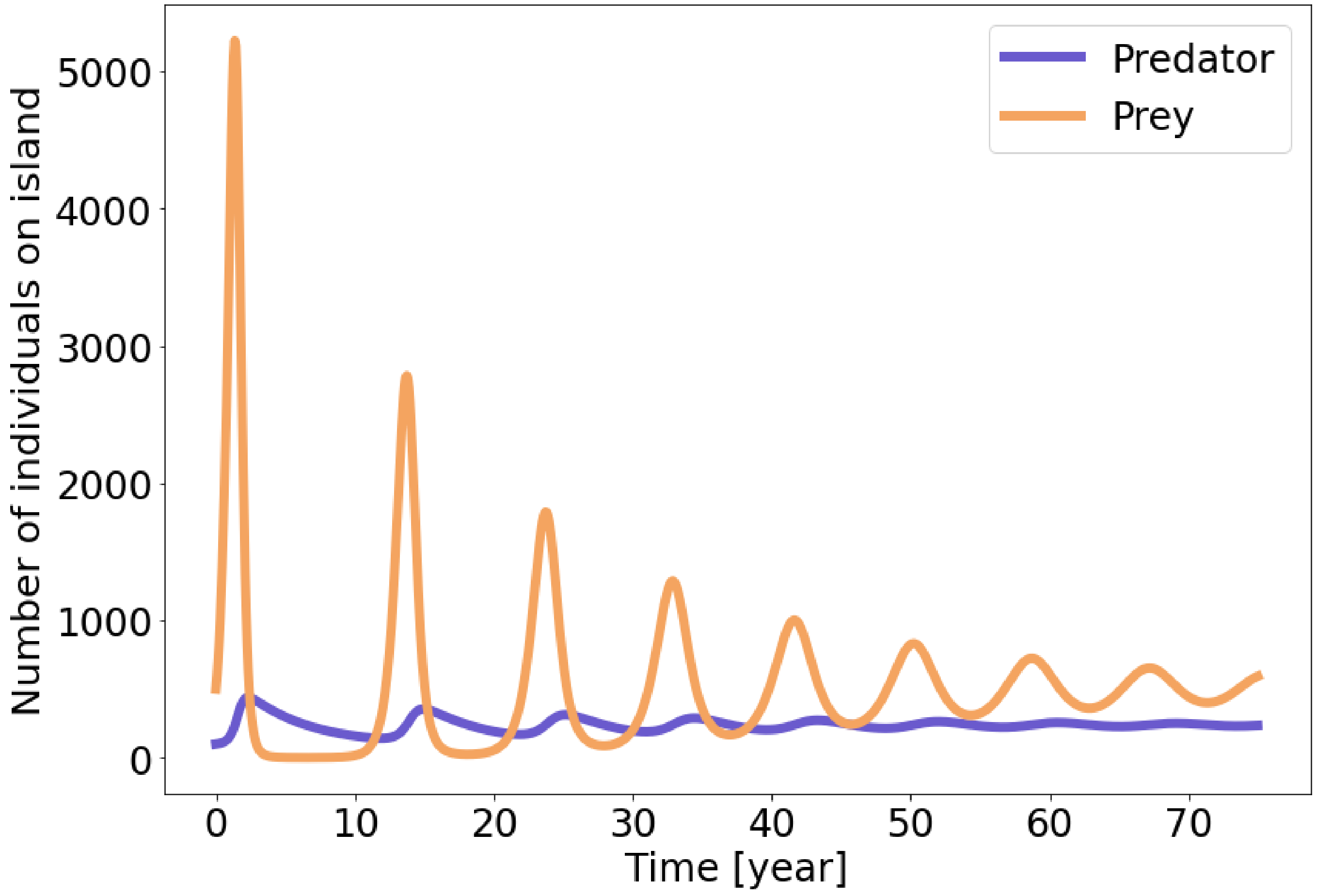
Populations of beetles and lizards oscillate every, approximately, 10 years, with the cycles becoming shorter and shorter in time.

The oscillations also become less acute.

Because of the constant immigration of predators, prey recovers to smaller total population number every cycle.

This prey population can support less predators, which makes the oscillations shorter in time.

Yet the constant predator immigration leads to the decreasing the number of prey in the next cycles.



Lotka-Volterra Predator-Prey: Immigration of predators

$$\frac{dP}{dt} = I + \varepsilon bNP - dP, \frac{dN}{dt} = -bNP + rN$$

The populations do not reach equilibrium in 75 years, but have the potential to reach it (if we would run the simulation for a longer period of time).

$$\bar{P} = {}^{r}/_{b} = 12\ [\#/\text{sqm}]{=}240[\#]$$

$$\bar{N} = \frac{-I + d \cdot r/d}{\varepsilon \cdot r} = 25[\#/\text{sqm}]{=}506[\#]$$