Selected Answers

CHAPTER 1

1. (b) $x_n = 10 \cdot 2^n$.

2. (b) $x_n = 1 + 2 \cdot (4)^n$; (c) $x_n = (-1)^n + 4$; (e) $x_n = 5 + (-2)^n$.

3. (b) (i) $A_1 + nA_2$; (ii) $(-1)^n[A_1 + nA_2]$; (iii) $3^n[A_1 + nA_2]$.

6. (a) $A(5)^n + B(2)^n$; (b) $A(\frac{1}{2})^n + B(-\frac{1}{2})^n$; (e) $A(\frac{1}{3})^n + B(-1)^n$.

7. (b) $c_1\begin{bmatrix}4\\1\end{bmatrix}(\frac{1}{2})^n + c_2\begin{bmatrix}4\\-3\end{bmatrix}(-\frac{1}{2})^n$;

(d) $c_1\begin{bmatrix}1\\-2\end{bmatrix}(-1)^n + c_2\begin{bmatrix}1\\1\end{bmatrix}(2)^n$;

(f) $c_1\begin{bmatrix}4\\1\end{bmatrix}(\frac{1}{2})^n + c_2\begin{bmatrix}6\\1\end{bmatrix}(\frac{1}{4})^n$.

8. (a) $(\sqrt{2})^n e^{i\pi n/4}$; (c) $(10)^n e^{i\pi n/2}$; (e) $\left(\frac{1}{\sqrt{2}}\right)^n e^{i5\pi n/4}$.

9. (a) $c_1 \cos\left(\frac{n\pi}{2}\right) + c_2 \sin\left(\frac{n\pi}{2}\right)$; (c) $\sqrt{2}^n\left[c_1 \cos\left(\frac{3\pi n}{4}\right) + c_2 \sin\left(\frac{3\pi n}{4}\right)\right]$.

10. (b) $f > 1/r(1 - m)$.

11. (b) $K = d/(a + b + c)$.

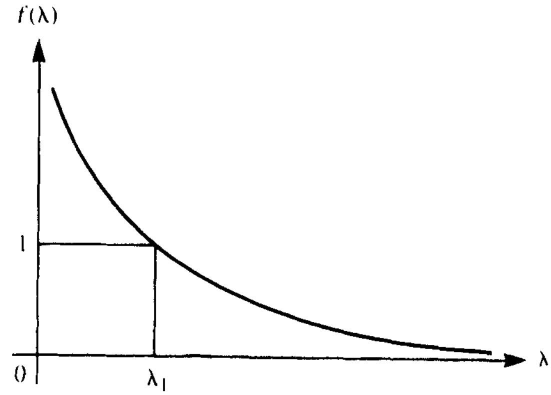
13. (a) 2,1,3,4,7,11,18,29,47,76,123.

14. (b) $R_n^0 = \frac{1}{\sqrt{5}}\left[\left(\frac{1 + \sqrt{5}}{2}\right)^{n+1} - \left(\frac{1 - \sqrt{5}}{2}\right)^{n+1}\right]$.

18. (a) $C_{n+1} = C_n - \beta V_n + m$,
$V_{n+1} = \alpha C_n$.

(c) $4\alpha\beta < 1 \Rightarrow$ amount of CO_2 lost per breath is less than $\frac{1}{4}$ (amount of CO_2 that induces a unit volume of breathing).
$4\alpha\beta > 1$: $\lambda = \frac{1}{2}\{1 \pm \gamma i\}$, $\gamma = (4\alpha\beta - 1)^{1/2}$. $|\lambda| \geq 1$ when
$\gamma \geq \sqrt{3} \Rightarrow \alpha\beta > 1$. Frequency $\phi = \frac{\pi}{3}$.

20. (c)



CHAPTER 2

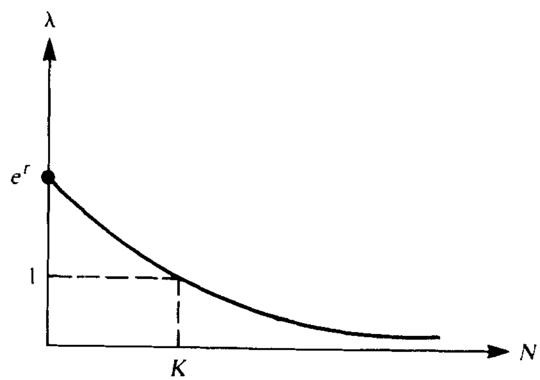
1. (a) Linear, $x_n = \left(\frac{1-\alpha}{1-\beta}\right)^n$, $\beta \neq 1$.

(c) Nonlinear, $\bar{x} = 0$.

(e) Nonlinear, $\bar{x} = (K - k_2)/k_1 \quad k_1 \neq 0$.

2. (a) Stable for $|r| < 1$. (b) Unstable. (c) Stable. (d) Unstable.

4. (a)



8. (b) $\bar{N}_1 = 0$ or $\bar{N}_2 = (\lambda^{1/b} - 1)/a$.

$\bar{N}_1$ stable iff $\lambda < 1$; $\bar{N}_2$ stable iff $0 < b(1 - \lambda^{-1/b}) < 2$.

11. Steady states: (0, 0) and $\left(\frac{Fk(F^{1/k} - 1)}{a(F-1)}, \frac{k}{a}(F^{1/k} - 1)\right)$.

14. $f'(x) = kb/(b+x)^2$, $f'(0) = k/b > 1$.

16. (b) $C_{t+1} = fC_tS_t$; $S_{t+1} = S_t - fC_tS_t + B$.

17. (b) $C_{n+1} = C_n - \beta C_n V_n + m$, $V_{n+1} = \alpha C_n$.

(c) Need $(m\alpha\beta) < 1$ for stability.

(d) yes, for $|x - 3| < 2\sqrt{2}$ where $x = (m\alpha\beta)^{1/2}$.

(h) $\bar{C} = \frac{1}{2}\{\gamma \pm \sqrt{\gamma^2 + 4k\gamma}\}$ where $\gamma = m/\beta\alpha$,

$\bar{V} = \alpha\bar{C}/(K + \bar{C})$.

CHAPTER 3

4. (c) $\bar{N}$ stable for $|1 - b(\lambda^{-1/b} - 1)| < 1$.

5. (b) 1-stable, 2-stable, 3-stable, 4-unstable, 5-stable.

7. (a) $n_{t+1} = \lambda n_t e^{-p_t}$, $p_{t+1} = n_t(1 - e^{-p_t})$ for $\bar{N} = \dfrac{1}{(ac)}$, $\bar{P} = \dfrac{1}{a}$.

9. (b) $a_{11} = (1 - r\bar{N}/K)$, $a_{12} = -\bar{N}a$, $a_{21} = \bar{P}/\bar{N}$, $a_{22} = a(\bar{N} - \bar{P})$.

10. (b) $N_{t+1} = \lambda N_t \exp[-(aP_t)^{1-m}]$, $P_{t+1} = N_t(1 - \exp[-(aP_t)^{1-m}])$.
(c) $\bar{P} = (\ln \lambda/a)^s$, $\bar{N} = \lambda\bar{P}/(\lambda - 1)$.

15. (a) Steady states $\bar{h} = (\ln f)/a$, $\bar{q} = \bar{h}\Big/\left(\delta - \dfrac{1}{r}\right)$.
(b) $k = \ln f$, $b = r\left[\delta - \dfrac{1}{r}\right]$.
(d) For $F(Q, H) = Qe^{k(1-H)}$, $G(Q, H) = bH\left(1 + \dfrac{1}{b} - \dfrac{H}{Q}\right)$.
$F_Q(\bar{Q}, \bar{H}) = 1$, $F_H(\bar{Q}, \bar{H}) = -k$, $R_x(\bar{Q}, \bar{H}) = -b$
(where $R = G/H$, $x = H/Q$).

19. $u_{n+1} = u_n^2 + \frac{1}{4}v_n^2$, $\quad v_{n+1} = \frac{1}{2}v_n^2$, $\quad w_{n+1} = \frac{1}{4}v_n^2 + w_n^2$.

CHAPTER 4

5. (a) r = intrinsic growth rate. B = carrying capacity.
(d) For $t \to \infty$, $e^{-rt} \to 0$ so $N(t) \to N_0B/N_0 = B$.
For N_0 small, $N(t) \approx N_0B/Be^{-rt} = N_0e^{rt}$.

6. (b) (mass nutrient)/(number of bacteria).

8. $\alpha_1 = \dfrac{v}{F}K_{\max}$ = ratio of [emptying time of chamber] to [$(1/\ln 2)$ times bacterial doubling time].
$\alpha_2 = c_0/K_n$ = ratio of [stock nutrient concentration] to [concentration which produces a half-maximal bacterial growth rate].

10. (a) $\dfrac{dN}{dt} = \left(\dfrac{C}{a + C}\right)N - bN$, $\dfrac{dC}{dt} = -a\left(\dfrac{C}{a + C}\right) - bC + 1$.
for $a = K_nVK_{\max}/FC_0$, $\quad b = F/K_{\max}V$.

11. Increase C_0, V, decrease F. (No local maximum).

15. (a) $y'' = -\cot(x)$: linear, order 2, not homogeneous, constant coefficients.
(d) $(2y + 2)y' - y = 0$; nonlinear, order 1, homogeneous, nonconstant coefficients.
(i) $\dfrac{dy}{dt} + y = \dfrac{1}{t}$ $(t \neq 0)$; linear, order 1, nonhomogeneous, constant coefficients.

16. (d) Steady states $(0, 0)$, $(1, 1)$; $\quad J(0, 0) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, $\quad J(1, 1) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

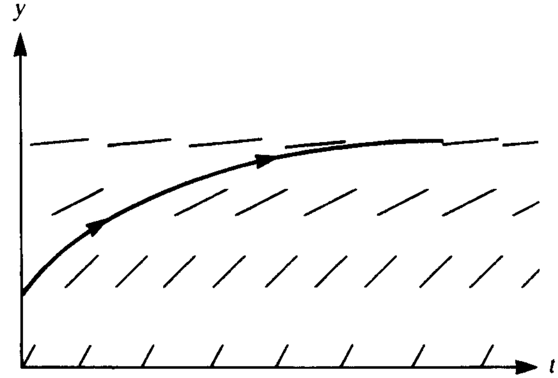
18. (b) $y(t) = .001e^{10t}$, $\quad$ (d) $y(t) = \frac{5}{2}(e^{3t} + e^{-3t})$, $\quad$ (e) $y(t) = \frac{1}{5}(3 + 2e^{5t})$.

22. (a) $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = c_1\begin{bmatrix} 1 \\ 0 \end{bmatrix}e^{-t} + c_2\begin{bmatrix} 0 \\ 1 \end{bmatrix}e^{t}$,
(c) $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = c_1\begin{bmatrix} 7 \\ -2 \end{bmatrix}e^{-4t} + c_2\begin{bmatrix} 1 \\ 1 \end{bmatrix}e^{5t}$,
(f) $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = c_1\begin{bmatrix} 1 \\ 2 - \sqrt{7} \end{bmatrix}e^{-(2+\sqrt{7})t} + c_2\begin{bmatrix} 1 \\ 2 + \sqrt{7} \end{bmatrix}e^{(-2+\sqrt{7})t}$.

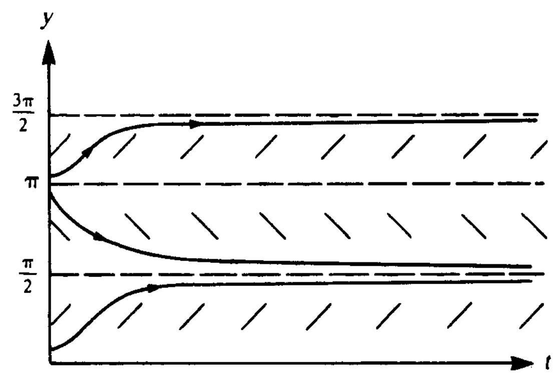
29. (b) $\bar{x}_1 = \dfrac{B}{F}\Big/\left[\dfrac{C}{F - E} - \dfrac{A}{F}\right]$, $\bar{x}_3 = \dfrac{C\bar{x}_1}{F - E}$, $\bar{x}_2 = A\bar{x}_1 - B$.
where $A = (u + k_{12})/k_{21}$, $B = D/k_{21}$, $C = k_{12}/(k_{21} + s + k_{23})$,
$E = k_{32}/(k_{21} + s + k_{23})$, $F = k_{32}/k_{23}$.

CHAPTER 5

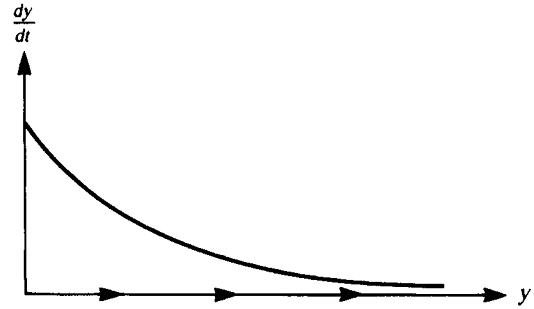
1. (b)



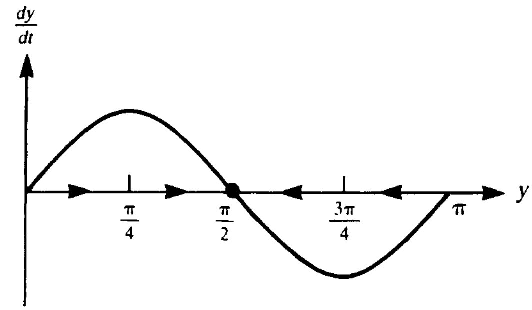
(e)



2. (b)



(e)

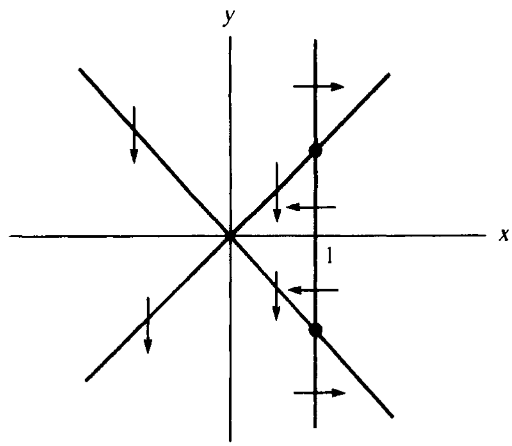


4. (b) (1) $(x, y) = (t, t(t-1)), \left(\frac{dx}{dt}, \frac{dy}{dt}\right) = (1, 2t-1).$

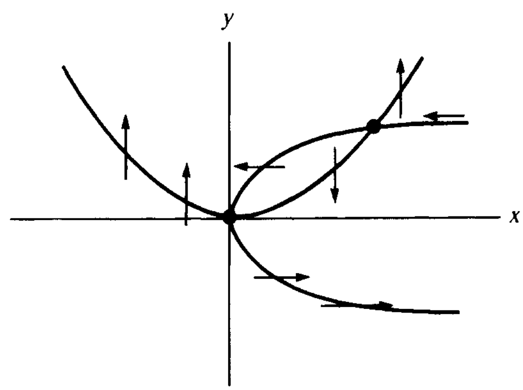
(4) $(x, y) = (\cos t, \sin t), \left(\frac{dx}{dt}, \frac{dy}{dt}\right) = (-\sin t, \cos t).$

(6) $(x, y) = (t, 4t^2), \left(\frac{dx}{dt}, \frac{dy}{dt}\right) = (1, 8t).$

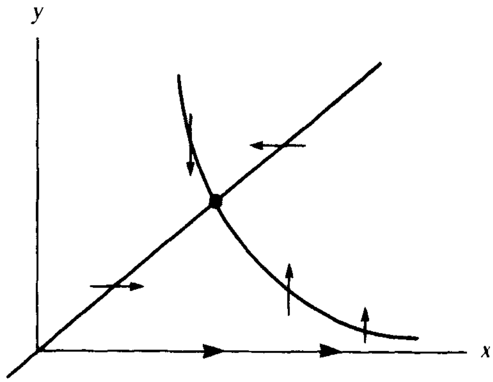
5. (a)



(e)



(g)



6. (a) $J(1, 1) = \begin{bmatrix} -2 & 2 \\ 1 & 0 \end{bmatrix}$, saddle; $J(1, -1) = \begin{bmatrix} -2 & -2 \\ 1 & 0 \end{bmatrix}$ stable spiral.

(d) $J(0, 1) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ stable node; $J(-1, 0) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ saddle point.

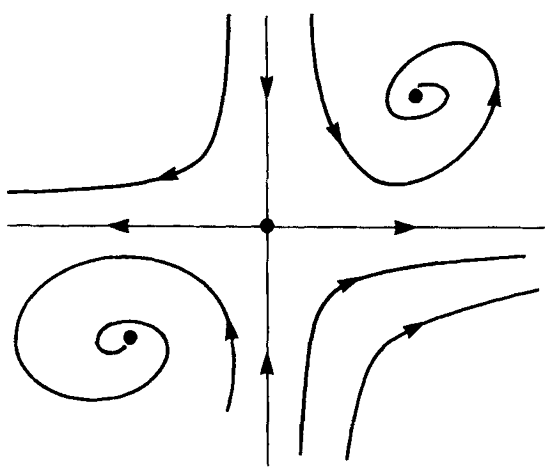
7. (a) neutral center, (b) saddle, (c) unstable node,
(d) saddle, (e) stable spiral, (f) unstable spiral.

12. (a) $A = \dfrac{\alpha_1}{\alpha_2}(\alpha_1 - 1)^2 - \dfrac{(\alpha_1 - 1)}{\alpha_1}$.

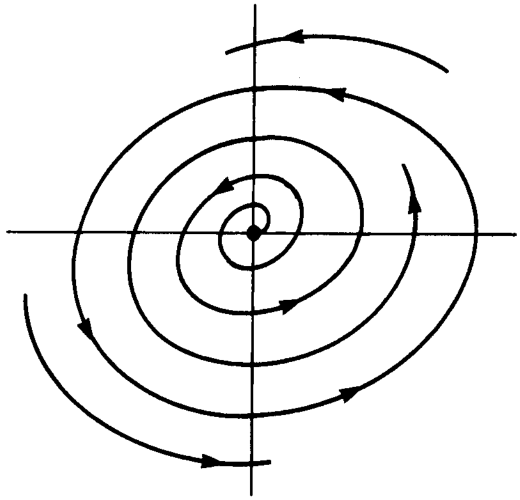
(b) $\beta = -(A + 1)$, $\gamma = A$, $\lambda_{12} = \frac{1}{2}\{-(A + 1) \pm \sqrt{((A + 1)^2 - 4A)}\} = -A, -1$.

$$\frac{N - \alpha_1(\alpha_2 - \overline{C}_1)}{C - \overline{C}_1} = -\alpha_1 \Rightarrow N - \alpha_1\alpha_2 = -\alpha_1 C.$$

15. (c)



(e)



20. (b) $K = K_2K_4I_0^3$, $\alpha = K_3K_4I_0$.

(c) steady state: $\bar{q} = \bar{I}$, $\bar{I}^2(\bar{I} - 1) = \frac{1}{K}$, $(\Rightarrow \bar{I} > 1)$.

$$J = \begin{bmatrix} -KI(I-1) & -Kq(2I-1) \\ \alpha & -\alpha \end{bmatrix}.$$

$Tr(J) < 0 \quad Det(J) > 0$ for $\bar{I} > 1 \therefore$ stable.

21. (a) $\epsilon = 1/\Delta t$.

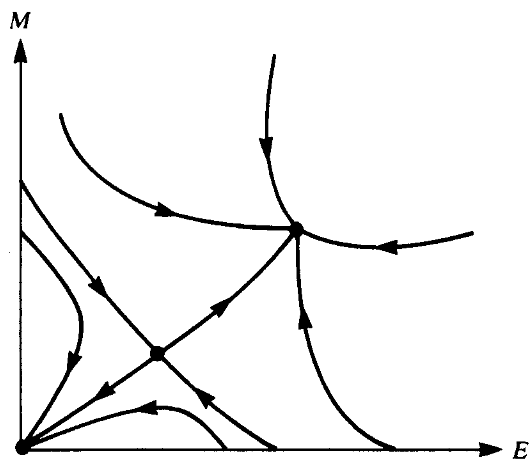
(b) $\frac{dC}{dt} = -\beta V + m$, $\frac{dV}{dt} = \alpha C - \epsilon V$.

steady state: $\bar{V} = m/\beta$, $\bar{C} = \frac{\epsilon m}{\alpha\beta}$ (stable).

(c) steady state: $\bar{V} = \frac{m}{\beta \bar{C}}$, $\bar{C} = \left(\frac{\epsilon m}{\alpha\beta}\right)^{1/2}$ (stable).

Decaying oscillations if $\left[\epsilon + \frac{\delta}{\epsilon}\right]^2 < 8\delta$ for $\delta^2 = (\epsilon m\beta\alpha)$.

22. (e) $m = 2$, $2\alpha\beta < 1$.



CHAPTER 6

2. (a) $a_1 = -rKM$, $a_2 = r(K + M)$, $a_3 = -r$.

(c) $\dfrac{1}{N^{1/KM}}\dfrac{|N - M|^C}{|K - N|^B} = Pe^{rt}$, P = constant.

3. (b) Beverton-Holt solution: $N^\alpha e^N = P\,e^{rt}$, P = constant.
(steady state $N = O$ is unstable).

4. (a) not stabilizing. (d) not stabilizing.

6. (d) $N = K$ is a stable steady state.

9. (b) $\dfrac{dx}{dt} = (a - \phi)x - bxy$, $\dfrac{dy}{dt} = -(c + \phi)y + dxy$;
steady states: (0, 0) and $\left(\dfrac{c + \phi}{d}, \dfrac{a - \phi}{d}\right)$.

14. Oscillations when $ac < (2dK)^2\left(1 - \dfrac{c}{dK}\right)$.

17. (a) $(K_1 + \alpha N_2)$ is the carrying capacity of species 1. Thus the presence of species 2 contributes positively to the carrying capacity of species 1.

(b) steady states: (0, 0) (unstable node), $(K_1, 0)$, $(0, K_2)$ (saddle points).
$\left(\dfrac{K_1 + \alpha K_2}{1 - \alpha\beta}, \dfrac{K_2 + \beta K_1}{1 - \alpha\beta}\right)$ (stable node).

(c) (Last steady state exists only if $\alpha\beta < 1$).

18. $H_1 = (a_1)$, $H_2 = \begin{bmatrix} a_1 & 1 \\ a_3 & a_2 \end{bmatrix}$, $H_3 = \begin{bmatrix} a_1 & 1 & 0 \\ a_3 & a_2 & a_1 \\ 0 & 0 & a_3 \end{bmatrix}$

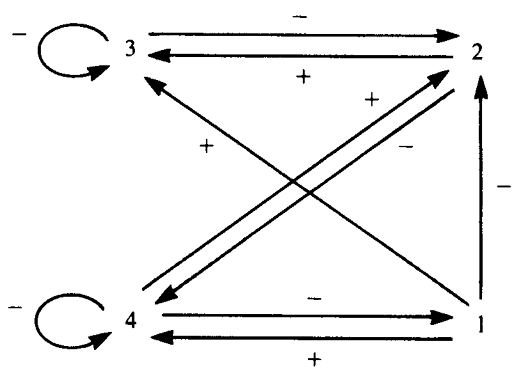
$\det H_1 = a_1$, $\det H_2 = a_1a_2 - a_3$, $\det H_3 = a_3(\det H_2)$.

23. (1) (a) $\begin{bmatrix} 0 & - & 0 & 0 \\ + & 0 & 0 & 0 \\ 0 & + & 0 & - \\ 0 & 0 & - & - \end{bmatrix}$. (e) $\begin{bmatrix} 0 & + & 0 & 0 & 0 \\ - & 0 & 0 & 0 & 0 \\ 0 & + & 0 & 0 & 0 \\ 0 & 0 & + & 0 & + \\ 0 & 0 & 0 & - & 0 \end{bmatrix}$;

(2) (a) {1, 2}. (e) {1, 2}, {4, 5}.

(3) (a) not qualitatively stable; (e) not qualitatively stable.

24. (b)



predation community:
{1, 4, 2, 3}.
Not qualitatively stable.

29. For $\lambda > 0$, the only equilibrium of S, I equations is $(S, I) = (0, 0)$.
But $J(0, 0) = \begin{bmatrix} -\lambda & 0 \\ 0 & -\gamma \end{bmatrix} \Rightarrow (0, 0)$ stable.

32. Steady states: (K, 0) (saddle point), $(\bar{x}_2, b\bar{x}_2)$ where $\bar{x}_2 = r\Big/\left(ab + \frac{r}{K}\right)$ (stable).

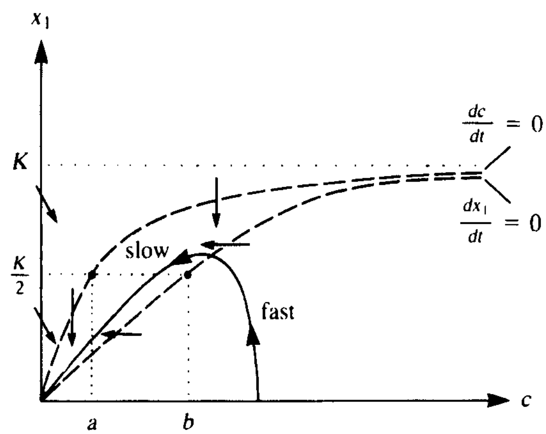
CHAPTER 7

2. (a) $x_1 = \dfrac{k_1 rc}{(k_1 + k_2) + k_1 c}$.

4. $K \ln c + c = -\lambda t + \text{constant}$.

6. $x_1(t) = (1 - e^{-(K+1)t})/(K + 1)$, $x_1(t) \uparrow \dfrac{1}{(K + 1)}$ for $t \to \infty$.

7. (a) $K = r$, $a = (k_{-1}/k_1)$, $b = (k_{-1} + k_2)/k_1$.
(b)



12. $\dfrac{da}{dt} = -K_1 ab + K_{-1}x$, $\dfrac{db}{dt} = -K_1 ab + K_{-1}x$.
$\dfrac{dx}{dt} = -K_{-1}x + K_1 ab$. At equilibrium $\dfrac{da}{dt} = \dfrac{db}{dt} = \dfrac{dx}{dt} = 0 \Rightarrow x = \dfrac{K_1}{K_{-1}}(ab)$.

16. (b) $J = \begin{bmatrix} ds_1 & -b \\ -ds_2 & d \end{bmatrix}$.
(c) $Tr\, J = bs_1 + d < 0$, $det\, J = bd(s_1 - s_2) > 0$.

19. (b) $J = \begin{bmatrix} -(k + \delta^2) & \dfrac{-2\delta^2}{k + \delta^2} \\ (k + \delta^2) & \dfrac{\delta^2 - k}{\delta^2 + k} \end{bmatrix}$; sign pattern $\begin{bmatrix} - & - \\ + & + \end{bmatrix}$ if $k < \delta^2$.

20. (a) Let $k_1 = k_2 = k_3 = k_4 = 1$.
(b) $\bar{x} = A$, $\bar{y} = B/A$.
(c) $J = \begin{bmatrix} B - 1 & A^2 \\ -B & -A^2 \end{bmatrix}$.

21. (a) Steady state: $(\rho + \gamma, (\rho + \gamma)^2/\gamma)$.

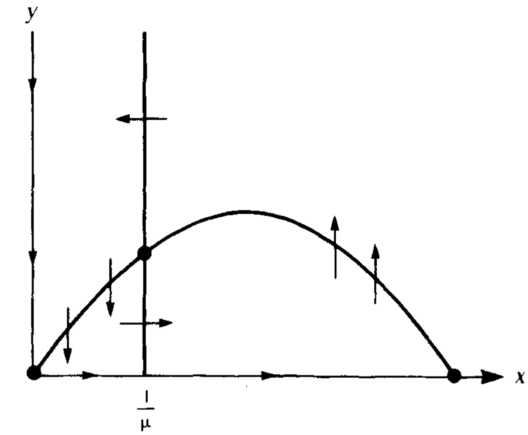
23. (b) $k_{11} = -F_x$, $k_{21} = \alpha F_x$, $k_{12} = -F_y$, $k_{22} = \alpha(F_y - G_y)$.

CHAPTER 8

2. (b) $\left[\dfrac{\partial f}{\partial x} + \dfrac{\partial g}{\partial y}\right] = b > 0.$

4. (a–c) No limit cycle possible.
(d) Cannot rule out limit cycle.

6. (b)



Flow can escape to (or emanate from) arbitrarily large y values. Poincaré–Bendixson theory inconclusive.

7. (b) No limit cycle.
(c) limit cycle exists.

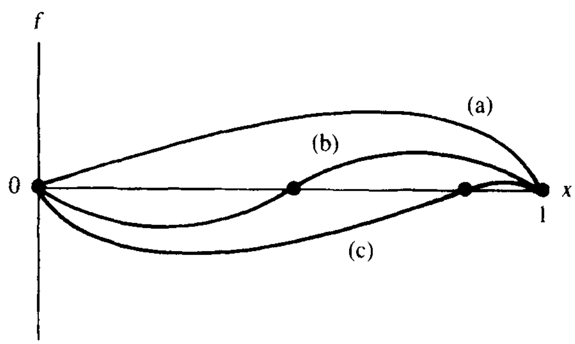
10. (a) Functions have no maxima or minima, only an inflection point.

13. (b) Figure b. (c) Cannot solve cubic equation.

17. (c) Condition guarantees $f = 0$ nullcline intersects x-axis to the right of the intersection of $g = 0$ with x-axis.

18. (a) $J = \begin{bmatrix} xf_x + f & xf_y \\ yg_x & yg_y + g \end{bmatrix}$. But $f = g = 0$ at the nontrivial steady state.

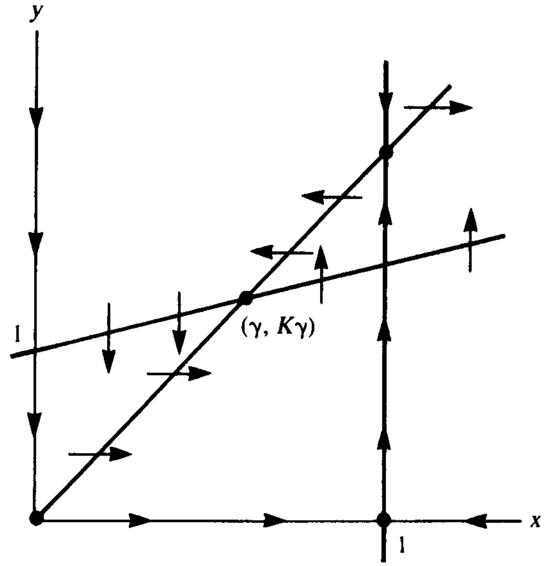
21. (a)



(a) $\alpha(y - 1) > 1$.
(b), (c) $0 < \alpha(y - 1) < 1$.

(b) $g(x, y) = \beta(Kx - y)$

(c, d)



(h) periodic solution about an unstable steady state $(\gamma, K\gamma)$ will exist.

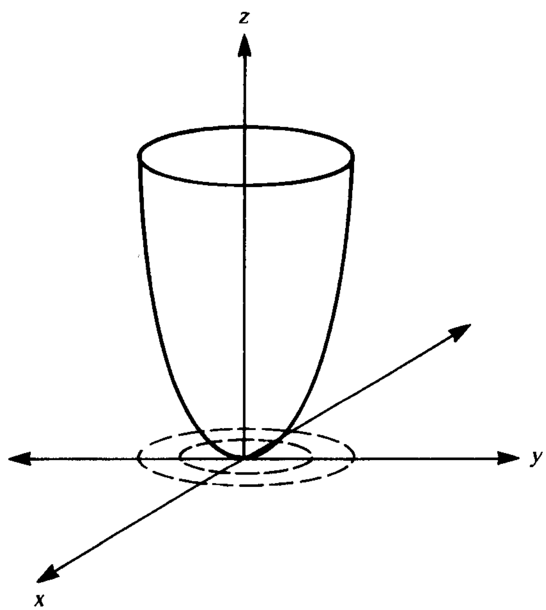
26. (a) Let $r^2 = x^2 + y^2$. $\dfrac{dr^2}{dt} = 2x\dfrac{2x}{dt} + 2y\dfrac{dy}{dt} = 2xy - 2xy = 0$

so r^2 = Constant is a solution (neutrally stable).

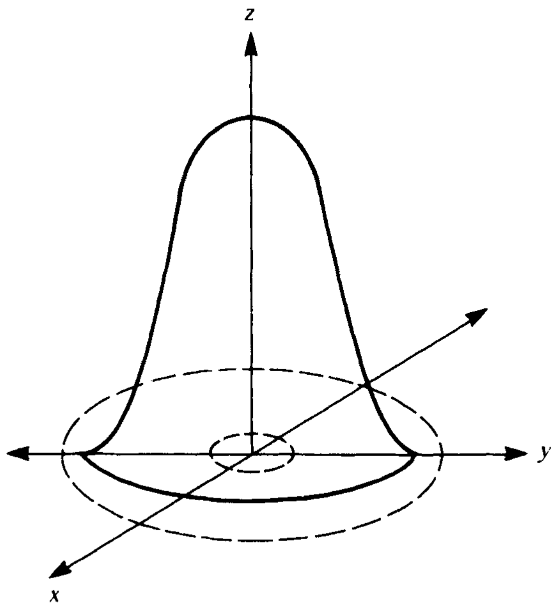
30. (b) $\dfrac{dr^2}{dt} = 2r^2[1 - r^2]$. $\dfrac{d\theta}{dt} = 1$.

CHAPTER 9

1. (a) Level curves are circles. Surface is a paraboloid



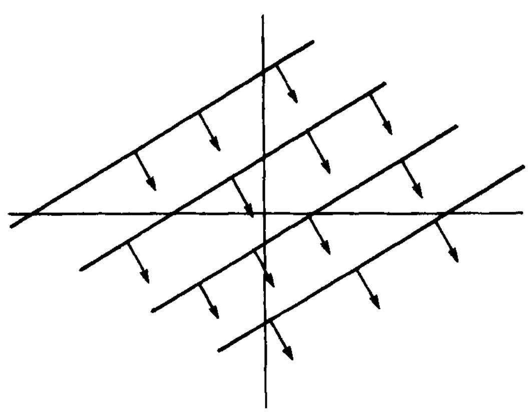
(b) Level curves are circles. Surface is a Gaussian, with maximum at (0, 0).



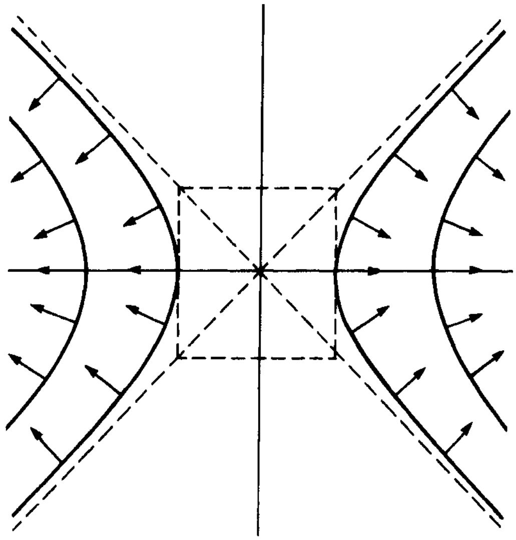
2.

	$\dfrac{\partial f}{\partial x}$	$\dfrac{\partial^2 f}{\partial x\,\partial y}$	∇f	**3.** critical points
(a)	$2x$	0	$(2x, 2y)$	minimum at (0, 0).
(c)	$-xe^{-R^2/2}$	$yxe^{-R^2/2}$	$(-x, -y)e^{-R^2/2}$	maximum at (0, 0), $R^2 = x^2 + y^2$.
(e)	y	1	(y, x)	(0, 0) is a saddle point.

4. (c)



(d)



5. (b) $\nabla \times F = (-2, -2, -2)$, $\nabla \cdot F = 0$.
(c) $\nabla \times F = (-2y, 1, -3)$, $\nabla \cdot F = 2x + 2y + 2z$.

6. (a) $\phi = \frac{1}{2}(x^2 + y^2) + C$.
(d) Not a gradient field.
(e) $\phi = e^{xy} + C$.

8. (b) $\dfrac{\partial c}{\partial t} = -\alpha c - \dfrac{a}{2A(x, t)}([2 - \sin(x - vt)] + c[2 + v \sin(x - vt)])$.

12. (a) Hint: Let $A = r\Delta\theta\Delta h$ (where $\Delta\theta$, Δh are constant).
(b) Let $A = R^2(\Delta\theta\Delta\phi)$.

14. (b) sphere: $\gamma = 36\pi$,
cylinder: $\gamma = 8\pi\dfrac{h}{r}$.

16. (a) $C(0.4R) \approx 3 \times 10^{-3} \times (C_0/s) = .663\mu$g/ml (see Figure 9.7b).

17. (b) $\dfrac{\partial c}{\partial t} = \mathfrak{D}\dfrac{\partial^2 c}{\partial x^2} - \dfrac{c}{\tau}$, $c(x, 0) = \begin{cases} c_0 & x < a \\ 0 & x \geq a \end{cases}$,
$\dfrac{\partial c}{\partial x} = 0$ for $x = L$ where L is length of tube.
$c(0, t) = c_0$.

19. (a) $c(r)\ln\left(\dfrac{L}{a}\right) = c_0 \ln\left(\dfrac{r}{a}\right)$.

22. (b) (i) $s_0 = 2s \Rightarrow \tau_0 = \dfrac{1}{4}\tau$.
(ii) $a_0 = 2a \Rightarrow \tau_0 = \tau \ln\left(\dfrac{L}{2a}\right) \Big/ \ln\left(\dfrac{L}{a}\right) = \tau\left(1 - \dfrac{\ln 2}{\ln(L/a)}\right)$

CHAPTER 10

1. (a) For v = prey (victim), e = predator (exploiter)
$\dfrac{\partial v}{\partial t} = \mathfrak{D}_v\dfrac{\partial^2 v}{\partial x^2} + F(v, e)$, $\dfrac{\partial e}{\partial t} = \mathfrak{D}_e\dfrac{\partial^2 e}{\partial x^2} + G(v, e)$,
with F, G any predator-prey kinetic terms.

2. (b) Two species competition with random motion of each of the species.

6. (b) $\mu \approx 0.2\ \text{cm}^2 h^{-1}$.

7. (b) Use $F = kv\eta$.

(c) The mean time between turns is $\tau = \lambda/v$.

8. (a) $\dfrac{\partial b}{\partial x} = 0,\ s = s_0$ at $x = L$,

$\dfrac{\partial b}{\partial x} = 0,\ \dfrac{\partial s}{\partial x} = 0$ at $x = 0$.

(e) $$\frac{\partial v}{\partial \tau} = \lambda \frac{\partial^2 v}{\partial \xi^2} + [KF(u) - \theta]v,$$

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial \xi^2} - F(u)v.$$

[with boundary conditions $\dfrac{\partial v}{\partial \xi} = 0,\ u = 1$ at $\xi = 1$, and $\dfrac{\partial v}{\partial \xi} = 0,\ \dfrac{\partial u}{\partial \xi} = 0$ at $\xi = 0$].

11. (c) $\rho = \dfrac{\mu_b}{\mu_c},\ \delta = \dfrac{\chi K_i}{\mu_c},\ \alpha = \dfrac{gK_i}{k_d c_0},\ \sigma = \dfrac{h_1 K_i}{k_d c_0},\ \kappa = K_b/K_i$.

12. (b) $-\gamma\rho$ = rate of branch mortality.

(e) dichotomous: $\sigma_{br} = \alpha n$;

lateral: $\sigma_{br} = \alpha\rho$.

14. (a) k_1 = rate of molecular binding to free sites,

k_2 = rate of unbinding.

D = effective diffusivity of free molecules.

v = speed of buffer through column.

(c) $\bar{u}_2 = k_1 B\bar{u}/(k_1\bar{u}_1 + k_2),\ \bar{u}_1 > 0$.

(d) Traveling Waves:

Require $U_1,\ U_2 \to 0,\ \dfrac{dU_1}{dz} \to 0$ for $z \to \infty$

waves satisfy $$\frac{dU_1}{dz} = [(v - c)/D]U_1 - (c/D)U_2,$$

$$\frac{dU_2}{dz} = -(k_1/c)(B - U_2)(U_1) + (k_2/c)U_2.$$

24. (a) $$\frac{\partial n}{\partial t} = -\frac{\partial n}{\partial \alpha} - \mu n.$$

(c) $$\frac{\partial n}{\partial t} = -\partial\frac{(kn\alpha)}{\partial \alpha} - \mu n,\ k \text{ a constant.}$$

25. (e) $v = 1/\tau = .0694\ \text{hr}^{-1},\ a = v \ln 2 = .048$,

$\mu \approx 0.25\ \text{hr}^{-1},\ dN/dt \approx -.2N$,

$t_{10^{-3}} \approx 34.5$ hr.

27. (b) Biomass of vegetation whose quality is within the range $(q,\ q+\Delta q)$ at time t.

(c) $\bar{Q} = Q/P$.

(d) $$\frac{\partial p}{\partial t} = -\frac{\partial pf}{\partial q}.$$

(e) $$\frac{dh}{dt} = h\, r(\bar{Q}, h).$$

(g) Take $\dfrac{dQ}{dt} = \displaystyle\int q \frac{\partial p}{\partial t}\, dq = -\int q \frac{\partial fp}{\partial q}\, dq$.

Integrate *RHS* by parts.

CHAPTER 11

2. (a) No. (b) Yes. (c) Yes. (d) inconclusive. (e) No.

3. (b) $\left[\dfrac{l}{f}\right]$ = time to build up a significant local cAMP concentration,

$\left[\dfrac{1}{k + D(\pi/L)^2}\right]$ = time to efface a cAMP perturbation by chemical decay and diffusion.

7. (b) $J(1, 0) = \begin{bmatrix} pq^2 + \gamma - (1/K) & 0 \\ \alpha\sigma & -q^2 - \alpha \end{bmatrix}$

15. (a) No diffusive instability possible.
(b) No diffusive instability possible.
(f) Diffusive instability if $b_1 b_2 > de$, $b_2 > e$
$D_2/D_1 > [(b_1/d)^{1/2} - (b_1/d - e/b_2)^{1/2}]^{-2}$.

19. (b) Increasing γ tends to stretch domain in the y direction.
(c) $E^2 = 32$, assumed fixed.

m	n	γ
4	4	1
5	3	1.28
4	5	1.56
4	6	2.25
5	4	2.28

20. (a) $C_i' = \alpha_i e^{\sigma t} \sin(q_1 x) \sin(q_2 y)$.
(b) Same form of wavenumbers.
(c) $C_i' = \alpha_i e^{\sigma t} \cos(q_1 x) \sin(q_2 y)$,
$q_1 = m\pi/L$, $q_2 = n\pi/\gamma L$.

21. (e) $\alpha = D_A'/D_S'$, $\beta = D_A/D_S$, $\gamma = L^2 D_S'/L_1 L_2 D_S$,
$\rho = L_1 L_2 V_m K_m / D_S'$.

22. (a) Nullclines: $Q = f(P)$, $P = g(Q)$.
(b) $J = \begin{bmatrix} Pf'(P) & P \\ Q & -Qg'(Q) \end{bmatrix} = \begin{bmatrix} + & - \\ + & - \end{bmatrix}$ for P to the left of hump.
(d) $Pf' - Qg' < 0$, $f'g' < 1$, and
$(Pf' D_2 - Qg' D_1) > 2\sqrt{D_1 D_2}\,(PQ(1 - f'g'))^{1/2} > 0$.

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