Week 1: Logistic growth

Christian Berrig   Rasmus Kristoffer Pedersen   Viggo Andreasen

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Model description:

The general model for the population growth is tha following:

$$\frac{dN}{dt} = Nr(N,t)$$

where the function $r(N,t)$ is the charecteristic growthrate of the population. We see that if this growthrate is constant, we end up with simple exponential growth. In general it depends on both populationsize and time explicitly, but we can investigate simple cases in which r depends on only N or or the t-dependency is simple:

0.1 logistic growth equation

assuming a linear decline in growth as a function of population-size, that is:

$$r(N,t) = r\left(1 - \frac{N}{K}\right)$$

we see that the population eqn. becomes:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

which is recognised as trhe logistic equation. This eqn. can be solved analytically, by separation of variables:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

0.2 rescaling/normal-form of logistic eqn.

The logistic equation can be brought into a "normal-form", which is dimension-less, by rescaling:

$$\tilde{N} = \frac{N}{K} \Rightarrow \frac{dN}{dt} = \frac{dN}{d\tilde{N}}\frac{d\tilde{N}}{dt} = K\frac{d\tilde{N}}{dt}$$
$$\tilde{t} = rt \Rightarrow \frac{d\tilde{N}}{dt} = \frac{d\tilde{t}}{dt}\frac{d\tilde{N}}{d\tilde{t}} = r\frac{d\tilde{N}}{d\tilde{t}}$$

whereby we find the relation:

$$\frac{dN}{dt} = rK\frac{d\tilde{N}}{d\tilde{t}}$$

And plugging this into the logistic equation:

$$\begin{aligned}
\frac{dN}{dt} = rK\frac{d\tilde{N}}{d\tilde{t}} &= rN\left(1 - \frac{N}{K}\right) \\
\frac{d\tilde{N}}{d\tilde{t}} &= \frac{N}{K}\left(1 - \frac{N}{K}\right) \\
\frac{d\tilde{N}}{d\tilde{t}} &= \tilde{N}\left(1 - \tilde{N}\right)
\end{aligned}$$

Thus all model parameters have been absorbed by the dimentionless population and dimensionless time.