Tragedy of the commons

This exercise builds and analyses a model of a freely exploited natural resource. The model should make it possible to answer the question: which statement in the comic on the following page is wrong?

First, consider an exploited population modeled by a logistic growth equation.

1. Choose an animal species and select relevant parameter values for the logistic growth.
2. Consider a population at carrying capacity ($N = K$). Suddenly, the population is harvested with a constant effort E. How does the population size vary as a function of time?
3. How does yield vary with time?
4. Does equilibrium population size and yield converge to the exact analytic predictions at equilibrium?

Consider now that the population is freely exploitable such that anybody can harvest the resource. We model the exploitation by agents with a density A and a clearance rate b. The catch of an agent is then bN and the total yield (per area) is bAN. Depending on the profit, the agents exploiting the resource will either increase or decrease their effort. The profit per agent is ($/time/agent):

$$P = pbN - c$$

where p is the price per catch and c is the costs of exploitation per agent per time. If the profit per cost P/c is positive there is an impetus for the agent to increase the effort (or for new agents to enter the exploitation):

$$\frac{\mathrm{d}A}{\mathrm{d}t} = k\frac{P}{c} = k(pbN/c - 1)$$

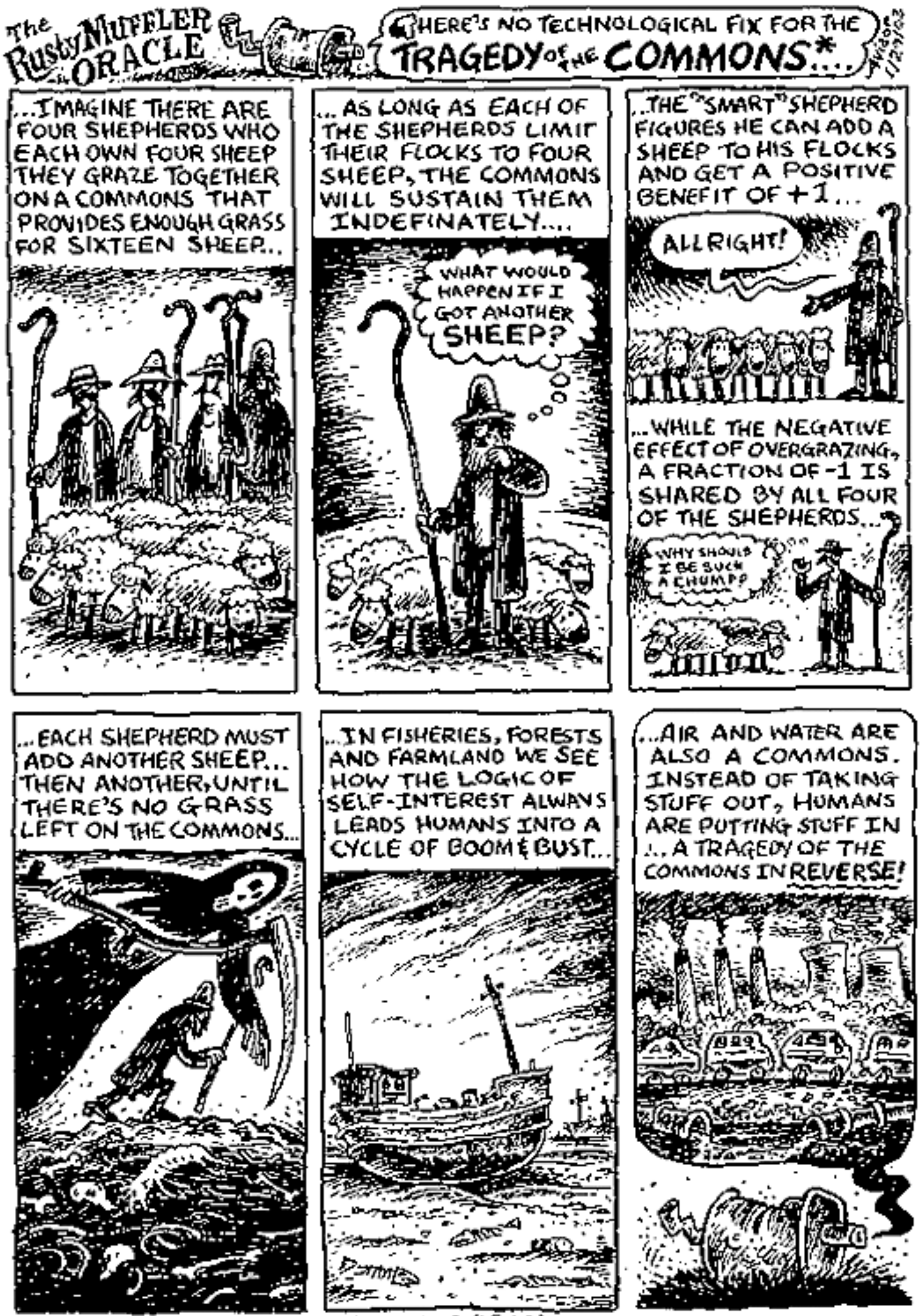
where k is the rate at which effort is increased.

5. What are the dimensions of the all parameters and variables in the model? What are reasonable values of the parameters?
6. Find the equilibrium solution for the population and effort (i.e. by setting $\mathrm{d}/\mathrm{d}t = 0$) using pen and paper. How does it compare to the MSY situation?
7. Solve the system of equations numerically and verify the numerical solutions of yield, revenue, effort and population size by comparing with the equilibrium solutions. Note: take care to avoid that the effort becomes negative.
8. How does the cost of harvesting, the price of the harvest, and the speed of introduction of effort influence the solution?

9. How are the results related to the “tragedy of the commons”?

Examine one of the following questions, or invent your own:

- What if yield is independent of abundance, i.e., $Y = \text{const} \cdot E$?
- How does the result change if the effort is varied on a time-scale faster or slower than the time scale of the population growth rate? What is a realistic rate of the change in effort?
- What happens if the cost is not proportional to the effort but proportional to the catch?
- What if the population has an Allee effect?
- What happens if it is harder to decrease effort than to increase it?
- What happens if exploitation is taxed? -- Or if the catch is taxed?
- What if the price of the resource depends on the available catch such that a smaller catch leads to a higher price?



*GARRETT HARDIN - SCIENCE 162 (1968)