# Spatial models: advection, reaction, and diffusion

The advection, reaction, diffusion model is:

$$\frac{\partial C(t,x)}{\partial t} = -\frac{\partial J(t,x)}{\partial x} + r(t,x)C(t,x) \qquad (1).$$

Here $C(t,x)$ is the concentration (organisms per volume) of an organism, $J(t,x)$ is the *flux* of matter (organisms per area per time), and $r(t,x)C(t,x)$ is the *reaction* term that describes population growth. The flux is composed of an advective and a diffusive component:

$$J(t,x) = J_A + J_D = v(t,x) \cdot C(t,x) - D(t,x)\frac{\partial C(t,x)}{\partial x},$$

Where $v$ is the advective velocity (length per time), and $D$ is the diffusivity (length$^2$ per time). Inserting the flux into (1) gives:

$$\frac{\partial C(t,x)}{\partial t} = \frac{\partial}{\partial x}\left(D(t,x)\frac{\partial C(t,x)}{\partial x}\right) - \frac{\partial v(t,x)C(t,x)}{\partial x} + r(t,x)C(t,x).$$

1. Check the `solvePDE` function. Try `help solvePDE` and run the examples.

2. Think about a case of a population of organisms that diffuses, and possibly moves (is being advected) in a direction. Determine the size of the diffusion and the advection and simulate the spatio-temporal population dynamics.

3. Extend the model with a reaction term and possibly with complications on the other terms. Ideas:
   a. Invading species that grows logistically ( $r(t,x,C) = r(1 - C/N)$ ). How fast does the species invade? How is invasion modified by an Allee effect?
   b. Model of a phytoplankton in a water column. Add a growth rate that declines with light: $\propto \exp(-kx)$, where the damping coefficient is in the order of 0.2 m$^{-1}$; add a sinking velocity of the phytoplankton; use a turbulent diffusion of the water column on the order of 10 m$^2$/day; make the mortality proportional to concentration to get density dependence. You can easily make extensions, such as light or diffusion that varies over the season, etc.
   c. Invent your own.