

Mathematical Biology, exercise 1

“...the wise Creator who commanded men in the beginning to be fruitful and multiply, did not intend, since He set limits to their habitation and sustenance, that multiplication should not continue without limits”

Otto Diederich Lütken, 1758

We will work with the logistic equation:

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K}\right)$$

which has the analytical solution:

$$N(t) = \frac{N(0)Ke^{rt}}{K + N(0)(e^{rt} - 1)}$$

- 1) Consider a population of animals of your own choice. Decide reasonable values of r and K for that population. What are the units of r and K ?
- 2) Solve the equation numerically and construct a plot that compares the numerical solution with the analytical solution given above.

Additional topics. Choose one or more from the list below, or invent your own:

- The “density dependence” in the logistic growth equation is a linear reduction in growth rate as $1 - N/K$. Try to introduce other forms of density dependence that do not just change linearly with N .
- How can the model be used to describe the population dynamics of a human population following a technological breakthrough?
- Modify the density dependence to make the model describe an “Allee effect”. An Allee effect is a negative growth rate for very small population sizes (note: the description of the Allee effect in many of the online pages is wrong).
- How can seasonal variations in birth or death rates be introduced into the model?
- Introduced a scaled variable of time as $\tilde{t} = tr$ and scaled abundance $\tilde{N} = N/K$. Write the logistic equation in terms of the scaled variables. What are the dimensions (“units”) of the scaled variables?
- ...

Present your results on slides (powerpoint or pdf) and upload them as an assignment at latest Wednesday. The first slides should be intended for presentation to the other students. They should contain:

- A description of the organisms and the case

- The model: a sketch and the equation(s) that are solved
- Parameters, with units and argument for their value
- Results

Further slides should contain additional results and explanations intended for the teacher. The last slide should contain your source code for your models. You are welcome to make group uploads, preferably groups of 2 and no larger than 3.